Hiroyasu Koizumi has proposed a new theory of superconductivity (SC) based on the notion of Berry phase related with an effective magnetic field assignable to adiabatically evolving systems. The model shares similarities with the TGD inspired view about SC. The article also mentioned   anomalies that were new to me. This motivated a fresh look in the TGD inspired model. The outcome was an integration of two separate ideas about supraphases.

\begin{enumerate}

\item Space-time surfaces as preferred extremals with CP_2 projection of dimension $D=2$ or $D=3$ would naturally correspond to 4-D generalizations of so called Beltrami flows, which are integrable flows defined by the flow lines of the induced K\"ahler field. The existence of a global coordinate z varying along flow lines requires the integrability of the flow. Classical dissipation is absent so that these surfaces are excellent candidates for the space-time correlates of supra flows. The exponential of z gives a phase factor associated with the complex order parameter of a coherent state of Cooper pairs as a counterpart of the Berry phase. K\"ahler magnetic monopole flux defines the TGD counterpart of "novel" magnetic field.

\item The identification of supra phases as dark matter as $h_{eff}>h$ phases at magnetic flux quanta (tubes and sheets)   implies that Cooper pairs correspond to dark fermions associated with the members of flux tube pair, which actually combine to form a closed flux tube.   Also single electrons can define supraflow.

\item The Cooper pairs must be created by bosonic oscillator operators constructed from fermionic oscillator operators by bosonization. This is possible only in 1+1-dimensional situations. Thanks to the Beltrami flow the situation is effectively 1+1-dimensional.   Bosonization makes it possible to identify SU(2) Kac-Moody algebra, which has   an interpretation in the TGD framework.

\end{enumerate}

The assumption that Cooper pairs reside at the magnetic flux quanta solves the 4 problems of standard framework mentioned by Koizumi: high-Tc SCs have two transition temperatures; electron mass m_e instead of its effective mass m_e^* appears in Thomson moment; the reversible phase transition in an external magnetic field inducing a splitting of Cooper pairs does not involve dissipation; why the erratic calculation of the Josephson frequencies in standard model neglecting the chemical potentials gives a correct result?.

The formation of the Cooper pairs appears as a condition stabilizing the space-time sheets carrying dark matter and all preferred extremals could satisfy the conditions guaranteeing integrable flow and existence of a phase factor varying along flow lines. Could supra phases exist in all scales? Could the breaking of supra phases be only due to the finite size of the space-time sheets? Could even hydrodynamic flow involve super-fluidity of some kind - perhaps based on neutrino Cooper pairs as speculated earlier?