Topological Geometrodynamics: What Might Be the Basic Principles

M. Pitkänen
Email: matpitka@mappi.helsinki.fi.
http://www.helsinki.fi/~matpitka/.

July 31, 2008

Contents

Abstract

A brief summary of various competing visions about the basic principles of quantum Topological Geometrodynamics (TGD) and about tensions between them is given with emphasis on the recent developments. These visions are following. Quantum physics as as classical spinor field geometry of the "world of classical worlds" consisting or light-like 3-surfaces of the 8-D imbedding space $H = M^4 \times CP_2$; zero energy ontology in which physical states correspond to physical events; TGD as almost topological quantum field theory for light-like 3-surfaces; physics as a generalized number theory with associativity defining the fundamental dynamical principle and involving a generalization of the number concept based on the fusion of real and p-adic number fields to a larger book like structure, the identification of real and various p-adic physics as algebraic completions of rational physics, and the notion of infinite prime; the identification of configuration space Clifford algebra elements as hyper-octonionic conformal fields with associativity condition implying what might be called number theoretic compacticitation; a generalization of quantum theory based on the introduction of hierarchy of Planck constants realized geometrically via a generalization of the notion of imbedding space H to a book like structure with pages which are coverings and orbifolds of H; the notion of finite measurement resolution realized in terms of inclusions of hyperfinite factors as the fundamental dynamical principle implying a generalization of S-matrix to M-matrix identified as Connes tensor product for positive and negative energy parts of zero energy states; two different kinds of extended super-conformal symmetries assignable to the light-cone of H and to the light-like 3-surfaces leading to a concrete construction recipe of M-matrix in terms of generalized Feynman diagrams having light-like 3-surfaces as lines and allowing to formulate generalized Einstein's equations in terms of coset construction.

Keywords: Topological Geometrodynamics, unified theories, symmetries, dynamical principles.

1 Introduction

While pondering how to make the writing of an overall view about basic principles and applications of quantum TGD (see [16, 17, 18] and the chapters *An Overview about the Evolution of Quantum TGD* and *An Overview about Quantum TGD* of [1]) a more interesting task than mere updating of what I have written earlier, I realized that the tension between two profoundly different approaches to physics might bring in a new perspective making the writing process and perhaps even its outcome more interesting. Einstein characterizes the difference of these two approaches in the following manner.

We can distinguish various kind of theories in physics. Most of them are constructive. They attempt to build up a picture of the more complex phenomena out of the materials of a relatively simple formal scheme from which they start out. Thus the kinetic theory of gases seeks to reduce mechanical, thermal, and diffusional processes to movements of molecules - i.e., to build them up out of the hypothesis of molecular motion. When we say that we have succeeded in understanding a group of natural processes, we invariably mean that a constructive theory has been found which covers the processes in question.

Along with this most important class of theories there exists a second, which I will call "principle-theories." These employ the analytic, not the synthetic, method. The elements which form their basis and starting-point are not hypothetically constructed but empirically discovered ones, general characteristics of natural processes, principles that give rise to mathematically formulated criteria which the separate processes or the theoretical representations of them have to satisfy. Thus the science of thermodynamics seeks by analytical means to deduce necessary conditions, which separate events have to satisfy, from the universally experienced fact that perpetual motion is impossible.

The advantages of the constructive theory are completeness, adaptability, and clearness, those of the principle theory are logical perfection and security of the foundations. The theory of relativity belongs to the latter class.

Theoretician probably agrees with Einstein that the identification of first principles is the basic goal. Unfortunately, the first principle approach sooner or later leads to astray if some principle happens to be wrong. Einstein himself was not able to accept the non-deterministic aspect of quantum physics and used rest of his life to fruitless attempts to get rid of it. I believe that the almost non-existence of theoretical biology and neuroscience is due to the adherence to the materialistic and reductionistic dogmas and to the stubborn belief that biology cannot teach anything to physicist at the level of fundamental principles. I also believe that the degeneration of superstring theory - once thought to be the ultimate victory of the reductionistic approach - to landscape misery is due to the refusal to realize that something is badly wrong at the level of first principles.

Therefore it seems that one most tolerate the tension created by the simultaneous application of both the analytic approach deducing predictions from believed-to-be first principles and the synthetic approach which I understand as a continual challenging of these principles. I have felt very intensively the tension between these approaches during the three decades that I have spent in attempts to identify the fundamental principles that would allow to build around the basic idea of TGD a computational machinery allowing to make precise quantitative predictions. What looked for some years to be nothing but a technical challenge of developing a perturbative approach based on path integral formalism has led to a profound restructuring of the basic ontology. Mention only the new view about time, the notion of many-sheeted space-time, p-adic physics, zero energy ontology, a generalization of quantum physics itself based on the hierarchy of Planck constants involving also a generalization of the notion of 8-D imbedding space $M^4 \times CP_2$, and extension of quantum physics to a quantum theory of consciousness.

This process is by no means over yet. I do not have explicit formulas for S-matrix elements and even during last two years several new ideas have emerged at fundamental level. A more precise basic formulation of quantum TGD in terms of M-matrix generalizing the notion of S-matrix in the framework of zero energy ontology relying on the notion of finite resolution of quantum measurement represents perhaps the most important step of progress (chapter *Construction of Quantum Theory: S-matrix* of [4]). The improved understanding of the relationship between experienced time and geometric time (chapter *Quantum Model of Memory* of [10]) has led to a more detailed picture about p-adic coupling constant evolution and p-adic length scale hypothesis can be now deduced from the first principles (chapter *Construction of Quantum Theory: S-matrix* of [4]). The surprise was that p-adic length scale hypothesis assigns with every elementary particle a fundamental macroscopic time scale characterizing the temporal span of the zero energy space-time sheet associated with the particle. This time scale has nothing to do with the lifetime of the particle. In the case of electron the time scale .1 seconds which happens to be a fundamental biorhythm. An improved understanding of Higgs mechanism and quantum classical correspondence has also emerged (chapter *Is it Possible to Understand Coupling Constant Evolution at Space-Time Level?* of [4]).

In this article I will discuss and compare different first principle approaches to quantum TGD. The following list of various competing visions about the basic principles of quantum Topological Geometrodynamics (TGD) might give some idea about how TGD relates to standard model, general relativity and super-string models and how it generalizes their conceptual frameworks.

1. Physics as the classical spinor field geometry of the "world of classical worlds" consisting of light-like 3-surfaces in certain 8-D imbedding space (to be referred as configuration space CH in the sequel) is the oldest and best developed approach to TGD and means generalization of Einstein's program of geometrizing classical physics so that it applies to entire quantum physics (chapters *Construction of Configuration Space Kähler Geometry from Symmetry Principles: Part I/II* and *Configuration Space Spinor Structure* of [2]).

2. Parton level formulation of quantum TGD as an almost topological quantum field theory [40] using light-like 3-surfaces as fundamental objects allows a detailed understanding of super-conformal symmetries generalizing those of super string models (chapters *Construction of Quantum Theory: Symmetries* and *Construction of Quantum Theory: S-matrix* of [4]). A category theoretical interpretation of M-matrix as a functor is possible. This picture has tight connections to the physics as configuration space geometry approach and implies it.

3. Zero energy ontology and the vision that finite measurement resolution formulated in terms of inclusions of certain von Neumann algebras known as hyperfinite factors of type II_1 (briefly HFFs in the sequel) allows to fix quantum dynamics completely and interpret quantum theory as a square root of thermodynamics (chapter *Construction of Quantum Theory: S-matrix* of [4]). Zero energy state corresponds geometrically to a causal diamond defined by pair of future and past directed light-cones and the temporal distance T between the tips of these light-cones defines a new physical time scale characterizing also given elementary particle. This picture leads to a justification for p-adic length scale hypothesis.

4. Physics as generalized number theory represents a further vision about TGD [5]. Number theoretic universality meaning a fusion of real and p-adic physics to single coherent whole forces a formulation in terms of so called number theoretic braids (chapter *TGD as a Generalized Number Theory: p-Adicization Program* of [5]). The symmetries of classical number fields strongly suggest the interpretation in terms of standard model symmetries and a number theoretic interpretation of the imbedding space $H = M^4 \times CP_2$ (chapter *TGD as a Generalized Number Theory: Quaternions, Octonions, and their Hyper Counterparts* of [5]). Associativity condition would define the laws of classical and quantum physics. The notion of infinite prime (chapter *TGD as a Generalized Number Theory: Infinite Primes* of [5]) defines a third thread in the braid of number theoretical ideas and it is now possible to give a rather detailed realization for what I have called number theoretic Brahman=Atman identity (or

algebraic holography) based on the generalization of the number concept by allowing infinite number of real units representable as ratios of infinite integers having interpretation as representations for physical states of a super-symmetric arithmetic QFT. The infinitely rich number theoretic anatomy for the points of number theoretic braids allow to represent that information about zero energy states which remains below the measurement resolution (chapter *TGD as a Generalized Number Theory: Infinite Primes* of [5]).

5. The identification of the elements of the configuration space Clifford algebra (in particular gamma matrices) associated with its spinor structure (chapter *Configuration Space Spinor Structure* of [2]) as hyper-octonion valued conformal fields having values in HFF (chapter *TGD as a Generalized Number Theory: Quaternions, Octonions, and their Hyper Counterparts* of [5]) allows to deduce most of the speculative "must-be-true's" of quantum TGD ([16] and the chapter *TGD as a Generalized Number Theory: Quaternions, Octonions, and their Hyper Counterparts* of [5]) and gives a tight connection to super string models allowing among other things a purely number theoretic interpretation of gauge symmetries.

6. The idea about hierarchy of Planck constants (chapter *Does TGD Predict the Spectrum of Planck Constants?* of [4]) was inspired by certain empirical facts [28]. The hierarchy leads to a generalization of the notion of imbedding space emerging from the requirement that the choice of quantization axes has a geometric correlate also at the level of imbedding space H and configuration space CH. The physical implication is the identification of dark matter in terms of a hierarchy of macroscopically quantum coherent phases with quantized values of Planck constant having arbitrarily large values and playing a key role, not only in biology but also in astrophysics and cosmology of TGD Universe (chapter *Quantum Astrophysics* of [3]). The hierarchy of Planck constants can be seen as necessary for the realization of quantum criticality. The generalization of imbedding space is also essential for the construction of the Kähler function of configuration space chapters (chapters *Construction of Configuration Space Kähler Geometry from Symmetry Principles: Part I/II*).

7. A further vision about quantum TGD is that the mere finiteness of measurement resolution fixes the scattering matrix of quantum TGD (chapter *Construction of Quantum Theory: S-matrix* of [4]). In zero energy ontology S-matrix must be generalized to M-matrix whose matrix elements are identified as time-like entanglement coefficients between positive and negative energy parts of zero energy states. M-matrix can be regarded as a "complex square root" of density matrix expressible as a product of a real square root of density matrix and unitary S-matrix: thermodynamics thus becomes part of quantum theory. HFFs emerge naturally through the Clifford algebra of CH and allow a formulation of quantum measurement theory with a finite measurement resolution. The notion of finite measurement resolution expressed in terms of inclusion $\mathcal{N} \subset \mathcal{M}$ of HFFs with included algebra $\mathcal{N}$ defining the measurement resolution leads to an identification of M-matrix in terms of Connes tensor product. A simple argument (chapter *Construction of Quantum Theory: S-matrix* of [4]) shows that M-matrix is unique apart from the presence of "complex" square root of density matrix needed by thermodynamics. Coupling constant evolution corresponds to a hierarchy of measurement resolutions and p-adic coupling constant evolution and p-adic length scale hypothesis $p \simeq 2^n$ follow as a consequence (chapter *TGD as a Generalized Number Theory: p-Adicization Program* of [5]) with an additional prediction for the temporal distance T between the tips of future and past directed light-cones defining causal diamond as secondary p-adic time scale $T(2,p) = \sqrt{p}L_p/c$ coming as a power of two multiple of CP_2 time scale. For electron this time scale is .1 seconds defining a fundamental biorhythm. Thus zero energy ontology implies

a direct connection between elementary particle physics and biology.

8. Generalized super-conformal symmetries associated with $\delta M^4_\pm \times CP_2$ (super-canonical algebra) and light-like 3-surfaces (Kac-Moody type algebra) define basic symmetries of quantum TGD (chapter *Construction of Quantum Theory: Symmetries* of [4]), and one can argue that these symmetries alone dictate to high degree the physics. A symplectic variant of conformal field theory emerges very naturally in TGD framework (symplectic symmetries acting on $\delta M^4_\pm \times CP_2$ are in question) and this leads to a concrete proposal for how to to construct n-point functions needed to calculate M-matrix (chapter *Construction of Quantum Theory: S-matrix* of [4]). The mechanism guaranteeing the predicted absence of divergences in M-matrix elements can be understood in terms of vanishing of symplectic invariants as two arguments of n-point function coincide. The recent progress in the understanding of the representations of super-conformal symmetries leads to a beautiful generalization of Equivalence Principle and Einstein's equations in terms of Super Virasoro conditions for the coset construction involving the super-canonical algebras associated with conformal symmetries of the light-cone of Minkowski space and super Kac-Moody symmetries associated with light-like 3-surfaces (chapter *Construction of Quantum Theory: S-matrix* of [4]).

9. Consciousness theory interpreted as a generalization of quantum measurement theory ([17] and chapter *Matter, Mind, Quantum* of [10]) provides the most plausible vision about quantum TGD and has already shown its power and brought into theory notions which cannot be imagined in the standard conceptual framework of quantum physics.

2 What might be the first principles of quantum TGD

In the following I want to summarize different visions about first principles of quantum TGD emphasizing the tension between analytic and synthetic approaches. The first principle approaches have been already listed in the introduction. There is of course also the tension between these different visions. How these approaches relate to each other and can one build bridges between them?

2.1 From Equivalence Principle to zero energy ontology

The tension between analytic and constructive approaches is present even in Einstein's own theory and was the basic stimulus leading to TGD. Equivalence Principle states that gravitational and inertial masses are identical. This statement is however rather problematic as Einstein himself was first to admit. Einstein's equations express the identity for gravitational and inertial energy momentum densities as a consequence of a variational principle. There is however no global version of this statement because one cannot define the notions of inertial and gravitational four-momenta without adherence to perturbative approach.

The hypothesis that space-times are 4-D surfaces of a higher-dimensional space-time of form $H = M^4 \times S$ resolves the problem: since Poincare symmetries become symmetries of H rather than space-time itself. Inertial four-momentum can be defined as a conserved Noether charge and also gravitational four-momentum can be regarded as a Noether charge albeit non-conserved. Equivalence Principle can hold true only under some additional conditions. For instance, for the imbeddings of Robertson-Walker cosmologies inertial four-momentum density vanishes unlike gravitational four-momentum density, which for a long time remained quite a mystery. The real understanding of the situation became possible only after the introduction of what I call zero energy ontology (chapter *Construction of Quantum Theory: S-matrix* of [4]).

In zero energy ontology one replaces positive energy states with zero energy states with positive and negative energy parts of the state at the boundaries of future and past direct light-cones forming a causal diamond. All conserved quantum numbers of the positive and negative energy states are of opposite sign so that these states can be created from vacuum. "Any physical state is creatable from vacuum" becomes thus a basic principle of quantum TGD and together with the notion of quantum jump resolves several philosophical problems (What was the initial state of universe?, What are the values of conserved quantities for Universe, Is theory building completely useless if only single solution of field equations is realized?).

At the level of elementary particle physics positive and negative energy parts of zero energy state are interpreted as initial and final states of a particle reaction so that quantum states become physical events. Equivalence Principle would hold true in the sense that the classical gravitational four-momentum of the vacuum extremal whose small deformations appear as the argument of configuration space spinor field is equal to the positive energy of the positive energy part of the zero energy quantum state. Equivalence Principle is expected to hold true for elementary particles and their composites but not for the quantum states defined around non-vacuum extremals.

More precisely, the inertial four-momentum assignable to the 3-D Chern-Simons action is non-vanishing only if one adds to the CP_2 Kähler form a pure gauge part $A_a = constant$, where a denotes light cone proper time (chapter *Configuration Space Spinor Structure* of [2]). A breaking of Poincare invariance is implied which is however compensated by the fact that configuration space corresponds to the union of configuration spaces associated with future and past directed light-cones. If the vacuum extremal is also an extremal of the curvature scalar, gravitational four-momentum is conserved.

In the case of CP_2 type vacuum extremal gravitational stationarity transforms the M^4 projection of the extremal from a random light-like curve to a light-like geodesic allowing an interpretation as incoming or outgoing on mass shell particle. General vacuum extremal corresponds to a virtual particle. At the classical level Equivalence Principle requires that the light-like gravitational four-momentum of CP_2 vacuum extremal co-incides with the light-like inertial four-momentum associated with Chern-Simons action in this situation. This condition relates the value of A_a to gravitational constant G and CP_2 radius R. G would thus appear as a fundamental constant and quantum criticality should dictate the ratio G/R^2.

The strong form of Equivalence Principle would require that the classical 4-momentum associated with Kähler action of allowed small deformations co-incides with the conserved gravitational four-momentum of the vacuum extremal extremizing curvature scalar. This might have a natural interpretation in terms of Bohr orbitology but is not be consistent with zero energy ontology inspired picture unless one has double sheeted structure with sheets possessing opposite energies such that double sheeted structure is approximated by single sheet with Robertson-Walker cosmology in GRT framework. The identification of gauge bosons as wormhole contacts and gravitons as pairs of wormhole contacts supports double sheeted structure with sheets possessing opposite arrows of geometric time.

A rather promising first principle formulation of Equivalence Principle relies on the generalized conformal invariance. The general view about particle massivation is based on the generalized coset construction allowing to understand the p-adic thermal contribution to mass squared as a thermal expectation value of the conformal weight for super Kac-Moody Virasoro algebra ($SKMV$) or equivalently super-canonical Virasoro algebra (SCV) (chapter *Construction of Quantum Theory: Symmetries* of [4]). Conformal invariance holds true only for the differences of $SKMV$ and SCV generators which annihilate physical states. In the case of SCV and $SKMV$ only the generators G_n,L_n, $n > 0$, annihilate the physical states. The actions of SCV generators and $SKMV$ generators on physical states are identical by coset construction. The interpretation is in terms of Equivalence Principle since $SKMV$ corresponds rather naturally to gravitational four-momentum and SCV to inertial four-momentum (this applies also to inertial and gravitational color quantum numbers). The conditions stating the vanishing of the differences of the generators become the TGD counterpart for Einstein's equations. The mathematical justification for this picture comes from the possibility to lift the SC algebra from $\delta M^4_\pm \times CP_2$ and SKM algebra from the partonic 3-surface X^3 to the level of imbedding space to hyper-complex and perhaps even hyper-quaternionic algebra. Here the basic prerequisite for number theoretic compactification to be discussed later plays a key role.

2.2 Physics as a physics of classical spinor fields in the world of classical worlds?

The oldest vision relies on the identification of quantum physics as a unique completely classical physics of spinor fields in the "world of classical worlds" (configuration space) with quantum jump being the only genuinely quantal element in this approach. This means a generalization of Einstein's geometrization program to the level of quantum theory. Fermi statistics finds geometrization: the anti-commutation relations for the configuration space gamma matrices correspond to the anticommutation relations for fermionic oscillator operators associated with free induced spinor fields defined at light-like 3-surfaces. Gamma matrices define also fermionic generators of a super-conformal algebra.

The needed infinite-dimensional Kähler geometry exists only if it has maximal group of isometries, and a generalization of super conformal symmetries of super string models emerges naturally in this approach and dictates the dynamics to a high degree (chapters *Construction of Configuration Space Geometry from Symmetry Principles: Part I/II* and *Configuration Space Spinor Structure* of [2]). One might even hope that physics is completely unique from the mere requirement that this geometry exists mathematically. General coordinate invariance requires that the definition of Kähler geometry in terms of Kähler function assigns to 3-surface a highly unique space-time surface: the interpretation is as the analog of Bohr orbit so that classical theory becomes exact part of quantum theory. Kähler function is identified as a preferred extremal for so called Kähler action which is essentially Maxwell action for the induced Kähler form of CP_2.

Quantum criticality is the basic dynamical principle in this approach and implies that the only coupling constant of the theory - Kähler coupling strength - is analogous to a critical temperature and follows as a prediction (chapter *Construction of Configuration Space Kähler Geometry from Symmetry Principles: Part II* of [2]).

2.3 Is the dynamics of quantum TGD fixed from lightlikeness of 3-surfaces?

The 4-D general coordinate invariance leads naturally to the identification of light-like 3-D surfaces as fundamental dynamical objects. These light-like 3-surfaces correspond to light-like orbits of throats of CP_2 sized wormhole contacts connecting so called CP_2 type extremal to a space-time sheet or CP_2 extremals connecting two space-time sheets to each other. In the case of fermions there is only single wormhole throat carrying fermion number. In the case of bosons connecting two space-time sheets throats carry opposite fermion numbers. Locally one can interpret light-like 3-surfaces as random orbits of partonic 2-surfaces moving with local light-velocity.

The first implication of the light-likeness is metric two-dimensionality meaning a generalization (chapter *Construction of Quantum Theory: Symmetries* of [4]) of the super-conformal symmetries of super-string models [32, 25]. Conformal invariance in turn means that the basic objects are in a well-defined sense 1-D strings. This does not however mean that TGD would degenerate to a theory of 2-D or 1-D fundamental objects (this fear generated one of the long lasting tensions!). The generalized conformal invariance is not global: light-like 3-surfaces decompose into 3-D regions inside which generalized conformal invariance holds true so that 3-dimensionality remains in a discrete sense. Global 2-D conformal invariance for partonic 2-surface fails in a similar manner which means discretized 2-dimensionality. Local 1-dimensionality in turn reduces to discreteness at the fundamental level by the finiteness of quantum measurement resolution. Discrete 4-dimensionality is implied in an analogous manner.

This vision leads to a profound understanding of quantum TGD as almost topological quantum field theory (TQFT [40]) based on the analog of Chern-Simons action and its fermionic counterpart (chapter *Configuration Space Spinor Structure* of [2]). The attribute "almost" is forced by the light-likeness condition involving induced metric and means that metric related quantum numbers such as energy and momentum characterize physical states. This theory should have conformal field theories as basic building blocks. Also a connection with category theoretical ideas emerges (chapter *Construction of Quantum Theory: S-matrix* of [4]).

Concerning the tension with geometric approach can be resolved by deducing from 3-D description the 4-D description based on Kähler function from the parton level description. Quantum classical correspondence should justify the basic assumption of the quantum measurement theory that quantum transitions have classical space-time correlates (for instance, measurement of spin component leads to a splitting of electron beam in Stern-Gerlach experiment realized as a splitting of 3-D surfaces to two parts as analog of stringy diagram). In other words, the 4-D description based on geometrized classical fields should provide 4-D space-time correlate for the fundamental quantum description in terms of 3-D light-like surfaces. This correspondence is in some sense dual of the holographic principle in M-theory.

Consistency requirement allows to guess concrete formulas for the exponent of Kähler function in terms of Dirac determinants associated with the modified Dirac operator defined at light-like partonic 3-surfaces (chapter *Configuration Space Spinor Structure* of [2]). Also proposals for explicit recipes for how to construct the 4-D space-time sheets as preferred extremals of Kähler action emerge. Discretization is present in terms of what I call number theoretical braids. The tension however remains: these formulas are educated guesses inspired by the general principles.

2.4 Physics as a generalized number theory?

Physics as a generalized number theory vision involves actually three threads: p-adic ideas (chapter *TGD as a Generalized Number Theory: p-Adicization Program* of [5]), the ideas related to classical number fields (chapter *TGD as a Generalized Number Theory: Quaternions, Octonions, and their Hyper Counterparts* of [5]), and ideas related to the notion of infinite prime (chapter *TGD as a Generalized Number Theory: Infinite Primes* of [5]).

2.4.1 Fusion of real and p-adic physics to a coherent whole as guiding principle

p-Adic number fields were not present in the original approach to TGD. The success of the p-adic mass calculations (summarized in the first part [6]) made however clear that one must generalize the notion of topology also at the infinitesimal level from that defined by real numbers so that the attribute "topological" in TGD gains much more profound meaning than intended originally. It took a decade to get convinced that the identification of p-adic physics as a correlate of cognition and intentionality is the only plausible interpretation discovered hitherto (chapter *p-Adic Physics as Physics of Cognition and Intention* of [10]) and that p-adic topology of p-adic space-time sheets induces the effective p-adic topology of real space-time sheets.

The original view about physics as the geometry of the world of the classical worlds is not enough to meet the challenge of unifying real and p-adic physics to a single coherent whole. This inspired "physics as a generalized number theory" approach [5] relying on a generalization of the notion of number obtained by "gluing" reals and various p-adic number fields and their algebraic extensions along common rationals and algebraics. The same gluing procedure for of real p-adic physics to a larger structure forces to introduce a discretization at space-time level in terms of rational and algebraic numbers.

The interpretation is in terms of cognitive, sensory, and measurement resolutions rather than fundamental discreteness of the space-time. What looks rather counter intuitive first is that transcendental points of p-adic space-time sheets are at spatiotemporal infinity in real sense so that the correlates of cognition and intentionality cannot be localized to any finite spatiotemporal volume unlike those of sensory experience. This description of intentionality and cognition in this manner predicts p-adic fractality of real physics meaning chaos in short scales combined with long range correlations: p-adic mass calculations represent one example of p-adic fractality.

There is also a tension between p-adication program and physics as infinite-D configuration space geometry and TGD as almost TQFT approaches. They seem to be consistent. The discretization forced by p-adicization leads to the notion of number theoretic braid - braids are indeed fundamental objects of 3-D TQFTs- and also to a guess for the formula for the exponent of Kähler function in terms of data associated with number theoretic braids associated with a given collection of light-like 3-surfaces.

2.4.2 Classical number fields and associativity and commutativity as fundamental law of physics

The dimensions of classical number fields appear as dimensions of basic objects in quantum TGD. Imbedding space has dimension 8, space-time has dimension 4, light-like 3-surfaces are orbits of 2-D partonic surfaces, and by conformal invariance one-dimensional structures are basic objects. The lowest level corresponds to discrete sets of points identifiable as intersections of real and p-adic space-time sheets. This suggests that besides p-adic number fields also classical number fields (reals, complex numbers, quaternions, octonions [26]) are involved (chapter *TGD as a Generalized Number Theory: Quaternions, Octonions, and their Hyper Counterparts* of [5]) and the notion of geometry generalizes considerably. In the recent view about quantum TGD the dimensional hierarchy defined by classical number field indeed plays a key role. $H = M^4 \times CP_2$ has a number theoretic interpretation and standard model symmetries can be understood number theoretically as symmetries of hyper-quaternionic planes of hyper-octonionic space.

The associativity condition $A(BC) = (AB)C$ suggests itself as a fundamental physical law of both classical and quantum physics. Commutativity can be considered as an additional condition. In conformal field theories associativity condition indeed fixes the n-point functions of the theory. At the level of classical TGD space-time surfaces could be identified as maximal associative (hyper-quaternionic) submanifolds of the imbedding space whose points contain a preferred hyper-complex plane M^2 in their tangent space and the hierarchy finite fields-rationals-reals-complex numbers-quaternions-octonions could have direct quantum physical counterpart (chapter *TGD as a Generalized Number Theory: Quaternions, Octonions, and their Hyper Counterparts* of [5]). This leads to the notion of number theoretic compactification analogous to the dualities of M-theory: one can interpret space-time surfaces either as hyper-quaternionic 4-surfaces of M^8 or as 4-surfaces in $M^4 \times CP_2$. As a matter fact, commutativity in number theoretic sense is a further natural condition and leads to the notion of number theoretic braid naturally as also to direct connection with super string models.

The tension with the vision about physics as infinite-D configuration space geometry might be resolvable. It would not be surprising if the uniqueness of infinite-dimensional Kähler geometric existence would require that the isometries and holonomies of imbedding space defining standard model symmetries correspond to a group having a number theoretic interpretation so that $X^4 \subset M^4 \times CP_2$ would be preferred (chapter *TGD as a Generalized Number Theory: Quaternions, Octonions, and their Hyper Counterparts* of [5]).

2.4.3 Infinite primes and quantum physics

The hierarchy of infinite primes (and of integers and rationals) (chapter *TGD as a Generalized Number Theory: Infinite Primes* of [5]) was the first mathematical notion stimulated by TGD inspired theory of consciousness. The construction recipe is equivalent with a repeated second quantization of super-symmetric arithmetic quantum field theory with bosons and fermions labeled by primes such that the many-particle states of previous level become the elementary particles of new level. The hierarchy of space-time sheets with many particle states of space-time sheet becoming elementary particles at the next level of hierarchy and also the hierarchy of n:th order logics are also possible correlates for this hierarchy. For instance, the description of proton as an elementary fermion would be in a well defined sense exact in TGD Universe.

This construction leads also to a number theoretic generalization of space-time point since a given real number has infinitely rich number theoretical structure not visible at the level of the real norm of the number a due to the existence of real units expressible in terms of ratios of infinite integers. This number theoretical anatomy suggest a kind of number theoretical Brahman=Atman principle stating that the set consisting of number theoretic variants of single point of the imbedding space (equivalent in real sense) is able to represent the points of the world of classical worlds or maybe even quantum states assignable to causal diamond[19]. Also a formulation in terms of number theoretic holography is possible.

Just for fun and to test these ideas one can consider a model for the representation of the configuration space spinor fields in terms of algebraic holography. I have considered guesses for this kind of map earlier (chapter *Intentionality, Cognition, and Physics as Number theory or Space-Time Point as Platonia* of [5]) and it is interesting to find whether additional constraints coming from zero energy ontology and finite measurement resolution might give. The identification of quantum corrections as insertion of zero energy states in time scale below measurement resolution to positive or negative energy part of zero energy state and the identification of number theoretic braid as a space-time correlate for the finite measurement resolution give considerable additional constraints.

1. The fundamental representation space consists of wave functions in the Cartesian power U^8 of space U of real units associated with any point of H. That there are 8 real units rather than one is somewhat disturbing: this point will be discussed below. Real units are ratios of infinite integers having interpretation as positive and negative energy states of a super-symmetric arithmetic QFT at some level of hierarchy of second quantizations. Real units have vanishing net quantum numbers so that only zero energy states defining the basis for configuration space spinor fields should be mapped to them. In the general case quantum superpositions of these basis states should be mapped to the quantum superpositions of real units. The first guess is that real units represent a basis for configuration space spinor fields constructed by applying bosonic and fermionic generators of super-canonical and super Kac-Moody type algebras to the vacuum state.

2. What can one say about this map bringing in mind Gödel numbering? Each pair of bosonic and corresponding fermionic generator at the lowest level must be mapped to its own finite prime. If this map is specified, the map is fixed at the higher levels of the hierarchy. There exists an infinite number of this kind of correspondences. To achieve some uniqueness, one should have some natural ordering which one might hope to reflect real physics. The irreps of the (non-simple) Lie group involved can be ordered almost uniquely. For simple group this ordering would be with respect to the sum $N = N_F + N_{F,c}$ of the numbers N_F *resp.* $N_{F,c}$ of the fundamental representation *resp.* its conjugate appearing in the minimal tensor product giving the irrep. The generalization to non-simple case should use the sum of the integers N_i for different factors for factor groups. Groups themselves could be ordered by some criterion, say dimension. The states of a given representation could be mapped to subsequent finite primes in an order respecting some natural ordering of the states by the values of quantum numbers from negative to positive (say spin for $SU(2)$ and color isospin and hypercharge for $SU(3)$). This would require the ordering of the Cartesian factors of non-simple group, ordering of quantum numbers for each simple group, and ordering of values of each quantum number from positive to negative.

The presence of conformal weights brings in an additional complication. One cannot use conformal as a primary orderer since the number of $SO(3) \times SU(3)$ irreps in the super-canonical sector is infinite. The requirement that the probabilities predicted by p-adic thermodynamics are rational numbers or equivalently that there is a length scale cutoff, implies a cutoff in conformal weight. The vision about M-matrix forces to conclude that different values of the total conformal weight n for the quantum state correspond to summands in a direct sum of HFFs. If so, the introduction of the conformal weight would mean for a given summand only the assignment n conformal weights to a given Lie-algebra generator. For each representation of the Lie group one would have n copies ordered with respect to the value of n and mapped to primes in this order.

3. Cognitive representations associated with the points in a subset, call it P, of the discrete intersection of p-adic and real space-time sheets, defining number theoretic braids, would be in question. Large number of partonic surfaces can be involved and only few of them need to contribute to P in the measurement resolution used. The fixing of P means measurement of N positions of H and each point carries fermion or anti-fermion numbers. A more general situation corresponds to plane wave type state obtained as superposition of these states. The condition of rationality or at least algebraicity means that discrete variants of plane waves are in question.

4. By the finiteness of the measurement resolution configuration space spinor field decomposes into a product of two parts or in more general case, to their superposition. The part Ψ_+, which is above measurement resolution, is representable using the information contained by P, coded by the product of second quantized induced spinor field at points of P, and provided by physical experiments. Configuration space "orbital" degrees of freedom should not contribute since these points are fixed in H.

5. The second part of the configuration space spinor field, call it Ψ_-, corresponds to the information below the measurement resolution and assignable with the complement of P and mappable to the structure of real units associated with the points of P. This part has vanishing net quantum numbers and is a superposition over the elements of the basis of CH spinor fields and mapped to a quantum superposition of real units. The representation of Ψ_- as a Schrödinger amplitude in the space of real units could be highly unique. Algebraic holography principle would state that the information below measurement resolution is mapped to a Schrödinger amplitude in space of real units associated with the points of P.

6. This would be also a representation for perceiver-external world duality. The correlation function in which P appears would code for the information appearing in M-matrix representing the laws of physics as seen by conscious entity about external world as an outsider. The quantum superposition of real units would represent the purely subjective information about the part of universe below measurement resolution.

There is an objection against this picture. One obtains an 8-plet of arithmetic zero energy states rather than one state only. What this strange 8-fold way could mean?

1. The crucial observation is that hyper-finite factor of type II_1 (HFF) creates states for which center of mass degrees of freedom of 3-surface in H are fixed. One should somehow generalize the operators creating local HFF states to fields in H, and an octonionic generalization of conformal field suggests itself. I have indeed proposed a quantum octonionic generalization of HFF extending to an HFF valued field Ψ in 8-D quantum octonionic space with the property that maximal quantum commutative sub-space corresponds to hyper-octonions (chapter *Was von Neumann Right After All* of [4]). This construction raises $X^4 \subset M^8$ and by number theoretic compactification also $X^4 \subset H$ in a unique position since non-associativity of hyper-octonions

does not allow to identify the algebra of HFF valued fields in M^8 with HFF itself.

2. The value of Ψ in the space of quantum octonions restricted to a maximal commutative subspace can be expressed in terms of 8 HFF valued coefficients of hyper-octonion units. By the hyper-octonionic generalization of conformal invariance all these 8 coefficients must represent zero energy HFF states. The restriction of Ψ to a given point of P would give a state, which has 8 HFF valued components and Brahman=Atman identity would map these components to U^8 associated with P. One might perhaps say that 8 zero energy states are needed in order to code the information about the H positions of points P. The condition that Ψ represents a state with vanishing quantum numbers gives additional constraints. The interpretation inspired by finite measurement resolution is that the coordinate h associated with Ψ corresponds to a zero energy insertion to a positive or negative energy state localizable to a causal diamond inside the upper or lower half of the causal diamond of observer. Below measurement resolution for imbedding space coordinates $\Psi(h)$ would correspond to a nonlocal operator creating a zero energy state. This would mean that Brahman=Atman would apply to the mini-worlds below the measurement resolution rather than to entire Universe but by algebraic fractality of HFFs this would would not be a dramatic loss.

2.5 Configuration Clifford algebra elements as hyper-octonionic conformal fields having values in HFF?

The fantastic properties of HFFs inspire the question whether a localized hyper-octonionic version of Clifford algebra of configuration space interpreted as an abstract HFF might allow to see space-time, embedding space, and configuration space as emergent structures. The following arguments do not prove this. Commutativity and associativity conditions however imply most of the speculative "must-be-true's" of quantum TGD.

The starting point is that configuration space gamma matrices act only in vibrational degrees of freedom of 3-surface. One must also include center of mass degrees of freedom which appear as zero modes. The natural idea is that the resulting local gamma matrices define a local version of HFF as a generalization of conformal field of gamma matrices appearing super string models obtained by replacing complex numbers with hyper-octonions identified as a subspace of complexified octonions. As a matter fact, one can generalize octonions to quantum octonions for which quantum commutativity means restriction to a hyper-octonionic subspace of quantum octonions (chapter *Was von Neumann Right After All* of [4]). Non-associativity is essential for obtaining something non-trivial: otherwise the resulting algebra reduces to HFF of type II_1 since matrix algebra as a tensor factor would give an algebra isomorphic with the original one. The octonionic variant of conformal invariance fixes the dependence of local gamma matrix field on the coordinate of HO. The coefficients of Laurent expansion of this field must commute with hyper-octonions.

The world of classical worlds has been identified as a union of configuration spaces associated with $M^4_{\pm}$ labeled by points of H or equivalently $HO = M^8$. The choice of quantization axes certainly fixes a point of H (HO) as a point remaining fixed under $SO(1,3) \times U(2)$ ($SO(1,3) \times SO(4)$). The condition that hyper-quaternionic inverses of $M^4 \subset HO$ points exist suggest a restriction of arguments of the n-point function to the interior of $M^4_{\pm}$.

Associativity condition for the n-point functions forces to restrict the arguments to a hyper-quaternionic plane $HQ = M^4$ of HO. One can also consider the commutativity condition by requiring that arguments belong to a preferred commutative sub-space $HC = M^2$ of HO. Fixing preferred real and imaginary units means a choice of M^2 interpreted as a partial choice of quantization axes. This has quite strong implications.

1. The hyper-quaternionic planes with a fixed choice of M^2 are labeled by points of CP_2. If the condition $M^2 \subset T^4$ characterizes the tan-

gent planes of all points of $X^4 \subset HO$ it is possible to map $X^4 \subset HO$ to $X^4 \subset H$ so that $HO - H$ duality ("number theoretic compactification") emerges. $X^4 \subset H$ should correspond to a preferred extremal of Kähler action. The physical interpretation would be as a global fixing of the plane M^2 of non-physical polarizations in M^8: it is not quite clear whether this choice of polarization need not have direct counterpart for $X^4 \subset H$. Standard model symmetries emerge naturally. The resulting surface in $X^4 \subset H$ would be analogous to a warped plane in E^3. This new result suggests rather direct connection with super string models. In super string models one can choose the polarization plane freely and one expects also now that the generalized choice $M^2 \subset M^4 \subset M^8$ of polarization plane can be made freely without losing Poincare invariance with a reasonable assumption about zero energy states.

2. One would like to fix local tangent planes T^4 of X^4 at 3-D light-like surfaces X^3_l fixing the preferred extremal of Kähler action defining the Bohr orbit. An additional direction t should be added to the tangent plane T^3 of X^3_l to give T^4. This might be achieved if t belongs to M^2 and perhaps corresponds to a light-like vector in M^2.

3. Assume that partonic 2-surfaces X belong to $\delta M^4_\pm \subset HO$ defining ends of the causal diamond. This is obviously an additional boundary condition. Hence the points of partonic 2-surfaces are associative and can appear as arguments of n-point functions. One thus finds an explanation for the special role of partonic 2-surfaces and a reason why for the role of light-cone boundary. Note that only the 2-D ends of light-like 3-surfaces need intersect $M^4_\pm \subset HO$. A stronger condition is that the pre-images of light-like 3-surfaces in H belong to $M^4_\pm \subset HO$.

4. Commutativity condition is satisfied if the arguments of the n-point function belong to an intersection $X^2 \cap M^2 \subset HQ$ and this gives a discrete set of points as intersection of light-like radial geodesic and X^2 perhaps identifiable in terms of points in the intersection of number theoretic braids with $\delta H_\pm$. One should show that this set of points consists of rational or at most algebraic points. Here the possibility to choose X^2 to some degree could be essential. As a matter fact, any radial light ray from the tip of light-cone allows commutativity and one can consider the possibility of integrating over n-point functions with arguments at light ray to obtain maximal information. For the pre-images of light-like 3-surfaces commutativity would allow one-dimensional curves having interpretation as braid strands.

To sum up, this picture implies HO-H duality with a choice of a preferred imaginary unit fixing the plane of non-physical polarizations globally, standard model symmetries, and number theoretic braids. The introduction of hyper-octonions could be however criticized: could octonions and quaternions be enough after all? Could HO-H duality be replaced with O-H duality and be interpreted as the analog of Wick rotation? This would mean that quaternionic 4-surfaces in E^8 containing global polarization plane E^2 in their tangent spaces would be mapped by essentially by the same map to their counterparts in $M^4 \times CP_2$,and the time coordinate in E^8 would be identified as the real coordinate. Also light-cones in E^8 would make sense as the inverse images of $M^4_\pm$.

2.6 Hierarchy of Planck constants and quantum criticality

The hypothesis about a hierarchy of Planck constants was motivated by anomalies of biophysics (chapters *Dark Matter Hierarchy and Hierarchy of EEGs* and *Quantum Model for Nerve Pulse and EEG* of [12] and *About the New Physics Behind Qualia* of [9]) and of astrophysics (chapters *TGD and Astrophysics* and *Quantum Astrophysics* of [3]) and led to a further generalization of the notion of imbedding space providing a connection with quantum groups and a fundamental description of anyons and quantum Hall effect (chapter *Does TGD Predict the Spectrum of Planck Constants?* of [4]). One can say that imbedding space has a book like structure obtained by gluing together almost copies of imbedding space along 4-dimensional submanifolds playing the role of the back of the book. These almost copies are in well-defined sense constructed from products of factor and covering spaces of M^4 and CP_2. Different pages of book would in general have different values of Planck constant. In the vertices of Feynman diagrams only particles of given sector with same Planck constant would appear so that the particles at different pages would be dark relative to each other. They would however interact via classical gauge fields and also by exchange of particles since the particles can tunnel between different pages. It seems that this notion of darkness -much weaker than the standard one - is consistent with empirical facts.

This approach to quantum TGD has been developing vigorously during last years and led to a general vision about the basic mechanisms of quantum biology (chapters *Dark Matter Hierarchy and Hierarchy of EEGs* and *Quantum Model for Nerve Pulse and EEG* of [12] and chapters *DNA as Topological Quantum Computer* and *Evolution in Many-Sheeted Space-Time* of [11]) and has also provided deep insights to TGD inspired theory of consciousness. In particular, certain mysterious looking findings about ionic currents through cell membrane can be understood if considerable fraction of ions are dark in the proposed sense (chapter *About the New Physics Behind Qualia* of [9]).

The idea about hierarchy of Planck constants forces to consider the possibility that the notions of space-time and the generalization of the imbedding space $M^4 \times CP_2$ might emerge from something more fundamental. As already suggests a unique octonionic generalization of HFF might be the unique fundamental structure (chapter *Was von Neumann Right After All* of [4]).

There are tensions present also now. Is the generalization of imbedding space a mere ad hoc addition or does quantum TGD require it? The hierarchy seems to be necessary in order to realize quantum criticality mathematically. Quantum criticality would mean criticality with respect to phase transitions changing the value of Planck constant interpreted as a tunneling between different pages of the big book. In the recent parton level formulation of quantum TGD the Kähler function of the configuration space emerges as a Dirac determinant and the construction also assigns 4-D space-time surface to a given collection of light-like 3-surfaces. Also this construction relies in an essential manner on the book like structure of the imbedding space.

2.7 Does the finiteness of measurement resolution dictate the laws of physics?

The hypothesis that the mere finiteness of measurement resolution could determine the laws of quantum physics (chapter *Construction of Quantum Theory: S-matrix* of [4]) completely belongs to the category of not at all obvious first principles. The basic observation is that the Clifford algebra spanned by the gamma matrices of the "world of classical worlds" represents HFF (chapters *Was von Neumann Right After All* and *Construction of Quantum Theory: S-matrix* of [4]). HFF [30, 33, 29] is an algebraic fractal having infinite hierarchy of included subalgebras isomorphic to the algebra itself [35]. The structure of HFF is closely related to several notions of modern theoretical physics such as integrable statistical physical systems [41], anyons [42, 37], quantum groups and conformal field theories[34, 36], and knots and topological quantum field theories [43, 40].

Zero energy ontology is second key element. In zero energy ontology these inclusions allow an interpretation in terms of a finite measurement resolution: in the standard positive energy ontology this interpretation is not possible. Inclusion hierarchy defines in a natural manner the notion of coupling constant evolution and p-adic length scale hypothesis follows as a prediction. In this framework the extremely heavy machinery of renormalized quantum field theory involving the elimination of infinities is replaced by a precisely defined mathematical framework. More concretely, the included algebra creates states which are equivalent in the measurement resolution used. Zero energy states are associated with a causal diamond formed by a pair of future and past directed light-cones having positive and negative energy parts of state at their boundaries. Zero energy state can be modified in a time scale shorter than the time scale of the zero energy state itself.

On can imagine two kinds of measurement resolutions. The element of the included algebra can leave the quantum numbers of the positive and negative energy parts of the state invariant, which means that the action of subalgebra leaves M-matrix invariant. The action of the included algebra can also modify the quantum numbers of the positive and negative energy parts of the state such that the zero energy property is respected. In this case the Hermitian operators subalgebra must commute with M-matrix.

The temporal distance between the tips of light-cones corresponds to the secondary p-adic time scale $T_{p,2} = \sqrt{p}T_p$ by a simple argument based on the observation that light-like randomness of light-like 3-surface is analogous to Brownian motion. This gives the relationship $T_p = L_p^2/Rc$, where R is CP_2 size. The action of the included algebra corresponds to an addition of zero energy parts to either positive or negative energy part of the state and is like addition of quantum fluctuation below the time scale of the measurement resolution. The natural hierarchy of time scales is obtained as $T_n = 2^{-n}T$ since these insertions must belong to either upper or lower half of the causal diamond. This implies that preferred p-adic primes are near powers of 2. For electron the time scale in question is .1 seconds defining the fundamental biorhythm of 10 Hz.

M-matrix representing a generalization of S-matrix and expressible as a product of a positive square root of the density matrix and unitary S-matrix would define the dynamics of quantum theory (chapter *Construction of Quantum Theory: S-matrix* of [4]). The notion of thermodynamical state would cease to be a theoretical fiction and in a well-defined sense quantum theory could be regarded as a square root of thermodynamics. M-matrix is identifiable in terms of Connes tensor product [29] and therefore exists and is almost unique. Connes tensor product implies that the Hermitian elements of the included algebra commute with M-matrix and hence act like infinitesimal symmetries. A connection with integrable quantum field theories is suggestive. The remaining challenge is the calculation of M-matrix and the needed machinery might already exist.

The tension is present also now. The connection with other visions should come from the discretization in terms of number theoretic braids providing a space-time correlate for the finite measurement resolution and making p-adicization in terms of number theoretic braids possible. Number theoretic braids give a connection with the construction of configuration space geometry in terms of Dirac determinant and with TGD as almost TQFT and with conformal field theory approach. The mathematics for the inclusions of HFFs is also closely related to that for conformal field theories including quantum groups relating closely to Connes tensor product and non-commutativity.

2.8 Are both symplectic and conformal field theories be needed?

Symplectic (or canonical as I have called them) symmetries of $\delta M^4_+ \times CP_2$ (light-cone boundary briefly) act as isometries of the "world of classical worlds". One can see these symmetries as analogs of Kac-Moody type symmetries with symplectic transformations of $S^2 \times CP_2$, where S^2 is $r_M = constant$ sphere of lightcone boundary, made local with respect to the light-like radial coordinate r_M taking the role of complex coordinate. Thus finite-dimensional Lie group G is replaced with infinite-dimensional group of symplectic transformations. This inspires the question whether a symplectic analog of conformal field theory at $\delta M^4_+ \times CP_2$ could be relevant for the construction of n-point functions in quantum TGD and what general properties these n-point functions would have. T

2.8.1 Symplectic QFT at sphere

The notion of symplectic QFT emerged as I tried to understand the properties of cosmic microwave background [27] which comes from the sphere of last scattering which corresponds roughly to the age of 5×10^5 years (chapter *Quantum Astrophysics* of [3]). In this situation vacuum extremals of Kähler action around almost unique critical Robertson-Walker cosmology imbeddable in $M^4 \times S^2$, where s^2 is a homologically trivial geodesic sphere of CP_2. Vacuum extremal property is satisfied for any space-time surface which is surface in $M^4 \times Y^2$, Y^2 a Lagrangian sub-manifold of CP_2 with vanishing induced Kähler form. Symplectic transformations of CP_2 and general coordinate transformations of M^4 are dynamical symmetries of the vacuum extremals so that the idea of symplectic QFT emerges natural. Therefore I shall consider first symplectic QFT at the sphere S^2 of last scattering with temperature fluctuation $\Delta T/T$ proportional to the fluctuation of the metric component g_{aa} in Robertson-Walker coordinates.

1. In quantum TGD the symplectic transformation of the light-cone boundary would induce action in the "world of classical worlds" (light-like 3-surfaces). In the recent situation it is convenient to regard perturbations of CP_2 coordinates as fields at the sphere of last scattering (call it S^2) so that symplectic transformations of CP_2 would act in the field space whereas those of S^2 would act in the coordinate space just like conformal transformations. The deformation of the metric would be a symplectic field in S^2. The symplectic dimension would be induced by the tensor properties of R-W metric in R-W coordinates: every S^2 coordinate index would correspond to one unit of symplectic dimension. The symplectic invariance in CP_2 degrees of freedom is guaranteed if the integration measure over the vacuum deformations is symplectic invariant. This symmetry does not play any role in the sequel.

2. For a symplectic scalar field $n \geq 3$-point functions with a vanishing anomalous dimension would be functions of the symplectic invariants defined by the areas of geodesic polygons defined by subsets of the arguments as points of S^2. Since n-polygon decomposes to 3-polygons these invariants can be expressed as sums of the areas of 3-polygons expressible in terms of symplectic form. n-point functions would be constant if arguments are along geodesic circle since the areas of all sub-polygons would vanish in this case. The decomposition of n-polygon to 3-polygons brings in mind the decomposition of the n-point function of conformal field theory to products of 2-point functions by using the fusion algebra of conformal fields (very symbolically $\Phi_k\Phi_l = c_{kl}^m\Phi_m$). This intuition seems to be correct.

3. Fusion rules stating the associativity of the products of fields at different points should generalize. In the recent case it is natural to assume a non-local form of fusion rules given in the case of symplectic scalars by the equation

$$\Phi_k(s_1)\Phi_l(s_2) = c_{kl}^m \int f(A(s_1,s_2,s_3))\Phi_m(s)d\mu_s . \quad (1)$$

Here the coefficients c_{kl}^m are constants and $A(s_1,s_2,s_3)$ is the area of the geodesic triangle of S^2 defined by the sympletic measure and integration is over S^2 with symplectically invariant measure $d\mu_s$ defined by symplectic form of S^2. Fusion rules pose powerful conditions on n-point functions and one can hope that the coefficients are fixed completely.

4. The application of fusion rules gives at the last step an expectation value of 1-point function of the product of the fields involves unit operator term $\int c_{kl} f(A(s_1,s_2,s))d\mu_s \times Id$ so that one has

$$\langle \Phi_k(s_1)\Phi_l(s_2)\rangle = c_{kl} \int f(A(s_1,s_2,s))d\mu_s . \quad (2)$$

Hence 2-point function is the average of a 3-point function over the third argument. The absence of non-trivial symplectic invariants for 1-point function means that they are constant, most naturally vanishinging, unless some kind of spontaneous symmetry breaking occurs. Since the function $f(A(s_1,s_2,s_3))$ is arbitrary, 2-point correlation function can have both signs. 2-point correlation function is invariant under rotations and reflections.

5. The condition that the symplectic invariants do not diverge for large distances requires that they are dimensionless and given as the ratio of the product of areas for some minimum number of triangles divided by a suitable power for the area of the entire polygon and vanishing as any pair of arguments of n-point function co-incide.

2.8.2 Symplectic QFT with spontaneous breaking of rotational and reflection symmetries

CMB data suggest breaking of rotational and reflection symmetries of S^2. A possible mechanism of spontaneous symmetry breaking is based on the observation that in TGD framework the hierarchy of Planck constants assigns to each sector of the generalized imbedding space a preferred quantization axes. The selection of the quantization axis is coded also to the geometry of "world of classical worlds", and to the quantum fluctuations of the metric in particular. Clearly, symplectic QFT with spontaneous symmetry breaking would provide the sought-for really deep reason for the quantization of Planck constant in the proposed manner.

1. The coding of angular momentum quantization axis to the generalized imbedding space geometry allows to select South and North poles as preferred points of S^2. To the three arguments s_1, s_2, s_3 of the 3-point function one can assign two squares with the added point being either North or South pole. The difference

$$\Delta A(s_1, s_2, s_3) \equiv A(s_1, s_2, s_3, N) - A(s_1, s_2, s_3, S) \quad (3)$$

of the corresponding areas defines a simple symplectic invariant breaking the reflection symmetry with respect to the equatorial plane. Note that ΔA vanishes if arguments lie along a geodesic line or if any two arguments co-incide. Quite generally, symplectic QFT differs from conformal QFT in that correlation functions do not possess singularities.

2. The reduction to 2-point correlation function gives a consistency conditions on the 3-point functions

$$\langle \Phi_k(s_1)\Phi_l(s_2))\Phi_m(s_3)\rangle = c^r_{kl}\int f(\Delta A(s_1, s_2, s))\langle \Phi_r(s)\Phi_m(s_3)\rangle d\mu_s = c^r_{kl}c_{rm}\int f(\Delta A(s_1, s_2, s))f(\Delta A(s, s_3, t))d\mu_s d\mu_t. \quad (4)$$

Associativity requires that this expression equals to $\langle \Phi_k(s_1)(\Phi_l(s_2)\Phi_m(s_3))\rangle$ and this gives additional conditions. Associativity conditions apply to $f(\Delta A)$ and could fix it highly uniquely.

3. 2-point correlation function would be given by

$$\langle \Phi_k(s_1)\Phi_l(s_2)\rangle = c_{kl}\int f(\Delta A(s_1, s_2, s))d\mu_s. \quad (5)$$

4. There is a clear difference between $n > 3$ and $n = 3$ cases: for $n > 3$ also non-convex polygons are possible: this means that the interior angle associated with some vertices of the polygon is larger than π. $n = 4$ theory is certainly well-defined, but one can argue that so are also $n > 4$ theories and skeptic would argue that this leads to an inflation of theories. TGD however allows only finite number of preferred points and fusion rules could eliminate the hierarchy of theories.

5. To sum up, the general predictions are following. Quite generally, for $f(0) = 0$ n-point correlation functions vanish if any two arguments co-incide which conforms with the spectrum of temperature fluctuations. It also implies that symplectic QFT is free of the usual singularities. For symmetry breaking scenario 3-point functions and thus also 2-point functions vanish also if s_1 and s_2 are at equator. All these are testable predictions using ensemble of CMB spectra.

2.8.3 Generalization to quantum TGD

Since number theoretic braids are the basic objects of quantum TGD, one can hope that the n-point functions assignable to them could code the properties of ground states and that one could separate from n-point functions the parts which correspond to the symplectic degrees of freedom acting as symmetries of vacuum extremals and isometries of the 'world of classical worlds'.

1. This approach indeed seems to generalize also to quantum TGD proper and the n-point functions associated with partonic 2-surfaces can be decomposed in such a manner that one obtains coefficients which are symplectic invariants associated with both S^2 and CP_2 Kähler form.

2. Fusion rules imply that the gauge fluxes of respective Kähler forms over geodesic triangles associated with the S^2 and CP_2 projections of the arguments of 3-point function serve basic building blocks of the correlation functions. The North and South poles of S^2 and three poles of CP_2 can be used to construct symmetry breaking n-point functions as symplectic invariants. Non-trivial 1-point functions vanish also now.

3. The important implication is that n-point functions vanish when some of the arguments coincide. This might play a crucial role in taming of the singularities: the basic general prediction of TGD is that standard infinities of local field theories should be absent and this mechanism might realize this expectation.

Next some more technical but elementary first guesses about what might be involved.

1. It is natural to introduce the moduli space for n-tuples of points of the symplectic manifold as the space of symplectic equivalence classes of n-tuples. In the case of sphere S^2 convex n-polygon allows $n+1$ 3-sub-polygons and the areas of these provide symplectically invariant coordinates for the moduli space of symplectic equivalence classes of n-polygons (2^n-D space of polygons is reduced to $n+1$-D space). For non-convex polygons the number of 3-sub-polygons is reduced so that they seem to correspond to lower-dimensional sub-space. In the case of CP_2 n-polygon allows besides the areas of 3-polygons also 4-volumes of 5-polygons as fundamental symplectic invariants. The number of independent 5-polygons for n-polygon can be obtained by using induction: once the numbers $N(k,n)$ of independent $k \leq n$-simplices are known for n-simplex, the numbers of $k \leq n+1$-simplices for $n+1$-polygon are obtained by adding one vertex so that by little visual gymnastics the numbers $N(k,n+1)$ are given by $N(k,n+1) = N(k-1,n) + N(k,n)$. In the case of CP_2 the allowance of 3 analogs $\{N,S,T\}$ of North and South poles of S^2 means that besides the areas of polygons (s_1,s_2,s_3), (s_1,s_2,s_3,X), (s_1,s_2,s_3,X,Y), and (s_1,s_2,s_3,N,S,T) also the 4-volumes of 5-polygons (s_1,s_2,s_3,X,Y), and of 6-polygon (s_1,s_2,s_3,N,S,T), $X,Y \in \{N,S,T\}$ can appear as additional arguments in the definition of 3-point function.

2. What one really means with symplectic tensor is not clear since the naive first guess for the n-point function of tensor fields is not manifestly general coordinate invariant. For instance, in the model of CMB, the components of the metric deformation involving S^2 indices would be symplectic tensors. Tensorial n-point functions could be reduced to those for scalars obtained as inner products of tensors with Killing vector fields of $SO(3)$ at S^2. Again a preferred choice of quantization axis would be introduced and special points would correspond to the singularities of the Killing vector fields.

   The decomposition of Hamiltonians of the "world of classical worlds" expressible in terms of Hamiltonians of $S^2 \times CP_2$ to irreps of $SO(3)$ and $SU(3)$ could define the notion of symplectic tensor as the analog of spherical harmonic at the level of configuration space. Spin and gluon color would have natural interpretation as symplectic spin and color. The infinitesimal action of various Hamiltonians on n-point functions defined by Hamiltonians and their super counterparts is well-defined and group theoretical arguments allow to deduce general form of n-point functions in terms of symplectic invariants.

3. The need to unify p-adic and real physics by requiring them to be completions of rational physics, and the notion of finite measurement resolution suggest that discretization of also fusion algebra is necessary. The set of points appearing as arguments of n-point functions could be finite in a given resolution so that the p-adically troublesome integrals in the formulas for the fusion rules would be replaced with sums. Perhaps rational/algebraic variants of $S^2 \times CP_2 = SO(3)/SO(2) \times SU(3)/U(2)$ obtained by replacing these groups with their rational/algebraic variants are involved. Tedrahedra, octahedra, and dodecahedra suggest themselves as simplest candidates for these discretized spaces. Also the symplectic moduli space would be discretized to contain only n-tuples for which the symplectic invariants are numbers in the allowed algebraic extension of rationals. This would provide an abstract looking but actually very concrete operational approach to the discretization involving only areas of n-tuples as internal coordinates of symplectic equivalence classes of n-tuples. The best that one could achieve would be a formulation involving nothing below measurement resolution.

4. This picture based on elementary geometry might make sense also in the case of conformal symmetries. The angles associated with the vertices of the S^2 projection of n-polygon could define conformal invariants appearing in n-point functions and the algebraization of the corresponding phases would be an operational manner to introduce the space-time correlates for the roots of unity introduced at quantum level. In CP_2 degrees of freedom the projections of n-tuples to the homologically trivial geodesic sphere S^2 associated with the particular sector of CH would allow to define similar conformal invariants. p-Adic length scale hypothesis and hierarchy of Planck constants would bring in the fundamental units of length and time in terms of CP_2 length.

2.8.4 Generalized Feynman diagrams

The recent view about M-matrix described in (chapter *Construction of Quantum Theory: S-matrix* of [4]) is something almost unique determined by Connes tensor product providing a formal realization for the statement that complex rays of state space are replaced with $\mathcal{N}$ rays where $\mathcal{N}$ defines the hyper-finite sub-factor of type II_1 defining the measurement resolution. M-matrix defines time-like entanglement coefficients between positive and negative energy parts of the zero energy state and need not be unitary. It is identified as square root of density matrix with real expressible as product of of real and positive square root and unitary S-matrix. This S-matrix is what is measured in laboratory. There is also a general vision about how vertices are realized: they correspond to light-like partonic 3-surfaces obtained by gluing incoming and outgoing partonic 3-surfaces along their ends together just like lines of Feynman diagrams. Note that in string models string world sheets are non-singular as 2-manifolds whereas 1-dimensional vertices are singular as 1-manifolds. These ingredients we should be able to fuse together. So we try once again!

1. *Bottom-up* starting from vertices and propagators is the basic approach in the construction of n-point function in standard QFT. This approach does not work in quantum TGD. Symplectic and conformal field theories suggest that *top-down* approach replaces bottom-up approach in the construction. One starts from an n-point function and reduces it step by step to a vacuum expectation value of a 2-point function using fusion rules. Associativity becomes the fundamental dynamical principle in this process. Associativity in the sense of classical number fields has already shown its power and led to a hyper-octoninic formulation of quantum TGD promising a unification of various visions about quantum TGD (chapter *TGD as a Generalized Number Theory: Quaternions, Octonions, and their*

Hyper Counterparts of [5]).

2. Let us start from the representation of a zero energy state in terms of a causal diamond defined by future and past directed light-cones. Zero energy state corresponds to a quantum superposition of light-like partonic 3-surfaces each of them representing possible particle reaction. These 3-surfaces are very much like generalized Feynman diagrams with lines replaced by light-like 3-surfaces coming from the upper and lower light-cone boundaries and glued together along their ends at smooth 2-dimensional surfaces defining the generalized vertices.

3. It must be emphasized that the generalization of ordinary Feynman diagrammatics arises and conformal and symplectic QFTs appear only in the calculation of single generalized Feynman diagram. Therefore one could still worry about loop corrections. The fact that no integration over loop momenta is involved and there is always finite cutoff due to discretization together with top-down approach instead of bottom-up approach gives however good hopes that everything works. Note that this picture is in conflict with one of the earlier approaches based on positive energy ontology in which the hope was that only single generalized Feynman diagram could define the U-matrix thought to correspond to physical S-matrix at that time (chapter *Intentionality, Cognition, and Physics as Number theory or Space-Time Point as Platonia* of [5]).

4. One can actually simplify things by identifying generalized Feynman diagrams as maxima of Kähler function with functional integration carried over perturbations around it. Thus one would have conformal field theory in both fermionic and configuration space degrees of freedom. The light-like time coordinate along light-like 3-surface is analogous to the complex coordinate of conformal field theories restricted to some curve. If it is possible continue the light-like time coordinate to a hyper-complex coordinate in the interior of 4-D space-time sheet, the correspondence with conformal field theories becomes rather concrete. Same applies to the light-like radial coordinates associated with the light-cone boundaries. At light-cone boundaries one can apply fusion rules of a symplectic QFT to the remaining coordinates. Conformal fusion rules are applied only to point pairs which are at different ends of the partonic surface and there are no conformal singularities since arguments of n-point functions do not co-incide. By applying the conformal and symplectic fusion rules one can eventually reduce the n-point function defined by the various fermionic and bosonic operators appearing at the ends of the generalized Feynman diagram to something calculable.

5. Finite measurement resolution defining the Connes tensor product is realized by the discretization applied to the choice of the arguments of n-point functions so that discretion is not only a space-time correlate of finite resolution but actually defines it. No explicit realization of the measurement resolution algebra $\mathcal{N}$ seems to be needed. Everything should boil down to the fusion rules and integration measure over different 3-surfaces defined by exponent of Kähler function and by imaginary exponent of Chern-Simons action. The continuation of the configuration space Clifford algebra for 3-surfaces with cm degrees of freedom fixed to a hyper-octonionic variant of gamma matrix field of super-string models defined in M^8 (hyper-octonionic space) and $M^8 \leftrightarrow M^4 \times CP_2$ duality leads to a unique choice of the points, which can contribute to n-point functions as intersection of M^4 subspace of M^8 with the counterparts of partonic 2-surfaces at the boundaries of light-cones of M^8. Therefore there are hopes that the resulting theory is highly unique. Symplectic fusion algebra reduces to a finite algebra for each space-time surface if this picture is correct.

6. Consider next some of the details of how the light-like 3-surface codes for the fusion rules associated with it. The intermediate partonic 2-surfaces must be involved since otherwise the construction would carry no information about the properties of the light-like 3-surface, and one

would not obtain perturbation series in terms of the relevant coupling constants. The natural assumption is that partonic 2-surfaces belong to future/past directed light-cone boundary depending on whether they are on lower/upper half of the causal diamond. Hyper-octonionic conformal field approach fixes the n_{int} points at intermediate partonic two-sphere for a given light-like 3-surface representing generalized Feynman diagram, and this means that the contribution is just N-point function with $N = n_{out} + n_{int} + n_{in}$ calculable by the basic fusion rules. Coupling constant strengths would emerge through the fusion coefficients, and at least in the case of gauge interactions they must be proportional to Kähler coupling strength since n-point functions are obtained by averaging over small deformations with vacuum functional given by the exponent of Kähler function. The first guess is that one can identify the spheres $S^2 \subset \delta M^4_{\pm}$ associated with initial, final and, and intermediate states so that symplectic n-points functions could be calculated using single sphere.

These findings raise the hope that quantum TGD is indeed a solvable theory. Even if one is not willing to swallow any bit of TGD, the classification of the symplectic QFTs remains a fascinating mathematical challenge in itself. A further challenge is the fusion of conformal QFT and symplectic QFT in the construction of n-point functions. One might hope that conformal and symplectic fusion rules can be treated separately.

2.8.5 Still more detailed view about the construction of M-matrix elements

After three decades there are excellent hopes of building an explicit recipe for constructing M-matrix elements but the devil is in the details.

1. Elimination of infinities and coupling constant evolution

The elimination of infinities would follow from the symplectic QFT part of the theory. The symplectic contribution to n-point functions vanishes when two arguments co-incide. The UV cancellation mechanism has nothing to do with the finite measurement resolution which corresponds to the size of the causal diamonds inside which the space-time sheets representing radiative corrections are. There is also IR cutoff due to the presence of largest causal diamond.

On can decompose the radiative corrections two two types. First kind of corrections appear both at the level of positive/and negative energy parts of zero energy states. Second kind of corrections appear at the level of interactions between them. This decomposition is standard in quantum field theories and corresponds to the renormalization constants of fields *resp.* renormalization of coupling constants. The corrections due to the increase of measurement resolution in time comes as very specific corrections to positive and negative energy states involving gluing of smaller causal diamonds to the upper and lower boundaries of causal diamonds along any radial light-like ray. The radiative corresponds to the interactions correspond to the addition of smaller causal diamonds in the interior of the larger causal diamond. Scales for the corrections come as scalings in powers of 2 rather than as continuous scaling of measurement resolution.

2. Conformal symmetries

The basic questions are the following ones. How hyper-octonionic/-quaternionic/-complex super-conformal symmetry relates to the super-canonical conformal symmetry at the imbedding space level and the super Kac-Moody symmetry associated with the light-like 3-surfaces? How do the dual $HO = M^8$ and $H = M^4 \times CP_2$ descriptions (number theoretic compactifcation) relate?

Concerning the understanding of these issues, the earlier construction of physical states poses strong constraints (chapter *Construction of Quantum Theory: Symmetries* of [4]).

1. The state construction utilizes both super-canonical and super Kac-Moody algebras. Super-canonical algebra has negative conformal weights and creates tachyonic ground states from which Super Kac-Moody algebra generates states with non-negative conformal weight de-

termining the mass squared value of the state. The commutator of these two algebras annihilates the physical states. This requires that both super conformal algebras must allow continuation to hyper-octonionic algebras, which are independent.

2. The light-like radial coordinate at $\delta M^4_\pm$ can be continued to a hyper-complex coordinate in $M^2_\pm$ defined the preferred commutative plane of non-physical polarizations, and also to a hyper-quaternionic coordinate in $M^4_\pm$. Hence it would seem that super-canonical algebra can be continued to an algebra in $M^2_\pm$ or perhaps in the entire $M^4_\pm$. This would allow to continue also the operators G, L and other super-canonical operators to operators in hyper-quaternionic $M^4_\pm$ needed in stringy perturbation theory.

3. Also the super KM algebra associated with the light-like 3-surfaces should be continueable to hyper-quaternionic $M^4_\pm$. Here $HO-H$ duality comes in rescue. It requires that the preferred hyper-complex plane M^2 is contained in the tangent plane of the space-time sheet at each point, in particular at light-like 3-surfaces. We already know that this allows to assign a unique space-time surface to a given collection of light-like 3-surfaces as hyper-quaternionic 4-surface of HO hypothesized to correspond to (an obviously preferred) extremal of Kähler action. An equally important implication is that the light-like coordinate of X^3 can be continued to hyper-complex coordinate M^2 coordinate and thus also to hyperquaternionic M^4 coordinate.

4. The four-momentum appears in super generators G_n and L_n. It seems that the formal Fourier transform of four-momentum components to gradient operators to $M^4_\pm$ is needed and defines these operators as particular elements of the CH Clifford algebra elements extended to fields in imbedding space.

3. What about stringy perturbation theory?

The analog of stringy perturbation theory does not seems only a highly attractive but also an unavoidable outcome since a generalization of massless fermionic propagator is needed. The inverse for the sum of super Kac-Moody and super-canonical super-Virasoro generators G (L) extended to an operator acting on the difference of the M^4 coordinates of the end points of the propagator line connecting two partonic 2-surfaces should appear as fermionic (bosonic) propagator in stringy perturbation theory. Virasoro conditions imply that only G_0 and L_0 appear as propagators. Momentum eigenstates are not strictly speaking possible since since discretization is present due to the finite measurement resolution. One can however represent these states using Fourier transform as a superposition of momentum eigenstates so that standard formalism can be applied.

Symplectic QFT gives an additional multiplicative contribution to n-point functions and there would be also braiding S-matrices involved with the propagator lines in the case that partonic 2-surface carriers more than 1 point. This leaves still modular degrees of freedom of the partonic 2-surfaces describable in terms of elementary particle vacuum functionals and the proper treatment of these degrees of freedom remains a challenge.

4. What about non-hermiticity of the CH super-generators carrying fermion number?

TGD represents also a rather special challenge, which actually represents the fundamental difference between quantum TGD and super string models. The assignment of fermion number to CH gamma matrices and thus also to the super-generator G is unavoidable. Also M^4 and H gamma matrices carry fermion number. This has been a long-standing interpretational problem in quantum TGD and I have been even ready to give up the interpretation of four-momentum operator appearing in G_n and L_n as actual four-momenta. The manner to get rid of this problem would be the assumption of Majorana property but this would force to give up the interpretation of different imbedding space chiralities in terms of conserved lepton and quark numbers and would also lead to super-string theory with critical dimension 10 or 11. A further problem is how to obtain amplitudes which respect fermion number conservation using string perturbation theory if $1/G = G^{\dagger}/L_0$ carries fermion number.

The recent picture does not leave many choices so that I was forced to face the truth and see how everything falls down to this single nasty detail! It became as a total surprise that gamma matrices carrying fermion number do not cause any difficulties in zero energy ontology and make sense even in the ordinary Feynman diagrammatics.

1. Non-hermiticity of G means that the center of mass terms CH gamma matrices must be distinguished from their Hermitian conjugates. In particular, one has $\gamma_0 \neq \gamma_0^{dagger}$. One can interpret the fermion number carrying M^4 gamma matrices of the complexified quaternion space.

2. One might think that $M^4 \times CP_2$ gamma matrices carrying fermion number is a catastrophe but this is not the case in massless theory. Massless momentum eigen states can be created by the operator $p^k \gamma_k^{\dagger}$ from a vacuum annihilated by gamma matrices and satisfying massless Dirac equation. The conserved fermion number defined by the integral of $\overline{\Psi}\gamma^0\Psi$ over 3-space gives just its standard value. A further experimentation shows that Feynman diagrams with non-hermitian gamma matrices give just the standard results since fermionic propagator and boson-emission vertices give compensating fermion numbers.

3. If the theory would contain massive fermions or a coupling to a scalar Higgs, a catastrophe would result. Hence ordinary Higgs mechanism is not possible in this framework. Of course, also the quantization of fermions is totally different. In TGD fermion mass is not a scalar in H. Part of it is given by CP_2 Dirac operator, part by p-adic thermodynamics for L_0, and part by Higgs field which behaves like vector field in CP_2 degrees of freedom, so that the catastrophe is avoided.

4. In zero energy ontology zero energy states are characterized by M-matrix elements constructed by applying the combination of stringy and symplectic Feynman rules and fermionic propagator is replaced with its super-conformal generalization reducing to an ordinary fermionic propagator for massless states. The norm of a single fermion state is given by a propagator connecting positive energy state and its conjugate with the propagator G_0/L_0 and the standard value of the norm is obtained by using Dirac equation and the fact that Dirac operator appears also in G_0.

5. The hermiticity of super-generators G would require Majorana property and one would end up with superstring theory with critical dimension $D = 10$ or $D = 11$ for the imbedding space. Hence the new interpretation of gamma matrices, proposed already years ago, has very profound consequences and convincingly demonstrates that TGD approach is indeed internally consistent.

3 Where are we now?

These three decades have made it clear to me how slow the progress in physics really is and how small the contribution of individual is bound to be. I have been able to identify these visions about basic principles and perhaps even demonstrate that M-matrix exists and is unique to a high degree. About how to calculate M-matrix I cannot say much: I simply lack the technical know-how about HFFs. I can only hope that some young mathematically oriented colleague had the patience to penetrate deep enough to this this strange jungle of ideas that I call TGD so that it could provide inspiration if not anything else. The mathematical work should be however guided by a continual application of already existing theory in order to avoid the degeneration to a production of dead formalism. Also the tension between different visions should continue to serve as a very effective idea generator.

References

Online books and articles about TGD, TGD inspired theory of consciousness and of quantum biology

[1] M. Pitkänen. Topological Geometrodynamics: Overview. Online book 2006.
http://www.helsinki.fi/∼matpitka/tgdview/tgdview.html.

[2] M. Pitkänen. Quantum Physics as Infinite-Dimensional Geometry. Online book 2006.
http://www.helsinki.fi/∼matpitka/tgdgeom/tgdgeom.html.

[3] M. Pitkänen. Physics in Many-Sheeted Space-Time. Online book 2006.
http://www.helsinki.fi/∼matpitka/tgdclass/tgdclass.html.

[4] M. Pitkänen. Quantum TGD. Online book 2006.
http://www.helsinki.fi/∼matpitka/tgdquant/tgdquant.html.

[5] M. Pitkänen. TGD as a Generalized Number Theory. Online book 2006.
http://www.helsinki.fi/∼matpitka/tgdnumber/tgdnumber.html.

[6] M. Pitkänen. p-Adic length Scale Hypothesis and Dark Matter Hierarchy. Online book 2006.
http://www.helsinki.fi/∼matpitka/paddark/paddark.html.

[7] M. Pitkänen. TGD and Fringe Physics. Online book 2006.
http://www.helsinki.fi/∼matpitka/freenergy/freenergy.html.

[8] M. Pitkänen. Bio-Systems as Self-Organizing Quantum Systems. Online book 2006.
http://www.helsinki.fi/∼matpitka/bioselforg/bioselforg.html.

[9] M. Pitkänen. Quantum Hardware of Living Matter. Online book 2006.
http://www.helsinki.fi/∼matpitka/bioware/bioware.html.

[10] M. Pitkänen. TGD Inspired Theory of Consciousness. Online book 2006.
http://www.helsinki.fi/∼matpitka/tgdconsc/tgdconsc.html.

[11] M. Pitkänen. Genes and Memes. Online book 2006.
http://www.helsinki.fi/∼matpitka/genememe/genememe.html.

[12] M. Pitkänen. TGD and EEG. Online book 2006.
http://www.helsinki.fi/∼matpitka/tgdeeg/tgdeeg/tgdeeg.html.

[13] M. Pitkänen. Bio-Systems as Conscious Holograms. Online book 2006.
http://www.helsinki.fi/∼matpitka/hologram/hologram.html.

[14] M. Pitkänen. Magnetospheric Consciousness. Online book 2006.
http://www.helsinki.fi/∼matpitka/magnconsc/magnconsc.html.

[15] M. Pitkänen. Mathematical Aspects of Consciousness Theory. Online book 2006. Online book 2006.
http://www.helsinki.fi/∼matpitka/mathconsc/mathconsc.html.

[16] M. Pitkänen. Topological Geometrodynamics: an Overall View. Online article 2008. http://www.helsinki.fi/∼matpitka/articles/TGDlong.pdf.

[17] M. Pitkänen. TGD Inspired Theory of Consciousness. Online article 2008. http://www.helsinki.fi/∼matpitka/articles/tgdconsc.pdf.

[18] M. Pitkänen. TGD Inspired Quantum Model of Living Matter. Online article 2008. http://www.helsinki.fi/∼matpitka/articles/quantumbio.pdf.

[19] M. Pitkänen. About Correspondence Between Infinite Primes, Space-time Surfaces, and Configuration Space Spinor Fields. Online article 2007. http://www.helsinki.fi/∼matpitka/articles/brahma.pdf.

[20] M. Pitkänen. DNA as Topological Quantum Computer. Online article 2008. http://www.helsinki.fi/∼matpitka/articles/dnatqcart.pdf.

[21] M. Pitkänen. Quantum Model for Nerve Pulse and EEG. Online article 2008. http://www.helsinki.fi/∼matpitka/articles/pulseeg.pdf.

[22] M. Pitkänen. Evolution in Many-Sheeted Space-Time. Online article 2008. http://www.helsinki.fi/∼matpitka/articles/prebiotic.pdf.

[23] M. Pitkänen. A Model for Protein Folding and Bio-catalysis. Online article 2008. http://www.helsinki.fi/∼matpitka/articles/prebiotic.pdf.

[24] M. Pitkänen. The Notion of Wave-Genome and DNA as Topological Quantum Computer. Online article 2008. http://www.helsinki.fi/∼matpitka/articles/gari.pdf.

Other references

[25] Ali A. Types of 2-dimensional $N = 4$ superconformal field theories, Pramana 2003; 61, 6: 1065-1078.

[26] Baez JC. The Octonions. Bull. Amer. Math. Soc. 2002; 39: 145-205. http://math.ucr.edu/home/baez/Octonions/octonions.html.

[27] Bennett CL et al. First Year Wilkinson Microwave Anisotropy Probe (WMAP1) Observations: Preliminary Maps and Basic Results. Astrophys J Suppl 2003; 148: 1-27.
Cosmic microwave background. http://en.wikipedia.org/wiki/Cosmic_microwave_background.

[28] Da Roacha D and Nottale L, Gravitational Structure Formation in Scale Relativity, astro-ph/0310036 2003.

[29] Connes A. Non-commutative Geometry. San Diego: Academic Press, 1994. http://www.noncommutativegeometry.net/article.php3?id_article=229.

[30] Dixmier J Von Neumann Algebras. Amsterdam: North-Holland Publishing Company, 1981. [First published in French in 1957: Les Algebres d'Operateurs dans l'Espace Hilbertien, Paris: Gauthier-Villars.]

[31] Freed DS. The Geometry of Loop Groups (Thesis). Berkeley: University of California, 1985.

[32] Gunaydin M. $N = 4$ superconformal algebras and gauges Wess-Zumino-Witten models. Phys Rev D 1993; 47, 8.

[33] Jones VFR. Braid groups, Hecke algebras and type II_1 factors. Geometric methods in operator algebras. Proceedings of the US-Japan Seminar, Kyoto, July 1983.

[34] Jones VFR. In and around the origin of quantum groups. arXiv:math.OA/0309199 2003.

[35] Jones VFR. Index for Subfactors. Invent Math 1983; 72: 1-25.

[36] Gomez C, Ruiz-Altaba M, Sierra G. Quantum Groups and Two-Dimensional Physics. Cambridge University Press, 1996.

[37] Laughlin RB. Phys Rev Lett 1990; 50: 1395.

[38] Libet B, Wright EW Jr, B. Feinstein B, and Pearl DK. Subjective referral of the timing for a conscious sensory experience. Brain 1979; 102: 193-224.

[39] Prochaska J, Howk JC, Wolfe AM. The elemental abundance pattern in a galaxy at z = 2.626. Nature 2003; 423: 57-59.
See also "Distant elements of surprise" at http://physicsworld.com/cws/article/print/17750.

[40] Sawin S. Links, Quantum Groups, and TQFT's. q-alg/9506002 1995.

[41] Temperley NHV and Lieb EH. Relations between the percolation and colouring problem and other graph-theoretical problems associated with regular planar lattices: some exact results for the percolation problem. Proc Roy Soc London 1971; 322: 251-280.

[42] Wilzek F. Fractional Statistics and Anyon Super-Conductivity. World Scientific, 1990.

[43] E. Witten 1989, Quantum field theory and the Jones polynomial. Comm Math Phys 1989; 121: 351-399.