The physical interpretation of the K\"ahler function and   the TGD based space-time concept are the basic themes of this book. The aim is to develop what might be called \index{classical TGD}classical TGD at fundamental level.   The strategy is simple: try to guess the general physical consequences of the geometry of the \blockquote{world of classical worlds} (WCW) and of the TGD based gauge field concept and study the simplest extremals of K\"ahler action and try to abstract general truths from their properties.

The fundamental underlying assumptions are the following:

\begin{enumerate}

\item The notion of preferred extremals emerged during the period when I believed that positive energy ontology applies in TGD. In this framework the 4-surface associated with given 3-surface defined by K\"ahler function K as a preferred extremal of the K\"ahler action is identifiable as a classical space-time. Number theoretically preferred   extremals would decompose to associative and co-associative regions. \index{associative surface}\index{co-associative surface} The reduction of the classical theory to the level of the K\"ahler-Dirac action implies that the preferred extremals are critical in the sense of allowing infinite number of deformations for which the second variation of K\"ahler action vanishes \cite{Dirac}. It is not clear whether criticality and associativeity are consistent with each other. A further natural conjecture is that these critical deformations should act as conformal symmetries of light-like wormhole contacts at which the signature of the induced metric changes and preserve their light-likeness.

Due to the preferred extremal property classical space-time

\index{space-time surface as generalized Bohr orbit} can be also regarded
as a generalized Bohr orbit - at least in positive energy ontology - so that the quantization of the various
parameters associated with a typical extremal of the K\"ahler action is
expected to take place in general. In TGD quantum states corresponds to
quantum superpositions of these classical space-times so that this
classical space-time is certainly not some kind of effective quantum
average space-time.

\item In ZEO one can also consider the possibility that there is no selection of
preferred extremal at all! The two space-like 3-surfaces at the ends of CD
define the space-time surface connecting them apart from conformal
symmetries acting as critical deformations. If 3-surface is identified as
union of both space-like 3-surfaces and the light-like surfaces defining
parton orbits connecting then, the conformal equivalence class of the
preferred extremal is unique without any additional conditions! This
conforms with the view about hierarchy of Planck constants requiring that
the conformal equivalence classes of light-like surfaces must be counted as
physical degrees of freedom and also with the idea that these surface
together define analog for the Wilson loop. Actually all the discussions
of this chapter are about extremals in general so that the attribute \blockquote{preferred}
is not relevant for them.

\item The \index{vacuum functional as exponent of K\"ahler function}
bosonic vacuum functional of the theory is the exponent of the K\"ahler
function $\Omega_B = exp(K)$. This assumption is the only assumption about
the dynamics of the theory and is necessitated by the requirement of
\index{divergence cancellation} divergence cancellation in perturbative
approach.

\item \index{spin glass analogy}\index{Renormalization group
invariance}Renormalization group invariance and spin glass analogy. The
value of the K\"ahler coupling strength is such that the vacuum

functional
$exp(K)$ is analogous to the exponent $exp(H/T)$ defining the partition
function of a statistical system at critical temperature. This allows
K\"ahler coupling strength to depend on zero modes of the configuration
space metric and as already found there is very attractive hypothesis
determining completely the dependence of the K\"ahler coupling strength on
the zero modes based on p-adic considerations motivated by the spin glass
analogy. Coupling constant evolution would be replaced by effective discrete
evolution with respect to p-adic length scale and angle variable defined by
the phases appearing in the algebraic extension of p-adic numbers in question.

\item In spin degrees of freedom the massless Dirac equation for the
induced spinor fields with K\"ahler-Dirac action defines classical theory:
this is in complete accordance with the proposed definition of the
WCW spinor structure.

\end{enumerate}

The geometrization of the classical gauge fields in terms of the induced
gauge field concept is also important concerning the physical
interpretation. Electro-weak gauge potentials correspond to the space-time
projections of the spinor connection of CP_2, gluonic gauge potentials
to the projections of the Killing vector fields of CP_2 and gravitational
field to the induced metric. The topics to be discussed in this part of the
book are summarized briefly in the following.

What the selection of preferred extremals of K\"ahler action might mean

has remained a long standing problem and real progress occurred only quite
recently (I am writing this towards the end of year 2003).

\begin{enumerate}\item The \index{vanishing of Lorentz-K\"ahler 4-force}
vanishing of Lorentz 4-force for the induced K\"ahler field means that the
vacuum 4-currents are in a mechanical equilibrium. Lorentz 4-force vanishes
for all known solutions of field equations which inspires the hypothesis
that all preferred extremals of K\"ahler action satisfy the condition. The
vanishing of the Lorentz 4-force in turn implies local conservation of the
ordinary energy momentum tensor. The corresponding condition is implied by
Einstein's equations in General Relativity. The hypothesis would mean that
the solutions of field equations are what might be called generalized
Beltrami fields. The condition implies that vacuum currents can be
non-vanishing only provided the dimension D_{CP_2} of the CP_2
projection of the space-time surface is less than four so that in the
regions with $D_{CP_2}=4$, Maxwell's vacuum equations are satisfied.

\item The hypothesis that \index{K\"ahler current} K\"ahler current is
proportional to a product of an arbitrary function ψ of CP_2
coordinates and of the \index{instanton current}instanton current
generalizes Beltrami condition and reduces to it when electric field
vanishes. Instanton current has a vanishing divergence for $D_{CP_2}<4$,
and Lorentz 4-force indeed vanishes. Four 4-dimensional projection the
scalar function multiplying the instanton current can make it
divergenceless. The remaining task would be the explicit construction of
the imbeddings of these fields and the demonstration that field equations
can be satisfied.

\item By \index{quantum classical correspondence}quantum classical
correspondence the non-deterministic space-time dynamics should mimic the
dissipative dynamics of the quantum jump sequence. \index{Beltrami

fields}
Beltrami fields appear in physical applications as asymptotic self organization patterns for which Lorentz force and dissipation vanish. This suggests that preferred extemals of K\"ahler action correspond to space-time sheets which at least asymptotically satisfy the generalized Beltrami conditions so that one can indeed assign to the final 3-surface a unique 4-surface apart from effects related to non-determinism. Preferred extremal property abstracted to purely algebraic generalized Beltrami conditions makes sense also in the p-adic context.

\end{enumerate}

This chapter is mainly devoted to the study of the basic extremals of the K\"ahler action besides the detailed arguments supporting the view that the preferred extrema satisfy \index{generalized Beltrami conditions}generalized Beltrami conditions at least asymptotically.

The newest results discussed in the last section about the weak form of electric-magnetic duality suggest strongly that Beltrami property is general and together with the weak form of electric-magnetic duality allows a reduction of quantum TGD to almost topological field theory with K\"ahler function allowing expression as a Chern-Simons term.

The surprising implication of the duality is that K\"ahler form of CP_2 must be replaced with that for $S^2\times CP_2$ in order to obtain a WCW metric which is non-trivial in M^4 degrees of freedom. This modification implies much richer vacuum structure than the original K\"ahler action which is a good news as far as the description of classical gravitational fields in terms of small deformations of vacuum extremals with the four-momentum density of the topologically condensed matter given by Einstein's equations is considered. The breaking of Lorentz invariace from $SO(3,1)$ to $SO(3)$ is implied already by the geometry of CD but is extremely small for a given causal diamond (CD). Since a wave function over the Lorentz boosts and translates of CD is allowed, there is no actual breaking of Poincare invariance at the level of the basic theory. Beltrami property leads to a rather explicit construction of the general solution of field equations based on the hydrodynamic picture implying that single particle quantum numbers are conserved along flow lines defined by the instanton current. The construction generalizes also to the fermionic sector.