Category Theory and TGD Inspired Theory of Consciousness

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Contents

1 Introduction **3**
- 1.1 Category theory as a purely technical tool 4
- 1.2 Category theory based formulation of the ontology of TGD Universe 4
- 1.3 Other applications 5

2 What categories are? **6**
- 2.1 Basic concepts 6
- 2.2 Presheaf as a generalization for the notion of set 7
- 2.3 Generalized logic defined by category 8

3 Category theory and consciousness **9**
- 3.1 The ontology of TGD is tripartistic 9
- 3.2 The new ontology of space-time 11
- 3.3 The new notion of sub-system and notions of quantum presheaf and quantum logic 12
- 3.4 Does quantum jump allow space-time description? 14
- 3.5 Brief summary of the basic categories relating to the self hierarchy 15
- 3.6 The category of light cones, the construction of the configuration space geometry, and the problem of psychological time 16

4 More precise characterization of the basic categories and possible applications **17**
- 4.1 Intuitive picture about the category formed by the geometric correlates of selves 18
- 4.2 Categories related to self and quantum jump 19

4.2.1 The categories defined by moments of consciousness and the notion of self 19
4.2.2 The category associated with quantum jump sequences 20
4.3 Communications in TGD framework 21
4.3.1 What communications are? 21
4.3.2 Communication as quantum measurement? 22
4.3.3 Communication as sharing of mental images 22
4.3.4 Comparison with Goro Kato's approach 24
4.4 Cognizing about cognition 24

5 Logic and category theory 25
5.1 Is the logic of conscious experience based on set theoretic inclusion or topological condensation? 25
5.2 Do configuration space spinor fields define quantum logic and quantum topos 26
5.2.1 Finite-dimensional spinors define quantum logic 26
5.2.2 Quantum logic for finite-dimensional spinor fields . . . 27
5.2.3 Quantum logic and quantum topos defined by the prepared configuration space spinor fields 28
5.3 Category theory and the modelling of aesthetic and ethical judgements 30

6 Appendix: Category theory and construction of construction of S-matrix 30

Abstract

Category theory has been proposed as a new approach to the deep problems of modern physics, in particular quantization of General Relativity. Category theory might provide the desired systematic approach to fuse together the bundles of general ideas related to the construction of quantum TGD proper. Category theory might also have natural applications in the general theory of consciousness and the theory of cognitive representations.

a) The ontology of quantum TGD and TGD inspired theory of consciousness based on the trinity of geometric, objective and subjective existences could be expressed elegantly using the language of the category theory. Quantum classical correspondence might allow a mathematical formulation in terms of structure respecting functors mapping the categories associated with the three kinds of existences to each other. Basic results are following.

i) Self hierarchy has indeed functorial map to the hierarchy of space-time sheets and also configuration space spinor fields reflect it. Thus

the self referentiality of conscious experience has a functorial formulation (it is possible to be conscious about what one *was* conscious).

ii) The inherent logic for category defined by Heyting algebra must be modified in TGD context. Set theoretic inclusion is replaced with the topological condensation. The resulting logic is two-valued but since same space-time sheet can simultaneously condense at two disjoint space-time sheets the classical counterpart of quantum superposition has a space-time correlate so that also quantum jump should have space-time correlate in many-sheeted space-time.

iii) The category of light cones with inclusion as an arrow defining time ordering appears naturally in the construction of the configuration space geometry and realizes the cosmologies within cosmologies scenario. In particular, the notion of the arrow of psychological time finds a nice formulation unifying earlier two different explanations.

iv) The category of light cones with inclusion as an arrow defining time ordering appears naturally in the construction of the configuration space geometry and realizes the cosmologies within cosmologies scenario. In particular, the notion of the arrow of psychological time finds a nice formulation unifying earlier two different explanations.

b) Cognition is categorizing and category theory suggests itself as a tool for understanding cognition and self hierarchies and the abstraction processes involved with conscious experience.

c) Categories possess inherent generalized logic based on set theoretic inclusion which in TGD framework is naturally replaced with topological condensation: the outcome is quantum variants for the notions of sieve, topos, and logic. This suggests the possibility of geometrizing the logic of both geometric, objective and subjective existences and perhaps understand why ordinary consciousness experiences the world through Boolean logic and Zen consciousness experiences universe through three-valued logic. Also the right-wrong logic of moral rules and beautiful-ugly logic of aesthetics seem to be too naive and might be replaced with a more general quantum logic.

1 Introduction

Goro Kato has proposed an ontology of consciousness relying on category theory [5, 6]. Physicist friendly summary of the basic concepts of category theory can be found in [2]) whereas the books [7, 8] provide more mathematically oriented representations. Category theory has been proposed as a new approach to the deep problems of modern physics, in particular quantization of General Relativity. To mention only one example, C. J. Isham [2] has proposed that topos theory could provide a new approach to quantum gravity in which space-time points would be replaced by regions of space-time and that category theory could geometrize and dynamicize even logic by replacing the standard Boolean logic with a dynamical logic dictated by the structure of the fundamental category purely geometrically [1].

Although I am an innocent novice in this field and know nothing about the horrible technicalities of the field, I have a strong gut feeling that category theory might provide the desired systematic approach to quantum TGD proper [TGD, padTGD], the general theory of consciousness, and the theory of cognitive representations [cbookI, cbookII].

1.1 Category theory as a purely technical tool

Category theory could help to disentangle the enormous technical complexities of the quantum TGD and to organize the existing bundle of ideas into a coherent conceptual framework. The construction of the geometry of the configuration space ("world of classical worlds")[A1, A2], of classical configuration space spinor fields [A3], and of S-matrix [A4, A5] using a generalization of the quantum holography principle are especially natural applications. Category theory might also help in formulating the new TGD inspired view about number system as a structure obtained by "gluing together" real and p-adic number fields and TGD as a quantum theory based on this generalized notion of number [A1, A7, A6].

1.2 Category theory based formulation of the ontology of TGD Universe

It is interesting to find whether also the ontology of quantum TGD and TGD inspired theory of consciousness based on the trinity of geometric, objective and subjective existences [D1] could be expressed elegantly using the language of the category theory.

There are indeed natural and non-trivial categories involved with many-sheeted space-time and the geometry of the configuration space ("the world of classical worlds"); with configuration space spinor fields; and with the notions of quantum jump, self and self hierarchy. Functors between these categories could express more precisely the quantum classical correspondences and self-referentiality of quantum states allowing them to express information about quantum jump sequence.

i) Self hierarchy has a structure of category and corresponds functorially to the hierarchical structure of the many-sheeted space-time.
ii) Quantum jump sequence has a structure of category and corresponds functorially to the category formed by a sequence of maximally deterministic

regions of space-time sheet.
iii) Even the quantum jump could have space-time correlates made possible by the generalization of the Boolean logic to what might be space-time correlate of quantum logic and allowing to identify space-time correlate for the notion of quantum superposition.

1.3 Other applications

One can imagine also other applications.

a) Categories posses inherent logic [1] based on the notion of sieves relying on the notion of presheaf which generalizes Boolean logic based on inclusion. In TGD framework inclusion is naturally replaced by topological condensation and this leads to a two-valued logic realizing space-time correlate of quantum logic based on the notions of quantum sieve and quantum topos.

This suggests the possibility to geometrize the logic of both geometric, objective and subjective existences and perhaps understand why ordinary consciousness experiences the world through Boolean logic and Zen consciousness experiences universe through logic in which the law of excluded middle is not true. Interestingly, the p-adic logic of cognition is naturally 2-valued whereas the real number based logic of sensory experience allows excluded middle (is the person at the door in or out, in and out, or neither in nor out?). The quantum logic naturally associated with spinors (in the "world of classical worlds") is consistent with the logic based on quantum sieves.

b) Simple Boolean logic of right and wrong does not seem to be ideal for understanding moral rules. Same applies to the beauty-ugly logic of aesthetic experience. The logic based on quantum sieves would perhaps provide a more flexible framework.

c) Cognition is categorizing and category theory suggests itself as a tool for understanding cognition and self hierarchies and the abstraction processes involved with conscious experience. Here the new elements associated with the ontology of space-time due to the generalization of number concept would be central. Category theory could be also helpful in the modelling of conscious communications, in particular the telepathic communications based on sharing of mental images involving the same mechanism which makes possible space-time correlates of quantum logic and quantum superposition. These aspects are discussed in [C2].

2 What categories are?

In the following the basic notions of category theory are introduced and the notion of presheaf and category induced logic are discussed.

2.1 Basic concepts

Categories [7, 8, 2] are roughly collections of objects A, B, C... and morphisms $f(A \rightarrow B)$ between objects A and B such that decomposition of two morphisms is always defined. Identity morphisms map objects to objects. Topological/linear spaces form a category with continuous/linear maps acting as morphisms. Also algebraic structures of a given type form a category: morphisms are now homomorphisms. Practically any collection of mathematical structures can be regarded as a category. Morphisms can can be very general: for instance, partial ordering $a \leq b$ can define morphism $f(A \rightarrow B)$.

Functors between categories map objects to objects and morphisms to morphisms so that a product of morphisms is mapped to the product of the images and identity morphism is mapped to identity morphism. Group representation is example of this kind of a functor: now group action in group is mapped to a linear action at the level of the representations. Commuting square is an easy visual manner to understand the basic properties of a functor, see Fig. 2.1.

The product $C = AB$ for objects of categories is defined by the requirement that there are projection morphisms π_A and π_B from C to A and B and that for any object D and pair of morphisms $f(D \rightarrow A)$ and $g(D \rightarrow B)$ there exist morphism $h(D \rightarrow C)$ such that one has $f = \pi_A h$ and $g = \pi_B h$. Graphically (see Fig. 2.1) this corresponds to a square diagram in which pairs A,B and C,D correspond to the pairs formed by opposite vertices of the square and arrows DA and DB correspond to morphisms f and g, arrows CA and CB to the morphisms π_A and π_B and the arrow h to the diagonal DC.

Examples of product categories are Cartesian products of topological and linear spaces, of differentiable manifolds, groups, etc. Also tensor products of linear spaces satisfies these axioms. One can define also more advanced concepts such as limits and inverse limits. Also the notions of sheafs, presheafs, and topos are important.

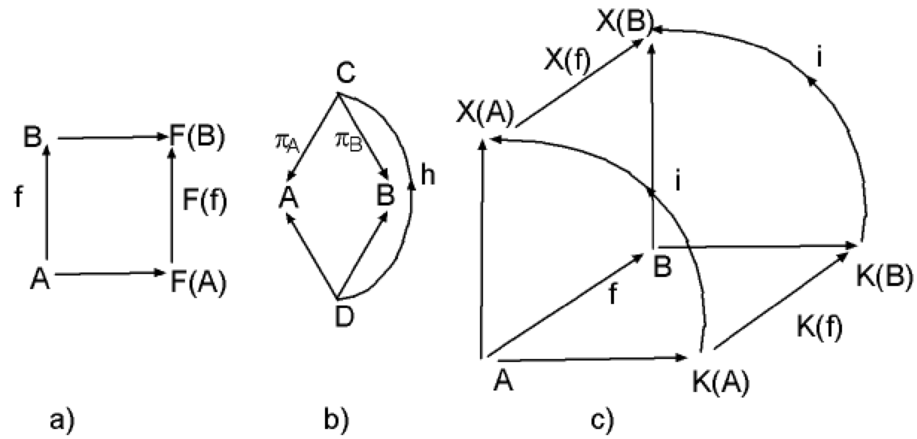

Figure 1: Commuting diagram associated with the definition of a) functor, b) product of objects of category, c) presheaf K as sub-object of presheaf X ("two pages of book".)

2.2 Presheaf as a generalization for the notion of set

Presheafs can be regarded as a generalization for the notion of set. Presheaf is a functor X that assigns to any object of a category **C** an object in the category **Set** (category of sets) and maps morphisms to morphisms (maps between sets for **C**). In order to have a category of presheafs, also morphisms between presheafs are needed. These morphisms are called natural transformations $N : X(A) \rightarrow Y(A)$ between the images $X(A)$ and $Y(A)$ of object A of **C**. They are assumed to obey the commutativity property $N(B)X(f) = Y(f)N(A)$ which is best visualized as a commutative square diagram. Set theoretic inclusion $i : X(A) \subset Y(A)$ is obviously a natural transformation.

An easy manner to understand and remember this definition is commuting diagram consisting of two pages of book with arrows of natural transformation connecting the corners of the pages: see Fig. 2.1.

As noticed, presheafs are generalizations of sets and a generalization for the notion of subset to a sub-object of presheaf is needed and this leads to the notion of topos [1, 2]. In the classical set theory a subset of given sets X can be characterized by a mapping from set X to the set $\Omega = \{true, false\}$ of Boolean statements. Ω itself belongs to the category **C**. This idea generalizes to sub-objects whose objects are collections of sets: Ω is only replaced with its Cartesian power. It can be shown that in the case of presheafs associated with category **C** the sub-object classifier Ω can be replaced with a more general algebra, so called Heyting algebra [2, 1] possessing the same basic operations as Boolean algebra (and, or, implication arrow, and negation) but is not in general equivalent with any Boolean algebra. What is important is that this generalized logic is inherent to the category **C** so that many-valued logic ceases to be an ad hoc construct in category theory.

In the theory of presheafs sub-object classifier Ω, which belongs to **Set**, is defined as a particular presheaf. Ω is defined by the structure of category **C** itself so that one has a geometrization of the notion of logic implied by the properties of category. The notion of sieve is essential here. A sieve for an object A of category **C** is defined as a collection of arrows $f(A \rightarrow ...)$ with the property that if $f(A \rightarrow B)$ is an arrow in sieve and if $g(B \rightarrow C)$ is any arrow then $gf(A \rightarrow C)$ belongs to sieve.

In the case that morphism corresponds to a set theoretic inclusion the sieve is just either empty set or the set of all sets of category containing set A so that there are only two sieves corresponding to Boolean logic. In the case of a poset (partially ordered set) sieves are sets for which all elements are larger than some element.

2.3 Generalized logic defined by category

The presheaf $\Omega : \mathbf{C} \rightarrow \mathbf{Set}$ defining sub-object classifier and a generalization of Boolean logic is defined as the map assigning to a given object A the set of all sieves on A. The generalization of maps $X \rightarrow \Omega$ defining subsets is based on the the notion of sub-object K. K is sub-object of presheaf X in the category of presheaves if there exist natural transformation $i : K \rightarrow X$ such that for each A one has $K(A) \subset X(A)$ (so that sub-object property is reduced to subset property).

The generalization of the map $X \rightarrow \Omega$ defining subset is achieved as follows. Let K be a sub-object of X. Then there is an associated characteristic arrow $\chi^K : X \rightarrow \Omega$ generalizing the characteristic Boolean valued map defining subset, whose components $\chi^K_A : X(A) \rightarrow \Omega(A)$ in **C** is defined as

$$\chi^K_A(x) = \{f(A \rightarrow B)|X(f)(x) \in K(B)\} \ .$$

By using the diagrammatic representation of Fig. 2.1 for the natural transformation i defining sub-object, it is not difficult to see that by the basic properties of the presheaf K $\chi^K_A(x)$ is a sieve. When morphisms f are inclusions in category Set, only two sheaves corresponding to all sets containing

X and empty sheaf result. Thus binary valued maps are replaced with sieve-valued maps and sieves take the role of possible truth values. What is also new that truths and logic are in principle context dependent since each object A of $\mathbf{C}$ serves as a context and defines its own collection of sieves.

The generalization for the notion of point of set X exists also and corresponds to a selection of single element γ_A in the set $X(A)$ for each A object of $\mathbf{C}$. This selection must be consistent with the action of morphisms $f(A \rightarrow B)$ in the sense that the matching condition $X(f)(\gamma_A) = \gamma_B$ is satisfied. It can happen that category of presheaves has no points at all since the matching condition need not be satisfied globally.

It turns out that TGD based notion of subsystem leads naturally to what might be called quantal versions of topos, presheaves, sieves and logic.

3 Category theory and consciousness

Category theory is basically about relations between objects, rather than objects themselves. Category theory is not about Platonic ideas, only about relations between them. This suggests a possible connection with TGD and TGD inspired theory of consciousness where the sequences quantum jumps between quantum histories defining selves have a role similar to morphisms and quantum states themselves are like Platonic ideas not conscious as such. Also the fact that it is not possible to write any formula for the contents of conscious experience although one can say a lot about its general structure bears a striking similarity to the situation in category theory.

3.1 The ontology of TGD is tripartistic

The ontology of TGD involves a trinity of existences.

a) Geometric existence or existence in the sense of classical physics. Objects are 3-surfaces in 8-D imbedding space, matter as res extensa. Quantum gravitational holography assigns to a 3-surface X^3 serving as a causal determinant space-time sheet $X^4(X^3)$ defining the classical physics associated with X^3 as a generalization of Bohr orbit. X^3 can be seen as a 3-D hologram representing the information about this 4-D space-time sheet

The geometry of configuration space of 3-surfaces, "the world of classical worlds" corresponds to a higher level geometric existence serving as the fixed arena for the quantum dynamics. The basic vision is that the existence requirement for Kähler geometry in the infinite-dimensional context fixes the infinite-dimensional geometric existence uniquely.

b) Quantum states defined as classical spinor fields in the world of classical worlds, and provide the quantum descriptions of possible physical realities that the probably never-reachable ultimate theory gives as solutions of field equations. The solutions *are* the objective realities in the sense of quantum theory: theory and theory about world are one and the same thing: there is no separate 'reality' behind the solutions of the field equations.

c) Subjective existence corresponds to quantum jumps between the quantum states identified as moment of consciousness. Just as quantum numbers characterize physical states, the increments of quantum numbers in quantum jump are natural candidates for qualia, and this leads to a concrete quantum model for sensory qualia and sensory perception [D3].

Quantum jump has a complex anatomy: counterpart for the unitary U process of Penrose followed by a counterpart of the state function reduction followed by a counterpart of the state preparation process yielding a classical state in Boolean and geometrical sense. State function preparation and reduction are nondeterministic processes and preparation is analogous to analysis since it decomposes at each step the already existing unentangled subsystems to unentangled subsystems if possible.

Quantum jump is the elementary particle of consciousness and selves are like atoms, molecules,... built from these. Self is by definition a system able to not develop bound state quantum entanglement with environment and loses consciousness when this occurs. Selves form a hierarchy very much analogous to the hierarchy of states formed from elementary particles. Self experiences its sub-selves as mental images. Selves form objects of a category in which arrows connect sub-selves to selves.

Macro-temporal and macroscopic quantum coherence corresponds to the formation of bound states [D2]: in this process state function reduction and preparation effective cease in appropriate degrees of freedom. In TGD framework one can assign to bound state entanglement negative entropy identifiable as a genuine measure for information [C1]. The bound state entanglement stable against state function preparation would thus serve as a correlate for the experience of understanding, and one could compare quantum jump to a brainstorm followed by an analysis leading to an experience of understanding.

Quantum classical correspondence relates the three levels of existence to each other. It states that both quantum states and quantum jump sequences have space-time correlates. This is made possible by p-adic and classical non-determinism, which are characteristic features of TGD space-time. p-Adic non-determinism makes it possible to map quantum jump sequences to p-adic space-time sheets: this gives rise to cognitive representations. The non-determinism of Kähler action makes possible symbolic sensory representations of quantum jump sequences of which language is the basic example.

The natural identification of the correlates of quantum states is as maximal deterministic regions of space-time sheet. The final states of quantum jump define a sequence of quantum states so that quantum jump sequence (contents of consciousness) has the decomposition of space-time sheet to maximal deterministic regions as a space-time correlate. Thus space-time surface can be said to define a symbolic (and unfaithful) representation for the contents of consciousness. Since configuration space spinor field is defined in the world of classical worlds, this means that quantum states carry information about quantum jump sequence and self reference becomes possible. System can become conscious about what it *was* (not "is") conscious of.

The possibility to represent quantum jump sequences at space-time level is what makes possible practical mathematics, cognition, and symbolic representations. The generation of these representations in turn means generation of reflective levels of consciousness and thus explains self-referential nature of consciousness. This feedback makes also possible the evolution of mathematical consciousness: mathematician without paper and pencil (or computer keyboard!) cannot do very much.

Category theory might help to formulate more precisely the quantum classical correspondence and self referentiality as structure respecting functors from the categories associated with subjective existence to the categories of quantum and classical existence and from the category of quantum existence to that of classical existence.

3.2 The new ontology of space-time

Classical worlds are space-time surfaces and have much richer ontology than the space-time of general relativity. Space-time is many-sheeted possessing a hierarchy of parallel space-time sheets topologically condensed at larger space-time sheets and identifiable as geometric correlates for physical objects in various length scales (see Fig. 3.3). Topological field quantization allows to assign to any material system "field body": this has important implications for quantum biology in TGD Universe [D1].

TGD leads to a generalization of the notion of real numbers obtained by gluing real number field and p-adic number fields R_p, labelled by primes $p = 2, 3, 5, ...$ and their extensions together along common rationals (very roughly) to form a "book like" structure [A1, A6, D1]. p-Adic space-time sheets are interpreted as space-time correlates of cognition and intentionality. The transformation of intention to action corresponds to a quantum jump replacing p-adic space-time sheet with a real one.

The p-adic notion of distance differs dramatically from its real counterpart. Two rationals infinitesimally near p-adically are infinitely distance in real sense. This means that p-adic space-time sheets have literally infinite size in the real sense and cognition and intentionality cannot be localized in brain. Biological body serves only as a sensory receptor and motor instrument utilizing symbolic representations built by brain.

The notion of infinite numbers (primes, rationals, reals, complex numbers and also quaternions and octonions)[A7, A6] inspired by TGD inspired theory of consciousness leads to a further generalization. One can form ratios of infinite rationals to get ordinary rational numbers in the real sense and division by its inverse gives numbers which are units in the real sense but not in various p-adic senses ($p = 2, 3, 5, ...$).

This means that each space-time point is infinitely structured (note also that configuration space points are 3-surfaces and infinitely structure too!) but this structure is not seen at the level of real physics. The infinite hierarchy of infinite primes implies that single space-time point is in principle able to represent the physical quantum state of the entire universe in its structure cognitively. There are several interpretations: space-time points are algebraic holograms realizing Brahman=Atman identity; the Platonia of mathematical ideas resides at every space-time point, space-time points are the monads of Leibniz or the nodes of Indra's web...

One might hope that category theory could be of help in formulating more precisely this intuitive view about space-time which generalizes also to the other two levels of ontology.

3.3 The new notion of sub-system and notions of quantum presheaf and quantum logic

TGD based notion subsystem differs from the standard one already at the classical level [C1]. The relationship of having wormhole contacts to a larger space-time sheet would correspond to the basic morphism and would correspond to inclusion in category Set. Note that same space-time sheet can have wormhole contacts to several larger space-time sheets (see Fig. 3.3). The wormhole contacts are surrounded by light like 3-surfaces somewhat analogous to black hole horizons. They act as causal determinants and define 3-dimensional quantum gravitational holograms. Also other causal determinants are possible but light-likeness seems to a common feature of them.

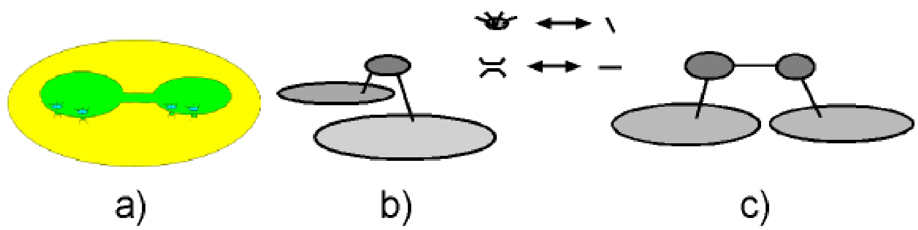

Figure 2: a) Wormhole contacts connect interiors space-time parallel space-time sheets (at a distance of about 10^4 Planck lengths) and join along boundaries bonds of possibly macroscopic size connect boundaries of space-time sheets. b) Wormhole contacts connecting space-time sheet to several space-time sheets could represent space-time correlate of quantum superposition. c) Space-time correlate for bound state entanglement making possible sharing of mental images.

Subsystem does not correspond to a mere subset geometrically as in standard physics and the functors mapping quantum level to space-time level are not maps to the category of sets but to that of space-time sheets, and thus pre-sheafs are replaced with what might be called quantum pre-sheafs. Boolean algebra and also Heyting algebra are replaced with their quantum variants.

a) The set theoretic inclusion $\subset$ in the definition of Heyting algebra is replaced by the arrow $A \rightarrow B$ representing a sequence of topological condensations connecting the space-time sheet A to B. The arrow from A to B is possible only if A is smaller than B, more precisely: if the p-adic prime $p(A)$ characterizing A is larger (or equal) than $p(B)$. The relation $\in$ of being a point of the space-time sheet A is not utilized at all.

b) Sieves at A are defined, not in terms of arrow sequences $f(A \rightarrow B)$, but as arrow sequences $f(B \rightarrow A)$: the wormhole contact roads leading from sheet B down to A. If there is a road from B to A then all roads to $C \rightarrow B$ combine with roads $B \rightarrow A$ to give roads $C \rightarrow A$ and thus define elements of the sieve.

c) X is quantum presheaf if it is a functor from the a category C to the category of space-time sheets. A sub-object of X is presheaf K such that for every A there is a road from $K(A)$ to $X(A)$.

d) Let K be a sub-object of the pre-sheaf X. The elements of the corresponding quantum Heyting algebra at A are defined as the collections of roads $f(B, A)$ leading via $K(A)$ to $K(X)$. This collection is either empty or contains all the roads via $K(A)$ to $K(X)$. A two-valued logic results trivially.

e) The difference with respect to Boolean logic comes from the fact space-time sheet can condense simultaneously to several disjoint space-time sheets whereas a given set cannot be a subset of two disjoint sets (see Fig. 3.3).

One can ask whether this property of "quantum logic" allows a space-time correlate even for the superposition of orthogonal quantum states as simultaneous topological condensation at several space-time sheets. This interpretation is consistent with the hypothesis that bound state entanglement has the formation of join along boundaries bonds (JABs) as a space-time correlate. Topologically condensed JAB-connected space-time sheets could indeed condense simultaneously on several space-time sheets.

The new notion of subsystem at space-time level forces to modify the notion of subsystem at quantum level. The subsystem defined by smaller space-time sheet is not describable as a simple tensor factor but the relation is given by the morphism representing the property of being subsystem.

3.4 Does quantum jump allow space-time description?

Quantum jump consists of a unitary process, state function reduction and state preparation. The geometrical realization of "quantum logic" suggests that simultaneous topological condensation to several space-time sheets could be a space-time correlate for the maximally entangled superposition of quantum states created in the U-process. Quantal multi-verse states would functorially correspond to classical multi-verse states: something which obviously came in my mind for long time ago but seemed stupid. State function reduction would lead to the splitting of the wormhole contacts and as a result maximally reduced state would result: one cannot however exclude bound state entanglement due to interactions mediated by wormhole contacts.

State function preparation would correspond to a sequence of splittings for join along boundaries bonds serving as prerequisites for entanglement in the degrees of freedom associated with second quantized induced spinor fields at space-time sheets. An equivalent process is the decay of 3-sheet to two pieces interpretable as de-coherence. For instance, the splitting of photon beam in the modified double slit experiment by Afshar [10, D1], which challenges the existing interpretations of quantum theory and provides support for TGD based theory of quantum measurement relying on classical non-determinism, would correspond to this process.

State preparation yields states in which no dissipation occurs. The space-time correlates are asymptotic solutions of field equations for which classical counterpart of dissipation identified as Lorentz 4-force vanishes: this hypothesis indeed leads to very general solutions of field equations [A9]. The non-determinism at quantum level would correspond to the non-determinism for the evolution of induced spinor fields at space-time level.

3.5 Brief summary of the basic categories relating to the self hierarchy

Category theory suggests the identification of space-time sheets as basic objects of the space-time category. Space-time sheets are natural correlates for selves and the arrow describing sub-self property is mapped to the arrow of being topologically condensed space-time sheet. Category theoretically this would mean the existence of a functor from the the category defined by self hierarchy to the hierarchy of space-time sheets.

The highly non-trivial implication of the new notion of sub-system is that same sub-self can be sub-self of several selves: mental images can be shared so that consciousness would not be so private as usually believed. Sharing involves also fusion of mental images. Sub-selves of different selves form a bound state and fuse to single sub-self giving rise to stereo consciousness (fusion of right and left visual fields is the basic example).

The formation of join along boundaries bonds connecting the boundaries of a sub-self space-time sheets is the space-time correlate for this process. The ability of subsystems to entangle when systems remain un-entangled is completely new and due to the new notion of subsystem (subsystem is separated by elementary particle horizon from system). Sharing of mental images and the possibility of time-like entanglement also possible telepathic quantum communications: for instance, TGD based model of episodal memories relies on this mechanism [D1].

The hierarchy of space-time sheets functorially replicates itself at the level of quantum states and of subjective existence. Quantum states have a hierarchical structure corresponding to the decomposition of space-time to space-time sheets. The sequence of quantum jumps decomposes into parallel sequences of quantum jumps occurring at different parallel space-time sheets characterized by p-adic length scales. The possibility of quantum parallel dissipation (quarks inside hadrons) is one important implication: although dissipation and de-coherence occur in short length and time scales, quantum coherence is preserved in longer length and time scales. This is of utmost importance for understanding how wet and hot brain can be macroscopic quantum system [D2].

The self hierarchy has also counterpart at the level of Platonia made possible by infinitely structured points of space-time. The construction of infinite primes is analogous to a repeated second quantization of an arithmetic quantum field theory such that the many particle states of previous level representing infinite primes at that level become elementary particles at the next level of construction. This hierarchy reflect itself as the hierarchy of units and as a hierarchy of levels of mathematical consciousness.

The steps in quantum jump, or equivalently the sequence of final states of individual steps would define the objects of the category associated with the quantum jump. The first step would be the formation of a larger number of wormhole contacts during U process followed by their splitting to minimum in the state function reduction. Formation and splitting of contacts would define arrows now. During the state preparation each decay to separate 3-sheets would define arrow from connecting initial state to both final states.

3.6 The category of light cones, the construction of the configuration space geometry, and the problem of psychological time

Light-like 7-surfaces of imbedding space are central in the construction of the geometry of the world of classical worlds. The original hypothesis was that space-times are 4-surfaces of $H = M^4_+ \times CP_2$, where M^4_+ is the future light cone of Minkowski space with the moment of big bang identified as its boundary $\delta H = \delta M^4_+ \times CP_2$: "the boundary of light-cone". The naive quantum holography would suggest that by classical determinism everything reduces to the light cone boundary. The classical non-determinism of Kähler action forces to give up this naive picture which also spoils the full Poincare invariance.

The new view about energy and time forces to conclude that space-time surfaces approach vacua at the boundary of the future light cone. The world of classical worlds, call it CH, would consist of classical universes having a vanishing inertial 4-momentum and other conserved quantities and being created from vacuum: big bang would be replaced with a "silent whisper amplified to a big bang". The net gravitational mass density can be non-vanishing since gravitational momentum is difference of inertial momenta of positive and negative energy matter: Einstein's Equivalence Principle is exact truth only at the limit when the interaction between positive and negative energy matter can be neglected [A10].

Poincare invariant theory results if one replaces CH with the union of its copies $CH(a)$ associated with the light cones $M^4_+(a)$ with a specifying the position of the dip of $M^4_+(a)$ in M^4. Also past directed light-cones $M^4_-(a)$ are allowed. The unions and intersections of the light cones with inclusion as a basic arrow would form category analogous to the category Set with inclusion defining the arrow of time. This category formalizes the ideas that cosmology has a fractal Russian doll like structure, that the cosmologies inside cosmologies are singularity free, and that cosmology is analogous to an organic evolution and organic evolution to a mini cosmology [A10].

The view also unifies the proposed two explanations for the arrow of psychological time [D1].

a) The mind like space-time sheets representing conscious self drift quantum jump by quantum jump towards geometric future whereas the matter like space-time sheets remain stationary. The self of the organism presumably consisting mostly of topological field quanta, would be like a passenger in a moving train seeing the changing landscape. The organism would be a mini cosmology drifting quantum jump to the geometric future. Also selves living in the reverse direction of time are possible.

b) Psychological time corresponds to a phase transition front in which intentions represented by p-adic space-time sheets transform to actions represented by real space-time sheets moving to the direction of geometric future. The motion would be due to the drift of $M^4_+(a)$. The very fact that the mini cosmology is created from vacuum, implies that space-time sheets of both negative and positive field energy are abundantly generated as realizations of intentions. The intentional resources are richest near the boundary of $M^4_+(a)$ and depleted during the ageing with respect to subjective time as asymptotic self-organization patterns are reached. Interestingly, mini cosmology can be seen as a fractally scaled up variant of quantum jump. The realization of intentions as negative energy signals (phase conjugate light) sent to the geometric past and inducing a positive energy response (say neural activity) is consistent with the TGD based models for motor action and long term memory [D1].

4 More precise characterization of the basic categories and possible applications

In the following the categories associated with self and quantum jump are discussed in more precise manner and applications to communications and cognition are considered.

4.1 Intuitive picture about the category formed by the geometric correlates of selves

Space-time surface $X^4(X^3)$ decomposes into regions obeying either real or p-adic topology and each region of this kind corresponds to an unentangled subsystem or self lasting at least one quantum jump. By the localization in the zero modes these decompositions are equivalent for all 3-surfaces X^3 in the quantum superposition defined by the prepared configuration space spinor fields resulting in quantum jumps. There is a hierarchy of selves since selves can contain sub-selves. The entire space-time surface $X^4(X^3)$ represents the highest level of the self hierarchy.

This structure defines in a natural manner a category. Objects are all possible sub-selves contained in the self hierarchy: sub-self is set consisting of lower level sub-selves, which in turn have a further decomposition to sub-selves, etc... The naive expectation is that geometrically sub-self belongs to a self as a subset and this defines an inclusion map acting as a natural morphism in this category. This expectation is not quite correct. More natural morphisms are the arrows telling that self as a set of sub-selves contains sub-self as an element. These arrows define a structure analogous to a composite of hierarchy trees.

To be more precise, for a single space-time surface $X^4(X^3)$ this hierarchy corresponds to a subjective time slice of the self hierarchy defined by a single quantum jump. The sequence of hierarchies associated with a sequence of quantum jumps is a natural geometric correlate for the self hierarchy. This means that the objects are now sequences of submoments of consciousness. Sequences are not arbitrary. Self must survive its lifetime although sub-selves at various levels can disappear and reappear (generation and disappearence of mental images). Geometrically this means typically a phase transition transforming real or p_1-adic to p_2-adic space-time region with same topology as the environment. Also sub-selves can fuse to single sub-self. The constraints on self sequences must be such that it takes these processes into account. Note that these constraints emerge naturally from the fact that quantum jumps sequences define the sequences of surfaces $X^4(X^3)$.

By the rich anatomy of the quantum jump there is large number of quantum jumps leading from a given initial quantum history to a given final quantum history. One could envisage quantum jump also as a discrete path in the space of configuration space spinor fields leading from the initial state to the final state. In particular, for given self there is an infinite number of closed elementary paths leading from the initial quantum history back to the initial quantum history and these paths in principle give all possible conscious information about a given quantum history/idea: kind of self morphisms are in question (analogous to, say, group automorhisms). Information about point of space is obtained only by moving around and coming back to the point, that is by studying the surroundings of the point. Self in turn can be seen as a composite of elementary paths defined by the quantum jumps. Selves can define arbitrarily complex composite closed paths giving information about a given quantum history.

4.2 Categories related to self and quantum jump

4.2.1 The categories defined by moments of consciousness and the notion of self

Since quantum jump involves state reduction and the sequence of self measurement reducing all entanglement except bound state entanglement, it defines a hierarchy of unentangled subsystems allowing interpretation as objects of a category. Arrows correspond to subsystem-system relationship and the two subsystems resulting in self measurement to the system. What subsystem corresponds mathematically is however not at all trivial and the naive description as a tensor factor does not work. Rather, a definition relying on the notion of p-adic length scale cutoff identified as a fundamental aspect of nature and consciousness is needed.

It is not clear what the statement that self corresponds to a subsystem which remains unentangled in subsequent quantum jump means concretely since subsystem can certainly change in some limits. What is clear that bound state entanglement between selves means a loss of consciousness. Category theory suggests that there should exist a functor between categories defined by two subsequent moments of consciousness. This functor maps submoments of consciousness to submoments of consciousness and arrows to arrows. Two subsequent submoments of consciousness belong to same sub-self is the functor maps the first one to the latter one. Thus category theory would play essential role in the precise definition of the notion of self.

The sequences of moments of consciousness form a larger category containing sub-selves as sequences of unentangled subsystems mapped to each other by functor arrows functoring subsequent quantum jumps to each other.

What might then be the ultimate characterizer of the self-identity? The theory of infinite primes suggests that space-time surface decomposes into regions labelled by finite p-adic primes. These primes must label also real regions rather than only p-adic ones, and one could understand this as resulting from a resonant transformation of intention to action. A p-adic space-time region characterized by prime p can transform to a real one or vice versa in quantum jump if the sizes of real and p-adic regions are characterized by the p-adic length scale L_p (or n-ary p-adic length scale $L_p(n)$. One can also consider the possibility that real region is accompanied by a p-adic region characterized by a definite prime p and providing a cognitive self-representation of the real region.

If this view is correct, the p-adic prime characterizing a given real or p-adic space-time sheet is the ultimate characterizer of the self-identity. Self identity is lost in bound state entanglement with another space-time sheet (at least when a space-time sheet with smaller value of the p-adic prime joins by join along boundaries bond to a one with a higher value of the p-adic prime). Self identity is also lost if a space-time sheet characterized by a given p-adic prime disappears in quantum jump.

4.2.2 The category associated with quantum jump sequences

There are several similarities between the ontologies and epistemologies of TGD and of category theory. Conscious experience is always determined by the discrete paths in the space of configuration space spinor fields defined by a quantum jump connecting two quantum histories (states) and is never determined by single quantum history as such (quantum states are unconscious). Also category theory is about relations between objects, not about objects directly: self-morphisms give information about the object of category (in case of group composite paths would correspond to products of group automorphisms). Analogously closed paths determined by quantum jump sequences give information about single quantum history. The point is however that it is impossible to have direct knowledge about the quantum histories: they are not conscious.

One can indeed define a natural category, call it **QSelf**, applying to this situation. The objects of the category **QSelf** are initial quantum histories of quantum jumps and correspond to prepared quantum states. The discrete path defining quantum jump can be regarded as an elementary morphism. Selves are composites of elementary morphisms of the initial quantum history defined by quantum jumps: one can characterize the morphisms by the number of the elementary morphisms in the product. Trivial self contains no quantum jumps and corresponds to the identity morphism, null path. Thus the collection of all possible sequences of quantum jumps, that is collections of selves allows a description in terms of category theory although the category in question is not a subcategory of the category **Set**.

Category **QSelf** does not possess terminal and initial elements (for terminal (initial) element T there is exactly one arrow $A \rightarrow T$ ($T \rightarrow A$) for every A: now there are always many paths between quantum histories involved).

4.3 Communications in TGD framework

Goro Kato identifies communications between conscious entities as natural maps between them whereas in TGD natural maps bind submoments of consciousness to selves. In TGD framework quantum measurement and the sharing of mental images are the basic candidates for communications. The problem is that the identification of communications as sharing of mental images is not consistent with the naive view about subsystem as a tensor factor. Many-sheeted space-time however forces length scale dependent notion of subsystem at space-time level and this saves the situation.

4.3.1 What communications are?

Communication is essentially generation of desired mental images/sub-selves in receiver. Communication between selves need not be directly conscious: in this case communication would generate mental images at some lower level of self hierarchy of receiver: for instance generate large number of sub-sub-selves of similar type. This is like communications between organizations. Communication can be also vertical: self can generate somehow sub-self in some sub-sub....sub-self or sub-sub...sub-self can generate sub-self of self somehow. This is communication from boss to the lower levels organization or vice versa.

These communications should have direct topological counterparts. For instance, the communication between selves could correspond to an exchange of mental image represented as a space-time region of different topology inside sender self space-time sheet. The sender self would simply throw this space-time region to a receiver self like a ball. This mechanism applies also to vertical communications since the ball could be also thrown from a boss to sub...sub-self at some lower level of hierarchy and vice versa.

The sequence of space-time surfaces provides a direct topological counterpart for communication as throwing balls representing sub-selves. Quantum jump sequence contains space-time surfaces in which the regions corresponding to receiver and sender selves are connected by a join along boundaries bond (perhaps massless extremal) representing classically the communication: during the communication the receiver and sender would form single self. The cartoon vision about rays connecting the eyes of communication persons would make sense quite concretely.

More refined means of communication would generate sub-selves of desired type directly at the end of receiver. In this case it is not so obvious how the sequence $X^(X^3)$ of space-time surfaces could represent communication. Of course, one can question whether communication is really what happens in this kind of situation. For instance, sender can affect the environment of receiver to be such that receiver gets irritated (computer virus is good manner to achieve this!) but one can wonder whether this is real communication.

4.3.2 Communication as quantum measurement?

Quantum measurement generates one-one map between the states of the entangled systems resulting in quantum measurement. Both state function reduction and self measurement give rise to this kind of map. This map could perhaps be interpreted as quantum communication between unentangled subsystems resulting in quantum measurement. For the state reduction process the space-time correlates are the values of zero modes. For state preparation the space-time correlates should correspond to classical spinor field modes correlating for the two subsystems generated in self measurement.

4.3.3 Communication as sharing of mental images

It has become clear that the sharing of mental images induced by quantum entanglement of sub-selves of two separate selves represents genuine conscious communication which is analogous telepathy and provides general mechanism of remote mental interactions making possible even molecular recognition mechanisms.

a) The sharing of mental images is not possible unless one assumes that self hierarchy is defined by using the notion of length scale resolution defined by p-adic length scale. The notion of scale of resolution is indeed fundamental for all quantum field theories (renormalization group invariance) for all quantum field theories and without it the practical modelling of physics would not be possible. The notion reflects directly the length scale resolution of conscious experience. For a given sub-self of self the resolution is given by the p-adic length scale associated with the sub-self space-time sheet.

b) Length scale resolution emerges naturally from the fact that sub-self space-time sheets having Minkowskian signature of metric are separated from the one representing self by wormhole contacts with Euclidian signature of metric. The signature of the induced metric changes from Minkowskian signature to Euclidian signature at 'elementary particle horizons' surrounding the throats of the wormhole contacts and having degenerate induced metric. Elementary particle horizons are thus metrically two-dimensional light like surfaces analogous to the boundary of the light cone and allow conformal invariance. Elementary particle horizons act as causal horizons. Topologically condensed space-time sheets are analogous to black hole interiors and due to the lack of the causal connectedness the standard description of sub-selves as tensor factors of the state space corresponding to self is not appropriate.

Hence systems correspond, not to the space-time sheets plus entire hierarchy of space-time sheets condensed to it, but rather, to space-time sheets with holes resulting when the space-time sheets representing subsystems are spliced off along the elementary particle horizons around wormhole contacts. This does not mean that all information about subsystem is lost: subsystem space-time sheet is only replaced by the elementary particle horizon. In analogy with the description of the black hole, some parameters (mass, charges,...) characterizing the classical fields created by the sub-self space-time sheet characterize sub-self.

One can say that the state space of the system contains 'holes'. There is a hierarchy of state spaces labelled by p-adic primes defining length scale resolutions. This picture resolves a longstanding puzzle relating to the interpretation of the fact that particle is characterize by both classical and quantum charges. Particle cannot couple simultaneously to both and this is achieved if quantum charge is associated with the lowest level description of the particle as CP_2 extremal and classical charges to its description at higher levels of hierarchy.

b) The immediate implication indeed is that it is possible to have a situation in which two selves are unentangled although their sub-selves (mental images) are entangled. This corresponds to the fusion and sharing of mental images. The sharing of the mental images means that union of disjoint hierarchy trees with levels labelled by p-adic primes p is replaced by a union of hierarchy threes with horizontal lines connecting subsystems at the same level of hierarchy. Thus the classical correspondence defines a category of presheaves with both vertical arrows replaced by sub-self-self relationship, horizontal arrows representing sharing of mental images, and natural maps representing binding of submoments of consciousness to selves.

4.3.4 Comparison with Goro Kato's approach

It is of interest to compare Goro Kato's approach with TGD approach. The following correspondence suggests itself.

a) In TGD each quantum jumps defines a category analogous to the Goro Kato's category of open sets of some topological space but set theoretic inclusion replaced by topological condensation. The category defined by a moment of consciousness is dynamical whereas the category of open sets is non-dynamical.

b) The assignment of a 3-surface acting as a causal determinant to each unentangled subsystem defined by a moment of consciousness defines a unique "quantum presheaf" which is the counterpart of the presheaf in Goro Kato's theory. The conscious entity of Kato's theory corresponds to the classical correlate for a moment of consciousness.

c) Natural maps between the causal determinants correspond to the space-time correlates for the functor arrows defining the threads connecting submoments of consciousness to selves. In Goro Kato's theory natural maps are interpreted as communications between conscious entities. The sharing of mental images by quantum entanglement between subsystems of unentangled systems defines horizontal bi-directional arrows between subsystems associated with same moment of consciousness and is counterpart of communication in TGD framework. It replaces the union of disjoint hierarchy trees associated with various unentangled subsystems with hierarchy trees having horizontal connections defining the bi-directional arrows. The sharing of mental images is not possible if subsystem is identified as a tensor factor and thus without taking into account length scale resolution.

4.4 Cognizing about cognition

There are close connections with basic facts about cognition.

a) Categorization means classification and abstraction of common features in the class formed by the objects of a category. Already quantum jump defines category with hierarchical structure and can be regarded as consciously experienced analysis in which totally entangled entire universe $U\Psi_i$ decomposes to a product of maximally unentangled subsystems. The sub-selves of self are like elements of set and are experienced as separate objects whereas sub-sub-selves of sub-self self experiences as an average: they belong to a class or category formed by the sub-self. This kind of averaging occurs also for the contributions of quantum jumps to conscious experience of self.

b) The notions of category theory might be useful in an attempt to construct a theory of cognitive structures since cognition is indeed to high degree classification and abstraction process. The sub-selves of a real self indeed have p-adic space-time sheets as geometric correlates and thus correspond to cognitive sub-selves, thoughts. A meditative experience of empty mind means in case of real self the total absence of thoughts.

c) Predicate logic provides a formalization of the natural language and relies heavily on the notion of n-ary relation. Binary relations $R(a, b)$ corresponds formally to the subset of the product set $A \times B$. For instance, statements like 'A does something to B' can be expressed as a binary relation, particular kind of arrow and morphism ($A \leq B$ is a standard example). For sub-selves this relation would correspond to a dynamical evolution at space-time level modelling the interaction between A and B. The dynamical path defined by a sequence of quantum jumps is able to describe this kind of relationships too at level of conscious experience. For instance, 'A touches B' would involve the temporary fusion of sub-selves A and B to sub-self C.

5 Logic and category theory

Category theory allows naturally more general than Boolean logics inherent to the notion of topos associated with any category. Basic question is whether the ordinary notion of topos algebra based on set theoretic inclusion or the notion of quantum topos based on topological condensation is physically appropriate. Starting from the quasi-Boolean algebra of open sets one ends up to the conclusion that quantum logic is more natural. Also configuration space spinor fields lead naturally to the notion of quantum logic.

5.1 Is the logic of conscious experience based on set theoretic inclusion or topological condensation?

The algebra of open sets with intersections and unions and complement defined as the interior of the complement defines a modification of Boolean algebra having the peculiar feature that the points at the boundary of the closure of open set cannot be said to belong to neither interior of open set or of its complement. There are two options concerning the interpretation.

a) 3-valued logic could be in question. It is however not possible to understand this three-valuedness if one defines the quasi-Boolean algebra of open sets as Heyting algebra. The resulting logic is two-valued and the points at boundaries of the closure do not correspond neither to the statement or its negation. In p-adic context the situation changes since p-adic open sets are also closed so that the logic is strictly Boolean. That our ordinary cognitive mind is Boolean provides a further good reason for why cognition is p-adic.

b) These points at the boundary of the closure belong to both interior and exterior in which case a two-valued "quantum logic" allowing superposition of opposite truth values is in question. The situation is indeed exactly the same as in the case of space-time sheet having wormhole contacts to several space-time sheets.

The quantum logic brings in mind Zen consciousness [9] (which I became fascinated of while reading Hofstadter's book "Gödel, Escher,Bach" [3]) and one can wonder whether selves having real space-time sheets as geometric correlates and able to live simultaneously in many parallel worlds correspond to Zen consciousness and Zen logic. Zen logic would be also logic of sensory experience whereas cognition would obey strictly Boolean logic.

The causal determinants associated with space-time sheets correspond to light like 3-surfaces which could elementary particle horizons or space-time boundaries and possibly also 3-surfaces separating two maximal deterministic regions of a space-time sheet. These surfaces act as 3-dimensional quantum holograms and have the strange Zen property that they are neither space-like nor time-like so that they represent both the state and the process. In the TGD based model for topological quantum computation (TQC) light-like boundaries code for the computation so that TQC program code would be equivalent with the running program [C3].

5.2 Do configuration space spinor fields define quantum logic and quantum topos

I have proposed already earlier that configuration space spinor fields define what might be called quantum logic. One can wonder whether configuration space spinors could also naturally define what might be called quantum topos since the category underlying topos defines the logic appropriate to the topos. This question remains unanswered in the following: I just describe the line of though generalizing ordinary Boolean logic.

5.2.1 Finite-dimensional spinors define quantum logic

Spinors at a point of an $2N$-dimensional space span 2^N-dimensional space and spinor basis is in one-one correspondence with Boolean algebra with N

different truth values (N bits). 2N=2-dimensional case is simple: Spin up spinor= true and spin-dow spinor=false. The spinors for $2N$-dimensional space are obtained as an N-fold tensor product of 2-dimensional spinors (spin up,spin down): just like in the case of Cartesian power of Ω.

Boolean spinors in a given basis are eigen states for a set N mutually commuting sigma matrices providing a representation for the tangent space group acting as rotations. Boolean spinors define N Boolean statements in the set Ω^N so that one can in a natural manner assign a set with a Boolean spinor. In the real case this group is $SO(2N)$ and reduces to $SU(N)$ for Kähler manifolds. For pseudo-euclidian metric some non-compact variant of the tangent space group is involved. The selections of N mutually commuting generators are labelled by the flag-manifold $SO(2N)/SO(2)^N$ in real context and by the flag-manifold $U(N)/U(1)^N$ in the complex case. The selection of these generators defines a collection of N 2-dimensional linear subspaces of the tangent space.

Spinors are in general complex superpositions of spinor basis which can be taken as the product spinors. The quantum measurement of N spins representing the Cartan algebra of $SO(2N)$ ($SU(N)$) leads to a state representing a definite Boolean statement. This suggests that quantum jumps as moments of consciousness quite generally make universe classical, not only in geometric but also in logical sense. This is indeed what the state preparation process for the configuration space spinor field seems to do.

5.2.2 Quantum logic for finite-dimensional spinor fields

One can generalize the idea of the spinor logic also to the case of spinor fields. For a given choice of the local spinor basis (which is unique only modular local gauge rotation) spinor field assigns to each point of finite-dimensional space a quantum superposition of Boolean statements decomposing into product of N statements.

Also now one can ask whether it is possible to find a gauge in which each point corresponds to definite Boolean statement and is thus an eigen state of a maximal number of mutually commuting rotation generators Σ_{ij}. This is not trivial if one requires that Dirac equation is satisfied. In the case of flat space this is certainly true and constant spinors multiplied by functions which solve d'Alembert equation provide a global basis.

The solutions of Dirac equation in a curved finite-dimensional space do not usually possess a definite spin direction globally since spinor curvature means the presence of magnetic spin-flipping interaction and since there need not exist a global gauge transformation leading to an eigen state of the local Cartan algebra everywhere. What might happen is that the local gauge transformation becomes singular at some point: for instance, the direction of spin would be radial around given point and become ill defined at the point. This is much like the singularities for vector fields on sphere. The spinor field having this kind of singularity should vanish at singularity but the local gauge rotation rotating spin in same direction everywhere is necessarily ill-defined at the singularity.

In fact, this can be expressed using the language of category theory. The category in question corresponds to a presheaf which assigns to the points of the base space the fiber space of the spinor bundle. The presence of singularity means that there are no global section for this presheaf, that is a continuous choice of a non-vanishing spinor at each point of the base space. The so called Kochen-Specker theorem discussed in [2] is closely related to a completely analogous phenomenon involving non-existence of global sections and thus non-existence of a global truth value.

Thus in case of curved spaces is not necessarily possible to have spinor field basis representing globally Boolean statements and only the notion of locally Boolean logic makes sense. Indeed, one can select the basis to be eigen state of maximal set of mutually commuting rotation generators in single point of the compact space. Any such choice does.

5.2.3 Quantum logic and quantum topos defined by the prepared configuration space spinor fields

The prepared configuration space spinor fields occurring as initial and final states of quantum jumps are the natural candidates for defining quantum logic. The outcomes of the quantum jumps resulting in the state preparation process are maximally unentangled states and are as close to Boolean states as possible.

Configuration space spinors correspond to fermionic Fock states created by infinite number of fermionic (leptonic and quarklike) creation and annihilation operators. The spin degeneracy is replaced by the double-fold degeneracy associated with a given fermion mode: given state either contains fermion or not and these two states represent true and false now. If configuration space were flat, the Fock state basis with definite fermion and anti-fermion numbers in each mode would be in one-one correspondence with Boolean algebra.

Situation is however not so simple. Finite-dimensional curved space is replaced with the fiber degrees of freedom of the configuration space in which the metric is non-vanishing. The precise analogy with the finite-dimensional case suggests that if the curvature form of the configuration space spinor connection is nontrivial, it is impossible to diagonalize even the prepared maximally unentangled configuration space spinor fields Ψ_i in the entire fiber of the configuration space (quantum fluctuating degrees of freedom) for given values of the zero modes. Local singularities at which the spin quantum numbers of the diagonalized but vanishing configuration space spinor field become ill-defined are possible also now.

In the infinite-dimensional context the presence of the fermion-anti-fermion pairs in the state means that it does not represent a definite Boolean statement unless one defines a more general basis of configuration space spinors for which pairs are present in the states of the state basis: this generalization is indeed possible. The sigma matrices of the configuration space appearing in the spinor connection term of the Dirac operator of the configuration space indeed create fermion-fermion pairs. What is decisive, is not the absence of fermion-anti-fermion pairs, but the possibility that the spinor field basis cannot be reduced to eigen states of the local Cartan algebra in fiber degrees of freedom globally.

Also for bound states of fermions (say leptons and quarks) it is impossible to reduce the state to a definite Boolean statement even locally. This would suggest that fermionic logic does not reduce to a completely Boolean logic even in the case of the prepared states.

Thus configuration space spinor fields could have interpretation in terms of non-Boolean quantum logic possessing Boolean logics only as sub-logics and define what might be called quantum topos. Instead of Ω^N-valued maps the values for the maps are complex valued quantum superpositions of truth values in Ω^N.

An objection against the notion of quantum logic is that Boolean algebra operations AND and OR do not preserve fermion number so that quantum jump sequences leading from the product state defined by operands to the state representing the result of operation are therefore not possible. One manner to circumvent the objection is to consider the sub-algebra spanned by fermion and anti-fermion pairs for given mode so that fermion number conservation is not a problem. The objection can be also circumvented for pairs of space-time sheets with opposite time orientations and thus opposite signs of energies for particles. One can construct the algebra in question as pairs of many fermion states consisting of positive energy fermion and negative energy anti-fermion so that all states have vanishing fermion number and logical operations become possible. Pairs of MEs with opposite time orientations are excellent candidates for carries of these fermion-anti-fermion pairs.

5.3 Category theory and the modelling of aesthetic and ethical judgements

Consciousness theory should allow to model model the logics of ethics and aesthetics. Evolution (representable as p-adic evolution in TGD framework) is regarded as something positive and is a good candidate for defining universal ethics in TGD framework. Good deeds are such that they support this evolution occurring in statistical sense in any case. Moral provides a practical model for what good deeds are and moral right-wrong statements are analogous to logical statements. Often however the two-valued right-wrong logic seems to be too simplistic in case of moral statements. Same applies to aesthetic judgements. A possible application of the generalized logics defined by the inherent structure of categories relates to the understanding of the dilemmas associated with the moral and aesthetic rules.

As already found, quantum versions of sieves provide a formal generalization of Boolean truth values as a characteristic of a given category. Generalized moral rules could perhaps be seen as sieve valued statements about deeds. Deeds are either right or wrong in what might be called Boolean moral code. One can also consider Zen moral in which some deeds can be said to be right and wrong simultaneously. Some deeds could also be such that there simply exists no globally consistent moral rule: this would correspond to the nonexistence of what is called global section assigning to each object of the category consisting of the pairs formed by a moral agents and given deed) a sieve simultaneously.

6 Appendix: Category theory and construction of construction of S-matrix

The construction of configuration space geometry, spinor structure and of S-matrix involve difficult technical and conceptual problems and category theory might be of help here. As already found, the application of category theory to the construction of configuration space geometry allows to understand how the arrow of psychological time emerges.

The construction of the S-matrix involves several difficult conceptual and technical problems in which category theory might help. The incoming states of the theory are what might be called free states and are constructed as products of the configuration space spinor fields. One can effectively regard them as being defined in the Cartesian power of the configuration space divided by an appropriate permutation group. Interacting states in turn are defined in the configuration space.

Cartesian power of the configuration space of 3-surfaces is however in geometrical sense more or less identical with the configuration space since the disjoint union of N 3-surfaces is itself a 3-surface in configuration space. Actually it differs from configuration space itself only in that the 3-surfaces of many particle state can intersect each other and if one allows this, one has paradoxical self-referential identification $CH = \overline{CH^2}/S_2 = ... = \overline{CH^N}/S_N ...$, where over-line signifies that intersecting 3-surfaces have been dropped from the product.

Note that arbitrarily small deformation can remove the intersections between 3-surfaces and four-dimensional general coordinate invariance allows always to use non-intersecting representatives. In case of the spinor structure of the Cartesian power this identification means that the tensor powers SCH^N of the configuration space spinor structure are in some sense identical with the spinor structure SCH of the configuration space. Certainly the oscillator operators of the tensor factors must be assumed to be mutually anti-commuting.

The identities $CH = \overline{CH^2}/S_2 = ..$ and corresponding identities $SCH = SCH^2 = ...$ for the space SCH of configuration space spinor fields might imply very deep constraints on S-matrix. What comes into mind are counterparts for the Schwinger-Dyson equations of perturbative quantum field theory providing defining equations for the n-point functions of the theory [4]. The isomorphism between SCH^2 and SCH is actually what is needed to calculate the S-matrix elements. Category theory might help to understand at a general level what these self-referential and somewhat paradoxical looking identities really imply and perhaps even develop TGD counterparts of Schwinger-Dyson equations.

There is also the issue of bound states. The interacting states contain also bound states not belonging to the space of free states and category theory might help also here. It would seem that the state space must be constructed by taking into account also the bound states as additional 'free' states in the decomposition of states to product states.

A category naturally involved with the construction of the S-matrix (or U-matrix) is the space of the absolute minima $X^4(X^3)$ of the Kähler action which might be called interacting category. The canonical transformations acting as isometries of the configuration space geometry act naturally as the morphisms of this category. The group $Diff^4$ of general coordinate transformations in turn acts as gauge symmetries.

S-matrix relates free and interacting states and is induced by the classical interactions induced by the absolute minimization of Kähler action.

S-matrix elements are essentially Glebch-Gordan coefficients relating the states in the tensor power of the interacting supercanonical representation with the interacting supercanonical representation itself. More concretely, N-particle free states can be seen as configuration space spinor fields in CH^N obtained as tensor products of ordinary CH spinor fields. Free states correspond classically to the unions of space-time surfaces associated with the 3-surfaces representing incoming particles whereas interacting states correspond classically to the space-time surfaces associated with the unions of the 3-surfaces defining incoming states. These two states define what might be called free and interacting categories with canonical transformations acting as morphisms.

The classical interaction is represented by a functor $S : \overline{CH^N}/S_N \rightarrow CH$ mapping the classical free many particle states, that is objects of the product category defined by $\overline{CH^N}/S_N$ to the interacting category CH. This functor assigns to the union $\cup_i X^4(X_i^3)$ of the absolute minima $X^4(X_i^3)$ of Kähler action associated with the incoming, free states X_i^3 the absolute minimum $X^4(\cup X_i^3)$ associated with the union of three-surfaces representing the outgoing interacting state. At quantum level this functor maps the state space SCH^N associated with $\cup_i X^4(X_i^3)$ to SCH in a unitary manner. An important constraint on S-matrix is that it acts effectively as a flow in zero modes correlating the quantum numbers in fiber degrees of freedom in one-to-one manner with the values of zero modes so that quantum jump $U\Psi_i \rightarrow \Psi_0$... gives rise to a quantum measurement.

In [A5] category theory is applied to formulate generalized Feynman rules in terms of what might be called Hopf category. In this formulation morphisms between the configuration space spinor fields in different sectors of the configuration space correspond to algebra and co-algebra products. The cancellation of loops reduces to the requirement that any generalized Feynman diagram is equivalent with a tree diagram so that loops can be eliminated. The highly non-trivial outcome is that these requirements can be formulated in terms of Hopf algebra axioms. This approach is genuinely non-perturbative.

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