TOPOLOGICAL GEOMETRODYNAMICS: AN OVERVIEW: PART I

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0.1 PREFACE

This book belongs to a series of online books summarizing the recent state Topological Geometrodynamics (TGD) and its applications. TGD can be regarded as a unified theory of fundamental interactions but is not the kind of unified theory as so called GUTs constructed by graduate students at seventies and eighties using detailed recipes for how to reduce everything to group theory. Nowadays this activity has been completely computerized and it probably takes only a few hours to print out the predictions of this kind of unified theory as an article in the desired format. TGD is something different and I am not ashamed to confess that I have devoted the last 37 years of my life to this enterprise and am still unable to write The Rules.

If I remember correctly, I got the basic idea of Topological Geometrodynamics (TGD) during autumn 1977, perhaps it was October. What I realized was that the representability of physical space-times as 4-dimensional surfaces of some higher-dimensional space-time obtained by replacing the points of Minkowski space with some very small compact internal space could resolve the conceptual difficulties of general relativity related to the definition of the notion of energy. This belief was too optimistic and only with the advent of what I call zero energy ontology the understanding of the notion of Poincare invariance has become satisfactory. This required also the understanding of the relationship to General Relativity.

It soon became clear that the approach leads to a generalization of the notion of space-time with particles being represented by space-time surfaces with finite size so that TGD could be also seen as a generalization of the string model. Much later it became clear that this generalization is consistent with conformal invariance only if space-time is 4-dimensional and the Minkowski space factor of imbedding space is 4-dimensional. During last year it became clear that 4-D Minkowski space and 4-D complex projective space CP_2 are completely unique in the sense that they allow twistor space with Kähler structure.

It took some time to discover that also the geometrization of also gauge interactions and elementary particle quantum numbers could be possible in this framework: it took two years to find the unique internal space (CP_2) providing this geometrization involving also the realization that family replication phenomenon for fermions has a natural topological explanation in TGD framework and that the symmetries of the standard model symmetries are much more profound than pragmatic TOE builders have believed them to be. If TGD is correct, main stream particle physics chose the wrong track leading to the recent deep crisis when people decided that quarks and leptons belong to same multiplet of the gauge group implying instability of proton.

There have been also longstanding problems.

- Gravitational energy is well-defined in cosmological models but is not conserved. Hence the conservation of the inertial energy does not seem to be consistent with the Equivalence Principle. Furthermore, the imbeddings of Robertson-Walker cosmologies turned out to be vacuum extremals with respect to the inertial energy. About 25 years was needed to realize that the sign of the inertial energy can be also negative and in cosmological scales the density of inertial energy vanishes: physically acceptable universes are creatable from vacuum. Eventually this led to the notion of zero energy ontology (ZEO) which deviates dramatically from the standard ontology being however consistent with the crossing symmetry of quantum field theories. In this framework the quantum numbers are assigned with zero energy states located at the boundaries of so called causal diamonds defined as intersections of future and past directed light-cones. The notion of energy-momentum becomes length scale dependent since one has a scale hierarchy for causal diamonds. This allows to understand the non-conservation of energy as apparent.

  Equivalence Principle as it is expressed by Einstein's equations follows from Poincare invariance once it is realized that GRT space-time is obtained from the many-sheeted space-time of TGD by lumping together the space-time sheets to a regionof Minkowski space and endowing it with an effective metric given as a sum of Minkowski metric and deviations of the metrices of space-time sheets from Minkowski metric. Similar description relates classical gauge potentials identified as components of induced spinor connection to Yang-Mills gauge potentials in GRT space-time. Various topological inhomogenities below resolution scale identified as particles are described using energy momentum tensor and gauge currents.

- From the beginning it was clear that the theory predicts the presence of long ranged classical electro-weak and color gauge fields and that these fields necessarily accompany classical electromagnetic fields.

  It took about 26 years to gain the maturity to admit the obvious: these fields are classical correlates for long range color and weak interactions assignable to dark matter. The only possible conclusion is that TGD physics is a fractal consisting of an entire hierarchy of fractal copies of standard model physics. Also the understanding of electro-weak massivation and screening of weak charges has been a long standing problem, and 32 years was needed to discover that what I call weak form of electric-magnetic duality gives a satisfactory solution of the problem and provides also surprisingly powerful insights to the mathematical structure of quantum TGD.

  The latest development was the realization that the well- definedness of electromagnetic charge as quantum number for the modes of the induced spinors field requires that the CP_2 projection of the region in which they are non-vanishing carries vanishing W boson field and is 2-D. This implies in the generic case their localization to 2-D surfaces: string world sheets and possibly also partonic 2-surfaces. This localization applies to all modes except covariantly constant right handed neutrino generating supersymmetry and mplies that string model in 4-D space-time is part of TGD. Localization is possible only for Kähler-Dirac assigned with Kähler action defining the dynamics of space-time surfaces. One must however leave open the question whether W field might vanish for the space-time of GRT if related to many-sheeted space-time in the proposed manner even when they do not vanish for space-time sheets.

I started the serious attempts to construct quantum TGD after my thesis around 1982. The original optimistic hope was that path integral formalism or canonical quantization might be enough to construct the quantum theory but the first discovery made already during first year of TGD was that these formalisms might be useless due to the extreme non-linearity and enormous vacuum degeneracy of the theory. This turned out to be the case.

- It took some years to discover that the only working approach is based on the generalization of Einstein's program. Quantum physics involves the geometrization of the infinite-dimensional "world of classical worlds" (WCW) identified as 3-dimensional surfaces. Still few years had to pass before I understood that general coordinate invariance leads to a more or less unique solution of the problem and in positive energyontology implies that space-time surfaces are analogous to Bohr orbits. This in positive energy ontology in which space-like 3-surface is basic object. It is not clear whether Bohr orbitology is necessary also in ZEO in which space-time surfaces connect space-like 3-surfaces at the light-like boundaries of causal diamond CD obtained as intersection of future and past directed light-cones (with CP_2 factor included). The reason is that the pair of 3-surfaces replaces the boundary conditions at single 3-surface involving also time derivatives. If one assumes Bohr orbitology then strong correlations between the 3-surfaces at the ends of CD follow. Still a couple of years and I discovered that quantum states of the Universe can be identified as classical spinor fields in WCW. Only quantum jump remains the genuinely quantal aspect of quantum physics.

- During these years TGD led to a rather profound generalization of the space-time concept. Quite general properties of the theory led to the notion of many-sheeted space-time with sheets representing physical subsystems of various sizes. At the beginning of 90s I became dimly aware of the importance of p-adic number fields and soon ended up with the idea that p-adic thermodynamics for a conformally invariant system allows to understand elementary particle massivation with amazingly few input assumptions. The attempts to understand p-adicity from basic principles led gradually to the vision about physics as a generalized number theory as an approach complementary to the physics as an infinite-dimensional spinor geometry of WCW approach. One of its elements was a generalization of the number concept obtained by fusing real numbers and various p-adic numbers along common rationals. The number theoretical trinity involves besides p-adic number fields also quaternions and octonions and the notion of infinite prime.

- TGD inspired theory of consciousness entered the scheme after 1995 as I started to write a book about consciousness. Gradually it became difficult to say where physics ends and consciousness theory begins since consciousness theory could be seen as a generalization of quantum measurement theory by identifying quantum jump as a moment of consciousness and by replacing the observer with the notion of self identified as a system which is conscious as long as it can avoid entanglement with environment. The somewhat cryptic statement "Everything is conscious and consciousness can be only lost" summarizes the basic philosophy neatly.

The idea about p-adic physics as physics of cognition and intentionality emerged also rather naturally and implies perhaps the most dramatic generalization of the space-time concept in which most points of p-adic space-time sheets are infinite in real sense and the projection to the real imbedding space consists of discrete set of points. One of the most fascinating outcomes was the observation that the entropy based on p-adic norm can be negative. This observation led to the vision that life can be regarded as something in the intersection of real and p-adic worlds. Negentropic entanglement has interpretation as a correlate for various positively colored aspects of conscious experience and means also the possibility of strongly correlated states stable under state function reduction and different from the conventional bound states and perhaps playing key role in the energy metabolism of living matter.

If one requires consistency of Negentropy Mazimization Pronciple with standard measurement theory, negentropic entanglement defined in terms of number theoretic negentropy is necessarily associated with a density matrix proportional to unit matrix and is maximal and is characterized by the dimension n of the unit matrix. Negentropy is positive and maximal for a p-adic unique prime dividing n.

- One of the latest threads in the evolution of ideas is not more than nine years old. Learning about the paper of Laurent Nottale about the possibility to identify planetary orbits as Bohr orbits with a gigantic value of gravitational Planck constant made once again possible to see the obvious. Dynamical quantized Planck constant is strongly suggested by quantum classical correspondence and the fact that space-time sheets identifiable as quantum coherence regions can have arbitrarily large sizes. Second motivation for the hierarchy of Planck constants comes from bio-electromagnetism suggesting that in living systems Planck constant could have large values making macroscopic quantum coherence possible. The interpretation of dark matter as a hierarchy of phases of ordinary matter characterized by the value of Planck constant is very natural.

  During summer 2010 several new insights about the mathematical structure and interpretation of TGD emerged. One of these insights was the realization that the postulated hierarchy of Planck constants might follow from the basic structure of quantum TGD. The point is that due to the extreme non-linearity of the classical action principle the correspondence between canonical momentum densities and time derivatives of the imbedding space coordinates is one-to-many and the natural description of the situation is in terms of local singular covering spaces of the imbedding space. One could speak about effective value of Planck constant $h_{eff} = n \times h$ coming as a multiple of minimal value of Planck constant. Quite recently it became clear that the non-determinism of Kähler action is indeed the fundamental justification for the hierarchy: the integer n can be also interpreted as the integer characterizing the dimension of unit matrix characterizing negentropic entanglement made possible by the many-sheeted character of the space-time surface.

  Due to conformal invariance acting as gauge symmetry the n degenerate space-time sheets must be replaced with conformal equivalence classes of space-time sheets and conformal transformations correspond to quantum critical deformations leaving the ends of space-time surfaces invariant. Conformal invariance would be broken: only the sub-algebra for which conformal weights are divisible by n act as gauge symmetries. Thus deep connections between conformal invariance related to quantum criticality, hierarchy of Planck constants, negentropic entanglement, effective p-adic topology, and non-determinism of Kähler action perhaps reflecting p-adic non-determinism emerges.

  The implications of the hierarchy of Planck constants are extremely far reaching so that the significance of the reduction of this hierarchy to the basic mathematical structure distinguishing between TGD and competing theories cannot be under-estimated.

From the point of view of particle physics the ultimate goal is of course a practical construction recipe for the S-matrix of the theory. I have myself regarded this dream as quite too ambitious taking into account how far reaching re-structuring and generalization of the basic mathematical structure of quantum physics is required. It has indeed turned out that the dream about explicit formula is unrealistic before one has understood what happens in quantum jump. Symmetries and general physical principles have turned out to be the proper guide line here. To give some impressions about what is required some highlights are in order.

- With the emergence of ZEO the notion of S-matrix was replaced with M-matrix defined between positive and negative energy parts of zero energy states. M-matrix can be interpreted as a complex square root of density matrix representable as a diagonal and positive square root of density matrix and unitary S-matrix so that quantum theory in ZEO can be said to define a square root of thermodynamics at least formally. M-matrices in turn bombine to form the rows of unitary U-matrix defined between zero energy states.

- A decisive step was the strengthening of the General Coordinate Invariance to the requirement that the formulations of the theory in terms of light-like 3-surfaces identified as 3-surfaces at which the induced metric of space-time surfaces changes its signature and in terms of space-like 3-surfaces are equivalent. This means effective 2-dimensionality in the sense that partonic 2-surfaces defined as intersections of these two kinds of surfaces plus 4-D tangent space data at partonic 2-surfaces code for the physics. Quantum classical correspondence requires the coding of the quantum numbers characterizing quantum states assigned to the partonic 2-surfaces to the geometry of space-time surface. This is achieved by adding to the modified Dirac action a measurement interaction term assigned with light-like 3-surfaces.

- The replacement of strings with light-like 3-surfaces equivalent to space-like 3-surfaces means enormous generalization of the super conformal symmetries of string models. A further generalization of these symmetries to non-local Yangian symmetries generalizing the recently discovered Yangian symmetry of $\mathcal{N} = 4$ supersymmetric Yang-Mills theories is highly suggestive. Here the replacement of point like particles with partonic 2-surfaces means the replacement of conformal symmetry of Minkowski space with infinite-dimensional super-conformal algebras. Yangian symmetry provides also a further refinement to the notion of conserved quantum numbers allowing to define them for bound states using non-local energy conserved currents.

- A further attractive idea is that quantum TGD reduces to almost topological quantum field theory. This is possible if the Kähler action for the preferred extremals defining WCW Kähler function reduces to a 3-D boundary term. This takes place if the conserved currents are so called Beltrami fields with the defining property that the coordinates associated with flow lines extend to single global coordinate variable. This ansatz together with the weak form of electric-magnetic duality reduces the Kähler action to Chern-Simons term with the condition that the 3-surfaces are extremals of Chern-Simons action subject to the constraint force defined by the weak form of electric magnetic duality. It is the latter constraint which prevents the trivialization of the theory to a topological quantum field theory. Also the identification of the Kähler function of WCW as Dirac determinant finds support as well as the description of the scattering amplitudes in terms of braids with interpretation in terms of finite measurement resolution coded to the basic structure of the solutions of field equations.

- In standard QFT Feynman diagrams provide the description of scattering amplitudes. The beauty of Feynman diagrams is that they realize unitarity automatically via the so called Cutkosky rules. In contrast to Feynman's original beliefs, Feynman diagrams and virtual particles are taken only as a convenient mathematical tool in quantum field theories. QFT approach is however plagued by UV and IR divergences and one must keep mind open for the possibility that a genuine progress might mean opening of the black box of the virtual particle.

  In TGD framework this generalization of Feynman diagrams indeed emerges unavoidably. Light-like 3-surfaces replace the lines of Feynman diagrams and vertices are replaced by 2-D partonic 2-surfaces. The approximate localization of the nodes of induced spinor fields to 2-D string world sheets (and possibly also to partonic 2-surfaces) implies a stringy formulation of the theory analogous to stringy variant of twistor formalism with string world sheets having interpretation as 2-braids. In TGD framework fermionic variant of twistor Grassmann formalism leads to a stringy variant of twistor diagrammatics in which basic fermions can be said to be on mass-shell but carry non-physical helicities in the internal lines. This suggests the generalization of the Yangian symmetry to infinite-dimensional super-conformal algebras.

What I have said above is strongly biased view about the recent situation in quantum TGD. This vision is single man's view and doomed to contain unrealistic elements as I know from experience. My dream is that young critical readers could take this vision seriously enough to try to demonstrate that some of its basic premises are wrong or to develop an alternative based on these or better premises. I must be however honest and tell that 32 years of TGD is a really vast bundle of thoughts and quite a challenge for anyone who is not able to cheat himself by taking the attitude of a blind believer or a light-hearted debunker trusting on the power of easy rhetoric tricks.

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Neither TGD nor these books would exist without the help and encouragement of many people. The friendship with Heikki and Raija Haila and their family and Kalevi and Ritva Tikkanen and their family have been kept me in contact with the everyday world and without this friendship I would not have survived through these lonely 45 lonely years most of which I have remained unemployed as a scientific dissident. I am happy that my children have understood my difficult position and like my friends have believed that what I am doing is something valuable although I have not received any official recognition for it.

During the last decade Tapio Tammi has helped me quite concretely by providing the necessary computer facilities and being one of the few persons in Finland with whom to discuss my work. Pertti Kärkkäinen is my old physicist friend and has provided continued economic support for a long time. I have also had stimulating discussions with Samuli Penttinen who has also helped to get through the economical situations in which there seemed to be no hope. The continual updating of fifteen online books means quite a heavy bureaucracy at the level of bits and without a systemization one ends up with endless copying and pasting and internal consistency is soon lost. Tommi Ullgren has provided both economic support and encouragement during years. Pekka Rapinoja has offered his help in this respect and I am especially grateful to him for my Python skills.

During the last five years I have had inspiring discussions with many people in Finland interested in TGD. We have had video discussions with Sini Kunnas and had podcast discussions with Marko Manninen related to the TGD based view of physics and consciousness. Marko has also helped in the practical issues related to computers and quite recently he has done a lot of testing of chatGPT helping me to get an overall view of what it is. The discussions in a Zoom group involving Marko Manninen, Tuomas Sorakivi and Rode Majakka have given me the valuable opportunity to clarify my thoughts.

The collaboration with Lian Sidorov was extremely fruitful and she also helped me to survive economically through the hardest years. The participation in CASYS conferences in Liege has been an important window to the academic world and I am grateful for Daniel Dubois and Peter Marcer for making this participation possible. The discussions and collaboration with Eduardo de Luna and Istvan Dienes stimulated the hope that the communication of new vision might not be a mission impossible after all. Also blog discussions have been very useful. During these years I have received innumerable email contacts from people around the world. I am grateful to Mark McWilliams, Paul Kirsch, Gary Ehlenberg, and Ulla Matfolk and many others for providing links to possibly interesting websites and articles. We have collaborated with Peter Gariaev and Reza Rastmanesh. These contacts have helped me to avoid the depressive feeling of being some kind of Don Quixote of Science and helped me to widen my views: I am grateful for all these people.

In the situation in which the conventional scientific communication channels are strictly closed it is important to have some loop hole through which the information about the work done can at least in principle leak to the public through the iron wall of academic censorship. Without any exaggeration I can say that without the world wide web I would not have survived as a scientist nor as an individual. Homepage and blog are however not enough since only the formally published result is a result in recent day science. Publishing is however impossible without direct support from power holders- even in archives like arXiv.org.

Situation changed as Andrew Adamatsky proposed the writing of a book about TGD when I had already gotten used to the thought that my work would not be published during my lifetime. The Prespacetime Journal and two other journals related to quantum biology and consciousness - all of them founded by Huping Hu - have provided this kind of loophole. In particular, Dainis Zeps, Phil Gibbs, and Arkadiusz Jadczyk deserve my gratitude for their kind help in the preparation of an article series about TGD catalyzing a considerable progress in the understanding of quantum TGD. Also the viXra archive founded by Phil Gibbs and its predecessor Archive Freedom have been of great help: Victor Christianto deserves special thanks for doing the hard work needed to run Archive Freedom. Also the Neuroquantology Journal founded by Sultan Tarlaci deserves a special mention for its publication policy.

And last but not least: there are people who experience as a fascinating intellectual challenge to spoil the practical working conditions of a person working with something which might be called unified theory: I am grateful for the people who have helped me to survive through the virus attacks, an activity which has taken roughly one month per year during the last half decade and given a strong hue of grey to my hair.

For a person approaching his 73th birthday it is somewhat easier to overcome the hard feelings due to the loss of academic human rights than for an inpatient youngster. Unfortunately the economic situation has become increasingly difficult during the twenty years after the economic depression in Finland which in practice meant that Finland ceased to be a constitutional state in the strong sense of the word. It became possible to depose people like me from society without fear about public reactions and the classification as dropout became a convenient tool of ridicule to circumvent the ethical issues. During the period when the right wing held political power this trend was steadily strengthening and the situation is the same as I am writing this. In this kind of situation the concrete help from individuals has been and will be of utmost importance. Against this background it becomes obvious that this kind of work is not possible without the support from outside and I apologize for not being able to mention all the people who have helped me during these years.

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Matti Pitkänen

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Chapter 1

Introduction

1.1 Basic Ideas of Topological Geometrodynamics (TGD)

Standard model describes rather successfully both electroweak and strong interactions but sees them as totally separate and contains a large number of parameters which it is not able to predict. For about four decades ago unified theories known as Grand Unified Theories (GUTs) trying to understand electroweak interactions and strong interactions as aspects of the same fundamental gauge interaction assignable to a larger symmetry group emerged. Later superstring models trying to unify even gravitation and strong and weak interactions emerged. The shortcomings of both GUTs and superstring models are now well-known. If TGD - whose basic idea emerged towards the end of 1977 - would emerge now it would be seen as an attempt to solve the difficulties of these approaches to unification.

The basic physical picture behind the geometric vision of TGD corresponds to a fusion of two rather disparate approaches: namely TGD as a Poincare invariant theory of gravitation and TGD as a generalization of the old-fashioned string model. After 1995 number theoretic vision started to develop and was initiated by the success of mass calculations based on p-adic thermodynamics. Number theoretic vision involves all number fields and is complementary to the geometric vision: one can say that this duality is analogous to momentum-position duality of wave mechanics. TGD can be also regarded as topological quantum theory in a very general sense as already the attribute "Topological" in "TGD" makes clear. Space-time surfaces as minimal surfaces can be regarded as representatives of homology equivalence classes and p-adic topologies generalize the notion of local topology and apply to the description of correlates of cognition.

1.1.1 Geometric Vision Very Briefly

T(opological) G(eometro)D(ynamics) is one of the many attempts to find a unified description of basic interactions. The development of the basic ideas of TGD to a relatively stable form took time of about half decade [K2].

The basic vision and its relationship to existing theories is now rather well understood.

1. Space-times are representable as 4-surfaces in the 8-dimensional embedding space $H = M^4 \times CP_2$, where M^4 is 4-dimensional (4-D) Minkowski space and CP_2 is 4-D complex projective space (see Appendix).

2. Induction procedure (a standard procedure in fiber bundle theory, see Appendix) allows to geometrize various fields. Space-time metric characterizing gravitational fields corresponds to the induced metric obtained by projecting the metric tensor of H to the space-time surface. Electroweak gauge potentials are identified as projections of the components of CP_2 spinor connection to the space-time surface, and color gauge potentials as projections of CP_2 Killing vector fields representing color symmetries. Also spinor structure can be induced: induced spinor gamma matrices are projections of gamma matrices of H and induced spinor fields just H spinor fields restricted to space-time surface. Spinor connection is also projected. The interpretation is that distances are measured in embedding space metric and parallel translation using spinor connection of embedding space.

Twistor lift of TGD means that one can lift space-time surfaces in H to 6-D surfaces a analogs of twistor space of space-time surface in the Cartesian product of the twistor spaces of M^4 and CP_2, which are the only 4-manifolds allowing twistor space with Kähler structure [A61]. The twistor structure would be induced in some sense, and should coincide with that associated with the induced metric. Clearly, the 2-spheres defining the fibers of twistor spaces of M^4 and CP_2 must allow identification: this 2-sphere defines the S^2 fiber of the twistor space of the space-time surface. This poses a constraint on the embedding of the twistor space of space-time surfaces as sub-manifold in the Cartesian product of twistor spaces. The existence of Kähler structure allows to lift 4-D Kähler action to its 6-D counterparts and the 6-D counterpart of twistor space is obtained by its dimensional reduction so that one obtains a sphere bundle. This makes possible twistorialization for all space-time surfaces: in general relativity the general metric does not allow this.

3. A geometrization of quantum numbers is achieved. The isometry group of the geometry of CP_2 codes for the color gauge symmetries of strong interactions. Vierbein group codes for electroweak symmetries, and explains their breaking in terms of CP_2 geometry so that standard model gauge group results. There are also important deviations from the standard model: color quantum numbers are not spin-like but analogous to orbital angular momentum: this difference is expected to be seen only in CP_2 scale. In contrast to GUTs, quark and lepton numbers are separately conserved and family replication has a topological explanation in terms of topology of the partonic 2-surface carrying fermionic quantum numbers.

   M^4 and CP_2 are unique choices for many other reasons. For instance, they are the unique 4-D space-times allowing twistor space with Kähler structure. M^4 light-cone boundary allows a huge extension of 2-D conformal symmetries. M^4 and CP_2 allow quaternionic structures. Therefore standard model symmetries have number theoretic meaning.

4. Induced gauge potentials are expressible in terms of embedding space coordinates and their gradients and general coordinate invariance implies that there are only 4 field-like variables locally. Situation is thus extremely simple mathematically. The objection is that one loses linear superposition of fields. The resolution of the problem comes from the generalization of the concepts of particle and space-time.

   Space-time surfaces can be also particle like having thus finite size. In particular, space-time regions with Euclidian signature of the induced metric (temporal and spatial dimensions in the same role) emerge and have interpretation as lines of generalized Feynman diagrams. Particles in space-time can be identified as a topological inhomogeneities in background space-time surface which looks like the space-time of general relativity in long length scales.

   One ends up with a generalization of space-time surface to many-sheeted space-time with space-time sheets having extremely small distances of about 10^4 Planck lengths (CP_2 size). As one adds a particle to this kind of structure, it touches various space-time sheets and thus interacts with the associated classical fields. Their effects superpose linearly in good approximation and linear superposition of fields is replaced with that for their effects.

   This resolves the basic objection. It also leads to the understanding of how the space-time of general relativity and quantum field theories emerges from TGD space-time as effective space-time when the sheets of many-sheeted space-time are lumped together to form a region of Minkowski space with metric replaced with a metric identified as the sum of empty Minkowski metric and deviations of the metrics of sheets from empty Minkowski metric. Gauge potentials are identified as sums of the induced gauge potentials. TGD is therefore a microscopic theory from which the standard model and general relativity follow as a topological simplification, however forcing a dramatic increase of the number of fundamental field variables.

5. A further objection is that classical weak fields identified as induced gauge fields are long ranged and should cause large parity breaking effects due to weak interactions. These effects are indeed observed but only in living matter. The basic problem is that one has long ranged classical electroweak gauge fields. The resolution of the problem is that the quantum averages of induced weak and color gauge fields vanish due to the fact that color rotations affect both space-time surfaces and induced weak and color fields. Only the averages of

electromagnetic fields are nonvanishing. The correlations functions for weak fields are non-vanishing below Compton lengths of weak bosons. In living matter large values of effective Planck constant labelling phases of ordinary matter identified as dark matter make possible long ranged weak fields and color fields.

6. General coordinate invariance requires holography so that space-time surfaces are analogous to Bohr orbits for particles identified as 3-surfaces. Bohr orbit property would be naturally realized by a 4-D generalization of holomorphy of string world sheets and implies that the space-time surfaces are minimal surfaces apart from singularities. This holds true for any action as long as it is general coordinate invariant and constructible in terms of the induced geometry. String world sheets and light-like orbits of partonic 2-surfaces correspond to singularities at which the minimal surface property of the space-time surfaces realizing the preferred extremal property fails. Preferred extremals are not completely deterministic, which implies what I call zero energy ontology (ZEO) meaning that the Bohr orbits are the fundamental objects. This leads to a solution of the basic paradox of quantum measurement theory. Also the mathematically ill-defined path integral disappears and leaves only the well-defined functional integral over the Bohr orbits.

7. A string model-like picture emerges from TGD and one ends up with a rather concrete view about the topological counterpart of Feynman diagrammatics. The natural stringy action would be given by the string world sheet area, which is present only in the space-time regions with Minkowskian signature. Gravitational constant could be present as a fundamental constant in string action and the ratio $\hbar/G/R^2$ would be determined by quantum criticality conditions. The hierarchy of Planck constants $h_{eff}/h = n$ assigned to dark matter in TGD framework would allow to circumvent the objection that only objects of length of order Planck length are possible since string tension given by $T = 1/\hbar_{eff}G$ apart from numerical factor could be arbitrary small. This would make possible gravitational bound states as partonic 2-surfaces as structures connected by strings and solve the basic problem of superstring theories. This option allows the natural interpretation of M^4 type vacuum extremals with CP_2 projection, which is Lagrange manifold as good approximations for space-time sheets at macroscopic length scales. String area does not contribute to the Kähler function at all.

   Whether induced spinor fields associated with Kähler-Dirac action and de-localized inside the entire space-time surface should be allowed remains an open question: super-conformal symmetry strongly suggests their presence. A possible interpretation for the corresponding spinor modes could be in terms of dark matter, sparticles, and hierarchy of Planck constants.

It is perhaps useful to make clear what TGD is not and also what new TGD can give to physics.

1. TGD is *not* just General Relativity made concrete by using embeddings: the 4-surface property is absolutely essential for unifying standard model physics with gravitation and to circumvent the incurable conceptual problems of General Relativity. The many-sheeted space-time of TGD gives rise only at the macroscopic limit to GRT space-time as a slightly curved Minkowski space. TGD is *not* a Kaluza-Klein theory although color gauge potentials are analogous to gauge potentials in these theories.

   TGD space-time is 4-D and its dimension is due to completely unique conformal properties of light-cone boundary and 3-D light-like surfaces implying enormous extension of the ordinary conformal symmetries. Light-like 3-surfaces represent orbits of partonic 2-surfaces and carry fundamental fermions at 1-D boundaries of string world sheets. TGD is *not* obtained by performing Poincare gauging of space-time to introduce gravitation and is plagued by profound conceptual problems.

2. TGD is *not* a particular string model although string world sheets emerge in TGD very naturally as loci for spinor modes: their 2-dimensionality makes among other things possible quantum deformation of quantization known to be physically realized in condensed matter, and conjectured in TGD framework to be crucial for understanding the notion of finite measurement resolution. Hierarchy of objects of dimension up to 4 emerge from TGD: this obviously means analogy with branes of super-string models.

TGD is *not* one more item in the collection of string models of quantum gravitation relying on Planck length mystics. Dark matter becomes an essential element of quantum gravitation and quantum coherence in astrophysical scales is predicted just from the assumption that strings connecting partonic 2-surfaces are responsible for gravitational bound states.

TGD is *not* a particular string model although AdS/CFT duality of super-string models generalizes due to the huge extension of conformal symmetries and by the identification of WCW gamma matrices as Noether super-charges of super-symplectic algebra having a natural conformal structure.

3. TGD is *not* a gauge theory. In TGD framework the counterparts of also ordinary gauge symmetries are assigned to super-symplectic algebra (and its Yangian [A27] [B25, B17, B18]), which is a generalization of Kac-Moody algebras rather than gauge algebra and suffers a fractal hierarchy of symmetry breakings defining hierarchy of criticalities. TGD is *not* one more quantum field theory like structure based on path integral formalism: path integral is replaced with functional integral over 3-surfaces, and the notion of classical space-time becomes an exact part of the theory. Quantum theory becomes formally a purely classical theory of WCW spinor fields: only state function reduction is something genuinely quantal.

4. TGD view about spinor fields is *not* the standard one. Spinor fields appear at three levels. Spinor modes of the embedding space are analogs of spinor modes characterizing incoming and outgoing states in quantum field theories. Induced second quantized spinor fields at space-time level are analogs of stringy spinor fields. Their modes are localized by the well-definedness of electro-magnetic charge and by number theoretic arguments at string world sheets. Kähler-Dirac action is fixed by supersymmetry implying that ordinary gamma matrices are replaced by what I call Kähler-Dirac gamma matrices - this something new. WCW spinor fields, which are classical in the sense that they are not second quantized, serve as analogs of fields of string field theory and imply a geometrization of quantum theory.

5. TGD is in some sense an extremely conservative geometrization of entire quantum physics: *no* additional structures such as gauge fields as independent dynamical degrees of freedom are introduced: Kähler geometry and associated spinor structure are enough. "Topological" in TGD should not be understood as an attempt to reduce physics to torsion (see for instance [B13]) or something similar. Rather, TGD space-time is topologically non-trivial in all scales and even the visible structures of the everyday world represent non-trivial topology of space-time in the TGD Universe.

6. Twistor space - or rather, a generalization of twistor approach replacing masslessness in 4-D sense with masslessness in 8-D sense and thus allowing description of also massive particles - emerged originally as a technical tool, and its Kähler structure is possible only for $H = M^4 \times CP_2$. It however turned out that much more than a technical tool is in question. What is genuinely new is the infinite-dimensional character of the Kähler geometry making it highly unique, and its generalization to p-adic number fields to describe correlates of cognition. Also the hierarchy of Planck constants $h_{eff} = n \times h$ reduces to the quantum criticality of the TGD Universe and p-adic length scales and Zero Energy Ontology represent something genuinely new.

The great challenge is to construct a mathematical theory around these physically very attractive ideas and I have devoted the last 45 years to the realization of this dream and this has resulted in 26 online books about TGD and nine online books about TGD inspired theory of consciousness and of quantum biology.

A collection of 30 online books is now (August 2023) under preparation. The goal is to minimize overlap between the topics of the books and make the focus of a given book sharper.

1.1.2 Two Visions About TGD as Geometrization of Physics and Their Fusion

As already mentioned, TGD as a geometrization of physics can be interpreted both as a modification of general relativity and generalization of string models.

TGD as a Poincare Invariant Theory of Gravitation

The first approach was born as an attempt to construct a Poincare invariant theory of gravitation. Space-time, rather than being an abstract manifold endowed with a pseudo-Riemannian structure, is regarded as a surface in the 8-dimensional space $H = M^4_\times CP_2$, where M^4 denotes Minkowski space and $CP_2 = SU(3)/U(2)$ is the complex projective space of two complex dimensions [A49, A60, A40, A56].

The identification of the space-time as a sub-manifold [A50, A71] of $M^4 \times CP_2$ leads to an exact Poincare invariance and solves the conceptual difficulties related to the definition of the energy-momentum in General Relativity.

It soon however turned out that sub-manifold geometry, being considerably richer in structure than the abstract manifold geometry, leads to a geometrization of all basic interactions. First, the geometrization of the elementary particle quantum numbers is achieved. The geometry of CP_2 explains electro-weak and color quantum numbers. The different H-chiralities of H-spinors correspond to the conserved baryon and lepton numbers. Secondly, the geometrization of the field concept results. The projections of the CP_2 spinor connection, Killing vector fields of CP_2 and of H-metric to four-surface define classical electro-weak, color gauge fields and metric in X^4.

The choice of H is unique from the condition that TGD has standard model symmetries. Also number theoretical vision selects $H = M^4 \times CP_2$ uniquely. M^4 and CP_2 are also unique spaces allowing twistor space with Kähler structure.

TGD as a Generalization of the Hadronic String Model

The second approach was based on the generalization of the mesonic string model describing mesons as strings with quarks attached to the ends of the string. In the 3-dimensional generalization 3-surfaces correspond to free particles and the boundaries of the 3- surface correspond to partons in the sense that the quantum numbers of the elementary particles reside on the boundaries. Various boundary topologies (number of handles) correspond to various fermion families so that one obtains an explanation for the known elementary particle quantum numbers. This approach leads also to a natural topological description of the particle reactions as topology changes: for instance, two-particle decay corresponds to a decay of a 3-surface to two disjoint 3-surfaces.

This decay vertex does not however correspond to a direct generalization of trouser vertex of string models. Indeed, the important difference between TGD and string models is that the analogs of string world sheet diagrams do not describe particle decays but the propagation of particles via different routes. Particle reactions are described by generalized Feynman diagrams for which 3-D light-like surface describing particle propagating join along their ends at vertices. As 4-manifolds the space-time surfaces are therefore singular like Feynman diagrams as 1-manifolds.

Quite recently, it has turned out that fermionic strings inside space-time surfaces define an exact part of quantum TGD and that this is essential for understanding gravitation in long length scales. Also the analog of AdS/CFT duality emerges in that the Kähler metric can be defined either in terms of Kähler function identifiable as Kähler action assignable to Euclidian space-time regions or Kähler action + string action assignable to Minkowskian regions.

The recent view about construction of scattering amplitudes is very “stringy”. By strong form of holography string world sheets and partonic 2-surfaces provide the data needed to construct scattering amplitudes. Space-time surfaces are however needed to realize quantum-classical correspondence necessary to understand the classical correlates of quantum measurement. There is a huge generalization of the duality symmetry of hadronic string models.

The proposal is that scattering amplitudes can be regarded as sequences of computational operations for the Yangian of super-symplectic algebra. Product and co-product define the basic vertices and realized geometrically as partonic 2-surfaces and algebraically as multiplication for the elements of Yangian identified as super-symplectic Noether charges assignable to strings. Any computational sequences connecting given collections of algebraic objects at the opposite boundaries of causal diamond (CD) produce identical scattering amplitudes.

Fusion of the Two Approaches via a Generalization of the Space-Time Concept

The problem is that the two approaches to TGD seem to be mutually exclusive since the orbit of a particle like 3-surface defines 4-dimensional surface, which differs drastically from the topologically trivial macroscopic space-time of General Relativity. The unification of these approaches forces a considerable generalization of the conventional space-time concept. First, the topologically trivial 3-space of General Relativity is replaced with a "topological condensate" containing matter as particle like 3-surfaces "glued" to the topologically trivial background 3-space by connected sum operation. Secondly, the assumption about connectedness of the 3-space is given up. Besides the "topological condensate" there could be "vapor phase" that is a "gas" of particle like 3-surfaces and string like objects (counterpart of the "baby universes" of GRT) and the non-conservation of energy in GRT corresponds to the transfer of energy between different sheets of the space-time and possible existence vapour phase.

. What one obtains is what I have christened as many-sheeted space-time (see **Fig.** `http://tgdtheory.fi/appfigures/manysheeted.jpg` or **Fig. 2.2** in the appendix of this book). One particular aspect is topological field quantization meaning that various classical fields assignable to a physical system correspond to space-time sheets representing the classical fields to that particular system. One can speak of the field body of a particular physical system. Field body consists of topological light rays, and electric and magnetic flux quanta. In Maxwell's theory the physical system does not possess this kind of field identity. The notion of the magnetic body is one of the key players in TGD inspired theory of consciousness and quantum biology. The existence of monopole flux tubes requiring no current as a source of the magnetic field makes it possible to understand the existence of magnetic fields in cosmological and astrophysical scales.

This picture became more detailed with the advent of zero energy ontology (ZEO). The basic notion of ZEO is causal diamond (CD) identified as the Cartesian product of CP_2 and of the intersection of future and past directed light-cones and having scale coming as an integer multiple of CP_2 size is fundamental. CDs form a fractal hierarchy and zero energy states decompose to products of positive and negative energy parts assignable to the opposite boundaries of CD defining the ends of the space-time surface. The counterpart of zero energy state in positive energy ontology is the pair of initial and final states of a physical event, say particle reaction.

At space-time level ZEO means that 3-surfaces are pairs of space-like 3-surfaces at the opposite light-like boundaries of CD. Since the extremals of Kähler action connect these, one can say that by holography the basic dynamical objects are the space-time surface connecting these 3-surfaces and identifiable as analogs of Bohr orbits. This changes totally the vision about notions like self-organization: self-organization by quantum jumps does not take for a 3-D system but for the entire 4-D field pattern associated with it.

General Coordinate Invariance (GCI) allows to identify the basic dynamical objects as space-like 3-surfaces at the ends of space-time surface at boundaries of CD: this means that space-time surface is analogous to Bohr orbit. An alternative identification of the lines of generalized Feynman diagrams is as light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian . Also the Euclidian 4-D regions can have a similar interpretation. The requirement that the two interpretations are equivalent, leads to a strong form of General Coordinate Invariance. The outcome is effective 2-dimensionality stating that the partonic 2-surfaces identified as intersections of the space-like ends of space-time surface and light-like wormhole throats are the fundamental objects. That only effective 2-dimensionality is in question is due to the effects caused by the failure of strict determinism of Kähler action. In finite length scale resolution these effects can be neglected below UV cutoff and above IR cutoff. One can also speak about a strong form of holography.

The understanding of the super symplectic invariance leads to the proposal that super symplectic algebra and other Kac-Moody type algebras labelled by non-negative multiples of basic conformal weights allow a hierarchy of symmetry breakings in which the analog of gauge symmetry breaks down to a genuine dynamical symmetry. This gives rise to fractal hierarchies of algebras and symmetry breakings. This breaking can occur also for ordinary conformal algebras if one restricts the conformal weights to be non-negative integers.

1.1.3 Basic Objections

Objections are the most powerful tool in theory building. The strongest objection against TGD is the observation that all classical gauge fields are expressible in terms of four embedding space coordinates only- essentially CP_2 coordinates. The linear superposition of classical gauge fields taking place independently for all gauge fields is lost. This would be a catastrophe without many-sheeted space-time. Instead of gauge fields, only the effects such as gauge forces are superposed. Particles topologically condense to several space-time sheets simultaneously and experience the sum of gauge forces. This transforms the weakness to extreme economy: in a typical unified theory the number of primary field variables is countered in hundreds if not thousands, now it is just four.

Second objection is that TGD space-time is quite too simple as compared to GRT space-time due to the embeddability to 8-D embedding space. One can also argue that Poincare invariant theory of gravitation cannot be consistent with General Relativity. The above interpretation makes it possible to understand the relationship to GRT space-time and how the Equivalence Principle (EP) follows from Poincare invariance of TGD. The interpretation of GRT space-time is as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrics of the space-time sheets from Minkowski metric. Poincare invariance strongly suggests classical EP for the GRT limit in long length scales at least. One can also consider other kinds of limits such as the analog of GRT limit for Euclidian space-time regions assignable to elementary particles. In this case deformations of CP_2 metric define a natural starting point and CP_2 indeed defines a gravitational instanton with a very large cosmological constant in Einstein-Maxwell theory. Also gauge potentials of the standard model correspond classically to superpositions of induced gauge potentials over space-time sheets.

Topological Field Quantization

Topological field quantization distinguishes between TGD based and more standard - say Maxwellian - notion of field. In Maxwell's fields created by separate systems superpose and one cannot tell which part of field comes from which system except theoretically. In TGD these fields correspond to different space-time sheets and only their effects on test particle superpose. Hence physical systems have well-defined field identifies - field bodies - in particular magnetic bodies.

The notion of magnetic body carrying dark matter with non-standard large value of Planck constant has become central concept in TGD inspired theory of consciousness and living matter, and by starting from various anomalies of biology one ends up to a rather detailed view about the role of magnetic body as intentional agent receiving sensory input from the biological body and controlling it using EEG and its various scaled up variants as a communication tool. Among other thins this leads to models for cell membrane, nerve pulse, and EEG.

1.1.4 Quantum TGD as Spinor Geometry of World of Classical Worlds

A turning point in the attempts to formulate a mathematical theory was reached after seven years from the birth of TGD. The great insight was "Do not quantize". The basic ingredients to the new approach have served as the basic philosophy for the attempt to construct Quantum TGD since then and have been the following ones.

World of Classical Worlds

The notion of WCW reduces the interacting quantum theory to a theory of free WCW spinor fields.

1. Quantum theory for extended particles is free(!), classical(!) field theory for a generalized Schrödinger amplitude identified as WCW spinor in the configuration space CH ("world of classical worlds", WCW) consisting of all possible 3-surfaces in H. "All possible" means that surfaces with arbitrary many disjoint components and with arbitrary internal topology and also singular surfaces topologically intermediate between two different manifold topologies are included.

2. 4-D general coordinate invariance forces holography and replaces the ill-defined path integral over all space-time surfaces with a discrete sum over 4-D analogs of Bohr orbits for particles identified as 3-surfaces. Holography means that basic objects are these analogs of Bohr orbits. Since there is no quantization at the level of WCW, one has an analog of wave mechanics with point-like particles replaced with 4-D Bohr orbits.

3. One must geometrize WCW as the space of Bohr orbits. In an infinite-dimensional situation the existence of geometry requires maximal symmetries already in the case of loop spaces. Physics is unique from its mathematical existence.

   WCW is endowed with metric and spinor structure so that one can define various metric related differential operators, say Dirac operators, appearing in the field equations of the theory [1]

Identification of Kähler function

The evolution of these basic ideas has been rather slow but has gradually led to a rather beautiful vision. One of the key problems has been the definition of Kähler function. Kähler function is Kähler action for a preferred extremal assignable to a given 3-surface but what this preferred extremal is? The obvious first guess was as absolute minimum of Kähler action but could not be proven to be right or wrong. One big step in the progress was boosted by the idea that TGD should reduce to almost topological QFT in which braids would replace 3-surfaces in finite measurement resolution, which could be inherent property of the theory itself and imply discretization at partonic 2-surfaces with discrete points carrying fermion number.

It took long time to realize that there is no discretization in 4-D sense - this would lead to difficulties with basic symmetries. Rather, the discretization occurs for the parameters characterizing co-dimension 2 objects representing the information about space-time surface so that they belong to some algebraic extension of rationals. These 2-surfaces - string world sheets and partonic 2-surfaces - are genuine physical objects rather than a computational approximation. Physics itself approximates itself, one might say! This is of course nothing but strong form of holography.

1. TGD as almost topological QFT vision suggests that Kähler action for preferred extremals reduces to Chern-Simons term assigned with space-like 3-surfaces at the ends of space-time (recall the notion of causal diamond (CD)) and with the light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian. Minkowskian and Euclidian regions would give at wormhole throats the same contribution apart from coefficients and in Minkowskian regions the $\sqrt{g_4}$ factorc coming from metric would be imaginary so that one would obtain sum of real term identifiable as Kähler function and imaginary term identifiable as the ordinary Minkowskian action giving rise to interference effects and stationary phase approximation central in both classical and quantum field theory.

   Imaginary contribution - the presence of which I realized only after 33 years of TGD - could also have topological interpretation as a Morse function. On physical side the emergence of Euclidian space-time regions is something completely new and leads to a dramatic modification of the ideas about black hole interior.

2. The way to achieve the reduction to Chern-Simons terms is simple. The vanishing of Coulomb contribution to Kähler action is required and is true for all known extremals if one makes a general ansatz about the form of classical conserved currents. The so called weak form of electric-magnetic duality defines a boundary condition reducing the resulting 3-D terms to Chern-Simons terms. In this way almost topological QFT results. But only "almost" since the Lagrange multiplier term forcing electric-magnetic duality implies that Chern-Simons action for preferred extremals depends on metric.

WCW spinor fields

Classical WCW spinor fields are analogous to Schrödinger amplitudes and the construction of WCW Kähler geometry reduces to the second quantization of free spinor fields of H.

[1] There are four kinds of Dirac operators in TGD. The geometrization of quantum theory requires Kähler metric definable either in terms of Kähler function identified as a the bosonic action for Euclidian space-time regions or as anti-commutators for WCW gamma matrices identified as conformal Noether super-charges associated with the second quantized modified Dirac action consisting of string world sheet term and possibly also modified Dirac action in Minkowskian space-time regions. These two possible definitions reflect a duality analogous to AdS/CFT duality.

1. The WCW metric is given by anticommutators of WCW gamma matrices which also have interpretation as supercharges assignable to the generators of WCW isometries and allowing expression as non-conserved Noether charges. Holography implies zero energy ontology (ZEO) meaning that zero energy states are superpositions of Bohr orbits connecting boundaries of causal diamond (CD). CDs form a fractal hierarchy and their space forming the spine of WCW is finite-dimensional and can be geometrized. The alternative interpretation is as a superposition of pairs of ordinary 3-D fermionic states assignable to the ends of the space-time surfaces.

2. There are several Dirac operators. WCW Dirac operator D_{WCW} appears in Super-symplectic gauge conditions analogous to Super Virasoro conditions. The algebraic variant of the H Dirac operator D_H appears in fermionic correlation functions: this is due to the fact that free fermions appearing as building bricks of WCW gamma matrices are modes of D_H. The modes of $_DH$ define the ground states of super-symplectic representations. There is also the modified Dirac operator D_{X^4} acting on the induced spinors at space-time surfaces and it is dictated by symmetry one the action fixing the space-time surfaces as Bohr orbits is fixed. D_H is needed since it determines the expressions of WCW gamma matrices as Noether charges assignable to 3-surfaces at the ends of WCW.

The role of modified Dirac action

1. By quantum classical correspondence, the construction of WCW spinor structure in sectors assignable to CDs reduces to the second quantization of the induced spinor fields of H. The basic action is so called modified Dirac action in which gamma matrices are replaced with the modified) gamma matrices defined as contractions of the canonical momentum currents of the bosonic action defining the space-time surfaces with the embedding space gamma matrices. In this way one achieves super-conformal symmetry and conservation of fermionic currents among other things and a consistent Dirac equation.

   Modified Dirac action is needed to define WCW gamma matrices as super charges assignable to WCW isometry generators identified as generators of symplectic transformations and by holography are needed only at the 3-surface at the boundaries of WCW. It is important to notice that the modified Dirac equation does not determine propagators since induced spinor fields are obtained from free second quantized spinor fields of H. This means enormous simplification and makes the theory calculable.

2. An important interpretational problem relates to the notion of the induced spinor connection. The presence of classical W boson fields is in conflict with the classical conservation of em charge since the coupling to classical W fields changes em charge.

   One way out of the problem is the fact that the quantum averages of weak and gluon fields vanish unlike the quantum average of the em field. This leads to a rather precise understanding of electroweak symmetry breaking as being due the fact that color symmetries rotate space-time surfaces and also affect the induced weak fields.

   One can also consider a stronger condition. If one requires that the spinor modes have well-defined em charge, one must assume that the modes in the generic situation are localized at 2-D surfaces - string world sheets or perhaps also partonic 2-surfaces - at which classical W boson fields vanish. Covariantly constant right handed neutrinos generating super-symmetries forms an exception. The vanishing of the Z^0 field is possible for Kähler-Dirac action and should hold true at least above weak length scales. This implies that the string model in 4-D space-time becomes part of TGD. Without these conditions classical weak fields can vanish above weak scale only for the GRT limit of TGD for which gauge potentials are sums over those for space-time sheets.

   The localization would simplify the mathematics enormously and one can solve exactly the Kähler-Dirac equation for the modes of the induced spinor field just like in super string models.

   At the light-like 3-surfaces the signature of the induced metric changes from Euclidian to Minkowskian so that $\sqrt{g_4}$ vanishes. One can pose the condition that the algebraic analog of the massless Dirac equation is satisfied by the modes of the modified-Dirac action assignable to the Chern-Simons-Kähler action.

1.1.5 Construction of scattering amplitudes

Reduction of particle reactions to space-time topology

Particle reactions are identified as topology changes [A66, A73, A82]. For instance, the decay of a 3-surface to two 3-surfaces corresponds to the decay $A \to B + C$. Classically this corresponds to a path of WCW leading from 1-particle sector to 2-particle sector. At quantum level this corresponds to the dispersion of the generalized Schrödinger amplitude localized to 1-particle sector to two-particle sector. All coupling constants should result as predictions of the theory since no nonlinearities are introduced.

During years this naïve and very rough vision has of course developed a lot and is not anymore quite equivalent with the original insight. In particular, the space-time correlates of Feynman graphs have emerged from theory as Euclidian space-time regions and the strong form of General Coordinate Invariance has led to a rather detailed and in many respects un-expected visions. This picture forces to give up the idea about smooth space-time surfaces and replace space-time surface with a generalization of Feynman diagram in which vertices represent the failure of manifold property. I have also introduced the word "world of classical worlds" (WCW) instead of rather formal "configuration space". I hope that "WCW" does not induce despair in the reader having tendency to think about the technicalities involved!

Construction of the counterparts of S-matrices

What does one mean with the counterpart of S-matrix in the TGD framework has been a long standing problem. The development of ZEO based quantum measurement theory has led to a rough overall view of the situation.

1. There are two kinds of state function reductions (SFRs). "Small" SFRs (SSFRs) following the TGD counterpart of a unitary time evolution defines a sequence of SFRs, which is analogous to a sequence of repeated quantum measurements associated with the Zeno effect. In wave mechanics nothing happens in these measurements. In quantum optics these measurements correspond to weak measurements. In TGD SSFR affects the zero energy state but leaves the 3-D state at the passive boundary of CD unaffected.

2. In TGD framework each SSFR is preceded by a counterpart of a unitary time evolution, which means dispersion in the space of CDs and unitary time evolution in fermionic degrees of freedom such that the passive boundary of CDs and 3-D states at it are unaffected but a superposition of CDs with varying active boundaries in the space of CDs is formed. In SSFR a localization in the space of CDs occurs such that the active is fixed. In a statistical sense the size of the CD increases and the increasing distance between the tips of the CD gives rise to the arrow of geometric time.

3. Also "big" SFRS (BSFRs) can occur and they correspond to ordinary SFRs. In BSFR the roles of the active and passive boundary are changed and this means that the arrow of time is changed. Big SFR occurs when the SSFR corresponds to a quantum measurement, which does not commute with the operators, which define the states at the passive boundary of CD as their eigenstates. This means a radical deviation from standard quantum measurement theory and has predictions in all scales.

4. One can assign the counterpart of S-matrix to the unitary time evolution between two subsequent SSFRs and also to the counterpart of S-matrix associated with BSFR. At least in the latter case the dimension of the state space can increase since at least BSFRs lead to the increase of the dimension of algebraic extension of rationals assignable to the space-time surface by $M^8 - H$ duality. Unitarity is therefore replaced with isometry.

5. I have also considered the possibility that unitary S-matrix could be replaced in the fermionic degrees of freedom with Kähler metric of the state space satisfying analogs of unitarity conditions but it seems that this is un-necessary and also too outlandish an idea.

The notion of M-matrix

1. The most ambitious dream is that zero energy states correspond to a complete solution basis for the Dirac operators associated with WCWs associated with the spaces of CDs with fixed passive boundary: this would define an S-matrix assignable to SFR. Also the analog of S-matrix for the localizations of the states to the active boundary assignable to the BSFR changing the state at the passive boundary of CD is needed.

2. If one allows entanglement between positive and energy parts of the zero energy state but assumes that the states at the passive boundary are fixed, one must introduce the counterpart of the density matrix, or rather its square root. This classical free field theory would dictate what I have called M-matrices defined between positive and negative energy parts of zero energy states which form orthonormal rows of what I call U-matrix as a matrix defined between zero energy states. A biven M-matrix in turn would decompose to a product of a hermitian square root of density matrix and unitary S-matrix.

3. M-matrix would define time-like entanglement coefficients between positive and negative energy parts of zero energy states (all net quantum numbers vanish for them) and can be regarded as a hermitian square root of density matrix multiplied by a unitary S-matrix. Quantum theory would be in a well-defined sense a square root of thermodynamics. The orthogonality and hermiticity of the M-matrices commuting with S-matrix means that they span infinite-dimensional Lie algebras acting as symmetries of the S-matrix. Therefore quantum TGD would reduce to group theory in a well-defined sense.

4. In fact the Lie algebra of Hermitian M-matrices extends to Kac-Moody type algebra obtained by multiplying hermitian square roots of density matrices with powers of the S-matrix. Also the analog of Yangian algebra involving only non-negative powers of S-matrix is possible and would correspond to a hierarchy of CDs with the temporal distances between tips coming as integer multiples of the CP_2 time.

   The M-matrices associated with CDs are obtained by a discrete scaling from the minimal CD and characterized by integer n are naturally proportional to a representation matrix of scaling: $S(n) = S^n$, where S is unitary S-matrix associated with the minimal CD [K69]. This conforms with the idea about unitary time evolution as exponent of Hamiltonian discretized to integer power of S and represented as scaling with respect to the logarithm of the proper time distance between the tips of CD.

5. I have also considered the notion of U-matrix. U-matrix elements between M-matrices for various CDs are proportional to the inner products $Tr[S^{-n_1} \circ H^i H^j \circ S^{n_2} \lambda]$, where λ represents unitarily the discrete Lorentz boost relating the moduli of the active boundary of CD and H^i form an orthonormal basis of Hermitian square roots of density matrices. $\circ$ tells that S acts at the active boundary of CD only. I have proposed a general representation for the U-matrix, reducing its construction to that of the S-matrix.

1.1.6 TGD as a generalized number theory

Quantum T(opological)D(ynamics) as a classical spinor geometry for infinite-dimensional configuration space ("world of classical worlds", WCW), p-adic numbers and quantum TGD, and TGD inspired theory of consciousness, have been for last ten years the basic three strongly interacting threads in the tapestry of quantum TGD. The fourth thread deserves the name "TGD as a generalized number theory". It involves three separate threads: the fusion of real and various p-adic physics to a single coherent whole by requiring number theoretic universality discussed already, the formulation of quantum TGD in terms of complexified counterparts of classical number fields, and the notion of infinite prime. Note that one can identify subrings such as hyper-quaternions and hyper-octonions as sub-spaces of complexified classical number fields with Minkowskian signature of the metric defined by the complexified inner product.

The Threads in the Development of Quantum TGD

The development of TGD has involved several strongly interacting threads: physics as infinite-dimensional geometry; TGD as a generalized number theory, the hierarchy of Planck constants interpreted in terms of dark matter hierarchy, and TGD inspired theory of consciousness. In the following these threads are briefly described.

1. Quantum T(opological) G(eometro)D(ynamics) as a classical spinor geometry for infinite-dimensional WCW, p-adic numbers and quantum TGD, and TGD inspired theory of consciousness and of quantum biology have been for last decade of the second millenium the basic three strongly interacting threads in the tapestry of quantum TGD.

2. The discussions with Tony Smith initiated a fourth thread which deserves the name "TGD as a generalized number theory". The basic observation was that classical number fields might allow a deeper formulation of quantum TGD. The work with Riemann hypothesis made time ripe for realization that the notion of infinite primes could provide, not only a reformulation, but a deep generalization of quantum TGD. This led to a thorough and rather fruitful revision of the basic views about what the final form and physical content of quantum TGD might be. Together with the vision about the fusion of p-adic and real physics to a larger coherent structure these sub-threads fused to the "physics as generalized number theory" thread.

3. A further thread emerged from the realization that by quantum classical correspondence TGD predicts an infinite hierarchy of macroscopic quantum systems with increasing sizes, that it is not at all clear whether standard quantum mechanics can accommodate this hierarchy, and that a dynamical quantized Planck constant might be necessary and strongly suggested by the failure of strict determinism for the fundamental variational principle. The identification of hierarchy of Planck constants labelling phases of dark matter would be natural. This also led to a solution of a long standing puzzle: what is the proper interpretation of the predicted fractal hierarchy of long ranged classical electro-weak and color gauge fields. Quantum classical correspondences allows only single answer: there is infinite hierarchy of p-adically scaled up variants of standard model physics and for each of them also dark hierarchy. Thus TGD Universe would be fractal in very abstract and deep sense.

The chronology based identification of the threads is quite natural but not logical and it is much more logical to see p-adic physics, the ideas related to classical number fields, and infinite primes as sub-threads of a thread which might be called "physics as a generalized number theory". In the following I adopt this view. This reduces the number of threads to three corresponding to geometric, number theoretic and topological views of physics.

TGD forces the generalization of physics to a quantum theory of consciousness, and TGD as a generalized number theory vision leads naturally to the emergence of p-adic physics as physics of cognitive representations.

Number theoretic vision very briefly

Number theoretic vision about quantum TGD involves notions like adelic physics, M^8-H duality and number theoretic universality. A short review of the basic ideas that have developed during years is in order.

1. The physical interpretation of M^8 is as an analog of momentum space and M^8-H duality is analogous to momentum-position duality of ordinary wave mechanics.

2. Adelic physics means that all classical number fields, all p-adic number fields and their extensions induced by extensions of rationals and defining adeles, and also finite number fields are basic mathematical building bricks of physics.

   The complexification of M^8, identified as complexified octonions, would provide a realization of this picture and M^8-H duality would map the algebraic physics in M^8 to the ordinary physics in $M^4\times CP_2$ described in terms of partial differential equations.

3. Negentropy Maximization Principle (NMP) states that the conscious information assignable with cognition representable measured in terms of p-adic negentropy increases in statistical sense.

   NMP is mathematically completely analogous to the second law of thermodynamics and number theoretic evolution as an unavoidable statistical increase of the dimension of the algebraic extension of rationals characterizing a given space-time region implies it. There is no paradox involved: the p-adic negentropy measures the conscious information assignable to the entanglement of two systems regarded as a conscious entity whereas ordinary entropy measures the lack of information about the quantums state of either entangled system.

4. Number theoretical universality requires that space-time surfaces or at least their $M^8 - H$ duals in M^8_c are defined for both reals and various p-adic number fields. This is true if they are defined by polynomials with integer coefficients as surfaces in M^8 obeying number theoretic holography realized as associativity of the normal space of 4-D surface using as holographic data 3-surfaces at mass shells identified in terms of roots of a polynomial. A physically motivated additional condition is that the coefficients of the polynomials are smaller than their degrees.

5. Galois confinement is a key piece of the number theoretic vision. It states that the momenta of physical states are algebraic integers in the extensions of rationals assignable to the space-time region considered. These numbers are in general complex and are not consistent with particle in box quantization. The proposal is that physical states satisfy Galois confinement being thus Galois singlets and having therefore total momenta, whose components are ordinary integers, when momentum unit defined by the scale of causal diamond (CD) is used.

6. The notion of p-adic prime was introduced in p-adic mass calculations that started the developments around 1995. p-Adic length scale hypothesis states that p-adic primes near powers of 2 have a special physical role (as possibly also the powers of other small primes such as $p = 3$).

   The proposal is that p-adic primes correspond to ramified primes assignable to the extension and identified as divisors of the polynomial defined by the products of the root differences for the roots of the polynomial defining space-time space and having interpretation as values of, in general complex, virtual mass squared.

p-Adic TGD and fusion of real and p-adic physics to single coherent whole

The p-adic thread emerged for roughly ten years ago as a dim hunch that p-adic numbers might be important for TGD. Experimentation with p-adic numbers led to the notion of canonical identification mapping reals to p-adics and vice versa. The breakthrough came with the successful p-adic mass calculations using p-adic thermodynamics for Super-Virasoro representations with the super-Kac-Moody algebra associated with a Lie-group containing standard model gauge group. Although the details of the calculations have varied from year to year, it was clear that p-adic physics reduces not only the ratio of proton and Planck mass, the great mystery number of physics, but all elementary particle mass scales, to number theory if one assumes that primes near prime powers of two are in a physically favored position. Why this is the case, became one of the key puzzles and led to a number of arguments with a common gist: evolution is present already at the elementary particle level and the primes allowed by the p-adic length scale hypothesis are the fittest ones.

It became very soon clear that p-adic topology is not something emerging in Planck length scale as often believed, but that there is an infinite hierarchy of p-adic physics characterized by p-adic length scales varying to even cosmological length scales. The idea about the connection of p-adics with cognition motivated already the first attempts to understand the role of the p-adics and inspired "Universe as Computer" vision but time was not ripe to develop this idea to anything concrete (p-adic numbers are however in a central role in TGD inspired theory of consciousness). It became however obvious that the p-adic length scale hierarchy somehow corresponds to a hierarchy of intelligences and that p-adic prime serves as a kind of intelligence quotient. Ironically, the almost obvious idea about p-adic regions as cognitive regions of space-time providing cognitive representations for real regions had to wait for almost a decade for the access into my consciousness.

In string model context one tries to reduces the physics to Planck scale. The price is the inability to say anything about physics in long length scales. In TGD p-adic physics takes care of this shortcoming by predicting the physics also in long length scales.

There were many interpretational and technical questions crying for a definite answer.

1. What is the relationship of p-adic non-determinism to the classical non-determinism of the basic field equations of TGD? Are the p-adic space-time region genuinely p-adic or does p-adic topology only serve as an effective topology? If p-adic physics is direct image of real physics, how the mapping relating them is constructed so that it respects various symmetries? Is the basic physics p-adic or real (also real TGD seems to be free of divergences) or both? If it is both, how should one glue the physics in different number field together to get *the* Physics? Should one perform p-adicization also at the level of the WCW? Certainly the p-adicization at the level of super-conformal representation is necessary for the p-adic mass calculations.
2. Perhaps the most basic and most irritating technical problem was how to precisely define p-adic definite integral which is a crucial element of any variational principle based formulation of the field equations. Here the frustration was not due to the lack of solution but due to the too large number of solutions to the problem, a clear symptom for the sad fact that clever inventions rather than real discoveries might be in question. Quite recently I however learned that the problem of making sense about p-adic integration has been for decades central problem in the frontier of mathematics and a lot of profound work has been done along same intuitive lines as I have proceeded in TGD framework. The basic idea is certainly the notion of algebraic continuation from the world of rationals belonging to the intersection of real world and various p-adic worlds.

Despite various uncertainties, the number of the applications of the poorly defined p-adic physics has grown steadily and the applications turned out to be relatively stable so that it was clear that the solution to these problems must exist. It became only gradually clear that the solution of the problems might require going down to a deeper level than that represented by reals and p-adics.

The key challenge is to fuse various p-adic physics and real physics to single larger structure. This has inspired a proposal for a generalization of the notion of number field by fusing real numbers and various p-adic number fields and their extensions along rationals and possible common algebraic numbers. This leads to a generalization of the notions of embedding space and space-time concept and one can speak about real and p-adic space-time sheets. One can talk about adelic space-time, embedding space, and WCW.

The corresponds of real 4-surfaces with the p-adic ones is induced by number theoretical discretization using points of 4-surfaces $Y^4 \subset M_c^8$ identifiable as 8-momenta, whose components are assumed to be algebraic integers in an extension of rationals defined by the extension of rationals associated with a polynomial P with integer coefficients smaller than the degree of P. These points define a cognitive representation, which is universal in the sense that it exists also in the algebraic extensions of p-adic numbers. The points of the cognitive representations associated with the mass shells with mass squared values identified as roots of P are enough since $M^8 - H$ duality can be used at both M^8 and H sides and also in the p-adic context. The mass shells are special in that they allow for Minkowski coordinates very large cognitive representations unlike the interiors of the 4-surfaces determined by holography by using the data defined by the 3-surfaces at the mass shells. The higher the dimension of the algebraic extension associated with P, the better the accuracy of the cognitive representation.

Adelization providing number theoretical universality reduces to algebraic continuation for the amplitudes from this intersection of reality and various p-adicities - analogous to a back of a book - to various number fields. There are no problems with symmetries but canonical identification is needed: various group invariant of the amplitude are mapped by canonical identification to various p-adic number fields. This is nothing but a generalization of the mapping of the p-adic mass squared to its real counterpart in p-adic mass calculations.

This leads to surprisingly detailed predictions and far reaching conjectures. For instance, the number theoretic generalization of entropy concept allows negentropic entanglement central for the applications to living matter (see **Fig.** `http://tgdtheory.fi/appfigures/cat.jpg` or **Fig. ??** in the appendix of this book). One can also understand how preferred p-adic primes could emerge as so called ramified primes of algebraic extension of rationals in question and characterizing string world sheets and partonic 2-surfaces. Preferred p-adic primes would be ramified primes for extensions for which the number of p-adic continuations of two-surfaces to space-time surfaces (imaginations) allowing also real continuation (realization of imagination) would be especially large. These ramifications would be winners in the fight for number theoretical survival. Also a generalization of p-adic length scale hypothesis emerges from NMP [K65].

The characteristic non-determinism of the p-adic differential equations suggests strongly that p-adic regions correspond to “mind stuff”, the regions of space-time where cognitive representations reside. This interpretation implies that p-adic physics is physics of cognition. Since Nature is probably a brilliant simulator of Nature, the natural idea is to study the p-adic physics of the cognitive representations to derive information about the real physics. This view encouraged by TGD inspired theory of consciousness clarifies difficult interpretational issues and provides a clear interpretation for the predictions of p-adic physics.

Infinite primes

The discovery of the hierarchy of infinite primes and their correspondence with a hierarchy defined by a repeatedly second quantized arithmetic quantum field theory gave a further boost for the speculations about TGD as a generalized number theory.

After the realization that infinite primes can be mapped to polynomials possibly representable as surfaces geometrically, it was clear how TGD might be formulated as a generalized number theory with infinite primes forming the bridge between classical and quantum such that real numbers, p-adic numbers, and various generalizations of p-adics emerge dynamically from algebraic physics as various completions of the algebraic extensions of complexified quaternions and octonions. Complete algebraic, topological and dimensional democracy would characterize the theory.

The infinite primes at the first level of hierarchy, which represent analogs of bound states, can be mapped to irreducible polynomials, which in turn characterize the algebraic extensions of rationals defining a hierarchy of algebraic physics continuable to real and p-adic number fields. The products of infinite primes in turn define more general algebraic extensions of rationals. The interesting question concerns the physical interpretation of the higher levels in the hierarchy of infinite primes and integers mappable to polynomials of $n > 1$ variables.

1.1.7 An explicit formula for $M^8 - H$ duality

$M^8 - H$ duality is a generalization of momentum-position duality relating the number theoretic and geometric views of physics in TGD and, despite that it still involves poorly understood aspects, it has become a fundamental building block of TGD. One has 4-D surfaces $Y^4 \subset M^8_c$, where M^8_c is complexified M^8 having interpretation as an analog of complex momentum space and 4-D spacetime surfaces $X^4 \subset H = M^4 \times CP_2$. M^8_c, equivalently E^8_c, can be regarded as complexified octonions. M^8_c has a subspace M^4_c containing M^4.

Comment: One should be very cautious with the meaning of "complex". Complexified octonions involve a complex imaginary unit i commuting with the octonionic imaginary units I_k. i is assumed to also appear as an imaginary unit also in complex algebraic numbers defined by the roots of polynomials P defining holographic data in M^8_c.

In the following $M^8 - H$ duality and its twistor lift are discussed and an explicit formula for the dualities are deduced. Also possible variants of the duality are discussed.

Holography in H

$X^4 \subset H$ satisfies holography and is analogous to the Bohr orbit of a particle identified as a 3-surface. The proposal is that holography reduces to a 4-D generalization of holomorphy so that X^4 is a simultaneous zero of two functions of complex CP_2 coordinates and of what I have called Hamilton-Jacobi coordinates of M^4 with a generalized Kähler structure.

The simplest choice of the Hamilton-Jacobi coordinates is defined by the decomposition $M^4 = M^2 \times E^2$, where M^2 is endowed with hypercomplex structure defined by light-like coordinates (u, v), which are analogous to z and $\overline{z}$. Any analytic map $u \rightarrow f(u)$ defines a new set of light-like coordinates and corresponds to a solution of the massless d'Alembert equation in M^2. E^2 has some complex coordinates with imaginary unit defined by i.

The conjecture is that also more general Hamilton-Jacobi structures for which the tangent space decomposition is local are possible. Therefore one would have $M^4 = M^2(x) \times E^2(x)$. These would correspond to non-equivalent complex and Kähler structures of M^4 analogous to those possessed by 2-D Riemann surfaces and parametrized by moduli space.

Number theoretic holography in M^8_c

$Y^4 \subset M^8_c$ satisfies number theoretic holography defining dynamics, which should reduce to associativity in some sense. The Euclidian complexified normal space $N^4(y)$ at a given point y of Y^4 is required to be associative, i.e. quaternionic. Besides this, $N^4(i)$ contains a preferred complex Euclidian 2-D subspace $Y^2(y)$. Also the spaces $Y^2(x)$ define an integrable distribution. I have assumed that $Y^2(x)$ can depend on the point y of Y^4.

These assumptions imply that the normal space $N(y)$ of Y^4 can be parameterized by a point of $CP_2 = SU(3)/U(2)$. This distribution is always integrable unlike quaternionic tangent space distributions. $M^8 - H$ duality assigns to the normal space $N(y)$ a point of CP_2. M^4_c point y is mapped to a point $x \in M^4 \subset M^4 \times CP_2$ defined by the real part of its inversion (conformal transformation): this formula involves effective Planck constant for dimensional reasons.

The 3-D holographic data, which partially fixes 4-surfaces Y^4 is partially determined by a polynomial P with real integer coefficients smaller than the degree of P. The roots define mass squared values which are in general complex algebraic numbers and define complex analogs of mass shells in $M^4_c \subset M^8_c$, which are analogs of hyperbolic spaces H^3. The 3-surfaces at these mass shells define 3-D holographic data continued to a surface Y^4 by requiring that the normal space of Y^4 is associative, i.e. quaternionic. These 3-surfaces are not completely fixed but an interesting conjecture is that they correspond to fundamental domains of tessellations of H^3.

What does the complexity of the mass shells mean? The simplest interpretation is that the space-like M^4 coordinates (3-momentum components) are real whereas the time-like coordinate (energy) is complex and determined by the mass shell condition. One would have $Re^2(E) - Im(E)^2 - p^2 = Re(m^2)$ and $2Re(E)Im(E) = Im(m^2)$. The condition for the real parts gives H^3 when $\sqrt{Re^2(E) - Im(E)^2}$ is taken as a time coordinate. The second condition allows to solve $Im(E)$ in terms of $Re(E)$ so that the first condition reduces to an equation of mass shell when $\sqrt{(Re(E)^2 - Im(E)^2)}$, expressed in terms of $Re(E)$, is taken as new energy coordinate $E_{eff} = \sqrt{(Re(E)^2 - Im(E)^2)}$. Is this deformation of H^3 in imaginary time direction equivalent with a region of the hyperbolic 3-space H^3?

One can look at the formula in more detail. Mass shell condition gives $Re^2(E) - Im(E)^2 - p^2 = Re(m^2)$ and $2Re(E)Im(E) = Im(m^2)$. The condition for the real parts gives H^3, when $\sqrt{Re^2(E) - Im(E)^2}$ is taken as an effective energy. The second condition allows to solve $Im(E)$ in terms of $Re(E)$ so that the first condition reduces to a dispersion relation for $Re(E)^2$.

$$Re(E)^2 = \frac{1}{2}(Re(m^2) - Im(m^2) + p^2)(1 \pm \sqrt{1 + \frac{2Im(m^2)^2}{(Re(m^2) - Im(m^2) + p^2)^2}} \; . \tag{1.1.1}$$

Only the positive root gives a non-tachyonic result for $Re(m^2) - Im(m^2) > 0$. For real roots with $Im(m^2) = 0$ and at the high momentum limit the formula coincides with the standard formula. For $Re(m^2) = Im(m^2)$ one obtains $Re(E)^2 \rightarrow Im(m^2)/\sqrt{2}$ at the low momentum limit $p^2 \rightarrow 0$. Energy does not depend on momentum at all: the situation resembles that for plasma waves.

Can one find an explicit formula for $M^8 - H$ duality?

The dream is an explicit formula for the $M^8 - H$ duality mapping $Y^4 \subset M^8_c$ to $X^4 \subset H$. This formula should be consistent with the assumption that the generalized holomorphy holds true for X^4.

The following proposal is a more detailed variant of the earlier proposal for which Y^4 is determined by a map g of $M^4_c \rightarrow SU(3)_c \subset G_{2,c}$, where $G_{2,c}$ is the complexified automorphism group of octonions and $SU(3)_c$ is interpreted as a complexified color group.

This map defines a trivial $SU(3)_c$ gauge field. The real part of g however defines a non-trivial real color gauge field by the non-linearity of the non-abelian gauge field with respect to the gauge potential. The quadratic terms involving the imaginary part of the gauge potential give an additional condition to the real part in the complex situation and cancel it. If only the real part of g contributes, this contribution would be absent and the gauge field is non-vanishing.

How could the automorphism $g(x) \subset SU(3) \subset G_2$ give rise to $M^8 - H$ duality?

1. The interpretation is that $g(y)$ at given point y of Y^4 relates the normal space at y to a fixed quaternionic/associative normal space at point y_0, which corresponds is fixed by some subgroup $U(2)_0 \subset SU(3)$. The automorphism property of g guarantees that the normal space is quaternionic/associative at y. This simplifies the construction dramatically.
2. The quaternionic normal sub-space (which has Euclidian signature) contains a complex sub-space which corresponds to a point of sphere $S^2 = SO(3)/O(2)$, where $SO(3)$ is the quaternionic automorphism group. The interpretation could be in terms of a selection of spin quantization axes. The local choice of the preferred complex plane would not be unique and is analogous to the possibility of having non-trivial Hamilton Jacobi structures in M^4 characterized by the choice of $M^2(x)$ and equivalently its normal subspace $E^2(x)$.

   These two structures are independent apart from dependencies forced by the number theoretic dynamics. Hamilton-Jacobi structure means a selection of the quantization axis of spin and energy by fixing a distribution of light-like tangent vectors of M^4 and the choice of the quaternionic normal sub-space fixes a choice of preferred quaternionic imaginary unit defining a quantization axis of the weak isospin.
3. The real part $Re(g(y))$ defines a point of $SU(3)$ and the bundle projection $SU(3) \rightarrow CP_2$ in turn defines a point of $CP_2 = SU(3)/U(2)$. Hence one can assign to g a point of CP_2 as $M^8 - H$ duality requires and deduce an explicit formula for the point. This means a realization of the dream.
4. The construction requires a fixing of a quaternionic normal space N_0 at y_0 containing a preferred complex subspace at a single point of Y^4 plus a selection of the function g. If M^4 coordinates are possible for Y^4, the first guess is that g as a function of complexified M^4 coordinates obeys generalized holomorphy with respect to complexified M^4 coordinates in the same sense and in the case of X^4. This might guarantee that the $M^8 - H$ image of Y^4 satisfies the generalized holomorphy.
5. Also space-time surfaces X^4 with M^4 projection having a dimension smaller than 4 are allowed. I have proposed that they might correspond to singular cases for the above formula: a kind of blow-up would be involved. One can also consider a more general definition of Y^4 allowing it to have a M^4 projection with dimension smaller than 4 (say cosmic strings). Could one have implicit equations for the surface Y^4 in terms of the complex coordinates of $SU(3)_c$ and M^4? Could this give for instance cosmic strings with a 2-D M^4 projection and CP_2 type extremals with 4-D CP_2 projection and 1-D light-like M^4 projection?

What could the number theoretic holography mean physically?

What could be physical meaning of the number theoretic holography? The condition that has been assumed is that the CP_2 coordinates at the mass shells of $M^4_c \subset M^8_c$ mapped to mass shells H^3 of $M^4 \subset M^4 \times CP_2$ are constant at the H^3. This is true if the $g(y)$ defines the same CP_2 point for a given component X^3_i of the 3-surface at a given mass shell. g is therefore fixed apart from a local $U(2)$ transformation leaving the CP_2 point invariant. A stronger condition would be that the CP_2 point is the same for each component of X^3_i and even at each mass shell but this condition seems to be unnecessarily strong.

Comment: One can o criticize this condition as too strong and one can consider giving up this condition. The motivation for this condition is that the number of algebraic points at the 3-surfaces associated with H^3 explodes since the coordinates associated with normal directions vanish. Kind of cognitive explosion would be in question.

$SU(3)$ corresponds to a subgroup of G_2 and one can wonder what the fixing of this subgroup could mean physically. G_2 is 14-D and the coset space $G_2/SU(3)$ is 6-D and a good guess is that it is just the 6-D twistor space $SU(3)/U(1) \times U(1)$ of CP_2: at least the isometries are the same.

The fixing of the $SU(3)$ subgroup means fixing of a CP_2 twistor. Physically this means the fixing of the quantization axis of color isospin and hypercharge.

Twistor lift of the holography

What is interesting is that by replacing $SU(3)$ with G_2, one obtains an explicit formula form the generalization of M^8-H duality to that for the twistorial lift of TGD!

One can also consider a twistorial generalization of the above proposal for the number theoretic holography by allowing local G_2 automorphisms interpreted as local choices of the color quantization axis. G_2 elements would be fixed apart from a local $SU(3)$ transformation at the components of 3-surfaces at mass shells. The choice of the color quantization axes for a connected 3-surface at a given mass shell would be the same everywhere. This choice is indeed very natural physically since 3-surface corresponds to a particle.

Is this proposal consistent with the boundary condition of the number theoretical holography mean in the case of 4-surfaces in M^8_c and $M^4\times CP_2$?

1. The selection of $SU(3)\subset G_2$ for ordinary M^8-H duality means that the $G_{2,c}$ gauge field vanishes everywhere and the choice of color quantization axis is the same at all points of the 4-surface. The fixing of the CP_2 point to be constant at H^3 implies that the color gauge field at $H^3\subset M^8_c$ and its image $H^3\subset H$ vanish. One would have color confinement at the mass shells H^3_i, where the observations are made. Is this condition too strong?
2. The constancy of the G_2 element at mass shells makes sense physically and means a fixed color quantization axis. The selection of a fixed $SU(3)\subset G_2$ for entire space-time surface is in conflict with the non-constancy of G_2 element unless G_2 element differs at different points of 4-surface only by a multiplication of a local $SU(3)_0$ element, that is local $SU(3)$ transformation. This kind of variation of the G_2 element would mean a fixed color group but varying choice of color quantization axis.
3. Could one consider the possibility that the local $G_{2,c}$ element is free and defines the twistor lift of M^8-H duality as something more fundamental than the ordinary M^8-H duality based on $SU(3)_c$. This duality would make sense only at the mass shells so that only the spaces $H^3\times CP_2$ assignable to mass shells would make sense physically? In the interior CP_2 would be replaced with the twistor space $SU(3)/U(1)\times U(1)$. Color gauge fields would be non-vanishing at the mass shells but outside the mass shells one would have G_2 gauge fields.

   There is also a physical objection against the G_2 option. The 14-D Lie algebra representation of G_2 acts on the imaginary octonions which decompose with respect to the color group to $1\oplus 3\oplus \overline{3}$. The automorphism property requires that 1 can be transformed to 3 or $\overline{3}$ to themselves: this requires that the decomposition contains $3\oplus\overline{3}$. Furthermore, it must be possible to transform 3 and $\overline{3}$ to themselves, which requires the presence of 8. This leaves only the decomposition $8\oplus 3\oplus\overline{3}$. G_2 gluons would both color octet and triplets. In the TDG framework the only conceivable interpretation would be in terms of ordinary gluons and leptoquark-like gluons. This does not fit with the basic vision of TGD.

The choice of twistor as a selection of quantization axes should make sense also in the M^4 degrees of freedom. M^4 twistor corresponds to a choice of light-like direction at a given point of M^4. The spatial component of the light-like vector fixes the spin quantization axis. Its choice together with the light-likeness fixes the time direction and therefore the rest system and energy quantization axis. Light-like vector fixes also the choice of M^2 and of E^2 as its orthogonal complement. Therefore the fixing of M^4 twistor as a point of $SU(4)/SU(3)\times U(1)$ corresponds to a choice of the spin quantization axis and the time-like axis defining the rest system in which the energy is measured. This choice would naturally correspond to the Hamilton-Jacobi structure fixing the decompositions $M^2(x)\times E^2(x)$. At a given mass shell the choice of the quantization axis would be constant for a given X^3_i.

1.1.8 Hierarchy of Planck Constants and Dark Matter Hierarchy

By quantum classical correspondence space-time sheets can be identified as quantum coherence regions. Hence the fact that they have all possible size scales more or less unavoidably implies that

Planck constant must be quantized and have arbitrarily large values. If one accepts this then also the idea about dark matter as a macroscopic quantum phase characterized by an arbitrarily large value of Planck constant emerges naturally as does also the interpretation for the long ranged classical electro-weak and color fields predicted by TGD. Rather seldom the evolution of ideas follows simple linear logic, and this was the case also now. In any case, this vision represents the fifth, relatively new thread in the evolution of TGD and the ideas involved are still evolving.

Dark Matter as Large $\hbar$ Phases

D. Da Rocha and Laurent Nottale [E1] have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive.

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests that astrophysical systems are at some levels of the hierarchy of space-time sheets macroscopic quantum systems. The space-time sheets in question would carry dark matter.

Nottale's hypothesis would predict a gigantic value of h_{gr}. Equivalence Principle and the independence of gravitational Compton length on mass m implies however that one can restrict the values of mass m to masses of microscopic objects so that h_{gr} would be much smaller. Large h_{gr} could provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale [K97].

It is natural to assign the values of Planck constants postulated by Nottale to the space-time sheets mediating gravitational interaction and identifiable as magnetic flux tubes (quanta) possibly carrying monopole flux and identifiable as remnants of cosmic string phase of primordial cosmology. The magnetic energy of these flux quanta would correspond to dark energy and magnetic tension would give rise to negative "pressure" forcing accelerate cosmological expansion. This leads to a rather detailed vision about the evolution of stars and galaxies identified as bubbles of ordinary and dark matter inside magnetic flux tubes identifiable as dark energy.

Certain experimental findings suggest the identification $h_{eff} = n\times = h_{gr}$. The large value of h_{gr} can be seen as a way to reduce the string tension of fermionic strings so that gravitational (in fact all!) bound states can be described in terms of strings connecting the partonic 2-surfaces defining particles (analogous to AdS/CFT description). The values $h_{eff}/h = n$ can be interpreted in terms of a hierarchy of breakings of super-conformal symmetry in which the super-conformal generators act as gauge symmetries only for a sub-algebras with conformal weights coming as multiples of n. Macroscopic quantum coherence in astrophysical scales is implied. If also Kähler-Dirac action is present, part of the interior degrees of freedom associated with the Kähler-Dirac part of conformal algebra become physical. A possible is that tfermionic oscillator operators generate super-symmetries and sparticles correspond almost by definition to dark matter with $h_{eff}/h = n > 1$. One implication would be that at least part if not all gravitons would be dark and be observed only through their decays to ordinary high frequency graviton ($E = hf_{high} = h_{eff}f_{low}$) of bunch of n low energy gravitons.

Hierarchy of Planck Constants from the Anomalies of Neuroscience and Biology

The quantal ELF effects of ELF em fields on vertebrate brain have been known since seventies. ELF em fields at frequencies identifiable as cyclotron frequencies in magnetic field whose intensity is about 2/5 times that of Earth for biologically important ions have physiological effects and affect also behavior. What is intriguing that the effects are found only in vertebrates (to my best knowledge). The energies for the photons of ELF em fields are extremely low - about 10^{-10} times lower than thermal energy at physiological temperatures- so that quantal effects are impossible in the framework of standard quantum theory. The values of Planck constant would be in these situations large but not gigantic.

This inspired the hypothesis that these photons correspond to so large a value of Planck constant that the energy of photons is above the thermal energy. The proposed interpretation was as dark photons and the general hypothesis was that dark matter corresponds to ordinary matter with non-standard value of Planck constant. If only particles with the same value of Planck constant can appear in the same vertex of Feynman diagram, the phases with different value of Planck constant are dark relative to each other. The phase transitions changing Planck constant can however make possible interactions between phases with different Planck constant but these interactions do not manifest themselves in particle physics. Also the interactions mediated by classical fields should be possible. Dark matter would not be so dark as we have used to believe.

The hypothesis $h_{eff} = h_{gr}$ - at least for microscopic particles - implies that cyclotron energies of charged particles do not depend on the mass of the particle and their spectrum is thus universal although corresponding frequencies depend on mass. In bio-applications this spectrum would correspond to the energy spectrum of bio-photons assumed to result from dark photons by h_{eff} reducing phase transition and the energies of bio-photons would be in visible and UV range associated with the excitations of bio-molecules.

Also the anomalies of biology (see for instance [K85, K86, K82]) support the view that dark matter might be a key player in living matter.

Dark Matter as a Source of Long Ranged Weak and Color Fields

Long ranged classical electro-weak and color gauge fields are unavoidable in TGD framework. The smallness of the parity breaking effects in hadronic, nuclear, and atomic length scales does not however seem to allow long ranged electro-weak gauge fields. The problem disappears if long range classical electro-weak gauge fields are identified as space-time correlates for massless gauge fields created by dark matter. Also scaled up variants of ordinary electro-weak particle spectra are possible. The identification explains chiral selection in living matter and unbroken $U(2)_{ew}$ invariance and free color in bio length scales become characteristics of living matter and of bio-chemistry and bio-nuclear physics.

The recent view about the solutions of Kähler- Dirac action assumes that the modes have a well-defined em charge and this implies that localization of the modes to 2-D surfaces (right-handed neutrino is an exception). Classical W boson fields vanish at these surfaces and also classical Z^0 field can vanish. The latter would guarantee the absence of large parity breaking effects above intermediate boson scale scaling like h_{eff}.

1.1.9 Twistors in TGD and connection with Veneziano duality

The twistorialization of TGD has two aspects. The attempt to generalize twistor Grassmannian approach emerged first. It was however followed by the realization that also the twistor lift of TGD at classical space-time level is needed. It turned out that the progress in the understanding of the classical twistor lift has been much faster - probably this is due to my rather limited technical QFT skills.

Twistor lift at space-time level

8-dimensional generalization of ordinary twistors is highly attractive approach to TGD [L8]. The reason is that M^4 and CP_2 are completely exceptional in the sense that they are the only 4-D manifolds allowing twistor space with Kähler structure [A61]. The twistor space of $M^4 \times CP_2$ is Cartesian product of those of M^4 and CP_2. The obvious idea is that space-time surfaces allowing twistor structure if they are orientable are representable as surfaces in H such that the properly induced twistor structure co-incides with the twistor structure defined by the induced metric.

In fact, it is enough to generalize the induction of spinor structure to that of twistor structure so that the induced twistor structure need not be identical with the ordinary twistor structure possibly assignable to the space-time surface. The induction procedure reduces to a dimensional reduction of 6-D Kähler action giving rise to 6-D surfaces having bundle structure with twistor sphere as fiber and space-time as base. The twistor sphere of this bundle is imbedded as sphere in the product of twistor spheres of twistor spaces of M^4 and CP_2.

This condition would define the dynamics, and the original conjecture was that this dynamics is equivalent with the identification of space-time surfaces as preferred extremals of Kähler action. The dynamics of space-time surfaces would be lifted to the dynamics of twistor spaces, which are sphere bundles over space-time surfaces. What is remarkable that the powerful machinery of complex analysis becomes available.

It however turned out that twistor lift of TGD is much more than a mere technical tool. First of all, the dimensionally reduction of 6-D Kähler action contained besides 4-D Kähler action also a volume term having interpretation in terms of cosmological constant. This need not bring anything new, since all known extremals of Kähler action with non-vanishing induced Kähler form are minimal surfaces. There is however a large number of embeddings of twistor sphere of space-time surface to the product of twistor spheres. Cosmological constant has spectrum and depends on length scale, and the proposal is that coupling constant evolution reduces to that for cosmological constant playing the role of cutoff length. That cosmological constant could transform from a mere nuisance to a key element of fundamental physics was something totally new and unexpected.

1. The twistor lift of TGD at space-time level forces to replace 4-D Kähler action with 6-D dimensionally reduced Kähler action for 6-D surface in the 12-D Cartesian product of 6-D twistor spaces of M^4 and CP_2. The 6-D surface has bundle structure with twistor sphere as fiber and space-time surface as base.

   Twistor structure is obtained by inducing the twistor structure of 12-D twistor space using dimensional reduction. The dimensionally reduced 6-D Kähler action is sum of 4-D Kähler action and volume term having interpretation in terms of a dynamical cosmological constant depending on the size scale of space-time surface (or of causal diamond CD in zero energy ontology (ZEO)) and determined by the representation of twistor sphere of space-time surface in the Cartesian product of the twistor spheres of M^4 and CP_2.

2. The preferred extremal property as a representation of quantum criticality would naturally correspond to minimal surface property meaning that the space-time surface is separately an extremal of both Kähler action and volume term almost everywhere so that there is no coupling between them. This is the case for all known extremals of Kähler action with non-vanishing induced Kähler form.

   Minimal surface property could however fail at 2-D string world sheets, their boundaries and perhaps also at partonic 2-surfaces. The failure is realized in minimal sense if the 3-surface has 1-D edges/folds (strings) and 4-surface 2-D edges/folds (string world sheets) at which some partial derivatives of the embedding space coordinates are discontinuous but canonical momentum densities for the entire action are continuous.

   There would be no flow of canonical momentum between interior and string world sheet and minimal surface equations would be satisfied for the string world sheet, whose 4-D counterpart in twistor bundle is determined by the analog of 4-D Kähler action. These conditions allow the transfer of canonical momenta between Kähler- and volume degrees of freedom at string world sheets. These no-flow conditions could hold true at least asymptotically (near the boundaries of CD).

   $M^8 - H$ duality suggests that string world sheets (partonic 2-surfaces) correspond to images of complex 2-sub-manifolds of M^8 (having tangent (normal) space which is complex 2-plane of octonionic M^8).

3. Cosmological constant would depend on p-adic length scales and one ends up to a concrete model for the evolution of cosmological constant as a function of p-adic length scale and other number theoretic parameters (such as Planck constant as the order of Galois group): this conforms with the earlier picture.

   Inflation is replaced with its TGD counterpart in which the thickening of cosmic strings to flux tubes leads to a transformation of Kähler magnetic energy to ordinary and dark matter. Since the increase of volume increases volume energy, this leads rapidly to energy minimum at some flux tube thickness. The reduction of cosmological constant by a phase transition however leads to a new expansion phase. These jerks would replace smooth cosmic expansion of GRT. The discrete coupling constant evolution predicted by the number theoretical vision could be understood as being induced by that of cosmological constant taking the role of cutoff parameter in QFT picture [L36].

Twistor lift at the level of scattering amplitudes and connection with Veneziano duality

The classical part of twistor lift of TGD is rather well-understood. Concerning the twistorialization at the level of scattering amplitudes the situation is much more difficult conceptually - I already mentioned my limited QFT skills.

1. From the classical picture described above it is clear that one should construct the 8-D twistorial counterpart of theory involving space-time surfaces, string world sheets and their boundaries, plus partonic 2-surfaces and that this should lead to concrete expressions for the scattering amplitudes.

   The light-like boundaries of string world sheets as carriers of fermion numbers would correspond to twistors as they appear in twistor Grassmann approach and define the analog for the massless sector of string theories. The attempts to understand twistorialization have been restricted to this sector.

2. The beautiful basic prediction would be that particles massless in 8-D sense can be massive in 4-D sense. Also the infrared cutoff problematic in twistor approach emerges naturally and reduces basically to the dynamical cosmological constant provided by classical twistor lift.

   One can assign 4-momentum both to the spinor harmonics of the embedding space representing ground states of super-conformal representations and to light-like boundaries of string world sheets at the orbits of partonic 2-surfaces. The two four-momenta should be identical by quantum classical correspondence: this could be seen as a concretization of Equivalence Principle. Also a connection with string model emerges.

3. As far as symmetries are considered, the picture looks rather clear. Ordinary twistor Grassmannian approach boils down to the construction of scattering amplitudes in terms of Yangian invariants for conformal group of M^4. Therefore a generalization of super-symplectic symmetries to their Yangian counterpart seems necessary. These symmetries would be gigantic but how to deduce their implications?

4. The notion of positive Grassmannian is central in the twistor approach to the scattering amplitudes in $calN = 4$ SUSYs. TGD provides a possible generalization and number theoretic interpretation of this notion. TGD generalizes the observation that scattering amplitudes in twistor Grassmann approach correspond to representations for permutations. Since 2-vertex is the only fermionic vertex in TGD, OZI rules for fermions generalizes, and scattering amplitudes are representations for braidings.

   Braid interpretation encourages the conjecture that non-planar diagrams can be reduced to ordinary ones by a procedure analogous to the construction of braid (knot) invariants by gradual un-braiding (un-knotting).

This is however not the only vision about a solution of non-planarity. Quantum criticality provides different view leading to a totally unexpected connection with string models, actually with the Veneziano duality, which was the starting point of dual resonance model in turn leading via dual resonance models to super string models.

1. Quantum criticality in TGD framework means that coupling constant evolution is discrete in the sense that coupling constants are piecewise constant functions of length scale replaced by dynamical cosmological constant. Loop corrections would vanish identically and the recursion formulas for the scattering amplitudes (allowing only planar diagrams) deduced in twistor Grassmann would involve no loop corrections. In particular, cuts would be replaced by sequences of poles mimicking them like sequences of point charge mimic line charges. In momentum discretization this picture follows automatically.

2. This would make sense in finite measurement resolution realized in number theoretical vision by number-theoretic discretization of the space-time surface (cognitive representation) as points with coordinates in the extension of rationals defining the adele [L30]. Similar discretization would take place for momenta. Loops would vanish at the level of discretization but what would happen at the possibly existing continuum limit: does the sequence of poles integrate to cuts? Or is representation as sum of resonances something much deeper?

3. Maybe it is! The basic idea of behind the original Veneziano amplitudes (see http://tinyurl.com/yyhwvbqb) was Veneziano duality. This 4-particle amplitude was generalized by Yoshiro Nambu, Holber-Beck Nielsen, and Leonard Susskind to N-particle amplitude (see http://tinyurl.com/yyvkx7as) based on string picture, and the resulting model was called dual resonance model. The model was forgotten as QCD emerged. Later came superstring models and led to M-theory. Now it has become clear that something went wrong, and it seems that one must return to the roots. Could the return to the roots mean a careful reconsideration of the dual resonance model?

4. Recall that Veneziano duality (1968) was deduced by assuming that scattering amplitude can be described as sum over s-channel resonances or t-channel Regge exchanges and Veneziano duality stated that hadronic scattering amplitudes have representation as sums over s- or t-channel resonance poles identified as excitations of strings. The sum over exchanges defined by t-channel resonances indeed reduces at larger values of s to Regge form.

   The resonances had zero width, which was not consistent with unitarity. Further, there were no counterparts for the *sum* of s-, t-, and u-channel diagrams with continuous cuts in the kinematical regions encountered in QFT approach. What puts bells ringing is the u-channel diagrams would be non-planar and non-planarity is the problem of twistor Grassmann approach.

5. Veneziano duality is true only for s- and t- channels but not been s- and u-channel. Stringy description makes t-channel and s-channel pictures equivalent. Could it be that in fundamental description u-channels diagrams cannot be distinguished from s-channel diagrams or t-channel diagrams? Could the stringy representation of the scattering diagrams make u-channel twist somehow trivial if handles of string world sheet representing stringy loops in turn representing the analog of non-planarity of Feynman diagrams are absent? The permutation of external momenta for tree diagram in absence of loops in planar representation would be a twist of π in the representation of planar diagram as string world sheet and would not change the topology of the string world sheet and would not involve non-trivial world sheet topology.

   For string world sheets loops would correspond to handles. The presence of handle would give an edge with a loop at the level of 3-surface (self energy correction in QFT). Handles are not allowed if the induced metric for the string world sheet has Minkowskian signature. If the stringy counterparts of loops are absent, also the loops in scattering amplitudes should be absent.

   This argument applies only inside the Minkowskian space-time regions. If string world sheets are present also in Euclidian regions, they might have handles and loop corrections could emerge in this manner. In TGD framework strings (string world sheets) are identified to 1-D edges/folds of 3-surface at which minimal surface property and topological QFT property fails (minimal surfaces as calibrations). Could the interpretation of edge/fold as discontinuity of some partial derivatives exclude loopy edges: perhaps the branching points would be too singular?

A reduction to a sum over s-channel resonances is what the vanishing of loops would suggest. Could the presence of string world sheets make possible the vanishing of continuous cuts even at the continuum limit so that continuum cuts would emerge only in the approximation as the density of resonances is high enough?

The replacement of continuous cut with a sum of *infinitely* narrow resonances is certainly an approximation. Could it be that the stringy representation as a sum of resonances with *finite* width is an essential aspect of quantum physics allowing to get rid of infinities necessarily accompanying loops? Consider now the arguments against this idea.

1. How to get rid of the problems with unitarity caused by the zero width of resonances? Could *finite* resonance widths make unitarity possible? Ordinary twistor Grassmannian approach predicts that the virtual momenta are light-like but complex: obviously, the imaginary part of the energy in rest frame would have interpretation as resonance with.

   In TGD framework this generalizes for 8-D momenta. By quantum-classical correspondence (QCC) the classical Noether charges are equal to the eigenvalues of the fermionic charges in Cartan algebrable (maximal set of mutually commuting observables) and classical TGD indeed predicts complex momenta (Kähler coupling strength is naturally complex). QCC thus supports this proposal.

2. Sum over resonances/exchanges picture is in conflict with QFT picture about scattering of particles. Could *finite* resonance widths due to the complex momenta give rise to the QFT type scattering amplitudes as one develops the amplitudes in Taylor series with respect to the resonance width? Unitarity condition indeed gives the first estimate for the resonance width.

   QFT amplitudes should emerge in an approximation obtained by replacing the discrete set of finite width resonances with a cut as the distance between poles is shorter than the resolution for mass squared.

   In superstring models string tension has single very large value and one cannot obtain QFT type behavior at low energies (for instance, scattering amplitudes in hadronic string model are concentrated in forward direction). TGD however predicts an entire hierarchy of p-adic length scales with varying string tension. The hierarchy of mass scales corresponding roughly to the lengths and thickness of magnetic flux tubes as thickened cosmic strings and characterized by the value of cosmological constant predicted by twistor lift of TGD. Could this give rise to continuous QCT type cuts at the limit when measurement resolution cannot distinguish between resonances?

   The dominating term in the sum over sums of resonances in t-channel gives near forward direction approximately the lowest mass resonance for strings with the smallest string tension. This gives the behavior $1/(t - m_{min}^2)$, where m_{min} corresponds to the longest mass scale involved (the largest space-time sheet involved), approximating the $1/t$-behavior of massless theories. This also brings in IR cutoff, the lack of which is a problem of gauge theories. This should give rise to continuous QFT type cuts at the limit when measurement resolution cannot distinguish between resonances.

1.1.10 Organization of "TGD: an Overview: Part I"

"TGD: an Overview: Part I" tries to give an overall view about quantum TGD as it stands now. The book consists of 3 parts.

1. In the 1st part I will try to give an overall view about the evolution of TGD and about quantum TGD in its recent form. I cannot avoid the use of various concepts without detailed definitions and my hope is that reader only gets a bird's eye of view about TGD. Two visions about physics are discussed. According to the first vision physical states of the Universe correspond to classical spinor fields in the world of the classical worlds identified as 3-surfaces or equivalently as corresponding 4-surfaces analogous to Bohr orbits and identified as special extrema of Kähler action. TGD as a generalized number theory vision leading naturally also to the emergence of p-adic physics as physics of cognitive representations is the second vision.
2. The 2nd part is devoted to the vision about physics as infinite-dimensional configuration space geometry. The basic idea is that classical spinor fields in infinite-dimensional "world of classical worlds", space of 3-surfaces in $M^4 \times CP_2$ describe the quantum states of the Universe. Quantum jump remains the only purely quantal aspect of quantum theory in this approach since there is no quantization at the level of the configuration space. Space-time surfaces correspond to special extremals of the Kähler action analogous to Bohr orbits and define what might be called classical TGD discussed in the first chapter. The construction of the configuration space geometry and spinor structure are discussed in remaining chapters.
3. The 3rd part of the book describes physics as generalized number theory vision. Number theoretical vision involves three loosely related approaches: fusion of real and various p-adic physics to a larger whole as algebraic continuations of what might be called rational physics; space-time as a hyper-quaternionic surface of hyper-octonion space, and space-time surfaces as a representations of infinite primes.

1.2 Sources

The eight online books about TGD [K117, K112, K91, K75, K22, K70, K49, K100] and nine online books about TGD inspired theory of consciousness and quantum biology [K108, K18, K81, K16, K45, K57, K60, K99, K107] are warmly recommended for the reader willing to get overall view about what is involved.

My homepage (http://tinyurl.com/ybv8dt4n) contains a lot of material about TGD. In particular, a TGD glossary at http://tinyurl.com/yd6jf3o7).

I have published articles about TGD and its applications to consciousness and living matter in *Journal of Non-Locality* (http://tinyurl.com/ycyrxj4o founded by Lian Sidorov and in *Prespacetime Journal* (http://tinyurl.com/ycvktjhn), *Journal of Consciousness Research and Exploration* (http://tinyurl.com/yba4f672), and *DNA Decipher Journal* (http://tinyurl.com/y9z52khg), all of them founded by Huping Hu. One can find the list about the articles published at http://tinyurl.com/ybv8dt4n. I am grateful for these far-sighted people for providing a communication channel, whose importance one cannot overestimate.

1.3 The contents of the book

1.3.1 PART I: PHYSICS AS INFINITE-DIMENSIONAL GEOMETRY

Why TGD and What TGD is?

This piece of text was written as an attempt to provide a popular summary about TGD. This is of course mission impossible since TGD is something at the top of centuries of evolution which has led from Newton to standard model. This means that there is a background of highly refined conceptual thinking about Universe so that even the best computer graphics and animations fail to help. One can still try to create some inspiring impressions at least. This chapter approaches the challenge by answering the most frequently asked questions. Why TGD? How TGD could help to solve the problems of recent day theoretical physics? What are the basic princples of TGD? What are the basic guidelines in the construction of TGD?

These are examples of this kind of questions which I try to answer in using the only language that I can talk. This language is a dialect of the language used by elementary particle physicists, quantum field theorists, and other people applying modern physics. At the level of practice involves technically heavy mathematics but since it relies on very beautiful and simple basic concepts, one can do with a minimum of formulas, and reader can always to to Wikipedia if it seems that more details are needed. I hope that reader could catch the basic principles and concepts: technical details are not important. And I almost forgot: problems! TGD itself and almost every new idea in the development of TGD has been inspired by a problem.

Topological Geometrodynamics: Three Visions

In this chapter I will discuss three basic visions about quantum Topological Geometrodynamics (TGD). It is somewhat matter of taste which idea one should call a vision and the selection of these three in a special role is what I feel natural just now.

1. The first vision is generalization of Einstein's geometrization program based on the idea that the Kähler geometry of the world of classical worlds (WCW) with physical states identified as classical spinor fields on this space would provide the ultimate formulation of physics.
2. Second vision is number theoretical and involves three threads. The first thread relies on the idea that it should be possible to fuse real number based physics and physics associated with various p-adic number fields to single coherent whole by a proper generalization of number concept. Second thread is based on the hypothesis that classical number fields could allow to understand the fundamental symmetries of physics and and imply quantum TGD from purely number theoretical premises with associativity defining the fundamental dynamical principle both classically and quantum mechanically. The third thread relies on the notion of infinite primes whose construction has amazing structural similarities with second quantization of super-symmetric quantum field theories. In particular, the hierarchy of infinite primes and integers allows to generalize the notion of numbers so that given real number has infinitely rich number theoretic anatomy based on the existence of infinite number of real units.
3. The third vision is based on TGD inspired theory of consciousness, which can be regarded as an extension of quantum measurement theory to a theory of consciousness raising observer from an outsider to a key actor of quantum physics.

TGD Inspired Theory of Consciousness

The basic ideas and implications of TGD inspired theory of consciousness are briefly summarized.

The quantum notion of self solved several key problems of TGD inspired theory of consciousness but the precise definition of self has also remained a long standing problem and I have been even ready to identify self with quantum jump. Zero energy ontology allows what looks like a final solution of the problem. Self indeed corresponds to a sequence of quantum jumps integrating to single unit, but these quantum jumps correspond state function reductions to a fixed boundary of CD leaving the corresponding parts of zero energy states invariant. In positive energy ontology these repeated state function reductions would have no effect on the state but in TGD framework there occurs a change for the second boundary and gives rise to the experienced flow of time and its arrow and gives rise to self. The first quantum jump to the opposite boundary corresponds to the act of free will or wake-up of self.

p-Adic physics as correlate for cognition and intention leads to the notion of negentropic entanglement possible in the intersection of real and p-adic worlds involves experience about expansion of consciousness. Consistency with standard quantum measurement theory forces negentropic entanglement to correspond to density matrix proportional to unit matrix. Unitary entanglement typical for quantum computing systems gives rise to unitary entanglement.

With the advent of the hierarchy of Planck constants realized in terms of generalized embedding space and of zero energy ontology emerged the idea that self hierarchy could be reduced to a fractal hierarchy of quantum jumps within quantum jumps. It seems now clear that the two definitions of self are consistent with each other. The identification of the imbedding space correlate of self as causal diamond (CD) of the embedding space combined with the identification of space-time correlates as space-time sheets inside CD solved also the problems concerning the relationship between geometric and subjective time. A natural conjecture is that the the integer n in $h_{eff} = n \times h$ corresponds to the dimension of the unit matrix associated with negentropic entanglement. Also a connection with quantum criticality made possible by non-determinism of Kähler action and extended conformal invariance emerges so that there is high conceptual coherence between the new concepts inspired by TGD.

Negentropy Maximization Principle (NMP) serves as a basic variational principle for the dynamics of quantum jump. The new view about the relation of geometric and subjective time leads to a new view about memory and intentional action. The quantum measurement theory based on finite measurement resolution and realized in terms of hyper-finite factors of type II_1 justifies the notions of sharing of mental images and stereo-consciousness deduced earlier on basis of quantum classical correspondence. Qualia reduce to quantum number increments associated with quantum jump. Self-referentiality of consciousness can be understood from quantum classical correspondence implying a symbolic representation of contents of consciousness at space-time level updated in each quantum jump. p-Adic physics provides space-time correlates for cognition and intentionality.

TGD and M-Theory

In this chapter a critical comparison of M-theory and TGD as two competing theories is carried out. Dualities and black hole physics are regarded as basic victories of M-theory.

1. The counterpart of electric magnetic duality plays an important role also in TGD and it has become clear that it might change the sign of Kähler coupling strength rather than leaving it invariant. The different signs would be related to different time orientations of the space-time sheets. This option is favored also by TGD inspired cosmology but unitarity seems to exclude it.

2. The AdS/CFT duality of Maldacena involved with the quantum gravitational holography has a direct counterpart in TGD with 3-dimensional causal determinants serving as holograms so that the construction of absolute minima of Kähler action reduces to a local problem.

3. The attempts to develop further the nebulous idea about space-time surfaces as associative (co-associative) sub-manifolds of an octonionic embedding space led to the realization of duality which could be called number theoretical spontaneous compactification. Space-time region can be regarded equivalently as a associative (co-associative) space-time region in M^8

with octonionic structure or as a 4-surface in $M^4 \times CP_2$. If the map taking these surface to each other preserves associtiativity in octonionic structure of H then the generalization to $H - H$ duality becomes natural and would make preferred extremals a category.

4. The notion of cotangent bundle of configuration space of 3-surfaces (WCW) suggests the interpretation of the number-theoretical compactification as a wave-particle duality in infinite-dimensional context. These ideas generalize at the formal level also to the M-theory assuming that stringy configuration space is introduced. The existence of Kähler metric very probably does not allow dynamical target space.

In TGD framework black holes are possible but putting black holes and particles in the same basket seems to be mixing of apples with oranges. The role of black hole horizons is taken in TGD by 3-D light like causal determinants, which are much more general objects. Black hole-elementary particle correspondence and p-adic length scale hypothesis have already earlier led to a formula for the entropy associated with elementary particle horizon.

In TGD framework the interior of blackhole is naturally replaced with a region of Euclidian signature of induced metric and can be seen as analog for the line of Feynman diagram. Blackholes appear only in GRT limit of TGD which lumps together the sheets of many-sheeted space-time to a piece of Minkowski space and provides it with an effective metric determined as sum of Minkowski metric and deviations of the metrics of space-time sheets from Minkowski metric.

The recent findings from RHIC have led to the realization that TGD predicts black hole like objects in all length scales. They are identifiable as highly tangled magnetic flux tubes in Hagedorn temperature and containing conformally confined matter with a large Planck constant and behaving like dark matter in a macroscopic quantum phase. The fact that string like structures in macroscopic quantum states are ideal for topological quantum computation modifies dramatically the traditional view about black holes as information destroyers.

The discussion of the basic weaknesses of M-theory is motivated by the fact that the few predictions of the theory are wrong which has led to the introduction of anthropic principle to save the theory. The mouse as a tailor history of M-theory, the lack of a precise problem to which M-theory would be a solution, the hard nosed reductionism, and the censorship in Los Alamos archives preventing the interaction with competing theories could be seen as the basic reasons for the recent blind alley in M-theory.

Can one apply Occam's razor as a general purpose debunking argument to TGD?

Occarm's razor have been used to debunk TGD. The following arguments provide the information needed by the reader to decide himself. Considerations are at three levels.

The level of "world of classical worlds" (WCW) defined by the space of 3-surfaces endowed with Kähler structure and spinor structure and with the identification of WCW space spinor fields as quantum states of the Universe: this is nothing but Einstein's geometrization program applied to quantum theory. Second level is space-time level.

Space-time surfaces correspond to preferred extremals of Käction in $M^4 \times CP_2$. The number of field like variables is 4 corresponding to 4 dynamically independent embedding space coordinates. Classical gauge fields and gravitational field emerge from the dynamics of 4-surfaces. Strong form of holography reduces this dynamics to the data given at string world sheets and partonic 2-surfaces and preferred extremals are minimal surface extremals of Kähler action so that the classical dynamics in space-time interior does not depend on coupling constants at all which are visible via boundary conditions only. Continuous coupling constant evolution is replaced with a sequence of phase transitions between phases labelled by critical values of coupling constants: loop corrections vanish in given phase. Induced spinor fields are localized at string world sheets to guarantee well-definedness of em charge.

At embedding space level the modes of embedding space spinor fields define ground states of super-symplectic representations and appear in QFT-GRT limit. GRT involves post-Newtonian approximation involving the notion of gravitational force. In TGD framework the Newtonian force correspond to a genuine force at embedding space level.

I was also asked for a summary about what TGD is and what it predicts. I decided to add this summary to this chapter although it is goes slightly outside of its title.

1.3.2 PART II: PHYSICS AS INFINITE-DIMENSIONAL SPINOR GEOMETRY

The geometry of the world of classical worlds

The topics of this chapter are the purely geometric aspects of the vision about physics as an infinite-dimensional Kähler geometry of configuration space or the "world of classical worlds"(WCW), with "classical world" identified either as 3-D surface of the unique Bohr orbit like 4-surface traversing through it. The non-determinism of Kähler action forces to generalize the notion of 3-surfaces so that unions of space-like surfaces with time like separations must be allowed. The considerations are restricted mostly to real context and the problems related to the p-adicization are discussed later.

There are two separate tasks involved.

1. Provide WCW with Kähler geometry which is consistent with 4-dimensional general coordinate invariance so that the metric is Diff^4 degenerate. General coordinate invariance implies that the definition of metric must assign to a give 3-surface X^3 a 4-surface as a kind of Bohr orbit $X^4(X^3)$.
2. Provide the WCW with a spinor structure. The great idea is to identify WCW gamma matrices in terms of super algebra generators expressible using second quantized fermionic oscillator operators for induced free spinor fields at the space-time surface assignable to a given 3-surface. The isometry generators and contractions of Killing vectors with gamma matrices would thus form a generalization of Super Kac-Moody algebra.

From the experience with loop spaces one can expect that there is no hope about existence of well-defined Riemann connection unless this space is union of infinite-dimensional symmetric spaces with constant curvature metric and simple considerations requires that Einstein equations are satisfied by each component in the union. The coordinates labeling these symmetric spaces are zero modes having interpretation as genuinely classical variables which do not quantum fluctuate since they do not contribute to the line element of the WCW. The construction of WCW Kähler geometry requires also the identification of complex structure and thus complex coordinates of WCW. A natural identification of symplectic coordinates is as classical symplectic Noether charges and their canonical conjugates.

There are three approaches to the construction of the Kähler metric.

1. Direct construction of Kähler function as action associated with a preferred Bohr orbit like extremal for some physically motivated action action leads to a unique result using standard formula once complex coordinates of WCW have been identified. The realiation in practice is not easy-
2. Second approach is group theoretical and is based on a direct guess of isometries of the infinite-dimensional symmetric space formed by 3-surfaces with fixed values of zero modes. The group of isometries is generalization of Kac-Moody group obtained by replacing finite-dimensional Lie group with the group of symplectic transformations of $\delta M^4_+ \times CP_2$, where δM^4_+ is the boundary of 4-dimensional future light-cone. The guesses for the Kähler metric rely on the symmetry considerations but have suffered from ad hoc character.
3. The third approach identifies the elements of WCW Kähler metric as anti-commutators of WCW gamma matrices identified as super-symplectic super-generators defined as Noether charges for Kähler- Dirac action. This approach leads to explicit formulas and to a natural generalization of the super-symplectic algebra to Yangian giving additional poly-local contributions to WCW metric. Contributions are expressible as anticommutators of super-charges associated with strings and one ends up to a generalization of AdS/CFT duality stating in the special case that the expression for WCW Kähler metric in terms of Kähler function is equivalent with the expression in terms of fermionic super-charges associated with strings connecting partonic 2-surfaces.

Classical TGD

In this chapter the classical field equations associated with the Kähler action are studied.

1. Are all extremals actually "preferred"?

The notion of preferred extremal has been central concept in TGD but is there really compelling need to pose any condition to select preferred extremals in zero energy ontology (ZEO) as there would be in positive energy ontology? In ZEO the union of the space-like ends of space-time surfaces at the boundaries of causal diamond (CD) are the first guess for 3-surface. If one includes to this 3-surface also the light-like partonic orbits at which the signature of the induced metric changes to get analog of Wilson loop, one has good reasons to expect that the preferred extremal is highly unique without any additional conditions apart from non-determinism of Kähler action proposed to correspond to sub-algebra of conformal algebra acting on the light-like 3-surface and respecting light-likeness. One expects that there are finite number n of conformal equivalence classes and n corresponds to n in $h_{eff} = nh$. These objects would allow also to understand the assignment of discrete physical degrees of freedom to the partonic orbits as required by the assignment of hierarchy of Planck constants to the non-determinism of Kähler action.

2. Preferred extremals and quantum criticality

The identification of preferred extremals of Kähler action defining counterparts of Bohr orbits has been one of the basic challenges of quantum TGD. By quantum classical correspondence the non-deterministic space-time dynamics should mimic the dissipative dynamics of the quantum jump sequence.

The space-time representation for dissipation comes from the interpretation of regions of space-time surface with Euclidian signature of induced metric as generalized Feynman diagrams (or equivalently the light-like 3-surfaces defining boundaries between Euclidian and Minkowskian regions). Dissipation would be represented in terms of Feynman graphs representing irreversible dynamics and expressed in the structure of zero energy state in which positive energy part corresponds to the initial state and negative energy part to the final state. Outside Euclidian regions classical dissipation should be absent and this indeed the case for the known extremals.

The non-determinism should also give rose to space-time correlate for quantum criticality. The study of Kähler-Dirac equations suggests how to define quantum criticality. Noether currents assignable to the Kähler-Dirac equation are conserved only if the first variation of Kähler-Dirac operator D_K defined by Kähler action vanishes. This is equivalent with the vanishing of the second variation of Kähler action - at least for the variations corresponding to dynamical symmetries having interpretation as dynamical degrees of freedom which are below measurement resolution and therefore effectively gauge symmetries.

It became later clear that the well-definedness of em charge forces in the generic case the localization of the spinor modes to 2-D surfaces - string world sheets. This would suggest that the equations stating the vanishing of the second variation of Kähler action hold true only at string world sheets.

The vanishing of second variations of preferred extremals suggests a generalization of catastrophe theory of Thom, where the rank of the matrix defined by the second derivatives of potential function defines a hierarchy of criticalities with the tip of bifurcation set of the catastrophe representing the complete vanishing of this matrix. In zero energy ontology (ZEO) catastrophe theory would be generalized to infinite-dimensional context. Finite number of sheets for catastrophe would be replaced with finite number of conformal equivalence classes of space-time surfaces connecting given space-like 3-surfaces at the boundaries causal diamond (CD).

3. Hamilton-Jacobi structure

Most known extremals share very general properties. One of them is Hamilton-Jacobi structure meaning the possibility to assign to the extremal so called Hamilton-Jacobi coordinates. This means dual slicings of M^4 by string world sheets and partonic 2-surfaces. Number theoretic compactification led years later to the same condition. This slicing allows a dimensional reduction of quantum TGD to Minkowskian and Euclidian variants of string model. Also holography in the sense that the dynamics of 3-dimensional space-time surfaces reduces to that for 2-D partonic surfaces in a given measurement resolution follows. The construction of quantum TGD relies in essential manner to this property. CP_2 type vacuum extremals do not possess Hamilton-Jaboci structure but have holomorphic structure.

4. Specific extremals of Kähler action

The study of extremals of Kähler action represents more than decade old layer in the development of TGD.

1. The huge vacuum degeneracy is the most characteristic feature of Kähler action (any 4-surface having CP_2 projection which is Legendre sub-manifold is vacuum extremal, Legendre sub-manifolds of CP_2 are in general 2-dimensional). This vacuum degeneracy is behind the spin glass analogy and leads to the p-adic TGD. As found in the second part of the book, various particle like vacuum extremals also play an important role in the understanding of the quantum TGD.
2. The so called CP_2 type vacuum extremals have finite, negative action and are therefore an excellent candidate for real particles whereas vacuum extremals with vanishing Kähler action are candidates for the virtual particles. These extremals have one dimensional M^4 projection, which is light like curve but not necessarily geodesic and locally the metric of the extremal is that of CP_2: the quantization of this motion leads to Virasoro algebra. Space-times with topology $CP_2\#CP_2\#...CP_2$ are identified as the generalized Feynmann diagrams with lines thickened to 4-manifolds of "thickness" of the order of CP_2 radius. The quantization of the random motion with light velocity associated with the CP_2 type extremals in fact led to the discovery of Super Virasoro invariance, which through the construction of the WCW geometry, becomes a basic symmetry of quantum TGD.
3. There are also various non-vacuum extremals.
   (a) String like objects, with string tension of same order of magnitude as possessed by the cosmic strings of GUTs, have a crucial role in TGD inspired model for the galaxy formation and in the TGD based cosmology.
   (b) The so called massless extremals describe non-linear plane waves propagating with the velocity of light such that the polarization is fixed in given point of the space-time surface. The purely TGD:eish feature is the light like Kähler current: in the ordinary Maxwell theory vacuum gauge currents are not possible. This current serves as a source of coherent photons, which might play an important role in the quantum model of bio-system as a macroscopic quantum system.

1.3.3 PART III: PHYSICS AS GENERALIZED NUMBER THEORY

Physics as a generalized number theory

There are two basic approaches to the construction of quantum TGD. The first approach relies on the vision of quantum physics as infinite-dimensional Kähler geometry for the "world of classical worlds" identified as the space of 3-surfaces in in certain 8-dimensional space. Essentially a generalization of the Einstein's geometrization of physics program is in question.

The second vision identifies physics as a generalized number theory and involves three threads: various p-adic physics and their fusion together with real number based physics to a larger structure, the attempt to understand basic physics in terms of classical number fields (in particular, identifying associativity condition as the basic dynamical principle), and infinite primes whose construction is formally analogous to a repeated second quantization of an arithmetic quantum field theory.

1. p-Adic physics and their fusion with real physics

The basic technical problems of the fusion of real physics and various p-adic physics to single coherent whole relate to the notion of definite integral both at space-time level, embedding space level and the level of WCW (the "world of classical worlds"). The expressibility of WCW as a union of symmetric spacesleads to a proposal that harmonic analysis of symmetric spaces can be used to define various integrals as sums over Fourier components. This leads to the proposal the p-adic variant of symmetric space is obtained by a algebraic continuation through a common intersection of these spaces, which basically reduces to an algebraic variant of coset space involving algebraic extension of rationals by roots of unity. This brings in the notion of angle measurement resolution coming as $\Delta\phi = 2\pi/p^n$ for given p-adic prime p. Also a proposal how one can complete the discrete version of symmetric space to a continuous p-adic versions emerges and means that each point is effectively replaced with the p-adic variant of the symmetric space identifiable as a p-adic counterpart of the real discretization volume so that a fractal p-adic variant of symmetric space results.

If the Kähler geometry of WCW is expressible in terms of rational or algebraic functions, it can in principle be continued the p-adic context. One can however consider the possibility that that the integrals over partonic 2-surfaces defining flux Hamiltonians exist p-adically as Riemann sums. This requires that the geometries of the partonic 2-surfaces effectively reduce to finite sub-manifold geometries in the discretized version of $\delta M^4_+ \times CP_2$. If Kähler action is required to exist p-adically same kind of condition applies to the space-time surfaces themselves. These strong conditions might make sense in the intersection of the real and p-adic worlds assumed to characterized living matter.

2. TGD and classical number fields

The basis vision is that the geometry of the infinite-dimensional WCW ("world of classical worlds") is unique from its mere existence. This leads to its identification as union of symmetric spaces whose Kähler geometries are fixed by generalized conformal symmetries. This fixes space-time dimension and the decomposition $M^4 \times S$ and the idea is that the symmetries of the Kähler manifold S make it somehow unique. The motivating observations are that the dimensions of classical number fields are the dimensions of partonic 2-surfaces, space-time surfaces, and embedding space and M^8 can be identified as hyper-octonions- a sub-space of complexified octonions obtained by adding a commuting imaginary unit. This stimulates some questions.

Could one understand $S = CP_2$ number theoretically in the sense that M^8 and $H = M^4 \times CP_2$ be in some deep sense equivalent ("number theoretical compactification" or $M^8 - H$ duality)? Could associativity define the fundamental dynamical principle so that space-time surfaces could be regarded as associative or co-associative (defined properly) sub-manifolds of M^8 or equivalently of H.

One can indeed define the associative (co-associative) 4-surfaces using octonionic representation of gamma matrices of 8-D spaces as surfaces for which the Kähler-Dirac gamma matrices span an associate (co-associative) sub-space at each point of space-time surface. In fact, only octonionic structure is needed. Also $M^8 - H$ duality holds true if one assumes that this associative sub-space at each point contains preferred plane of M^8 identifiable as a preferred commutative or co-commutative plane (this condition generalizes to an integral distribution of commutative planes in M^8). These planes are parametrized by CP_2 and this leads to $M^8 - H$ duality.

WCW itself can be identified as the space of 4-D local sub-algebras of the local Clifford algebra of M^8 or H which are associative or co-associative. An open conjecture is that this characterization of the space-time surfaces is equivalent with the preferred extremal property of Kähler action with preferred extremal identified as a critical extremal allowing infinite-dimensional algebra of vanishing second variations.

3. Infinite primes

The construction of infinite primes is formally analogous to a repeated second quantization of an arithmetic quantum field theory by taking the many particle states of previous level elementary particles at the new level. Besides free many particle states also the analogs of bound states appear. In the representation in terms of polynomials the free states correspond to products of first order polynomials with rational zeros. Bound states correspond to n^{th} order polynomials with non-rational but algebraic zeros at the lowest level. At higher levels polynomials depend on several variables.

The construction might allow a generalization to algebraic extensions of rational numbers, and also to classical number fields and their complexifications obtained by adding a commuting imaginary unit. Special class corresponds to hyper-octonionic primes for which the imaginary part of ordinary octonion is multiplied by the commuting imaginary unit so that one obtains a sub-space M^8 with Minkowski signature of metric. Also in this case the basic construction reduces to that for rational or complex rational primes and more complex primes are obtained by acting using elements of the octonionic automorphism group which preserve the complex octonionic integer property.

Can one map infinite primes/integers/rationals to quantum states? Do they have space-time surfaces as correlates? Quantum classical correspondence suggests that if infinite rationals can be mapped to quantum states then the mapping of quantum states to space-time surfaces automatically gives the map to space-time surfaces. The question is therefore whether the mapping to quantum states defined by WCW spinor fields is possible. A natural hypothesis is that number theoretic fermions can be mapped to real fermions and number theoretic bosons to WCW ("world of classical worlds") Hamiltonians.

The crucial observation is that one can construct infinite hierarchy of rational units by forming ratios of infinite integers such that their ratio equals to one in real sense: the integers have interpretation as positive and negative energy parts of zero energy states. One can generalize the construction to quaternionic and octonionic units. One can construct also sums of these units with complex coefficients using commuting imaginary unit and these sums can be normalized to unity and have interpretation as states in Hilbert space. These units can be assumed to possess well defined standard model quantum numbers. It is possible to map the quantum number combinations of WCW spinor fields to these states. Hence the points of M^8 can be said to have infinitely complex number theoretic anatomy so that quantum states of the universe can be mapped to this anatomy. One could talk about algebraic holography or number theoretic Brahman=Atman identity.

Also the question how infinite primes might relate to the p-adicization program and to the hierarchy of Planck constants is discussed.

1.3.4 Unified Number Theoretical Vision

An updated view about M^8-H duality is discussed. M^8-H duality allows to deduce $M^4\times CP_2$ via number theoretical compactification. One important correction is that octonionic spinor structure makes sense only for M^8 whereas for $M^4\times CP_2$ complefixied quaternions characterized the spinor structure.

Octonions, quaternions associative and co-associative space-time surfaces, octonionic spinors and twistors and twistor spaces are highly relevant for quantum TGD. In the following some general observations distilled during years are summarized.

There is a beautiful pattern present suggesting that $H = M^4\times CP_2$ is completely unique on number theoretical grounds. Consider only the following facts. M^4 and CP_2 are the unique 4-D spaces allowing twistor space with Kähler structure. Octonionic projective space OP_2 appears as octonionic twistor space (there are no higher-dimensional octonionic projective spaces). Octotwistors generalise the twistorial construction from M^4 to M^8 and octonionic gamma matrices make sense also for H with quaternionicity condition reducing OP_2 to to 12-D $G_2/U(1)\times U(1)$ having same dimension as the the twistor space $CP_3\times SU(3)/U(1)\times U(1)$ of H assignable to complexified quaternionic representation of gamma matrices.

A further fascinating structure related to octo-twistors is the non-associated analog of Lie group defined by automorphisms by octonionic imaginary units: this group is topologically six-sphere. Also the analogy of quaternionicity of preferred extremals in TGD with the Majorana condition central in super string models is very thought provoking. All this suggests that associativity indeed could define basic dynamical principle of TGD.

Number theoretical vision about quantum TGD involves both p-adic number fields and classical number fields and the challenge is to unify these approaches. The challenge is non-trivial since the p-adic variants of quaternions and octonions are not number fields without additional conditions. The key idea is that TGD reduces to the representations of Galois group of algebraic numbers realized in the spaces of octonionic and quaternionic adeles generalizing the ordinary adeles as Cartesian products of all number fields: this picture relates closely to Langlands program. Associativity would force sub-algebras of the octonionic adeles defining 4-D surfaces in the space of octonionic adeles so that 4-D space-time would emerge naturally. M^8-H correspondence in turn would map the space-time surface in M^8 to $M^4\times CP_2$.

A long-standing question has been the origin of preferred p-adic primes characterizing elementary particles. I have proposed several explanations and the most convincing hitherto is related to the algebraic extensions of rationals and p-adic numbers selecting naturally preferred primes as those which are ramified for the extension in question.

Part I

GENERAL OVERVIEW

Chapter 2

Why TGD and What TGD is?

2.1 Introduction

This text was written as an attempt to provide a popular summary about TGD. This is of course mission impossible as such since TGD is something at the top of centuries of evolution which has led from Newton to standard model. This means that there is a background of highly refined conceptual thinking about Universe so that even the best computer graphics and animations do not help much. One can still try - at least to create some inspiring impressions. This chapter approaches the challenge by answering the most frequently asked questions. Why TGD? How TGD could help to solve the problems of recent day theoretical physics? What are the basic principles of TGD? What are the basic guidelines in the construction of TGD?

These are examples of this kind of questions which I try to answer in this chapter using the only language that I can talk. This language is a dialect used by elementary particle physicists, quantum field theorists, and other people applying modern physics. At the level of practice involves technically heavy mathematics but since it relies on very beautiful and simple basic concepts, one can do with a minimum of formulas, and reader can always to Wikipedia if it seems that more details are needed. I hope that reader could catch the basic idea: technical details are not important, it is principles and concepts which really matter. And I almost forgot: problems! TGD itself and almost every new idea in the development of TGD has been inspired by a problem.

2.1.1 Why TGD?

The first question is "Why TGD?". The attempt to answer this question requires overall view about the recent state of theoretical physics.

Obviously standard physics plagued by some problems. These problems are deeply rooted in basic philosophical - one might even say ideological - assumptions which boil down to -isms like reductionism, materialism, determinism, and locality.

Thermodynamics, special relativity, and general relativity involve also postulates, which can be questioned. In thermodynamics second law in its recent form and the assumption about fixed arrow of thermodynamical time can be questions since it is hard to understand biological evolution in this framework. Clearly, the relationship between the geometric time of physics and experienced time is poorly understood. In general relativity the beautiful symmetries of special relativity are in principle lost and by Noether's theorem this means also the loss of classical conservation laws, even the definitions of energy and momentum are in principle lost. In quantum physics the basic problem is that the non-determinism of quantum measurement theory is in conflict with the determinism of Schrödinger equation.

Standard model is believed to summarize the recent understanding of physics. The attempts to extrapolate physics beyond standard model are based on naive length scale reductionism and have products Grand Unified Theories (GUTs), supersymmetric gauge theories (SUSYs). The attempts to include gravitation under same theoretical umbrella with electroweak and strong interactions has led to super-string models and M-theory. These programs have not been successful, and the recent dead end culminating in the landscape problem of super string theories and M-theory could have its origins in the basic ontological assumptions about the nature of space-time and quantum.

2.1.2 How Could TGD Help?

The second question is "Could TGD provide a way out of the dead alley and how?". The claim is that is the case. The new view about space-time as 4-D surface in certain fixed 8-D space-time is the starting point motivated by the energy problem of general relativity and means in certain sense fusion of the basic ideas of special and general relativities.

This basic idea has gradually led to several other ideas. Consider only the identification of dark matter as phases of ordinary matter characterized by non-standard value of Planck constant, extension of physics by including physics in p-adic number fields and assumed to describe correlates of cognition, and zero energy ontology (ZEO) in which quantum states are identified as counterparts of physical events. These new elements generalize considerably the view about space-time and quantum and give good hopes about possibility to understand living systems and consciousness in the framework of physics.

2.1.3 Two Basic Visions About TGD

There are two basic visions about TGD as a mathematical theory. The first vision is a generalization of Einstein's geometrization program from space-time level to the level of "world of classical worlds" identified as space of 4-surfaces. There are good reasons to expect that the mere mathematical existence of this infinite-dimensional geometry fixes it highly uniquely and therefore also physics. This hope inspired also string model enthusiasts before the landscape problem forcing to give up hopes about predictability.

Second vision corresponds to a vision about TGD as a generalized number theory having three separate threads.

1. The inspiration for the first thread came from the need to fuse various p-adic physics and real physics to single coherent whole in terms of principle that might be called number theoretical universality.
2. Second thread was based on the observation that classical number fields (reals, complex numbers, quaternions, and octonions) have dimensions which correspond to those appearing in TGD. This led to the vision that basic laws of both classical and quantum physics could reduce to the requirements of associativity and commutativity.
3. Third thread emerged from the observation that the notion of prime (and integer, rational, and algebraic number) can be generalized so that infinite primes are possible. One ends up to a construction principle allowing to construct infinite hierarchy of infinite primes using the primes of the previous level as building bricks at new level. Rather surprisingly, this procedure is structurally identical with a repeated second quantization of supersymmetric arithmetic quantum field theory for which elementary bosons and fermions are labelled by primes. Besides free many-particle states also the analogs of bound states are obtained and this means the situation really fascinating since it raises the hope that the really hard part of quantum field theories - understanding of bound states - could have number theoretical solution.

It is not yet clear whether both great visions are needed or whether either of them is in principle enough. In any case their combination has provided a lot of insights about what quantum TGD could be.

2.1.4 Guidelines In The Construction Of TGD

The construction of new physical theory is slow and painful task but leads gradually to an identification of basic guiding principles helping to make quicker progress. There are many such guiding principles.

> "Physics is uniquely determined by the existence of WCW" is a conjecture but motivates highly interesting questions. For instance: "Why $M^4 \times CP_2$ a unique choice for the embedding space?", "Why space-time dimension must be 4?", etc...

- Number theoretical Universality is a guiding principle in attempts to realize number theoretical vision, in particular the fusion of real physics and various p-adic physics to single structure.
- The construction of physical theories is nowadays to a high degree guesses about the symmetries of the theory and deduction of consequences. The very notion of symmetry has been generalized in this process. Super-conformal symmetries play even more powerful role in TGD than in super-string models and gigantic symmetries of WCW in fact guarantee its existence.
- Quantum classical correspondence is of special importance in TGD. The reason is that where classical theory is not anymore an approximation but in well-defined sense exact part of quantum theory.

There are also more technical guidelines.

- Strong form of General Coordinate invariance (GCI) is very strong assumption. Already GCI leads to the assumption that Kähler function is Kähler action for a preferred extremal defining the counterpart of Bohr orbit. Even in a form allowing the failure of strict determinism this assumption is very powerful. Strong form of general coordinate invariance requires that the light-like 3-surfaces representing partonic orbits and space-like 3-surfaces at the ends of causal diamonds are physically equivalent. This implies effective 2-dimensionality: the intersections of these two kinds of 3-surfaces and 4-D tangent space data at them should code for quantum states.
- Quantum criticality states that Universe is analogous to a critical system meaning that it has maximal structural richness. One could also say that Universe is at the boundary line between chaos and order. The original motivation was that quantum criticality fixes the basic coupling constant dictating quantum dynamics essentially uniquely.
- The notion of finite measurement resolution has also become an important guide-line. Usually this notion is regarded as ugly duckling of theoretical physics which must be tolerated but the mathematics of von Neumann algebras seems to raise its status to that of beautiful swan.
- What I have used to call weak form of electric-magnetic duality is a TGD version of electric-magnetic duality discovered by Olive and Montonen [B3]. It makes it possible to realize strong form of holography implied actually by strong for of General Coordinate Invariance. Weak form of electric magnetic duality in turn encourages the conjecture that TGD reduces to almost topological QFT. This would mean enormous mathematical simplification.
- TGD leads to a realization of counterparts of Feynman diagrams at the level of space-time geometry and topology: I talk about generalized Feynman diagrams. The highly non-trivial challenge is to give them precise mathematical content. Twistor revolution has made possible a considerable progress in this respect and led to a vision about twistor Grassmannian description of stringy variants of Feynman diagrams. In TGD context string like objects are not something emerging in Planck length scale but already in scales of elementary particle physics. The irony is that although TGD is not string theory, string like objects and genuine string world sheets emerge naturally from TGD in all length scales. Even TGD view about nuclear physics predicts string like objects.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L9].

2.2 The Great Narrative Of Standard Physics

Narratives allow a simplified understanding of very complex situations. This is why they are so powerful and this is why we love narratives. Unfortunately, narrative can also lead to the wrong track when one forgets that only a rough simplification of something very complex is in question.

2.2.1 Philosophy

In the basic philosophy of physics reductionism, materialism, determinism, and locality are four basic dogmas forming to which the great narrative relies.

Reductionism

Reductionism can be understood in many ways. One can imagine reduction of physics to few very general principles, which is of course just the very idea of science as an attempt to understand rather than only measure. This reductionism is naïve length scale reductionism. Physical systems consist of smaller building bricks which consist of even smaller building bricks... The entire physics would reduce to the dance of quarks and this would reduce to the dynamics of super strings in the scale of Planck length. The brief summary about the reductionistic story would describe physics as a march from macroscopic to increasingly microscopic length scales involving a series of invasions:

Biology → biochemistry → chemistry → atomic physics as electrodynamics for nuclei and electrons. Nuclear physics for nuclei → hadronic physics for nuclei and their excitations → strong and weak interactions for quarks and and leptons.

One can of course be skeptic about the first steps in the sequence of conquests. Is biology really in possession? Physicists cannot give definition of life and can say even less about consciousness. Even the physics based definition of the notion of information central for living systems is lacking and only entropy has physics based definition. Do we really understand the extreme effectiveness of bio-catalysts and miracle like replication of DNA, transcription of DNA to mRNA, and translation of mRNA to aminoacids. It is yet impossible to test numerically whether phenomenological notions like chemical bond really emerge from Schrödinger equation.

The reduction step from nuclear physics to hadron physics is purely understand as is the reduction step from hadron physics to the physics of quarks and gluons. Here one can blame mathematics: the perturbative approach to quantum chromodynamics fails at low energies and one cannot realize deduce hadrons from basic principle by analytical calculations and must resort to non-perturbative approaches like QCD involving dramatic approximations.

The standard model is regarded as the recent form of reductionism. The generalization of standard model: Grand Unified Theories (GUTs), Supersymmetric gauge theories (SUSYs), and super string models and M-theory are attempts to continue reductionistic program beyond standard model making an enormous step in terms of length scales directly to GUT scale or Planck scale. These approaches have been followed during last forty years and one must admit that they have not been very successful. This point will be discussed in detail later.

Therefore reductionistic dogma involves many bridges assumed to exist but about whose existence we do not really know. Further, reductionistic dogma cannot be tested. This untestability might be the secret of its success besides the natural human laziness and temptations of groupthink, which could quite generally explain the amazing success of great narratives even when they have been obviously wrong.

Materialism

Materialism is another big chunk in the great narrative of physics. What it states is that only the physically measurable properties matter. One cannot measure the weight of the soul, so that there is no such thing as soul. The physical state of the brain at given moment determines completely the contents of conscious experience. In principle all sensory qualia, say experience of redness, must have precise correlates at the level of brain state.

At what level does life and consciousness appear. What makes matter conscious and behaving as if would have goals and intentions and need to survive? This is difficult question for the materialistic approach one postulates the fuzzy notion of emergence. When the system becomes complex enough, something genuinely new - be it consciousness or life - emerges. The notion of emergence seems to be in obvious conflict with that of naïve length scale reductionism and a lot of handwaving is needed to get rid of unpleasant questions. What this something new really is is very difficult or even impossible to define in in the framework reductionistic physics.

The problems culminate in neuroscience and consciousness theory which has become a legitimate field of science during last decade. The hard problem is the coding of the properties of the physical state of the brain to conscious experience. Recent day physics does not provide a slightest clue regarding this correspondence. One has of course a lot of correlations. Light with certain wavelength creates the sensation of red but a blow in the head can produce the same sensation. EEG and nerve pulse activity correlate with the contents of conscious experience and EEG seems to even code for contents of conscious experience. Only correlates are however in question. It is also temporal patterns of EEG rather than EEG at given moment of time which matters from the point of view of conscious experience. This relates closely to another dogma of standard quantum physics stating that time=constant slice of time evolution contains all information about the state of the system.

Determinism

The successes of Newtonian mechanism were probable the main reason for why determinism became a basic dogma of physics. Determinism implies a romantic vision: theoretician working with mere paper and pencil can predict the future. This leads also to the idea that Nature can be governed: this idea has dominated western thinking for centuries and led to the various crises that human kind is suffering. Ironically, this idea is actually in conflict with the belief in strict determinism! Also the narrative provided by Darwinism assumes survival as a goal, which means that organisms behave like intentional agents: something in conflict with strict determinism predicting clockwork Universe. On the other hand, genetic determinism assumes that genes determine everything. The great narrative is by no means free of contradictions. They are present and one must simply put them under the rug in order to keep the faith. The situation is same as in religions: everyone realizes that Bible is full of internal contradictions and one must just forget them in to not lose the great narrative provided by it.

In quantum theory one is forced to give up the notion of strict determinism at the level of individual systems. The outcome of state function reduction occurring in quantum measurement is not predictable at the level of individual systems. For ensembles one can predict probabilities of various outcomes so that classical determinism is replaced with statistical determinism, which of course involves the idealized notion of ensemble consisting of large number of identical copies of the system under consideration.

In consciousness theory strict determinism means denial of free will. One could ask whether the non-determinism of state function reduction could be interpreted in terms of free will so that even elementary particles would be conscious systems. It seems that this identification cannot explain intentional goal directed free will. State function reductions produce entropy and this provides deeper justification for the second law and quantum mechanism makes it possible to calculate various parameters like viscosity and diffusion constants needed in the phenomenological description of macroscopic systems. Living systems however produce and store information and experience it consciously. Quantum theory in its recent form does not have the descriptive power to describe this. Something more is needed: one should bring the notion of information to physics.

Locality

Locality is fourth basic piece of great narrative. What locality says that physical systems can be split into basic units and that understanding the behavior of this units and the interaction between them is enough to understand the system. This is very much akin to naïve length scale reductionism stating that everything can be reduced to the level of elementary particles or even to the level of superstrings.

Already in quantum theory one must give up the notion of locality although Schrödinger equation is still local. Standard quantum theory tells that in macroscopic scales entanglement has no implications. Quantum entanglement is now experimentally demonstrated to be possible between systems with macroscopic distance and even between macroscopic and microscopic systems. What does this mean: is the standard quantum theory really all that is needed or should we try to generalize it?

Locality dogma becomes especially problematic in living systems. Living systems behave as coherent units behaving very “quantally” and it is very difficult to understand how sacks of water containing some chemicals could climb in trees and even compose symphonies. The attempts to produce something which would look like living from a soup of chemicals have not been successful.

The proposed cure is macroscopic quantum coherence and macroscopic entanglement. There exist macroscopically quantum coherent systems such as suprafluids and super-conductors but these systems are very simple all particles are in same state- Bose Einstein condensate and quite different from living matter. Standard quantum theory is also unable to explain macroscopic quantum coherence and preservation of entanglement at physical temperatures.

Evidence for quantum coherence in cell scales and at physiological temperatures is however accumulating. Photosynthesis, navigation behavior of some birds and fishes, and olfaction represent examples of this kind. The recent finding that microtubules carry quantum waves should be also mentioned. Does this mean that something is missing from standard quantum theory. The small value of Planck constant characterizes the sizes of quantum effects and tells that spatial and temporal scales of quantum coherence are typically rather short. Is Planck constant really constant. One can of course ask whether this problem could relate to another mystery of recent day physics: the dark matter. We know that it exists but there is no generally accepted idea about what it is. Could living systems involve dark matter in an essential manner and could it be that Planck constant does not have only its standard value?

Locality postulate has far reaching implications for science policy. There is a lot of anecdotal evidence for various remote mental interactions such as telepathy, clairvoyance, psychokinesis of various kinds, remote healing, etc... The common feature of these phenomena is non-locality so that standard science denies them as impossible. Fr this reason people trying to study these phenomena have automatically earned the label of crackpot. Therefore experimental demonstration of these phenomena is very difficult since we do not have any theory of consciousness. Situation is not helped by the fact that skeptics deny in reflex like manner all evidence.

2.2.2 Classical Physics

Classical physics began with the advent of Newton's mechanics and brought the dogma of determinism to physics. In the following only thermodynamics and special and general relativities are discussed as examples about classical physics because they are most relevant from the TGD viewpoint.

Thermodynamics

Second law is the basic pillar of thermodynamics. It states that the entropy of a closed system tends to increase and achieve maximum in thermodynamical equilibrium. This law does not tell about the detailed evolution but only poses the eventual goal of evolution. This means irreversibility: one cannot reverse the arrow of thermodynamical time. For instance, one one can live life in the reverse direction of time.

The physical justification for the second law comes from quantum theory. Again one must however make clear that the basic assumption that time characteristic time scale for interactions involved is short as compared to the time scale one monitors the system. In time scales shorter the quantum coherence time the situation changes. If quantum coherence is possible in macroscopic time scales, one cannot apply thermodynamics.

The thermodynamical time has a definite arrow and is believe to be the same always. Living matter might form exception to this belief and Fantappie has proposed that this is indeed the case and proposed the notion of syntropy to characterize systems which seem to have non-standard arrow of time. Also phase conjugate laser rays seem to dissipate in wrong direction of time so that entropy seems to decrease from them when they are viewed in standard time direction.

The basic equations of physics are not believed posses arrow of time. Therefore the relationship between thermodynamical time and the geometric time of Einstein is problematic. Thermodynamical arrow of time relates closely to that of experienced/psychological arrow of time. Is the identification of experienced time and geometric time really acceptable? They certainly look different notions: experienced time has not future unlike geometric time, and experienced time is irreversible unlike geometric time. Certainly the notion of geometric time is well-understood. The notion of experienced time is not. Are we hiding ourselves behind the back of Einstein when we identify these two times. Should we bravely face the reality and ask what experienced time really is? Is it something different from geometric time and why these two times have also many common aspects - so many that we have identified them.

Second law provides a rather pessimistic view about future: Universe is unavoidably approaching heat death as it approaches thermodynamical equilibrium. Thermodynamics provides a measure for entropy but not for information. Is biological evolution really a mere thermodynamical fluctuation in which entropy in some space-time volume is reduced? Can one really understand information created and stored by living matter as a mere thermodynamical fluctuation? The attempt to achieve this has been formulated as non-equilibrium thermodynamics for open systems. One can however wonder whether could go wrong in the basic premises of thermodynamics?

Special Relativity

Relativity principle is the basic pillar of special relativity. It states that all system with respect to each other with constant relativity are physically equivalent: in other worlds the physics looks the same in these systems. Light velocity is absolute upper limit for signal velocity.

This kind of principle holds true also in Newton's mechanics and is known as Galilean relativity. Now there is however not upper bound for signal velocity. The difference between these principles follows from different meaning for what it is to move with constant relativity velocity. In special relativity time is not absolute anymore but the time shown by the clocks of two systems are different: time and spatial coordinates are mixed by the transformation between the systems.

Maxwell's electrodynamics satisfied the Relavity Principle and in modern terminology Poincare group generated by rotations, Lorentz transformations (between systems moving with respect to each other with constant velocity), translations in spatial and time drections act as symmetries of Maxwell's equations. In particle physics and quantum theory the formulation of relativity principle in terms of symmetries has become indispensable.

The essence of Special theory of relativity is geometric. Minkowski space is four-dimensional analog of Riemannian geometry with metric which characterizes what length and angle measurement mean mathematically. The metric is characterized in terms of generalization of the law of Pythagoras stating $ds^2 = dt^2 - dx^2 - dy^2 - dz^2$ in Minkowski coordinates. What is special is that time and space are in different positions in this infinitesimal expression for line element telling the length of the diameter of 4-dimensional infinitesimal cube.

Time dilation and Lorentz contraction are two effects predicted by special relativity. Time dilation day-to-day phenomenon in particle physics: particles moving with high velocity live longer in the laboratory system. Lorentz contraction must be also taken into account. Lorentz himself believed for long that Lorentz contraction is a physical rather than purely geometric effect but finally admitted that Einstein was right.

There are some pseudo paradoxes associated with Special Relativity and regularly some-one comes and claims that is some horrible logical error in the formulation of the theory. One paradox is twin paradox. One consider twins. Second goes for a long space-time travel moving very near to light-velocity and experiences time dilation. When he arrives at home he finds that his twin brother is very old. One can however argue that by relativity principle it is the second twin who has made the travel and should look older. The solution of the paradox is trivial. The situation is not symmetric since the second brother is not entire time in motion with constant velocity since he must turn around during the travel and spend this period in accelerated motion.

General Relativity

Einstein based his theories on general principles and maybe this is why they have survived all the tests. The theoretical physics has become very technical since the time of Einstein and the formulation of theories in terms of principles has not been in fashion. Instead, concrete equations and detailed models have replaced this approach. Super string models provide a good example. Maybe this explains why the modest success.

In general relativity there are two basic principles. General Coordinate Invariance and Equivalence Principle.

General Coordinate Invariance (GCI) states that the formulation of physics must be such that the basic equations are same in all coordinate systems. This is very powerful principle when formulated in terms of space-time geometry which is assumed to be generalization of Riemannian geometry from that for the Minkowski space of special relativity. Now line element is expressed as $ds^2 = g_{ij}dx^idx^j$ and it can be reduced to Minkowskian form only in vacuum regions far enough from massive bodies. Another new element is curvature of space-time which can be concretized in terms of spherical geometry. For triangles at the surface of sphere having as sides pieces of big circles (geodesic lines, which now represent the analog of free rectilinear motion) the sum of angles is larger than 180 degrees. For geodesic triangles at the surface of saddle like surface the sum is smaller than 180 degrees. This holds for arbitrarily small geodesic triangles and is therefore a local property of Riemann geometry.

Quite often one encounters the belief that GCI is generalization of Relativity Principle. This is not the case. Relativity Principle states that the isometries of Minkowski space consisting of Poincare transformations leave the physics invariant. General Coordinate transformations are not in general isometries of space-time and in the case of general space-time there are not isometries. Therefore GCI is only a constraint on the form of field equations: they just remain invariant under general coordinate transformations. Tensor analysis is the mathematical tool making it possible to expresss this universality. Tensor analysis allows to express the space-time geometry algebraically in terms of metric tensor, curvature tensor, Ricci tensor and Einstein tensor, and Ricci scalar associated with it. In particular, the notion of angle defect can be expressed in terms of curvature tensor.

In the case of Equivalence Principle (EP) the starting point is the famous thought experiment involving lift. In stationary elevator material objects fall down with accelerated velocity. One can however study the situation in freely falling life and in this case the material objects remain stationary as if there were not gravitational force. The idea is therefore that gravitational force is not a genuine force but only apparent coordinate forces which vanishes locally in suitable coordinates known as geodesic coordinates for which coordinate lines are geodesic lines. Gravitational force would be analogous to apparent forces like centripetal forces and Coriolis force appearing in rotating coordinate systems already in Newton's mechanics. The characteristic signature is that the associated acceleration does not depend on the mass of the particle. This leads to the postulate that the motion of particles occurs along geodesic lines in absence of other than gravitational interactions. Equivalence Principle is already present in Newton's theory of gravitation and states that inertial masses appearing in $F = ma$ can be chosen to be same as the gravitational mass appearing in the expression of gravitational forces $F_{gr} = GmM/r^2$ between bodies with gravitational masses m and M. Equivalence Principle looks rather innocent and almost trivial but its formulation in competing theories is surprisingly difficult and the situation is not made easier by the fact that the mathematics involved is highly non-linear.

Tensor analysis allows the tools to deduce the implications of EP. The starting point is the equality of inertial and gravitational masses but made a local statement for the corresponding mass densities or more generally corresponding tensors. For inertial mass energy momentum tensor characterizing the density and currents of four-momentum components is the notion needed. For gravitational energy the only tensor quantities to be considered are Einstein tensor and metric tensor because they satisfy the conservation of energy and momentum locally in the sense that their covariant divergence is vanishing. Also energy momentum tensor should be conserved and thus have vanishing divergence. The manner to achieve this is to assume that the two tensor are proportional to each other. This identification actually realizes EP and gives Einstein's equations. Cosmological term proportional to the metric tensor can be present and Einstein consider also this possibility since otherwise cosmology was predicted to be expanding and this did not fit with the prevailing wisdom. The cosmological expansion was observed and Einstein regarded his proposal as the worst blunder of this professional life. Ironically, the recently observed acceleration of cosmic expansion might be understood if cosmological term is present after all albeit with sign different than in Einstein's proposal. Einstein's equations state that matter serves as a source of gravitational fields and gravitational fields tell for matter how to move in presence of gravitational interaction. These equations have been amazingly successful.

There is however a problem relating to the difference between GCI and Principle of Relativity already mentioned. Noether's theorem states that symmetries and conservation laws correspond to each other. In quantum theory this theorem has become the guiding principle and construction of new theories is to high degree postulation of various kinds of symmetries and deducing the consequences. In generic curved space-time the presence of massive bodies makes space-time curved (see **Fig. 2.1**) and Poincare symmetries of empty Minkowski space are lost. This does not imply not only non-conservation of otherwise conserved quantities. These quantities do not even exist mathematically. This is a very serious conceptual drawback and the only manner to circumvent the problem is to make an appeal to the extreme weakness of gravitational interaction and say that gravitational four-momentum can be assigned to a system in regions very far from it because gravitational field is very weak.

This difficulty might explain why the quantization of gravitation by starting from Einstein's equations has been so difficult. It must be however noticed that the perturbative quantization of super-symmetric variant of Einstein's equation works amazingly well in flat Minkowski background and it has been even conjectured that divergences which plague practically every quantum field theory might be absent. Here the twistor Grassmann approach has allowed to overcome the formidable technical difficulties due to the extreme non-linearity the action principle involved. Still the question remains: could it be possible to modify general relativity in such a way that the symmetries of special relativity would not be lost?

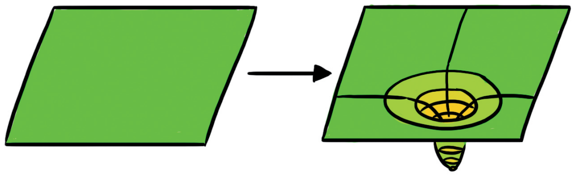

Figure 2.1: Matter makes space-time curved and leads to the loss of Poincare invariance so that momentum and energy are not well-defined notions in GRT.

2.2.3 Quantum Physics

Quantum physics forces to change both the ontology and epistemology of classical physics dramatically.

Quantum theory

In the following I just list the basic aspects of quantum theory which distinguish it from classical physics.

1. Point like particle is replaced in quantum physics by wave function. This is rather radical abstraction in ontology. For mathematician this looks almost trivial transition from space to function space: the 3-D configuration for particle is replaced by the space of complex valued functions in this space - Schrödinger amplitudes. From the point of view of physical interpretation this is big step since wave function means abstraction which cannot be visualized in terms of sensory experience. This transition is repeated in second quantization whether the function space is replaced with functional space consisting of functions defined classical fields. Also the proper interpretation of Schrödinger amplitudes is found to be in terms of classical fields. The new exotic elements are spinor fields, which are anti-commuting already at the classical level. They are introduced to describe fermions: this element is however not absolutely necessary.

   The interpretation is as probability amplitudes - square roots of probability densities familiar from probability theory applied in kinetic theory.

2. Schrödinger amplitude is mathematically analogous to a classical field, say classical electromagnetic fields fields appearing in Maxwell's theory. Interference for probability amplitudes leads to completely analogous effects such as interference and diffraction. The classical experiment demonstrating diffraction is double slit experiment in which electron bream travels along double slit system and is made visible at screen behind it. What one observes a distribution reflecting interference pattern for Schrödinger waves from the two slits just as for classical electromagnetic fields. The modulus square for probability amplitude inhibits the interference pattern. As the other slit is closed, interference pattern disappears. One cannot explain the interference pattern using ordinary probability theory: in this case electrons of the beam would not "know" which slits are open and destructive interference would be impossible. In quantum world they "know" and behave accordingly. Physics is not anymore completely local.

3. The model of electrons in atoms relies on Schrödinger amplitude and this might suggests that Schrödinger amplitude is classical field. This is however not the case. To understand what is involved one must introduce the notion of state function reduction and Uncertainty Principle.

   It was learned basically by doing experiments that quantum measurements differ from classical ones. First of all, even ideal quantum measurement typically changes the system, which does not happen in ideal classical measurement. The outcome of the measurement is non-deterministic and there are several outcomes, whose number is typically finite. One can predict only the probability of particular outcome and it is dictated by the state of the system and the measured observables.

   Uncertainty Principle is a further new element and dramatic restriction to ontology. For instance, one cannot measure momentum and position of the particle simultaneously in arbitrary accuracy. Ideal momentum measurement delocalizes the particle completely and vice versa. This is very difficult to understand in the framework of classical mechanics were particle is point of space. If one accepts the mathematician's view that particle states are elements of function space, Uncertainty Principle can be understood and is present already in Fourier analysis. One also can get rid of ontological un-easiness created by statements like "electron can exist simultaneously in many places". Also the construction of more complex systems using simpler ones as building bricks (second quantization) is easy to understand in this framework: in classical particle picture second quantization looks rather mysterious procedure. It is however not at all easy for even mathematical physicist to think that function space could be something completely real rather than only a figment of mathematical imagination.

4. What remains something irreducibly quantal is the occurrence of the non-deterministic state function reduction. This seems to be the core of quantum physics. The rest might reduce to deterministic physics in some function space characterizing physical states.

   The real problem is that the non-determinism of state function is not consistent with the determinism of Schrödinger equation. It seems that the laws of physics cease to hold temporarily and this has motivated the statements about craziness of quantum theory. More plausible view is that something in our view about time - or more precisely, about the relation between the geometric time of physicist and experienced time is wrong. These times are identified but we know that they are different: geometric time as no intrinsic arrow whereas subjective time has and future does not exist for subjective time but for geometric time it exists.

   There have been several attempts to reduce also state function reduction to deterministic classical physics or change the ontology so that it does not exist, but these attempts have not been successful. Ironically the core of quantum physics has remained also the taboo of quantum physics. The formulation is as "shut and calculate" paradigm which has dominated academic theoretical physics for century. One can only imagine where we could be without this professional taboo.

5. Quantum entanglement is a phenomenon without any classical counterpart. Schrödinger cat has become the standard manner to illustrate what is involved. One considers cat and bottle of poison which can be either open or closed. Classically one has two states: cat alive-bottle closed and cat dead-bottle open. Quantum mechanically also the superposition of these two states is possible and this obviously does not make sense in classical ontology. We cannot however observe quantum entanglement. When we want to know whether cat is dead or alive we induce state function reduction selecting either of these two states and the situation become completely classical. This suggests epistemological restriction: the character of conscious experience is that it produces always classical world as an outcome. One should of course not take this as dogma. The so called interaction free measurement allows to get information about system without destroying entanglement.

Standard model

Standard model summarizes our recent official understanding about physics. The attribute "official" is important here: there exists a lot of claims for anomalies, which are simply denied by the mainstream as impossible. Reductionists believe standard model to summarize even physics accessible to us. Standard model has been extremely successful in elementary particle physics. Even Higgs particle was found at LCH with predicted properties.

There are however issues related to the Higgs mechanism. Higgs particle has mass that it should not have and SUSY particles are too heavy to help in the problem. Stabilization of Higgs mass by cancelling radiative corrections to Higgs mass from heavy particles was one of the basic motivations for postulating SUSY in TeV energy scaled studied at LHC. Therefore one has what is called fine tuning problem for the parameters characterizing the interactions of Higgs and theory loses its predictivity.

Even worse, RHIC and LCH provide data telling that perturbative QCD does not seem to work at high energies where it should work. What was though to be quark gluon plasma - something behaving in very simple manner - was something different and one cannot exclude that there is some new physics there.

Neutrinos are the black sheep of the standard model. Each of the three leptons is accompanied by neutrino and in the most standard standard model they are massless. This has turned out to be not the case. Neutrinos also mix with each other as do also quarks. This phenomenon relates closely to the massivation. There are also indications that neutrinos could have several states with different mass values. The experimental neutrino physics is however extremely difficult since neutrinos are so weakly interaction so that the experimental progress is slow and plagued by uncertainties.

Therefore there are excellent reasons to be skeptical about standard model: one should continue to ask questions about the basics of the standard model. The attempt to answer this kind of fundamental questions concerning standard model could lead to re-awakening of particle physics from its recent stagnation. In particular, one could wonder what might be the origin of standard model quantum numbers and what is the origin of quark and gluon color. Standard model gauge group has very special and apparently un-elegant structure - something not suggested by GUT ideology. Why this Could this reflect some deeper principles?

This kind of questions were possible at sixties, and they led to the amazingly fast evolution of standard model. This hippie era in theoretical physics continued to the beginning of eighties but then the super string revolution around 1984 changed suddenly everything. Comparison with the revolution leading to birth of Soviet Union might be very rewarding. For me hippie era meant the possibility to make my thesis at Helsinki Technological University receiving even little salary: officially the goal was to make me a citizen able to take care of myself. Nowadays the idea about a person writing thesis about his own theory of everything is something totally unthinkable.

Grand Unified Theories

According to the great narrative the next step was huge: something like 13 orders of magnitude from the length scale of electroweak bosons (10^{-17} meters) to the length scale of extremely have gauge bosons of GUTs. At the time when I was preparing my thesis, GUTs were the highest fashion and every graduate student in particle physics had the opportunity to become the new Einstein and pick up his/her own gauge group and build up the GUT. All the needed formulas could be found easily and there was even a thick article containing all the recipes ranging from formulas for tensor products of group representations to beta functions for given group.

Both leptons and quarks form single family belonging to same multiplet of the big GUT gauge symmetry. The new gauge interactions predicted that and lepton and baryon number are not separately conserved so that proton is not stable. The theory allowed to predict its lifetime. The disappointing fact has been that no decays of proton have been however observed and this has led to a continual fine tuning of coupling parameters to keep proton alive for long time enough. This of course should put bells ringing since the stability of proton is extremely powerful guideline in theory building would suggest totally different track to follow based on question "Can one imagine any scenario in which B and L are separately conserved?".

The mass splittings between different fermions (quarks and leptons) believed to be related by gauge symmetries are huge: the mass ratio for top quark and neutrinos would be of the order 10^{12}, which is a huge number. Quite generally, the mass scales between symmetry related particles would be huge, which suggests that the notion of mass scale is part of physics. Also could serve as extremely powerful hint for a theory builder who is not afraid for becoming kicked out from the academic community.

GUT approach predicts a huge desert without any new physics ranging from electroweak scale to GUT length scale! So many orders of magnitude without any new physics looks like an incredible prediction when one recalls that 2 orders of magnitude separating electron and nuclei is the record hitherto. This assumption is of course just a scaled up variant of the child's assumption that the world ends at the backyard, and its basic virtue is that it makes theorist's life simple. There is nothing bad in this kind of assumption when taken as simplifying working hypothesis. The problem is that people have forgot that GUT hypothesis is only a pragmatic working hypothesis and believe that it represent an established piece of physics. Nothing could be farther from truth.

Super Symmetric Yang Mills theories

GUTs were followed by supersymmetric Yang-Mills theories - briefly SUSYs. The ambitious idea was to extend the unification program even further. Also fermions and bosons - particles with different statistics - would belong to same multiplet of some big symmetry group replaced with something even more general- super symmetry group. This required generalization of the very notion of symmetry by extending the notion of infinitesimal symmetry. One manner to achieve this is to replace space-time with a more general structure - superspace - possessing fermionic dimensions. This is however not necessarily and many mathematicians would regard this structure highly artificial. As a mathematical idea the generalization of symmetry is however extremely beautiful and shows how powerful just the need to identify bigger patterns is. One can indeed generalize of the various GUTs to supersymmetric gauge theories.

The number $\mathcal{N}$ of independent super-symmetries characterizes SUSY, and there are arguments suggesting that physically $\mathcal{N} = 1$ theories are the only possible ones. Certainly they are the simplest ones, and it is mostly these theories that particle phenomwnologists have studied. $\mathcal{N} = 4$ SUSYs possesses in certain sense maximal SUSY in four-dimensions. It is unrealistic as a physical model but because of its exceptional simplicity has led to a mathematical breakthrough in theoretical physics. The twistor Grassmannian approach has been applied to these theories and led to a totally new view about how to calculate in quantum field theory. The earlier approach based on Feynman diagrams suffered from combinatorial explosion so that only few lowest orders could be calculated numerically. The new approach strongly advocated by Nima Arkani Hamed and his coworkers allows to sum up huge numbers of Feynman diagrams and write the answer which took earlier ten pages with few lines. Also a lot of new mathematics developed by leading Russian mathematicians has been introduced.

$\mathcal{N} = 1$ SUSY, whose particles would have mass scale of order TeV, the energy scale studied at LHC, was motivated by several reasons. One reason was that in that ideal situation that all particles remain massless the contributions of ordinary and supersymmetric particles to many kinds of radiative corrections in particle reactions cancel each other. In the case of Higgs this would mean stability of the parameters characterizing the interactions of Higgs with other particles. In particular, Higgs vacuum expectation value determining the masses of leptons and quarks and gauge bosons would be stable. All this depends sensitively on precise values of particle masses and unfortunately it happens that the mechanism does not stabilize the parameters of Higgs.

Second motivation was that SUSY might provide solution to the dark matter mystery. The called lightest super-symmetric particle is predicted to be stable by so called R-parity symmetry which naturally accompanies SUSY but can be also broken. This particle is fermion and super partner of photon or weak boson Z^0 or mixture of these. This particle would provide an explanation for the mysterious dark matter about which we recently know only its existence. Dark matter would be a remnant from early cosmology - those lightest supersymmetric particles which failed annihilate with their antiparticles to bosons because cosmic expansion reduced their densities and made annihilation rate too small.

The results from LCH were however a catastrophic event in the life of SUSY phenomenologists. Not a slightest shred of evidence for SUSY has been found. There is still hope that some fine tuned SUSY scenarios might survive but if SUSY is there it cannot satisfy the basic hopes put on it. The results from LHC arriving during 2005 will be decisive for the fate of SUSY.

The results of LHC do not of course exclude the notion of supersymmetry. There are lots of variants of supersymmetry and $\mathcal{N} = 1$ SUSYs represents only one particular, especially simple variant in some respects and involving ad hoc assumptions such as straightforward generalization of Higgs mechanism as origin of particle massivation, which can be questioned already in standard model context. Furthermore, $\mathcal{N} = 1$ SUSY forces to give up separate conservation of lepton and baryon numbers for which there is no experimental evidence. For higher values of $\mathcal{N}$ this is not necessary.

Superstrings and M-theory

Super-strings mean a further extension for the notion of symmetry and thus reductionism at conceptual level. Conformal symmetries define infinite-dimensional symmetries and were first discovered in attempts to understand 2-dimensional critical systems. Critical system is a system in phase transition. There are two phases present that and the regions of given phase can have arbitrary large sizes. This means scale invariance and long range fluctuations: system does not behave as if it would consist of billiard balls having only contact interactions. The discovery was that the notion of scale invariance generalizes to local scale invariance. The transformations of plane (or sphere or any 2-D space) known as conformal transformations preserve the angle between two curves and introduce local scaling of distances. These transformations appear in complex analysis as holomorphic maps.

In string model which emerged first as hadronic string model, hadrons are identified as strings. Their orbits define 2-D surfaces and conformal transformations for these surfaces appear as symmetries of the theory. One could say that strings physics resembles that of 2-D critical systems. Hadronic string model did not evolve to a real theory of hadrons: for instance, the critical dimension in which worked was 26 for bosonic strings and 10 for their super counterparts. Therefore hadronic string model was largely given up as quantum chromodynamics trying to reduce hadronic physics to that of point-like quarks and gluons emerged. This approach worked nicely at high energies but at low energies the problem is that perturbative approach fails. The already mentioned unexpected behavior of what was expected to be quark gluon plasma challenges also QCD.

String model contained also graviton like states possessing spin 2 and the description for their interactions resemble that for the description of gravitons with matter according to the lowest order predictions of quantized general relativity. This eventually led to the idea that maybe super-symmetric variants of string might provide the long sough solution to the problem of quantizing gravitation. Perhaps even more: maybe they could allow to unify all known fundamental interactions with framework of single notion: super string.

In superstring approach the last step in the reductionistic sequence of conquests would be directly to the Planck length scale making about 16 orders of magnitudes. The first superstring revolution shook physics world around 1984. During the first years gurus believed that proton mass would be calculated within few years and first Nobels would be received within decade. Gradually the optimism began to fade as it turned out that superstring theory is not so unique as it was believed to be. Also the building or the bridge to the particle phenomenology was not at all so easy as was believed first.

Superstring exists in mathematically acceptable manner only in dimension $D = 10$ and this was of course a big problem. The notion of spontaneous compactification was needed and brought in an ugly ad hoc trick to the otherwise so beautiful vision. This mechanism would compactify 6 large dimensions of the 10-D Minkowski space so that they would become very small - the scale would be of the order of Planck length. For all practical purposes the 10-D space would look 4-dimensional. The 6 large dimensions would curl up to so called Calabi-Yau space and the finding of the correct Calabi-Yau was thought to be a simple procedure.

This was not the case. It turned out that there are very many Calabi-Yau manifolds [A3] to begin with: the number 10^{500} was introduced to give some idea about how many of them are - the number could be quite well infinite. The simple Calabi-Yau spaces did not produce the standard model physics at low energies. This problem became known as landscape problem. Landscape inspired in cosmology to the notion of multiverse: universe would split to regions which can have practically any imaginable laws of physics. There is no empirical support for this vision but this has not bothered the gurus.

Gradually it became clear that landscape problem spoils the predictivity of the theory and eventually many leading gurus turned they coat. The original idea was that string models are so wonderful because they predict unique physics. Now they were so beautiful because they force us to give up completely the belief that physical theories can predict something. In this framework antropic principle remains the only guideline in attempts to relate theory to the real world. This means that we can deduce the properties of the particular physics we happen to live from our own existence and by scanning through this huge repertoire of possible physics.

Around 1995 so called second superstring revolution took place. Five very different looking super string models had emerged. The great vision advocated especially by Witten was that they are limiting cases of one theory christened as M-theory. The 10-D target space for superstrings was replaced with 11-dimensional one. Besides this higher dimensional objects - branes- of varying dimension entered the picture and made it even more complex. This gave of course and enormous flexibility. For instance, the 4-D observed space-time could be understood as brane rather than the effectively 4-D target space obtained by spontaneous compactification. This gave for particle phenomenologists wanting to reproduce standard model an endless number of alternatives and the theory degenerated to endless variety of attempts to reproduce standard model by suitable configurations of branes. Around 2005 the situation in M-theory began to become public and so called string wars began. At this moment the funding of super-strings has reduced dramatically and the talks in string conferences hardly mention superstrings.

One can conclude that the forty years of unification based on naïve length scale reductionism was a failure. What was thought to become the brightest jewel in the crown of reductionistic vision was a complete failure. If history could teach something, it should teach us that we should perhaps follow Einstein and his co-temporaries and be asking questions about fundamentals. The shut-up and calculate approach forbidding all discussion about the basic assumptions has leads nowhere during these four decades.

As one looks this process in the light of after wisdom, one realizes that there are two kinds of reductionisms involved. The naïve length scale reductionism has not been successful. Time might be ripe for its replacement with the notion of fractality which postulates that similar looking structures appear in all length scales. Fractality is also a central aspect of the renormalization group approach to quantum field theory.

A second kind of reductionistic sequence has been realized at conceptual level. The notion of symmetry has evolved from ordinary symmetry to supersymmetry to super-conformal symmetry and even created new mathematical notions. The size of the postulated symmetry groups has steadily increased: note that already Einstein initiated this trend by postulating general coordinate invariance as a symmetry analogous to gauge symmetry. In superstring type approaches one can ask whether one should put all particles to same symmetry multiplet in the ultimate theory.

Symmetry breaking is what remains poorly understood in gauge theories and GUTS. Conformal field theories however provide a very profound and deep mechanism involving now ad hoc elements as Higgs mechanism does. Maybe one should try to understand particle massivation in terms of breaking of superconformal symmetries rather than blindly following the reductionistic approach and trying to reproduce SUSY and GUT approaches and Higgs mechanism as intermediate steps in the imagined reductionistic ladder leading from standard model to the ultimate theory. Maybe we should try to understand symmetry breaking as reflecting the limitations of the observer. For instance, in thermodynamical systems we can observe only thermodynamical averages of the properties of particles, such as energy.

2.2.4 Summary Of The Problems In Nutshell

New theory must solve the problems of the old theory. The old theory indeed has an impressive list of problems. The last 30 or 40 years have been an Odysseia in theoretical physics. When did this Odysseia begin?

Did the discovery of super strings initiate the misery for thirty years ago? Or can we blame SUSY approach? Was the SUSY perhaps too simple - or perhaps better to say, too simplistic? Did already the invention of GUTs lead to a side track: is it too simplistic to force quarks and leptons to multiplets of single symmetry group? This forcing of the right leg to the left hand shoe predicts proton decay, which has not been observed?

Or is there something badly wrong even with the cherished standard model: do particles really get their masses through Higgs mechanism: is the fact that Higgs is too light indication that something went wrong? Do we really understand quark and gluon color and neutrinos? What about family replication and standard model quantum numbers in general? What about dark matter and dark energy? The only thing we know is that they exist and naïve identifications for dark matter have turned out to be wrong. There is also the energy problem of General Relativity. Did we go choose a wrong track already almost century ago?

And even at the level of the basic theory - quantum mechanics - taken usually as granted we have the same problem that we had almost century ago.

2.3 Could TGD Provide A Way Out Of The Dead End?

The following gives a concise summary of the basic ontology and epistemology of TGD followed by a more detailed discussion of the basic ideas.

2.3.1 What New Ontology And Epistemology Of TGD Brings In?

TGD based ontology and epistemology involves several elements, which might help to solve the listed problems.

1. The new view about space-time as 4-D surface in certain 8-D embedding space leads to the notion of many-sheeted space-time and to geometrization and topological quantization of classical fields replacing the notion of superposition for fields with superposition for their effect.
2. ZEO means new view about quantum state. Quantum states as states with positive energy are replaced with zero energy states which are pairs of states with opposite quantum numbers and "live" at opposite boundaries of causal diamond (CD) which could be seen as spotlight of consciousness at the level of 8-D embedding space.
3. ZEO leads to a new view about state function reduction identified as moment of consciousness. Consciousness is not anymore property of physical states but something between two physical states, in the moment of recreation. One ends up to ask difficult questions: how the experience flow of time experience in this picture, how the arrow of geometric time emerges from that of subjective time, is the arrow of geometric time same always, etc...
4. Hierarchy of Planck constants is also a new element in ontology and means extension of quantum theory. It is somewhat matter of taste whether one speaks about hierarchy of effective or real Planck constants and whether one introduces only coverings of space-time surface or also those of embedding space to describe what is involved. What however seems clear that hierarchy of Planck constants follows from fundamental TGD naturally. The matter forms phases with different values of $h_{eff}(h = n$ and for large values of n this means macroscopic quantum coherence so that application to living matter is obvious challenge. The identification of these new phases as dark matter is the natural first working hypothesis.
5. p-Adic physics is a further new ontological and epistemological element. p-Adic numbers fields are completions of rational numbers in many respects analogous to reals and one can ask whether the notion of p-adic physics might make sense. The first success comes from elementary particle mass calculations based on p-adic thermodynamics combined with very general symmetry arguments. It turned out that the most natural interpretation of p-adic physics is as physics describing correlates of cognition. This brings to the vocabulary p-adic space-time sheets, p-adic counterparts of field equations, p-adic quantum theory, etc.. The need to fuse real and various p-adic physics to gain by number-theoretical universality becomes a powerful constraint on the theory.

   The notion of negentropic entanglement is natural outcome of p-adic physics. This entanglement is very special: all entanglement probabilities are identical and an entanglement matrix proportional to a unitary matrix gives rise to this kind of entanglement automatically. The U-matrix characterizing interactions indeed consists of unitary building blocks giving rise to negentropic entanglement. Negentropic entanglement tends to be respected by Negentropy Maximization Principle (NMP) which defines the basic variational principle of TGD inspired theory of consciousness and negentropic entanglement defines kind of Akaschic records which are approximate quantum invariants. They form kind of universal potentially conscious data basis, universal library. This obviously represents new epistemology.
6. Strong form of holography implied by the strong form of general coordinate invariance (GCI) states that both classical and quantum physics are coded by string world sheets and partonic

2-surfaces. This principle means co-dimension 2-rule: instead of 0-dimensional discretization replacing geometric object with a discrete set of points discretization is realized by co-dimension two surfaces. This allows to avoid problems with symmetries since discrete point set is replaced with a set of co-dimension 2-surfaces parameterized by parameters in an algebraic extension of rationals- conformal moduli of these surfaces are natural general coordinate invariant parameters.

Fermions are localized to string world sheets and partonic 2-surfaces also by the well-definedness of em charges. One can say that fermions as correlates of Boolean cognition reside at these 2-surfaces and cognition and sensory experience are basically 2-dimensional. One can also roughly say that the degrees of freedom in the exterior of 2-surfaces corresponds to conformal gauge degrees of freedom. 4-D space-time is however necessary to interpret quantum experiments.

2.3.2 Space-Time As 4-Surface

Energy problem of GRT as starting point

The physical motivation for TGD was what I have christened the energy problem of General Relativity, which has been already mentioned. The notion of energy is ill-defined because the basic symmetries of empty space-time are lost in the presence of gravity. The presence of matter curves empty Minkowski space M^4 so that its rotational, translational and Lorentz symmetries realized as transformations leaving the distances between points and thus shapes of 4-D objects invariant. Noether's theorem states that symmetries and conservation laws correspond to each other so that conservations laws are lost: energy, momentum, and angular momentum are not only non-conserved but even ill-defined. The mathematical expression for this is that the energy momentum tensor is 2-tensor so that it is impossible to assign with it any conserved energy and momentum mathematically except in empty Minkowski space. Usually it is argued that this is not a practical problem since gravitation is so weak interaction. When one however tries to quantized general relativity, this kind of sloppiness cannot be allowed, and the problem reason for the continual failure of the attempts to build a theory of quantum gravity might be tracked down to this kind of conceptual sloppiness.

The way out of the problem is based on assumption that space-times are imbeddable as 4-surfaces to certain 8-dimensional space by replacing the points of 4-D empty Minkowski space with 4-D very small internal space. This space -call it S- is unique from the requirement that the theory has the symmetries of standard model: $S = CP_2$, where CP_2 is complex projective space with 4 real dimensions [L7] , is the unique choice. Symmetries as isometries of space-time are lifted to those of embedding space. Symmetry transformation does not move point of space-time along it but moves entire space-time surface. Space-time surface is like rigid body rotated, translated, and Lorentz boosted by symmetries. This means that Noether's theorem predicts the classical conserved charges once general coordinate action principle is written down.

Also now the curvature of space-time codes for gravitation. Now however the number of solutions to field equations is dramatically smaller than in Einstein's theory. An unexpected bonus was that a geometrization classical fields of standard model for $S = CP_2$. Later it turned out that also the counterparts for field quanta emerge naturally but this requires profound generalization of the notion of space-time so that topological inhomogenities of space-time surface are identified as particles. This meant a further huge reduction in dynamical field like variables. By general coordinate invariance only four embedding space coordinates appear as variables analogous to classical fields: in a typical gut their number is hundreds.

CP_2 also codes for the standard model quantum numbers in its geometry in the sense that electromagnetic charge and weak isospin emerge from CP_2 geometry : the corresponding symmetries are not isometries so that electroweak symmetry breaking is coded already at this level. Color quantum numbers which correspond to the isometries of CP_2 and are unbroken symmetry: this also conforms with empirical facts. The color of TGD however differs from that in standard model in several aspects and LHC has began to exhibit these differences via the unexpected behavior of what was believed to be quark gluon plasma. The conservation of baryon and lepton number follows as a prediction. Leptons and quarks correspond to opposite chiralities for fermions at the level of embedding space.

What remains to be explained is family replication phenomenon for leptons and quarks which means that both quarks and leptons appear as three families which are identical except that they have different masses. Here the identification of particles as 2-D boundary components of 3-D surface inspired the conjecture that fermion families correspond to different topologies for 2-D surfaces characterized by genus telling the number g (genus) of handles attached to sphere to obtain the surfae: sphere, torus, ... The identification as boundary component turned out to be too simplistic but can be replaced with partonic 2-surface assignable to light-like 3-surface at which the signature of the induced metric of space-time surface transforms from Minkowskian to Euclidian. This 3-D surfaces replace the lines of Feynman diagrams in TGD Universe in accordance with the replacement of point-like particle with 3-surface.

The problem was that only three lowest genera are observed experimentally. Are the genera $g > 2$ very heavy or don't they exist. One ends up with a possible explanation in terms of conformal symmetries: the genera $g \leq 2$ allow always two element group as subgroup of conformal symmetries (this is called hyper-ellipticity) whereas higher genera in general do not. Observed 3 particle families would have especially high conformal symmetries. This could explain why higher genera are very massive or not realized as elementary particles in the manner one would expect.

The surprising outcome is that $M^4 \times CP_2$ codes for the standard model. Much later further arguments in favor of this choice have emerged. The latest one relates to twistorialization. 4-D Minkowski space is unique space-time with Minkowskian signature of metric in the sense that it allows twistor structure. This is a big problem in attempts to introduce twistors to General Relativity Theory (GRT) and very serious obstacle in quantization based on twistor Grassmann approach which has demonstrate its enormous power in the quantization of gauge theories. The obvious idea in TGD framework is whether one could lift also the twistor structure to the level of embedding space $M^4 \times CP_2$. M^4 has twistor structure and so does also CP_2: which is the only Euclidian 4-manifold allowing twistor space which is also Kähler manifold!

It soon became clear that TGD can be seen as a generalization of hadronic string model - not yet superstring model since this model became fashionable two years after the thesis about TGD. Later it has become clear that string like objects, which look like strings but are actually 3-D are basic stuff of TGD Universe and appear in all scales. Also strictly 2-D string world sheets pop up in the formulation of quantum TGD so that one can say that string model in 4-D space-time is part of TGD.

One can say that TGD generalizes standard model symmetries and provides a proposal for a dynamics which is incredibly simple as compared to the competing theories: only 4 classical field variables and in fermionic sector only quark and lepton like spinor fields. The basic objection against TGD looks rather obvious in the light of afterwisdom. One loses linear superposition of fields which holds in good approximation in ordinary field theories, which are almost linear. The solution of the problem relies on the notion many-sheeted space-time to be discussed below.

Many-sheeted space-time

The replacement of the abstract manifold geometry of general relativity with the geometry of surfaces brings the shape of surface as seen from the perspective of 8-D space-time and this means additional degrees of freedom giving excellent hopes of realizing the dream of Einstein about geometrization of fundamental interactions.

The work with the generic solutions of the field equations assignable to almost any general coordinate invariant variational principle led soon to the realization that the space-time in this framework is much more richer than in general relativity.

1. Space-time decomposes into space-time sheets with finite size (see **Fig. 2.2**): this lead to the identification of physical objects that we perceive around us as space-time sheets. For instance, the outer boundary of the table is where that particular space-time sheet ends. We can directly see the complex topology of many-sheeted space-time! Besides sheets also string like objects and elementary particle like objects appear so that TGD can be regarded also as a generalization of string models obtained by replacing strings with 3-D surfaces.

   What does one mean with space-time sheet? Originally it was identified as a piece of slightly deformed M^4 in $M^4 \times CP_2$ having boundary. It however became gradually clear that boundaries are probably not allowed since boundary conditions cannot be satisfied. Rather, it seems that sheet in this sense must be glued along its boundaries together with its deformed copy to get double covering. Sphere can be seen as simplest example of this kind of covering: northern and southern hemispheres are glued along equator together.

So: what happens to the identification of family replication in terms of genus of boundary of 3-surface and to the interpretation of the boundaries of physical objects as space-time boundaries? Do they correspond to the surfaces at which the gluing occurs? Or do they correspond to 3-D light-like surfaces at which the signature of the induced metric changes. My educated guess is that the latter option is correct but one must keep mind open since TGD is not an experimentally tested theory.

2. Elementary particles are roughly speaking identified as topological inhomogenities glued to these space-time sheets using topological sum contacts. This means roughly drilling a hole to both sheets and connecting with a cylinder. 2-dimensional illustration should give the idea. In this conceptual framework material structures and shapes are not due to some mysterious substance in slightly curved space-time but reduce to space-time topology just as energy-momentum currents reduce to space-time curvature in general relativity.

   This view has gradually evolved to much more detailed picture. Without going to details one can say that particles have wormhole contacts as basic building bricks. Wormhole contact is very small Euclidian connecting two Minkowskian space-time sheets with light-like boundaries carrying spinor fields and there particle quantum numbers. Wormhole contact carries magnetic monopole flux through it and there must be second wormhole contact in order to have closed lines of magnetic flux. One might describe particle as a pair of magnetic monopoles with opposite charges. With some natural assumptions the explanation for the family replication phenomenon is not affected and nothing new is predicted. Bosons emerge as fermion anti-fermion pairs with fermion and anti-fermion at the opposite throats of the wormhole contact. In principle family replication phenomenon should have bosonic analog. This picture assigns to particles strings connecting the two wormhole throats at each space-time sheet so that string model mathematics becomes part of TGD.

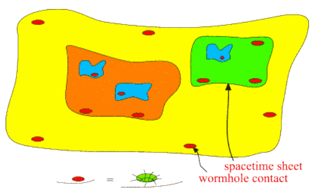

Figure 2.2: Many-sheeted space-time.

The notion of classical field differs in TGD framework in many respects from that in Maxwellian theory.

1. In TGD framework fields do not obey linear superposition and all classical fields are expressible in terms of four embedding space coordinates in given region of space-time surface. Superposition for classical fields is replaced with superposition of their effects. Particle can topologically condensed simultaneously to several space-time sheets by generating topological sum contacts. Particle experiences the superposition of the *effects* of the classical fields at various space-time sheets rather than the superposition of the fields. It is also natural to expect that at macroscopic length scales the physics of classical fields (to be distinguished from that for field quanta) can be explained using only four fields since only four primary field like variables are present. Electromagnetic gauge potential has only four components and classical electromagnetc fields give and excellent description of physics. This relates directly to electroweak symmetry breaking in color confinement which in standard model imply the effective absence of weak and color gauge fields in macroscopic scales. TGD however predicts that copies of hadronic physics and electroweak physics could exist in arbitrary long scales and

there are indications that just this makes living matter so different as compared to inanimate matter.

2. The notion of induced field means that one induces electroweak gauge potentials defining so called spinor connection to space-time surface. Induction means (see Appendx) locally a projection for the embedding space vectors representing the spinor connection locally. This is essentially dynamics of shadows! The classical fields at the embedding space level are non-dynamical and fixed and extremely simple: one can say that one has generalization of constant electric field and magnetic fields in CP_2. The dynamics of the 3-surface however implies that induced fields can form arbitrarily complex field patterns.

   Induced fields are not however equivalent with ordinary free fields. In particular, the attempt to represent constant magnetic or electric field as a space-time time surface has a limited success. Only a finite portion of space-time carrying this field allows realization as 4-surface. I call this topological field quantization. The magnetization of electric and magnetic fluxes is the outcome. Also gravitational field patterns allowing embedding are very restricted: one implication is that topological with over-critical mass density are not globally imbeddable. This would explain why the mass density in cosmology can be at most critical. This solves one of the mysteries of GRT based cosmology. Quite generally the field patterns are extremely restricted: not only due to imbeddability constraint but also due to the fact that only very restricted set of space-time surfaces can appear solutions of field equations: I speak of preferred extremals. One might speak about archetypes at the level of physics: they are in quite strict sense analogies of Bohr orbits in atomic physics: this is implies by the realization of general coordinate invariance (GCI).

   One might of course argue that this kind of simplicity does not conform with what we observed. The way out is many-sheeted space-time. Particles experience superposition of effects from the archetypal field configurations. Basic field patterns are simple but effects are complex!

   The important implication is that one can assign to each material system a field identity since electromagnetic and other fields decompose to topological field quanta. Examples are magnetic and electric flux tubes and flux sheets and topological light rays representing light propagating along tube like structure without dispersion and dissipation making em ideal tool for communications [K77] . One can speak about field body or magnetic body of the system.

3. Field body indeed becomes the key notion distinguishing TGD inspired model of quantum biology from competitors but having applications also in particle physics since also leptons and quarks possess field bodies. The is evidence for the Lamb shift anomaly of muonic hydrogen [C2] and the color magnetic body of u quark whose size is somewhat larger than the Bohr radius could explain the anomaly [K66] .

2.3.3 The Hierarchy Of Planck Constants

The motivations for the hierarchy of Planck constants come from both astrophysics and biology [K88, K35] . In astrophysics the observation of Nottale [E1] that planetary orbits in solar system seem to correspond to Bohr orbits with a gigantic gravitational Planck constant motivated the proposal that Planck constant might not be constant after all [K97, K78] .

This led to the introduction of the quantization of Planck constant as an independent postulate. It has however turned that quantized Planck constant in effective sense could emerge from the basic structure of TGD alone. Canonical momentum densities and time derivatives of the embedding space coordinates are the field theory analogs of momenta and velocities in classical mechanics. The extreme non-linearity and vacuum degeneracy of Kähler action imply that the correspondence between canonical momentum densities and time derivatives of the embedding space coordinates is 1-to-many: for vacuum extremals themselves 1-to-infinite.

TGD Universe is assumed to be quantum critical so that Kähler coupling constant strength is analogous to critical temperature. This raises the hope that quantum TGD as a "square root" of thermodynamics is uniquely fixed. Quantum criticality implies that TGD Universe is like a ball at the top of hill on the top of hill at... Conformal invariance characterizes 2-D critical systems and generalizes in TGD framework to its 4-D counterpart and includes super-symplectic symmetry acting as isometries of WCW. Therefore the proposal is that the sub-algebras of super-symplectic algebra with conformal weights coming as n-ples of those for the full algebra define a fractal hierarchy of isomorphic sub-algebras acting as gauge conformal symmetries: n would be identifiable as $n = h_{eff}/h$. The phase transitions increasing n would scale n by integer and occur spontaneously so that the generation of dark phases of matter would be a spontaneous process. This has far reaching implications in the dark matter model for living systems.

A convenient technical manner to treat the situation is to replace embedding space with its n-fold singular covering. Canonical momentum densities to which conserved quantities are proportional would be same at the sheets corresponding to different values of the time derivatives. At each sheet of the covering Planck constant is effectively $\hbar = n\hbar_0$. This splitting to multi-sheeted structure can be seen as a phase transition reducing the densities of various charges by factor $1/n$ and making it possible to have perturbative phase at each sheet (gauge coupling strengths are proportional to $1/\hbar$ and scaled down by $1/n$). The connection with fractional quantum Hall effect [D1] is almost obvious [K80]. It must be emphasize that this description has become only an auxiliary tool allowing to understand easily some aspects of what it is to be dark matter.

Nottale [E1] introduced originally so called gravitational Planck constant as $\hbar_{gr} = GMm/v_0$, where v_0 has dimensions of velocity and characterizing the system: h_{gr} is assigned with magnetic flux tubes carrying dark gravitons mediating gravitational interaction between masses M and m. The identification $h_{eff} = h_{gr}$ [K79] turns out to be natural and implies a deep connection with quantum gravity. The recent formulation of TGD involving fermions localized at string world sheets in space-time regions with Minkowskian signature of induced metric suggests to consider the inclusion of string world sheet area as an additional contribution to the bosonic action in Minkowskian regions. String tension would be given by $T \propto 1/\hbar_{eff}G$ as in string models. The condition that in gravitationally bound states partonic 2-surfaces are connected by strings makes sense only if one has $T \propto \hbar_{eff}^2$. This excludes area action.

The remaining possibility is that the bosonic part of the action is just the Kähler action reducing to stringy contributions with effective metric defined by the anticommutators of the K-D gamma matries predicting $T \propto \hbar_{eff}^2$. Large values of $\hbar_{eff}$ are necessary for the formation of gravitationally bound states: ordinary quantum theory would be simply not enough for quantum gravitation. Macroscopic quantum coherence in astrophysical scales is predicted and the fountain effect of superfluidity serves could be seen as an example about gravitational quantum coherence [?].

This has many profound implications, which are welcome from the point of view of quantum biology but the implications would be profound also from particle physics perspective and one could say that living matter represents zoom up version of quantum world at elementary particle length scales.

1. Quantum coherence and quantum superposition become possible in arbitrary long length scales. One can speak about zoomed up variants of elementary particles and zoomed up sizes make it possible to satisfy the overlap condition for quantum length parameters used as a criterion for the presence of macroscopic quantum phases. In the case of quantum gravitation the length scale involved are astrophysical. This would conform with Penrose's intuition that quantum gravity is fundamental for the understanding of consciousness and also with the idea that consciousness cannot be localized to brain.

2. Photons with given frequency can in principle have arbitrarily high energies by $E = hf$ formula, and this would explain the strange anomalies associated with the interaction of ELF em fields with living matter [J3] . Quite generally the cyclotron frequencies which correspond to energies much below the thermal energy for ordinary value of Planck constant could correspond to energies above thermal threshold.

3. The value of Planck constant is a natural characterizer of the evolutionary level and biological evolution would mean a gradual increase of the largest Planck constant in the hierarchy characterizing given quantum system. Evolutionary leaps would have interpretation as phase transitions increasing the maximal value of Planck constant for evolving species. The space-time correlate would be the increase of both the number and the size of the sheets of the covering associated with the system so that its complexity would increase.

4. The question of experimenter is obvious: How could one create dark matter as large h_{eff} phases? The surprising answer is that in (quantum) critical systems this could take places automatically [?]. The long range correlations characterizing criticality would correspond to the scaled up quantal lengths for dark matter.

5. The phase transitions changing Planck constant change also the length of the magnetic flux tubes. The natural conjecture is that biomolecules form a kind of Indra's net connected by the flux tubes and $\hbar$ changing phase transitions are at the core of the quantum bio-dynamics. The contraction of the magnetic flux tube connecting distant biomolecules would force them near to each other making possible for the bio-catalysis to proceed. This mechanism could be central for DNA replication and other basic biological processes. Magnetic Indra's net could be also responsible for the coherence of gel phase and the phase transitions affecting flux tube lengths could induce the contractions and expansions of the intracellular gel phase. The reconnection of flux tubes would allow the restructuring of the signal pathways between biomolecules and other subsystems and would be also involved with ADP-ATP transformation inducing a transfer of negentropic entanglement [**?**] . The braiding of the magnetic flux tubes could make possible topological quantum computation like processes and analog of computer memory realized in terms of braiding patterns [K4] .

6. p-Adic length scale hypothesis - which can be now justified by very general arguments - and the hierarchy of Planck constants suggest entire hierarchy of zoomed up copies of standard model physics with range of weak interactions and color forces scaling like $\hbar$. This is not conflict with the known physics for the simple reason that we know very little about dark matter (partly because we might be making misleading assumptions about its nature). One implication is that it might be someday to study zoomed up variants particle physics at low energies using dark matter.

   Dark matter would make possible the large parity breaking effects manifested as chiral selection of bio-molecules [C6] . What is required is that classical Z^0 and W fields responsible for parity breaking effects are present in cellular length scale. If the value of Planck constant is so large that weak scale is some biological length scale, weak fields are effectively massless below this scale and large parity breaking effects become possible.

   For the solutions of field equations which are almost vacuum extremals Z^0 field is non-vanishing and proportional to electromagnetic field. The hypothesis that cell membrane corresponds to a space-time sheet near a vacuum extremal (this corresponds to criticality very natural if the cell membrane is to serve as an ideal sensory receptor) leads to a rather successful model for cell membrane as sensory receptor with lipids representing the pixels of sensory qualia chart. The surprising prediction is that bio-photons [I5] and bundles of EEG photons can be identified as different decay products of dark photons with energies of visible photons. Also the peak frequencies of sensitivity for photoreceptors are predicted correctly [K88] .

2.3.4 P-Adic Physics And Number Theoretic Universality

p-Adic physics [K70, K105] has become gradually a key piece of TGD inspired biophysics. Basic quantitative predictions relate to p-adic length scale hypothesis and to the notion of number theoretic entropy. Basic ontological ideas are that life resides in the intersection of real and p-adic worlds and that p-adic space-time sheets serve as correlates for cognition. Number theoretical universality requires the fusion of real physics and various p-adic physics to single coherent whole. On implication is the generalization of the notion of number obtained by fusing real and p-adic numbers to a larger adelic structure allowing in turn to define adelic variants of embedding space and space-time and even WCW.

p-Adic number fields

p-Adic number fields Q_p [A38] -one for each prime p- are analogous to reals in the sense that one can speak about p-adic continuum and that also p-adic numbers are obtained as completions of the field of rational numbers. One can say that rational numbers belong to the intersection of real and p-adic numbers. p-Adic number field Q_p allows also an infinite number of its algebraic extensions. Also transcendental extensions are possible. For reals the only extension is complex numbers.

p-Adic topology defining the notions of nearness and continuity differs dramatically from the real topology. An integer which is infinite as a real number can be completely well defined and finite as a p-adic number. In particular, powers p^n of prime p have p-adic norm (magnitude) equal to p^{-n} in Q_p so that at the limit of very large n real magnitude becomes infinite and p-adic magnitude vanishes.

p-Adic topology is rough since p-adic distance $d(x,y) = d(x-y)$ depends on the lowest pinary digit of $x-y$ only and is analogous to the distance between real points when approximated by taking into account only the lowest digit in the decimal expansion of $x-y$. A possible interpretation is in terms of a finite measurement resolution and resolution of sensory perception. p-Adic topology looks somewhat strange. For instance, p-adic spherical surface is not infinitely thin but has a finite thickness and p-adic surfaces possess no boundary in the topological sense. Ultrametricity is the technical term characterizing the basic properties of p-adic topology and is coded by the inequality $d(x-y) \leq Min\{d(x), d(y)\}$. p-Adic topology brings in mind the decomposition of perceptive field to objects.

Motivations for p-adic number fields

The physical motivations for p-adic physics came from the observation that p-adic thermodynamics - not for energy but infinitesimal scaling generator of so called super-conformal algebra [A21] acting as symmetries of quantum TGD [K112] - predicts elementary particle mass scales and also masses correctly under very general assumptions [K70]. In particular, the ratio of proton mass to Planck mass, the basic mystery number of physics, is predicted correctly. The basic assumption is that the preferred primes characterizing the p-adic number fields involved are near powers of two: $p \simeq 2^k$, k positive integer. Those nearest to power of two correspond to Mersenne primes $M_n = 2^n - 1$. One can also consider complex primes known as Gaussian primes, in particular Gaussian Mersennes $M_{G,n} = (1+i)^n - 1$.

It turns out that Mersennes and Gaussian Mersennes are in a preferred position physically in TGD based world order. What is especially interesting that the length scale range 10 nm-5 μm contains as many as four scaled up electron Compton lengths $L_e(k) = \sqrt{5}L(k)$ assignable to Gaussian Mersennes $M_k = (1+i)^k - 1$, $k = 151, 157, 163, 167$, [K88] . This number theoretical miracle supports the view that p-adic physics is especially important for the understanding of living matter.

The philosophicaljustification for p-adic numbers fields come from the question about the possible physical correlates of cognition [K73]. Cognition forms representations of the external world which have finite cognitive resolution and the decomposition of the perceptive field to objects is an essential element of these representations. Therefore p-adic space-time sheets could be seen as candidates of thought bubbles, the mind stuff of Descartes. The longheld idea that p-adic space-time sheets could serve as correlates of intentions transformed to real space-time sheets in quantum jumps has turned out to be mathematically awkward and also un-necessary.

Rational numbers belong to the intersection of real and p-adic continua. Also algebraic extensions of rationals inducing those of p-dic numbers have similar role so that a hierarchy suggesting interpretation in terms of evolution of complexity is suggestive. An obvious generalization of this statement applies to real manifolds and their p-adic variants. When extensions of p-adic numbers are allowed, also some algebraic numbers can belong to the intersection of p-adic and real worlds. The notion of intersection of real and p-adic worlds has actually two meanings.

1. The minimal guess is that the intersection consists of discretion intersections of real and p-adic partonic 2-surfaces at the ends of CD. The interpretation could be as discrete cognitive representations.
2. The intersection could have a more abstract meaning at the evel of WCW. The parameters of the surfaces in the intersection would belong to the extension of rationals and intersection would consist of discrete set of surfaces. One could say that life resides in the intersection of real and p-adic worlds in this abstract sense.

It turns out that the abstract meaning is the correct interpretation [K119]. The reason is that map of reals to p-adics and vice versa is highly desirable. I have made an attempt to realize this map in terms of so called p-adic manifold concept allowing to map real space-time surfaces as preferred extremals of Kähler action to their p-adic counterparts and vice versa. This forces discretization at space-time level since the correspondence between real and p-adic worlds would be local. General coordinate invariance (GCI) however raises a problem and symmetries in general are respected at most in finite measurement resolution.

Strong form of holography allowing to identify string world sheets and partonic 2-surfaces as "space-time genes" plus non-local correspondence between realities and p-adicities allows to circumvent the problem. These 2-surfaces can be said to be in the intersection of realities and p-adicities having charactrizing parameters in an algebraic extension of rationals and allowing continuation to real and various p-adic sectors. This vision has a powerful and highly desirable implication. The so called ramified primes characterizing the algebraic extension of rationals assign preferred primes assignable to these 2-surfaces identifiable as preferred p-adic primes.

In strong form of holography p-adic continuations of 2-surfaces to preferred extrmals identifiable as imaginations would be easy due to the existence of p-adic pseudo-constants. The continuation could fail for most configurations of partonic 2-surfaces and string world sheets in the real sector: the interpretation would be that some space-time surfaces can be imagined but not realized [K73]. For certain extensions the number of realizable imaginations could be exceptionally large. These extensions would be winners in the number theoretic fight for survivalandcorresponding ramified primes would be preferred p-adic primes. One could understand even p-adic length scale hypothesis using Negentropy Maximization Principle in weak form [K65].

Additional support for the idea comes from the observation that Shannon entropy $S = -\sum p_n log(p_n)$ allows a p-adic generalization if the probabilities are rational numbers by replacing $log(p_n)$ with $-log(|p_n|_p)$, where $|x|_p$ is p-adic norm. Also algebraic numbers in some extension of p-adic numbers can be allowed. The unexpected property of the number theoretic Shannon entropy is that it can be negative and its unique minimum value as a function of the p-adic prime p it is always negative. Entropy transforms to information!

In the case of number theoretic entanglement entropy there is a natural interpretation for this. Number theoretic entanglement entropy would measure the information carried by the entanglement whereas ordinary entanglement entropy would characterize the uncertainty about the state of either entangled system. For instance, for p maximally entangled states both ordinary entanglement entropy and number theoretic entanglement negentropy are maximal with respect to R_p norm. Entanglement carries maximal information. The information would be about the relationship between the systems, a rule. Schrödinger cat would be dead enough to know that it is better to not open the bottle completely.

Negentropy Maximization Principle (NMP) [K65] coding the basic rules of quantum measurement theory implies that negentropic entanglement can be stable against the effects of quantum jumps unlike entropic entanglement. Therefore living matter could be distinguished from inanimate matter also by negentropic entanglement possible in the intersection of real and p-adic worlds. In consciousness theory negentropic entanglement could be seen as a correlate for the experience of understanding or any other positively colored experience, say love.

Negentropically entangled states are stable but binding energy and effective loss of relative translational degrees of freedom is not responsible for the stability. Therefore bound states are not in question. The distinction between negentropic and bound state entanglement could be compared to the difference between unhappy and happy marriage. The first one is a social jail but in the latter case both parties are free to leave but do not want to. The special characterics of negentropic entanglement raise the question whether the problematic notion of high energy phosphate bond [I2] central for metabolism could be understood in terms of negentropic entanglement. This would also allow an information theoretic interpretation of metabolism since the transfer of metabolic energy would mean a transfer of negentropy [?].

The recent form of NMP is an outcome of a long evolution. Quantum measurement theory requires that the outcome of quantum jump corresponds to an eigenspace of density matrix - in standard physics it is typically 1-D ray of Hilbert space and is assumed to be such. In TGD quantum criticality allows also higher-dimensional eigenspaces characterized by n-dimensional projector. Strong form of NMP would state that the outcome of measurement is such that negentropy of the final state is maximal. The weak form would say that also any lower-dimensional sub-space of n-dimensional eigenspace is possible. Weak form allows free will: self can choose also the non-optimal outcome. Weak form allows to improve negentropy gain when n consists several prime factors, predicts a generalization of p-adic length scale hypothesis, and also suggest quantum correlates for ethics and moral. For these reasons it seems to be the only reasonable choice.

2.3.5 ZEO

Zero energy state as counterpart of physical event

In standard ontology of quantum physics physical states are assumed to have positive energy. In zero energy ontology (ZEO) physical states decompose to pairs of positive and negative energy states such that all net values of the conserved quantum numbers vanish. The interpretation of these states in ordinary ontology would be as transitions between initial and final states, physical events.

ZEO conforms with the crossing symmetry of quantum field theories meaning that the final states of the quantum scattering event are effectively negative energy states. As long as one can restrict the consideration to either positive or negative energy part of the state ZEO is consistent with positive energy ontology. This is the case when the observer characterized by a particular CD studies the physics in the time scale of much larger CD containing observer's CD as a sub-CD. When the time scale sub-CD of the studied system is much shorter that the time scale of sub-CD characterizing the observer, the interpretation of states associated with sub-CD is in terms of quantum fluctuations.

ZEO solves the problem which results in any theory assuming symmetries giving rise to conservation laws. The problem is that the theory itself is not able to characterize the values of conserved quantum numbers of the initial state. In ZEO this problem disappears since in principle any zero energy state is obtained from any other state by a sequence of quantum jumps without breaking of conservation laws. The fact that energy is not conserved in general relativity based cosmologies can be also understood since each CD is characterized by its own conserved quantities. As a matter fact, one must be speak about average values of conserved quantities since one can have a quantum superposition of zero energy states with the quantum numbers of the positive energy part varying over some range.

At the level of principle the implications are quite dramatic. In quantum jump as recreation replacing the quantum Universe with a new one it is possible to create entire sub-universes from vacuum without breaking the fundamental conservation laws. Free will is consistent with the laws of physics. This makes obsolete the basic arguments in favor of materialistic and deterministic world view.

Zero energy states are located inside causal diamond (CD)

By quantum classical correspondence zero energy states must have space-time and embedding space correlates.

1. Positive and negative energy parts reside at future and past light-like boundaries of causal diamond (CD) defined as intersection of future and past directed light-cones and visualizable as double cone (see **Fig. ??**). The analog of CD in cosmology is big bang followed by big crunch. CDs for a fractal hierarchy containing CDs within CDs. Disjoint CDs are possible and CDs can also intersect.

   The interpretation of CD in TGD inspired theory of consciousness is as an embedding space correlate for the spot-light of consciousness: the contents of conscious experience is about the region defined by CD. At the level of space-time sheets the experience come from space-time sheets restricted to the interior of CD. Whether the sheets can continue outside CD is still unclear.

2. By number theoretical universality the temporal distances between the tips of the intersecting light-cones are assumed to come as integer multiples $T = m \times T_0$ of a fundamental time scale T_0 defined by CP_2 size R as $T_0 = R/c$. p-Adic length scale hypothesis [K71, K119] motivates the stonger hypotheswis that the distances tend to come as as octaves of T_0: $T = 2^n T_0$. One prediction is that in the case of electron this time scale is .1 seconds defining the fundamental biorhythm. Also in the case u and d quarks the time scales correspond to biologically important time scales given by 10 ms for u quark and by and 2.5 ms for d quark [K9] . This means a direct coupling between microscopic and macroscopic scales.

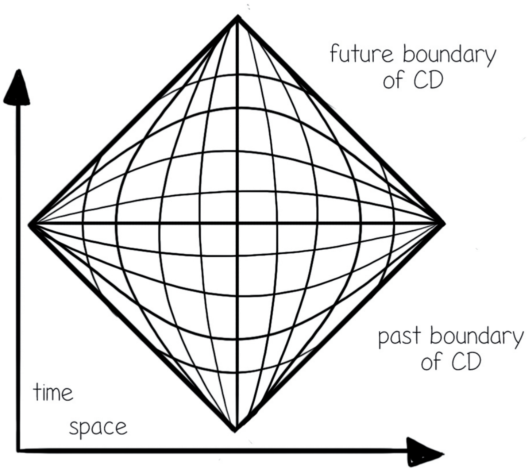

Figure 2.3: The 2-D variant of CD is equivalent with Penrose diagram in empty Minkowski space although interpretation is different.

Quantum theory as square root of thermodynamics

Quantum theory in ZEO can be regarded as a "complex square root" of thermodynamics obtained as a product of positive diagonal square root of density matrix and unitary S-matrix. M-matrix defines time-like entanglement coefficients between positive and negative energy parts of the zero energy state and replaces S-matrix as the fundamental observable. Various M-matrices define the rows of the unitary U matrix characterizing the unitary process part of quantum jump.

The fact that M-matrices are products of Hermitian square roots (operator analog for real variable) of Hermitian density matrix multiplied by a unitary S-matrix S with they commutte implies that possible U-matrices for an algebra generalizing Kac-Moody algebra defining Kac-Moody type symmetries of the S-matrix. This might mean final step in the reduction of theories to their symmetries since the states predicted by the theory would generate its symmetries!

State function reduction,arrow of time in ZEO, and Akaschic records

From the point of view of consciousness theory the importance of ZEO is that conservation laws in principle pose no restrictions for the new realities created in quantum jumps: free will is maximal. In standard quantum measurement theory this time-like entanglement would be reduced in quantum measurement and regenerated in the next quantum jump if one accepts Negentropy Maximization Principle (NMP) [K65] as the fundamental variational principle.

CD as two light-like boundaries corresponding to the positive and negative energy parts of zero energy states which correspond to initial and final states of physical event. State function reduction can occur to both of these boundaries.

1. If state function reductions occur alternately- one at time- then it is very difficult to understand why we experience same arrow of time continually: why not continual flip-flop at the level of perceptions. Some people claim to have actually experienced a temporary change of the arrow of time: I belong to them and I can tell that the experience is frightening. Why we experience the arrow of time as constant?

2. One possible way to solve this problem - perhaps the simplest one - is that state function reduction to the same boundary of CD can occur many times repeatedly. This solution is so absolutely trivial that I could perhaps use this triviality to defend myself for not realizing it immediately! I made this totally trivial observation only after I had realized that also in this process the wave function in the moduli space of CDs could change in these reductions.

Zeno effect in ordinary measurement theory relies on the possibility of repeated state function reductions. In the ordinary quantum measurement theory repeated state function reductions don't affect the state in this kind of sequence but in ZEO the wave function in the moduli space labelling different CDs with the same boundary could change in each quantum jump. It would be natural that this sequence of quantum jumps give rise to the experience about flow of time?

3. This option would allow the size scale of CD associated with human consciousness be rather short, say .1 seconds. It would also allow to understand why we do not observe continual change of arrow of time. Maybe living systems are working hardly to keep the personal arrow of time changed and that it would be extremely difficult to live against the collective arrow of time.

NMP implies that negentropic entanglement generated in state function reductions tends to increase. This tendency is mirror image of entropy growth for ensembles and would provide a natural explanation for evolution as something real rather than just thermodynamical fluctuation as standard thermodynamics suggests. Quantum Universe is building kind of Akashic records. The history would be recorded in a huge library and these books could might be read by interaction free quantum measurements giving conscious information about negentropically entangled states and without changing them: as a matter fact, this is an idealization. Conscious information would require also now state function reduction but it would occur for another system. Elitzur-Vaidman bomb tester (see `http://tinyurl.com/kx2jsyu`) is a down-to-earth representation for what is involved.

2.4 Different Visions About TGD As Mathematical Theory

There are two basic vision about Quantum TGD: physics as infinite-dimensional geometry and physics as generalized number theory.

2.4.1 Quantum TGD As Spinor Geometry Of World Of Classical Worlds

A turning point in the attempts to formulate a mathematical theory was reached after seven years from the birth of TGD. The great insight was "Do not quantize". The basic ingredients to the new approach have served as the basic philosophy for the attempt to construct Quantum TGD since then and have been the following ones:

1. Quantum theory for extended particles is free(!), classical(!) field theory for a generalized Schrödinger amplitude in the WCW CH consisting of all possible 3-surfaces in H. "All possible" means that surfaces with arbitrary many disjoint components and with arbitrary internal topology and also singular surfaces topologically intermediate between two different manifold topologies are included. Particle reactions are identified as topology changes [A66, A73, A82]. For instance, the decay of a 3-surface to two 3-surfaces corresponds to the decay $A \rightarrow B + C$. Classically this corresponds to a path of WCW leading from 1-particle sector to 2-particle sector. At quantum level this corresponds to the dispersion of the generalized Schrödinger amplitude localized to 1-particle sector to two-particle sector. All coupling constants should result as predictions of the theory since no nonlinearities are introduced.

2. During years this naïve and very rough vision has of course developed a lot and is not anymore quite equivalent with the original insight. In particular, the space-time correlates of Feynman graphs have emerged from theory as Euclidian space-time regions and the strong form of General Coordinate Invariance has led to a rather detailed and in many respects un-expected visions. This picture forces to give up the idea about smooth space-time surfaces and replace space-time surface with a generalization of Feynman diagram in which vertices represent the failure of manifold property. I have also introduced the word "world of classical worlds" (WCW) instead of rather formal "WCW". I hope that "WCW" does not induce despair in the reader having tendency to think about the technicalities involved!

3. WCW is endowed with metric and spinor structure so that one can define various metric related differential operators, say Dirac operator, appearing in the field equations of the

theory. The most ambitious dream is that zero energy states correspond to a complete solution basis for the Dirac operator of WCW so that this classical free field theory would dictate M-matrices which form orthonormal rows of what I call U-matrix. Given M-matrix in turn would decompose to a product of a hermitian density matrix and unitary S-matrix.

M-matrix would define time-like entanglement coefficients between positive and negative energy parts of zero energy states (all net quantum numbers vanish for them) and can be regarded as a hermitian square root of density matrix multiplied by a unitary S-matrix. Quantum theory would be in well-defined sense a square root of thermodynamics. The orthogonality and hermiticity of the complex square roots of density matrices commuting with S-matrix means that they span infinite-dimensional Lie algebra acting as symmetries of the S-matrix. Therefore quantum TGD would reduce to group theory in well-defined sense: its own symmetries would define the symmetries of the theory. In fact the Lie algebra of Hermitian M-matrices extends to Kac-Moody type algebra obtained by multiplying hermitian square roots of density matrices with powers of the S-matrix. Also the analog of Yangian algebra involving only non-negative powers of S-matrix is possible.

4. U-matrix realizes in ZEO unitary time evolution in the space for zero energy states realized geometrically as dispersion in the moduli space of causal diamonds (CDs) leaving second boundary (passive boundary) of CD and states at it fixed [K69].

   This process can be seen as the TGD counterpart of repeated state function reductions leaving the states at passive boundary unaffected and affecting only the member of state pair at active boundary (Zeno effect). In TGD inspired theory of consciousness self corresponds to the sequence these state function reductions. M-matrix describes the entanglement between positive and negative energy parts of zero energy states and is expressible as a hermitian square root H of density matrix multiplied by a unitary matrix S, which corresponds to ordinary S-matrix, which is universal and depends only the size scale n of CD through the formula $S(n) = S^n$. M-matrices and H-matrices form an orthonormal basis at given CD and H-matrices would naturally correspond to the generators of super-symplectic algebra.

   The first state function reduction to the opposite boundary corresponds to what happens in quantum physics experiments. The relationship between U- and S-matrices has remained poorly understood. In this article this relationship is analyzed by starting from basic principles. One ends up to formulas allowing to understand the architecture of U-matrix and to reduce its construction to that for S-matrix having interpretation as exponential of the generator L_{-1} of the Virasoro algebra associated with the super-symplectic algebra.

5. By quantum classical correspondence the construction of WCW spinor structure reduces to the second quantization of the induced spinor fields at space-time surface. The basic action is so called modified Dirac action in which gamma matrices are replaced with the modified gamma matrices defined as contractions of the canonical momentum currents with the embedding space gamma matrices. In this manner one achieves super-conformal symmetry and conservation of fermionic currents among other things and consistent Dirac equation. This Kähler-Dirac gamma matrices define as anticommutators effective metric, which might provide geometrization for some basic observables of condensed matter physics. The conjecture is that Dirac determinant for the Kähler-Dirac action gives the exponent of Kähler action for a preferred extremal as vacuum functional so that one might talk about bosonic emergence in accordance with the prediction that the gauge bosons and graviton are expressible in terms of bound states of fermion and antifermion.

The evolution of these basic ideas has been rather slow but has gradually led to a rather beautiful vision. One of the key problems has been the definition of Kähler function. Kähler function is Kähler action for a preferred extremal assignable to a given 3-surface but what this preferred extremal is? The obvious first guess was as absolute minimum of Kähler action but could not be proven to be right or wrong. One big step in the progress was boosted by the idea that TGD should reduce to almost topological QFT in which braids wold replace 3-surfaces in finite measurement resolution, which could be inherent property of the theory itself and imply discretization at partonic 2-surfaces with discrete points carrying fermion number.

1. TGD as almost topological QFT vision suggests that Kähler action for preferred extremals reduces to Chern-Simons term assigned with space-like 3-surfaces at the ends of space-time

(recall the notion of causal diamond (CD)) and with the light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian. Minkowskian and Euclidian regions would give at wormhole throats the same contribution apart from coefficients and in Minkowskian regions the $\sqrt{g_4}$ factor would be imaginary so that one would obtain sum of real term identifiable as Kähler function and imaginary term identifiable as the ordinary action giving rise to interference effects and stationary phase approximation central in both classical and quantum field theory. Imaginary contribution - the presence of which I realized only after 33 years of TGD - could also have topological interpretation as a Morse function. On physical side the emergence of Euclidian space-time regions is something completely new and leads to a dramatic modification of the ideas about black hole interior.

2. The manner to achieve the reduction to Chern-Simons terms is simple. The vanishing of Coulombic contribution to Kähler action is required and is true for all known extremals if one makes a general ansatz about the form of classical conserved currents. The so called weak form of electric-magnetic duality defines a boundary condition reducing the resulting 3-D terms to Chern-Simons terms. In this manner almost topological QFT results. But only "almost" since the Lagrange multiplier term forcing electric-magnetic duality implies that Chern-Simons action for preferred extremals depends on metric.

3. A further quite recent hypothesis inspired by effective 2-dimensionality is that Chern-Simons terms reduce to a sum of two 2-dimensional terms. An imaginary term proportional to the total area of Minkowskian string world sheets and a real term proportional to the total area of partonic 2-surfaces or equivalently strings world sheets in Euclidian space-time regions. Also the equality of the total areas of strings world sheets and partonic 2-surfaces is highly suggestive and would realize a duality between these two kinds of objects. String world sheets indeed emerge naturally for the proposed ansatz defining preferred extremals. Therefore Kähler action would have very stringy character apart from effects due to the failure of the strict determinism meaning that radiative corrections break the effective 2-dimensionality.

The definition of spinor structure - in practice definition of so called gamma matrices of WCW- and WCW Kähler metric define by their anti-commutators has been also a very slow process. The progress in the physical understanding of the theory and the wisdom that has emerged about preferred extremals of Kähler action and about general solution of the field equations for Kähler-Dirac operator during last decade have led to a considerable progress in this respect quite recently.

1. Preferred extremals of Kähler action [K14] seem to have slicing to string world sheets and partonic 2-surfaces such that points of partonic 2-surface slice parametrize different world sheets. I have christened this slicing as Hamilton-Jacobi structure. This slicing brings strongly in mind string models.

2. The modes of the Kähler-Dirac action - fixed uniquely by Kähler action by the requirement of super-conformal symmetry and internal consistency - must be localized to 2-dimensional string world sheets with one exception: the modes of right handed neutrino which do not mix with left handed neutrino, which are delocalized into entire space-time sheet. The localization follows from the condition that modes have well-defined em charge in presence of classical W boson fields. This implies that string model in 4-D space-time becomes part of TGD.

This input leads to a modification of the earlier construction allowing to overcome its features vulnerable to critics. The earlier proposal forced strong form of holography in sense which looked too strong. The data about WCW geometry was localized at partonic 2-surfaces rather than 3-surfaces. The new formulations uses data also from interior of 3-surfaces and this is due to replacement of point-like particle with string: point of partonic 2-surface -wormhole throat- is replaced with a string connecting it to another wormhole throat. The earlier approach used only single mode of induced spinor field: right-handed neutrino. Now all modes of induced spinor field are used and one obtains very concrete connection between elementary particle quantum numbers and WCW geometry.

2.4.2 TGD As A Generalized Number Theory

Quantum T(opological)D(ynamics) as a classical spinor geometry for infinite-dimensional WCW, p-adic numbers and quantum TGD, and TGD inspired theory of consciousness, have been for last ten years the basic three strongly interacting threads in the tapestry of quantum TGD. The fourth thread deserves the name "TGD as a generalized number theory". It involves three separate threads: the fusion of real and various p-adic physics to a single coherent whole by requiring number theoretic universality discussed already, the formulation of quantum TGD in terms of hyper-counterparts of classical number fields identified as sub-spaces of complexified classical number fields with Minkowskian signature of the metric defined by the complexified inner product, and the notion of infinite prime.

p-Adic TGD and fusion of real and p-adic physics to single coherent whole

The p-adic thread emerged for roughly ten years ago as a dim hunch that p-adic numbers might be important for TGD. Experimentation with p-adic numbers led to the notion of canonical identification mapping reals to p-adics and vice versa. The breakthrough came with the successful p-adic mass calculations using p-adic thermodynamics for Super-Virasoro representations with the super-Kac-Moody algebra associated with a Lie-group containing standard model gauge group. Although the details of the calculations have varied from year to year, it was clear that p-adic physics reduces not only the ratio of proton and Planck mass, the great mystery number of physics, but all elementary particle mass scales, to number theory if one assumes that primes near prime powers of two are in a physically favored position. Why this is the case, became one of the key puzzles and led to a number of arguments with a common gist: evolution is present already at the elementary particle level and the primes allowed by the p-adic length scale hypothesis are the fittest ones.

It became very soon clear that p-adic topology is not something emerging in Planck length scale as often believed, but that there is an infinite hierarchy of p-adic physics characterized by p-adic length scales varying to even cosmological length scales. The idea about the connection of p-adics with cognition motivated already the first attempts to understand the role of the p-adics and inspired "Universe as Computer" vision but time was not ripe to develop this idea to anything concrete (p-adic numbers are however in a central role in TGD inspired theory of consciousness). It became however obvious that the p-adic length scale hierarchy somehow corresponds to a hierarchy of intelligences and that p-adic prime serves as a kind of intelligence quotient. Ironically, the almost obvious idea about p-adic regions as cognitive regions of space-time providing cognitive representations for real regions had to wait for almost a decade for the access into my consciousness.

There were many interpretational and technical questions crying for a definite answer.

1. What is the relationship of p-adic non-determinism to the classical non-determinism of the basic field equations of TGD? Are the p-adic space-time region genuinely p-adic or does p-adic topology only serve as an effective topology? If p-adic physics is direct image of real physics, how the mapping relating them is constructed so that it respects various symmetries? Is the basic physics p-adic or real (also real TGD seems to be free of divergences) or both? If it is both, how should one glue the physics in different number field together to get *The Physics*? Should one perform p-adicization also at the level of the WCW of 3-surfaces? Certainly the p-adicization at the level of super-conformal representation is necessary for the p-adic mass calculations.

2. Perhaps the most basic and most irritating technical problem was how to precisely define p-adic definite integral which is a crucial element of any variational principle based formulation of the field equations. Here the frustration was not due to the lack of solution but due to the too large number of solutions to the problem, a clear symptom for the sad fact that clever inventions rather than real discoveries might be in question. Quite recently I however learned that the problem of making sense about p-adic integration has been for decades central problem in the frontier of mathematics and a lot of profound work has been done along same intuitive lines as I have proceeded in TGD framework. The basic idea is certainly the notion of algebraic continuation from the world of rationals belonging to the intersection of real world and various p-adic worlds.

Despite these frustrating uncertainties, the number of the applications of the poorly defined p-adic physics grew steadily and the applications turned out to be relatively stable so that it was clear that the solution to these problems must exist. It became only gradually clear that the solution of the problems might require going down to a deeper level than that represented by reals and p-adics.

The key challenge is to fuse various p-adic physics and real physics to single larger structures. This has inspired a proposal for a generalization of the notion of number field by fusing real numbers and various p-adic number fields and their extensions along rationals and possible common algebraic numbers. This leads to a generalization of the notions of embedding space and space-time concept and one can speak about real and p-adic space-time sheets. The quantum dynamics should be such that it allows quantum transitions transforming space-time sheets belonging to different number fields to each other. The space-time sheets in the intersection of real and p-adic worlds are of special interest and the hypothesis is that living matter resides in this intersection. This leads to surprisingly detailed predictions and far reaching conjectures. For instance, the number theoretic generalization of entropy concept allows negentropic entanglement (see **Fig.** `http://tgdtheory.fi/appfigures/cat.jpg` or **Fig. ??** in the appendix of this book) central for the applications to living matter.

The basic principle is number theoretic universality stating roughly that the physics in various number fields can be obtained as completion of rational number based physics to various number fields. Rational number based physics would in turn describe physics in finite measurement resolution and cognitive resolution. The notion of finite measurement resolution has become one of the basic principles of quantum TGD and leads to the notions of braids as representatives of 3-surfaces and inclusions of hyper-finite factors as a representation for finite measurement resolution.

The proposal for a concrete realization of this program at space-time level is in terms of the notion of p-adic manifold [K122] generalising the notion of real manifold. Chart maps of p-adic manifold are however not p-adic but real and mediated by a variant of canonical correspondence between real and p-adic numbers. This modification of the notion of chart map allows to circumvent the grave difficulties caused by p-adic topology. Also p-adic manifolds can serve as charts for real manifolds and now the interpretation is as cognitive representation. The coordinate maps are characterized by finite measurement/cognitive resolution and are not completely unique. The basic principle reducing part of the non-uniqueness is the condition that preferred extremals are mapped to preferred extremals: actually this requires finite measurement resolution (see **Fig.** `http://tgdtheory.fi/appfigures/padmanifold.jpg` or **??** in the appendix of this book).

The role of classical number fields

The vision about the physical role of the classical number fields relies on the notion of number theoretic compactification stating that space-time surfaces can be regarded as surfaces of either M^8 or $M^4 \times CP_2$. As surfaces of M^8 identifiable as space of hyper-octonions they are hyper-quaternionic or co-hyper-quaternionic- and thus maximally associative or co-associative. This means that their tangent space is either hyper-quaternionic plane of M^8 or an orthogonal complement of such a plane. These surface can be mapped in natural manner to surfaces in $M^4 \times CP_2$ [K105] provided one can assign to each point of tangent space a hyper-complex plane $M^2(x) \subset M^8$ [K105, K119]. One can also speak about $M^8 - H$ duality.

This vision has very strong predictive power. It predicts that the extremals of Kähler action correspond to either hyper-quaternionic or co-hyper-quaternionic surfaces such that one can assign to tangent space at each point of space-time surface a hyper-complex plane $M^2(x) \subset M^4$. As a consequence, the M^4 projection of space-time surface at each point contains $M^2(x)$ and its orthogonal complement. These distributions are integrable implying that space-time surface allows dual slicings defined by string world sheets Y^2 and partonic 2-surfaces X^2. The existence of this kind of slicing was earlier deduced from the study of extremals of Kähler action and christened as Hamilton-Jacobi structure. The physical interpretation of $M^2(x)$ is as the space of non-physical polarizations and the plane of local 4-momentum.

One can fairly say, that number theoretical compactification is responsible for most of the understanding of quantum TGD that has emerged during last years. This includes the realization of Equivalence Principle at space-time level, dual formulations of TGD as Minkowskian and Euclidian string model type theories, the precise identification of preferred extremals of Kähler action as extremals for which second variation vanishes (at least for deformations representing dynamical symmetries) and thus providing space-time correlate for quantum criticality, the notion of number theoretic braid implied by the basic dynamics of Kähler action and crucial for precise construction of quantum TGD as almost-topological QFT, the construction of WCW metric and spinor structure in terms of second quantized induced spinor fields with Kähler-Dirac action defined by Kähler action realizing automatically the notion of finite measurement resolution and a connection with inclusions of hyper-finite factors of type II_1 about which Clifford algebra of WCW represents an example.

The two most important number theoretic conjectures relate to the preferred extremals of Kähler action. The general idea is that classical dynamics for the preferred extremals of Kähler action should reduce to number theory: space-time surfaces should be either associative or co-associative in some sense.

1. The first meaning for associativity (co-associativity) would be that tangent (normal) spaces of space-time surfaces are quaternionic in some sense and thus associative. This can be formulated in terms of octonionic representation of the embedding space gamma matrices possible in dimension $D = 8$ and states that induced gamma matrices generate quaternionic sub-algebra at each space-time point. It seems that induced rather than Kähler-Dirac gamma matrices must be in question.

2. Second meaning for associative (co-associativity) would be following. In the case of complex numbers the vanishing of the real part of real-analytic function defines a 1-D curve. In oct-nionic case one can decompose octonion to sum of quaternion and quaternion multiplied by an octonionic imaginary unit. Quaternionicity could mean that space-time surfaces correspond to the vanishing of the imaginary part of the octonion real-analytic function. Co-quaternionicity would be defined in an obvious manner. Octonionic real analytic functions form a function field closed also with respect to the composition of functions. Space-time surfaces would form the analog of function field with the composition of functions with all operations realized as algebraic operations for space-time surfaces. Co-associativity could be perhaps seen as an additional feature making the algebra in question also co-algebra.

3. The third conjecture is that these conjectures are equivalent.

Infinite primes

The discovery of the hierarchy of infinite primes and their correspondence with a hierarchy defined by a repeatedly second quantized arithmetic quantum field theory gave a further boost for the speculations about TGD as a generalized number theory. The work with Riemann hypothesis led to further ideas.

After the realization that infinite primes can be mapped to polynomials representable as surfaces geometrically, it was clear how TGD might be formulated as a generalized number theory with infinite primes forming the bridge between classical and quantum such that real numbers, p-adic numbers, and various generalizations of p-adics emerge dynamically from algebraic physics as various completions of the algebraic extensions of rational (hyper-)quaternions and (hyper-)octonions. Complete algebraic, topological and dimensional democracy would characterize the theory.

What is especially satisfying is that p-adic and real regions of the space-time surface could emerge automatically as solutions of the field equations. In the space-time regions where the solutions of field equations give rise to in-admissible complex values of the embedding space coordinates, p-adic solution can exist for some values of the p-adic prime. The characteristic non-determinism of the p-adic differential equations suggests strongly that p-odic regions correspond to "mind stuff", the regions of space-time where cognitive representations reside. This interpretation implies that p-adic physics is physics of cognition. Since Nature is probably an extremely brilliant simulator of Nature, the natural idea is to study the p-adic physics of the cognitive representations to derive information about the real physics. This view encouraged by TGD inspired theory of consciousness clarifies difficult interpretational issues and provides a clear interpretation for the predictions of p-adic physics.

2.5 Guiding Principles

2.5.1 Physics Is Unique From The Mathematical Existence Of WCW

1. The conjecture inspired by the geometry of loop spaces [A43] is that H is fixed from the mere requirement that the infinite-dimensional Kähler geometry exists. WCW must reduce to a union of symmetric spaces having infinite-dimensional isometry groups and labeled by zero modes having interpretation as classical dynamical variables.

   This requires infinite-dimensional symmetry groups. At space-time level super-conformal symmetries are possible only if the basic dynamical objects can be identified as light-like or space-like 3-surfaces. At embedding space level there are extended super-conformal symmetries assignable to the light-cone of H if the Minkowski space factor is four-dimensional.

2. The great vision has been that the second quantization of the induced spinor fields can be understood geometrically in terms of the WCW spinor structure in the sense that the anti-commutation relations for WCW gamma matrices require anti-commutation relations for the oscillator operators for free second quantized induced spinor fields defined at space-time surface. This means geometrization of Fermi statistics usually regarded as one of the purely quantal features of quantum theory.

2.5.2 Number Theoretical Universality

The original view about physics as the geometry of WCW is not enough to meet the challenge of unifying real and p-adic physics to a single coherent whole. This inspired "physics as a generalized number theory" approach [K75].

Fusion of real and padic physics to single coherent whole

Fusion of real and p-adic physics to single coherent whole is the first part in the program aiming to realize number theoretical universality.

1. The first element is a generalization of the notion of number obtained by "gluing" reals and various p-adic number fields and their algebraic extensions along common points defined by algebraic extension of rationals defining also extension of p-adics to form a larger structure (see **Fig.** `http://tgdtheory.fi/appfigures/book.jpg` or **Fig. ??** in the appendix of this book). This vision leads to what might be called adelic space-time [K119] identifiable as a book like structure having space-time surfaces in various number fields glued along common back to form a book-like structure. What this back is, is far from clear.

2. Reality-p-adicity correspondence could be local or only global. Local correspondence at the level of embedding space would correspond to a gluing of real and p-adic variants of the embedding space together along rational and common algebraic points (the number of which depends on algebraic extension of p-adic numbers used) to what could be seen as a book like structure. General Coordinate Invariance (GCI) restricted to rationals or their extension requires preferred coordinates for $CD \times CP_2$ and this kind coordinates can be fixed by isometries of H. The coordinates are however not completely unique since non-rational isometries produce new equally good choices. This can be seen as an objection against the local correspondence.

3. Global correspondence is weaker and would make sense at the level of WCW. The fact that p-adic variants of field equations make sense allows to ask what are the common points of WCWs associated with real and various p-adic worlds and whether one can speak about WCWs in various number fields forming a book like structure.

   Strong form of holography suggests a formulation in terms of string world sheets and partonic 2-surfaces so that real and p-adic space-time surfaces would be obtained by holography from them and one could circumvent the problems with GCI.

   What it is to be a 2-surface belonging to the intersection of real and p-adic variants of WCW? The natural answer is that partonic 2-surfaces which have a mathematical representation making sense both for real numbers and p-adic numbers or their algebraic extensions can be regarded as "common" or "identifiable" points of p-adicity and reality.

By conformal invariance one could argue that only the conformal moduli of the 2-surfaces matter, and that these moduli, which are in general coordinate invariants belong to the algebraic extension of rationals in the intersection. Situation would become finite-dimensional and tractable using the mathematics applied already in string models.

4. By the strong form of holography scattering amplitudes should allow a formulation using only the data assignable to the 2-surfaces in the intersection. An almost trivial looking algebraic continuation of the parameters of the amplitudes from the extension of rationals to various number fields would give the amplitudes in various number fields.

   Note however that one must always make approximations for the parameters of the scattering amplitudes (say Lorentz invariants formed from momenta and other four-vectors) in an algebraic extension of rationals. Even a smallest change of rational in real sense can induce large change of corresponding p-adic number. In order to achieve stability one must map numbers of extension of rationals regarded as real numbers to the corresponding extension of p-adic numbers. Here some form of canonical identification could be involved. It would not however break symmetries if the parameters in question are Lorentz invariant and general coordinate invariant. In p-adic mass calculations mass squared eigenvalues are mapped in this manner.

5. Note that the number theoretical universality of Boolean cognition having fermions as physical correlates demands that fermions reside at the two-surfaces in the intersection. The same result follows from many other constraints.

Classical number fields and associativity and commutativity as fundamental law of physics

The dimensions of classical number fields appear as dimensions of basic objects in quantum TGD. Embedding space has dimension 8, space-time has dimension 4, light-like 3-surfaces are orbits of 2-D partonic surfaces. If conformal QFT applies to 2-surfaces (this is questionable), one-dimensional structures would be the basic objects. The lowest level would correspond to discrete sets of points identifiable as intersections of real and p-adic space-time sheets. This suggests that besides p-adic number fields also classical number fields (reals, complex numbers, quaternions, octonions [A67]) are involved [K105] and the notion of geometry generalizes considerably. In the recent view about quantum TGD the dimensional hierarchy defined by classical number field indeed plays a key role. $H = M^4 \times CP_2$ has a number theoretic interpretation and standard model symmetries can be understood number theoretically as symmetries of hyper-quaternionic planes of hyper-octonionic space.

The associativity condition $A(BC) = (AB)C$ suggests itself as a fundamental physical law of both classical and quantum physics. Commutativity can be considered as an additional condition. In conformal field theories associativity condition indeed fixes the n-point functions of the theory. At the level of classical TGD space-time surfaces could be identified as maximal associative (hyper-quaternionic) sub-manifolds of the embedding space whose points contain a preferred hyper-complex plane M^2 in their tangent space and the hierarchy finite fields-rationals-reals-complex numbers-quaternions-octonions could have direct quantum physical counterpart [K105]. This leads to the notion of number theoretic compactification analogous to the dualities of M-theory: one can interpret space-time surfaces either as hyper-quaternionic 4-surfaces of M^8 or as 4-surfaces in $M^4 \times CP_2$. As a matter fact, commutativity in number theoretic sense is a further natural condition and leads to the notion of number theoretic braid naturally as also to direct connection with super string models.

At the level of Kähler-Dirac action the identification of space-time surface as an associative (co-associative) submanifold of H means that the Kähler-Dirac gamma matrices of the space-time surface defined in terms of canonical momentum currents of Kähler action using octonionic representation for the gamma matrices of H span a associative (co-associative) sub-space of hyper-octonions at each point of space-time surface (hyper-octonions are the subspace of complexified octonions for which imaginary units are octonionic imaginary units multiplied by commutating imaginary unit). Hyper-octonionic representation leads to a proposal for how to extend twistor program to TGD framework [K121, K119, L8].

2.5.3 Symmetries

Magic properties of light cone boundary and isometries of WCW

The special conformal, metric and symplectic properties of the light cone of four-dimensional Minkowski space: δM^4_+, the boundary of four-dimensional light cone is metrically 2-dimensional(!) sphere allowing infinite-dimensional group of conformal transformations and isometries(!) as well as Kähler structure. Kähler structure is not unique: possible Kähler structures of light cone boundary are parameterized by Lobatchevski space $SO(3,1)/SO(3)$. The requirement that the isotropy group $SO(3)$ of S^2 corresponds to the isotropy group of the unique classical 3-momentum assigned to $X^4(Y^3)$ defined as a preferred extremum of Kähler action, fixes the choice of the complex structure uniquely. Therefore group theoretical approach and the approach based on Kähler action complement each other.

1. The allowance of an infinite-dimensional group of isometries isomorphic to the group of conformal transformations of 2-sphere is completely unique feature of the 4-dimensional light cone boundary. Even more, in case of $\delta M^4_+ \times CP_2$ the isometry group of δM^4_+ becomes localized with respect to CP_2! Furthermore, the Kähler structure of δM^4_+ defines also symplectic structure.

   Hence any function of $\delta M^4_+ \times CP_2$ would serve as a Hamiltonian transformation acting in both CP_2 and δM^4_+ degrees of freedom. These transformations obviously differ from ordinary local gauge transformations. This group leaves the symplectic form of $\delta M^4_+ \times CP_2$, defined as the sum of light cone and CP_2 symplectic forms, invariant. The group of symplectic transformations of $\delta M^4_+ \times CP_2$ is a good candidate for the isometry group of the WCW.

2. The approximate symplectic invariance of Kähler action is broken only by gravitational effects and is exact for vacuum extremals. If Kähler function were exactly invariant under the symplectic transformations of CP_2, CP_2 symplectic transformations would correspond to zero modes having zero norm in the Kähler metric of WCW. This does not make sense since symplectic transformations of $\delta M^4 \times CP_2$ actually parameterize the quantum fluctuation degrees of freedom.

3. The groups G and H, and thus WCW itself, should inherit the complex structure of the light cone boundary. The diffeomorphisms of M^4 act as dynamical symmetries of vacuum extremals. The radial Virasoro localized with respect to $S^2 \times CP_2$ could in turn act in zero modes perhaps inducing conformal transformations: note that these transformations lead out from the symmetric space associated with given values of zero modes.

Symplectic transformations of $\delta M^4_+ \times CP_2$ as isometries of WCW

The symplectic transformations of $\delta M^4_+ \times CP_2$ are excellent candidates for inducing symplectic transformations of the WCW acting as isometries. There are however deep differences with respect to the Kac Moody algebras.

1. The conformal algebra of the WCW is gigantic when compared with the Virasoro + Kac Moody algebras of string models as is clear from the fact that the Lie-algebra generator of a symplectic transformation of $\delta M^4_+ \times CP_2$ corresponding to a Hamiltonian which is product of functions defined in δM^4_+ and CP_2 is sum of generator of δM^4_+-local symplectic transformation of CP_2 and CP_2-local symplectic transformations of δM^4_+. This means also that the notion of local gauge transformation generalizes.

2. The physical interpretation is also quite different: the relevant quantum numbers label the unitary representations of Lorentz group and color group, and the four-momentum labelling the states of Kac Moody representations is not present. Physical states carrying no energy and momentum at quantum level are predicted. The appearance of a new kind of angular momentum not assignable to elementary particles might shed some light to the longstanding problem of baryonic spin (quarks are not responsible for the entire spin of proton). The possibility of a new kind of color might have implications even in macroscopic length scales.

3. The central extension induced from the natural central extension associated with $\delta M^4_+ \times CP_2$ Poisson brackets is anti-symmetric with respect to the generators of the symplectic algebra rather than symmetric as in the case of Kac Moody algebras associated with loop spaces. At

first this seems to mean a dramatic difference. For instance, in the case of CP_2 symplectic transformations localized with respect to δM^4_+ the central extension would vanish for Cartan algebra, which means a profound physical difference. For $\delta M^4_+ \times CP_2$ symplectic algebra a generalization of the Kac Moody type structure however emerges naturally.

The point is that δM^4_+-local CP_2 symplectic transformations are accompanied by CP_2 local δM^4_+ symplectic transformations. Therefore the Poisson bracket of two δM^4_+ local CP_2 Hamiltonians involves a term analogous to a central extension term symmetric with respect to CP_2 Hamiltonians, and resulting from the δM^4_+ bracket of functions multiplying the Hamiltonians. This additional term could give the entire bracket of the WCW Hamiltonians at the maximum of the Kähler function where one expects that CP_2 Hamiltonians vanish and have a form essentially identical with Kac Moody central extension because it is indeed symmetric with respect to indices of the symplectic group.

How the extended super-conformal symmetries act?

The basic question is whether the extended super-conformal symmetries act as gauge symmetries or as genuine dynamical symmetries generating new physical states. Both alternatives are in some sense correct and in some sense wrong.

The huge vacuum degeneracy manifesting itself as CP_2 type vacuum extremals and as M^4 type vacuum extremals of Kähler action allows both symplectic transformations of $\delta M^{\pm} \times CP_2$ and Kac-Moody type super-conformal symmetries are gauge transformations. This motivates the hypothesis that symplectic transformations act as isometries of WCW.

The proposal inspired by quantum criticality of TGD Universe is that there is a hierarchy of breakings for super-conformal symmetries acting as gauge symmetries. One has sequences of symmetry breakings of various super-conformal algebras to sub-algebras for which conformal weights are integer multiples of some integer n. For a given sequence one would have $n_{i+1} = m_i n_i$ giving $n_i = \prod_{k \leq i} m_k$. These symmetry breaking hierarchies would correspond to hierarchies of inclusions for hyper-finite factors of type II_1 and describe measurement resolution [K120]. The larger the value of n, the better the resolution. Also the numbers of string world sheets and partonic 2-surfaces of would correlate with the resolution. In each breaking identifiable as emergence of criticality new super-conformal generators creating originally zero norm states begin to create genuine physical states and new physical degrees of freedom emerge.

The classical space-time correlate would be that the space-time surfaces the conformal charges for the sub-algebra characterized by n would correspond to vanishing symplectic Noether charges: this would give the long sought for precise condition characterizing the notion of preferred extremal in ZEO. Interior degrees of freedom of 3-surfaces are almost totally gauge degrees of freedom in accordance with strong form of holography implied by strong form of General Coordinate Invariance and stating that partonic 2-surfaces and their 4-D tangent space data code for quantum physics. Dark phase might be perhaps seen as breaking of this property. Similar hierarchy would appear in fermionic degrees of freedom.

This hierarchy would also correspond to the hierarchy of Planck constants $h_{eff} = n \times h$ giving rise to a hierarchy of phases behaving like dark matter with respect to each other (relative darkness). Naturally, the evolution assignable to the increase of n would correspond to the increase of measurement resolution. Living systems would be quantum critical as I proposed long time ago with inspiration coming from the quantum criticality of TGD Universe itself.

Attempts to identify WCW Kähler metric

The construction of the Kähler metric of WCW has been one of the hard problems of TGD. I have considered three approaches.

1. The first approach is based on Kähler function identified as Kähler action for the Euclidian regions of space-time surface identified as wormhole contacts with 4-D CP_2 projection. The general formula for the Kähler metric remains however only a formal expression.

2. Second approach relies on huge group of WCW isometries, which fix the WCW metric apart from a conformal factor depending on zero modes (non-quantum fluctuating degrees

of freedom not contributing to differentials in WCW line element) identifiable as symplectic invariants. I have even considered a formula for WCW Hamiltonians in terms "half-Poisson-brackets" for the fluxes of the Hamiltonians of $\delta M^4_\pm \times CP_2$ symplectic transformations [K28, K121]. I am the first one to admit that this does not give a totally convincing formula for the matrix elements of the Kähler metric.

3. In the third approach the construction of WCW metric reduces to that for complexified WCW gamma matrices expressible in terms of fermionic oscillator operators for second quantized induced spinor fields. The isometry generators at the level of WCW correspond to the symplectic algebra at the boundary of CD that is at $\delta M^4 \pm \times CP_2$ defining WCW Hamiltonians. WCW gamma matrices are identified as super-symplectic Noether charges assignable to the fermionic part of the action and completely well-defined if fermionic anti-commutation relations can be fixed as seems to be the case. In the most general case there is a contribution from both the fermions in the interior associated with Kähler-Dirac action (they might be absent by associativity condition) and fermions at string world sheets. This would give the desired explicit formula for the WCW Kähler metric. There are still some options to be considered but this approach seems to be the practical one.

2.5.4 Quantum Classical Correspondence

Quantum classical correspondence (QCC) has been the basic guiding principle in the construction of TGD. Below are some basic examples about its application.

1. QCC led to the idea that Kähler function for poitn X^3 of WCW must have interpretation as classical action for a preferred extremal $X^4(X^3)$ assignable to Kähler action assumed to be unique: this assumption can of course be criticized because the dynamics is not strictly deterministic. This criticism led to ZEO. The interpretation of preferred extremal is as analog of Bohr orbit so that Bohr orbitology usually believed to be an outcome of stationary phase approximations would be an exact part of quantum TGD.

2. QCC suggests a correlation between 4-D geometry of space-time sheet and quantum numbers. This could result if the classical charges in Cartan algebra are identical with the quantal ones. This would give very powerful constraint on the allowed space-time sheets in the superposition of space-time sheets defining WCW spinor field. An even strong condition would be that classical correlation functions are equal to quantal ones.

   The equality of quantal and classical Cartan charges could be realized by adding constraint terms realized using Lagrange multipliers at the space-like ends of space-time surface at the boundaries of CD. This procedure would be very much like the thermodynamical procedure used to fix the average energy or particle number of the the system using Lagrange multipliers identified as temperature or chemical potential. Since quantum TGD can be regarded as square root of thermodynamics in ZEO (ZEO), the procedure looks logically sound.

   One aspect of quantum criticality is the condition that the eigenvalues of quantal Noether charges in Cartan algebra associated with the Kähler Dirac action have correspond to the Noether charges for Kähler action in the sense that for given eigenvalue the space-time surfaces have same Kähler Noether charge.

3. A stronger form of QCC requires that classical correlation functions for general coordinate invariance observables as functions of two points of embedding space are equal to the quantal ones - at least in the length cale resolution considered. This would give a very powerful - maybe too powerful - constraint on the zero energy states.

The strong form of QCC is of course a rather speculative hypothesis. What seems clear is that the notion of preferred extremal is defined naturally by posing the vanishing of conformal Noether charges at the ends of space-time surfaces at the boundaries of CD. These conditions are extremely restrictive in ZEO. Whether they imply the proposed strong form of QCC remains an open question.

2.5.5 Quantum Criticality

The notion of quantum criticality of TGD Universe was originally inspired by the question about how to make TGD unique if Kähler function $K(X^3)$ in WCW is defined by the Kähler action for a preferred extremal $X^4(X^3)$ assignable to a given 3-surface. Vacuum functional defined by the exponent of Kähler function is analogous to thermodynamical weight and the obvious idea with Kähler coupling strength taking the role of temperature. The obvious idea was that the value of Kähler coupling strength α_K is analogous to critical temperature so that TGD would be more or less uniquely defined. One cannot exclude the possibility that α_K has several values, and the doomsday scenario is that there is infinite number of critical values converging towards $\alpha_K = 0$, which corresponds to vanishing temperature).

Various variations of Kähler action

To understand the delicacies it is convenient to consider various variations of Kähler action first.

1. The variation can leave 3-surface invariant but modify space-time surface in such a manner that Kähler action remains invariant. In this case infinitesimal deformation reduces to a diffeomorphism at space-like 3-surface X^3 and perhaps also at light-like 3-surfaces representing partonic orbits. The correspondence between X^3 and $X^4(X^3)$ would not be unique. Actually this is suggested by that the non-deterministic dynamics characteristic for critical systems. Also the failure of the strict classical determinism implying spin glass type vacuum degeneracy forces to consider this possibility. This criticality would correspond to criticality of Kähler action at X^3 but not that of Kähler function. Note that the original working hypothesis was that $X^4(X^3)$ is unique.
2. The variation could act on zero modes which do not affect Kähler metric, which corresponds to (1, 1) part of Hessian in complex coordinates for WCW. Only the zero modes characterizing 3-surface appearing as parameters in the metric of WCW would be affected, and the result would be a generalization of modification of conformal scaling factor. Kähler function would change but only due to the change in zero modes. These transformations do not correspond to critical transformations since Kähler function changes.
3. The variation could act on 3-surface both in zero modes and dynamical degrees of freedom represented by complex coordinates. It would affect also the space-time surface. Criticality for Kähler function would mean that Kähler metric has zero modes at X^3 meaning that (1, 1) part of Hessian is degenerate. This would mean that in the vicinity of X^3 the Hessian has non-definite signature: same could be true also for the (1, 1) part. Physically this is unacceptable since the inner product in Hilbert space should be positive definite.

Critical deformations

Consider now critical deformations (the first option). Critical deformations are expected to relate closely to the coset space decomposition of WCW to a union of coset spaces G/H labelled by zero modes.

1. Critical deformations leave 3-surface X^3 invariant as do also the transformations of H associated with X^3. If H affects $X^4(X^3)$ and corresponds to critical deformations then critical they would allow to extend WCW to a bundle for which 3-surfaces X^3 would be base points and preferred extremals $X^4(X^3)$ would define the fiber. Gauge invariance with respect to H would generalize the assumption that $X^4(X^3)$ is unique.
2. Critical deformations could correspond to H or sub-group of H (which depends on X^3). For other 3-surfaces than X^3 the action of H is non-trivial: to see this consider the simple finite-dimensional case $CP_2 = SU(3)/U(2)$. The groups $H(X^3)$ are symplectic conjugates of each other for given values of zero modes which are symplectic invariants.
3. A possible identification of Lie-algebra of H is as a sub-algebra of Virasoro algebra associated with the symplectic transformations of $\delta M^4 \times CP_2$ and acting as diffeomorphisms for the light-like radial coordinate of δM^4_+. The sub-algebras of Virasoro algebra have conformal weights coming as integer multiplies $= km$, $k \in Z$, of given conformal weight m and form inclusion hierarchies suggesting a direct connection with finite measurement resolution realized in terms of inclusions of hyperfinite factors of type II_1.

   For $m > 1$ one would have breaking of maximal conformal symmetry. The action of these Virasoro algebra on symplectic algebra would make the corresponding sub-algebras gauge degrees of freedom so that the number of symplectic generators generating non-gauge transformations would be finite. This result is not surprising since also for 2-D critical systems criticality corresponds to conformal invariance acting as local scalings.

Vanishing of the second variation at criticality

The vanishing of the second variation for some deformations means that the system is critical, in the recent case quantum critical [K28]. Basic example of criticality is the bifurcation diagram for cusp catastrophe [A4]. Quantum criticality realized as the vanishing of the second variation gives hopes about identification of preferred extremals. One must however give up hopes about uniqueness. The natural expectation is that the number of critical deformations is infinite and corresponds to conformal symmetries naturally assignable to criticality. The number n of conformal equivalence classes of the deformations can be finite and n would naturally relate to the hierarchy of Planck constants $h_{eff} = n \times h$. In each breaking of conformal symmetry some number of conformal gauge degrees of freedom would transform to physical degrees of freedom and the measurement resolution would improve. The hierarchies of criticality defined by sequences of integers n_i dividing n_{i+1} would correspond to hierarchies for the inclusions of hyper-finite factors and both n and numbers of string world sheet and partonic 2-surfaces would correlate with measurement resolution.

Alternative identification of preferred extremals

Quantum criticality provides a very natural identification of the preferred extremal property I have considered also alternative identifications such as absolute minimization of Kähler action, which is just the opposite of criticality (see **Fig.** `http://tgdtheory.fi/appfigures/planckhierarchy.jpg` or **Fig. ??** in the appendix of this book).

One must also remember that space-time surface decomposes to regions with Euclidian and Minkowskian signature of the induced metric and it is not quite clear whether the conformal symmetries giving rise to quantum criticality appear in both regions. In fact, Kähler action is non-negative in Eudlidian space-time regions, so that absolute minimization could make sense in Euclidian regions and therefore for Kähler function. Criticality could be purely Minkowskian notion.

Symplectic Noether charges vanish for both M^4 and CP_2 type vacuum extremals identically, which suggests that the hierarchy of quantum criticalities brings in non-vanishing symplectic Noether charges associated with the deformations of these extremals. These charges would be actually natural coordinates in WCW.

One must be very cautious here: there are two criticalities: one for the extremals of Kähler action with respect to the deformations of four-surface and second for the Kähler function itself with respect to the deformations of 3-surface: these criticalities are not equivalent since in the latter case variation respects preferred extremal property unlike in the first case.

1. The criticality for preferred extremals (G/H option) would make 4-D criticality a property of all physical systems. Conformal symmetry breaking would however break criticality below some scale.

2. The criticality for Kähler function would be 3-D and might hold only for very special systems. In fact, the criticality means that some eigenvalues for the Hessian of Kähler function vanish and for nearby 3-surfaces some eigenvalues are negative. On the other hand the Kähler metric defined by (1, 1) part of Hessian in complex coordinates must be positive definite. Thus criticality might therefore imply problems.

   This allows and suggests non-criticality of Kähler function coming from Kähler action for Euclidian space-time regions: this is mathematically the simplest situation since in this case there are no troubles with Gaussian approximation to the functional integral. The Morse function coming from Kähler action in Minkowskian as imaginary contribution analogous to that appearing in path integral could however be critical and allow non-definite signature in principle. In fact this is expected by the defining properties of Morse function. Kähler function would make WCW integral mathematically existing and Morse function would imply the typical quantal interference effects.

3. The almost 2-dimensionality implied by strong form of holography suggests that the interior degrees of freedom of 3-surface can be regarded as almost gauge degrees of freedom and that this relates directly to generalised conformal symmetries associated with symplectic isometries of WCW. These degrees of freedom are not critical in the sense inspired by G/H decomposition. The only plausible interpretation seems to be that these degrees of freedom correspond to deformations in zero modes.

The hierarchy of quantum criticalities as a hierarchy of breakings of super-symplectic symmetry

The latest step in progress is an astonishingly simple formulation of quantum criticality at space-time level. At given level of hierarchy of criticalities the classical symplectic charges for preferred extremals vanish for a sub-algebra of symplectic algebra with conformal weights coming as n-ples of those for the full algebra.

This gives also a connection with the hierarchy of Planck constants. It conforms also with the strong form of holography and the adelic vision about preferred extemals and the construction of scattering amplitudes.

This is a brief summary about quantum criticality in bosonic degrees of freedom. One must formulate quantum criticality for the Kähler-Dirac action [K121]. The new element is that critical deformations with vanishing second variation of Kähler action define vanishing first variation of Kähler Dirac action so that second order Noether charges correspond to first order Noether charges in fermionic sector. It seems that the formulation in terms of hierarchy of broken conformal symmetries is the most promising one mathematically and also correspond to physical intuition. Also in the fermionic sector the vanishing of conformal Noether super charges for sub-algebra of super-symplectic algebra serves as a criterion for quantum criticality.

2.5.6 The Notion Of Finite Measurement Resolution

Finite measurement resolution has become one of the basic principles of quantum TGD. Finite measurement resolution has two realizations: the quantal realization in terms of inclusions of von Neumann algebras and the classical realization in terms of discretization having a nice description in number theoretic approach.

The notion of p-adic manifold (see the appendix of the book) relying on the canonical correspondence between real and p-adic physics would force finite cognitive and measurement resolution automatically and imply that p-adic preferred extremals are cognitive representations for real preferred extremals in finite cognitive representations [K122]. GCI is the problem of this approach and it seems that the correct formulation is at at the level of WCW so that one gives up local correspondence between preferred extremals in various number fields. Finite measurement resolution would be defined in terms of the parameters characterizing string world sheets and partonic 2-surfaces in turn defining space-time surfaces by strong form of holography [K119].

Von Neumann introduced three types of algebras as candidates for the mathematics of quantum theory. These algebras are known as von Neumann algebras and the three factors (kind of basic building bricks) are known as factors of type I, II, and III. The factors of type I are simplest and apply in wave mechanics where classical system has finite number of degrees of freedom. Factors of type III apply to quantum field theory where the number of degrees of freedom is infinite. Von Neumann himself regarded factors of type III somehow pathological.

Factors of type II contains as sub-class hyper-finite factors of type II_1 (HFFs). The naïve definition of trace of unit matrix as infinite dimension of the Hilbert space involved is replaced with a definition in which unit matrix has finite trace equal to 1 in suitable normalization. One cannot anymore select single ray of Hilbert space but one must always consider infinite-dimensional sub-space. The interpretation is in terms of finite measurement resolution: the sub-Hilbert space representing non-detectable degrees of freedom is always infinite-dimensional and the inclusion to larger Hilbert space is accompanied by inclusion of corresponding von Neumann algebras.

HFFs are between factors of type I and III in the sense that approximation of the system as a finite-dimensional system can be made arbitrary good: this motivates the term hyper-finite.

The realization that HFFs [K120] are tailor made for quantum TGD has led to a considerable progress in the understanding of the mathematical structure of the theory and these algebras provide a justification for several ideas introduced earlier on basis of physical intuition.

HFF has a canonical realization as an infinite-dimensional Clifford algebra and the obvious guess is that it corresponds to the algebra spanned by the gamma matrices of WCW. Also the local Clifford algebra of the embedding space $H = M^4 \times CP_2$ in octonionic representation of gamma matrices of H is important and the entire quantum TGD emerges from the associativity or co-associativity conditions for the sub-algebras of this algebra which are local algebras localized to maximal associative or co-associate sub-manifolds of the embedding space identifiable as space-time surfaces.

The notion of inclusion for hyper-finite factors provides an elegant description for the notion of measurement resolution absent from the standard quantum measurement theory.

1. The included sub-factor creates in ZEO states not distinguishable from the original one and the formally the coset space of factors defining quantum spinor space defines the space of physical states modulo finite measurement resolution.
2. The quantum measurement theory for hyperfinite factors differs from that for factors of type I since it is not possible to localize the state into single ray of state space. Rather, the ray is replaced with the sub-space obtained by the action of the included algebra defining the measurement resolution. The role of complex numbers in standard quantum measurement theory is taken by the non-commutative included algebra so that a non-commutative quantum theory is the outcome.
3. The inclusions of HFFs are closely related to quantum groups studied in recent modern physics but interpreted in terms of Planck length scale exotics formulated in terms of non-commutative space-time. The formulation in terms of finite measurement resolution brings this mathematics to physics in all scales.

   For instance, the finite measurement resolution means that the components of spinor do not commute anymore and it is not possible to reduce the state to a precise eigenstate of spin. It is however perform a reduction to an eigenstate of an observable which corresponds to the probability for either spin state.
4. The realization for quantum measurement theory modulo finite measurement resolution is in terms of M-matrices defined in terms of Connes tensor product which essentially means that the included hyper-finite factor N takes the role of complex numbers.

Discretization at the level of partonic 2-surfaces defines the lowest level correlate for the finite measurement resolution.

1. The dynamics of TGD itself might realize finite measurement resolution automatically in the sense that the quantum states at partonic 2-surfaces are always defined in terms of fermions localized at discrete points defined the ends of braids defined as the ends of string world sheets.
2. The condition that these selected points are common to reals and some algebraic extension of p-adic numbers for some p allows only algebraic points. GCI requires the special coordinates and natural coordinate systems are possible thanks to the symmetries of WCW. A restriction of GCI to discrete subgroup might well occur and have interpretation in terms of the constraints from the presence of cognition. One might say that the world in which mathematician uses Cartesian coordinates is different from the world in mathematician uses spherical coordinates.
3. The realization at the level of WCW would be number theoretical. In given resolution all parameters characterizing the mathematical representation of partonic 2-surfaces would belong to some algebraic extension of rational numbers. Same would hold for their 4-D tangent space data. This would imply that WCW would be effectively discrete space so that finite measurement resolution would be realized.

The recent view about the realization of finite measurement resolution is surprisingly concrete.

1. Also the hierarchy of Planck constants giving rise to a hierarchy of criticalities defines a hierarchy of measurement resolutions since each breaking of conformal symmetries transforms some gauge degrees of freedom to physical ones.

2. The numbers of partonic 2-surfaces and string world sheets connecting them, would give rise to a physical realization of the finite measurement resolution since fermions at string world sheets represent the space-time geometry physically in finite measurement resolution realized also as a hierarchy of geometries for WCW (via the representation of WCW Kähler metric in terms of anti-commutators of super charges). Finite measurement resolution is a property of physical system formed by the observer and system studied: the system studied changes when the resolution changes.
3. This representation is automatically discrete the level of partonic 2-surfaces, 1-D at their light-like orbits and 4-D at space-time interior. For $D > 0$ the discretization would take place for the parameters characterizing the functions (say coefficients of polynomials) characterizing string boundaries, string world sheets and partonic 2-surfaces, 3-surfaces and space-time surfaces. Clearly, an abstraction hierarchy is involved. p-Adicization suggests that rational numbers and their algebraic extensions are naturally involved.

2.5.7 Weak Form Of Electric Magnetic Duality

The notion of electric-magnetic duality [B3] was proposed first by Olive and Montonen and is central in $\mathcal{N} = 4$ supersymmetric gauge theories. It states that magnetic monopoles and ordinary particles are two different phases of theory and that the description in terms of monopoles can be applied at the limit when the running gauge coupling constant becomes very large and perturbation theory fails to converge.

The notion of electric-magnetic self-duality is more natural in TGD since for CP_2 geometry Kähler form is self-dual and Kähler magnetic monopoles are also Kähler electric monopoles and Kähler coupling strength is by quantum criticality renormalization group invariant rather than running coupling constant.

In TGD framework one must adopt a weaker form of the self-duality applying at partonic 2-surfaces [K121]. The principle is statement about boundary values of the induced Kähler form analogous to Maxwell field at the light-like 3-surfaces, at which the situation is singular since the induced metric for four-surface has a vanishing determinant because the signature of the induced metric changes from Minkowskian to Euclidian. What the principle says is that Kähler electric field in the normal space is the dual of Kähler magnetic field in the 4-D tangent space of the light-like 3-surface. One can consider even weaker formulation assuming this only at partonic 2-surfaces at the intersection of light-like 3-surfaces and space-like 3-surfaces at the boundaries of CD.

Every new idea must be taken with a grain of salt but the good sign is that this concept leads to precise predictions.

1. Elementary particles do not generate monopole fields in macroscopic length scales: at least when one considers visible matter. The first question is whether elementary particles could have vanishing magnetic charges: this turns out to be impossible. The next question is how the screening of the magnetic charges could take place and leads to an identification of the physical particles as string like objects identified as pairs magnetic charged wormhole throats connected by magnetic flux tubes. The string picture was later found to emerge naturally from Kähler Dirac action.
2. Second implication is a new view about electro-weak massivation reducing it to weak confinement in TGD framework. The second end of the string contains particle having electroweak isospin neutralizing that of elementary fermion and the size scale of the string is electro-weak scale would be in question. Hence the screening of electro-weak force takes place via weak confinement realized in terms of magnetic confinement.
3. This picture generalizes to the case of color confinement. Also quarks correspond to pairs of magnetic monopoles but the charges need not vanish now. Rather, valence quarks would be connected by flux tubes of length of order hadron size such that magnetic charges sum up to zero. For instance, for baryonic valence quarks these charges could be $(2, -1, -1)$ and could be proportional to color hyper charge.
4. The highly non-trivial prediction making more precise the earlier stringy vision is that elementary particles are string like objects in electro-weak scale: this should become manifest at LHC energies. Stringy character is manifested in two ways: as string like objects defined by Kähler magnetic flux tubes and 2-D string world sheets.

5. The weak form electric-magnetic duality together with Beltrami flow property of Kähler leads to the reduction of Kähler action to Chern-Simons action so that TGD reduces to almost topological QFT and that Kähler function is explicitly calculable. This has enormous impact concerning practical calculability of the theory.

6. One ends up also to a general solution ansatz for field equations from the condition that the theory reduces to almost topological QFT. The solution ansatz is inspired by the idea that all isometry currents are proportional to Kähler current which is integrable in the sense that the flow parameter associated with its flow lines defines a global coordinate. The proposed solution ansatz would describe a hydrodynamical flow with the property that isometry charges are conserved along the flow lines (Beltrami flow). A general ansatz satisfying the integrability conditions is found.

   The solution ansatz applies also to the extremals of Chern-Simons actionandto the conserved currents associated with the Kähler-Dirac equation defined as contractions of the Kähler-Dirac gamma matrices between the solutions of the Kähler-Dirac equation. The strongest form of the solution ansatz states that various classical and quantum currents flow along flow lines of the Beltrami flow defined by Kähler current (Kähler magnetic field associated with Chern-Simons action). Intuitively this picture is attractive. A more general ansatz would allow several Beltrami flows meaning multi-hydrodynamics. The integrability conditions boil down to two scalar functions: the first one satisfies massless d'Alembert equation in the induced metric and the the gradients of the scalar functions are orthogonal. The interpretation in terms of momentum and polarization directions is natural.

7. In order to obtain non-trivial fermion propagator one must add to Dirac action 1-D Dirac action in induced metric with the boundaries of string world sheets at the light-like parton orbits. Its bosonic counterpart is line-length in induced metric. Field equations imply that the boundaries are light-like geodesics and fermion has light-like 8-momentum. This suggests strongly a connection with quantum field theory and an 8-D generalization of twistor Grassmannian approach. By field equations the bosonic part of this action does not contribute to the Kähler action. Chern-Simons Dirac terms to which Kähler action reduces could be responsible for the breaking of CP and T symmetries as they appear in CKM matrix.

2.5.8 TGD As Almost Topological QFT

Topological QFTs (TQFTs) represent examples of the very few quantum field theories which exist in mathematically rigorous manner. TQFTs are of course physically non-realistic since the notion of distance is lacking and one cannot assign to the particles observables like mass. This raises the hope that TGD could be as near as possible to TQFT.

The vision about TGD as almost topological QFT is very attractive. Almost topological QFT property would naturally correspond to the reduction of Kähler action for preferred extremals to Chern-Simons form integrated over boundary of space-time and over the light-like 3-surfaces means. This is achieved if weak form of em duality vanishes and $j \cdot A$ term in the decomposition of Kähler action to 4-D integral and 3-D boundary term vanishes. Almost topological QFT would suggests conformal field theory at partonic 2-surface or at their light-like orbits. Strong form of holography states that also conformal field theory associated with space-like 3-surfaces at the ends of CDs describes the physics. These facts suggest that almost 2-dimensional QFT coded by data given at partonic 2-surfaces and their 4-D tangent space is enough to code for physics.

Topological QFT property would mean description in terms of braids. Braids would correspond to the orbits of fermions at partonic 2-surfaces identifiable as ends of string world sheets at which the modes of induced spinor field are localized with one exception: right-handed neutrino. This follows from well-definedness of electromagnetic charge in presence of induce W boson fields. The first guess is that induced W boson field must vanish at string world sheet. "Almost" could mean the replacement of the ends of strings defining braids with strings and duality for the descriptions based on string world sheets resp. partonic 2-surfaces analogous to AdS/CFT duality.

2.5.9 Three good reasons for the localization of spinor modes at string world sheets

There are three good reasons for the modes of the induced spinor fields to be localized to 2-D string world sheets and partonic 2-surfaces - in fact, to the boundaries of string world sheets at them defining fermionic world lines. I list these three good reasons in the same order as I became aware of them.

1. The first good reason is that this condition allows spinor modes to have well-defined electromagnetic charges - the induced classical W boson fields and perhaps also Z field vanish at string world sheets so that only em field and possibly Z field remain and one can have eigenstates of em charge.
2. Second good reason actually a set of closely related good reasons. First, strong form of holography implied by the strong form of general coordinate invariance demands the ocalization: string world sheets and partonic 2-surfaces are "space-time genes". Also twistorial picture follows naturally if the locus for the restriction of spinor modes at the light-like orbits of partonic 2-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian is 1-D fermion world line. Thanks to holography fermions behave like point like particles, which are massless in 8-D sense. Thirdly, conformal invariance in the fermionic sector demands the localization.
3. The third good reason emerges from the mathematical problem of field theories involving fermions: also in the models of condensed matter systems this problem is also encountered - in particular, in the models of high T_c superconductivity. For instance, AdS/CFT correspondence involving 10-D blackholes has been proposed as a solution - the reader can decide whether to take this seriously.

   Fermionic path integral is the source of problems. It can be formally reduced to the analog of partition function but the Boltzman weights (analogous to probabilities) are not necessary positive in the general case and this spoils the stability of the numerical computation. One gets rid of the sign problem if one can diagonalize the Hamiltonian, but this problem is believed to be NP-hard in the generic case. A further reason to worry in QFT context is that one must perform Wick rotation to transform action to Hamiltonian and this is a trick. It seems that the problem is much more than a numerical problem: QFT approach is somehow sick.

   The crucial observation giving the third good reason is that this problem is encountered *only in dimensions* $D \geq 3$ - not in dimensions $D = 1, 2$! No sign problem in TGD where second quantized fundamental fermions are at string world sheets!

A couple of comments are in order.

1. Although the assumption about localization 2-D surfaces might have looked first a desperate attempt to save em charge, it now seems that it is something very profound. In TGD approach standard model and GRT emerge as an approximate description obtained by lumping the sheets of the many-sheeted space-time together to form a slightly curved region of Minkowski space and by identifying gauge potentials and gravitational field identified as sums of those associated with the sheets lumped together. The more fundamental description would not be plagued by the mathematical problem of QFT approach .
2. Although fundamental fermions as second quantized induced spinor fields are 2-D character, it is the modes of the classical embedding space spinor fields - eigenstates of four-momentum and standard model quantum numbers - that define the ground states of the super-conformal representations. It is these modes that correspond to the 4-D spinor modes of QFT limit. What goes wrong in QFT is that one assigns fermionic oscillator operators to these modes although second quantization should be carried out at deeper level and for the 2-D modes of the induced spinor fields: 2-D conformal symmetry actually makes the construction of these modes trivial.

To conclude, the condition that the theory is computable would pose a powerful condition on the theory. As a matter fact, this is not a new finding. The mathematical existence of Kähler geometry of WCW fixes its geometry more or less uniquely and therefore also the physics: one obtains a union of symmetric spaces labelled by zero modes of the metric and for symmetric space all points (now 3-surfaces) are geometrically equivalent meaning a gigantic simplification allowing to handle the infinite-dimensional case. Even for loop spaces the Kähler geometry is unique and has infinite-dimensional isometry group (Kac-Moody symmetries).

Chapter 3

Topological Geometrodynamics: Three Visions

3.1 Introduction

Originally Topological Geometrodynamics (TGD) was proposed as a solution of the problems related to the definition of conserved four-momentum in General Relativity. It was assumed that physical space-times are representable as 4-D surfaces in certain higher-dimensional space-time having symmetries of the empty Minkowski space of Special Relativity. This is guaranteed by the decomposition $H = M^4 \times S$, where S is some compact internal space. It turned out that the choice $S = CP_2$ is unique in the sense that it predicts the symmetries of the standard model and provides a realization for Einstein's dream of geometrizing of fundamental interactions at classical level. TGD can be also regarded as a generalization of super string models obtained by replacing strings with light-like 3-surfaces or equivalently with space-like 3-surfaces: the equivalence of these identification implies quantum holography.

The construction of quantum TGD turned out to be much more than mere technical problem of deriving S-matrix from path integral formalism. A new ontology of physics (many-sheeted space-time, zero energy ontology, generalization of the notion of number, and generalization of quantum theory based on spectrum of Planck constants giving hopes to understand what dark matter and dark energy are) and also a generalization of quantum measurement theory leading to a theory of consciousness and model for quantum biology providing new insights to the mysterious ability of living matter to circumvent the constraints posed by the second law of thermodynamics were needed. The construction of quantum TGD involves a handful of different approaches consistent with a similar overall view, and one can say that the construction of M-matrix, which generalizes the S-matrix of quantum field theories, is understood to a satisfactory degree although it is not possible to write even in principle explicit Feynman rules except at quantum field theory limit [**?**].

In this chapter I will discuss three basic visions about quantum Topological Geometrodynamics (TGD). It is somewhat matter of taste which idea one should call a vision and the selection of these three in a special role is what I feel natural just now.

1. The first vision is generalization of Einstein's geometrization program based on the idea that the Kähler geometry of the world of classical worlds (WCW) with physical states identified as classical spinor fields on this space would provide the ultimate formulation of physics [K112].
2. Second vision is number theoretical [K75] and involves three threads.
   (a) The first thread [K104] relies on the idea that it should be possible to fuse real number based physics and physics associated with various p-adic number fields to single coherent whole by a proper generalization of number concept.
   (b) Second thread [K105] is based on the hypothesis that classical number fields could allow to understand the fundamental symmetries of physics and and imply quantum TGD from purely number theoretical premises with associativity defining the fundamental dynamical principle both classically and quantum mechanically.

(c) The third thread [K103] relies on the notion of infinite primes whose construction has amazing structural similarities with second quantization of super-symmetric quantum field theories. In particular, the hierarchy of infinite primes and integers allows to generalize the notion of numbers so that given real number has infinitely rich number theoretic anatomy based on the existence of infinite number of real units. This implies number theoretical Brahman=Atman identity or number theoretical holography when one consider hyper-octonionic infinite primes.

(d) The third vision is based on TGD inspired theory of consciousness [K108]. which can be regarded as an extension of quantum measurement theory to a theory of consciousness raising observer from an outsider to a key actor of quantum physics. The basic notions at quantum jump identified as as a moment of consciousness and self. Negentropy Maximization Principle (NMP) defines the fundamental variational principle and reproduces standard quantum measurement theory and predicts second law but also some totally new physics in the intersection of real and p-adic worlds where it is possible to define a hierarchy of number theoretical variants of Shannon entropy which can be also negative. In this case NMP favors the generation of entanglement and state function reduction does not mean generation of randomness anymore. This vision has obvious almost applications to biological self-organization.

My aim is to provide a bird's eye of view and my hope is that reader would take the attitude that details which cannot be explained in this kind of representation are not essential for the purpose of getting a feeling about the great dream behind TGD.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L9].

3.2 Quantum Physics As Infinite-Dimensional Geometry

The first vision in its original form is a the generalization of Einstein's program for the geometrization of physics by replacing space-time with the WCW identified roughly as the space of 4-surfaces in $H = M^4 \times CP_2$. Later generalization due to replacement of H with book like structures from by real and p-adic variants of H emerged. A further book like structure of embedding space emerged via the introduction of the hierarchy of Planck constants. These generalizations do not however add anything new to the basic geometric vision.

3.2.1 Geometrization Of Fermionic Statistics In Terms Of WCW Spinor Structure

The great vision has been that the second quantization of the induced spinor fields can be understood geometrically in terms of the WCW spinor structure in the sense that the anti-commutation relations for WCW gamma matrices require anti-commutation relations for the oscillator operators for free second quantized induced spinor fields defined at space-time surface.

1. One must identify the counterparts of second quantized fermion fields as objects closely related to the configuration space spinor structure. Ramond model [B39] has as its basic field the anti-commuting field $\Gamma^k(x)$, whose Fourier components are analogous to the gamma matrices of the configuration space and which behaves like a spin 3/2 fermionic field rather than a vector field. This suggests that the are analogous to spin 3/2 fields and therefore expressible in terms of the fermionic oscillator operators so that their naturally derives from the anti-commutativity of the fermionic oscillator operators.

   WCW spinor fields can have arbitrary fermion number and there are good hopes of describing the whole physics in terms of WCW spinor field. Clearly, fermionic oscillator operators would act in degrees of freedom analogous to the spin degrees of freedom of the ordinary spinor and bosonic oscillator operators would act in degrees of freedom analogous to the "orbital" degrees of freedom of the ordinary spinor field. One non-trivial implication is bosonic emergence: elementary bosons correspond to fermion anti-fermion bound states associated with the wormhole contacts (pieces of CP_2 type vacuum extremals) with throats carrying fermion and anti-fermion numbers. Fermions correspond to single throats associated with topologically condensed CP_2 type vacuum extremals.

2. The classical theory for the bosonic fields is an essential part of WCW geometry. It would be very nice if the classical theory for the spinor fields would be contained in the definition of the WCW spinor structure somehow. The properties of the associated with the induced spinor structure are indeed very physical. The modified massless Dirac equation for the induced spinors predicts a separate conservation of baryon and lepton numbers. The differences between quarks and leptons result from the different couplings to the CP_2 Kähler potential. In fact, these properties are shared by the solutions of massless Dirac equation of the embedding space.

3. Since TGD should have a close relationship to the ordinary quantum field theories it would be highly desirable that the second quantized free induced spinor field would somehow appear in the definition of the WCW geometry. This is indeed true if the complexified WCW gamma matrices are linearly related to the oscillator operators associated with the second quantized induced spinor field on the space-time surface and its boundaries. There is actually no deep reason forbidding the gamma matrices of WCW to be spin half odd-integer objects whereas in the finite-dimensional case this is not possible in general. In fact, in the finite-dimensional case the equivalence of the spinorial and vectorial vielbeins forces the spinor and vector representations of the vielbein group $SO(D)$ to have same dimension and this is possible for $D = 8$-dimensional Euclidian space only. This coincidence might explain the success of 10-dimensional super string models for which the physical degrees of freedom effectively correspond to an 8-dimensional Euclidian space.

4. It took a long time to realize that the ordinary definition of the gamma matrix algebra in terms of the anti-commutators $\{\gamma_A, \gamma_B\} = 2g_{AB}$ must in TGD context be replaced with

$$\{\gamma_A^\dagger, \gamma_B\} = iJ_{AB} \ ,$$

where J_{AB} denotes the matrix elements of the Kähler form of WCW. The presence of the Hermitian conjugation is necessary because WCW gamma matrices carry fermion number. This definition is numerically equivalent with the standard one in the complex coordinates. The realization of this delicacy is necessary in order to understand how the square of the WCW Dirac operator comes out correctly.

3.2.2 Construction Of WCW Clifford Algebra In Terms Of Second Quantized Induced Spinor Fields

The construction of WCW spinor structure must have a direct relationship to quantum physics as it is usually understood. The second quantization of the space-time spinor fields is needed to define the anti-commutative gamma matrices of WCW: this means a geometrization of Fermi statistics [K121] in the sense that free fermionic quantum fields at space-time surface correspond to purely classical Clifford algebra of WCW. This is in accordance with the idea that physics at WCW level is purely classical apart from the notion of quantum jump.

The identification of the correct variational principle for the dynamics of space-time spinor fields identified as induced spinor fields has involved many trials and errors. Ironically, the final outcome was almost the most obvious guess: the so called Kähler-Dirac action. What was difficult to discover was that the well-definedness of em charge requires that the modes of K-D equation are localized at 2-D string world sheets. The same condition results also from the condition that octonionic and ordinary spinor structures are equivalent for the modes of the induced spinor field and also from the condition that quantum deformations of fermionic oscillator operator algebra requiring 2-dimensionality can be realized as realization of finite measurement resolution. Fermionic string model therefore emerges from TGD.

The notion of measurement resolution realized in terms of the inclusions of hyper-finite factors of type II_1 and having discretization using rationals or algebraic extensions of rationals have been one of the key challenges of quantum TGD. Quantum classical correspondence suggests with measurement interaction term defined as Lagrange multiplier terms stating that classical charges belonging to Cartan algebra are equal to their quantal counterparts after state function reduction for space-time surfaces appearing in quantum superposition [K121]. This makes sense if classical charges parametrize zero modes. State function reduction would mean state function collapse in zero modes.

Kähler function equals to the real part of Kähler action coming from Euclidian space-time regions for a preferred extremal whereas Minkowski regions give an exponent of phase factor responsible for quantum interferences effects. The conjecture is that preferred extremals by internal consistency conditions are critical in the sense that they allows infinite number of vanishing second variations having interpretation as conformal deformations respecting light-likeness of the partonic orbits. Criticality is realize classically as vanishing of the super-symplectic charges for sub-algebra of the entire super-symplectic algebra. This realizes the notion of quantum criticality-one of guiding principles of quantum TGD-at space-time level.

Recently this idea has become very concrete.

1. There is an infinite hierarchy of quantum criticalities identified as a hierarchy of breakings of conformal symmetry in the sense that the gauge symmetry for the super-symplectic algebra having natural conformal structure is broken to a dynamical symmetry: gauge degrees of freedom are transformed to physical ones.
2. The sub-algebras of the supersymplectic algebra isomorphic with the algebra itself are parametrized by integer n: the conformal weights for the sub-algebra are n-multiples for those of the entire algebra. This predicts an infinite number of infinite hierarchies characterized by sequences of integers $n_{i+1} = \prod_{k\leq i} m_k$. The integer n_i characterizes the effective value of Planck constant $h_{eff} = n_i$ for a given level of hierarchy and the interpretation is in terms of dark matter. The increase of n_i takes place spontaneously since it means reduction of criticality. Both the value of n_i and the numbers of string world sheets associated with 3-surfaces at the ends of CD and connecting partonic 2-surfaces characterize measurement resolution.
3. The symplectic hierarchies correspond to hierarchies of inclusions for HFFs [K120] and finite measurement resolution is a property of both zero energy state and space-time surface. The original idea about addition of measurement interaction terms to the Kähler action does not seem to be needed.

Number theoretical approach in turn leads to the conclusion that space-time surfaces are either associative or co-associative in the sense that the induced gamma matrices at each point of space-time surface in their octonionic representation define a quaternionic or co-quaternionic algebra and therefore have matrix representation. The conjecture is that these identifications of space-time dynamics are consistent or even equivalent. The string sheets at which spinor modes are localized can be regarded as commutative surfaces.

The recent understanding of the Kähler-Dirac action has emerged through a painful process and has strong physical implications.

1. Kähler-Dirac equation at string world sheets can be solved exactly just as in string models. At the light-like boundaries the limit of K-D equation holds true and gives rise to the analog of massless Dirac equation but for K-D gamma matrices. One could have a 1-D boundary term defined by the induced Dirac equation at the light-like boundaries of string world sheet. If it is there, the modes are solutions with light-like 8-momentum which has light-like projection to space-time surface. This would give rise to a fermionic propagator in the construction of scattering amplitudes mimicking Feynman diagrammatics: note that the M^4 projection of the momentum need not be light-like.
2. The space-time super-symmetry generalizes to what might be called $\mathcal{N} = \infty$ supersymmetry whose least broken sub-symmetry reduces to $\mathcal{N} = 2$ broken super-symmetry generated by right-handed neutrino and ant-ineutrino [?] . The generators of the super-symmetry correspond to the oscillator operators of the induced spinor field at space-time sheet and to the super-symplectic charges. Bosonic emergence means dramatic simplifications in the formulation of quantum TGD.
3. It is also possible to generalize the twistor program to TGD framework if one accepts the use of octonionic representation of the gamma matrices of embedding space and hyper-quaternionicity of space-time surfaces [L8]: what one obtains is 8-D generalization of the twistor Grassmann approach allowing non-light-like M^4 momenta. Essential condition is that octonionic and ordinary spinor structures are equivalent at string world sheets.

3.2.3 ZEO And WCW Geometry

In the ZEO quantum states have vanishing net values of conserved quantum numbers and decompose to superposition of pairs of positive and negative energy states defining counterparts of initial and final states of a physical event in standard ontology.

ZEO

ZEO was forced by the interpretational problems created by the vacuum extremal property of Robertson-Walker cosmologies imbedded as 4-surfaces in $M^4 \times CP_2$ meaning that the density of inertial mass (but not gravitational mass) for these cosmologies was vanishing meaning a conflict with Equivalence Principle. The most feasible resolution of the conflict comes from the realization that GRT space-time is obtained by lumping the sheets of many-sheeted space-time to M^4 endowed with effective metric. Vacuum extremals could however serve as models for GRT space-times such that the effective metric is identified with the induced metric [K114]. This is true if space-time is genuinely single-sheeted. In the models of astrophysical objects and cosmology vacuum extremals have been used [K98].

In zero energy ontology physical states are replaced by pairs of positive and negative energy states assigned to the past *resp.* future boundaries of causal diamonds defined as pairs of future and past directed light-cones ($\delta M^4_\pm \times CP_2$). The net values of all conserved quantum numbers of zero energy states vanish. Zero energy states are interpreted as pairs of initial and final states of a physical event such as particle scattering so that only events appear in the new ontology. It is possible to speak about the energy of the system if one identifies it as the average positive energy for the positive energy part of the system. Same applies to other quantum numbers.

The matrix ("M-matrix") representing time-like entanglement coefficients between positive and negative energy states unifies the notions of S-matrix and density matrix since it can be regarded as a complex square root of density matrix expressible as a product of real squared of density matrix and unitary S-matrix. The system can be also in thermal equilibrium so that thermodynamics becomes a genuine part of quantum theory and thermodynamical ensembles cease to be practical fictions of the theorist. In this case M-matrix represents a superposition of zero energy states for which positive energy state has thermal density matrix.

ZEO combined with the notion of quantum jump resolves several problems. For instance, the troublesome questions about the initial state of universe and about the values of conserved quantum numbers of the Universe can be avoided since everything is in principle creatable from vacuum. Communication with the geometric past using negative energy signals and time-like entanglement are crucial for the TGD inspired quantum model of memory and both make sense in zero energy ontology. ZEO leads to a precise mathematical characterization of the finite resolution of both quantum measurement and sensory and cognitive representations in terms of inclusions of von Neumann algebras known as hyperfinite factors of type II_1. The space-time correlate for the finite resolution is discretization which appears also in the formulation of quantum TGD.

Causal diamonds

The embedding space correlates for ZEO are causal diamonds (CDs) CD serves as the correlate zero energy state at embedding space-level whereas space-time sheets having their ends at the light-like boundaries of CD are the correlates of the system at the level of 4-D space-time. Zero energy state can be regarded as a quantum superposition of space-time sheets with fermionic and other quantum numbers assignable to the partonic 2-surfaces at the ends of the space-time sheets.

1. The basic construct in the ZEO is the space $CD \times CP_2$, where the causal diamond CD is defined as an intersection of future and past directed light-cones with time-like separation between their tips regarded as points of the underlying universal Minkowski space M^4. In ZEO physical states correspond to pairs of positive and negative energy states located at the boundaries of the future and past directed light-cones of a particular CD.

2. CDs form a fractal hierarchy and one can glue smaller CDs within larger CDs. Also unions of CDs are possible.

3. Without any restrictions CDs would be parametrized by the position of say lower tip of CD and by the relative M^4 coordinates of the upper tip with respect to the lower one so that

the moduli space would be $M^4 \times M^4_+$. p-Adic length scale hypothesis follows if the values of temporal distance T between tips of CD come in powers of 2^n: $T = 2^n T_0$. This would reduce the future light-cone M^4_+ reduces to a union of hyperboloids with quantized value of light-cone proper time. A possible interpretation of this distance is as a quantized cosmic time. Also the quantization of the hyperboloids to a lattices of discrete points classified by discrete sub-groups of Lorentz group is an attractive proposal and the quantization of cosmic redshifts provides some support for it.

ZEO forces to replaced the original WCW by a union of WCWs associated with CDs and their unions. This does not however mean any problems of principle since Clifford algebras are simply tensor products of the Clifford algebras of CDs for the unions of CDs.

Generalization of S-matrix in ZEO

ZEO forces the generalization of S-matrix with a triplet formed by U-matrix, M-matrix, and S-matrix. The basic vision is that quantum theory is at mathematical level a complex square root of thermodynamics. What happens in quantum jump was already discussed.

1. M-matrices are matrices between positive and negative energy parts of the zero energy state and correspond to the ordinary S-matrix. M-matrix is a product of a hermitian square root - call it H - of density matrix ρ and universal S-matrix S. There is infinite number of different Hermitian square roots H_i of density matrices assumed to define orthogonal matrices with respect to the inner product defined by the trace: $Tr(H_i H_j) = 0$. One can interpret square roots of the density matrices as a Lie algebra acting as symmetries of the S-matrix. The most natural identification is in terms of super-symplectic algebra or as its sub-algebra. Since these operators should not change the vanishing quantum number of zero energy states, a natural identification would be as bilinears of the generators of super-symplectic generators associated with the opposite boundaries of CD and having vanishing net quantum numbers.
2. One can consider a generalization of M-matrices so that they would be analogous to the elements of Kac-Moody algebra. These M-matrices would involve all powers of S.
   (a) The orthogonality with respect to the inner product defined by $\langle A|B\rangle = Tr(AB)$ requires the conditions $Tr(H_1 H_2 S^n) = 0$ for $n \neq 0$ and H_i are Hermitian matrices appearing as square root of density matrix. $H_1 H_2$ is hermitian if the commutator $[H_1, H_2]$ vanishes. It would be natural to assign n:th power of S to the CD for which the scale is n times the CP_2 scale.
   (b) Trace - possibly quantum trace for hyper-finite factors of type II_1) is the analog of integration and the formula would be a non-commutative analog of the identity $\int_{S^1} exp(in\phi) d\phi = 0$ and pose an additional condition to the algebra of M-matrices.
   (c) It might be that one must restrict M matrices to a Cartan algebra and also this choice would be a process analogous to state function reduction. Since density matrix becomes an observable in TGD Universe, this choice could be seen as a direct counterpart for the choice of a maximal number of commuting observables which would be now hermitian square roots of density matrices. Therefore ZEO gives good hopes of reducing basic quantum measurement theory to infinite-dimensional Lie-algebra.

The collections of M-matrices defined as time reversals of each other define the sought for two natural state basis.

1. As for ordinary S-matrix, one can construct the states in such a way that either positive or negative energy part of the state has well defined particle numbers, spin, etc... resulting in state function preparation. Therefore one has two kinds of M-matrices: $M_K^{\pm}$ and for both of these the above orthogonality relations hold true. This implies also two kinds of U-matrices call them $U^{\pm}$. The natural assumption is that the two M-matrices differ only by Hermitian conjugation so that one would have $M_K^- = (M_K^+)^\dagger$.

   One can assign opposite arrows of geometric time to these states and the proposal is that the arrow of time is a result of a process analogous to spontaneous magnetization. The possibility that the arrow of geometric time could change in quantum jump has been already discussed.

2. Unitary U-matrix $U^{\pm}$ is induced from a projector to the zero energy state basis $|K^{\pm}\rangle$ acting on the state basis $|K^{\mp}\rangle$ and the matrix elements of U-matrix are obtained by acting with the representation of identity matrix in the space of zero energy states as $I = \sum_K |K^+\rangle\langle K^+|$ on the zero energy state $|K^-\rangle$ (the action on K^+ is trivial!) and gives

$$U^+_{KL} = Tr(M^+_K M^+_L) \ .$$

Note that finite measurement resolution requires that the trace operation is q-trace rather than ordinary trace.

3. As the detailed discussion of the anatomy of quantum jump demonstrated, the first step in state function reduction is the choice of $M^{\pm}_K$ meaning the choice of the hermitian square root of a density matrix. A quantal selection of the measured observable takes place. This step is followed by a choice of "initial" state analogous to state function preparation and a choice of the "final state" analogous to state function reduction. The net outcome is the transition $|K^{\pm}\rangle \rightarrow |L^{\pm}\rangle$. It could also happen that instead of state function reduction as third step unitary process $U^{\mp}$ (note the change of the sign factor!) takes place and induces the change of the arrow of geometric time.

4. As noticed, one can imagine even higher level choices and this would correspond to the choice of the commuting set of hermitian matrices H defining the allowed square roots of density matrices as a set of mutually commuting observables.

5. The original naïve belief that the unitary U-matrix has as its rows orthonormal M-matrices turned out to be wrong. One can deduce the general structure of U-matrix from first principles by identifying it as a time evolution operator in the space of moduli of causal diamonds relating to each other M-matrices. Inner product for M-matrices gives the matrix elements of U-matrix. S-matrix can be identified as a representation for the exponential of the Virasoro generator L_{-1} for the super-symplectic algebra. The detailed construction of U-matrix in terms of M-matrices and S-matrices depending on CD moduli is discussed in [K69].

3.2.4 Quantum Criticality, Strong Form Form of Holography, and WCW Geometry

Quantum TGD and WCW geometry in particular can be understood in terms of two principles: Quantum Criticality (QC) and Strong form of Holography (SH).

Quantum Criticality

In its original form QC stated that the Kähler couplings strength appearing in the exponent of vacuum functional identifiable uniquely as the exponent of Kähler function defining the Kähler metric of WCW defines the analog of partition function of a thermodynamical system. Later it became clear that Kähler action in Minkowskian space-time regions is imaginary (by $\sqrt{g}$ factor) so that the exponent become that of complex number. The interpretation in ZEO is in terms of quantum TGD as "square root of thermodynamics" vision. Minkowskian Kähler action is the analog of action of quantum field theories.

TGD should be unique. The analogy with thermodynamics implies that Kähler coupling strength α_K is analogous to temperature. The natural guess is that it corresponds to a critical temperature at which a phase transition between two phases occurs. It is of course possible that there are several critical values of α_K.

QC is physically very attractive since it would give maximally complex Universe. At quantum criticality long range fluctuations would be present and make possible macroscopic quantum coherence especially relevant for life.

In 2-D critical systems conformal symmetry provides the mathematical description of criticality and in TGD something similar but based on a huge generalization of the conformal symmetries is expected. Ordinary conformal symmetries are indeed replaced by super-symplectic isometries, by the generalized conformal symmetries acting on light-cone boundary and on light-like orbits of partonic 2-surfaces, and by the ordinary conformal symmetries at partonic 2-surfaces and string world sheets carrying spinors. Even a quaternionic generalization of conformal symmetries must be considered.

Strong Form of Holography

Strong form of holography (SH) is the second big principle. It is strongly suggested by the strong form of general coordinate invariance (SGCI) stating that the fundamental objects can be taken to be either the light-like orbits of partonic 2-surfaces or space-like 3-surfaces at the ends of causal diamonds (CDs). This would imply that partonic 2-surfaces at their intersection at the boundaries of CDs carry the data about quantum states.

As a matter fact, one must include also string world sheets at which fermions are localized - this for instance by the condition that em charge is well-defined. String world sheets carry vanishing induced W boson fields (they would mix different charge states) and the Kähler-Dirac gamma matrices are parallel to them. These conditions give powerful integrability conditions and it remains to be seen whether solutions to them indeed exist.

The best manner to proceed is to construct preferred extremals using SH - that is by assuming just string world sheets and partonic 2-surfaces intersecting by discrete point set as given, and finding the preferred extremals of Kähler action containing them and satisfying the boundary conditions at string world sheets and partonic 2-surfaces.

If this construction works, it must involve boundary conditions fixing the space-time surfaces to very high degree. Due to the non-determinism of Kähler action implied by its huge vacuum degeneracies, one however expects a gauge degeneracy. QC indeed suggests non-determinism. By 2-D analogy one expects the analogs of conformal symmetries acting as gauge symmetries. The proposal is that the fractal hierarchy of mutually isomorphic sub-algebras of super-symplectic algebra (and possibly of all conformal algebras involved) having conformal weights, which are n-ples of those for the entire algebra act as gauge symmetries so that the Noether charges for this sub-algebra would vanish. This would be the case at the ends of preferred extremals at both boundaries of CDs. This almost eliminates the classical degrees of freedom outside string world sheets and partonic 2-surfaces, and thus realizes the strong form of holography. In the fermionic sector the fermionic super-symplectic charges in the sub-algebra annihilate the physical states: this is a generalization of Super-Virasoro and Super Kac-Moody conditions.

In the phase transitions increasing the value of n the sub-algebra of gauge symmetries is reduced and gauge degrees of freedom become physical ones. By QC this transition occurs spontaneously. TGD Universe is like ball at the top of hill at the top of ....: ad infinitum and its evolution is endless dropping down. In TGD inspired theory of consciousness, one can understand living systems as systems fighting to stay at given level of criticality.

One could say that the conformal subalgebra is analogous to that defined by functions of $w = z^n$ act as conformal symmetries. One can also see the space-time surfaces at the level n as analogous to Riemann surface for function $f(z) = z^{1/n}$ conformal gauge symmetries as those defined by functions of z. This brings in n sheets not connected by conformal gauge symmetries. Hence the conformal equivalence classes of sheets give rise n-fold physical degeneracy. An effective description for this would be in terms of n-fold singular covering of the embedding space introduced originally but this is only an auxiliary concept.

A natural interpretation of the hierarchy of conformal criticalities is as a hierarchy of Planck constants $h_{eff} = n \times h$. The identification is suggested by the interpretation of n as the number of sheets in the singular covering of the space-time surface for which the sheets at the ends of space-time surface (the 3-surfaces at boundaries of CD) co-incide. The n sheets increase the action by a factor n and this is equivalent with the replacement $h \rightarrow h_{eff} = n \times h$.

The hierarchy of Planck constants allows to consider several interpretations.

1. If one regards the sheets of the covering as distinct, one has single critical value of g_K^2 and of h. This is the fundamental interpretation and justifies the subscript "$_{eff}$" in $h_{eff} = n \times h$.

2. If the sheets of the covering are are lumped to a single sheet (this is done for all sheets of the many-sheeted space-time in General Relativity approximation), there are two possible interpretations. There is single critical value of g_K^2 and a hierarchy of Planck constants $h_{eff} = n \times h$ giving rise to $\alpha_K(n) = g_K^2/2h_{eff}$. Alternatively, there is single value of Planck constant and a hierarchy of critical values $\alpha_K(n) = (g_K^2/2h)/n$ having an accumulation point at origin (zero temperature).

Non-commutative embedding space and strong form of holography

The precise formulation of strong form of holography (SH) is one of the technical problems in TGD. A comment in FB page of Gareth Lee Meredith led to the observation that besides the purely number theoretical formulation based on commutativity also a symplectic formulation in the spirit of non-commutativity of embedding space coordinates can be considered. One can however use only the notion of Lagrangian manifold and avoids making coordinates operators leading to a loss of General Coordinate Invariance (GCI).

Quantum group theorists have studied the idea that space-time coordinates are non-commutative and tried to construct quantum field theories with non-commutative space-time coordinates (see `http://tinyurl.com/z3m8sny`). My impression is that this approach has not been very successful. In Minkowski space one introduces antisymmetry tensor J_{kl} and uncertainty relation in linear M^4 coordinates m^k would look something like $[m^k, m^l] = l_P^2 J^{kl}$, where l_P is Planck length. This would be a direct generalization of non-commutativity for momenta and coordinates expressed in terms of symplectic form J^{kl}.

1+1-D case serves as a simple example. The non-commutativity of p and q forces to use either p or q. Non-commutativity condition reads as $[p, q] = \hbar J^{pq}$ and is quantum counterpart for classical Poisson bracket. Non-commutativity forces the restriction of the wave function to be a function of p or of q but not both. More geometrically: one selects Lagrangian sub-manifold to which the projection of J_{pq} vanishes: coordinates become commutative in this sub-manifold. This condition can be formulated purely classically: wave function is defined in Lagrangian sub-manifolds to which the projection of J vanishes. Lagrangian manifolds are however not unique and this leads to problems in this kind of quantization. In TGD framework the notion of "World of Classical Worlds" (WCW) allows to circumvent this kind of problems and one can say that quantum theory is purely classical field theory for WCW spinor fields. "Quantization without quantization" would have Wheeler stated it.

GCI poses however a problem if one wants to generalize quantum group approach from M^4 to general space-time: linear M^4 coordinates assignable to Lie-algebra of translations as isometries do not generalize. In TGD space-time is surface in embedding space $H = M^4 \times CP_2$: this changes the situation since one can use 4 embedding space coordinates (preferred by isometries of H) also as space-time coordinates. The analog of symplectic structure J for M^4 makes sense and number theoretic vision involving octonions and quaternions leads to its introduction. Note that CP_2 has naturally symplectic form.

Could it be that the coordinates for space-time surface are in some sense analogous to symplectic coordinates (p_1, p_2, q_1, q_2) so that one must use either (p_1, p_2) or (q_1, q_2) providing coordinates for a Lagrangian sub-manifold. This would mean selecting a Lagrangian sub-manifold of space-time surface? Could one require that the sum $J_{\mu\nu}(M^4) + J_{\mu\nu}(CP_2)$ for the projections of symplectic forms vanishes and forces in the generic case localization to string world sheets and partonic 2-surfaces. In special case also higher-D surfaces - even 4-D surfaces as products of Lagrangian 2-manifolds for M^4 and CP_2 are possible: they would correspond to homologically trivial cosmic strings $X^2 \times Y^2 \subset M^4 \times CP_2$, which are not anymore vacuum extremals but minimal surfaces if the action contains besides Käction also volume term.

But why this kind of restriction? In TGD one has strong form of holography (SH): 2-D string world sheets and partonic 2-surfaces code for data determining classical and quantum evolution. Could this projection of $M^4 \times CP_2$ symplectic structure to space-time surface allow an elegant mathematical realization of SH and bring in the Planck length l_P defining the radius of twistor sphere associated with the twistor space of M^4 in twistor lift of TGD? Note that this can be done without introducing embedding space coordinates as operators so that one avoids the problems with general coordinate invariance. Note also that the non-uniqueness would not be a problem as in quantization since it would correspond to the dynamics of 2-D surfaces.

The analog of brane hierarchy for the localization of spinors - space-time surfaces; string world sheets and partonic 2-surfaces; boundaries of string world sheets - is suggestive. Could this hierarchy correspond to a hierarchy of Lagrangian sub-manifolds of space-time in the sense that $J(M^4) + J(CP_2) = 0$ is true at them? Boundaries of string world sheets would be trivially Lagrangian manifolds. String world sheets allowing spinor modes should have $J(M^4)+J(CP_2) = 0$ at them. The vanishing of induced W boson fields is needed to guarantee well-defined em charge at string world sheets and that also this condition allow also 4-D solutions besides 2-D generic solutions.

This condition is physically obvious but mathematically not well-understood: could the condition $J(M^4) + J(CP_2) = 0$ force the vanishing of induced W boson fields? Lagrangian cosmic string type minimal surfaces $X^2 \times Y^2$ would allow 4-D spinor modes. If the light-like 3-surface defining boundary between Minkowskian and Euclidian space-time regions is Lagrangian surface, the total induced Kähler form Chern-Simons term would vanish. The 4-D canonical momentum currents would however have non-vanishing normal component at these surfaces. I have considered the possibility that TGD counterparts of space-time super-symmetries could be interpreted as addition of higher-D right-handed neutrino modes to the 1-fermion states assigned with the boundaries of string world sheets [K96].

Induced spinor fields at string world sheets could obey the "dynamics of avoidance" in the sense that *both* the induced weak gauge fields W, Z^0 and induced Kähler form (to achieve this U(1) gauge potential must be sum of M^4 and CP_2 parts) would vanish for the regions carrying induced spinor fields. They would couple only to the *induced em field (!)* given by the R_{12} part of CP_2 spinor curvature [K15] for $D = 2, 4$. For $D = 1$ at boundaries of string world sheets the coupling to gauge potentials would be non-trivial since gauge potentials need *not* vanish there. Spinorial dynamics would be extremely simple and would conform with the vision about symmetry breaking of weak group to electromagnetic gauge group.

An alternative - but of course not necessarily equivalent - attempt to formulate SH would be in terms of number theoretic vision. Space-time surfaces would be associative or co-associative depending on whether tangent space or normal space in embedding space is associative - that is quaternionic. These two conditions would reduce space-time dynamics to associativity and commutativity conditions. String world sheets and partonic 2-surfaces would correspond to maximal commutative or co-commutative sub-manifolds of embedding space. Commutativity (co-commutativity) would mean that tangent space (normal space as a sub-manifold of space-time surface) has complex tangent space at each point and that these tangent spaces integrate to 2-surface. SH would mean that data at these 2-surfaces would be enough to construct quantum states. String world sheet boundaries would in turn correspond to real curves of the complex 2-surfaces intersecting partonic 2-surfaces at points so that the hierarchy of classical number fields would have nice realization at the level of the classical dynamics of quantum TGD. The analogy with branes and super-symmetry force to consider two options.

Two options for fundamental variational principle

One ends up to two options for the fundamental variational principle.

Option I: The *fundamental* action principle for space-time surfaces contains besides 4-D action also 2-D action assignable to string world sheets, whose topological part (magnetic flux) gives rise to a coupling term to Kähler gauge potentials assignable to the 1-D boundaries of string world sheets containing also geodesic length part. Super-symplectic symmetry demands that modified Dirac action has 1-, 2-, and 4-D parts: spinor modes would exist at both string boundaries, string world sheets, and space-time interior. A possible interpretation for the interior modes would be as generators of space-time super-symmetries [K96].

This option is not quite in the spirit of SH and string tension appears as an additional parameter. Also the conservation of em charge forces 2-D string world sheets carrying vanishing induced W fields and this is in conflict with the existence of 4-D spinor modes unless they satisfy the same condition. This looks strange.

Option II: Stringy action and its fermionic counterpart are effective actions only and justified by SH. In this case there are no problems of interpretation. SH requires only that the induced spinor fields at string world sheets determine them in the interior much like the values of analytic function at curve determine it in an open set of complex plane. At the level of quantum theory the scattering amplitudes should be determined by the data at string world sheets. If the induced W fields at string world sheets are vanishing, the mixing of different charge states in the interior of X^4 would not make itself visible at the level of scattering amplitudes!

If string world sheets are generalized Lagrangian sub-manifolds, only the induced em field would be non-vanishing and electroweak symmetry breaking would be a fundamental prediction. This however requires that M^4 has the analog of symplectic structure suggested also by twistorialization. This in turn provides a possible explanation of CP breaking and matter-antimatter asymmetry. In this case 4-D spinor modes do not define space-time super-symmetries.

The latter option conforms with SH and would mean that the theory is amazingly simple. String world sheets together with number theoretical space-time discretization meaning small breaking of SH would provide the basic data determining classical and quantum dynamics. The Galois group of the extension of rationals defining the number-theoretic space-time discretization would act as a covering group of the covering defined by the discretization of the space-time surface, and the value of $h_{eff}/h = n$ would correspond to the dimension of the extension dividing the order of its Galois group. The phase transitions reducing n would correspond to spontaneous symmetry breaking leading from Galois group to a subgroup and the transition would replace n with its factor.

The ramified primes of the extension would be preferred primes of given extension. The extensions for which the number of p-adic space-time surfaces representable also as a real algebraic continuation of string world sheets to preferred extrenal is especially large would be physically favored as also corresponding ramified primes. In other words, maximal number of p-adic imaginations would be realizable so that these extensions and corresponding ramified primes would be winners in the number-theoretic fight for survival. Whether this conforms with p-adic length scale hypothesis, remains an open question.

Consequences

The outcome is a precise identification of preferred extremals and therefore also a precise definition of Kähler function as Kähler action in Euclidian space-time regions: the Kähler action in Minkowskian regions takes the role of action in quantum field theories and emerges because one has complex square root of thermodynamics. The outcome is a vision combining several big ideas thought earlier to be independent.

1. Effective 2-dimensionality, which was already 30 years ago realized to be unavoidable but meant a catastrophe with the physical understanding that I had at that time. Now it is the outcome of SH implied by SGCI.
2. QC is very naturally realized in terms of generalized conformal symmetries and implies a fractal hierarchy of quantum criticalities, and gives as a side product the hierarchy of Planck constants, which emerged originally from purely physical considerations rather than from TGD. Also the hierarchy of inclusions of hyper-finite factors is a natural outcome as well as the interpretation in terms of measurement resolutions (increasing when n increases by integer factor).
3. The reduction of quantum TGD proper by SH so that only data at partonic 2-surfaces and string world sheets are used to construct the scattering amplitudes. This allows to realized number theoretical universality both at the level of space-time and WCW using algebraic continuation of the physics from an algebraic extension of rationals to real and p-adic number fields. This adelic picture together with Negentropy Maximization Principle (NMP) allows to understand the preferred p-adic primes and deduce a generalization of p-adic length scale hypothesis.

3.2.5 Hyper-Finite Factors And The Notion Of Measurement Resolution

The work with TGD inspired model [K5, K4] for topological quantum computation [B30] led to the realization that von Neumann algebras [A65], in particular so called hyper-finite factors of type II_1 [A52], seem to provide the mathematics needed to develop a more explicit view about the construction of S-matrix. Later came the realization that the Clifford algebra of WCW defines a canonical representation of hyper-finite factors of type II_1 and that WCW spinor fields give rise to HFFs of type III_1 encountered also in relativistically invariant quantum field theories [K120].

Philosophical ideas behind von Neumann algebras

The goal of von Neumann was to generalize the algebra of quantum mechanical observables. The basic ideas behind the von Neumann algebra are dictated by physics. The algebra elements allow

Hermitian conjugation * and observables correspond to Hermitian operators. Any measurable function $f(A)$ of operator A belongs to the algebra and one can say that non-commutative measure theory is in question.

The predictions of quantum theory are expressible in terms of traces of observables. Density matrix defining expectations of observables in ensemble is the basic example. The highly non-trivial requirement of von Neumann was that identical a priori probabilities for a detection of states of infinite state system must make sense. Since quantum mechanical expectation values are expressible in terms of operator traces, this requires that unit operator has unit trace: $tr(Id) = 1$.

In the finite-dimensional case it is easy to build observables out of minimal projections to 1-dimensional eigen spaces of observables. For infinite-dimensional case the probably of projection to 1-dimensional sub-space vanishes if each state is equally probable. The notion of observable must thus be modified by excluding 1-dimensional minimal projections, and allow only projections for which the trace would be infinite using the straightforward generalization of the matrix algebra trace as the dimension of the projection.

The non-trivial implication of the fact that traces of projections are never larger than one is that the eigen spaces of the density matrix must be infinite-dimensional for non-vanishing projection probabilities. Quantum measurements can lead with a finite probability only to mixed states with a density matrix which is projection operator to infinite-dimensional subspace. The simple von Neumann algebras for which unit operator has unit trace are known as factors of type II_1 [A52].

The definitions of adopted by von Neumann allow however more general algebras. Type I_n algebras correspond to finite-dimensional matrix algebras with finite traces whereas I_∞ associated with a separable infinite-dimensional Hilbert space does not allow bounded traces. For algebras of type III non-trivial traces are always infinite and the notion of trace becomes useless being replaced by the notion of state which is generalization of the notion of thermodynamical state. The fascinating feature of this notion of state is that it defines a unique modular automorphism of the factor defined apart from unitary inner automorphism and the question is whether this notion or its generalization might be relevant for the construction of M-matrix in TGD.

Von Neumann, Dirac, and Feynman

The association of algebras of type I with the standard quantum mechanics allowed to unify matrix mechanism with wave mechanics. Note however that the assumption about continuous momentum state basis is in conflict with separability but the particle-in-box idealization allows to circumvent this problem (the notion of space-time sheet brings the box in physics as something completely real).

Because of the finiteness of traces von Neumann regarded the factors of type II_1 as fundamental and factors of type III as pathological. The highly pragmatic and successful approach of Dirac [K121] based on the notion of delta function, plus the emergence of generalized Feynman graphs [K41], the possibility to formulate the notion of delta function rigorously in terms of distributions [A68, A55], and the emergence of path integral approach [A74] meant that von Neumann approach was forgotten by particle physicists.

Algebras of type II_1 have emerged only much later in conformal and topological quantum field theories [A47, A77] allowing to deduce invariants of knots, links and 3-manifolds. Also algebraic structures known as bi-algebras, Hopf algebras, and ribbon algebras [A39, A81] relate closely to type II_1 factors. In topological quantum computation [B30] based on braid groups [A84] modular S-matrices they play an especially important role.

In algebraic quantum field theory [A59] defined in Minkowski space the algebras of observables associated with bounded space-time regions correspond quite generally to the type III_1 hyper-finite factor [A36, A62].

Hyper-finite factors in quantum TGD

The following argument suggests that von Neumann algebras known as hyper-finite factors (HFFs) of type II_1 and III_1- the latter appearing in relativistic quantum field theories provide also the proper mathematical framework for quantum TGD.

1. The Clifford algebra of the infinite-dimensional Hilbert space is a von Neumann algebra known as HFF of type II_1. There also the Clifford algebra at a given point (light-like 3-surface) of WCW is therefore HFF of type II_1. If the fermionic Fock algebra defined by the fermionic oscillator operators assignable to the induced spinor fields (this is actually not obvious!) is infinite-dimensional it defines a representation for HFF of type II_1. Super-conformal symmetry suggests that the extension of the Clifford algebra defining the fermionic part of a super-conformal algebra by adding bosonic super-generators representing symmetries of WCW respects the HFF property. It could however occur that HFF of type II_∞ results.
2. WCW is a union of sub-WCWs associated with causal diamonds (CD) defined as intersections of future and past directed light-cones. One can allow also unions of CDs and the proposal is that CDs within CDs are possible. Whether CDs can intersect is not clear.
3. The assumption that the M^4 proper distance a between the tips of CD is quantized in powers of 2 reproduces p-adic length scale hypothesis but one must also consider the possibility that a can have all possible values. Since $SO(3)$ is the isotropy group of CD, the CDs associated with a given value of a and with fixed lower tip are parameterized by the Lobatchevski space $L(a) = SO(3,1)/SO(3)$. Therefore the CDs with a free position of lower tip are parameterized by $M^4 \times L(a)$. A possible interpretation is in terms of quantum cosmology with a identified as cosmic time [K98]. Since Lorentz boosts define a non-compact group, the generalization of so called crossed product construction strongly suggests that the local Clifford algebra of WCW is HFF of type III_1. If one allows all values of a, one ends up with $M^4 \times M^4_+$ as the space of moduli for WCW.

Hyper-finite factors and M-matrix

HFFs of type III_1 provide a general vision about M-matrix [K120].

1. The factors of type III allow unique modular automorphism Δ^{it} (fixed apart from unitary inner automorphism). This raises the question whether the modular automorphism could be used to define the M-matrix of quantum TGD. This is not the case as is obvious already from the fact that unitary time evolution is not a sensible concept in ZEO.
2. Concerning the identification of M-matrix the notion of state as it is used in theory of factors is a more appropriate starting point than the notion modular automorphism but as a generalization of thermodynamical state is certainly not enough for the purposes of quantum TGD and quantum field theories (algebraic quantum field theorists might disagree!). ZEO requires that the notion of thermodynamical state should be replaced with its "complex square root" abstracting the idea about M-matrix as a product of positive square root of a diagonal density matrix and a unitary S-matrix. This generalization of thermodynamical state -if it exists- would provide a firm mathematical basis for the notion of M-matrix and for the fuzzy notion of path integral.
3. The existence of the modular automorphisms relies on Tomita-Takesaki theorem [A72], which assumes that the Hilbert space in which HFF acts allows cyclic and separable vector serving as ground state for both HFF and its commutant. The translation to the language of physicists states that the vacuum is a tensor product of two vacua annihilated by annihilation oscillator type algebra elements of HFF and creation operator type algebra elements of its commutant isomorphic to it. Note however that these algebras commute so that the two algebras are not hermitian conjugates of each other. This kind of situation is exactly what emerges in ZEO: the two vacua can be assigned with the positive and negative energy parts of the zero energy states entangled by M-matrix.
4. There exists infinite number of thermodynamical states related by modular automorphisms. This must be true also for their possibly existing "complex square roots". Physically they would correspond to different measurement interactions giving rise to Kähler functions of WCW differing only by a real part of holomorphic function of complex coordinates of WCW and arbitrary function of zero mode coordinates and giving rise to the same Kähler metric of WCW.

The concrete construction of M-matrix utilizing the idea of bosonic emergence (bosons as fermion anti-fermion pairs at opposite throats of wormhole contact) meaning that bosonic propagators reduce to fermionic loops identifiable as wormhole contacts leads to generalized Feynman rules for M-matrix in which Kähler-Dirac action containing measurement interaction term defines stringy propagators [K26]. This M-matrix should be consistent with the above proposal.

Connes tensor product as a realization of finite measurement resolution

The inclusions $\mathcal{N} \subset \mathcal{M}$ of factors allow an attractive mathematical description of finite measurement resolution in terms of Connes tensor product [A41] but do not fix M-matrix as was the original optimistic belief.

1. In ZEO $\mathcal{N}$ would create states experimentally indistinguishable from the original one. Therefore $\mathcal{N}$ takes the role of complex numbers in non-commutative quantum theory. The space $\mathcal{M}/\mathcal{N}$ would correspond to the operators creating physical states modulo measurement resolution and has typically fractal dimension given as the index of the inclusion. The corresponding spinor spaces have an identification as quantum spaces with non-commutative $\mathcal{N}$-valued coordinates.
2. This leads to an elegant description of finite measurement resolution. Suppose that a universal M-matrix describing the situation for an ideal measurement resolution exists as the idea about square root of state encourages to think. Finite measurement resolution forces to replace the probabilities defined by the M-matrix with their $\mathcal{N}$ averaged counterparts. The "averaging" would be in terms of the complex square root of $\mathcal{N}$-state and a direct analog of functionally or path integral over the degrees of freedom below measurement resolution defined by (say) length scale cutoff.
3. One can construct also directly M-matrices satisfying the measurement resolution constraint. The condition that $\mathcal{N}$ acts like complex numbers on M-matrix elements as far as $\mathcal{N}$ averaged probabilities are considered is satisfied if M-matrix is a tensor product of M-matrix in $\mathcal{M}(\mathcal{N}$ interpreted as finite-dimensional space with a projection operator to $\mathcal{N}$. The condition that $\mathcal{N}$ averaging in terms of a complex square root of $\mathcal{N}$ state produces this kind of M-matrix poses a very strong constraint on M-matrix if it is assumed to be universal (apart from variants corresponding to different measurement interactions).

Number theoretical braids as space-time correlates for finite measurement resolution

Finite measurement resolution has discretization as a space-time counterpart. In the intersection of real and p-adic worlds defines as partonic 2-surfaces with a mathematical representation allowing interpretation in terms of real or p-adic number fields one can identify points common to real and p-adic worlds as rational points and common algebraic points (in preferred coordinates dictated by symmetries of embedding space). Quite generally, one can identify rational points and algebraic points in some extension of rationals as points defining the initial points of what might be called number theoretical braid beginning from the partonic 2-surface at the past boundary of CD and connecting it with the future boundary of CD. The detailed definition of the braid inside light-like 3-surface is not relevant if only the information at partonic 2-surface is relevant for quantum physics.

Number theoretical braids are especially relevant for topological QFT aspect of quantum TGD. The topological QFT associated with braids accompanying light-like 3-surfaces having interpretation as lines of generalied Feynman diagrams should be important part of the definition of amplitudes assigned to generalized Feynman diagrams. The number theoretic braids relate also closely to a symplectic variant of conformal field theory emerges very naturally in TGD framework (symplectic symmetries acting on $\delta M^4_\pm \times CP_2$ are in question) and this leads to a concrete proposal for how to construct n-point functions needed to calculate M-matrix [K26]. The mechanism guaranteeing the predicted absence of divergences in M-matrix elements can be understood in terms of vanishing of symplectic invariants as two arguments of n-point function coincide.

Quantum spinors and fuzzy quantum mechanics

The notion of quantum spinor leads to a quantum mechanical description of fuzzy probabilities [K120]. For quantum spinors state function reduction to spin eigenstates cannot be performed unless quantum deformation parameter $q = exp(i2\pi/n)$ equals to $q = 1$. The reason is that the components of quantum spinor do not commute: it is however possible to measure the commuting operators representing moduli squared of the components giving the probabilities associated with "true" and "false". Therefore the probability for either spin state becomes a quantized observable. The universal eigenvalue spectrum for probabilities does not in general contain (1,0) so that quantum qbits are inherently fuzzy. State function reduction would occur only after a transition to q=1 phase and de-coherence is not a problem as long as it does not induce this transition.

Concrete realization of finite measurement resolution

The recent view about the realization of finite measurement resolution is surprisingly concrete.

1. The hierarchy of Planck constants $h_{eff} = n \times h$ relates to a hierarchy of criticalities and hierarchy of measurement resolutions since each breaking of symplectic conformal symmetries transforms some gauge degrees of freedom to physical ones making possible improved resolution. For the conformal symmetries associated with the spinor modes the identification as unbroken gauge symmetries is the natural one and conforms with the interpretation as counterparts of gauge symmetries. The hierarchies of conformal symmetry breakings can be identified as hierarchies of inclusions of HFFs. Criticality would generate dark matter phase characterized by n.

   The conformal sub-algebra realized as gauge transformations corresponds to the included algebra gets smaller as n increases so that the measurement resolution improves. The integer n would naturally characterize the inclusions of hyperfinite factors of type II_1 characterized by quantum phase $exp(2\pi/n)$. Finite measurement resolution is expected to give rise to the quantum group representations of symmetries, q-special functions, and q-derivative replacing ordinary derivative and reflecting the presence of discretization.

   In p-adic context representation of angle by phases coming as roots of unity corresponds to this as also the hierarchy of effective p-adic topologies reflecting the fact that finite measurement resolution makes well-orderedness of real numbers as un-necessary luxury and one can use much simpler p-adic mathematics. An excellent example is provided by p-adic mass calculations where number theoretical existence arguments fix the predictions of the model based on p-adic thermodynamics to a high degree.

2. Also the numbers of partonic 2-surfaces and string world sheets connecting them give rise to a physical realization of the finite measurement resolution since fermions at string world sheets represent the space-time geometry physically in finite measurement resolution realized also as a hierarchy of geometries for WCW (via the representation of WCW Kähler metric in terms of anti-commutators of super charges). Finite measurement resolution is a property of physical system formed by the observer and system studied: the system studied changes when the resolution changes.

3. This representation is automatically discrete at the level of partonic 2-surfaces, 1-D at their light-like orbits and 4-D in space-time interior. The discretization can be induced from discretization at the level of embedding space as is done in the definition of p-adic variants of space-time surfaces [K122].

   For $D > 0$ the discretization could also take place more abstractly for the parameters characterizing the functions (say coefficients of polynomials) characterizing string boundaries, string world sheets and partonic 2-surfaces, 3-surfaces, and 4-D space-time surfaces. Clearly, an abstraction hierarchy is involved. Similar discretization applied to the parameters characterizing the functions defining the 3-surfaces makes sense at the level of WCW. The discretization is obviously analogous to a choice of gauge and p-adicization suggests that rational numbers and their algebraic extensions give rise to a natural discretization allowing easy algebraic continuation of scattering amplitudes between different number fields.

3.3 Physics As A Generalized Number Theory

Physics as a generalized number theory vision involves actually three threads: p-adic ideas [K104], the ideas related to classical number fields [K105], and the ideas related to the notion of infinite prime [K103].

3.3.1 Fusion Of Real And P-Adic Physics To A Coherent Whole

p-Adic number fields were not present in the original approach to TGD. The success of the p-adic mass calculations (summarized in the first part of [K70]) made however clear that one must generalize the notion of topology also at the infinitesimal level from that defined by real numbers so that the attribute "topological" in TGD gains much more profound meaning than intended originally. It took a decade to get convinced that the identification of p-adic physics as a correlate of cognition is the most plausible interpretation [K73].

Another idea has been that that p-adic topology of p-adic space-time sheets somehow induces the effective p-adic topology of real space-time sheets. This idea could make physical sense but is not necessary in the recent adelic vision.

The discovery of the properties of number theoretic variants of Shannon entropy led to the idea that living matter could be seen as as something in the intersection of real and p-adic worlds and gave additional support for this interpretation. If even elementary particles reside in this intersection and effective p-adic topology applies for real partonic 2-surfaces, the success of p-adic mass calculations can be understood. The precise identification of this intersection has been a long-standing problem and only quite recently a definite progress has taken place [K119].

The original view about physics as the geometry of WCW is not enough to meet the challenge of unifying real and p-adic physics to a single coherent whole. This inspired "physics as a generalized number theory" approach [K75].

1. The first element is a generalization of the notion of number obtained by "gluing" reals and various p-adic number fields and their algebraic extensions along common rationals and algebraics to form a larger adelic structure (see **Fig. ??** in the appendix of this book).
2. At the level of embedding space this gluing could be seen as a gluing of real and p-adic variants of the embedding space together along common points in an algebraic extension of rationals inducing those for p-adic fields to what could be seen as a book like structure. General Coordinate Invariance (GCI) restricted to rationals or their extension requires preferred coordinates for $CD \times CP_2$ and this kind coordinates can be fixed by isometries of H. The coordinates are however not completely unique since non-rational isometries produce new equally good choices.
3. The manner to get rid of these problems is a more abstract formulation at the level of WCW: a discrete collection of space-time surface instead of a discrete collection of points of space-time surface. In the recent formulation based on strong form of holography identifying the back of the book as string world sheets and partonic 2-surfaces with parameters in some algebraic extension of rationals, the problems with GCI seem to disappear since the equations for the 2-surfaces in the intersection can be interpreted in any number field. One also gets rid of the ugly discretization at space-time level needed in the notion of p-adic manifold [K122] since it is performed at the level of parameters characterizing 2-D surfaces. By conformal invariance these parameters could be conformal moduli so that infinite-D WCW would effectively reduce to finite-D spaces.
4. The possibility to assign a p-adic prime to the real space-time sheets is required by the success of the elementary particle mass calculations and various applications of the p-adic length scale hypothesis. The original idea was that the non-determinism of Kähler action corresponds to p-adic non-determinism for some primes. It has been however difficult to make this more concrete.

   Rational numbers are common to reals and all p-adic number fields. One can actually assign to any algebraic extension of rationals extensions of p-adic numbers and construct corresponding adeles. These extensions can be arranged according to the complexity and I have already earlier proposed that this hierarchy gives rie to an evolutionary hierarchy.

   How the existence of preferred p-adic primes characterizing space-time surfaces emerge was solved only quite recently [K119]. The solution relies on p-adicization based on strong holography motivating the idea the idea that string world sheets and partonic surfaces with parameters in algebraic extensions of rationals define the intersection of reality and various p-adicities. The algebraic extension possesses preferred primes as primes, which are ramified meaning that their decomposition to a product of primes of the extension contains higher than first powers of its primes (prime ideals is the more precise notion).

These primes are obviously natural candidates for the primes characterizing string world sheets number theoretically and it might even happen that strong form of holography is possible only for these primes. The weak form of NMP [K65] allows also to justify a generalization of p-adic length scale hypothesis. Primes near but below powers of primes are favoured since they allow exceptionally large negentropy gain so that state function reductions to tend to select them. Therefore the adelic approach combined with strong form of holography seems to be a rather promising approach.

p-Adic continuations of 2-surfaces to 4-surfaces identifiable as imaginations would be due to the existence of p-adic pseudo-constants. The continuation could fail for most configurations of partonic 2-surfaces and string world sheets in the real sector: the interpretation would be that some space-time surfaces can be imagined but not realized [K73]. For certain extensions the number of realizable imaginations could be exceptionally large. These extensions would be winners in the number theoretic fight for survivalandcorresponding ramified primes would be preferred p-adic primes.

The interpretation for discretization the level of partonic 2-surfaces could be in terms of cognitive, sensory, and measurement resolutions rather than fundamental discreteness of the space-time. At the level of partonic 2-surface the discretization reduces to the naïvely expected one: the corners of string world sheets at partonic 2-surface defined the end points of string and orbits of string ends carrying fermion number. This discretization has concrete physical interpretation. Clearly a co-dimension rule holds. Discretization of n-D object consist of n-2-D objects.

What looks rather counter intuitive first is that transcendental points of p-adic space-time sheets are at spatiotemporal infinity in real sense so that the correlates of cognition cannot be localized to any finite spatiotemporal volume unlike those of sensory experience. The description of cognition in this manner predicts p-adic fractality of real physics meaning chaos in short scales combined with long range correlations: p-adic mass calculations represent one example of p-adic fractality.

The realization of this program at the level of WCW is far from trivial. Kähler-Dirac equation and classical field equations make sense but quantities expressible as space-time integrals - in particular Kähler action- do not make sense p-adically. Therefore one can ask whether only the partonic surfaces in the intersection of real and p-adic worlds should be allowed. Also this restricted theory would be highly non-trivial physically.

3.3.2 Classical Number Fields And Associativity And Commutativity As Fundamental Law Of Physics

The dimensions of classical number fields appear as dimensions of basic objects in quantum TGD. Embedding space has dimension 8, space-time has dimension 4, light-like 3-surfaces are orbits of 2-D partonic surfaces. If conformal QFT applies to 2-surfaces (this is questionable), one-dimensional structures would be the basic objects. The lowest level would correspond to discrete sets of points identifiable as intersections of real and p-adic space-time sheets. This suggests that besides p-adic number fields also classical number fields (reals, complex numbers, quaternions, octonions [A67]) are involved [K105] and the notion of geometry generalizes considerably. In the recent view about quantum TGD the dimensional hierarchy defined by classical number field indeed plays a key role. $H = M^4 \times CP_2$ has a number theoretic interpretation and standard model symmetries can be understood number theoretically as symmetries of hyper-quaternionic planes of hyper-octonionic space.

The associativity condition $A(BC) = (AB)C$ suggests itself as a fundamental physical law of both classical and quantum physics. Commutativity can be considered as an additional condition. In conformal field theories associativity condition indeed fixes the n-point functions of the theory. At the level of classical TGD space-time surfaces could be identified as maximal associative (hyper-quaternionic) sub-manifolds of the embedding space whose points contain a preferred hyper-complex plane M^2 in their tangent space and the hierarchy finite fields-rationals-reals-complex numbers-quaternions-octonions could have direct quantum physical counterpart [K105]. This leads to the notion of number theoretic compactification analogous to the dualities of M-theory: one can interpret space-time surfaces either as hyper-quaternionic 4-surfaces of M^8 or as 4-surfaces in $M^4 \times CP_2$. As a matter fact, commutativity in number theoretic sense is a further natural condition and leads to the notion of number theoretic braid naturally as also to direct connection with super string models.

At the level of Kähler-Dirac action the identification of space-time surface as a hyper-quaternionic sub-manifold of H means that the modified gamma matrices of the space-time surface defined in terms of canonical momentum currents of Kähler action using octonionic representation for the gamma matrices of H span a hyper-quaternionic sub-space of hyper-octonions at each point of space-time surface (hyper-octonions are the subspace of complexified octonions for which imaginary units are octonionic imaginary units multiplied by commutating imaginary unit). Hyper-octonionic representation leads to a proposal for how to extend twistor program to TGD framework [K121, L8].

How to achieve associativity in the fermionic sector?

In the fermionic sector an additional complication emerges. The associativity of the tangent- or normal space of the space-time surface need not be enough to guarantee the associativity at the level of Kähler-Dirac or Dirac equation. The reason is the presence of spinor connection. A possible cure could be the vanishing of the components of spinor connection for two conjugates of quaternionic coordinates combined with holomorphy of the modes.

1. The induced spinor connection involves sigma matrices in CP_2 degrees of freedom, which for the octonionic representation of gamma matrices are proportional to octonion units in Minkowski degrees of freedom. This corresponds to a reduction of tangent space group $SO(1,7)$ to G_2. Therefore octonionic Dirac equation identifying Dirac spinors as complexified octonions can lead to non-associativity even when space-time surface is associative or co-associative.

2. The simplest manner to overcome these problems is to assume that spinors are localized at 2-D string world sheets with 1-D CP_2 projection and thus possible only in Minkowskian regions. Induced gauge fields would vanish. String world sheets would be minimal surfaces in $M^4 \times D^1 \subset M^4 \times CP_2$ and the theory would simplify enormously. String area would give rise to an additional term in the action assigned to the Minkowskian space-time regions and for vacuum extremals one would have only strings in the first approximation, which conforms with the success of string models and with the intuitive view that vacuum extremals of Kähler action are basic building bricks of many-sheeted space-time. Note that string world sheets would be also symplectic covariants.

   Without further conditions gauge potentials would be non-vanishing but one can hope that one can gauge transform them away in associative manner. If not, one can also consider the possibility that CP_2 projection is geodesic circle S^1: symplectic invariance is considerably reduces for this option since symplectic transformations must reduce to rotations in S^1.

3. The fist heavy objection is that action would contain Newton's constant G as a fundamental dynamical parameter: this is a standard recipe for building a non-renormalizable theory. The very idea of TGD indeed is that there is only single dimensionless parameter analogous to critical temperature. One can of coure argue that the dimensionless parameter is $\hbar G/R^2$, R CP_2 "radius".

   Second heavy objection is that the Euclidian variant of string action exponentially damps out all string world sheets with area larger than $\hbar G$. Note also that the classical energy of Minkowskian string would be gigantic unless the length of string is of order Planck length. For Minkowskian signature the exponent is oscillatory and one can argue that wild oscillations have the same effect.

   The hierarchy of Planck constants would allow the replacement $\hbar \rightarrow \hbar_{eff}$ but this is not enough. The area of typical string world sheet would scale as h_{eff} and the size of CD and gravitational Compton lengths of gravitationally bound objects would scale as $\sqrt{h_{eff}}$ rather than $\hbar_{eff} = GMm/v_0$, which one wants. The only way out of problem is to assume $T \propto (\hbar/h_{eff})^2 \times (1/h_{bar}G)$. This is however un-natural for genuine area action. Hence it seems that the visit of the basic assumption of superstring theory to TGD remains very short.

Is super-symmetrized Kähler-Dirac action enough?

Could one do without string area in the action and use only K-D action, which is in any case forced by the super-conformal symmetry? This option I have indeed considered hitherto. K-D Dirac equation indeed tends to reduce to a lower-dimensional one: for massless extremals the K-D operator is effectively 1-dimensional. For cosmic strings this reduction does not however take place. In any case, this leads to ask whether in some cases the solutions of Kähler-Dirac equation are localized at lower-dimensional surfaces of space-time surface.

1. The proposal has indeed been that string world sheets carry vanishing W and possibly even Z fields: in this manner the electromagnetic charge of spinor mode could be well-defined. The vanishing conditions force in the generic case 2-dimensionality.

   Besides this the canonical momentum currents for Kähler action defining 4 embedding space vector fields must define an integrable distribution of two planes to give string world sheet. The four canonical momentum currents $\Pi_k\alpha = \partial L_K/\partial_{\partial_\alpha h^k}$ identified as embedding 1-forms can have only two linearly independent components parallel to the string world sheet. Also the Frobenius conditions stating that the two 1-forms are proportional to gradients of two embedding space coordinates Φ_i defining also coordinates at string world sheet, must be satisfied. These conditions are rather strong and are expected to select some discrete set of string world sheets.

2. To construct preferred extremal one should fix the partonic 2-surfaces, their light-like orbits defining boundaries of Euclidian and Minkowskian space-time regions, and string world sheets. At string world sheets the boundary condition would be that the normal components of canonical momentum currents for Kähler action vanish. This picture brings in mind strong form of holography and this suggests that might make sense and also solution of Einstein equations with point like sources.

3. The localization of spinor modes at 2-D surfaces would would follow from the well-definedness of em charge and one could have situation is which the localization does not occur. For instance, covariantly constant right-handed neutrinos spinor modes at cosmic strings are completely de-localized and one can wonder whether one could give up the localization inside wormhole contacts.

4. String tension is dynamical and physical intuition suggests that induced metric at string world sheet is replaced by the anti-commutator of the K-D gamma matrices and by conformal invariance only the conformal equivalence class of this metric would matter and it could be even equivalent with the induced metric. A possible interpretation is that the energy density of Kähler action has a singularity localized at the string world sheet.

   Another interpretation that I proposed for years ago but gave up is that in spirit with the TGD analog of AdS/CFT duality the Noether charges for Kähler action can be reduced to integrals over string world sheet having interpretation as area in effective metric. In the case of magnetic flux tubes carrying monopole fluxes and containing a string connecting partonic 2-surfaces at its ends this interpretation would be very natural, and string tension would characterize the density of Kähler magnetic energy. String model with dynamical string tension would certainly be a good approximation and string tension would depend on scale of CD.

5. There is also an objection. For M^4 type vacuum extremals one would not obtain any non-vacuum string world sheets carrying fermions but the successes of string model strongly suggest that string world sheets are there. String world sheets would represent a deformation of the vacuum extremal and far from string world sheets one would have vacuum extremal in an excellent approximation. Situation would be analogous to that in general relativity with point particles.

6. The hierarchy of conformal symmetry breakings for K-D action should make string tension proportional to $1/h_{eff}^2$ with $h_{eff} = h_{gr}$ giving correct gravitational Compton length $\Lambda_{gr} = GM/v_0$ defining the minimal size of CD associated with the system. Why the effective string tension of string world sheet should behave like $(\hbar/\hbar_{eff})^2$?

   The first point to notice is that the effective metric $G^{\alpha\beta}$ defined as $h^{kl}\Pi_k^\alpha\Pi_l^\beta$, where the canonical momentum current $\Pi_k\alpha = \partial L_K/\partial_{\partial_\alpha h^k}$ has dimension $1/L^2$ as required. Kähler action density must be dimensionless and since the induced Kähler form is dimensionless the canonical momentum currents are proportional to $1/\alpha_K$.

Should one assume that α_K is fundamental coupling strength fixed by quantum criticality to $\alpha_K = 1/137$? Or should one regard g_K^2 as fundamental parameter so that one would have $1/\alpha_K = \hbar_{eff}/4\pi g_K^2$ having spectrum coming as integer multiples (recall the analogy with inverse of critical temperature)?

The latter option is the in spirit with the original idea stating that the increase of h_{eff} reduces the values of the gauge coupling strengths proportional to α_K so that perturbation series converges (Universe is theoretician friendly). The non-perturbative states would be critical states. The non-determinism of Kähler action implying that the 3-surfaces at the boundaries of CD can be connected by large number of space-time sheets forming n conformal equivalence classes. The latter option would give $G^{\alpha\beta} \propto h_{eff}^2$ and $det(G) \propto 1/h_{eff}^2$ as required.

7. It must be emphasized that the string tension has interpretation in terms of gravitational coupling on only at the GRT limit of TGD involving the replacement of many-sheeted space-time with single sheeted one. It can have also interpretation as hadronic string tension or effective string tension associated with magnetic flux tubes and telling the density of Kähler magnetic energy per unit length.

   Superstring models would describe only the perturbative Planck scale dynamics for emission and absorption of $h_{eff}/h = 1$ on mass shell gravitons whereas the quantum description of bound states would require $h_{eff}/n > 1$ when the masses. Also the effective gravitational constant associated with the strings would differ from G.

   The natural condition is that the size scale of string world sheet associated with the flux tube mediating gravitational binding is $G(M + m)/v_0$, By expressing string tension in the form $1/T = n^2\hbar G_1$, $n = h_{eff}/h$, this condition gives $\hbar G_1 = \hbar^2/M_{red}^2$, $M_{red} = Mm/(M + m)$. The effective Planck length defined by the effective Newton's constant G_1 analogous to that appearing in string tension is just the Compton length associated with the reduced mass of the system and string tension equals to $T = [v_0/G(M + m)]^2$ apart from a numerical constant ($2G(M + m)$ is Schwartschild radius for the entire system). Hence the macroscopic stringy description of gravitation in terms of string differs dramatically from the perturbative one. Note that one can also understand why in the Bohr orbit model of Nottale [E1] for the planetary system and in its TGD version [K97] v_0 must be by a factor 1/5 smaller for outer planets rather than inner planets.

Are 4-D spinor modes consistent with associativity?

The condition that octonionic spinors are equivalent with ordinary spinors looks rather natural but in the case of Kähler-Dirac action the non-associativity could leak in. One could of course give up the condition that octonionic and ordinary K-D equation are equivalent in 4-D case. If so, one could see K-D action as related to non-commutative and maybe even non-associative fermion dynamics. Suppose that one does not.

1. K-D action vanishes by K-D equation. Could this save from non-associativity? If the spinors are localized to string world sheets, one obtains just the standard stringy construction of conformal modes of spinor field. The induce spinor connection would have only the holomorphic component A_z. Spinor mode would depend only on z but K-D gamma matrix Γ^z would annihilate the spinor mode so that K-D equation would be satisfied. There are good hopes that the octonionic variant of K-D equation is equivalent with that based on ordinary gamma matrices since quaternionic coordinated reduces to complex coordinate, octonionic quaternionic gamma matrices reduce to complex gamma matrices, sigma matrices are effectively absent by holomorphy.

2. One can consider also 4-D situation (maybe inside wormhole contacts). Could some form of quaternion holomorphy [A83] [L8] allow to realize the K-D equation just as in the case of super string models by replacing complex coordinate and its conjugate with quaternion and its 3 conjugates. Only two quaternion conjugates would appear in the spinor mode and the corresponding quaternionic gamma matrices would annihilate the spinor mode. It is essential that in a suitable gauge the spinor connection has non-vanishing components only

for two quaternion conjugate coordinates. As a special case one would have a situation in which only one quaternion coordinate appears in the solution. Depending on the character of quaternionion holomorphy the modes would be labelled by one or two integers identifiable as conformal weights.

Even if these octonionic 4-D modes exists (as one expects in the case of cosmic strings), it is far from clear whether the description in terms of them is equivalent with the description using K-D equation based ordinary gamma matrices. The algebraic structure however raises hopes about this. The quaternion coordinate can be represented as sum of two complex coordinates as $q = z_1 + Jz_2$ and the dependence on two quaternion conjugates corresponds to the dependence on two complex coordinates z_1, z_2. The condition that two quaternion complexified gammas annihilate the spinors is equivalent with the corresponding condition for Dirac equation formulated using 2 complex coordinates. This for wormhole contacts. The possible generalization of this condition to Minkowskian regions would be in terms Hamilton-Jacobi structure.

Note that for cosmic strings of form $X^2 \times Y^2 \subset M^4 \times CP_2$ the associativity condition for S^2 sigma matrix and without assuming localization demands that the commutator of Y^2 imaginary units is proportional to the imaginary unit assignable to X^2 which however depends on point of X^2. This condition seems to imply correlation between Y^2 and S^2 which does not look physical.

To summarize, the minimal and mathematically most optimistic conclusion is that Kähler-Dirac action is indeed enough to understand gravitational binding without giving up the associativity of the fermionic dynamics. Conformal spinor dynamics would be associative if the spinor modes are localized at string world sheets with vanishing W (and maybe also Z) fields guaranteeing well-definedness of em charge and carrying canonical momentum currents parallel to them. It is not quite clear whether string world sheets are present also inside wormhole contacts: for CP_2 type vacuum extremals the Dirac equation would give only right-handed neutrino as a solution (could they give rise to $N = 2$ SUSY?).

The construction of preferred extremals would realize strong form of holography. By conformal symmetry the effective metric at string world sheet could be conformally equivalent with the induced metric at string world sheets.

Dynamical string tension would be proportional to $\hbar/h_{eff}^2$ due to the proportionality $\alpha_K \propto 1/h_{eff}$ and predict correctly the size scales of gravitationally bound states for $\hbar_{gr} = \hbar_{eff} = GMm/v_0$. Gravitational constant would be a prediction of the theory and be expressible in terms of α_K and R^2 and $\hbar_{eff}$ ($G \propto R^2/g_K^2$).

In fact, all bound states - elementary particles as pairs of wormhole contacts, hadronic strings, nuclei [K68], molecules, etc. - are described in the same manner quantum mechanically. This is of course nothing new since magnetic flux tubes associated with the strings provide a universal model for interactions in TGD Universe. This also conforms with the TGD counterpart of AdS/CFT duality.

3.3.3 Infinite Primes And Quantum Physics

The hierarchy of infinite primes (and of integers and rationals) [K103] was the first mathematical notion stimulated by TGD inspired theory of consciousness. The construction recipe is equivalent with a repeated second quantization of a super-symmetric arithmetic quantum field theory with bosons and fermions labeled by primes such that the many-particle states of previous level become the elementary particles of new level. At a given level there are free many particles states plus counterparts of many particle states. There is a strong structural analogy with polynomial primes. For polynomials with rational coefficients free many-particle states would correspond to products of first order polynomials and bound states to irreducible polynomials with non-rational roots.

The hierarchy of space-time sheets with many particle states of space-time sheet becoming elementary particles at the next level of hierarchy. For instance, the description of proton as an elementary fermion would be in a well defined sense exact in TGD Universe. Also the hierarchy of n:th order logics are possible correlates for this hierarchy.

This construction leads also to a number theoretic generalization of space-time point since a given real number has infinitely rich number theoretical structure not visible at the level of the real norm of the number a due to the existence of real units expressible in terms of ratios of infinite integers. This number theoretical anatomy suggest a kind of number theoretical Brahman=Atman identity stating that the set consisting of number theoretic variants of single point of the embedding space (equivalent in real sense) is able to represent the points of WCW or maybe even quantum states assignable to causal diamond. One could also speak about algebraic holography.

The hierarchy of algebraic extensions of rationals is becoming a fundamental element of quantum TGD. This hierarchy would correspond to the hierarchy of quantum criticalities labelled by integer $n = h_{eff}/h$, and n could be interpreted as the product of ramified primes of the algebraic extension or its power so that number theoretic criticality would correspond to quantum criticality. The idea is that ramified primes are analogous to multiple roots of polynomial and criticality indeed corresponds to this kind of situation.

Infinite primes at the n:th level of hierarchy representing analogs of bound states correspond to irreducible polynomials of n-variables identifiable as polynomials of z_n with coefficients, which are polynomials of $z_1, .., z_{n-1}$. At the first level of hierarchy one has irreducible polynomials of single variable and their roots define irreducible algebraic extensions of rationals. Infinite integers in turn correspond to products of reducible polynomials defining reducible extensions. The infinite integers at the first level of hierarchy would define the hierarchy of algebraic extensions of rationals in turn defining a hierarchy of quantum criticalities. This observation could generalize to the higher levels of hierarchy of infinite primes so that infinite primes would be part of quantum TGD although in much more abstract sense as thought originally.

3.4 Physics As Extension Of Quantum Measurement Theory To A Theory Of Consciousness

TGD inspired theory of consciousness could be seen as a generalization of quantum measurement theory to make observer, which in standard quantum measurement theory remains an outsider, a genuine part of physical system subject to laws of quantum physics. The basic notions are quantum jump identified as moment of consciousness and the notion of self [K61]: in zero energy ontology these notions might however reduce to each other. Negentropy Maximization Principle [K65] defines the dynamics of consciousness and as a special case reproduces standard quantum measurement theory.

3.4.1 Quantum Jump As Moment Of Consciousness

TGD suggests that the quantum jump between quantum histories could identified as moment of consciousness and could therefore be for consciousness theory what elementary particle is for physics [K61].

This means that subjective time evolution corresponds to the sequence of quantum jumps $\Psi_i \rightarrow U\Psi_i \rightarrow \Psi_f$ consisting of unitary process followed by state function process. Originally U was thought to be the TGD counterpart of the unitary time evolution operator $U(-t,t)$, $t \rightarrow \infty$, associated with the scattering solutions of Schrödinger equation. It seems however impossible to assign any real Schrödinger time evolution with U. In zero energy ontology U defines a unitary matrix between zero energy states and is naturally assignable to intentional actions whereas the ordinary S-matrix telling what happens in particle physics experiment (for instance) generalizes to M-matrix defining time-like entanglement between positive and negative energy parts of zero energy states. One might say that U process corresponds to a fundamental act of creation creating a quantum superposition of possibilities and the remaining steps generalizing state function reduction process select between them.

3.4.2 Negentropy Maximization Principle And The Notion Of Self

Negentropy Maximization Principle (NMP [K65]) defines the variational principle of TGD inspired theory of consciousness. It has developed considerably during years. The notion of negentropic entanglement (NE) and Zero Energy Ontology (ZEO) have been main stimuli in this process.

1. U-process is followed by a sequence of state function reductions. Negentropy Maximization Principle (NMP [K65]) in its original form stated that in a given quantum state the most

quantum entangled subsystem-complement pair can perform the quantum jump to a state with vanishing entanglement. More precisely: the reduction of the entanglement entropy in the quantum jump is as large as possible. This selects the pair in question and in case of ordinary entanglement entropy leads the selected pair to a product state. The interpretation of the reduction of the entanglement entropy as a conscious information gain makes sense. The sequence of state function reductions decomposes at first step the entire system to two parts in such a way that the reduction entanglement entropy is maximal. This process repeats itself for subsystems. If the subsystem in question cannot be divided into a pair of entangled free system the process stops since energy conservation does not allow it to occur (binding energy).

The original definition of self was as a subsystem able to remain unentangled under state function reductions associated with subsequent quantum jumps. Everything is consciousness but consciousness can be lost if self develops bound state entanglement during U process so that state function reduction to smaller un-entangled pieces is impossible.

2. The existence of number theoretical entanglement entropies in the intersection of real and various p-adic worlds forced to modify this picture. These entropies can be negative and therefore are actually positive negentropies representing conscious or potentially conscious information.

   The reduction process can stop also if the self in question allows only decompositions to pairs of systems with negentropic entanglement (NE). This does not require that the system forms a bound state for any pair of subsystems so that the systems decomposing it can be free (no binding energy). This defines a new kind of bound state not describable as a jail defined by the bottom of a potential well. Subsystems are free but remain correlated by NE (see **Fig.** `http://tgdtheory.fi/appfigures/cat.jpg` or **Fig. ??** in the appendix of this book).

   The consistency with quantum measurement theory demands that quantum measurement leads to an eigen-space of the density matrix so that the outcome of the state function reduction would be characterized by a possibly higher-dimensional projection operator. This would define strong form of NMP. The condition that negentropy gain (rather than final state negentropy) is maximal fixed the sub-system complement pair for which the reduction occurs.

3. Strong form of NMP would mean very restricted form of free will: we would live in the best possible world. The weak form of NMP allows the outcome of state function reduction to be a lower-dimensional subspace of the space defined by the projector. This form of NMP allows free will, event also ethics and moral can be understood if one assumes that NE means experience with positive emotional coloring and has interpretation as information (Akashic records) [K116]. Weak form of NMP allows also to predict generalization of p-adic length scale hypothesis [K119]. Hence weak NMP is much more feasible than strong form of NMP.

It is not at all obvious that NMP is consistent with the second law and it is quite possible that second law holds true only if one restricts the consideration to the visible matter sector with ordinary value of Planck constant.

1. The ordinary state function reductions - as opposed to those generating negentropic entanglement - imply dissipation crucial for self organization and quantum jump could be regarded as the basic step of an iteration like process leading to the asympotic self-organization patterns. One could regard dissipation as a Darwinian selector as in standard theories of self-organization. NMP thus predicts that self organization and hence presumably also fractalization can occur inside selves. NMP would favor the generation of negentropic entanglement. This notion is highly attractive since it could allow to understand how quantum self-organization generates larger coherent structures.
2. State function reduction for NE is not deterministic for the weak form of NMP but on the average sense negentropy assignable to dark matter sectors increases. This could allow to understand how living matter is able to develop almost deterministic cellular automaton like behaviors.
3. A further implication of NMP is that Universe generates information about itself represented in terms of NE: if one is not afraid of esoteric associations one could call this information Akashic records. This ia not in obvious conflict with second law since the entropy in the case of second law is ensemble entropy assignable to single particle in thermodynamical description.

The simplest assumption is that the information measured by number theoretic negentropy is experienced during the state function reduction sequence at fixed boundary of CD defining self.

Weak NMP provides an understanding of life, which is the mirror image of that believed to be provided by the second law. Life in the standard Universe would be a thermodynamical fluctuation - the needed size of this fluctuation has been steadily increasing and it seems that it will eventually fill the entire Universe! Life in TGD Universe is a necessity implied by NMP and the attribute "weak" makes possible the analogs of thermodynamical fluctuations in opposite effects meaning that the world is not the best possible one. On the other hand, weak form of NMP implies evolution as selection of preferred p-adic primes since the free will allows also larger negentropy gains than strong form of NMP.

3.4.3 Life As Islands Of Rational/Algebraic Numbers In The Seas Of Real And P-Adic Continua?

NMP and negentropic entanglement demanding entanglement probabilities which are equal to inverse of integer, is the starting point. Rational and even algebraic entanglement coefficients make sense in the intersection of real and p-adic words, which suggests that in some sense life and conscious intelligence reside in the intersection of the real and p-adic worlds.

What could be this intersection of realities and p-adicities?

1. The facts that fermionic oscillator operators are correlates for Boolean cognition and that induced spinor fields are restricted to string world sheets and partonic 2-surfaces suggests that the intersection consists of these 2-surfaces.
2. Strong form of holography allows a rather elegant adelization of TGD by a construction of space-time surfaces by algebraic continuations of these 2-surfaces defined by parameters in algebraic extension of rationals inducing that for various p-adic number fields to real or p-adic number fields. Scattering amplitudes could be defined also by a similar algebraic contination. By conformal invariance the conformal moduli characterizing the 2-surfaces would defined the parameters.

This suggests a rather concrete view about the fundamental quantum correlates of life and intelligence.

1. For the minimal option life would be effectively 2-dimensional phenomenon and essentially a boundary phenomenon as also number theoretical criticality suggests. There are good reasons to expect that only the data from the intersection of real and p-adic string world sheets partonic two-surfaces appears in U-matrix so that the data localizable to strings connecting partonic 2-surfaces would dictate the scattering amplitudes.

A good guess is that algebraic entanglement is essential for quantum computation, which therefore might correspond to a conscious process. Hence cognition could be seen as a quantum computation like process, a more appropriate term being quantum problem solving [K4]. Living-dead dichotomy could correspond to rational-irrational or to algebraic-transcendental dichotomy: this at least when life is interpreted as intelligent life. Life would in a well defined sense correspond to islands of rationality/algebraicity in the seas of real and p-adic continua. Life as a critical phenomenon in the number theoretical sense would be one aspect of quantum criticality of TGD Universe besides the criticality of the space-time dynamics and the criticality with respect to phase transitions changing the value of Planck constant and other more familiar criticalities. How closely these criticalities relate remains an open question [K92].

The view about the crucial role of rational and algebraic numbers as far as intelligent life is considered, could have been guessed on very general grounds from the analogy with the orbits of a dynamical system. Rational numbers allow a predictable periodic decimal/pinary expansion and are analogous to one-dimensional periodic orbits. Algebraic numbers are related to rationals by a finite number of algebraic operations and are intermediate between periodic and chaotic orbits allowing an interpretation as an element in an algebraic extension of any p-adic number field. The projections of the orbit to various coordinate directions of the algebraic extension represent now periodic orbits. The decimal/pinary expansions of transcendentals are un-predictable being analogous to chaotic orbits. The special role of rational and algebraic numbers was realized already by Pythagoras, and the fact that the ratios for the frequencies of the musical scale are rationals supports the special nature of rational and algebraic numbers. The special nature of the Golden Mean, which involves $\sqrt{5}$, conforms the view that algebraic numbers rather than only rationals are essential for life.

Later progress in understanding of quantum TGD allows to refine and simplify this view dramatically. The idea about p-adic-to-real transition for space-time sheets as a correlate for the transformation of intention to action has turned out to be un-necessary and also hard to realize mathematically. In adelic vision real and p-adic numbers are aspects of existence in all length scales and mean that cognition is present at all levels rather than emerging. Intentions have interpretation in terms of state function reductions in ZEO and there is no need to identify p-adic space-time sheets as their correlates.

3.4.4 Two Times

The basic implication of the proposed view is that subjective time and geometric time of physicist are not the same [K61]. This is not a news actually. Geometric time is reversible, subjective time irreversible. Geometric future and past are in completely democratic position, subjective future does not exist at all yet. One can say that the non-determinism of quantum jump is completely outside space-time and Hilbert space since quantum jumps replaces entire 4-D time evolution (or rather, their quantum superposition) with a new one, re-creates it. Also conscious existence defies any geometric description. This new view resolves the basic problem of quantum measurement theory due to the conflict between determinism of Schrödinger equation and randomness of quantum jump. The challenge is to understand how these two times correlate so closely as to lead to their erratic identification.

With respect to geometric time the contents of conscious experience is naturally determined by the space-time region inside CD in zero energy ontology. This geometro-temporal integration should have subjecto-temporal counterpart. The experiences of self are determined partially by the mental images assignable to sub-selves (having sub-CDs as embedding space correlates) and the quantum jump sequences associated with sub-selves define a sequence of mental images.

The view about the experience of time has changed.

1. The original hypothesis was that self experiences these sequences of mental images as a continuous time flow. If the mental images define the contents fo consciousness completely, self would experience in absence of mental images experience of "timelessness". This could be seen to be in accordance with the reports of practitioners of various spiritual practices. One must be however extremely cautious and try to avoid naïve interpretations.
2. ZEO forces to modify this view: the experience about the flow of time and its arrow corresponds to a sequence of repeated state function reductions leaving the state at fixed boundary of CD invariant: in standard quantum theory the entire state would remain invariant but now the position of the upper boundary of CD and state at it changes. Perhaps the experiences of meditators are such that the upper boundary of CD is more or less stationary during them.

What happens when consciousness is lost?

1. The original vision was that self loses consciousness in quantum jump generating entropic entanglement and experience an expansion of consciousness if the resulting entanglement is negentropic.
2. The recent vision is that the first state function reduction to the opposite boundary of CD means for self death followed by re-incarnation at the opposite boundary.

The assumption that the integration of experiences of self involves a kind of averaging over sub-selves of sub-selves guarantees that the sensory experiences are reliable despite the fact that quantum nondeterminism is involved with each quantum jump.

The measurement of density matrix defined by the $MM^\dagger$, where M is the M-matrix between positive and negative energy parts of the zero energy state would correspond to the passive aspects of consciousness such as sensory experiencing. U would represent at the fundamental level volition as a creation of a quantum superposition of possibilities. What follows it would be a selection between them. The volitional choice between macroscopically differing space-time sheets representing different maxima of Kähler function could be basically responsible for the active aspect of consciousness. The fundamental perception-reaction feedback loop of biosystems would result from the combination of the active and passive aspects of consciousness represented by U and M.

3.4.5 How Experienced Time And The Geometric Time Of Physicist Relate To Each Other?

The relationship between experienced time and time of physicist is one of the basic puzzles of modern physics. In the proposed framework they are certainly two different things and the challenge is to understand why the correlation between them is so strong that it has led to their identification. One can imagine several alternative views explaining this correlation [K116, K7] and it is better to keep mind open.

Basic questions

The flow of subjective time corresponds to quantum jump sequences for sub-selves of self having interpretation as mental images. If mind is completely empty of mental images subjectively experienced time ceases to exists. This leaves however several questions to be answered.

1. Why the contents of conscious of self comes from a finite space-time region looks like an easy question. If the contents of consciousness for sub-selves representing mental images is localized to the sub-CDs with indeed have defined temporal position inside CD assigned with the self the contents of consciousness is indeed from a finite space-time volume. This implies a new view about memory. There is no need to store again and again memories to the "brain now" since the communications with the geometric past by negative energy signals and also time-like negentropic quantum entanglement allow the sharing of the mental images of the geometric past.
2. There are also more difficult questions. Subjective time has arrow and has only the recent and possibly also past. The subjective past could in principle reduce to subjective now if conscious experience is about 4-D space-time region so that memories would be always geometric memories. How these properties of subjective time are transferred to apparent properties of geometric time? How the arrow of geometric time is induced? How it is possible that the locus for the contents of conscious experience shifts or at least seems to be shifted quantum jump by quantum jump to the direction of geometric future? Why the sensory mental images are located in a narrow time interval of about .1 seconds in the usual states of consciousness (not that sensory memories are possible: scent memories and phantom pain in leg could be seen as examples of vivid sensory memory)?

The recent view about arrow of time

The basic intuitive idea about the explanation for the arrow of psychological time has been the same from the beginning - diffusion inside light-cone - but its detailed realization has required understanding of what quantum TGD really is. The replacement of ordinary positive energy ontology with zero energy ontology (ZEO) has played a crucial role in this development. The TGD based vision about how the arrow of geometric time is by no means fully developed and final. It however seems that the most essential aspects have been understood now.

1. What seems clear now is the decisive role of ZEO and hierarchy of CDs, and the fact that the quantum arrow of geometric time is coded into the structure of zero energy states to a high extent. The still questionable but attractively simple hypothesis is that U matrix two basis with opposite quantum arrows of geometric time: is this assumption really consistent with what we know about the arrow of time? If this is the case, the question is how the relatively well-defined quantum arrow of geometric time implies the experienced arrow of geometric time. Should one assume the arrow of geometric time separately as a basic property of the state function reduction cascade or more economically- does it follow from the arrow of time for zero energy states or only correlate with it?

2. The state function reductions can occur at both boundaries of CD. If the reduction occurs at given boundary is immediately followed by a reduction at the opposite boundary, the arrow of time alternates: this does not conform with intuitive expectations: for instance, this would imply that there are two selves assignable to the opposite boundaries!

   Zero energy states are however de-localized in the moduli space CDs (size of CD plus discrete subgroup of Lorentz group defining boosts of CD leaving second tip invariant). One has quantum superposition of CDs with difference scales but with fixed upper or lower boundary belonging to the same light-cone boundary after state function reduction. In standard quantum measurement theory the repetition of state function reduction does not change the state but now it would give rise to the experienced flow of time. Zeno effect indeed requires that state function reductions can occur repeatedly at the same boundary. In these reductions the wave function in moduli degrees of freedom of CD changes. This implies "dispersion" in the moduli space of CDs experienced as flow of time with definite arrow. This view lead to a precise definition of self as sequence of quantum jumps to the reducing to the same boundary of CD.

3. This approach codes also the arrow of time at the space-time level: the average space-time sheet in quantum superposition increases in size as the average position of the "upper boundaries" of CDs drift towards future state function reduction by state function reduction.

4. In principle the arrow of time can temporarily change but it would seem that this can occur in very special circumstances and probably takes place in living matter routinely. Phase conjugate laser beam is a non-biological example about reversal of the arrow of time. The act of volition would correspond to the first state function reduction to the opposite boundary so that the reversal of time arrow at some level of the hierarchy of selves would take place in the act of volition.

Usually it is thought that the increase of ensemble entropy implied by second law gives rise to the arrow of observed time. In TGD framework NMP replaces second law as a fundamental principle and at the level of ensembles implies it. The negentropy assignable to entanglement increases by NMP if one accept the number of number theoretic Shannon entropy.

Could the increase of entanglement negentropy define the arrow of time? Negentropy is assignable to the fixed boundary of CD and characterizes self. The sequence of repeated state function reductions cannot therefore increase negentropy. Negentropy would increase only in the state function reduction a the opposite boundary of CD and the increased negentropy would be associated the re-incarnated self. The increase of negentropy would be forced by NMP and also the size scale of CD would increase.

This would be certainly consistent with evolution. The prediction is that a given CD corresponds to an entire family CDs coming integer multiples $n = h_{eff}/h$ of a minimal size. During state function reduction sequence to fixed boundary of CD the average size defined by average value of n and p-adic length scale involved would increase in statistical sense. One can consider also the possibility that there is sharp localization to given value of n.

The periods of repeated state function reductions would be periods of coherence (sustained mental image, subself) and decoherence would be implied by the first state function to the opposite boundary of CD forced by NMP to eventually to occur. At the level of action principle the increase of h_{eff} means gradual reduction of string tension $T \propto 1/\hbar_{eff}G$ and generation of gravitationally bound states of increasing size with binding realized in terms of strings connecting the partonic 2-surfaces. Gravitation, biology, and evolution would be very intimately related.

Chapter 4

TGD Inspired Theory of Consciousness

4.1 Introduction

The conflict between the non-determinism of state function reduction and determinism of time evolution of Schrödinger equation is serious enough a problem to motivate the attempt to extend physics to a theory of consciousness by raising the observer from an outsider to a key notion also at the level of physical theory. Further motivations come from the failure of the materialistic and reductionistic dogmas in attempts to understand consciousness in neuroscience context. There are reasons to doubt that standard quantum physics could be enough to achieve this goal and the new physics predicted by TGD is indeed central in the proposed theory.

4.1.1 Quantum Jump As Moment Of Consciousness And The Notion Of Self

If quantum jump occurs between two different time evolutions of Schrödinger equation (understood here in very metaphorical sense) rather than interfering with single deterministic Schrödinger evolution, the basic problem of quantum measurement theory finds a resolution. The interpretation of quantum jump as a moment of consciousness means that volition and conscious experience are outside space-time and state space and that quantum states and space-time surfaces are "zombies". Quantum jump would have actually a complex anatomy corresponding to unitary process U, state function reduction and state preparation at least.

Quantum jump is expected to have a complex anatomy since it must include state preparation, state function reduction, and also unitary process characterized by U-matrix. Zero energy ontology means that one must distinguish between M-matrix and U-matrix. M-matrix characterizes the time like entanglement between positive and negative energy parts of zero energy state and is measured in particle scattering experiments. M-matrix need not be unitary and can be identified as a "complex" square root of density matrix representable as a product of its real and positive square root and of unitary S-matrix so that thermodynamics becomes part of quantum theory with thermodynamical ensemble being replaced with a zero energy state. The unitary U-matrix describes quantum transitions between zero energy states and is therefore something genuinely new. It is natural to assign the statistical description of intentional action with U-matrix since quantum jump occurs between zero energy states.

Negentropy Maximization Principle (NMP) codes for the dynamics of standard state function reduction and states that the state function reduction process following U-process gives rise to maximal reduction of entanglement entropy at each step. In the generic case this implies decomposition of the system to unique unentangled systems and the process repeats itself for these systems. The process stops when the resulting subsystem cannot be decomposed to a pair of free systems since energy conservation makes the reduction of entanglement kinematically impossible in the case of bound states.

Intuitively self corresponds to a sequence of quantum jumps which somehow integrates to a larger unit much like many-particle bound state is formed from more elementary building blocks. It also seems natural to assume that self stays conscious as long as it can avoid bound state entanglement with the environment in which case the reduction of entanglement is energetically impossible. One could say that everything is conscious and consciousness can be only lost when the system forms bound state entanglement with environment. Quite generally, an infinite self hierarchy with the entire Universe at the top is predicted.

The precise definition of self has remained a long standing problem and I have been even ready to identify self with quantum jump. Zero energy ontology allows what looks like a final solution of the problem. Self indeed corresponds to a sequence of quantum jumps integrating to single unit, but these quantum jumps correspond state function reductions to a fixed boundary of CD leaving the corresponding parts of zero energy states invariant. In positive energy ontology these repeated state function reductions would have no effect on the state but in TGD framework there occurs a change for the second boundary and gives rise to the experienced flow of time and its arrow and gives rise to self. The first quantum jump to the opposite boundary corresponds to the act of free will or wake-up of self. I would be forced by NMP since the increase of ordinary entropy inside self probably also means reduction of negentropy gain in state function reduction and eventually reduction to opposite boundary of CD is unavoidable by NMP.

Negentropy Maximization Principle (NMP) states that entanglement entropy tends to be reduced in state function reduction. In standard quantum measurement this would mean that reduction reduces the enganglement between the system and its complement. There is an important exception to this vision based on ordinary Shannon entropy. There exists an infinite hierarchy of number theoretical entropies making sense for rational or even algebraic entanglement probabilities. In this case the entanglement negentropy can be negative so that NMP favors the generation of negentropic entanglement, which need not be bound state entanglement in standard sense. Negentropic entanglement might serve as a correlate for emotions like love and experience of understanding. The reduction of ordinary entanglement entropy to random final state implies second law at the level of ensemble.

The generation of negentropic entanglement means that the outcome of the reduction is not random: the prediction is that second law is not universal truth holding true in all scales. Since number theoretic entropies are natural in the intersection of real and p-adic worlds, this suggests that life resides in this intersection. Negentropic entanglement need not involved binding energy. The existence effectively bound states with no binding energy might have important implications for the understanding the stability of basic bio-polymers and the key aspects of metabolism [**?**]. Generation of negentropic entanglement gives rise to what could be called Akashic records read consciously via interaction free quantum measurement: the Universe would be increasing its information resources.

The consistency with ordinary measurement theory requires that negentropic entanglement corresponds to a density matrix proportional to a unit matrix: this correspond to entanglement matrix proportional to a unitary matrix characterizing quantum computation. The negentropic entanglement of this kind corresponds naturally to the hierarchy of Planck constants made possible by the non-determinism of Kähler action. There is also a connection with quantum criticality.

Self is assumed to experience sub-selves as mental images identifiable as "averages" of their mental images. This implies the notion of ageing of mental images as being due to the growth of ensemble entropy as the ensemble consisting of quantum jumps (sub-sub-sub-selves) increases. That sequence of sub-selves are experienced as separate mental images explains why we can distinguish between digits of phone number. The irreducible component of self (pure awareness) would correspond to the highest level in the "personal" hierarchy of quantum jumps and the sequence of lower level quantum jumps would be responsible for the experience of time flow. Entire life cycle would correspond to self at the highest(?) level of the personal self hierarchy and pure awareness would prevail during sleep: this would make it possible to experience directly that I existed yesterday.

4.1.2 Sharing And Fusion Of Mental Images

The standard dogma about consciousness is that it is completely private. It seems that this cannot be the case in TGD Universe. Von Neumann algebras known as hyper-finite factors of type II_1 (HFF) [K120, K38] provide the basic mathematical framework for quantum TGD and this suggests important modifications of the standard measurement theory besides those implied by the zero energy ontology predicting that all physical states have vanishing net quantum numbers and are creatable from vacuum. The notion of measurement resolution characterized in terms of Jones inclusions $\mathcal{N} \subset \mathcal{M}$ of HFFs implies that entanglement is defined always modulo some resolution characterized by infinite-dimensional sub-Clifford algebra $\mathcal{N}$ playing a role analogous to that of gauge algebra.

This modification has also important implications for consciousness. For ordinary quantum measurement theory separate selves are by definition unentangled and the same applies to their sub-selves so that they cannot entangle and thus fuse and shared mental images are impossible: consciousness would be completely private.

Space-time sheets as correlates for selves however suggests that space-time sheets topologically condensed at larger space-time sheets and serving as space-time correlates for mental images can be connected by join along boundaries bonds so that mental images could fuse and be shared.

HFFs allow to realize mathematically this intuitive picture. The entanglement in $\mathcal{N}$ degrees of freedom between selves corresponding to $\mathcal{M}$ is below the measurement resolution so that these selves can be regarded as separate conscious entities. These selves can be said to be unentangled although their sub-selves corresponding to $\mathcal{N}$ (mental images at upper level) can entangle. Fusion and sharing of mental images becomes possible. For instance, in stereo vision right and left visual fields would fuse together. More generally, a pool of shared stereo mental images might be fundamental for evolution of social structures and development of social and moral rules and language (shared mental images make possible common meaning for symbols of language). A concrete realization for this would be in terms of hyper-genome making possible collective gene expression [K46, K58].

4.1.3 Qualia

Since physical states are labeled by quantum numbers, various qualia correspond naturally to the increments of quantum numbers in quantum jump which leads to a general classification of qualia in terms of the fundamental symmetries [K44]. One can speak also about geometric qualia assignable to the increments of zero modes which correspond to the classical variables in ordinary quantum measurement theory and non-quantum fluctuating degrees of freedom which do not contribute to the metric of world of classical worlds (WCW) in TGD framework. Dark matter hierarchy suggests that also qualia form a hierarchy with larger values of Planck constant identifiable as more refined qualia. Rather amusingly, visual colors would correspond to increments of color quantum numbers assignable to quarks and gluons in standard model physics. The term "color", originally introduced as an algebraic joke, would directly relate to visual color.

4.1.4 Self-Referentiality Of Consciousness

Quantum classical correspondence is the basic guiding principle of quantum TGD. Thanks to the failure of a complete determinism of classical dynamics, space-time surface can provide symbolic representations not only for quantum states (as maximal deterministic regions) but also for quantum jump sequences (sequences of quantum states) and thus for the contents of consciousness. These representations are regenerated in each quantum jump, and make possible the self referentiality of consciousness: self can be conscious of what it *was* conscious of.

The "Akashic records" realized in terms of negentropic entanglement are a natural candidate for self model.

4.1.5 Hierarchy Of Planck Constants And Consciousness

The hierarchy of Planck constants is realized in terms of a generalization of the causal diamond $CD \times CP_2$, where CD is defined as an intersection of the future and past directed light-cones of 4-D Minkowski space M^4. $CD \times CP_2$ is generalized by gluing singular coverings and factor spaces of both CD and CP_2 together like pages of book along common back, which is 2-D sub-manifold which is M^2 for CD and homologically trivial geodesic sphere S^2 for CP_2 [K38]. The value of the Planck constant characterizes partially given page and arbitrary large values of $\hbar$ are predicted so that macroscopic quantum phases are possible since the fundamental quantum scales scale like

$\hbar$. All particles in the vertices of Feynman diagrams have the same value of Planck constant so that particles at different pages cannot have local interactions. Thus one can speak about relative darkness in the sense that only the interactions mediated by the exchange of particles and by classical fields are possible between different pages. Dark matter in this sense can be observed, say through the classical gravitational and electromagnetic interactions. It is in principle possible to photograph dark matter by the exchange of photons which leak to another page of book, reflect, and leak back. This leakage corresponds to $\hbar$ changing phase transition occurring at quantum criticality and living matter is expected carry out these phase transitions routinely in bio-control. This picture leads to no obvious contradictions with what is really known about dark matter and to my opinion the basic difficulty in understanding of dark matter (and living matter) is the blind belief in standard quantum theory.

Dark matter hierarchy and p-adic length scale hierarchy would provide a quantitative formulation for the self hierarchy. To a given p-adic length scale one can assign a secondary p-adic time scale as the temporal distance between the tips of the causal diamond (pair of future and past directed light-cones in $H = M^4 \times CP_2$). For electron this time scale is.1 second, the fundamental biorhythm. For a given p-adic length scale dark matter hierarchy gives rise to additional time scales coming as $\hbar/\hbar_0$ multiples of this time scale. These two hierarchies could allow to get rid of the notion of self as a primary concept by reducing it to a quantum jump at higher level of hierarchy. Self would in general consists of quantum jumps inside quantum jumps inside... and thus experience the flow of time through sub-quantum jumps.

As already mentioned, it is possible to reduce the hierarchy of Planck constant to quantum criticality made possible by the non-determinism of Kähler action.

4.1.6 Zero Energy Ontology And Consciousness

Zero energy ontology was forced by the interpretational problems created by the vacuum extremal property of Robertson-Walker cosmologies imbedded as 4-surfaces in $M^4 \times CP_2$ meaning that the density of inertial mass (but not gravitational mass) for these cosmologies was vanishing meaning a conflict with Equivalence Principle. In zero energy ontology physical states are replaced by pairs of positive and negative energy states assigned to the past *resp.* future boundaries of causal diamonds defined as pairs of future and past directed light-cones ($\delta M^4_\pm \times CP_2$). The net values of all conserved quantum numbers of zero energy states vanish. Zero energy states are interpreted as pairs of initial and final states of a physical event such as particle scattering so that only events appear in the new ontology.

Zero energy ontology combined with the notion of quantum jump resolves several problems. For instance, the troublesome questions about the initial state of universe and about the values of conserved quantum numbers of the Universe can be avoided since everything is in principle creatable from vacuum. Communication with the geometric past using negative energy signals and time-like entanglement are crucial for the TGD inspired quantum model of memory and both make sense in zero energy ontology. Zero energy ontology leads to a precise mathematical characterization of the finite resolution of both quantum measurement and sensory and cognitive representations in terms of inclusions of von Neumann algebras known as hyperfinite factors of type II_1. The space-time correlate for the finite resolution is discretization which appears also in the formulation of quantum TGD.

At the embedding space-level CD is the correlate of self whereas space-time sheets having their ends at the light-like boundaries of CD are the correlates at the level of 4-D space-time. The hierarchy of CDs within CDs corresponds to the hierarchy of selves.

ZEO forces to generalize the quantum measurement theory since state function reduction is possible at either boundary of CD. This leads to a precise definition of self and allows to understand the arrow of time and the localization of the contents of sensory consciousness to such a narrow time interval (located near the future boundary of CD). Volition corresponds to the first quantum jump to opposite boundary of CD and thus reverses the arrow of time at some level of the self hierarchy.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L9].

4.2 Negentropy Maximization Principle

Negentropy Maximization Principle (NMP [K65]) stating that the reduction of entanglement entropy is maximal at a given step of state function reduction process following U-process is the basic variational principle for TGD inspired theory of consciousness and says that the information contents of conscious experience is maximal. Although this principle is diametrically opposite to the second law of thermodynamics it is structurally similar to the second law. NMP does not dictate the dynamics completely since in state function reduction any eigen state of the density matrix is allowed as final state. NMP need not be in contradiction with second law of thermodynamics which might relate as much to the ageing of mental images as to physical reality.

4.2.1 Number Theoretic Shannon Entropy As Information

The notion of number theoretic entropy obtained by can be defined by replacing in Shannon entropy the logarithms of probabilities p_n by the logarithms of their p-adic norms $|p_n|_p$. This replacement makes sense for algebraic entanglement probabilities if appropriate algebraic extension of p-adic numbers is used. What is new that entanglement entropy can be negative, so that algebraic entanglement can carry information and NMP can force the generation of bound state entanglement so that evolution could lead to the generation of larger coherent bound states rather than only reducing entanglement. A possible interpretation for algebraic entanglement is in terms of experience of understanding or some positive emotion like love.

Standard formalism of physics lacks a genuine notion of information and one can speak only about increase of information as a local reduction entropy. It seems strange that a system gaining wisdom should increase the entropy of the environment. Hence number theoretic information measures could have highly non-trivial applications also outside the theory consciousness.

NMP combined with number theoretic entropies leads to an important exception to the rule that the generation of bound state entanglement between system and its environment during U process leads to a loss of consciousness. When entanglement probabilities are rational (or even algebraic) numbers, the entanglement entropy defined as a number theoretic variant of Shannon entropy can be non-positive (actually is) so that entanglement carries information. NMP favors the generation of algebraic entanglement. The attractive interpretation is that the generation of algebraic entanglement leads to an expansion of consciousness ("fusion into the ocean of consciousness") instead of its loss.

State function reduction period of the quantum jumps involves much more than in wave mechanics. For instance, the choice of quantization axes realized at the level of geometric delicacies related to CDs is involved. U-process generates a superposition of states in which any sub-system can have both real and algebraic entanglement with the external world. If state function reduction involves also a choice between generic and negentropic entanglement (between real world, a particular p-adic world, or their intersection) it might be possible to identify a candidate for the physical correlate for the choice between good and evil. The hedonistic complete freedom resulting as the entanglement entropy is reduced to zero on one hand, and the algebraic bound state entanglement implying correlations with the external world and meaning giving up the maximal freedom on the other hand. The hedonistic option is risky since it can lead to non-algebraic bound state entanglement implying a loss of consciousness. The second option means expansion of consciousness - a fusion to the ocean of consciousness as described by spiritual practices. Note that if the total entanglement negentropy defined as sum of contributions from various levels of CD hierarchy up to the highest matters in NMP then also sub-selves should develop negentropic entanglement. For instance, the generation of entropic entanglement at cell level can lead to a loss of consciousness also at higher levels. Life would evolve from short to long scales.

4.2.2 About NMP And Quantum Jump

NMP is assumed to be the variational principle telling what can happen in quantum jump and says that the information content of conscious experience for the entire system is maximized. In zero energy ontology (ZEO) the definition of NMP is far from trivial and the recent progress - as I believe - in the understanding of structure of quantum jump forces to check carefully the details related to NMP. A very intimate connection between quantum criticality, life as something in the intersection of realities and p-adicities, hierarchy of effective values of Planck constant, negentropic entanglement (NE), and p-adic view about cognition emerges. One ends up also with an argument why p-adic sector is necessary if one wants to speak about conscious information. I will proceed by making questions.

What happens in single state function reduction?

State function reduction is a measurement of density matrix. The condition that a measurement of density matrix takes place implies standard measurement theory on both real and p-adic sectors: system ends to an *eigen-space* of density matrix. This is true in both real and p-adic sectors. NMP is stronger principle at the real side and implies state function reduction to 1-D subspace - its eigenstate.

The resulting N-dimensional space has however rational entanglement probabilities $p = 1/N$ so that one can say that it is the intersection of realities and p-adicities. If the number theoretic variant of entanglement entropy is used as a measure for the amount of entropy carried by entanglement rather than either entangled system, the state carries genuine information and is stable with respect to NMP if the p-adic prime p divides N. NMP allows only single p-adic prime for real $\rightarrow$ p-adic transition: the power of this prime appears is the largest power of prime appearing in the prime decomposition of N. Degeneracy means also criticality so that ordinary quantum measurement theory for the density matrix favors criticality and NMP fixes the p-adic prime uniquely.

If one - contrary to the above conclusion - assumes that NMP holds true in the entire p-adic sector, NMP gives in p-adic sector rise to a *reduction* of the negentropy in state function reduction if the original situation is negentropic and the eigen-spaces of the density matrix are 1-dimensional. This situation is avoided if one assumes that state function reduction cascade in real or genuinely p-adic sector occurs first (without NMP) and gives therefore rise to N-dimensional eigen spaces. The state is negentropic and stable if the p-adic prime p divides N. Negentropy is generated.

The real state can be transformed to a p-adic one in quantum jump (defining cognitive map) if the entanglement coefficients are rational or belong to an algebraic extension of p-adic numbers in the case that algebraic extension of p-adic numbers is allowed (number theoretic evolution gradually generates them). The density matrix can be expressed as sum of projection operators multiplied by probabilities for the projection to the corresponding sub-spaces. After state function reduction cascade the probabilities are rational numbers of form $p = 1/N$.

Number theoretic entanglement entropy also allows to avoid some objections related to fermionic and bosonic statistics. Fermionic and bosonic statistics require complete anti-symmetrization/symmetriza This implies entanglement which cannot be reduced away. By looking for symmetrized or antisymmetrized 2-particle state consisting of spin 1/2 fermions as the simplest example one finds that the density matrix for either particle is the simply unit 2×2 matrix. This is stable under NMP based on number theoretic negentropy. One expects that the same result holds true in the general case. The interpretation would be that particle symmetrization/antisymmetrization carries negentropy.

The degeneracy of the density matrix is of course not a generic phenomenon and one can argue that it corresponds to some very special kind of physics. The identification of space-time correlates for the hierarchy for the effective values $\hbar_{eff} = n\hbar$ of Planck constant as n-furcations of space-time sheet suggests strongly the identification of this physics in terms of this hierarchy. Hence quantum criticality, the essence of life as something in the rational intersection of realities and p-adicities, the hierarchy of effective values of $\hbar$, negentropic quantum entanglement, and the possibility to make real-p-adic transitions and thus cognition and intentionality would be very intimately related. This is a highly satisfactory outcome, since these ideas have been rather loosely related hitherto.

What happens in quantum jump?

Suppose that everything can be reduced to what happens for a given CD characterized by a scale. There are at least two questions to be answered.

1. There are two processes involved. State function reduction and quantum jump transforming real state to p-adic state (matter to cognition) and vice versa (intention to action). Do these transitions occur independently or not? Does the ordering of the processes matter? It has

turned out that the mathematical realization of this picture is very difficult and that these transformations are not even needed in the adelic vision where cognitionandsensory aspects realized as p-adic and real space-time sheets are both present in all scales.

2. State function reduction cascade in turn consists of two different kinds of state function reductions. The M-matrix characterizing the zero energy state is product of square root of density matrix and of unitary S-matrix and the first step means the measurement of the projection operator. It defines a density matrix for both upper and lower boundary of CD and these density matrices are essentially same.
   (a) At the first step a measurement of the density matrix between positive and negative energy parts of the quantum state takes place for CD. One can regard both the lower and upper boundary as an eigenstate of density matrix in absence of NE. The measurement is thus completely symmetric with respect to the boundaries of CDs. At the real sector this leads to a 1-D eigen-space of density matrix if NMP holds true. In the intersection of real and p-adic sectors this need not be the case if the eigenvalues of the density matrix have degeneracy. Zero energy state becomes stable against further state function reductions! The interactions with the external world can of course destroy the stability sooner or later. An interesting question is whether so called higher states of consciousness relate to this kind of states.
   (b) If the first step gave rise to 1-D eigen-space of the density matrix, a state function reduction cascade at either upper of lower boundary of CD proceeding from long to short scales. At given step divides the sub-system into two systems and the sub-system-complement pair which produces maximum negentropy gain is subject to quantum measurement maximizing negentropy gain. The process stops at given subsystem resulting in the process if the resulting eigen-space is 1-D or has NE (p-adic prime p divides the dimension N of eigenspace in the intersection of reality and p-adicity).

4.2.3 Life As Islands Of Rational/Algebraic Numbers In The Seas Of Real And P-Adic Continua?

NMP and negentropic entanglement demanding entanglement probabilities which are equal to inverse of integer, is the starting point. Rational and even algebraic entanglement coefficients make sense in the intersection of real and p-adic words, which suggests that in some sense life and conscious intelligence reside in the intersection of the real and p-adic worlds.

What could be this intersection of realities and p-adicities?

1. The facts that fermionic oscillator operators are correlates for Boolean cognition and that induced spinor fields are restricted to string world sheets and partonic 2-surfaces suggests that the intersection consists of these 2-surfaces.
2. Strong form of holography allows a rather elegant adelization of TGD by a construction of space-time surfaces by algebraic continuations of these 2-surfaces defined by parameters in algebraic extension of rationals inducing that for various p-adic number fields to real or p-adic number fields. Scattering amplitudes could be defined also by a similar algebraic contination. By conformal invariance the conformal moduli characterizing the 2-surfaces would defined the parameters.

This suggests a rather concrete view about the fundamental quantum correlates of life and intelligence.

1. For the minimal option life would be effectively 2-dimensional phenomenon and essentially a boundary phenomenon as also number theoretical criticality suggests. There are good reasons to expect that only the data from the intersection of real and p-adic string world sheets partonic two-surfaces appears in U-matrix so that the data localizable to strings connecting partonic 2-surfaces would dictate the scattering amplitudes.

A good guess is that algebraic entanglement is essential for quantum computation, which therefore might correspond to a conscious process. Hence cognition could be seen as a quantum computation like process, a more appropriate term being quantum problem solving [K4]. Living-dead dichotomy could correspond to rational-irrational or to algebraic-transcendental dichotomy:

this at least when life is interpreted as intelligent life. Life would in a well defined sense correspond to islands of rationality/algebraicity in the seas of real and p-adic continua. Life as a critical phenomenon in the number theoretical sense would be one aspect of quantum criticality of TGD Universe besides the criticality of the space-time dynamics and the criticality with respect to phase transitions changing the value of Planck constant and other more familiar criticalities. How closely these criticalities relate remains an open question [K92].

The view about the crucial role of rational and algebraic numbers as far as intelligent life is considered, could have been guessed on very general grounds from the analogy with the orbits of a dynamical system. Rational numbers allow a predictable periodic decimal/pinary expansion and are analogous to one-dimensional periodic orbits. Algebraic numbers are related to rationals by a finite number of algebraic operations and are intermediate between periodic and chaotic orbits allowing an interpretation as an element in an algebraic extension of any p-adic number field. The projections of the orbit to various coordinate directions of the algebraic extension represent now periodic orbits. The decimal/pinary expansions of transcendentals are un-predictable being analogous to chaotic orbits. The special role of rational and algebraic numbers was realized already by Pythagoras, and the fact that the ratios for the frequencies of the musical scale are rationals supports the special nature of rational and algebraic numbers. The special nature of the Golden Mean, which involves $\sqrt{5}$, conforms the view that algebraic numbers rather than only rationals are essential for life.

Later progress in understanding of quantum TGD allows to refine and simplify this view dramatically. The idea about p-adic-to-real transition for space-time sheets as a correlate for the transformation of intention to action has turned out to be un-necessary and also hard to realize mathematically. In adelic vision real and p-adic numbers are aspects of existence in all length scales and mean that cognition is present at all levels rather than emerging. Intentions have interpretation in terms of state function reductions in ZEO and there is no need to identify p-adic space-time sheets as their correlates.

4.2.4 Hyper-Finite Factors Of Type Ii_1 And NMP

Hyper-finite factors of type II_1 bring in additional delicacies to NMP. The basic implication of finite measurement resolution characterized by Jones inclusion is that state function reduction can never reduce entanglement completely so that entire universe can be regarded as an infinite living organism. It would seem that entanglement coefficients become $\mathcal{N}$ valued and the same is true for eigen states of density matrix. For quantum spinors associated with $\mathcal{M}/\mathcal{N}$ entanglement probabilities must be defined as traces of the operators $\mathcal{N}$. An open question is whether entanglement probabilities defined in this manner are algebraic numbers always (as required by the notion of number theoretic entanglement entropy) or only in special cases.

4.3 Time, Memory, And Realization Of Intentional Action

Quantum classical correspondence requires that the flow of subjective time identified as a sequence of quantum jumps should have the flow of geometric time as a space-time correlate. The understanding of the detailed relationship between these two times has however remained a long standing problem, and only the emergence of zero energy ontology allows an ad hoc free model for how the flow and arrow of geometric time emerge, and answers why the relationship between geometric past and future is so asymmetric and why sensory experience is about so narrow interval of geometric time. Also the notion of self reduces in well-defined sense to the notion of quantum jump with fractal structure.

4.3.1 Two Times

The basic implication of the proposed view is that subjective time and geometric time of physicist are not the same [K61]. This is not a news actually. Geometric time is reversible, subjective time irreversible. Geometric future and past are in completely democratic position, subject future does not exist at all yet. One can say that the non-determinism of quantum jump is completely outside space-time and Hilbert space since quantum jumps replaces entire 4-D time evolution (or rather, their quantum superposition) with a new one, re-creates it. Also conscious existence defies any geometric description. This new view resolves the basic problem of quantum measurement theory due to the conflict between determinism of Schödinger equation and randomness of quantum jump. The challenge is to understand how these two times correlate so closely as to lead to their erratic identification.

With respect to geometric time the contents of conscious experience is naturally determined by the space-time region inside CD in zero energy ontology. This geometro-temporal integration should have subjecto-temporal counterpart. The experiences of self are determined by the mental images assignable to subselves (having sub-CDs as embedding space correlates) and the quantum jump sequences associated with sub-selves define a sequence of mental images. The hypothesis is that self experiences these sequences of mental images as a continuous time flow. In absence of mental images self would have experience of "timelessness" in accordance with the reports of practitioners of various spiritual practices. Self would lose consciousness in quantum jump generating entropic entangelement and experience expansion of consciousness if the resulting entanglement is negentropic. The assumption that the integration of experiences of self involves a kind of averaging over sub-selves of sub-selves guarantees that the sensory experiences are reliable despite the fact that quantum nondeterminism is involved with each quantum jump.

Thus the measurement of density matrix defined by the $MM^\dagger$, where M is the M-matrix between positive and negative energy parts of the zero energy state would correspond to the passive aspects of consciousness such as sensory experiencing. U would represent at the fundamental level volition as a creation of a quantum superposition of possibilities. What follows it would be a selection between them. The volitional choice between macroscopically differing space-time sheets representing different maxima of Kähler function could be basically responsible for the active aspect of consciousness. The fundamental perception-reaction feedback loop of biosystems would result from the combination of the active and passive aspects of consciousness represented by U and M.

The fact that the contents of conscious experience is about 4-D rather than 3-D space-time region, motivates the notions of 4-D brain, body, and even society. In particular, conscious existence continues after biological death since 4-D body and brain continue to exist.

4.3.2 About The Arrow Of Psychological Time

Quantum classical correspondence predicts that the arrow of subjective time is somehow mapped to that for the geometric time. The detailed mechanism for how the arrow of psychological time emerges has however remained open. Also the notion of self is problematic.

Two earlier views about how the arrow of psychological time emerges

The basic question how the arrow of subjective time is mapped to that of geometric time. The common assumption of all models is that quantum jump sequence corresponds to evolution and that by quantum classical correspondence this evolution must have a correlate at space-time level so that each quantum jump replaces typical space-time surface with a more evolved one.

1. The earliest model assumes that the space-time sheet assignable to observer ("self") drifts along a larger space-time sheet towards geometric future quantum jump by quantum jump: this is like driving car in a landscape but in the direction of geometric time and seeing the changing landscape. There are several objections.

   i) Why this drifting?

   ii) If one has a large number of space-time sheets (the number is actually infinite) as one has in the hierarchy the drifting velocity of the smallest space-time sheet with respect to the largest one can be arbitrarily large (infinite).

   iii) It is alarming that the evolution of the background space-time sheet by quantum jumps, which must be the quintessence of quantum classical correspondence, is not needed at all in the model.

2. Second model relies on the idea that intentional action -understood as p-adic-to-real phase transition for space-time sheets and generating zero energy states and corresponding real space-time sheets - proceeds as a kind of wave front towards geometric future quantum jump by quantum jump. Also sensory input would be concentrated on this kind of wave front. The

difficult problem is to understand why the contents of sensory input and intentional action are localized so strongly to this wave front and rather than coming from entire life cycle.

There are also other models but these two are the ones which represent basic types for them.

The third option

The third explanation for the arrow of psychological time - which I have considered earlier but only half-seriously - looks to me the most elegant at this moment. This option is actually favored by Occam's razor since it uses only the assumption that space-time sheets are replaced by more evolved ones in each quantum jump. Also the model of topological quantum computation favors it. A more detailed discussion of this option can be found in [K7]. Here only a rough summary of the basic ideas is given.

1. In standard picture the attention would gradually shift towards geometric future and space-time in 4-D sense would remain fixed. Now however the fact that quantum state is quantum superposition of space-time surfaces allows to assume that the attention of the conscious observer is directed to a fixed volume of 8-D embedding space. Quantum classical correspondence is achieved if the evolution in a reasonable approximation means shifting of the space-time sheets and corresponding field patterns backwards backwards in geometric time by some amount per quantum jump so that the perceiver finds the geometric future in 4-D sense to enter to the perceptive field. This makes sense since the shift with respect to M^4 time coordinate is an exact symmetry of extremals of Kähler action. It is also an excellent approximate symmetry for the preferred extremals of Kähler action and thus for maxima of Kähler function spoiled only by the presence of light-cone boundaries. This shift occurs for both the space-time sheet that perceiver identifies itself and perceived space-time sheet representing external world: both perceiver and percept change.
2. Both the landscape and observer space-time sheet remain in the same position in embedding space but both are modified by this shift in each quantum jump. The perceiver experiences this as a motion in 4-D landscape. Perceiver (Mohammed) would not drift to the geometric future (the mountain) but geometric future (the mountain) would effectively come to the perceiver (Mohammed)!
3. There is an obvious analogy with Turing machine: what is however new is that the tape effectively comes from the geometric future and Turing machine can modify the entire incoming tape by intentional action. This analogy might be more than accidental and could provide a model for quantum Turing machine operating in TGD Universe. This Turing machine would be able to change its own program as a whole by using the outcomes of the computation already performed.
4. The concentration of the sensory input and the effects of conscious motor action to a narrow interval of time (.1 seconds typically, secondary p-adic time scale associated with the largest Mersenne M_{127} defining p-adic length scale which is not completely super-astronomical) can be understood as a concentration of sensory/motor attention to an interval with this duration: the space-time sheet representing sensory "me" would have this temporal length and "me" definitely corresponds to a zero energy state.
5. The fractal view about topological quantum computation strongly suggests an ensemble of almost copies of sensory "me" scattered along my entire life cycle and each of them experiencing my life as a separate almost copy.
6. The model of geometric and subjective memories would not be modified in an essential manner: memories would result when "me" is connected with my almost copy in the geometric past by braid strands or massless extremals (MEs) or their combinations (ME parallel to magnetic flux tube is the analog of Alfwen wave in TGD).

This argument leaves many questions open. What is the precise definition for the volume of attention? Is the attention of self doomed to be directed to a fixed volume or can quantum jumps change the volume of attention? What distinguishes between geometric future and past as far as contents of conscious experience are considered? How this picture relates to p-adic and dark matter hierarchies? Does this framework allow to formulate more precisely the notion of self? Zero energy ontology allows to give tentative answers to these questions.

4.3.3 Questions Related To The Notion Of Self

I have proposed two alternative notions of self and have not been able to choose between them. A further question is what happens during sleep: do we lose consciousness or is it that we cannot remember anything about this period? The work with the model of topological quantum computation has led to an overall view allowing to select the most plausible answer to these questions. But let us be cautious!

Can one choose between the two variants for the notion of self or are they equivalent?

I have considered two different notions of "self" and it is interesting to see whether the new view about time might allow to choose between them or to show that they are actually equivalent.

1. In the original variant of the theory "self" corresponds to a sequence of quantum jumps. "Self" would result through a binding of quantum jumps to single "string" in close analogy and actually in a concrete correspondence with the formation of bound states. Each quantum jump has a fractal structure: unitary process is followed by a sequence of state function reductions and preparations proceeding from long to short scales. Selves can have sub-selves and one has self hierarchy. The questionable assumption is that self remains conscious only as long as it is able to avoid entanglement with environment.

   Even slightest entanglement would destroy self unless on introduces the notion of finite measurement resolution applying also to entanglement. This notion is indeed central for entire quantum TGD also leads to the notion of sharing of mental images: selves unentangled in the given measurement resolution can experience shared mental images resulting as fusion of sub-selves by entanglement not visible in the resolution used.

2. According to the newer variant of theory, quantum jump has a fractal structure so that there are quantum jumps within quantum jumps: this hierarchy of quantum jumps within quantum jumps would correspond to the hierarchy of dark matters labeled by the values of Planck constant. Each fractal structure of this kind would have highest level (largest Planck constant) and this level would corresponds to the self. What might be called irreducible self would corresponds to a quantum jump without any sub-quantum jumps (no mental images). The quantum jump sequence for lower levels of dark matter hierarchy would create the experience of flow of subjective time.

It would be nice to reduce the original notion of self hierarchy to the hierarchy defined by quantum jumps. There are some objections against this idea. One can argue that fractality is a purely geometric notion and since subjective experience does not reduce to the geometry it might be that the notion of fractal quantum jump does not make sense. It is also not quite clear whether the reasonable looking idea about the role of entanglement as destroyer of self can be kept in the fractal picture.

These objections fail if one can construct a well-defined mathematical scheme allowing to understand what fractality of quantum jump at the level of space-time correlates means and showing that the two views about self are equivalent. The following argument represents such a proposal. Let us start from the causal diamond model as a lowest approximation for a model of zero energy states and for the space-time region defining the contents of sensory experience.

Let us make the following assumptions.

1. Assume the hierarchy of causal diamonds within causal diamonds in a sense to be specified more precisely below. Causal diamonds would represent the volumes of attention. Assume that the highest level in this hierarchy defines the quantum jump containing sequences of lower level quantum jumps in some sense to be specified. Assume that these quantum jumps integrate to single continuous stream of consciousness as long as the sub...-sub-self in question remains unentangled and that entangling means loss of consciousness or at least that it is not possible to remember anything about contents of consciousness during entangled state.

2. Assume that the contents of conscious experience come from the interior of the causal diamond. A stronger condition would be that the contents come from the boundaries of the two light-cones involved since physical states are defined at these in the simplest picture. In this case one could identify the lower light-cone boundary as giving rise to memory.

3. The time span characterizing the contents of conscious experience associated with a given quantum jump would correspond to the temporal distance T between the tips of the causal diamond. T would also characterize the average and approximate shift of the superposition of space-time surfaces backwards in geometric time in single quantum jump at a given level of hierarchy. This time scale naturally scales as $T_n = 2^n T_{CP_2}$ so that p-adic length scale hypothesis follows as a consequence. T would be essentially the secondary p-adic time scale $T_{2,p} = \sqrt{p}T_p$ for $p \simeq 2^k$. This assumption - absolutely essential for the hierarchy of quantum jumps within quantum jumps - would differentiate the model from the model in which T corresponds to either CP_2 time scale or p-adic time scale T_p. One would have hierarchy of quantum jumps with increasingly longer time span for memory and with increasing duration of geometric chronon at the highest level of fractal quantum jump. Without additional restrictions, the quantum jump at n^{th} level would contain 2^n quantum jumps at the lowest level of hierarchy. Note that in the case of sub-self - and without further assumptions which will be discussed next - one would have just two quantum jumps: mental image appears, disappears or exists all the time. At the level of sub-sub-selves 4 quantum jumps and so on. Maybe this kind of simple predictions might be testable.
4. We know that the contents of sensory experience comes from a rather narrow time interval of duration about.1 seconds, which corresponds to the time scale T_{127} associated with electron. We also know that there is asymmetry between positive and negative energy parts of zero energy states both physically and at the level of conscious experience. This asymmetry must have some space-time correlate. The simplest correlate for the asymmetry between positive and negative energy states would be that the upper light-like boundaries in the structure formed by light-cones within light-cones intersect along light-like radial geodesic. No condition of this kind would be posed on lower light-cone boundaries. The scaling invariance of this condition makes it attractive mathematically and would mean that arbitrarily long time scales T_n can be present in the fractal hierarchy of light cones. At all levels of the hierarchy all contribution from upper boundary of the causal diamond to the conscious experience would come from boundary of the same past directed light-cone so that the conscious experience would be sharply localized in time in the manner as we know it to be. The new element would be that content of conscious experience would come from arbitrarily large region of Universe and seing Milky Way would mean direct sensory contact with it.
5. These assumptions relate the hierarchy of quantum jumps to p-adic hierarchy. One can also include also dark matter hierarchy into the picture. For dark matter hierarchy the time scale hierarchy $\{T_n\}$ is scaled by the factor $r = \hbar/\hbar_0$ which can be also rational number. For $r = 2^k$ the hierarchy of causal diamonds generalizes without difficulty and there is a kind of resonance involved which might relate to the fact that the model of EEG favors the values of $k = 11n$, where $k = 11$ also corresponds in good approximation to proton-electron mass ratio. For more general values of $\hbar/\hbar_0$ the generalization is possible assuming that the position of the upper tip of causal diamond is chosen in such a way that their positions are always the same whereas the position of the lower light-cone boundary would correspond to $\{rT_n\}$ for given value of Planck constant. Geometrically this picture generalizes the original idea about fractal hierarchy of quantum jumps so that it contains both p-adic hierarchy and hierarchy of Planck constants.

The contributions from lower the boundaries identifiable in terms of memories would correspond to different time scales and for a given value of time scale T the net contribution to conscious experience would be much weaker than the sensory input in general. The asymmetry between geometric now and geometric past would be present for all contributions to conscious experience, not only sensory ones. What is nice that the contents of conscious experience would rather literally come from the boundary of the past directed light-cone along which the classical signals arrive. Hence the mystic feeling about telepathic connection with a distant object at distance of billions of light years expressed by an astrophysicist, whose name I have unfortunately forgotten, would not be romantic self deception.

This framework explains also the sharp distinction between geometric future and past (not surprisingly since energy and time are dual): this distinction has also been a long standing problem of TGD inspired theory of consciousness. Precognition is not possible unless one assumes that communications and sharing of mental images between selves inside disjoint causal diamonds is possible. Physically there seems to be no good reason to exclude the interaction between zero energy states associated with disjoint causal diamonds.

The mathematical formulation of this intuition is however a non-trivial challenge and can be used to articulate more precisely the views about what WCW and configurations space spinor fields actually are mathematically.

1. Suppose that the causal diamonds with tips at different points of $H = M^4 \times CP_2$ and characterized by distance between tips T define sectors CH_i of the full WCW CH ("world of classical worlds"). Precognition would represent an interaction between zero energy states associated with different sectors CH_i in this scheme and tensor factor description is required.

2. Inside given sector CH_i it is not possible to speak about second quantization since every quantum state correspond to a single mode of a classical spinor field defined in that sector.

3. The question is thus whether the Clifford algebras and zero energy states associated with different sectors CH_i combine to form a tensor product so that these zero energy states can interact. Tensor product is required by the vision about zero energy insertions assignable to CH_i which correspond to causal diamonds inside causal diamonds. Also the assumption that zero energy states form an ensemble in 4-D sense - crucial for the deduction of scattering rates from M-matrix - requires tensor product.

4. The argument unifying the two definitions of self requires that the tensor product is restricted when CH_i correspond to causal diamonds inside each other. The tensor factors in shorter time scales are restricted to the causal diamonds hanging from a light-like radial ray at the upper end of the common past directed light-cone. If the causal diamonds are disjoint there is no obvious restriction to be posed, and this would mean the possibility of also precognition and sharing of mental images.

This scenario allows also to answers the questions related to a more precise definition of volume of attention. Causal diamond - or rather - the associated light-like boundaries containing positive and negative energy states define the primitive volume of attention. The obvious question whether the attention of a given self is doomed to be fixed to a fixed volume can be also answered. This is not the case. Selves can delocalize in the sense that there is a wave function associated with the position of the causal diamond and quantum jumps changing this position are possible. Also many-particle states assignable to a union of several causal diamonds are possible. Note that the identification of magnetic flux tubes as space-time correlates of directed attention in TGD inspired quantum biology makes sense if these flux tubes connect different causal diamonds. The directedness of attention in this sense should be also understood: it could be induced from the ordering of p-adic primes and Planck constant: directed attention would be always from longer to shorter scale.

What after biological death?

Could the new option allow to speculate about the course of events at the moment of death? Certainly this particular sensory "me" would effectively meet the geometro-temporal boundary of the biological body: sensory input would cease and there would be no biological body to use anymore. "Me" might lose its consciousness (if it can!). "Me" has also other mental images than sensory ones and these could begin to dominate the consciousness and "me" could direct its attention to space-time sheets corresponding to much longer time scale, perhaps even to that of life cycle, giving a summary about the life.

What after that? The Tibetan Book of Dead gives some inspiration. A western "me" might hope (and even try use its intentional powers to guarantee) that quantum Turing tape sooner later brings into the volume of attention (which might also change) a living organism, be it human or cat or dog or at least some little bug. If this "me" is lucky, it could direct its attention to it and become one of the very many sensory "me's" populating this particular 4-D biological body. There would be room for a newcomer unlike in the alternative models. A "me" with Eastern/New-Ageish traits could however direct its attention permanently to the dark space-time sheets and achieve what she might call enlightenment.

Does sleep state involve a loss of consciousness?

The ability to avoid entropic entanglement with environment is essential for the original notion of self and in the case of sub-selves it would explain the finite life-time of mental images. Algebraic entanglement can be however negentropic and the idea that its generation does not lead to a loss of consciousness is attractive. If sleep really means a loss of consciousness it must lead to a generation of entropic entanglement. But does this really happen? Could sleep only lead to a loss of consciousness at those levels of self hiererachy responsible for conscious memories, which correspond to mental images and thus sub-CDs located in those space-time regions of CD, where the sleeping occurs?

Is the assumption about the loss of consciousness during sleep really necessary? Can one imagine good reasons for why we should remain conscious during sleep?

1. One could argue that if consciousness is really lost during sleep, we could not have the deep conviction that we existed yesterday.
2. Second argument is based on the assumption that brains are acting as topological quantum computers during sleep. During an ideal topological quantum computation the entanglement with the surrounding world is absent and thus topological quantum computation should correspond to a conscious experience with a vanishing entanglement entropy. Night time is the best time for topological quantum computation since sensory input and motor action do not take metabolic resources and we certainly do problem solving during sleep. Thus we should be conscious at some level during sleep and perform quite a long topological quantum computation. The problem with this argument is that the ideal topological quantum computation could be performed by a larger system than brain so that ability to perform topological quantum computation does not allow to conclude whether we are conscious during sleep or not. In fact, the idea that large number of brains entangle to a larger unit giving rise to a stereo consciousness about what it is to be human besides performing topological quantum computation like processes, is rather attractive.

Could it then be that we do not remember anything about the period of sleep because our attention is directed elsewhere and memory recall uses only copies of "me" assignable to brain manufacturing standardized mental images? Perhaps the communication link to the mental images during sleep experienced at dark matter levels of existence is lacking or sensory input and motor activities of busy westeners do not allow to use metabolic energy to build up this kind of communications. Hence one can at least half-seriously ask, whether self is actually eternal with respect to the subjective time and whether entangling with some system means only diving into the ocean of consciousness as someone has expressed it. Could we be Gods as also quantum classical correspondence in the reverse direction suggests (p-adic cognitive space-time sheets have literally infinite size in both temporal and spatial directions)?

4.3.4 Do Declarative Memories And Intentional Action Involve Communications With Geometric Past?

Communications with geometric past using time mirror mechanism (see **Fig.** `http://tgdtheory.fi/appfigures/timemirror.jpg` or **Fig. ??** in the appendix of this book) in which phase conjugate photons propagating to the geometric past are reflected back as ordinary photons (typically dark photons with energies above thermal threshold) make possible realization of declarative memories in the brain of the geometric past [K90].

This mechanism makes also possible realization of intentional actions as a process proceeding from longer to shorter time scales and inducing the desired action already in geometric past. This kind of realization would make living systems extremely flexible and able to react instantaneously to the changes in the environment. This model explains Libet's puzzling finding that neural activity seems to precede volition [J9].

Also a mechanism of remote metabolism ("quantum credit card") based on sending of negative energy signals to geometric past becomes possible [K56]: this signal could also serve as a mere control signal inducing much larger positive energy flow from the geometric past. For instance, population inverted system in the geometric past could allow this kind of mechanism. Remote metabolism could also have technological implications.

4.3.5 Episodal Memories As Time-Like Entanglement

Time-like entanglement explains episodal memories as sharing of mental images with the brain of geometric past [K90]. An essential element is the notion of magnetic body which serves as an intentional agent "looking" the brain of geometric past by allowing phase conjugate dark photons with negative energies to reflect from it as ordinary photons. The findings of Libet about time delays related to the passive aspects of consciousness [J5] support the view that the part of the magnetic body corresponding to EEG time scale has the same size scale as Earth's magnetosphere. The unavoidable conclusion would be that our field/magnetic bodies contain layers with astrophysical sizes.

p-Adic length scale hierarchy and number theoretically preferred hierarchy of values of Planck constants, when combined with the condition that the frequencies f of photons involved with the communications in time scale T satisfy the condition $f \sim 1/T$ and have energies above thermal energy, lead to rather stringent predictions for the time scales of long term memory. The model for the hierarchy of EEGs relies on the assumption that these time scales come as powers $n = 2^{11k}$, $k = 0, 1, 2,,$ and predicts that the time scale corresponding to the duration of human life cycle is ~ 50 years and corresponds to $k = 7$ (amusingly, this corresponds to the highest level in chakra hierarchy).

4.4 Cognition And Intentionality

4.4.1 Fermions And Boolean Cognition

Fermionic Fock state basis defines naturally a quantum version of Boolean algebra. In zero energy ontology predicting that physical states have vanishing net quantum numbers, positive and negative energy components of zero energy states with opposite fermion numbers define realizations of Boolean functions via time-like quantum entanglement. One can also consider an interpretation of zero energy states in terms of rules of form $A \rightarrow B$ with the instances of A and B represented as elements Fock state basis fixed by the diagonalization of the density matrix defined by $M-$-matrix. Hence Boolean consciousness would be basic aspect of zero energy states. Physical states would be more like memes than matter. Note also that the fundamental super-symmetric duality between bosonic degrees of freedom (size and shape of the 3-surface) and fermionic degrees of freedom would correspond to the sensory-cognitive duality.

This would explain why Boolean and temporal causalities are so closely related. Note that zero energy ontology is certainly consistent with the usual positive energy ontology if unitary process U associated with the quantum jump is more or less trivial in the degrees of freedom usually assigned with the material world. There are arguments suggesting that U is tensor product of of factoring S-matrices associated with 2-D integrable QFT theories [K26]: these are indeed almost trivial in momentum degrees of freedom. This would also imply that our geometric past is rather stable so that quantum jump of geometric past does not suddenly change your profession from that of musician to that of physicist.

4.4.2 Fuzzy Logic, Quantum Groups, And Jones Inclusions

Matrix logic [A33] emerges naturally when one calculates expectation values of logical functions defined by the zero energy states with positive energy fermionic Fock states interpreted as inputs and corresponding negative energy states interpreted as outputs. Also the non-commutative version of the quantum logic, with spinor components representing amplitudes for truth values replaced with non-commutative operators, emerges naturally. The finite resolution of quantum measurement generalizes to a finite resolution of Boolean cognition and allows description in terms of Jones inclusions $\mathcal{N} \subset \mathcal{M}$ of infinite-dimensional Clifford algebras of the world of classical worlds (WCW) identifiable in terms of fermionic oscillator algebras. $\mathcal{N}$ defines the resolution in the sense that quantum measurement and conscious experience does not distinguish between states differing from each other by the action of $\mathcal{N}$.

The finite-dimensional quantum Clifford algebra $\mathcal{M}/\mathcal{N}$ creates the physical states modulo the resolution. This algebra is non-commutative which means that corresponding quantum spinors have non-commutative components. The non-commutativity codes for the that the spinor components are correlated: the quantized fractal dimension for quantum counterparts of 2-spinors satisfying $d = 2cos(\pi/n) \leq 2$ expresses this correlation as a reduction of effective dimension.

The moduli of spinor components however commute and have interpretation as eigenvalues of truth and false operators or probabilities that the statement is true/false. They have quantized spectrum having also interpretation as probabilities for truth values and this spectrum differs from the spectrum $\{1, 0\}$ for the ordinary logic so that fuzzy logic results from the finite resolution of Boolean cognition [K120].

4.4.3 P-Adic Physics As Physics Of Cognition

p-Adic physics as physics of cognition provides a further element of TGD inspired theory of consciousness. At the fundamental level light-like 3-surfaces are basic dynamical objects in TGD Universe and have interpretation as orbits of partonic 2-surfaces. The generalization of the notion of number concept by fusing real numbers and various p-adic numbers to a more general structure makes possible to assign to real parton a p-adic prime p and corresponding p-adic partonic 3-surface obeying same algebraic equations. The almost topological QFT property of quantum TGD is an essential prerequisite for this. The intersection of real and p-adic 3-surfaces would consists of a discrete set of points with coordinates which are algebraic numbers. p-Adic partons would relate to both intentionality and cognition.

Real fermion and its p-adic counterpart forming a pair would represent matter and its cognitive representation being analogous to a fermion-hole pair resulting when fermion is kicked out from Dirac sea. The larger the number of points in the intersection of real and p-adic surfaces, the better the resolution of the cognitive representation would be. This would explain why cognitive representations in the real world are always discrete (discreteness of numerical calculations represent the basic example about this fundamental limitation).

All transcendental p-adic integers are infinite as real numbers and one can say that most points of p-adic space-time sheets are at spatial and temporal infinity in the real sense so that intentionality and cognition would be literally cosmic phenomena. If the intersection of real and p-adic space-time sheet contains large number of points, the continuity and smoothness of p-adic physics should directly reflect itself as long range correlations of real physics realized as p-adic fractality. It would be possible to measure the correlates of cognition and intention and in the framework of zero energy ontology [K26] the success of p-adic mass calculations can be seen as a direct evidence for the role of intentionality and cognition even at elementary particle level: all matter would be basically created by intentional action as zero energy states.

4.4.4 Algebraic Brahman=Atman Identity

The proposed view about cognition emerges from the notion of infinite primes [K103], which was actually the first genuinely new mathematical idea inspired by TGD inspired consciousness theorizing. Infinite primes, integers, and rationals have a precise number theoretic anatomy. For instance, the simplest infinite primes correspond to the numbers $P_{\pm} = X \pm 1$, where $X = \prod_k p_k$ is the product of all finite primes. Indeed, $P_{\pm}\ mod\ p = 1$ holds true for all finite primes. The construction of infinite primes at the first level of the hierarchy is structurally analogous to the quantization of super-symmetric arithmetic quantum field theory with finite primes playing the role of momenta associated with fermions and bosons. Also the counterparts of bound states emerge. This process can be iterated: at the second level the product of infinite primes constructed at the first level replaces X and so on.

The structural similarity with repeatedly second quantized quantum field theory strongly suggests that physics might in some sense reduce to a number theory for infinite rationals M/N and that second quantization could be followed by further quantizations. As a matter fact, the hierarchy of space-time sheets could realize this endless second quantization geometrically and have also a direct connection with the hierarchy of logics labeled by their order. This could have rather breathtaking implications.

1. One is forced to ask whether this hierarchy corresponds to a hierarchy of realities for which level below corresponds in a literal sense infinitesimals and the level next above to infinity.

2. Second implication is that there is an infinite number of infinite rationals behaving like real units ($M/N \equiv 1$ in real sense) so that space-time points could have infinitely rich number theoretical anatomy not detectable at the level of real physics. Infinite integers would correspond to positive energy many particle states and their inverses (infinitesimals with number theoretic structure) to negative energy many particle states and $M/N \equiv 1$ would be a counterpart for zero energy ontology to which oneness and emptiness are assigned in mysticism.

3. Single space-time point, which is usually regarded as the most primitive and completely irreducible structure of mathematics, would take the role of Platonia of mathematical ideas being able to represent in its number theoretical structure even the quantum state of entire Universe. Algebraic Brahman=Atman identity and algebraic holography would be realized in a rather literal sense.

This number theoretical anatomy should relate to mathematical consciousness in some manner. For instance, one can ask whether it makes sense to speak about quantum jumps changing the number theoretical anatomy of space-time points and whether these quantum jumps give rise to mathematical ideas. In fact, the identifications of Platonia as spinor fields in WCW on one hand and as the set number theoretical anatomies of point of embedding space force the conclusion that WCW spinor fields (recall also the identification as correlates for logical mind) can be realized in terms of the space for number theoretic anatomies of embedding space points. Therefore quantum jumps would be correspond to changes in anatomy of the space-time points. Embedding space would be experiencing genuine number theoretical evolution. The whole physics would reduce to the anatomy of numbers. All mathematical notions which are more than mere human inventions would be imbeddable to the Platonia realized as the number theoretical anatomies of single embedding space point.

In [K27, K103] a concrete realization of this vision is discussed by assuming hyper-octonionic infinite primes as a starting point. The simplest realization of infinite octonionic/quaternionic primes as products of infinite primes and octonions avoids the problems related to non-associativity and commutativity. Quantum states are required to be associative in the sense that they correspond to quantum super-positions of all possible associations for the products of finite primes (say $|A(BC)\rangle + |(AB)C\rangle$). The ground states of super conformal representations would correspond to infinite primes mappable to space-time surfaces (quantum classical correspondence). The excited states of super-conformal representations would be represented as quantum entangled states in the tensor product of state spaces $\mathcal{H}_{h_k}$ formed from Schrödinger amplitudes in discrete subsets of the space of 8 real units associated with embedding space 8 coordinates at point h_k: the interpretation is in terms of a 8-fold tensor power of basic super-conformal representation. Although the representations are not completely local at the level of embedding space, they involve only a discrete set of points identifiable as arguments of n-point function. The basic symmetries of the standard model reduce to number theory if hyper-octonionic infinite rationals are allowed. Color confinement reduces to rationality of infinite integers representing many particle states.

4.5 Quantum Information Processing In Living Matter

The notion of magnetic body leads to a dramatic modification of the views about functions of brain. In the following the discussion the the new vision about life as number theoretically critical phenomenon is not discussed separately.

4.5.1 Magnetic Body As Intentional Agent And Experiencer

In TGD Universe brain would be basically a builder of symbolic representations assigning a meaning to the sensory input by decomposing sensory field to objects and making possible effective motor control by magnetic body containing dark matter. A concrete model for how magnetic controls biological body and receives information from it is discussed in the model for the hierarchy of EEGs [K35].

Also magnetic body could have sensory qualia, which should be in a well-defined sense more refined than ordinary sensory qualia [K44]. The quantum number increments associated with cyclotron phase transitions of dark ion cyclotron condensates at magnetic body could correspond to emotional and cognitive content of sensory input and would indeed have interpretation as higher level sensory qualia. Right brain sings – left brain talks metaphor would characterize this emotional-cognitive distinction for higher level qualia and would correspond to coding of sensory input from brain by frequency patterns *resp.* temporal patterns (analogs of phonemes). These qualia would be somatosensory qualia at the level of magnetic body.

Remote mental interactions between magnetic body and biological body are a key element of this picture. Remote mental interactions in the usual sense of the world would occur between magnetic body and some other, not necessary biological, body. This would include receival of sensory input from and motor control of other than own body. Also "dead" matter possesses magnetic bodies so that also psychokinesis would be based on the same mechanism. Magnetic body for which dissipation is much smaller than for ordinary matter (proportional to $1/\hbar$, would presumably continue its conscious existence after biological death and find another biological body and use it as a tool of sensory perception and intentional action.

4.5.2 Summary About The Possible Role Of The Magnetic Body In Living Matter

The notion of magnetic/field body is probably the feature of TGD inspired theory of quantum biology which creates strongest irritation in standard model physicist. A ridicule as some kind of Mesmerism might be the probable reaction. The notion of magnetic/field body has however gradually gained more and more support and it is now an essential element of TGD based view about living matter. In the following I list the basic applications in the hope that the overall coherency of the picture might force some readers to take this notion seriously. I will talk only about magnetic body although it is clear that field body has also electric parts as well as radiative parts realized in terms of "massless extremals" or topological light rays.

In the following discussion the possible implications of the idea that living matter resides in the intersection of real and p-adic worlds is not taken into account. An attractive working hypothesis is that negentropic entanglement can be assigned to the magnetic bodies. For instance, the ends of the magnetic flux tubes connecting (say) biomolecules could be entangled negentropically. This idea has been already applied to explain the stability of high energy phosphate bond and of DNA polymers, which are highly charged [**?**].

Anatomy of magnetic body

Consider first the anatomy of the magnetic body.

1. Magnetic body has a fractal onion like structure with decreasing magnetic field strengths and the highest layers can have astrophysical sizes. Cyclotron wave length gives an estimate for the size of particular layer of magnetic body. $B = .2$ Gauss is the field strength associated with a particular layer of the magnetic body assignable to vertebrates and EEG. This value is not the same as the nominal value of the Earth's magnetic field equal to.5 Gauss. It is quite possible that the flux quanta of the magnetic body correspond to those of wormhole magnetic field and thus consist of two parallel flux quanta which have opposite time orientation. This is true for flux tubes assigned to DNA in the model of DNA as a topological quantum computer.

2. The layers of the magnetic body are characterized by the values of Planck constant and the matter at the flux quanta can be interpreted as macroscopically quantum coherent dark matter. This picture makes sense only if one accepts the generalization of the notion of embedding space.

3. In the case of wormhole magnetic fields it is natural to assign a definite temporal duration to the flux quanta and the time scales defined by EEG frequencies are natural. In particular, the inherent time scale.1 seconds assignable to electron as a duration of zero energy space-time sheet having positive and negative energy electron at its ends would correspond to 10 Hz cyclotron frequency for ordinary value of Planck constant. For larger values of Planck constants the time scale scales as $\hbar$. Quite generally, a connection between p-adic time scales of EEG and those of electron and lightest quarks is highly suggestive since light quarks play key role in the model of DNA as topological quantum computer.

4. TGD predicts also hierarchy of scaled variants of electro-weak and color physics so that ZXG, QXG, and GXG corresponding to Z^0 boson, W boson, and gluons appearing effectively as massless particles below some biologically relevant length scale suggest themselves. In this phase quarks and gluons are unconfined and electroweak symmetries are unbroken so that gluons, weak bosons, quarks and even neutrinos might be relevant to the understanding of living matter. In particular, long ranged entanglement in charge and color degrees of freedom becomes possible. For instance, TGD based model of atomic nucleus as nuclear string suggests that biologically important fermionic could be actually chemically equivalent bosons and form cyclotron Bose-Einstein condensates.

Functions of the magnetic body

The list of possible functions of the magnetic body is already now rather impressive.

1. Magnetic body controls biological body and receives sensory data from it. Together with zero energy ontology and new view about time explains Libet's strange findings about time lapses of consciousness. EEG, or actually fractal hierarchy of EXGs assignable to various body parts makes possible communications to and control by the various layers of the magnetic body. WXG could induce charge density gradients by the exchange of W boson.

2. The flux sheets of the magnetic body traverse through DNA strands. The hierarchy of Planck constants and quantization of magnetic flux predicts that the flux sheets can have arbitrarily large width. This leads to the idea that there is hierarchy of genomes corresponding to ordinary genome, supergenome consisting of genomes of several cell nuclei arranged along flux sheet like lines of text, and hypergenomes involving genomes of several organisms arranged in a similar manner. The prediction is coherent gene expression at the level of organ, and even of population. In this picture the big jumps in evolution, in particular, the emergence of EEG, could be seen as the emergence of a new larger layer of magnetic body characterized by a larger value of Planck constant. For instance, this would allow to understand why the quantal effects of ELF em fields requiring so large value of Planck constant that cyclotron energies are above thermal energy at body temperature are observed for vertebrates only.

3. Magnetic body makes possible information process in a way highly analogous to topological quantum computation. The model of DNA as topological quantum computer assumes that flux tubes of wormhole magnetic field connect DNA nucleotides with the lipids of the lipid layer of nuclear or cell membrane. The flux tubes would continue through the membrane and split during topological quantum computation. The time-like braiding of flux tubes makes possible topological quantum computation via time-like braiding and space-like braiding makes possible the representation of memories. The model allows general vision about the deeper meaning of the structure of cell and makes testable predictions about DNA.

   One prediction is the coloring of braid strands realized by an association of quark or antiquark to nucleotide. Color and spin of quarks and antiquarks would thus correspond to the quantum numbers assignable to braid ends. Color isospin could replace ordinary spin as a representation of qubit and quarks would naturally give rise to qutrit, with third quark would have interpretation as unspecified truth value. Fractionization of these quantum numbers takes place which increases the number of degrees of freedom. This prediction would relate closely to the discovery of topologist Barbara Shipman that the model for the honeybee dance suggests that quarks are in some manner involved with cognition. Also microtubules associated with axons connected to a space-time sheet outside axonal membrane via lipids could be involved with topological quantum computation and actually define an analog of a higher level programming language.

4. The strange findings about the behavior of cell membrane, in particular the finding that metabolic deprivation does not lead to the death of cell, the discovery that ionic currents through the cell membrane are quantal, and that these currents are essentially similar than those through an artificial membrane, suggest that the ionic currents are dark ionic Josephson currents along magnetic flux tubes. A high percent of biological ions would be dark and ionic channels and pumps would be responsible only for the control of the flow of ordinary ions through cell membrane.

5. These findings together with the discovery that also nerve pulse seems to involve only low dissipation lead to a model of nerve pulse in which dark ionic currents automatically return back as Josephson currents without any need for pumping. This does not exclude the possibility that ionic channels might be involved with the generation of nerve pulse so that the original view about quantal currents as controllers of the generation of nerve pulse would be turned upside down. Nerve pulse would result as a perturbation of kHz soliton sequence mathematically equivalent to a situation in which a sequence of gravitational penduli rotates with constant phase difference between neighbors except for one pendulum which oscillates and oscillation moves along the sequence with the same velocity as the kHz wave. The oscillation would be induced by a "kick" for which one can imagine several mechanisms.

   The model explains features of nerve pulse not explained by Hodkin-Huxley model. These include the mechanical changes associated with axon during nerve pulse, the outwards force generated by nerve pulse with a correct prediction for its order of magnitude, the adiabatic character of nerve pulse, and the small rise of temperature of membrane during pulse followed by a reduction slightly below the original temperature.

   The model predicts that the time taken to travel along any axon is a multiple of time dictated by the resting potential so that synchronization is an automatic prediction. Not only kHz waves but also a fractal hierarchy of EEG (and EXG) waves are induced as Josephson radiation by voltage waves along axons and microtubules and by standing waves assignable to neuronal (cell) soma. The value of Planck constant involved with flux tubes determines the frequency scale of EXG so that a fractal hierarchy results.

   The model forces to challenge the existing interpretation of nerve pulse patterns and the function of neural transmitters. Neural transmitters need not represent actual/only) signal but could be more analogous to links in quantum web. The transmitter would coding the address of the receiver, which could be gene inside neuronal nucleus. Nerve pulses would build a connection line between sender and receiver of nerve pulse along which actual signals would propagate. Also quantum entanglement between receiver and sender can be considered.

6. Acupuncture points, meridians, and Chi are key notions of Eastern medicine and find a natural identification in terms of magnetic body lacking from the western medicine. Also a connection with well established notions of DC currents and potentials discovered by Becker and with TGD based view about universal metabolic currencies as differences of zero point energies for pairs of space-time sheets with different p-adic length scale emerges.

   Chi would correspond to these fundamental metabolic energy quanta to which ordinary chemically stored metabolic energy would be transformed. Meridians would most naturally correspond to flux tubes with large $\hbar$ along which dark supra currents flow without dissipation and transfer the metabolic energy between distant cells. Acupuncture points would correspond to points between which metabolic energy is transferred and their high conductivity and semiconductor like behavior would conform with the interpretation in terms of metabolic energy storages. The energy gained in the potential difference between the points would help to kick the charge carrier to a smaller space-time sheet. It is possible that the main contribution to the of charge at magnetic flux tube is magnetic energy and slightly below the metabolic energy quantum and that the voltage difference gives only the lacking small energy increment making the transfer possible. Also direct kicking of charge carriers to smaller space-time sheets by photons is possible and the observed action spectrum for IR and red photons corresponds to the predicted increments of zero point kinetic energies.

7. Magnetic flux tubes could also play key role in bio-catalysis and explain the magic ability of biomolecules to find each other. The model of DNA as topological quantum computer [K4] suggest that not only DNA and its conjugate but also some amino-acid sequences acting as catalysts could be connected to DNA and other amino-acids sequences or more general biomolecules by flux tubes acting as colored braid strands. The shortening of the flux tubes in a phase transition reducing the value of Planck constant would make possible extremely selective mechanisms of catalysis allowing precisely defined locations of reacting molecules to attach to each other. With recently discovered mechanism for programming sequences of biochemical reactions this would make possible to understand the miraculous looking feats of bio-catalysis.

8. The ability to construct "stories", temporally scaled down or possible also scaled up representations about the dynamical processes of external world, might be one of the key aspects of intelligence. There is direct empirical evidence for this activity in hippocampus. The phase transitions reducing or increasing the value of Planck constant would indeed allow to achieve this by scaling the time duration of the zero energy space-time sheets providing cognitive representations.

Direct experimental evidence for the notion of magnetic body carrying dark matter

The list of nice things made possible by the magnetic body is impressive and one can ask whether there is any experimental support for this notion. The findings of Peter Gariaev and collaborators give evidence for the representation of DNA sequences based on the coding of nucleotide to a rotation angle of the polarization direction as photon travels through the flux tube and for the decoding of this representation to gene activation [I4], for the transformation of laser light to light at various radio-wave frequencies having interpretation in terms of phase transitions increasing $\hbar$ [I3, I1], and even for the possibility to photograph magnetic flux tubes containing dark matter by using ordinary light in UV-IR range scattered from DNA [I6].

4.5.3 Brain And Consciousness

In the proposed vision the role of brain for consciousness is not so central than in neuroscience view. Brain is not the seat of sensory mental images but builder of symbolic representations and magnetic body replaces brain as an intentional agent and higher level experiencer. Furthermore, p-adic view about cognition means that only cognitive representations but not cognition itself can be localized in a finite space-time region.

The simplest sensory qualia would be realized at the level of sensory organs so that one can avoid the problematic assignment of sensory qualia to the sensory pathways. The new view about time would allow to resolve the objections against this view. For instance, phantom leg phenomenon would result by sharing of sensory mental images of the geometric past by time like quantum entanglement. For instance, visual colors would correspond to increments of color quantum numbers in quantum jumps at the level of retina. Our sensory mental images do not correspond to the sensory input as such. Rather, the feedback from brain (or from magnetic body via brain) to sensory organs is an essential element in the construction of sensory mental images. For instance, during REM sleep rapid eye movements would reflect the presence of this feedback. The feedback would be also very important in the case of hearing. Visual mental images in absence of eye movements could be interpreted as sharing of visual mental images by quantum entanglement (in particular, time-like entanglement giving rise to episodal memories).

Chapter 5

TGD and M-Theory

5.1 Introduction

In this chapter a critical comparison of M-theory [B19] and TGD (see [K117, K112, K22, K70, K91, K75, K100] and [K108, K18, K81, K16, K45, K57, K60, K99]) as two competing theories is carried out. Also some comments about the sociology of Big Science are made.

The problem with this chapter is that it is almost by definition always out-of-date. I have recently (I am writing this in 2015) updated the file trying to mention the most recent steps of progress about which there is a summary [L10] at my homepage as an article with links to my blog where one can find links to books about TGD.

5.1.1 From Hadronic String Model To M-Theory

The evolution of string theories began 1968 from Veneziano formula realizing duality symmetry of hadronic interactions. It took two years to realize that Veneziano amplitude could be interpreted in terms of interacting strings: Nambu, Susskind and Nielsen made the discovery simultaneously 1970. The need to describe also fermions led to the discovery of super-symmetry [B50] and Ramond and Neveu-Schwartz type superstrings in the beginning of seventies.

Gradually it became however clear that the strings do not describe hadrons: for instance, the critical dimensions for strings *resp.* superstrings where 26 *resp.* 10, and the breakthrough of QCD at 1973 meant an end for the era of hadronic string theory. 1974 Schwartz and Scherk proposed that strings might provide a quantum theory of gravitation [B56] if one accepts that space-time has compactified dimensions.

The first superstring revolution was initiated around 1984 by the paper by Green and Schwartz demonstrating the cancellation of anomalies in certain superstring theories [B33, B34]. The proposal was that superstrings might provide a divergence-free and anomaly-free quantum theory of gravitation. A crucial boost was given by Witten's interest on superstrings. Also the highly effective use of media played a key role in establishing superstring hegemony.

It became clear that superstrings come in five basic types [B48]. There are type I strings (both open and closed) with $N = 1$ super-symmetry and gauge group $SO(32)$, type IIA and IIB closed strings with $N = 2$ super-symmetry, and heterotic strings, which are closed and possess $N = 1$ super-symmetry with gauge groups $SO(32)$ and $E^8 \times E^8$. There is an entire landscape of solutions associated with each superstring theory defined by the compactifications whose dynamics is partially determined by the vanishing of conformal anomalies. For a moment it was believed that it would be an easy task to find which of the superstrings would allow the compactification which corresponds to the observed Universe but it became clear that this was too much to hope. In particular, the number 4 for non-compact space-time dimensions is by no means in a special position.

Around 1995 came the second superstring revolution with the idea that various superstring species could be unified in terms of an 11-dimensional M-theory with M meaning membrane in the lowest approximation [B19]. M-theory allowed to see various superstrings as limiting situations when 11-D theory reduces to 10-D one so that very special kind of membranes reduce to strings. This allowed to justify heuristically the claimed dualities between various superstrings [B48].

Matrix Theory as a proposal for a non-perturbative formulation of M-theory appeared 2 years later [B26].

Now, almost a decade later, M-theory is in a deep crisis: the few predictions that the theory can make are definitely wrong and even anthropic principle is advocated as a means to save the theory [B45]. Despite this, very many people continue to work with M-theory and fill hep-th with highly speculative preprints proving that this is dual with that although the flow of papers dealing with strings and M-theory has reduced dramatically.

A reader interested in critical views about string theory can consult the article of Smolin [B44] criticizing anthropic principle, the web-lectures "Fantasy, Fashion, and Faith in Theoretical Physics" of Penrose [B55] as well as his article in New Scientist [B54] criticizing the notion of hidden space time dimensions, and the articles of Peter [C10] [B52]. Also the discussion group "Not Even Wrong" [B4] gives a critical perspective to the situation almost a decade after the birth of M-theory.

5.1.2 Evolution Of TGD Very Briefly

The first superstring revolution shattered the world at 1984, about two years after my own doctoral dissertation (1982), and four years after the Esalem conference in which the quantum consciousness movement started. Remarkably, David Finkelstein was one of the organizers of the conference besides being the chief editor of "International Journal of Theoretical Physics", in which I managed to publish first articles about TGD. The first and last contact with stars was Wheeler's review of my first article published in IJTP, and I cannot tell what my and TGD's fate had been without Wheeler's highly encouraging review.

During the 31 years after the discovery that space-times could be regarded as 4-surfaces as well as extended objects generalizing strings, I have devoted my time to the development of TGD. Without exaggeration I can say that life devoted to TGD has been much more successful project than I dared or even could dream and has led outside the very narrow realms of particle physics and quantum gravity. Indeed, without knowing anything about Finkelstein and Esalem at that time, I started to write a book about consciousness around 1995 when the second superstring revolution occurred. TGD inspired theory of consciousness has now materialized as 8 online books at my home page.

Altogether these 37 years boil down to eight online books [K117, K112, K22, K91, K70, K49, K75, K100] about TGD proper and eight online books about TGD inspired theory of consciousness and of quantum biology [K108, K18, K81, K16, K45, K57, K99, K107] plus one printed book about TGD [**?**] and second printed book about TGD inspired theory of consciousness and quantum biologys [K3].

This makes about more than 10,000 pages of TGD spanning everything between elementary particle physics and cosmology. One might expect that the sheer waste amount of material at my web site might have stirred some interest in the physics community despite the fact that it became impossible to publish anything and to get anything into Los Alamos archives after the second super-string revolution. The only visible reaction has been from my Finnish colleagues and guarantees that I will remain unemployed in the foreseeable future. I will discuss some reasons for this state of affairs after comparing string models and TGD, and considering the reasons for the failure of the theory formerly known as superstring model.

Before continuing, I hasten to admit that I am not a string specialist and I do not handle the technicalities of M-theory. On the other hand, TGD has given quite a good perspective about the real problems of TOEs and provides also solutions to them. Hence it is relatively easy to identify the heuristic and usually slippery parts of various arguments from the formula jungle. Also I want to express my deep admiration for the people living in the theory world but from my own experience I know how easy it is to fall on wishful thinking and how necessary but painful it is to lose face now and then.

My humble suggestion is that M-theorists might gain a lot by asking what "What possibly went wrong?". This chapter suggests answers to this question: see also [L17]. Perhaps M-theorists might also spend a few hours in the web to check whether M-theory is indeed the only viable approach to quantum gravity: the material at my own home page might provide a surprise in this respect.

Ironically, TGD seems to be predict more stringy physics than string model. For instance, the well-definedness of em charge localizes the modes of induced spinor fields in generic case to 2-D surfaces so that strings become genuine part of TGD. Furthermore, string like objects defined by magnetic flux tubes appear in all scales, even in nuclear physics. These two kinds of strings actually seem to accompany each other.

Also the AdS/CFT correspondence generalizes in TGD framework (the conformal symmetries in TGD are gigantic as compared to those in string models) and is made obvious by the fact that WCW Kähler metric can be expressed either in terms of Kähler function or as commutations of WCW gamma matrices identifiable as Noether super charges in the Yangian of super-symplectic algebra.

A further fascinating quite recent finding is that if one assumes that strings connecting partonic 2-surfaces serve as correlates for the formation of gravitationally bound states, string tension $T = 1/\hbar G$ allows only bound states of size of order Planck length - a fatal prediction. Even the replacement $h \rightarrow h_{eff} = n \times h$ does not help. The solution of the problem comes from the supersymmetry and generalization of AdS/CFT correspondence. By supersymmetry Kähler action must be expressible as bosonic string action defined by the total string world sheet area in the effective metric defined by the anti-commutator of Kähler-Dirac matrices at string world sheets. This predicts that string tension is proportional to $1/h_{eff}^2$ and allows to understand the formation of gravitationally bound states. Macroscopic quantum gravitational coherence even in astrophysical scales is predicted.

The most recent steps of progress relate to the formulation of scattering amplitudes based on the proposed 8-D variant of twistor approach involving octonionic representation of the embedding space gamma matrices in an essential way and light-like momenta in 8-D sense plus the lifting of space-time surfaces to their twistor spaces represented as surfaces in the twistor space of $M^4 \times CP_2$. The Yangian of super-symplectic algebra for which Noether charges are expressible as integrals over strings connecting partonic 2-surfaces defines the basic 3-vertices as product and co-product and the scattering amplitudes can be seen a sequences of algebraic manipulating connecting initial and final state identified as states at the opposite boundaries of CD. Universe would be doing Yangian arithmetics.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L9].

5.2 A Summary About The Evolution Of TGD

The basic idea about space-time as a 4-surface popped in my mind in autumn at 1978, I am not quite sure about the year, it might be also 1977. . The first implication was that I lost my job at Helsinki University. During the next 4 years this idea led to a thesis with the title "Topological GeometroDynamics" (TGD), which I think was suggested by David Finkelstein to distinguish TGD from Wheeler's GeometroDynamics.

5.2.1 Space-Times As 4-Surfaces

TGD can be seen as as a solution to the energy problem of General Relativity via the unification of special and general relativities by assuming that space-times are representable as 4-surfaces in certain 8-dimensional space-time with the symmetries of empty Minkowski space. An alternative interpretation is as a generalization of string models by replacing strings with 3-dimensional surfaces: depending on their size they would represent elementary particles or the space we live in and anything between these extremes. From this point of view superstring theories are unique candidates for a Theory of Everything if space-time were 2- rather than 4-dimensional.

The first superstring revolution made me happy since I was convinced that it would be a matter of few years before TGD would replace superstring models as a natural generalization allowing to understand the four-dimensionality of the space-time. After all, only a half-page argument, a simple exercise in the realization of standard model symmetries, leads to a unique identification of the higher-dimensional embedding space as a Cartesian product of Minkowski space and complex projective space CP_2 unifying electro-weak and color symmetries in terms of its holonomy and isometry groups. By the 4-dimensionality of the basic objects there was no need for the embedding space geometry to be dynamical. Theory realized the dream about the geometrization of fundamental interactions and predicted the observed quantum numbers. In particular, the horrors of spontaneous compactification to be crystallized in the notion of M-theory landscape two decades later can be circumvented completely.

5.2.2 Uniqueness Of The Embedding Space From The Requirement Of Infinite-Dimensional Kähler Geometric Existence

Later I discovered heuristic mathematical arguments suggesting but not proving that the choice of the embedding space is unique. The arguments relied on the uniqueness of the infinite-dimensional Kähler geometry of WCW of 3-surfaces. This uniqueness was discovered already in the context of loop spaces by Dan Freed [A43].

CH, the "world of the classical worlds" serves as the arena of quantum dynamics [K28], which reduces to the theory of classical spinor fields in CH and geometrizes fermionic anti-commutation relations and the notion of super-symmetry in terms of the gamma matrices of CH [K121]. Only quantum jump is the genuinely non-classical element of the theory in CH context. The heuristic argument states that CH geometry exists only for $H = M^4 \times CP_2$.

The strongest argument for the uniqueness of H emerged only rather recently (2014) [L8]. M^4 and CP_2 are the only 4-D manifolds allowing twistor space with Kähler structure. This fact has been discovered by Hitchin at about same time as I discovered the basic idea of TGD [A61] but had escaped my attention. This leads to a formulation of TGD using liftings of space-time surfaces to their twistor spaces: allowed space-time surfaces are those whose twistor spaces can be induced from the product of twistor spaces of M^4 and CP_2.

Also number theoretical arguments relating to quaternions and octonions fix the dimensions of space-time and embedding space to four and 8 respectively. The fact that the space of quaternionic sub-spaces of octonion space containing preferred plane complex plane is CP_2 suggest an explanation for the special role of CP_2.

This stimulated a development, which led to notion of number theoretic compactification. Space-time surfaces can be regarded either as hyper-quaternionic, and thus maximally associative, 4-surfaces in M^8 or as surfaces in $M^4 \times CP_2$ [K105]. What makes this duality possible is that CP_2 parameterizes different quaternionic planes of octonion space containing a fixed imaginary unit. Hyper-quaternions/-octonions form a sub-space of complexified quaternions/-octonions for which imaginary units are multiplied by $\sqrt{-1}$: they are needed in order to have a number theoretic norm with Minkowski signature.

The weakest form of number theoretical compactification states that light-like 3-surfaces $X_l^3 \subset HO$ are mapped to $X_l^3 \subset M^4 \times CP_2$ and requires only that one can assign preferred plane $M^2 \subset M^4$ to any connected component of X_l^3. This hyper-complex plane of hyper-quaternionic M^4 has interpretation as the plane of non-physical polarizations so that the gauge conditions of super string theories are obtained purely number theoretically. M^2 corresponds also to the degrees of freedom which do not contribute to the metric of WCW . The un-necessarily strong form would require that hyper-quaternionic 4-surfaces correspond to preferred extremals of Kähler action.

The requirement that M^2 belongs to the tangent space $T(X^4(X_l^3))$ at each point point of X_l^3 fixes also the boundary conditions for the preferred extremal of Kähler action. The construction of WCW spinor structure supports the conclusion that there must exist preferred coordinates of X^4 in which additional conditions $g_{ni} = 0$ and $J_{ni} = 0$ at X_l^3. The conditions state that induced metric and Kähler form are stationary at X_l^3. M^2 plays a key role also in many other constructions of quantum TGD, in particular the generalization of the embedding space needed to realize the idea about hierarchy of Planck constant allowing to identify dark matter as matter with a non-standard value of Planck constant.

The realization of 4-D general coordinate invariance forces to assume that Kähler function assigns a unique space-time surface to a given 3-surface: by the breakdown of the strict classical determinism of Kähler action unions of 3-surfaces with time like separations must be however allowed as 3-D causal determinants and quantum classical correspondence allows to interpret them as representations of quantum jump sequences at space-time level. Space-time surface defined as a preferred extremal [K105] of Kähler action is analogous to Bohr orbit so that classical physics becomes part of the definition of configuration space geometry rather than being a result of a stationary phase approximation.

What "preferred" has been a longstanding problem. In zero energy ontology (ZEO) 3-surfaces are pairs of 3-surfaces at the opposite light-like boundaries of causal diamond (CD), whose M^4 projection is an intersection of future and past directed light-cones. In spirit with what I call strong form of holography, the space-time surfaces connecting these two 3-surfaces are assumed to possess vanishing Noether charges in a sub-algebra of super-symplectic algebra with conformal weights coming as n-multiple of the weights of the entire algebra. This condition is extremely powerful. For the sub-algebra labelled by n super-symplectic generators act as conformal gauge symmetries, and one obtains infinite number of hierarchies of conformal gauge symmetry breakings. One can also interpret these conformal hierarchies in terms of gradually reduced quantum criticality. An attractive interpretation is that n corresponds the value of effective Planck constant $h_{eff}/h = n$, whose values label a hierarchy of dark matter phases. Also a connection with hierarchies of hyper-finite factors emerges. There are many other partial characterizations of blockquotepreferred to be discussed later but this looks to me the most attractive one now.

5.2.3 TGD Inspired Theory Of Consciousness

During the last decade a lot has happened in TGD and it is sad that only those colleagues with mind open enough to make a visit my home page have had opportunity to be informed about this. Knowing the fact that a typical theoretical physicist reads only the articles published in respected journals about his own speciality, one can expect that the number of these physicists is not very high. Some examples of the work done during this decade are in order.

I have developed quantum TGD in a considerable detail with highly non-trivial number theoretical speculations relating to Riemann hypothesis and Riemann Zeta in riema. One outcome is a proposal for the proof of Riemann hypothesis [L1].

During the same period I have constructed TGD inspired theory of consciousness [K108]. One outcome is a theory of quantum measurement and of observer having direct implications for the quantum TGD itself. The results of the modification of the double slit experiment carried out by Afshar [D6] , [J6] provides a difficult challenge for the existing interpretations of quantum theory and a support for the TGD view about quantum measurement in which space-time provides correlates for the non-deterministic process in question. The new views about energy and time have also profound technological implications.

The hierarchy of Planck constants, quantum criticality, and the notion of magnetic body inspired by the notions of many-sheeted space-time and topological field quantization have become central concepts in TGD inspired theory of consciousness. Also p-adic physics as physics of cognition is key element.

The new view about measurement theory based on Zero Energy Ontology (ZEO) and the notion of causal diamond (CD) forces a more detailed view about state function reduction. In quantum context one has quantum superposition of CDs and each CD carries zero energy state: it is assumed that the CDs in superposition have second boundary which belongs to common light-cone boundary. State function can occur at both boundaries of CD and self as a conscious entity can be identified as the sequence of repeated state function reductions occurring at fixed boundary doing nothing for the fixed boundary. The experience about flow of time and arrow of time can be understood and the latter can correspond to both arrow of geometric time. Volitional act corresponds to the first reduction at the opposite boundary.

Negentropy Maximization Principle (NMP) serves as the basic variational principle and implies ordinary quantum measurement theory and second law for a generic entanglement. There is however a notable exception. When the density matrix decomposes into direct sum containing $n \times n$ unit matrices with $n > 1$: this happens in two-particle system when the entanglement coefficients define a unitary matrix. One can assign number theoretic variant of Shannon entropy to a state with this kind of density matrix and the state is stable with respect to NMP. One can speak of negentropic entangleement since entanglement entropy is negative. NMP predicts that the amount of negentropic entanglement increases in the Universe. Negentropic entanglement has interpretation as abstraction: the state pairs in the superposition represent instances of a rule. An obvious conjecture is that n relates to $h_{eff}/h = n$ and to the hierarchy of quantum criticalities.

5.2.4 Number Theoretic Vision

Physics as infinite-D spinor geometry of WCW and physics as generalized number theory are the two basic visiona about TGD.

The number theoretic vision involves three threads.

1. The first thread involves the notion of number theoretic universality: quantum TGD should make sense in both real and p-adic number fields (and their algebraic extensions). p-Adic number fields would be needed to understand the space-time correlates of cognition and intentionality [K71, K42, K73]. .

   p-Adic number fields lead to the notion of a p-adic length scale hierarchy quantifying the notion of the many-sheeted space-time [K71, K42]. One of the first applications was the calculation of elementary particle masses [K59]. The basic predictions are only weakly model independent since only p-adic thermodynamics for Super Virasoro algebra is involved. Not only the fundamental mass scales reduce to number theory but also individual masses are predicted correctly under very mild assumptions. Also predictions such as the possibility of neutrinos to have several mass scales were made on the basis of number theoretical arguments and have found experimental support [K59].

2. Second thread is inspired by the dimensions of the basic objects of TGD and assumes that classical number fields are in a crucial role in TGD. 8-D embedding space would have octonionic structure and space-time surfaces would have associative (quaternionic) tangent space or normal space. String world sheets would correspond to commutative surfaces. Also the notion of M^8-H-duality is part of this thread and states that quaternionic 4-surfaces of M^8 containing preferred M^2 in its tangent space can be mapped to preferred extremals of Kähler action in H by assigning to the tangent space CP_2 point parametrizing it. M^2 could be replaced by integrable distribution of $M^2(x)$. If the preferred extremals are also quaternionic one has also $H-H$ duality allowing to iterate the map so that preferred extremals form a category.

3. The third thread corresponds to infinite primes [K103] leading to several speculations. The construction of infinite primes is structurally analogous to a repeated second quantization of a supersymmetric arithmetic quantum field theory with free particle states characterized by primes. The many-sheeted structure of TGD space-time could reflect directly the structure of infinite prime coding it. Space-time point would become infinitely structured in various p-adic senses but not in real sense (that is cognitively) so that the vision of Leibniz about monads reflecting the external world in their structure is realized in terms of algebraic holography. Space-time becomes algebraic hologram and realizes also Brahman=Atman idea of Eastern philosophies.

p-Adic physics as physics of cognition

p-Adic mass calculations relying on p-adic length scale hypothesis led to an understanding of elementary particle masses using only super-conformal symmetries and p-adic thermodynamics. The need to fuse real physics and various p-adic physics to single coherent whole led to a generalization of the notion of number obtained by gluing together reals and p-adics together along common rationals and algebraics (see **Fig.** `http://tgdtheory.fi/appfigures/book.jpg` or **Fig. ??** in the appendix of this book). The interpretation of p-adic space-time sheets is as correlates for cognition and intentionality. p-Adic and real space-time sheets intersect along common rationals and algebraics and the subset of these points defines what I call number theoretic braid in terms of which both WCW geometry and S-matrix elements should be expressible. Thus one would obtain number theoretical discretization which involves no ad hoc elements and is inherent to the physics of TGD.

Perhaps the most dramatic implication relates to the fact that points, which are p-adically infinitesimally close to each other, are infinitely distant in the real sense (recall that real and p-adic embedding spaces are glued together along rational embedding space points). This means that any open set of p-adic space-time sheet is discrete and of infinite extension in the real sense. This means that cognition is a cosmic phenomenon and involves always discretization from the point of view of the real topology. The testable physical implication of effective p-adic topology of real space-time sheets is p-adic fractality meaning characteristic long range correlations combined with short range chaos.

Also a given real space-time sheets should correspond to a well-defined prime or possibly several of them. The classical non-determinism of Kähler action should correspond to p-adic non-determinism for some prime(s) p in the sense that the effective topology of the real space-time sheet is p-adic in some length scale range.

An ideal realization of the space-time sheet as a cognitive representation results if the CP_2 coordinates as functions of M^4_+ coordinates have the same functional form for reals and various p-adic number fields and that these surfaces have discrete subset of rational numbers with upper and lower length scale cutoffs as common. The hierarchical structure of cognition inspires the idea that S-matrices form a hierarchy labeled by primes p and the dimensions of algebraic extensions.

The number-theoretic hierarchy of extensions of rationals appears also at the level of WCW spinor fields and allows to replace the notion of entanglement entropy based on Shannon entropy with its number theoretic counterpart having also negative values in which case one can speak about genuine information. In this case case entanglement is stable against Negentropy Maximization Principle (NMP) stating that entanglement entropy is minimized in the self measurement and can be regarded as bound state entanglement. Bound state entanglement makes possible macro-temporal quantum coherence. One can say that rationals and their finite-dimensional extensions define islands of order in the chaos of continua and that life and intelligence correspond to these islands.

TGD inspired theory of consciousness and number theoretic considerations inspired for years ago the notion of infinite primes [K103]. It came as a surprise, that this notion might have direct relevance for the understanding of mathematical cognition. The ideas is very simple. There is infinite hierarchy of infinite rationals having real norm one but different but finite p-adic norms. Thus single real number (complex number, (hyper-)quaternion, (hyper-)octonion) corresponds to an algebraically infinite-dimensional space of numbers equivalent in the sense of real topology. Space-time and embedding space points ((hyper-)quaternions, (hyper-)octonions) become infinitely structured and single space-time point would represent the Platonia of mathematical ideas. This structure would be completely invisible at the level of real physics but would be crucial for mathematical cognition and explain why we are able to imagine also those mathematical structures which do not exist physically. Space-time could be also regarded as an algebraic hologram. The connection with Brahman=Atman idea is also obvious.

One very interesting aspect of number theoretic vision is the possibility that scattering amplitude could be regarded as a representation for sequences of algebraic operations (product and co-product) in super-symplectic Yangian representing 3-vertices and leading from initial set of algebraic objects to a final set of them [L8]. The construction would have a gigantic symmetry: any sequence of operations connecting initial and final state would correspond to the same scattering amplitude.

Number theoretical symmetries

TGD as a generalized number theory vision leads to a highly speculative idea that also number theoretical symmetries are important for physics. Reader can decided whether the following should be taken with any seriousness. Also I try to do so.

1. There are good reasons to believe that the strands of number theoretical braids can be assigned with the roots of a polynomial with suggests the interpretation corresponding Galois groups as purely number theoretical symmetries of quantum TGD. Galois groups are subgroups of the permutation group S_∞ of infinitely manner objects acting as the Galois group of algebraic numbers. The group algebra of S_∞ is HFF which can be mapped to the HFF defined by WCW spinors. This picture suggest a number theoretical gauge invariance stating that S_∞ acts as a gauge group of the theory and that global gauge transformations in its completion correspond to the elements of finite Galois groups represented as diagonal groups of $G \times G \times ....$ of the completion of S_∞. The groups G should relate closely to finite groups defining inclusions of HFFs.

2. HFFs inspire also an idea about how entire TGD emerges from classical number fields, actually their complexifications. In particular, SU(3) acts as subgroup of octonion automorphisms

leaving invariant preferred imaginary unit and $M^4 \times CP_2$ can be interpreted as a structure related to hyper-octonions which is a subspace of complexified octonions for which metric has naturally Minkowski signature. This would mean that TGD could be seen also as a generalized number theory. This conjecture predicts the existence of two dual formulations of TGD based on the identification space-times as 4-surfaces in hyper-octonionic space M^8 *resp.* $M^4 \times CP_2$.

3. The vision about TGD as a generalized number theory involves also the notion of infinite primes. This notion leads to a further generalization of the ideas about geometry: this time the notion of space-time point generalizes so that it has an infinitely complex number theoretical anatomy not visible in real topology.

5.2.5 Hierachy Of Planck Constants And Dark Matter

TGD has lead to two proposals for how non-standard values of Planck constants might appear in physics.

Large Planck constant from neuroscience

The strange quantal effects of ELF em fields on vertebrate brain suggest that the energies $E = hf$ of ELF photons were above thermal energy at physiological temperature. This suggests the replacement $h \rightarrow h_{eff} = n \times h$ and the leads to the vision that bio-systems are macroscopic quantum systems with ordinary quantum scales scaled up by factor n.

The earlier work with topological quantum computation [K5] had already led to the idea that Planck constant could relate to the quantum phase $q = exp(i\pi/n)$. The improved understanding of Jones inclusions and their role in TGD [K120] allowed to deduce then extremely simple formula $h_{eff} = n \times h$. Much later came the realization that the hierarchy of Planck constants corresponds naturally to a hierarchy of gauge symmetry breakings assignable with the super-symplectic algebra possessing conformal structure and having also interpretation as a hierarchy of improved measurement resolutions suggested to have mathematical description in terms of inclusions of hyper-finite factors of type II_1. Since the inclusions are accompanied by quantum groups characterized by q the connection with the inclusions and h_{eff} can be understood. The localization of the induced spinor fields at string world sheets is also essential: their 2-D character is what makes possible to pose a quantum version of anti-commutation relations for the induced spinor fields. Hence it seems that the $h_{eff} = n \times h$ hypothesis fits naturally to the framework of physical principles and mathematical concepts underlying TGD.

One can speculate about the most probable values of n. I have suggested that the values of n for which the quantum phase is expressible using only iterated square root operation (corresponding polygon is obtained by ruler and compass construction) are of special interest since they correspond to the lowest evolutionary levels for cognition so that corresponding systems should be especially abundant in the Universe. One should be however extremely cautious with this kind of speculations.

The general philosophy would be that when the quantum system becomes non-perturbative, a phase transition increasing the value of $\hbar$ occurs to preserve the perturbative character. This would apply to QCD and to atoms with $Z > 137$ and to any other system. $q \neq 1$ quantum groups characterize non-perturbative phases. Macroscopic gravitation is second fundamental example: the coupling parameter $GMm/\hbar c$ exceeds unity for macroscopic systems. Here Nottale led to the hypothesis $\hbar_{gr} = GMm/v_0$ to be described in more detail below. The obvious conjecture $h_{gr} = h_{eff}$ has very interesting biological implications discussed in [K79] and [?].

Large Planck constant from astrophysics

Another step in the rapid evolution of quantum TGD [K97], [L5] was stimulated when I learned that D. Da Rocha and Laurent Nottale [E1] have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.82 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive.

Nottale and Da Rocha suggest that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests astrophysical systems are not only quantum systems at larger space-time sheets but correspond to a gigantic value of gravitational Planck constant assignable to the flux tubes mediating gravitational interaction so that there the dependence on both masses makes sense.

The gravitational (ordinary) Schrödinger equation - in TGD framework it is better to restrict to the Bohr orbitology version of it - would provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The basic objection is that astrophysical systems are extremely classical whereas TGD predicts macrotemporal quantum coherence in the scale of life time of gravitational bound states. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale.

TGD allows a reasonable estimate for the value of the velocity parameter v_0 assuming that cosmic strings and their decay remnants are responsible for the dark matter. The value of v_0 has interpretation as velocity of distant stars around galaxies in the gravitational field of long cosmic string like objects traversing through galactic plane. The harmonics of v_0 could be understood as corresponding to perturbations replacing cosmic strings with their n-branched coverings so that tension becomes n^2-fold: much like the replacement of a closed orbit with an orbit closing only after n turns. Sub-harmonics would result when cosmic strings decay to magnetic flux tubes: magnetic energy density per unit length is quantized by the preferred extremal property and the simplest possibility is the reduction of the energy density by a factor $1/n^2$.

That the value of h_{gr} is different for inner and outer planets is of course disturbing. In this aspect quite recent progress in the understand of basic quantum TGD comes in rescue. The generalization of AdS/CFT duality to TGD framework predicts that gravitational binding is mediated by strings connecting partonic 2-surfaces. If string world sheet area is define by the effective metric defined by the anti-commutators of Kähler-Dirac gamma matrices, it is proportional to $\alpha_K^2 \propto 1/h_{eff}^2$ if one assumes $\alpha_K = g_K^2/4\pi h_{eff}$ so that α_K would have a spectrum of critical values coming as inverses of integers. The size scale of bound state would scale like $\hbar_{gr} = GMm/v_0$ and would be of order GM/v_0: this make sense. The outer planets have much larger size than the 4 inner planets and the reduction of v_0 by factor 1/5 helps to understand their orbits. How $1/h_{eff}^2$ proportionality might be understood is discussed in [?] in terms electric-magnetic duality.

As noticed, ruler and compass rule suggests a spectrum of the most plausible values of $h_{eff}/h = n$. This quantization does not depend at all on the velocity parameter v_0 appearing in the formula of Nottale and this gives strong additional constraints to the ratios of planetary masses and also on the masses themselves if one assumes that the gravitational Planck constant corresponds to the values allowed by ruler and compass construction. Also correct prediction for the ratio of densities of visible and dark matter emerges.

The rather amazing coincidences between basic bio-rhythms and the periods associated with the states of orbits in solar system suggest that the frequencies defined by the energy values predicted by gravitational Bohr orbitology might entrain with various biological frequencies such as the cyclotron frequencies associated with the magnetic flux tubes. For instance, the period associated with n=1 orbit in the case of Sun is 24 hours within experimental accuracy for v_0. This would make sense of $h_{eff} = h_{gr}$ hypothesis holds true. Quantum gravitation would be crucial for life, as Penrose intuited, but in manner very different from what has been usually thought [K79, ?].

Needless to add, if the proposed general picture is correct, not much is left from the superstring/M-theory approach to quantum gravitation since perturbative quantum field theory as the fundamental corner stone must be given up and because the underlying physical picture about gravitational interaction is simply wrong.

Mathematical realization for the hierarchy of Planck constants

The work with hyper-finite factors of type II_1 (HFFs) combined with experimental input led to the notion of hierarchy of Planck constants interpreted in terms of dark matter [K38]. The original proposal was that the hierarchy is realized via a generalization of the notion of embedding space obtained by gluing infinite number of its variants along common lower-dimensional sub-manifolds to which are "quantum critical" in the sense that they are analogous to the back of a book having pages labelled by the values of Planck constant. These variants of embedding space would be characterized by discrete subgroups of $SU(2)$ acting in M^4 and CP_2 degrees of freedom as either symmetry groups or homotopy groups of covering. Among other things this picture implies a general model of fractional quantum Hall effect.

It is now clear that the coverings of embedding space can only serve as auxiliary tools only. TGD predicts the hierarchy of Planck constants without generalization of embedding space concept. At fundamental level n-coverings are realized for space-time surfaces connecting two 3-surfaces at the opposite boundaries of CD. They are analogous to singular coverings of plane defined by analytic functions $z^{1/n}$. Each sheet of covering corresponds to a gauge equivalence class of conformal symmetries defined by a sub-algebra of the symplectic algebra. What comes in mind first is that the radial light-like radial coordinate serving as the analog of complex coordinate is transformed from r_M to $u = r_M^{1/n}$ so that conformal gauge symmetry is true only for the symplectic generators proportional to u^n and the powers u^k, $k = 0, , n-1$ correspond to broken conformal symmetries and to the gauge equivalence classes -different sheets of singular covering.

What is especially remarkable is that the construction gives also the 4-D space-time sheets associated with the light-like orbits of the partonic 2-surfaces: it remains to be shown whether they correspond to preferred extremals of Kähler action. It is clear that the hierarchy of Planck constants has become an essential part of the construction of quantum TGD and of mathematical realization of the notion of quantum criticality rather than a possible generalization of TGD.

5.2.6 Von Neumann Algebras And TGD

The work with TGD inspired model for quantum computation led to the realization that von Neumann algebras, in particular hyper-finite factors of type II_1 could provide the mathematics needed to develop a more explicit view about the construction of S-matrix and its generalizations M-matrix and U-matrix suggested by ZEO.

It has been for few years clear that TGD could emerge from the mere infinite-dimensionality of the Clifford algebra of infinite-dimensional "world of classical worlds" and from number theoretical vision in which classical number fields play a key role and determine embedding space and space-time dimensions. This would fix completely the "world of classical worlds".

Infinite-dimensional Clifford algebra is a standard representation for von Neumann algebra known as a hyper-finite factor of type II_1 (HFF). In TGD framework the infinite tensor power of C(8), Clifford algebra of 8-D space would be the natural representation of this algebra.

The physical idea is following.

1. Finite measurement resolution could be represented as inclusion of HFFs - at classical level it would correspond to a discretization with some resolution defined by the algebraic extension of rationals used and by the p-adic length scale cutoffs. The included algebra would act like gauge group in the sense that its elements in zero energy ontology would generate states not distinguishable from the original one.
2. The space of physical states would be an analog of coset space but with fractal dimension given by the index of inclusion defined in terms of quantum phase. It might well be possible to act analog of gauge group with the inclusion.
3. An alternative view is that the hierarchy of inclusions is associated with the hierarchy of sub-algebras of supersymplectic algebra acting gauge transformations. The sub-algebra would be isomorphic to the entire algebra with conformal weights coming as n-multiples of those for the entire algebra. This subalgebra would define measurement resolution, and one would indeed have gauge group interpretation in a wide sense of the word. $n = h_{eff}/h$ identification would give a direct connection with the hierarchy of Planck constants and dark matter hierarchy.

This idea has led to speculations: two such speculations are discussed in this section. The first one is the extension of WCW Clifford algebra to a local algebra in Minkowski space. Second speculation is that Connes tensor product might help to understand interactions in TGD framework.

Unfortunately, the problem is that the understanding of Connes tensor product is for a physicist like me a tougher challenge than understanding of physics! What is obvious even for physicist like me that Connes tensor product differs from the ordinary tensor product in that it implies strong correlations between factors represented as entanglement and entanglement indeed represents interactions.

1. Quantum phase q is associated also with the Yangians of super-symplectic algebra. The localization of the induced spinor fields at string world sheets makes possible to introduced quantum phase directly at the level of anti-commutators of oscillator operators. Yangian realized in terms of super-symplectic Noether charges assignable to strings connecting partonic 2-surfaces leads to a concrete proposal for the construction of scattering amplitudes utilizing product and co-product as basic vertices [L8]. This construction of vertices could relate closely to Connes tensor product.

2. The construction of zero energy states implies strong correlations between the positive and negative energy parts of zero energy state at the boundaries of CD. One cannot just construct ordinary tensor product of state spaces. These correlations are expressed classically by preferred extremal property serving as the analog of Bohr orbit and at least partially realized by the condition that 3-surfaces carry vanishing symplectic Noether charges for the sub-algebra of symplectic algebra. These strong correlations could have mathematical representation in terms of Connes tensor product.

Quantum criticality and inclusions of factors

Quantum criticality fixes the value of Kähler coupling strength but is expected to have also an interpretation in terms of a hierarchies of broken super-symplectic gauge symmetries suggesting hierarchies of inclusions.

1. In ZEO 3-surfaces are unions of space-like 3-surfaces at the ends of causal diamond (CD). Space-time surfaces connect 3-surfaces at the boundaries of CD. The non-determinism of Kähler action allows the possibility of having several space-time sheets connecting the ends of space-time surface but the conditions that classical charges are same for them reduces this number so that it could be finite. Quantum criticality in this sense implies non-determinism analogous to that of critical systems since preferred extremals can co-incide and suffer this kind of bifurcation in the interior of CD. This quantum criticality can be assigned to the hierarchy of Planck constants and the integer n in $h_{eff} = n \times h$ [K38] corresponds to the number of degenerate space-time sheets with same Kähler action and conserved classical charges.

2. Also now one expects a hierarchy of criticalities and since criticality and conformal invariance are closely related, a natural conjecture is that the fractal hierarchy of sub-algebras of conformal algebra isomorphic to conformal algebra itself and having conformal weights coming as multiples of n corresponds to the hierarchy of Planck constants. This hierarchy would define a hierarchy of symmetry breakings in the sense that only the sub-algebra would act as gauge symmetries.

3. The assignment of this hierarchy with super-symplectic algebra having conformal structure with respect to the light-like radial coordinate of light-cone boundary looks very attractive. An interesting question is what is the role of the super-conformal algebra associated with the isometries of light-cone boundary $R_+ \times S^2$ which are conformal transformations of sphere S^2 with a scaling of radial coordinate compensating the scaling induced by the conformal transformation. Does it act as dynamical or gauge symmetries?

4. The natural proposal is that the inclusions of various superconformal algebras in the hierarchy define inclusions of hyper-finite factors which would be thus labelled by integers. Any sequences of integers for which n_i divides n_{i+1} would define a hierarchy of inclusions proceeding in reverse direction. Physically inclusion hierarchy would correspond to an infinite hierarchy of criticalities within criticalities: hill at the top of hill at the top....

How to localize infinite-dimensional Clifford algebra?

An interesting speculation is that one could make the WCW Clifford algebra *local*: local Clifford algebra as a generalization of gamma field of string models.

1. Represent Minkowski coordinate of M^d as linear combination of gamma matrices of D-dimensional space. This is the first guess. One fascinating finding is that this notion can be quantized and classical M^d is genuine quantum M^d with coordinate values eigenvalues of quantal commuting Hermitian operators built from matrix elements. Euclidian space is

not obtained in this manner. Minkowski signature is something quantal and the standard quantum group $Gl_(2,q)(C)$ with (non-Hermitian matrix elements) gives M^4.

2. Form power series of the M^d coordinate represented as linear combination of gamma matrices with coefficients in corresponding infinite-D Clifford algebra. You would get tensor product of two algebra.

3. There is however a problem: one cannot distinguish the tensor product from the original infinite-D Clifford algebra. $D = 8$ is however an exception! You can replace gammas in the expansion of M^8 coordinate by hyper-octonionic units which are non-associative (or octonionic units in quantum complexified-octonionic case). Now you cannot anymore absorb the tensor factor to the Clifford algebra and you get genuine M^8-localized factor of type II_1. Everything is determined by infinite-dimensional gamma matrix fields analogous to conformal super fields with z replaced by hyperoctonion.

4. Octonionic non-associativity actually reproduces whole classical and quantum TGD: space-time surface must be associative sub-manifolds hence hyper-quaternionic surfaces of M^8. Representability as surfaces in $M^4 \times CP_2$ follows naturally, the notion of WCW of 3-surfaces, etc....

Connes tensor product for free fields as a universal definition of interaction quantum field theory

This picture has profound implications. Consider first the construction of S-matrix.

1. A non-perturbative construction of S-matrix emerges. The deep principle is simple. The canonical outer automorphism for von Neumann algebras defines a natural candidate unitary transformation giving rise to propagator. This outer automorphism is trivial for II_1 factors meaning that all lines appearing in Feynman diagrams must be on mass shell states satisfying Super Virasoro conditions. You can allow all possible diagrams: all on mass shell loop corrections vanish by unitarity and what remains are diagrams with single N-vertex.

2. At 2-surface representing N-vertex space-time sheets representing generalized Bohr orbits of incoming and outgoing particles meet. This vertex involves von Neumann trace (finite!) of localized gamma matrices expressible in terms of fermionic oscillator operators and defining free fields satisfying Super Virasoro conditions.

3. For free fields ordinary tensor product would not give interacting theory. What makes S-matrix non-trivial is that *Connes tensor product* is used instead of the ordinary one. This tensor product is a universal description for interactions and we can forget perturbation theory! Interactions result as a deformation of tensor product. Unitarity of resulting S-matrix is unproven but I dare believe that it holds true.

4. The subfactor $\mathcal{N}$ defining the Connes tensor product has interpretation in terms of the interaction between experimenter and measured system and each interaction type defines its own Connes tensor product. Basically $\mathcal{N}$ represents the limitations of the experimenter. For instance, IR and UV cutoffs could be seen as primitive ways to describe what $\mathcal{N}$ describes much more elegantly. At the limit when $\mathcal{N}$ contains only single element, theory would become free field theory but this is ideal situation never achievable.

5. Large $\hbar$ phases provide good hopes of realizing topological quantum computation. There is an additional new element. For quantum spinors state function reduction cannot be performed unless quantum deformation parameter equals to $q = 1$. The reason is that the components of quantum spinor do not commute: it is however possible to measure the commuting operators representing moduli squared of the components giving the probabilities associated with "true" and "false". The universal eigenvalue spectrum for probabilities does not in general contain (1, 0) so that quantum qbits are inherently fuzzy. State function reduction would occur only after a transition to q=1 phase and de-coherence is not a problem as long as it does not induce this transition.

5.3 Quantum TGD In Nutshell

This section provides a very brief summary about quantum TGD. The discussions are based on the general vision that quantum states of the Universe correspond to the modes of *classical* spinor fields in the "world of the classical worlds" identified as the infinite-dimensional WCW of light-like 3-surfaces of $H = M^4 \times CP_2$ (more or less-equivalently, the corresponding 4-surfaces defining generalized Bohr orbits). This implies a radical deviation from path integral formalism, in which one integrates over all space-time surfaces. A second important deviation is due to Zero Energy Ontology. The properties of Kähler action imply a further crucial deviation, which in fact forced the introduction of WCW , and is behind the hierarchy of Planck constants, hierarchy of quantum criticalities, and hierarchy of inclusions of hyper-finite factors.

I include also an excerpt from [L8] representing the most recent view about how scattering amplitudes could be constructed in TGD using the notion of super-symplectic Yangian and generalization of the notion of twistor structure so that it applies at the level of 8-D embedding space.

5.3.1 Basic Physical And Geometric Ideas

TGD relies heavily on geometric ideas, which have gradually generalized during the years. Symmetries play a key role as one might expect on basis of general definition of geometry as a structure characterized by a given symmetry.

Physics as infinite-dimensional Kähler geometry

1. The basic idea is that it is possible to reduce quantum theory to WCW geometry and spinor structure. The geometrization of loop spaces inspires the idea that the mere existence of Riemann connection fixes WCW Kähler geometry uniquely. Accordingly, WCW can be regarded as a union of infinite-dimensional symmetric spaces labeled by zero modes labeling classical non-quantum fluctuating degrees of freedom.

   The huge symmetries of WCW geometry deriving from the light-likeness of 3-surfaces and from the special conformal properties of the boundary of 4-D light-cone would guarantee the maximal isometry group necessary for the symmetric space property. Quantum criticality is the fundamental hypothesis allowing to fix the Kähler function and thus dynamics of TGD uniquely. Quantum criticality leads to surprisingly strong predictions about the evolution of coupling constants.

2. WCW spinors correspond to Fock states and anti-commutation relations for fermionic oscillator operators correspond to anti-commutation relations for the gamma matrices of the WCW . WCW gamma matrices contracted with Killing vector fields give rise to a super-algebra which together with Hamiltonians of WCW forms what I have used to called super-symplectic algebra.

   WCW metric can be expressed in two ways. Either as anti-commutators of WCW gamma matrices identified as super-symplectic Noether super charges (this is highly non-trivial!) or in terms of the second derivatives of Kähler function expressible as Kähler action for the space-time regions with 4-D CP_2 projection and Euclidian signature of the induced metric (wormhole contacts).

   This leads to a generalization of AdS/CFT duality if one assumes that spinor modes are localized at string world sheets to guarantee well-definedness of em charge for the spinor modes following from the assumption that induced classical W fields vanish at string world sheets. Also number theoretic argument requiring that octonionic spinor structure for the embedding space is equivalent with ordinary spinor structure implies the localization. String model in space-time becomes part of TGD.

3. Super-symplectic degrees of freedom represent completely new degrees of freedom and have no electroweak couplings. In the case of hadrons super-symplectic quanta correspond to what has been identified as non-perturbative sector of QCD they define TGD correlate for the degrees of freedom assignable to hadronic strings. They could be responsible for the most of the mass of hadron and resolve spin puzzle of proton.

It has turned out that super-symplectic quanta would naturally give rise to a hierarchy of dark matters labelled by the value of effective Planck constant $h_{eff} = n \times h$. n would characterize the breaking of super-symplectic symmetry as gauge symmetry and for $n = 1$ (ordinary matter) there would be no breaking.

Besides super-symplectic symmetries there extended conformal symmetries associated with light-cone boundary and light-like orbits of partonic 2-surfaces and Super-Kac Moody symmetries assignable to light-like 3-surfaces. A further super-conformal symmetry is associated with the spinor modes at string world sheets and it corresponds to the ordinary super-conformal symmetry. The existence of quaternion conformal generalization of these symmetries is suggestive and the notion of quaternion holomorphy [A83] indeed makes sense [K93]. Together these algebras mean a gigantic extension of the conformal symmetries of string models [L17]. Some of these symmetries act as dynamical symmetries instead of mere gauge symmetries. The construction of the representations of these symmetries is one of the main challenges of quantum TGD.

The original proposal was that the commutator algebras of super-symplectic and super Kac-Moody algebra annihilate physical states. Recently the possibility that a sub-algebra of super-symplectic algebra (at least this algebra) with conformal weights coming as multiples of integer some integer n annihilates physical states at both boundaries of CD. This would correspond to broken gauge symmetry and would predict fractal hierarchies of quantum criticalities defined by sequences of integers $n_{i+1} = \prod_{k<i+1} m_k$. The conformal algebra of string world sheet could always correspond to $n = 1$. Super Virasoro conditions could be regarded as analogs of WCW Dirac equation. These sequences would define hierarchies of inclusions of hyper finite factors of type II_1 and the identification $n = h_{eff}/h$ would relate this hierarchy to the hierarchy of Planck constants. n would also characterize the non-determinism of Kähler action: there would be n conformal gauge equivalence classes connecting members of a pair of 3-surfaces at the boundaries of CD and defining the ends of space-time.

An intriguing possibility consistent with this picture is that the conformal weights of the super-symplectic algebra charactering the exponent $h$of the power r_M^h of the light-like radial coordinate r_M appearing in the Hamiltonian of the symplectic transformation of $\delta M^4_\pm \times CP_2$ is not an integer but a linear combination of zeros of Riemann Zeta with integer coefficients. For physical states the weights would be real integers (if mass squared corresponds to conformal weight): one would have conformal confinement in the sense that the sum of imaginary parts of conformal weights would be zero. This is an old idea that I already gave up but seems rather attractive in the recent framework.

Modular invariance is one aspect of conformal symmetries and plays a key role in the understanding of elementary particle vacuum functionals and the description of family replication phenomenon in terms of the topology of partonic 2-surfaces.

4. WCW spinors define a von Neumann algebra known as hyper-finite factor of type II_1 (HFFs). This realization has led also to a profound generalization of quantum TGD through a generalization of the notion of embedding space to characterize quantum criticality. The resulting space has a book like structure with various almost-copies of embedding space representing the pages of the book meeting at quantum critical sub-manifolds. The outcome of this approach is that the exponents of Kähler function and Chern-Simons action are not fundamental objects but reduce to the Dirac determinant associated with the Kähler-Dirac operator assigned to the light-like 3-surfaces.

5.3.2 The Notions Of Embedding Space, 3-Surface, And Configuration Space

The notions of embedding space, 3-surface (and 4-surface), and WCW (world of classical worlds (WCW)) are central to quantum TGD. The original idea was that 3-surfaces are space-like 3-surfaces of $H = M^4 \times CP_2$ or $H = M^4_+ \times CP_2$, and WCW consists of all possible 3-surfaces in H. The basic idea was that the definition of Kähler metric of WCW assigns to each X^3 a unique space-time surface $X^4(X^3)$ allowing in this manner to realize general coordinate invariance. During years these notions have however evolved considerably.

The notion of embedding space

Two generalizations of the notion of embedding space were forced by number theoretical vision [K104, K105, K103].

1. p-Adicization forced to generalize the notion of embedding space by gluing real and p-adic variants of embedding space together along rationals and common algebraic numbers. The generalized embedding space has a book like structure with reals and various p-adic number fields (including their algebraic extensions) representing the pages of the book.
2. With the discovery of zero energy ontology [K121, K27] it became clear that the so called causal diamonds (CDs) interpreted as intersections $M^4_+ \cap M^4_-$ of future and past directed light-cones of $M^4 \times CP_2$ define correlates for the quantum states. The position of the "lower" tip of CD characterizes the position of CD in H. If the temporal distance between upper and lower tip of CD is quantized in power-of-two multiples of CP_2 length, p-adic length scale hypothesis [K74] follows as a consequence. The upper *resp.* lower light-like boundary $\delta M^4_+ \times CP_2$ *resp.* $\delta M^4_- \times CP_2$ of CD can be regarded as the carrier of positive *resp.* negative energy part of the state. All net quantum numbers of states vanish so that everything is creatable from vacuum. Space-time surfaces assignable to zero energy states would would reside inside $CD \times CP_2$s and have their 3-D ends at the light-like boundaries of $CD \times CP_2$. Fractal structure is present in the sense that CDs can contains CDs within CDs, and measurement resolution dictates the length scale below which the sub-CDs are not visible.
3. The realization of the hierarchy of Planck constants [K38] suggests a further generalization of the notion of embedding space, which has however turned out to be an auxiliary tool only.

   Generalized embedding space would be obtained by gluing together Cartesian products of singular coverings and factor spaces of CD and CP_2 to form a book like structure. The particles at different pages of this book behave like dark matter relative to each other. This generalization also brings in the geometric correlate for the selection of quantization axes in the sense that the geometry of the sectors of the generalized embedding space with non-standard value of Planck constant involves symmetry breaking reducing the isometries to Cartan subalgebra. Roughly speaking, each CD and CP_2 is replaced with a union of CDs and CP_2s corresponding to different choices of quantization axes so that no breaking of Poincare and color symmetries occurs at the level of entire WCW .

   It is now clear that this generealization only provides a description for the non-determinism realized in terms of n conformal equivalences of preferred extremals connecting 3-surfaces at the opposite boundaries of CD.
4. The construction of quantum theory at partonic level brings in very important delicacies related to the Kähler gauge potential of CP_2. Kähler gauge potential must have what one might call pure gauge parts in M^4 in order that the theory does not reduce to mere topological quantum field theory. Hence the strict Cartesian product structure $M^4 \times CP_2$ breaks down in a delicate manner. These additional gauge components -present also in CP_2- play key role in the model of anyons, charge fractionization, and quantum Hall effect [K80].

The notion of 3-surface

The question what one exactly means with 3-surface turned out to be non-trivial.

1. The original identification of 3-surfaces was as arbitrary space-like 3-surfaces subject to equivalence believed to be implied by General Coordinate Invariance. There was a problem related to the realization of equivalence since it was not at all obvious why the preferred extremal $X^4(Y^3)$ for Y^3 at $X^4(X^3)$ and Diff^4 related X^3 should satisfy $X^4(Y^3) = X^4(X^3)$.
2. Much later it became clear that light-like 3-surfaces identified as boundaries between regions of Minkowskian and Euclidian signature (wormhole contacts and exterior) have unique properties for serving as basic dynamical objects, in particular for realizing the General Coordinate Invariance in 4-D sense (obviously the identification resolves the above mentioned problem) and understanding the conformal symmetries of the theory.

   The condition that light-like parton orbits and space-like 3-surfaces at the ends of CD are physically equivalent allows to conclude that partonic 2-surfaces and their tangent space data should be enough for physics. One would have strong form of General Coordinate Invariance (GCI) and strong form of holography. The condition that the symplectic Noether charges for the above mentioned sub-algebra of the symplectic algebra vanish for space-like 3-surfaces at the ends of CD would be natural in this framework.

It is however important to emphasize that this indeed holds true only locally. At the level of WCW metric this means that the components of the Kähler form and metric can be expressed in terms of data assignable to 2-D partonic surfaces. It is however essential that information about normal space of the 2-surface is needed.

3. An important step of progress was the realization that light-like 3-surfaces can have singular topology in the sense that they are analogous to Feynman diagrams. The light-like 3-surfaces representing lines of Feynman diagram can be glued along their 2-D ends playing the role of vertices to form what I call generalized Feynman diagrams ("Feynman" could be replaced with twistor, or braid, or something else). The ends of lines are located at boundaries of sub-CDs. This brings in also a hierarchy of time scales: the increase of the measurement resolution means introduction of sub-CDs containing sub-Feynman diagrams. As the resolution is improved, new sub-Feynman diagrams emerge so that effective 2-D character holds true in discretized sense and in given resolution scale only.

The notion of space-time surface

The basic vision has been that space-time surfaces correspond to preferred extremals $X^4(X^3)$ of Kähler action. Kähler function $K(X^3)$ defining the Kähler geometry of the world of classical worlds would correspond to the Kähler action for the preferred extremal. The precise identification of the preferred extremals turned out to be far from trivial. The recent discussion of this topic can be found at [K8].

1. The obvious first guess motivated by physical intuition was that preferred extremals correspond to the absolute minima of Kähler action for space-time surfaces containing X^3. This choice would have some nice implications. For instance, one can develop an argument for the existence of an infinite number of conserved charges. If X^3 is light-like surface- either light-like boundary of X^4 or light-like 3-surface assignable to a wormhole throat at which the induced metric of X^4 changes its signature- this identification circumvents the obvious objections.

   This choice might well be correct for (non-negative) Kähler function identifiable as Kähler action in Euclidian space-time regions (wormhole contacts). In Minkowskian regions Kähler action is imaginary ($\sqrt{g}$ factor is imaginary) and gives a complex phase to vacuum functional and clearly serves as the analog of action in quantum field theories. The identification as preferred extremal does not look natural now.

2. The recent identification has been already described: the vanishing of symplectic Noether charges in a sub-algebra isomorphic to the entire algebra would define the conformal gauge and fix the preferred extremals in ZEO highly uniquely. For a generic pair of 3-surfaces at the boundaries of CD it is not clear whether any preferred extremal exists. The non-determinism of Kähler action makes it difficult to make any conclusions in this respect.

3. I have consider many other identifications of preferred extremals during years. In Minkowskian regions the contraction $j \cdot A$ of Kähler current and Kähler gauge potential vanishes for the known extremals. Together with the weak form of electric-magnetic duality stating $\epsilon_{ijnt}J^{nt} = kJ_{ij}$, k proportionality constant, this condition would reduce Kähler action to 3-D Chern-Simons terms. This would realize TGD as almost topological QFT. Whether this condition makes sense in Euclidian regions and whether it is strong enough remains an open question.

   The construction of WCW geometry suggests also the strengthening the boundary conditions to the condition that there exists space-time coordinates in which the induced CP_2 Kähler form and induced metric satisfy the conditions $J_{ni} = 0$, $g_{ni} = 0$ hold at X^3_l (n denote normal direction). One could say that at X^3_l situation is static both metrically and for the Maxwell field defined by the induced Kähler form. There are reasons to hope that this is the final step in a long process.

4. One possible identification of preferred extremals would be as quaternionic sub-manifolds of embedding space with the property that quaternionic tangent space at given point contains a preferred M^2 identifiable as a commutative sub-space of quaternionic tangent spaces. One can also consider the possibility that M^2 depends on the point of space-time surface but that one has an integrable distribution defining string world sheet in M^4: this leads to the notion of Hamilton-Jacobi structure [K8]. M^8-H duality allowing to map surfaces of M^8 with this property to surfaces in M^8 by mapping the local tangent space to a point of CP_2 relates closely to this proposal.

5. The localization of the modes of Kähler-Dirac equation to string world sheets with vanishing W fields (to guarantee well-defined em charge for the modes) requires that Frobenius integrability conditions are satisfied for the 2-D tangent spaces and that the energy momentum currents as vectors of X^4 have no components normal to the string world sheet. I remains to be proven that these conditions can be satisfied.

   This suggests that one should construct preferred extremals as a concrete realization of holography. One would start from data given by string world sheets and partonic 2-surfaces and possibly also space-like 3-surface and the light-like orbits of partonic 2-surfaces by posing the conditions that sub-algebra of symplectic algebra acts as gauge algera. The reason for fixing of 3-surfaces apart from symplectic gauge transformation in an appropriate sub-algebra is that otherwise the possibility of strings and their orbits to get knotted and linked becomes impossible to describe. One clearly would have effective 2-dimensionality.

   According to the recent view about Kähler-Dirac action the boundaries of string world sheets are embedding space geodesics characterizing by light-like 8-momentum. This suggests that the braiding along partonic orbits is probably possible only if one allows intermediate partonic 2-surfaces in which the direction of four-momentum changes. The particle physics interpretation would be that braiding must respect conservation of momentum and thus occurs by exchange of say bosonic quanta. So that braiding diagram would be replaced by the analog of Feynman diagram.

6. One bundle of ideas relates is inpired by basic thinking about massless fields and relies on the observation that the known extremals seems to decompose in Minkowskian regions to pieces having interpretation as classical analogs of massless field quanta allowing local polarization vector and light-like 4-momentum vector orthogonal to each other. The simplest example is provided by massless extremals for which one has linear superposition of modes in the direction of four-momentum. One has therefore very quantal behavior already classically. In particular, linear superposition fails and can be realized only for effects experienced by a particle like 3-surface topologically condensed to several space-time sheets. At GRT-QFT limit superposition of effects becomes superposition of fields when the many-sheeted space-time is approximated with slightly curved M^4.

   Also number theoretical vision led to a related proposal that $X^4(X^3_{l,i})$, where $X^3_{l,i}$ denotes i^th connected component of the light-like 3-surface X^3_l, contain in their 4-D tangent space $T(X^4(X^3_{l,i}))$ a subspace $M^2_i \subset M^4$ having interpretation as the plane of non-physical polarizations. This means a close connection with super string models. Geometrically this would mean that the deformations of 3-surface in the plane of non-physical polarizations would not contribute to the line element of WCW . This is as it must be since complexification does not make sense in M^2 degrees of freedom.

   In number theoretical framework M^2_i has interpretation as a preferred hyper-complex subspace of hyper-octonions defined as 8-D subspace of complexified octonions with the property that the metric defined by the octonionic inner product has signature of M^8. A stronger condition would be that the condition holds true at all points of $X^4(X^3)$ for a global choice M^2 but this is un-necessary and leads to strong un-proven conjectures. The condition $M^2_i \subset T(X^4(X^3_{l,i}))$ in principle fixes the tangent space at $X^3_{l,i}$, and one has good hopes that the boundary value problem is well-defined and fixes $X^4(X^3)$ uniquely as a preferred extremal of Kähler action. This picture is rather convincing since the choice $M^2_i \subset M^3$ plays also other important roles.

7. The weakest form of number theoretic compactification states that light-like 3-surfaces $X^3 \subset X^4(X^3) \subset M^8$, where $X^4(X^3)$ hyper-quaternionic surface in hyper-octonionic M^8 can be

mapped to light-like 3-surfaces $X^3 \subset X^4(X^3) \subset M^4 \times CP_2$, where $X^4(X^3)$ is now preferred extremum of Kähler action. The natural guess is that $X^4(X^3) \subset M^8$ is a preferred extremal of Kähler action associated with Kähler form of E^4 in the decomposition $M^8 = M^4 \times E^4$, where M^4 corresponds to hyper-quaternions. The conjecture would be that the value of the Kähler action in M^8 is same as in $M^4 \times CP_2$.

The notion of WCW

From the beginning there was a problem related to the precise definition of WCW ("world of classical worlds" (WCW)). Should one regard CH as the space of 3-surfaces of $M^4 \times CP_2$ or $M^4_+ \times CP_2$ or perhaps something more delicate.

1. For a long time I believed that the question "M^4_+ or M^4?" had been settled in favor of M^4_+ by the fact that M^4_+ has interpretation as empty Roberson-Walker cosmology. The huge conformal symmetries assignable to $\delta M^4_+ \times CP_2$ were interpreted as cosmological rather than laboratory symmetries. The work with the conceptual problems related to the notions of energy and time, and with the symmetries of quantum TGD, however led gradually to the realization that there are strong reasons for considering M^4 instead of M^4_+.
2. With the discovery of zero energy ontology it became clear that the so called causal diamonds (CDs) define excellent candidates for the fundamental building blocks of WCW or "world of classical worlds" (WCW). The spaces $CD \times CP_2$ regarded as subsets of H defined the sectors of WCW .
3. This framework allows to realize the huge symmetries of $\delta M^4_\pm \times CP_2$ as isometries of WCW . The gigantic symmetries associated with the $\delta M^4_\pm \times CP_2$ are also laboratory symmetries. Poincare invariance fits very elegantly with the two types of super-conformal symmetries of TGD. The first conformal symmetry corresponds to the light-like surfaces $\delta M^4_\pm \times CP_2$ of the embedding space representing the upper and lower boundaries of CD. Second conformal symmetry corresponds to light-like 3-surface X^3_l, which can be boundaries of X^4 and light-like surfaces separating space-time regions with different signatures of the induced metric. This symmetry is identifiable as the counterpart of the Kac Moody symmetry of string models.

A rather plausible conclusion is that WCW is a union of sub- WCW s associated with the spaces $CD \times CP_2$. CDs can contain CDs within CDs so that a fractal like hierarchy having interpretation in terms of measurement resolution results. Since the complications due to p-adic sectors and hierarchy of Planck constants are not relevant for the basic construction, it reduces to a high degree to a study of a simple special case $\delta M^4_+ \times CP_2$.

5.3.3 Could The Universe Be Doing Yangian Arithmetics?

One of the old TGD inspired really crazy ideas about scattering amplitudes is that Universe is doing some sort of arithmetics so that scattering amplitude are representations for computational sequences of minimum length. The idea is so crazy that I have even given up its original form, which led to an attempt to assimilate the basic ideas about bi-algebras, quantum groups [K11], Yangians [L8], and related exotic things. The work with twistor Grassmannian approach inspired a reconsideration of the original idea seriously with the idea that super-symplectic Yangian could define the arithmetics. I try to describe the background, motivation, and the ensuing reckless speculations in the following.

Do scattering amplitudes represent quantal algebraic manipulations?

I seems that tensor product $\otimes$ and direct sum $\oplus$ - very much analogous to product and sum but defined between Hilbert spaces rather than numbers - are naturally associated with the basic vertices of TGD. I have written about this a highly speculative chapter - both mathematically and physically [K76]. The chapter [K11] is a remnant of earlier similar speculations.

1. In $\otimes$ vertex 3-surface splits to two 3-surfaces meaning that the 2 "incoming" 4-surfaces meet at single common 3-surface and become the outgoing 3-surface: 3 lines of Feynman diagram meeting at their ends. This has a lower-dimensional shadow realized for partonic 2-surfaces. This topological 3-particle vertex would be higher-D variant of 3-vertex for Feynman diagrams.

2. The second vertex is trouser vertex for strings generalized so that it applies to 3-surfaces. It does not represent particle decay as in string models but the branching of the particle wave function so that particle can be said to propagate along two different paths simultaneously. In double slit experiment this would occur for the photon space-time sheets.

3. The idea is that Universe is doing arithmetics of some kind in the sense that particle 3-vertex in the above topological sense represents either multiplication or its time-reversal co-multiplication.

The product, call it $\circ$, can be something very general, say algebraic operation assignable to some algebraic structure. The algebraic structure could be almost anything: a random list of structures popping into mind consists of group, Lie-algebra, super-conformal algebra quantum algebra, Yangian, etc.... The algebraic operation $\circ$ can be group multiplication, Lie-bracket, its generalization to super-algebra level, etc...). Tensor product and thus linear (Hilbert) spaces are involved always, and in product operation tensor product $\otimes$ is replaced with $\circ$.

1. The product $A_k \otimes A_l \to C = A_k \circ A_l$ is analogous to a particle reaction in which particles A_k and A_l fuse to particle $A_k \otimes A_l \to C = A_k \circ A_l$. One can say that $\otimes$ between reactants is transformed to $\circ$ in the particle reaction: kind of bound state is formed.

2. There are very many pairs A_k, A_l giving the same product C just as given integer can be divided in many ways to a product of two integers if it is not prime. This of course suggests that elementary particles are primes of the algebra if this notion is defined for it! One can use some basis for the algebra and in this basis one has $C = A_k \circ A_l = f_{klm} A_m$, f_{klm} are the structure constants of the algebra and satisfy constraints. For instance, associativity $A(BC) = (AB)C$ is a constraint making the life of algebraist more tolerable and is almost routinely assumed.

   For instance, in the number theoretic approach to TGD associativity is proposed to serve as fundamental law of physics and allows to identify space-time surfaces as 4-surfaces with associative (quaternionic) tangent space or normal space at each point of octonionic embedding space $M^4 \times CP_2$. Lie algebras are not associative but Jacobi-identities following from the associativity of Lie group product replace associativity.

3. Co-product can be said to be time reversal of the algebraic operation $\circ$. Co-product can be defined as $C = A_k \to \sum_{lm} f_k^{lm} A_l \otimes A_m$, where f_k^{lm} are the structure constants of the algebra. The outcome is quantum superposition of final states, which can fuse to C (the "reaction" $A_k \otimes A_l \to C = A_k \circ A_l$ is possible). One can say that $\circ$ is replaced with $\otimes$: bound state decays to a superposition of all pairs, which can form the bound states by product vertex.

There are motivations for representing scattering amplitudes as sequences of algebraic operations performed for the incoming set of particles leading to an outgoing set of particles with particles identified as algebraic objects acting on vacuum state. The outcome would be analogous to Feynman diagrams but only the diagram with minimal length to which a preferred extremal can be assigned is needed. Larger ones must be equivalent with it.

The question is whether it could be indeed possible to characterize particle reactions as computations involving transformation of tensor products to products in vertices and co-products to tensor products in co-vertices (time reversals of the vertices). A couple of examples gives some idea about what is involved.

1. The simplest operations would preserve particle number and to just permute the particles: the permutation generalizes to a braiding and the scattering matrix would be basically unitary braiding matrix utilized in topological quantum computation.

2. A more complex situation occurs, when the number of particles is preserved but quantum numbers for the final state are not same as for the initial state so that particles must interact. This requires both product and co-product vertices. For instance, $A_k \otimes A_l \to f_{kl}^m A_m$ followed by $A_m \to f_m^{rs} A_r \otimes A_s$ giving $A_k \to f_{kl}^m f_m^{rs} A_r \otimes A_s$ representing 2-particle scattering. State function reduction in the final state can select any pair $A_r \otimes A_s$ in the final state. This reaction is characterized by the ordinary tree diagram in which two lines fuse to single line and defuse back to two lines. Note also that there is a non-deterministic element involved. A given final state can be achieved from a given initial state after large enough number of

trials. The analogy with problem solving and mathematical theorem proving is obvious. If the interpretation is correct, Universe would be problem solver and theorem prover!

3. More complex reactions affect also the particle number. 3-vertex and its co-vertex are the simplest examples and generate more complex particle number changing vertices. For instance, on twistor Grassmann approach on can construct all diagrams using two 3-vertices. This encourages the restriction to 3-vertice (recall that fermions have only 2-vertices)
4. Intuitively it is clear that the final collection of algebraic objects can be reached by a large - maybe infinite - number of ways. It seems also clear that there is the shortest manner to end up to the final state from a given initial state. Of course, it can happen that there is no way to achieve it! For instance, if $\circ$ corresponds to group multiplication the co-vertex can lead only to a pair of particles for which the product of final state group elements equals to the initial state group element.
5. Quantum theorists of course worry about unitarity. How can avoid the situation in which the product gives zero if the outcome is element of linear space. Somehow the product should be such that this can be avoided. For instance, if product is Lie-algebra commutator, Cartan algebra would give zero as outcome.

Generalized Feynman diagram as shortest possible algebraic manipulation connecting initial and final algebraic objects

There is a strong motivation for the interpretation of generalized Feynman diagrams as shortest possible algebraic operations connecting initial and final states. The reason is that in TGD one does not have path integral over all possible space-time surfaces connecting the 3-surfaces at the ends of CD. Rather, one has in the optimal situation a space-time surface unique apart from conformal gauge degeneracy connecting the 3-surfaces at the ends of CD (they can have disjoint components).

Path integral is replaced with integral over 3-surfaces. There is therefore only single minimal generalized Feynman diagram (or twistor diagram, or whatever is the appropriate term). It would be nice if this diagram had interpretation as the shortest possible computation leading from the initial state to the final state specified by 3-surfaces and basically fermionic states at them. This would of course simplify enormously the theory and the connection to the twistor Grassmann approach is very suggestive. A further motivation comes from the observation that the state basis created by the fermionic Clifford algebra has an interpretation in terms of Boolean quantum logic and that in ZEO the fermionic states would have interpretation as analogs of Boolean statements $A \rightarrow B$.

To see whether and how this idea could be realized in TGD framework, let us try to find counterparts for the basic operations $\otimes$ and $\circ$ and identify the algebra involved. Consider first the basic geometric objects.

1. Tensor product could correspond geometrically to two disjoint 3-surfaces representing 3-particles. Partonic 2-surfaces associated with a given 3-surface represent second possibility. The splitting of a partonic 2-surface to two could be the geometric counterpart for co-product.
2. Partonic 2-surfaces are however connected to each other and possibly even to themselves by strings. It seems that partonic 2-surface cannot be the basic unit. Indeed, elementary particles are identified as pairs of wormhole throats (partonic 2-surfaces) with magnetic monopole flux flowing from throat to another at first space-time sheet, then through throat to another sheet, then back along second sheet to the lower throat of the first contact and then back to the thirst throat. This unit seems to be the natural basic object to consider. The flux tubes at both sheets are accompanied by fermionic strings. Whether also wormhole throats contain strings so that one would have single closed string rather than two open ones, is an open question.
3. The connecting strings give rise to the formation of gravitationally bound states and the hierarchy of Planck constants is crucially involved. For elementary particle there are just two wormhole contacts each involving two wormhole throats connected by wormhole contact. Wormhole throats are connected by one or more strings, which define space-like boundaries of corresponding string world sheets at the boundaries of CD. These strings are responsible for the formation of bound states, even macroscopic gravitational bound states.

Does super-symplectic Yangian define the arithmetics?

Super-symplectic Yangian would be a reasonable guess for the algebra involved.

1. The 2-local generators of Yangian would be of form $T_1^A = f^A_{BC} T^B \otimes T^C$, where f^A_{BC} are the structure constants of the super-symplectic algebra. n-local generators would be obtained by iterating this rule. Note that the generator T_1^A creates an entangled state of T^B and T^C with f^A_{BC} the entanglement coefficients. T_n^A is entangled state of T^B and T^C_{n-1} with the same coefficients. A kind replication of T^A_{n-1} is clearly involved, and the fundamental replication is that of T^A. Note that one can start from any irreducible representation with well defined symplectic quantum numbers and form similar hierarchy by using T^A and the representation as a starting point.

   That the hierarchy T_n^A and hierarchies irreducible representations would define a hierarchy of states associated with the partonic 2-surface is a highly non-trivial and powerful hypothesis about the formation of many-fermion bound states inside partonic 2-surfaces.

2. The charges T^A correspond to fermionic and bosonic super-symplectic generators. The geometric counterpart for the replication at the lowest level could correspond to a fermionic/bosonic string carrying super-symplectic generator splitting to fermionic/bosonic string and a string carrying bosonic symplectic generator T^A. This splitting of string brings in mind the basic gauge boson-gauge boson or gauge boson-fermion vertex.

   The vision about emission of virtual particle suggests that the entire wormhole contact pair replicates. Second wormhole throat would carry the string corresponding to T^A assignable to gauge boson naturally. T^A should involve pairs of fermionic creation and annihilation operators as well as fermionic and anti-fermionic creation operator (and annihilation operators) as in quantum field theory.

3. Bosonic emergence suggests that bosonic generators are constructed from fermion pairs with fermion and anti-fermion at opposite wormhole throats: this would allow to avoid the problems with the singular character of purely local fermion current. Fermionic and anti-fermionic string would reside at opposite space-time sheets and the whole structure would correspond to a closed magnetic tube carrying monopole flux. Fermions would correspond to superpositions of states in which string is located at either half of the closed flux tube.

4. The basic arithmetic operation in co-vertex would be co-multiplication transforming T_n^A to $T^A_{n+1} = f^A_{BC} T_n^B \otimes T^C$. In vertex the transformation of T^A_{n+1} to T_n^A would take place. The interpretations would be as emission/absorption of gauge boson. One must include also emission of fermion and this means replacement of T^A with corresponding fermionic generators F^A, so that the fermion number of the second part of the state is reduced by one unit. Particle reactions would be more than mere braidings and re-grouping of fermions and anti-fermions inside partonic 2-surfaces, which can split.

5. Inside the light-like orbits of the partonic 2-surfaces there is also a braiding affecting the M-matrix. The arithmetics involved would be therefore essentially that of measuring and "co-measuring" symplectic charges.

   Generalized Feynman diagrams (preferred extremals) connecting given 3-surfaces and many-fermion states (bosons are counted as fermion-anti-fermion states) would have a minimum number of vertices and co-vertices. The splitting of string lines implies creation of pairs of fermion lines. Whether regroupings are part of the story is not quite clear. In any case, without the replication of 3-surfaces it would not be possible to understand processes like e-e scattering by photon exchange in the proposed picture.

It is easy to hear the comments of the skeptic listener in the back row.

1. The attribute "minimal" - , which could translate to minimal value of Kähler function - is dangerous. It might be very difficult to determine what the minimal diagram is - consider only travelling salesman problem or the task of finding the shortest proof of theorem. It would be much nicer to have simple calculational rules.

   The original proposal might help here. The generalization of string model duality was in question. It stated that it is possible to move the positions of the vertices of the diagrams just as one does to transform s-channel resonances to t-channel exchange. All loops of generalized diagrams could be be eliminated by transforming the to tadpoles and snipped away so that only tree diagrams would be left. The variants of the diagram were identified as different continuation paths between different paths connecting sectors of WCW corresponding to different 3-topologies. Each step in the continuation procedure would involve product or co-product defining what continuation between two sectors means for WCW spinors. The continuations between two states require some minimal number of steps. If this is true, all computations connecting identical states are also physically equivalent. The value of the vacuum functional be same for all of them. This looks very natural.

That the Kähler action should be same for all computational sequences connecting the same initial and final states looks strange but might be understood in terms of the vacuum degeneracy of Kähler action closed related to quantum criticality, which means infinite gauge degeneracy associated with the Yangian of a sub-algebra of super-symplectic algebra.

2. QFT perturbation theory requires that should have superposition of computations/continuations. What could the superposition of QFT diagrams correspond to in TGD framework?

   Could it correspond to a superposition of generators of the Yangian creating the physical state? After all, already quantum computer perform superpositions of computations. The fermionic state would not be the simplest one that one can imagine. Could AdS/CFT analogy allow to identify the vacuum state as a superposition of multi-string states so that single super-symplectic generator would be replaced with a superposition of its Yangian counterparts with same total quantum numbers but with a varying number of strings? The weight of a given superposition would be given by the total effective string world sheet area. The sum of diagrams would emerge from this superposition and would basically correspond to functional integration in WCW using exponent of Kähler action as weight. The stringy functional integral ("functional" if also wormhole contacts contain string portion, otherwise path integral) would give the perturbation theory around given string world sheet. One would have effective reduction of string theory.

How does this relate to the ordinary perturbation theory?

One can of course worry about how to understand the basic results of the usual perturbation theory in this picture. How does one obtain a perturbation theory in powers of coupling constant, what does running coupling constant mean, etc...? I have already discussed how the superposition of diagrams could be understood in the new picture.

1. The QFT picture with running coupling constant is expected at QFT limit, when many-sheeted space-time is replaced with a slightly curved region of M^4 and gravitational field and gauge potentials are identified as sums of the deviations of induced metric from M^4 metric and classical induced gauge potentials associated with the sheets of the many-sheeted space-time. The running coupling constant would be due to the dependence of the size scale of CD, and p-adic coupling constant evolution would be behind the continuous one.

2. The notion of running coupling constant is very physical concept and should have a description also at the fundamental level and be due to a finite computational resolution, which indeed has very concrete description in terms of Noether charges of super-symplectic Yangian creating the states at the ends of space-time surface at the boundaries of CD. The space-time surface and the diagram associated with a given pair of 3-surfaces and stringy Noether charges associated with them can be characterized by a complexity measured in terms of the number of vertices (3-surface at which three 3-surfaces meet).

   For instance, 3-particle scattering can be possible only by using the simplest 3-vertex defined by product or co-product for pairs of 3-surfaces. In the generic case one has more complex diagram and what looks first 3-particle vertex has complex substructure rather than being simple product or co-product.

3. Complexity seems to have two separate aspects: the complexities of the positive and negative parts of zero energy state as many-fermion states and the complexity of associated 3-surfaces. The generalization of AdS/CFT however suggests that once the string world sheets and partonic 2-surfaces appearing in the diagram have been fixed, the space-time surface itself is fixed. The principle also suggests that the fixing partonic 2-surface and the strings connecting them

at the boundaries of CD fixes the 3-surface apart from the action of sub-algebra of Yangian acting as gauge algebra (vanishing classical Noether charges). If one can determine the minimal sequence of allowed algebraic operation of Yangian connecting initial and final fermion states, one knows the minimum number of vertices and therefore the topological structure of the connecting minimal space-time surface.

4. In QFT spirit one could describe the finite measurement resolution by introducing effective 3-point vertex, which is need not be product/co-produce anymore. 3-point scattering amplitudes in general involve microscopic algebraic structure involving several vertices. One can however give up the nice algebraic interpretation and just talk about effective 3-vertex for practical purposes. Just as the QFT vertex described by running coupling constant decomposes to sum of diagrams, product/co-product in TGD could be replaced with effective product/co-product expressible as a longer computation. This would imply coupling constant evolution.

   Fermion lines could however remain as such since they are massless in 8-D sense and mass renormalization does not make sense.

   Similar practical simplification could be done the initial and final states to get rid of superposition of the Yangian generators with different numbers of strings ("cloud of virtual particles"). This would correspond to wave function renormalization.

The number of vertices and wormhole contact orbits serves as a measure for the complexity of the diagram.

1. Since fermion lines are associated with wormhole throats assignable with wormhole contacts identifiable as deformations CP_2 type vacuum extremals, one expects that the exponent of the Kähler function defining vacuum functional is in the first approximation the total CP_2 volume of wormhole contacts giving a measure for the importance of the contribution in functional integral. If it converges very rapidly only Gaussian approximation around maximum is needed.

2. Convergence depends on how large the fraction of volume of CP_2 is associated with a given wormhole contact. The volume is proportional to the length of the wormhole contact orbit. One expects exponential convergence with the number of fermion lines and their lengths for long lines. For short distances the exponential damping is small so that diagrams with microscopic structure of diagrams are needed and are possible. This looks like adding small scale details to the algebraic manipulations.

3. One must be of course be very cautious in making conclusions. The presence of $1/\alpha_K \propto h_{eff}$ in the exponent of Kähler function would suggest that for large values of h_{eff} only the 3-surfaces with smallest possible number of wormhole contact orbits contribute. On the other hand, the generalization of AdS/CFT duality suggests that Kähler action reducible to area of string world sheet in the effective metric defined by canonical momentum currents of Kähler action behaves as $\alpha_K^2 \propto 1/h_{eff}^2$. How $1/h_{eff}^2$ proportionality might be understood is discussed in [?] in terms electric-magnetic duality.

To sum up, the identification of vertex as a product or co-product in Yangian looks highly promising approach. The Nother charges of the super-symplectic Yangian are associated with strings and are either linear or bilinear in the fermion field. The fermion fields associated with the partonic 2-surface defining the vertex are contracted with fermion fields associated with other partonic 2-surfaces using the same rule as in Wick expansion in quantum field theories. The contraction gives fermion propagator for each leg pair associated with two vertices. Vertex factor is proportional to the contraction of spinor modes with the operators defining the Noether charge or super charge and essentially Kähler-Dirac gamma matrix and the representation of the action of the symplectic generator on fermion realizable in terms of sigma matrices. This is very much like the corresponding expression in gauge theories but with gauge algebra replaced with symplectic algebra. The possibility of contractions of creation and annihilation operator for fermion lines associated with opposite wormhole throats at the same partonic 2-surface (for Noether charge bilinear in fermion field) gives bosonic exchanges as lines in which the fermion lines turns in time direction: otherwise only regroupings of fermions would take place.

This was not the whole story yet

The proposed amplitude represents only the value of WCW spinor field for single pair of 3-surfaces at the opposite boundaries of given CD. Hence Yangian construction does not tell the whole story.

1. Yangian algebra would give only the vertices of the scattering amplitudes. On basis of previous considerations, one expects that each fermion line carries propagator defined by 8-momentum. The structure would resemble that of super-symmetric YM theory. Fermionic propagators should emerge from summing over intermediate fermion states in various vertices and one would have integrations over virtual momenta which are carried as residue integrations in twistor Grassmann approach. 8-D counterpart of twistorialization would apply.

2. Super-symplectic Yangian would give the scattering amplitudes for single space-time surface and the purely group theoretical form of these amplitudes gives hopes about the independence of the scattering amplitude on the pair of 3-surfaces at the ends of CD near the maximum of Kähler function. This is perhaps too much to hope except approximately but if true, the integration over WCW would give only exponent of Kähler action since metric and poorly defined Gaussian and determinants would cancel by the basic properties of Kähler metric. Exponent would give a non-analytic dependence on α_K.

   The Yangian supercharges are proportional to $1/\alpha_K$ since covariant Kähler-Dirac gamma matrices are proportional to canonical momentum currents of Kähler action and thus to $1/\alpha_K$. Perturbation theory in powers of $\alpha_K = g_K^2/4\pi\hbar_{eff}$ is possible after factorizing out the exponent of vacuum functional at the maximum of Kähler function and the factors $1/\alpha_K$ multiplying super-symplectic charges.

   The additional complication is that the characteristics of preferred extremals contributing significantly to the scattering amplitudes are expected to depend on the value of α_K by quantum interference effects. Kähler action is proportional to $1/\alpha_K$. The analogy of AdS/CFT correspondence states the expressibility of Kähler function in terms of string area in the effective metric defined by the anti-commutators of K-D matrices. Interference effects eliminate string length for which the area action has a value considerably larger than one so that the string length and thus also the minimal size of CD containing it scales as h_{eff}. Quantum interference effects therefore give an additional dependence of Yangian super-charges on h_{eff} leading to a perturbative expansion in powers of α_K although the basic expression for scattering amplitude would not suggest this.

5.4 Victories Of M-Theory From TGD View Point

The basic victories of the M-theory relate to conformal symmetries and dualities and black hole physics and it is useful perform comparison with TGD.

5.4.1 Super-Conformal Symmetries Of String Theory

Space-time super-symmetries are regarded as one of the basic predictions of the super string model. Typically these super-symmetries appear at the level of effective quantum field theory limit derived from spontaneous compactification and predict that massless particles possess massless super partners, sparticles. The problem has been how to generalize Higgs mechanism to break the space-time super-symmetry. That sparticles have relatively low mass scale has been seen as one of the absolute predictions of M-theory and the ability to predict at least something has been counted as a success. Since sparticles have hitherto escaped the attempts to detect them, even this belief has been now challenged, and proposals has been made that perhaps M-theory might after all predict sparticles to be very massive.

How TGD view about supersymmetries differs from the standard view?

TGD and standard views about super-symmetry differ in many respects.

1. The standard view is inspired by the mathematically awkward and formal idea of assigning to the space-time coordinates anti-commuting super part. The belief is that string world sheet

super-symmetries give rise to the space-time super symmetries of the low energy effective quantum field theory assigned to the string model.

2. In TGD the super-symmetry generators of the spectrum generating super-conformal algebra act as gamma matrices of WCW ("world of classical worlds"). Anti-commuting infinitesimals are encountered nowhere. Majorana spinors are not possible in TGD framework were B and L are conserved separately.

   The gamma matrices of WCW are identified as super-symplectic Noether super charges for induced spinor field restricted at partonic 2-surfaces and can be expressed as integral over string [K121]. This identification can be extended to the Yangian defined by multi-stringy generators of symplectic algebra. This identification implies the analog of AdS/CFT duality: one can express WCW Kähler metric either in terms of Kähler function or in terms of anti-commutators of second quantized spinor fields. The super generators of Yangian are natural candidates to the role of oscillator operators creating physical states.

3. The counterparts of the word sheet super-symmetries act as gauge super-symmetries at space-time level but it is far from clear whether they give rise to global space-time super-symmetries at the level of embedding space. The second quantized oscillator operators of induced spinor field give rise to Clifford algebra having the structure of SUSY algebra.

   It seems that the anti-commutators of oscillator operators can be chosen so that one can have super-Poincare invariance. These super-symmetries are however broken. If conformal invariance holds true one obtains $\mathcal{N} = 4 + 4$ SUSY corresponding to quark and lepton generators. Right-handed neutrino generates the least broken $\mathcal{N} = 2$ SUSY. It is possible that sparticles have same p-adic mass scale as particles but are dark in TGD sense so that they have non-standard value of Planck constant.

Super-conformal symmetries at the space-time level

There have been a considerable progress in the understanding of super-conformal symmetries [K28, K121]. It must be however admitted that there are still several possible scenarios depending on whether these symmetries act as gauge symmetries or dynamical symmetries.

1. Super-symplectic algebra corresponds to the isometries of "world of classical worlds" (WCW) constructed in terms covariantly constant right handed neutrino mode and second quantized induced spinor field Ψ and the corresponding Super-Kac-Moody algebra restricted to symplectic isometries and realized in terms of all spinor modes and Ψ is the most plausible identification of the superconformal algebras when the constraints from p-adic mass calculations are taken into account. These algebras act as dynamical rather than gauge algebras and related to the isometries of WCW .

2. Light-cone boundary is expressible as $\delta M^4_+ = R_+ \times S^2$ and is metrically 2-D . This implies that the conformal transformations of S^2 depending parametrically on the light-like radial coordinate r_M of R_+ act as conformal transformations. By selecting the radial dependence so that conformal scaling factors cancel, one obtains infinite-dimensional group of isometries isomorphic to the group of conformal transformations. Similar group of conformal symmetries and isometries is associated with the light-like orbits of partonic 2-surfaces. The interpretation of these groups has remained unclear but they are expected to play a key role. One possibility is that they act as gauge symmetries.

3. One expects also Kac-Moody type gauge symmetries due to the non-determinism of Kähler action. They transform to each other preferred extremals having fixed 3-surfaces as ends at the boundaries of the causal diamond. They preserve the value of Kähler action and those of conserved charges. The assumption is that there are n gauge equivalence classes of these surfaces and that n defines the value of the effective Planck constant $h_{eff} = n \times h$ in the effective GRT type description replacing many-sheeted space-time with single sheeted one.

4. An interesting question is whether the symplectic isometries of $\delta M^4_\pm \times CP_2$ should be extended to include all isometries of $\delta M^4_\pm = S^2 \times R_+$ in one-one correspondence with conformal transformations of S^2.The S^2 local scaling of the light-like radial coordinate r_M of R_+ compensates the conformal scaling of the metric coming from the conformal transformation of S^2. Also light-like 3-surfaces allow the analogs of these isometries.

The interpretation of the bosonic Kac Moody symmetries is as deformations preserving the light likeness of the light like 3-D CD X_l^3. Since general coordinate invariance corresponds to gauge degeneracy of the metric it is possible to consider reduced WCW consisting of the light like 3-D CDs. The conformal symmetries in question suggests strongly a further degeneracy of the WCW metric and effective metric 2-dimensionality of 3-surfaces. These conformal symmetries could accompanied by super conformal symmetries defined by the solutions of the induced spinor fields.

Could these conformal symmetries allow a continuation to quaternion conformal super symmetries in the interior of the space-time surface realized as real analytic power series of a quaternionic space-time coordinate?

1. At first glance the answer seems to be "No". The reason is that these symmetries involve both transversal complex coordinate and light like coordinate as independent variables whereas quaternion conformal symmetries are algebraically one-dimensional.
2. Somewhat surprisingly, it has however turned out that quaternionic analog of Riemann conditions characterizing analyticity allows also a variant, which corresponds to analyticity in two complex variables, which in Minkowskian signature would correspond to hyper-complex and complex coordinate [A83] [L8]. Also quaternion analyticity realized as powers series of quaternion is possible. The crucial trick is that the Taylor coefficients multiply powers of quaternionion from right whereas derivatives act from left. An interesting question is whether right- and left- analyticity have physical meaning and what the reflection like operation taking right-analytic to left analytic series could mean. Could it relate somehow to time reversal?

5.4.2 Dualities Of String Theories

The starting point of duality physics was the classical paper of Montonen and Olive about electric-magnetic Montonen which was generalized to what are known as S and T dualities in superstring context. The notion of duality is central also in TGD framework.

Dualities as victories of M-theory

Dualities [B48] allowing to unify various superstring models are regarded as basic victories of M-theory. The heuristic proofs for various dualities between various variants of superstring model that I have seen apply what might be called M-logic. Consider special examples defined by 11-dimensional super-gravity using a particular background and particular spontaneous compactification and demonstrate that these examples are consistent with the duality. Then generalize from special to general. For a non-specialist, it is difficult to decide, whether all this is just wishful thinking and clever choices of compactifications.

Mirror symmetry of Calabi-Yau manifolds

String theory has stimulated very general conjectures about the properties of Calabi-Yau manifolds, which have turned out to be correct. Calabi-Yau manifolds are 3-dimensional Kähler manifolds with $SU(3)$ (rather than $U(3)$) holonomy group and thus satisfy empty space Einstein equations implied by the requirement of the vanishing of conformal anomaly in closed super string models. The prediction of the mirror symmetry for Calabi-Yau manifolds [B9] emerged before the era of M-theory from the study of $N = 2$ super-conformal sigma models with Calabi-Yau manifold as a target space and closed string world sheet as the "space-time". In the 11-dimensional M-theory context Calabi-Yau manifolds are obtained only by a special compactification for which 11^{th} dimension corresponds to a circle. The argument taken from [B9] written in a physicist friendly manner runs as follows.

1. In conformal field theories the so called marginal operators correspond to the deformations of the original conformal field theory respecting the property of being a conformal field theory, and thus the criticality of the physical system. In particular, the deformations of complex and Kähler structures of the target space, now Calabi-Yau space, induce this kind of deformations. The basic finding was that the operators inducing these two kinds of deformations differ only by the opposite sign of their U(1) charge associated with the $U(1)$ current of $\mathcal{N} = 2$ supersymmetry algebra.

2. The mere change of the sign of $U(1)$ charge would correspond to a permutation of the spaces of complex and Kähler moduli, which means a rather drastic geometric and even a topological change. On the other hand, the physical change must be marginal since the system remains critical. Both signs of $U(1)$ charge seem highly plausible so that the hypothesis is that the Calabi-Yau manifolds appear a mirror pairs so that in a rough sense the moduli for Kähler and complex structures are permuted for the members of the mirror pair by performing a change of sign of $U(1)$ charge for the left moving modes of string. Actually a generalization of the notion of Kähler moduli is necessary. This is achieved by combining the Kähler form and antisymmetric field B defining a generalization of $U(1)$ gauge potential to form a imaginary and complex parts of a more general structure for which Kähler moduli space (Kähler cone) is complexified and by introducing so called extended Kähler cone combining the Kähler moduli associated with several Calabi-Yau spaces so that single Calabi-Yau manifold can have several mirrors [B9].

There are two implications. First, two different Calabi-Yau geometries and even topologies give rise to the same conformally invariant physics: the physics $\leftrightarrow$ geometry identification of General Relativity is not strictly true anymore. Secondly, the continuous change of the complex moduli for the Calabi-Yau manifold corresponds to a topology change for the mirror manifold so that even topology change corresponds to a quite smooth change of physics, in fact a change respecting 2-dimensional criticality. Even the possibility that the change involves a temporary contraction of the Calabi-Yau to a point during the change cannot be excluded [B9], which looks really weird. Also singular Calabi-Yau manifolds are possible and not mere limiting cases of non-singular ones [B9].

These implications might be also seen as a failure of the theory basically due to the spontaneous compactification trick. In TGD embedding space is fixed and similar phenomenon does not occur. The moduli space of conformal structures of the metrically 2-dimensional light like causal determinants effectively corresponding to closed string word sheets is however involved also now, and implies naturally the concept of elementary particle vacuum functional defined in the moduli space of complex structures characterizing the effectively 2-D induced metrics at causal determinants [K25]. The notion is essential for p-adic mass calculations and predicts correct ratios for electron, muon, and tau lepton masses [K59].

To conclude, the discovery of the mirror symmetry is quite beautiful and impressive but as such does not provide support for the super string theory as a physical theory. The discovery could have been made by a conformal field theorist interested in two-dimensional critical statistical systems.

The enormous mathematical significance of Calabi-Yau spaces is that the tools of algebraic geometry and complex analysis become available. Only quite recently (2014) it turned out that a generalization of twistor approach makes similar tools available in TGD framework. The lift of space-time surfaces to their twistor spaces imbedded to the product of 6-D twistor spaces of M^4 and CP_2 - the only twistor spaces that are Kähler manifolds - assumed to receive their twistor structure by induction suggests one further formulation of the preferred extremal property [L8].

Mirror symmetry means non-uniqueness of geometry-physics correspondence: several Calabi-Yau spaces describe the same physics. The analog of this phenomenon would be the existence of several lifts of a preferred extremal to its induced twistor space: this of course assuming that space-time surface code for the physics and twistor spaces are only an auxiliary tool. Note that in TGD the space-time surfaces connecting 3-surfaces at the ends of CD are non-unique if the recent view about symplectic symmetry holds true so that geometry-physics correspondence fails to be 1-1 also in this sense.

5.4.3 Dualities And Conformal Symmetries In TGD Framework

The reason for discussing the rather speculative notion of dualities before considering the definition of the Kähler-Dirac action and discussing the proposal how to define Kähler function in terms of Dirac determinants, is that the duality thinking gives the necessary overall view about the complex situation: even wrong vision is better than no vision at all.

The first candidate for a duality in TGD is electric-magnetic duality appearing in the construction of WCW geometry.

Weak form of electric-magnetic duality

CP_2 Kähler form is self-dual. This does not however hold for the induced Kähler form and the duality is replaced with weak form of electric magnetic duality posed only as a boundary condition stating that at the light-like orbits of partonic 2-surfaces defining the boundaries between Minkowskian and Euclidian space-time regions Kähler magnetic field is the dual of Kähler electric field. If this duality holds true, the proposed vanishing of the Coulombic contribution $j \cdot A$ to Kähler action density for preferred extremals would reduce Kähler action to mere 3-D Chern-Simons terms. This would reduce enormously difficult problem of identifying preferred extremals of Kähler action and calculating corresponding Kähler action to a local data at light-light-like 3-surfaces. A concrete realization of holography would be in question and one could speak about TGD as almost topological QFT.

Quantum gravitational holography

The so called AdS/CFT duality of Maldacena [B40] correspondence relates to quantum-gravitational holography states roughly that the gravitational theory formulated in terms string model in 10-dimensional $AdS_{10-n} \times S^n$ manifold is equivalent with the conformal field theory at the boundary of AdS_D factor, which is $D-1$-dimensional Minkowski space. This duality has been seen as a manifestation of a duality between super-gravity with Kaluza-Klein quantum numbers (closed strings) and super Yang-Mills theories (open strings with quantum numbers at the ends of string).

In TGD framework this duality is not enough since the super-conformal symmetries of TGD are gigantic as compared to those in string models. One obtains an analog of this duality.

1. The condition that the value of electromagnetic charge is well-defined for the modes of the induced spinor field implies the localization of the modes to 2-D surfaces - string world sheets and possibly also partonic 2-surfaces - with the property that the induced W field and above weak scale also the induced Z^0 field vanish. The fermionic sector of TGD is very similar to string model.
2. One can express WCW Kähler metric in two ways. In terms of second derivatives of Kähler function, a purely bosonic object given as Kähler action for magnetically charged wormhole contacts having Euclidian signature of the induced metric, or as anti-commutators of WCW gamma matrices identified as super-symplectic Noether super charges assignable to strings connecting partonic 2-surfaces. One must generalize the super charges to those of Yangian since multi-stringy objects are physically unavoidable.
3. This is highly non-trivial duality between purely bosonic degrees of freedom assignable to elementary particles (wormhole contacts server as their building bricks) and fermionic degrees of freedom assignable to string world sheets. Note that it is still not clear whether string world sheets are present also inside wormhole contacts or only in Minkowskian regions. Depending on this TGD would rely on open or closed strings.
4. This duality is expected to generalize. For instance, Kähler action should be expressible as string world sheet area in the effective metric defined by the anti-commutators of the Kähler-Dirac gamma matrices at string world sheet. Dirac determinant should be expressible as exponent of Kähler action and by almost topological QFT property of TGD as an exponent of Chern-Simons terms -at least in Minkowskian space-time regions.

Perhaps the most practical form of the quantum gravitational holography is implied by the generalized conformal invariance implying effective 2-dimensionality. This means that X_l^3 represent generalized Feynman diagrams with lines representing by light-like 3-surfaces and vertices as 2-surfaces $X^2 \subset \delta CD \times CP_2$ at which these lines meet. Vertices can be expressed as N-point functions of super-conformal field theory at these 2-surfaces. Only effective two-dimensionality is in question since one has hierarchy of CDs within CDs and improvement of measurement resolution brings into consideration CDs with smaller size. Effective 2-dimensionality obvious means quantum holography in lower dimensional sense and this sequence of holographies continues down to the level of number theoretic braids with information about M-matrix coded by a set of discrete points at partonic 2-surfaces X^2.

Computationally TGD would reduce to almost string model type theory since light like 3-surfaces are analogous to closed string word sheets on one hand, and to the ends of open string on the other hand. A possible sketch based on twistors and the idea about scattering amplitudes as algebraic computations was already formulated. There is also an analogy with the Wess-Zumino-Witten model: light like causal determinants would correspond to the 2-D space of WZW model and 4-surface to the associated 3-D space defining the central extension of the Kac-Moody algebra.

5.4.4 Number Theoretic Compactification And $M^8 - H$ Duality

This section summarizes the basic vision about number theoretic compactification reducing the classical dynamics to associativity or co-associativity. Originally $M^8 - H$ duality was introduced as a number theoretic explanation for $H = M^4 \times CP_2$. Much later it turned out that the completely exceptional twistorial properties of M^4 and CP_2 are enough to justify $X^4 \subset H$ hypothesis. Skeptic could therefore criticize the introduction of M^8 as an un-necessary mathematical complication producing only unproven conjectures and bundle of new statements to be formulated precisely.

One can question the feasibility of $M^8 - H$ duality if the dynamics is purely number theoretic at the level of M^8 and determined by Kähler action at the level of H. Situation becomes more democratic if Kähler action defines the dynamics in both M^8 and H: this might mean that associativity could imply field equations for preferred extremals or vice versa or there might be equivalence between two. This means the introduction Kähler structure at the level of M^8, and motivates also the coupling of Kähler gauge potential to M^8 spinors characterized by Kähler charge or em charge. One could call this form of duality strong form of $M^8 - H$ duality.

The strong form $M^8 - H$ duality boils down to the assumption that space-time surfaces can be regarded either as surfaces of H or as surfaces of M^8 composed of associative and co-associative regions identifiable as regions of space-time possessing Minkowskian *resp.* Euclidian signature of the induced metric. They have the same induced metric and Kähler form and WCW associated with H should be essentially the same as that associated with M^8. Associativity corresponds to hyper-quaterniocity at the level of tangent space and co-associativity to co-hyper-quaternionicity - that is associativity/hyper-quaternionicity of the normal space. Both are needed to cope with known extremals. Since in Minkowskian context precise language would force to introduce clumsy terms like hyper-quaternionicity and co-hyper-quaternionicity, it is better to speak just about associativity or co-associativity.

For the octonionic spinor fields the octonionic analogs of electroweak couplings reduce to mere Kḧler or electromagnetic coupling and the solutions reduce to those for spinor d'Alembertian in 4-D harmonic potential breaking $SO(4)$ symmetry. Due to the enhanced symmetry of harmonic oscillator, one expects that partial waves are classified by $SU(4)$ and by reduction to $SU(3) \times U(1)$ by em charge and color quantum numbers just as for CP_2 - at least formally.

Harmonic oscillator potential defined by self-dual em field splits M^8 to $M^4 \times E^4$ and implies Gaussian localization of the spinor modes near origin so that E^4 effectively compactifies. The The resulting physics brings strongly in mind low energy physics, where only electromagnetic interaction is visible directly, and one cannot avoid associations with low energy hadron physics. These are some of the reasons for considering $M^8 - H$ duality as something more than a mere mathematical curiosity.

Remark: The Minkowskian signatures of M^8 and M^4 produce technical nuisance. One could overcome them by Wick rotation, which is however somewhat questionable trick.

1. The proper formulation is in terms of complexified octonions and quaternions involving the introduction of commuting imaginary unit j. If complexified quaternions are used for H, Minkowskian signature requires the introduction of two commuting imaginary units j and i meaning double complexification.

2. Hyper-quaternions/octonions define as subspace of complexified quaternions/octonions spanned by real unit and jI_k, where I_k are quaternionic units. These spaces are obviously not closed under multiplication. One can however however define the notion of associativity for the subspace of M^8 by requiring that the products and sums of the tangent space vectors generate complexified quaternions.

3. Ordinary quaternions Q are expressible as $q = q_0 + q^k I_k$. Hyper-quaternions are expressible as $q = q_0 + jq^k I_k$ and form a subspace of complexified quaternions $Q_c = Q \oplus jQ$. Similar formula applies to octonions and their hyper counterparts which can be regarded as subspaces of complexified octonions $O \oplus jO$. Tangent space vectors of H correspond hyper-quaternions

$q_H = q_0 + jq^k I_k + jiq_2$ defining a subspace of doubly complexified quaternions: note the appearance of two imaginary units.

The recent definitions of associativity and M^8 duality has evolved slowly from in-accurate characterizations and there are still open questions.

1. Kähler form for M^8 implies unique decomposition $M^8 = M^4 \times E^4$ needed to define $M^8 - H$ duality uniquely. This forces to introduce also Kähler action, induced metric and induced Kähler form. Could strong form of duality meant that the space-time surfaces in M^8 and H have same induced metric and induced Kähler form? Could the WCW s associated with M^8 and H be identical with this assumption so that duality would provide different interpretations for the same physics?

2. One can formulate associativity in M^8 by introducing octonionic structure in tangent spaces or in terms of the octonionic representation for the induced gamma matrices. Does the notion have counterpart at the level of H as one might expect if Kähler action is involved in both cases? The analog of this formulation in H might be as quaternionic "reality" since tangent space of H corresponds to complexified quaternions: I have however found no acceptable definition for this notion.

   The earlier formulation is in terms of octonionic flat space gamma matrices replacing the ordinary gamma matrices so that the formulation reduces to that in M^8 tangent space. This formulation is enough to define what associativity means although one can protest. Somehow H is already complex quaternionic and thus associative. Perhaps this just what is needed since dynamics has two levels: *embedding space level* and *space-time level*. One must have embedding space spinor harmonics assignable to the ground states of super-conformal representations and quaternionicity and octonionicity of H tangent space would make sense at the level of space-time surfaces.

3. Whether the associativity using induced gamma matrices works is not clear for massless extremals (MEs) and vacuum extremals with the dimension of CP_2 projection not larger than 2.

4. What makes this notion of associativity so fascinating is that it would allow to iterate duality as a sequence $M^8 \to H \to H...$ by mapping the space-time surface to $M^4 \times CP_2$ by the same recipe as in case of M^8. This brings in mind the functional composition of O_c-real analytic functions (O_c denotes complexified octonions: complexification is forced by Minkowskian signature) suggested to produced associative or co-associative surfaces. The associative (co-associative) surfaces in M^8 would correspond to loci for vanishing of imaginary (real) part of octonion-real-analytic function.

It might be possible to define associativity in H also in terms of Kähler-Dirac gamma matrices defined by Kähler action (certainly not M^8).

1. All known extremals are associative or co-associative in H in this sense. This would also give direct correlation with the variational principle. For the known preferred extremals this variant is successful partially because the Kähler-Dirac gamma matrices need not span the entire tangent space. The space spanned by the Kähler-Dirac gammas is not necessarily tangent space. For instance for CP_2 type vacuum extremals the Kähler-Dirac gamma matrices are CP_2 gamma matrices plus an additional light-like component from M^4 gamma matrices.

   If the space spanned by Kähler-Dirac gammas has dimension D smaller than 3 co-associativity is automatic. If the dimension of this space is $D = 3$ it can happen that the triplet of gammas spans by multiplication entire octonionic algebra. For $D = 4$ the situation is of course non-trivial.

2. For Kähler-Dirac gamma matrices the notion of co-associativity can produce problems since Kähler-Dirac gamma matrices do not in general span the tangent space. What does co-associativity mean now? Should one replace normal space with orthogonal complement of the space spanned by Kähler-Dirac gamma matrices? Co-associativity option must be considered for $D = 4$ only. CP_2 type vacuum extremals provide a good example. In this case the Kähler-Dirac gamma matrices reduce to sums of ordinary CP_2 gamma matrices and ligt-like M^4 contribution. The orthogonal complement for the Kähler-Dirac gamma matrices consists

of dual light-like gamma matrix and two gammas orthogonal to it: this space is subspace of M^4 and trivially associative.

Basic idea behind $M^8 - M^4 \times CP_2$ duality

If four-surfaces $X^4 \subset M^8$ under some conditions define 4-surfaces in $M^4 \times CP_2$ indirectly, the spontaneous compactification of super string models would correspond in TGD to two different ways to interpret the space-time surface. This correspondence could be called number theoretical compactification or $M^8 - H$ duality.

The hard mathematical facts behind the notion of number theoretical compactification are following.

1. One must assume that M^8 has unique decomposition $M^8 = M^4 \times E^4$. This would be most naturally due to Kähler structure in E^4 defined by a self-dual Kähler form defining parallel constant electric and magnetic fields in Euclidian sense. Besides Kähler form there is vector field coupling to sigma matrix representing the analog of strong isospin: the corresponding octonionic sigma matrix however is imaginary unit times gamma matrix - say ie_1 in M^4 - defining a preferred plane M^2 in M^4. Here it is essential that the gamma matrices of E^4 defined in terms of octonion units commute to gamma matrices in M^4. What is involved becomes clear from the Fano triangle illustrating octonionic multiplication table.
2. The space of hyper-complex structures of the hyper-octonion space - they correspond to the choices of plane $M^2 \subset M^8$ - is parameterized by 6-sphere $S^6 = G^2/SU(3)$. The subgroup $SU(3)$ of the full automorphism group G_2 respects the a priori selected complex structure and thus leaves invariant one octonionic imaginary unit, call it e_1. Fixed complex structure therefore corresponds to a point of S^6.
3. Quaternionic sub-algebras of M^8 are parametrized by $G_2/U(2)$. The quaternionic sub-algebras of octonions with fixed complex structure (that is complex sub-space defined by real and preferred imaginary unit and parametrized by a point of S^6) are parameterized by $SU(3)/U(2) = CP_2$ just as the complex planes of quaternion space are parameterized by $CP_1 = S^2$. Same applies to hyper-quaternionic sub-spaces of hyper-octonions. $SU(3)$ would thus have an interpretation as the isometry group of CP_2, as the automorphism sub-group of octonions, and as color group. Thus the space of quaternionic structures can be parametrized by the 10-dimensional space $G_2/U(2)$ decomposing as $S^6 \times CP_2$ locally.
4. The basic result behind number theoretic compactification and $M^8 - H$ duality is that associative sub-spaces $M^4 \subset M^8$ containing a fixed commutative sub-space $M^2 \subset M^8$ are parameterized by CP_2. The choices of a fixed hyper-quaternionic basis $1, e_1, e_2, e_3$ with a fixed complex sub-space (choice of e_1) are labeled by $U(2) \subset SU(3)$. The choice of e_2 and e_3 amounts to fixing $e_2 \pm \sqrt{-1}e_3$, which selects the $U(2) = SU(2) \times U(1)$ subgroup of $SU(3)$. $U(1)$ leaves 1 invariant and induced a phase multiplication of e_1 and $e_2 \pm e_3$. $SU(2)$ induces rotations of the spinor having e_2 and e_3 components. Hence all possible completions of $1, e_1$ by adding e_2, e_3 doublet are labeled by $SU(3)/U(2) = CP_2$.

Consider now the formulation of $M^8 - H$ duality.

1. The idea of the standard formulation is that associative manifold $X^4 \subset M^8$ has at its each point associative tangent plane. That is X^4 corresponds to an integrable distribution of $M^2(x) \subset M^8$ parametrized 4-D coordinate x that is map $x \to S^6$ such that the 4-D tangent plane is hyper-quaternionic for each x.
2. Since the Kähler structure of M^8 implies unique decomposition $M^8 = M^4 \times E^4$, this surface in turn defines a surface in $M^4 \times CP_2$ obtained by assigning to the point of 4-surface point $(m, s) \in H = M^4 \times CP_2$: $m \in M^4$ is obtained as *projection* $M^8 \to M^4$ (this is modification to the earlier definition) and $s \in CP_2$ parametrizes the quaternionic tangent plane as point of CP_2. Here the local decomposition $G_2/U(2) = S^6 \times CP_2$ is essential for achieving uniqueness.
3. One could also map the associative surface in M^8 to surface in 10-dimensional $S^6 \times CP_2$. In this case the metric of the image surface cannot have Minkowskian signature and one cannot assume that the induced metrics are identical. It is not known whether S^6 allows genuine complex structure and Kähler structure which is essential for TGD formulation.

4. Does duality imply the analog of associativity for $X^4 \subset H$? The tangent space of H can be seen as a sub-space of doubly complexified quaternions. Could one think that quaternionic sub-space is replaced with sub-space analogous to that spanned by real parts of complexified quaternions? The attempts to define this notion do not however look promising. One can however define associativity and co-associativity for the tangent space M^8 of H using octonionization and can formulate it also terms of induced gamma matrices.

5. The associativity defined in terms of induced gamma matrices in both in M^8 and H has the interesting feature that one can assign to the associative surface in H a new associative surface in H by assigning to each point of the space-time surface its M^4 projection and point of CP_2 characterizing its associative tangent space or co-associative normal space. It seems that one continue this series ad infinitum and generate new solutions of field equations! This brings in mind iteration which is standard manner to generate fractals as limiting sets. This certainly makes the heart of mathematician beat.

6. Kähler structure in $E^4 \subset M^8$ guarantees natural $M^4 \times E^4$ decomposition. Does associativity imply preferred extremal property or vice versa, or are the two notions equivalent or only consistent with each other for preferred extremals?

A couple of comments are in order.

1. This definition differs from the first proposal for years ago stating that each point of X^4 contains a *fixed* $M^2 \subset M^4$ rather than $M_2(x) \subset M^8$ and also from the proposal assuming integrable distribution of $M^2(x) \subset M^4$. The older proposals are not consistent with the properties of massless extremals and string like objects for which the counterpart of M^2 depends on space-time point and is not restricted to M^4. The earlier definition $M^2(x) \subset M^4$ was problematic in the co-associative case since for the Euclidian signature is is not clear what the counterpart of $M^2(x)$ could be.

2. The new definition is consistent with the existence of Hamilton-Jacobi structure meaning slicing of space-time surface by string world sheets and partonic 2-surfaces with points of partonic 2-surfaces labeling the string world sheets [K14]. This structure has been proposed to characterize preferred extremals in Minkowskian space-time regions at least.

3. Co-associative Euclidian 4-surfaces, say CP_2 type vacuum extremal do not contain integrable distribution of $M^2(x)$. It is normal space which contains $M^2(x)$. Does this have some physical meaning? Or does the surface defined by $M^2(x)$ have Euclidian analog?

   A possible identification of the analog would be as string world sheet at which W boson field is pure gauge so that the modes of the modified Dirac operator [K121] restricted to the string world sheet have well-defined em charge. This condition appears in the construction of solutions of Kähler-Dirac operator.

   For octonionic spinor structure the W coupling is however absent so that the condition does not make sense in M^8. The number theoretic condition would be as commutative or co-commutative surface for which imaginary units in tangent space transform to real and imaginary unit by a multiplication with a fixed imaginary unit! One can also formulate co-associativity as a condition that tangent space becomes associative by a multiplication with a fixed imaginary unit.

   There is also another justification for the distribution of Euclidian tangent planes. The idea about associativity as a fundamental dynamical principle can be strengthened to the statement that space-time surface allows slicing by hyper-complex or complex 2-surfaces, which are commutative or co-commutative inside space-time surface. The physical interpretation would be as Minkowskian or Euclidian string world sheets carrying spinor modes. This would give a connection with string model and also with the conjecture about the general structure of preferred extremals.

4. Minimalist could argue that the minimal definition requires octonionic structure and associativity *only* in M^8. There is no need to introduce the counterpart of Kähler action in M^8 since the dynamics would be based on associativity or co-associativity alone. The objection is that one must assumes the decomposition $M^8 = M^4 \times E^4$ without any justification.

   The map of space-time surfaces to those of $H = M^4 \times CP_2$ implies that the space-time surfaces in H are in well-defined sense quaternionic. As a matter of fact, the standard spinor structure of H can be regarded as quaternionic in the sense that gamma matrices are essentially tensor products of quaternionic gamma matrices and reduce in matrix representation for quaternions to ordinary gamma matrices. Therefore the idea that one should introduce octonionic gamma matrices in H is questionable. If all goes as in dreams, the mere associativity or co-associativity would code for the preferred extremal property of Kähler action in H. One could at least hope that associativity/co-associativity in H is consistent with the preferred extremal property.

5. One can also consider a variant of associativity based on modified gamma matrices - but only in H. This notion does not make sense in M^8 since the very existence of quaternionic tangent plane makes it possible to define M^8-H duality map. The associativity for modified gamma matrices is however consistent with what is known about extremals of Kähler action. The associativity based on induced gamma matrices would correspond to the use of the space-time volume as action. Note however that gamma matrices are *not* necessary in the definition.

Hyper-octonionic Pauli "matrices" and the definition of associativity

Octonionic Pauli matrices suggest an interesting possibility to define precisely what associativity means at the level of M^8 using gamma matrices (for background see [L8, K10]).

1. According to the standard definition space-time surface $X^4 \subset M^8$ is associative if the tangent space at each point of X^4 in $X^4 \subset M^8$ picture is associative. The definition can be given also in terms of octonionic gamma matrices whose definition is completely straightforward.
2. Could/should one define the analog of associativity at the level of H? One can identify the tangent space of H as M^8 and can define octonionic structure in the tangent space and this allows to define associativity locally. One can replace gamma matrices with their octonionic variants and formulate associativity in terms of them locally and this should be enough.

   Skeptic however reminds M^4 allows hyper-quaternionic structure and CP_2 quaternionic structure so that complexified quaternionic structure would look more natural for H. The tangent space would decompose as $M^8 = HQ + ijQ$, weher j is commuting imaginary unit and HQ is spanned by real unit and by units iI_k, where i second commutating imaginary unit and I_k denotes quaternionic imaginary units. There is no need to make anything associative.

   There is however far from obvious that octonionic spinor structure can be (or need to be!) defined globally. The lift of the CP_2 spinor connection to its octonionic variant has questionable features: in particular vanishing of the charged part and reduction of neutral part to photon. Therefore is is unclear whether associativity condition makes sense for $X^4 \subset M^4 \times CP_2$. What makes it so fascinating is that it would allow to iterate duality as a sequences $M^8 \rightarrow H \rightarrow H....$ This brings in mind the functional composition of octonion real-analytic functions suggested to produced associative or co-associative surfaces.

I have not been able to settle the situation. What seems the working option is associativity in both M^8 and H and Kähler-Dirac gamma matrices defined by appropriate Kähler action and correlation between associativity and preferred extremal property.

Are Kähler and spinor structures necessary in M^8?

If one introduces M^8 as dual of H, one cannot avoid the idea that hyper-quaternionic surfaces obtained as images of the preferred extremals of Kähler action in H are also extremals of M^8 Kähler action with same value of Kähler action defining Kähler function. As found, this leads to the conclusion that the M^8-H duality is Kähler isometry. Coupling of spinors to Kähler potential is the next step and this in turn leads to the introduction of spinor structure so that quantum TGD in H should have full M^8 dual.

1. Are also the 4-surfaces in M^8 preferred extremals of Kähler action?

It would be a mathematical miracle if associative and co-associative surfaces in M^8 would be in 1-1 correspondence with preferred extremals of Kähler action. This motivates the question whether Kähler action make sense also in M^8. This does not exclude the possibility that associativity implies or is equivalent with the preferred extremal property.

One expects a close correspondence between preferred extremals: also now vacuum degeneracy is obtained, one obtains massless extremals, string like objects, and counterparts of CP_2 type vacuum extremals. All known extremals would be associative or co-associative if modified gamma matrices define the notion (possible only in the case of H).

The strongest form of duality would be that the space-time surfaces in M^8 and H have same induced metric same induced Kähler form. The basic difference would be that the spinor connection for surfaces in M^8 would be however neutral and have no left handed components and only em gauge potential. A possible interpretation is that M^8 picture defines a theory in the phase in which electroweak symmetry breaking has happened and only photon belongs to the spectrum.

The question is whether one can define WCW also for M^8. Certainly it should be equivalent with WCW for H: otherwise an inflation of poorly defined notions follows. Certainly the general formulation of the WCW geometry generalizes from H to M^8. Since the matrix elements of symplectic super-Hamiltonians defining WCW gamma matrices are well defined as matrix elements involve spinor modes with Gaussian harmonic oscillator behavior, the non-compactness of E^4 does not pose any technical problems.

2. Spinor connection of M^8

There are strong physical constraints on M^8 dual and they could kill the hypothesis. The basic constraint to the spinor structure of M^8 is that it reproduces basic facts about electro-weak interactions. This includes neutral electro-weak couplings to quarks and leptons identified as different H-chiralities and parity breaking.

1. By the flatness of the metric of E^4 its spinor connection is trivial. E^4 however allows full S^2 of covariantly constant Kähler forms so that one can accommodate free independent Abelian gauge fields assuming that the independent gauge fields are orthogonal to each other when interpreted as realizations of quaternionic imaginary units. This is possible but perhaps a more natural option is the introduction of just single Kähler form as in the case of CP_2.
2. One should be able to distinguish between quarks and leptons also in M^8, which suggests that one introduce spinor structure and Kähler structure in E^4. The Kähler structure of E^4 is unique apart form $SO(3)$ rotation since all three quaternionic imaginary units and the unit vectors formed from them allow a representation as an antisymmetric tensor. Hence one must select one preferred Kähler structure, that is fix a point of S^2 representing the selected imaginary unit. It is natural to assume different couplings of the Kähler gauge potential to spinor chiralities representing quarks and leptons: these couplings can be assumed to be same as in case of H.
3. Electro-weak gauge potential has vectorial and axial parts. Em part is vectorial involving coupling to Kähler form and Z^0 contains both axial and vector parts. The naïve replacement of sigma matrices appearing in the coupling of electroweak gauge fields takes the left handed parts of these fields to zero so that only neutral part remains. Further, gauge fields correspond to curvature of CP_2 which vanishes for E^4 so that only Kähler form form remains. Kähler form couples to 3L and q so that the basic asymmetry between leptons and quarks remains. The resulting field could be seen as analog of photon.
4. The absence of weak parts of classical electro-weak gauge fields would conform with the standard thinking that classical weak fields are not important in long scales. A further prediction is that this distinction becomes visible only in situations, where H picture is necessary. This is the case at high energies, where the description of quarks in terms of $SU(3)$ color is convenient whereas $SO(4)$ QCD would require large number of E^4 partial waves. At low energies large number of $SU(3)$ color partial waves are needed and the convenient description would be in terms of $SO(4)$ QCD. Proton spin crisis might relate to this.

3. Dirac equation for leptons and quarks in M^8

Kähler gauge potential would also couple to octonionic spinors and explain the distinction between quarks and leptons.

1. The complexified octonions representing H spinors decompose to $1 + 1 + 3 + \overline{3}$ under SU(3) representing color automorphisms but the intepretation in terms of QCD color does not make sense. Rather, the triplet and single combine to two weak isospin doublets and quarks and

leptons corresponds to "spin" states of octonion valued 2-spinor. The conservation of quark and lepton numbers follows from the absence of coupling between these states.

2. One could modify the coupling so that coupling is on electric charge by coupling it to electromagnetic charge which as a combination of unit matrix and sigma matrix is proportional to $1 + kI_1$, where I_1 is octonionic imaginary unit in $M^2 \subset M^4$. The complexified octonionic units can be chosen to be eigenstates of Q_{em} so that Laplace equation reduces to ordinary scalar Laplacian with coupling to self-dual em field.

3. One expects harmonic oscillator like behavior for the modes of the Dirac operator of M^8 since the gauge potential is linear in E^4 coordinates. One possibility is Cartesian coordinates is $A(A_x, A_y, A_z, A_t) = k(-y, x, t, -z)$. Thhe coupling would make E^4 effectively a compact space.

4. The square of Dirac operator gives potential term proportional to $r^2 = x^2 + y^2 + z^2 + t^2$ so that the spectrum of 4-D harmonic oscillator operator and $SO(4)$ harmonics localized near origin are expected. For harmonic oscillator the symmetry enhances to $SU(4)$.

   If one replaces Kähler coupling with em charge symmetry breaking of $SO(4)$ to vectorial $SO(3)$ is expected since the coupling is proportional to $1 + ike_1$ defining electromagnetic charge. Since the basis of complexified quaternions can be chosen to be eigenstates of e_1 under multiplication, octonionic spinors are eigenstates of em charge and one obtains two color singles $1 \pm e_1$ and color triplet and antitriplet. The color triplets cannot be however interpreted in terms of quark color.

   Harmonic oscillator potential is expected to enhance $SO(3)$ to SU(3). This suggests the reduction of the symmetry to $SU(3) \times U(1)$ corresponding to color symmetry and em charge so that one would have same basic quantum numbers as tof CP_2 harmonics. An interesting question is how the spectrum and mass squared eigenvalues of harmonics differ from those for CP_2.

5. In the square of Dirac equation $J^{kl}\Sigma_{kl}$ term distinguishes between different em charges ($\Sigma_k l$ reduces by self duality and by special properties of octonionic sigma matrices to a term proportional to iI_1 and complexified octonionic units can be chosen to be its eigenstates with eigen value ± 1. The vacuum mass squared analogous to the vacuum energy of harmonic oscillator is also present and this contribution are expected to cancel themselves for neutrinos so that they are massless whereas charged leptons and quarks are massive. It remains to be checked that quarks and leptons can be classified to triality $T = \pm 1$ and $t = 0$ representations of dynamical $SU(3)$ respectively.

4. What about the analog of Kähler Dirac equation

Only the octonionic structure in $T(M^8)$ is needed to formulate quaternionicity of space-time surfaces: the reduction to O_c-real-analyticity would be extremely nice but not necessary (O_c denotes complexified octonions needed to cope with Minkowskian signature). Most importantly, there might be no need to introduce Kähler action (and Kähler form) in M^8. Even the octonionic representation of gamma matrices is un-necessary. Neither there is any absolute need to define octonionic Dirac equation and octonionic Kähler Dirac equation nor octonionic analog of its solutions nor the octonionic variants of embedding space harmonics.

It would be of course nice if the general formulas for solutions of the Kähler Dirac equation in H could have counterparts for octonionic spinors satisfying quaternionicity condition. One can indeed wonder whether the restriction of the modes of induced spinor field to string world sheets defined by integrable distributions of hyper-complex spaces $M^2(x)$ could be interpretated in terms of commutativity of fermionic physics in M^8. $M^8 - H$ correspondence could map the octonionic spinor fields at string world sheets to their quaternionic counterparts in H. The fact that only holomorphy is involved with the definition of modes could make this map possible.

How could one solve associativity/co-associativity conditions?

The natural question is whether and how one could solve the associativity/-co-associativity conditions explicitly. One can imagine two approaches besides $M^8 \to H \to H...$ iteration generating new solutions from existing ones.

1. Could octonion-real analyticity be equivalent with associativity/co-associativity?

Analytic functions provide solutions to 2-D Laplace equations and one might hope that also the field equations could be solved in terms of octonion-real-analyticity at the level of M^8 perhaps also at the level of H. Signature however causes problems - at least technical. Also the compactness of CP_2 causes technical difficulties but they need not be insurmountable.

For E^8 the tangent space would be genuinely octonionic and one can define the notion octonion-real analytic map as a generalization of real-analytic function of complex variables (the coefficients of Laurent series are real to guarantee associativity of the series). The argument is complexified octonion in $O \oplus iO$ forming an algebra but not a field. The norm square is Minkowskian as difference of two Euclidian octonionic norms: $N(o_1 + io_2) = N(o_1) - N(o_2)$ and vanishes at 15-D light cone boundary. Obviously, differential calculus is possible outside the light-cone boundary. Rational analytic functions have however poles at the light-cone boundary. One can wonder whether the poles at M^4 light-cone boundary, which is subset of 15-D light-cone boundary could have physical significance and relevant for the role of causal diamonds in ZEO.

The candidates for associative surfaces defined by O_c-real-analytic functions (I use O_c for complexified octonions) have Minkowskian signature of metric and are 4-surfaces at which the projection of $f(o_1 + io_2)$ to $Im(O_1)$, $iIm(O_2)$, and $iRe(Q_2) \oplus Im(Q_1)$ vanish so that only the projection to hyper-quaternionic Minkowskian sub-space $Re(Q_1) + iIm(Q_2)$ with signature (1, -1, -, 1-, 1) is non-vanishing. Co-associative surfaces would be surfaces for which the projections to $Re(O_1)$, $iRe(O_2)$, and to $Im(O_1)$ so that only the projection to $iIm(O_2)$ with signature $(-1-1-1-1)$ is non-vanishing.

These sub-manifolds are excellent candidate for associative and co-associative 4-surfaces if one believes on the intuition from complex analysis (the image of real axes under the map defined by O_c-real-analytic function is real axes in the new coordinates defined by the map). The possibility to solve field equations in this manner would be of enormous significance since besides basic arithmetic operations also the functional decomposition of O_c-real-analytic functions produces similar functions. One could speak of the algebra of space-time surfaces.

The alert reader has probably observed that the inverse image of the M^4 or E^4 as sub-space of O_c does not belong to $M^4 \times E^4$ sub-space of O_c. One can however assign to each point of this 4-surface a unique point of M^4 as projection and a unique point of CP_2 as characterization of the quaternionic tangent plane hence $O_c \to H$ correspondence holds true.

What is remarkable that the complexified octonion real analytic functions are obtained by analytic continuation from single real valued function of real argument. The real functions form naturally a hierarchy of polynomials (maybe also rational functions) and number theoretic vision suggests that there coefficients are rationals or algebraic numbers. Already for rational coefficients hierarchy of algebraic extensions of rationals results as one solves the vanishing conditions. There is a temptation to regard this hierarchy coding for space-time sheets as an analog of DNA.

Note that in the recent formulation there is no need to pose separately the condition about integrable distribution of $M^2(x) \subset M^4$.

2. Quaternionicity condition for space-time surfaces

Quaternionicity actually has a surprisingly simple formulation at the level of space-time surfaces. The following discussion applies to both M^8 and H with minor modifications if one accepts that also H can allow octonionic tangent space structure, which does not require gamma matrices.

1. Quaternionicity is equivalent with associativity guaranteed by the vanishing of the associator $A(a, b, c) = a(bc) - (ab)c$ for any triplet of imaginary tangent vectors in the tangent space of the space-time surface. The condition must hold true for purely imaginary combinations of tangent vectors.

2. If one is able to choose the coordinates in such a way that one of the tangent vectors corresponds to real unit (in the embedding map embedding space M^4 coordinate depends only on the time coordinate of space-time surface), the condition reduces to the vanishing of the octonionic product of remaining three induced gamma matrices interpreted as octonionic gamma matrices. This condition looks very simple - perhaps too simple!- since it involves only first derivatives of the embedding space vectors.

One can of course whether quaternionicity conditions replace field equations or only select preferred extremals. In the latter case, one should be able to prove that quaternionicity conditions are consistent with the field equations.

3. Field equations would reduce to tri-linear equations in in the gradients of embedding space coordinates (rather than involving embedding space coordinates quadratically). Sum of analogs of 3×3 determinants deriving from $a \times (b \times b)$ for different octonion units is involved.

4. Written explicitly field equations give in terms of vielbein projections e^A_α, vielbein vectors e^A_k, coordinate gradients $\partial_\alpha h^k$ and octonionic structure constants f_{ABC} the following conditions stating that the projections of the octonionic associator tensor to the space-time surface vanishes:

$$\begin{aligned} e^A_\alpha e^B_\beta e^C_\gamma A^E_{ABC} &= 0 \ , \\ A^E_{ABC} &= f_{AD}{}^E f_{BC}{}^D - f_{AB}{}^D f_{DC}{}^E \ , \\ e^A_\alpha &= \partial_\alpha h^k e^A_k \ , \\ \Gamma_k &= e^A_k \gamma_A \ . \end{aligned} \tag{5.4.1}$$

The very naïve idea would be that the field equations are indeed integrable in the sense that they reduce to these tri-linear equations. Tri-linearity in derivatives is highly non-trivial outcome simplifying the situation further. These equations can be formulated as the as purely algebraic equations written above plus integrability conditions

$$F^A_{\alpha\beta} = D_\alpha e^A_\beta - D_\beta e^A_\alpha = 0 \ . \tag{5.4.2}$$

One could say that vielbein projections define an analog of a trivial gauge potential. Note however that the covariant derivative is defined by spinor connection rather than this effective gauge potential which reduces to that in SU(2). Similar formulation holds true for field equations and one should be able to see whether the field equations formulated in terms of derivatives of vielbein projections commute with the associativity conditions.

5. The quaternionicity conditions can be formulated as vanishing of generalization of Cayley's hyperdeterminant for "hypermatrix" a_{ijk} with 2-valued indiced (see `http://tinyurl.com/ya7h3n9z`). Now one has 8 hyper-matrices with 3 8-valued indices associated with the vanishing $A^E_{BCD} x^B y^C z^D = 0$ of trilinear forms defined by the associators. The conditions say something only about the octonionic structure constants and since octonionic space allow quaternionic sub-spaces these conditions must be satisfied.

The inspection of the Fano triangle [A67] (see **Fig. 10.1**) expressing the multiplication table for octonionic imaginary units reveals that give any two imaginary octonion units e_1 and e_2 their product $e_1 e_2$ (or equivalently commutator) is imaginary octonion unit (2 times octonion unit) and the three units span together with real unit quaternionic sub-algebra. There it seems that one can generate local quaternionic sub-space from two imaginary units plus real unit. This generalizes to the vielbein components of tangent vectors of space-time surface and one can build the solutions to the quaternionicity conditions from vielbein projections e_1, e_2, their product $e_3 = k(x) e_1 e_2$ and real fourth "timelike" vielbein component which must be expressible as a combination of real unit and imaginary units:

$$e_0 = a \times 1 + b^i e_i$$

For static solutions this condition is trivial. Here summation over i is understood in the latter term. Besides these conditions one has integrability conditions and field equations for Kähler action. This formulation suggests that quaternionicity is additional - perhaps defining - property of preferred extremals.

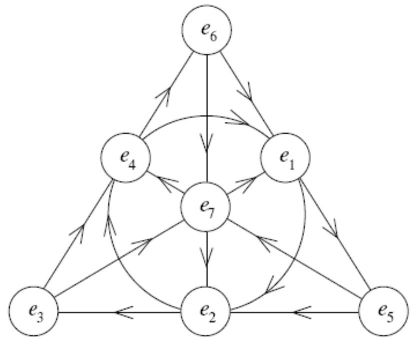

Figure 5.1: Octonionic triangle: the six lines and one circle containing three vertices define the seven associative triplets for which the multiplication rules of the ordinary quaternion imaginary units hold true. The arrow defines the orientation for each associative triplet. Note that the product for the units of each associative triplets equals to real unit apart from sign factor.

Quaternionicity at the level of embedding space quantum numbers

From the multiplication table of octonions as illustrated by Fano triangle [A67] one finds that all edges of the triangle, the middle circle and the three the lines connecting vertices to the midpoints of opposite side define triplets of quaternionic units. This means that by taking real unit and any imaginary unit in quaternionic M^4 algebra spanning $M^2 \subset M^4$ and two imaginary units in the complement representing CP_2 tangent space one obtains quaternionic algebra. This suggests an explanation for the preferred M^2 contained in tangent space of space-time surface (the M^2: s could form an integrable distribution). Four-momentum restricted to M^2 and I_3 and Y interpreted as tangent vectors in CP_2 tangent space defined quaternionic sub-algebra. This could give content for the idea that quantum numbers are quaternionic.

I have indeed proposed that the four-momentum belongs to M^2. If $M^2(x)$ form a distribution as the proposal for the preferred extremals suggests this could reflect momentum exchanges between different points of the space-time surface such that total momentum is conserved or momentum exchange between two sheets connected by wormhole contacts.

Questions

In following some questions related to $M^8 - H$ duality are represented.

1. Could associativity condition be formulated using modified gamma matrices?

Skeptic can criticize the minimal form of $M^8 - H$ duality involving no Kähler action in M^8 is unrealistic. Why just Kähler action? What makes it so special? The only defense that I can imagine is that Kähler action is in many respects unique choice.

An alternative approach would replace induced gamma matrices with the modified ones to get the correlation In the case of M^8 this option cannot work. One cannot exclude it for H.

1. For Kähler action the Kähler-Dirac gamma matrices $\Gamma^\alpha = \frac{\partial L_K}{\partial h^k_\alpha}\Gamma^k$, $\Gamma_k = e^A_k \gamma_A$, assign to a given point of X^4 a 4-D space which need not be tangent space anymore or even its sub-space.

   The reason is that canonical momentum current contains besides the gravitational contribution coming from the induced metric also the "Maxwell contribution" from the induced Kähler form not parallel to space-time surface. In the case of M^8 the duality map to H is therefore lost.

2. The space spanned by the Kähler-Dirac gamma matrices need not be 4-dimensional. For vacuum extremals with at most 2-D CP_2 projection Kähler-Dirac gamma matrices vanish identically. For massless extremals they span 1- D light-like subspace. For CP_2 vacuum extremals the modified gamma matrices reduces to ordinary gamma matrices for CP_2 and the

situation reduces to the quaternionicity of CP_2. Also for string like objects the conditions are satisfied since the gamma matrices define associative sub-space as tangent space of $M^2 \times S^2 \subset M^4 \times CP_2$. It seems that associativity is satisfied by all known extremals. Hence Kähler-Dirac gamma matrices are flexible enough to realize associativity in H.

3. Kähler-Dirac gamma matrices in Dirac equation are required by super conformal symmetry for the extremals of action and they also guarantee that vacuum extremals defined by surfaces in $M^4 \times Y^2$, Y^2 a Lagrange sub-manifold of CP_2, are trivially hyper-quaternionic surfaces. The modified definition of associativity in H does not affect in any manner $M^8 - H$ duality necessarily based on induced gamma matrices in M^8 allowing purely number theoretic interpretation of standard model symmetries. One can however argue that the most natural definition of associativity is in terms of induced gamma matrices in both M^8 and H.

Remark: A side comment not strictly related to associativity is in order. The anticommutators of the Kähler-Dirac gamma matrices define an effective Riemann metric and one can assign to it the counterparts of Riemann connection, curvature tensor, geodesic line, volume, etc... One would have two different metrics associated with the space-time surface. Only if the action defining space-time surface is identified as the volume in the ordinary metric, these metrics are equivalent. The index raising for the effective metric could be defined also by the induced metric and it is not clear whether one can define Riemann connection also in this case. Could this effective metric have concrete physical significance and play a deeper role in quantum TGD? For instance, AdS-CFT duality leads to ask whether interactions be coded in terms of the gravitation associated with the effective metric.

Remark added later: In fact, it has turned that this effective metric assignable to string world sheets plays a fundamental role in the recent formulation of TGD and allows to understand gravitational bound states in terms of string connecting partonic 2-surfaces: something impossible in string model.

Now skeptic can ask why should one demand $M^8 - H$ correspondence if one in any case is forced to introduced Kähler action also at the level of M^8? Does $M^8 - H$ correspondence help to construct preferred extremals or does it only bring in a long list of conjectures? I can repeat the questions of the skeptic.

2. Minkowskian-Euclidian $\leftrightarrow$ associative–co-associative?

The 8-dimensionality of M^8 allows to consider both associativity of the tangent space and associativity of the normal space- let us call this co-associativity of tangent space- as alternative options. Both options are needed as has been already found. Since space-time surface decomposes into regions whose induced metric possesses either Minkowskian or Euclidian signature, there is a strong temptation to propose that Minkowskian regions correspond to associative and Euclidian regions to co-associative regions so that space-time itself would provide both the description and its dual.

The proposed interpretation of conjectured associative-co-associative duality relates in an interesting manner to p-adic length scale hypothesis selecting the primes $p \simeq 2^k$, k positive integer as preferred p-adic length scales. $L_p \propto \sqrt{p}$ corresponds to the p-adic length scale defining the size of the space-time sheet at which elementary particle represented as CP_2 type extremal is topologically condensed and is of order Compton length. $L_k \propto \sqrt{k}$ represents the p-adic length scale of the wormhole contacts associated with the CP_2 type extremal and CP_2 size is the natural length unit now. Obviously the quantitative formulation for associative-co-associative duality would be in terms $p \rightarrow k$ duality.

3. Can $M^8 - H$ duality be useful?

Skeptic could of course argue that $M^8 - H$ duality generates only an inflation of unproven conjectures. This might be the case. In the following I will however try to defend the conjecture. One can however find good motivations for $M^8 - H$ duality: both theoretical and physical.

1. If $M^8 - H$ duality makes sense for induced gamma matrices also in H, one obtains infinite sequence if dualities allowing to construct preferred extremals iteratively. This might relate to octonionic real-analyticity and composition of octonion-real-analytic functions.
2. $M^8 - H$ duality could provide much simpler description of preferred extremals of Kähler action as hyper-quaternionic surfaces. Unfortunately, it is not clear whether one should introduce

the counterpart of Kähler action in M^8 and the coupling of M^8 spinors to Kähler form. Note that the Kähler form in E^4 would be self dual and have constant components: essentially parallel electric and magnetic field of same constant magnitude.

3. $M^8 - H$ duality provides insights to low energy physics, in particular low energy hadron physics. M^8 description might work when H-description fails. For instance, perturbative QCD which corresponds to H-description fails at low energies whereas M^8 description might become perturbative description at this limit. Strong $SO(4) = SU(2)_L \times SU(2)_R$ invariance is the basic symmetry of the phenomenological low energy hadron models based on conserved vector current hypothesis (CVC) and partially conserved axial current hypothesis (PCAC). Strong $SO(4) = SU(2)_L \times SU(2)_R$ relates closely also to electro-weak gauge group $SU(2)_L \times U(1)$ and this connection is not well understood in QCD description. $M^8 - H$ duality could provide this connection. Strong $SO(4)$ symmetry would emerge as a low energy dual of the color symmetry. Orbital $SO(4)$ would correspond to strong $SU(2)_L \times SU(2)_R$ and by flatness of E^4 spin like $SO(4)$ would correspond to electro-weak group $SU(2)_L \times U(1)_R \subset SO(4)$. Note that the inclusion of coupling to Kähler gauge potential is necessary to achieve respectable spinor structure in CP_2. One could say that the orbital angular momentum in $SO(4)$ corresponds to strong isospin and spin part of angular momentum to the weak isospin.

   This argument does not seem to be consistent with $SU(3) \times U(1) \subset SU(4)$ symmetry for Mx Dirac equation. One can however argue that $SU(4)$ symmetry combines $SO(4)$ multiplets together. Furthermore, $SO(4)$ represents the isometries leaving Kähler form invariant.

4. $M^8 - H$ duality in low energy physics and low energy hadron physics

$M^8 - H$ can be applied to gain a view about color confinement. The basic idea would be that $SO(4)$ and $SU(3)$ provide provide dual descriptions of quarks using E^4 and CP_2 partial waves and low energy hadron physics corresponds to a situation in which M^8 picture provides the perturbative approach whereas H picture works at high energies.

A possible interpretation is that the space-time surfaces vary so slowly in CP_2 degrees of freedom that can approximate CP_2 with a small region of its tangent space E^4. One could also say that color interactions mask completely electroweak interactions so that the spinor connection of CP_2 can be neglected and one has effectively E^4. The basic prediction is that $SO(4)$ should appear as dynamical symmetry group of low energy hadron physics and this is indeed the case.

Consider color confinement at the long length scale limit in terms of $M^8 - H$ duality.

1. At high energy limit only lowest color triplet color partial waves for quarks dominate so that QCD description becomes appropriate whereas very higher color partial waves for quarks and gluons are expected to appear at the confinement limit. Since WCW degrees of freedom begin to dominate, color confinement limit transcends the descriptive power of QCD.

2. The success of $SO(4)$ sigma model in the description of low lying hadrons would directly relate to the fact that this group labels also the E^4 Hamiltonians in M^8 picture. Strong $SO(4)$ quantum numbers can be identified as orbital counterparts of right and left handed electro-weak isospin coinciding with strong isospin for lowest quarks. In sigma model pion and sigma boson form the components of E^4 valued vector field or equivalently collection of four E^4 Hamiltonians corresponding to spherical E^4 coordinates. Pion corresponds to S^3 valued unit vector field with charge states of pion identifiable as three Hamiltonians defined by the coordinate components. Sigma is mapped to the Hamiltonian defined by the E^4 radial coordinate. Excited mesons corresponding to more complex Hamiltonians are predicted.

3. The generalization of sigma model would assign to quarks E^4 partial waves belonging to the representations of $SO(4)$. The model would involve also 6 $SO(4)$ gluons and their $SO(4)$ partial waves. At the low energy limit only lowest representations would be important whereas at higher energies higher partial waves would be excited and the description based on CP_2 partial waves would become more appropriate.

4. The low energy quark model would rely on quarks moving $SO(4)$ color partial waves. Left *resp.* right handed quarks could correspond to $SU(2)_L$ *resp.* $SU(2)_R$ triplets so that spin statistics problem would be solved in the same manner as in the standard quark model.

5. Family replication phenomenon is described in TGD framework the same manner in both cases so that quantum numbers like strangeness and charm are not fundamental. Indeed, p-adic mass calculations allowing fractally scaled up versions of various quarks allow to replace Gell-Mann mass formula with highly successful predictions for hadron masses [K72].

To my opinion these observations are intriguing enough to motivate a concrete attempt to construct low energy hadron physics in terms of $SO(4)$ gauge theory.

Summary

The overall conclusion is that the most convincing scenario relies on the associativity/co-associativity of space-time surfaces define by induced gamma matrices and applying both for M^8 and H. The fact that the duality can be continued to an iterated sequence of duality maps $M^8 \rightarrow H \rightarrow H...$ is what makes the proposal so fascinating and suggests connection with fractality.

The introduction of Kähler action and coupling of spinors to Kähler gauge potentials is highly natural. One can also consider the idea that the space-time surfaces in M^8 and H have same induced metric and Kähler form: for iterated duality map this would mean that the steps in the map produce space-time surfaces which identical metric and Kähler form so that the sequence might stop. M^8_H duality might provide two descriptions of same underlying dynamics: M^8 description would apply in long length scales and H description in short length scales.

5.4.5 Black Hole Physics

The hierarchy of Planck constants has forced to modify dramatically TGD based view about black holes. TGD black holes however have a lot of common with ordinary black holes.

M-theory and black holes

The reproduction of the formula for the black hole entropy [B57, B48] has been sold as a victory of M-theory. The first thing that has been forgotten is that GRT based formula has never been experimentally verified and could be even wrong.

One can also criticize the procedure leading to the formula.

1. First M-theory is replaced by 11-D super gravity in order to calculate something. What this effectively means that, although the aim was to replace General Relativity with something more fundamental, one ends up with 11-D classical super-gravity after all.
2. After this one finds black-hole type solutions and identifies them with M-branes. At this step one could protest by saying that the fundamental theory should replace black holes with something less singular.
3. Next quantum gravitational holography is assumed and a conformal field theory on brane identified as a black hole horizon leads to an estimate for the entropy and estimates for what are known as greyness factors. The last step is nice in the 4-D situation and also TGD would suggest something very similar.

In Matrix Theory based estimate things look even less elegant. In [B26] a matrix theory based estimate for the entropy is made producing the correct order of magnitude for the entropy estimate using conformal field theory. An essential step is the estimate for the number N of 0-branes (ordinary particles) and is ad hoc (in particular one does not take the limit $N \rightarrow \infty$). I do not whether the arguments are more rigorous in other estimates but, to put it mildly, I do not find this argument is not too convincing.

Black holes in TGD framework

Black holes in the standard sense are possible in TGD framework but would be basically astrophysical objects and putting black holes and elementary particles in the same basket would be mixing apples with oranges. The vision about dark matter as a macroscopic quantum phases with large value of Planck constant (the value of gravitational Planck constant is enormous) forces to reconsider the identification of black holes. One can view TGD counterparts of black hole horizons as light-like 3-surfaces at which the signature of the induced metric changes to Euclidian.

Black holes would be gigantic elementary particle (or rather parton-) like objects containing particles in anyonic phase with fractional charges guaranteeing confinement. Dark anyonic matter at light-like 3-surfaces of astrophysical size analogous to stringy black holes thought to be tightly tangled strings has several basic characteristics of black hole and would populate TGD Universe in all length scales.

In TGD Universe the role of black hole horizons is taken by light like 3-surfaces, which are fundamental objects of the theory whereas the role of big bang is taken by the boundary δM^4_+ of causal diamond (CD). The basic difference to black hole horizons is that the signature of induced metric changes at the wormhole throat.

1. The basic example is provided by elementary particle horizons surrounding the ends of the wormhole contacts having Euclidian signature of the induced metric and connecting with each other space-time sheets with Minkowskian signature of the induced metric. The light-like wormhole throats are carriers of fermion numbers. The interpretation of wormhole contacts is in terms of gauge bosons and Higgs bosons consisting of fermion and anti-fermion at the two wormhole throats. By its spin the only possible identification of graviton is as a pair of wormhole contacts connected by a flux tube carrying various gauge fluxes. Elementary fermions correspond to wormhole throats associated with CP_2 type vacuum extremals (note Euclidian signature of induced metric) glued to the background space-time with Minkowskian signature of metric.

2. Second example is provided by light-like surfaces separating maximal deterministic regions of the space-time sheet. Light-like boundaries is a further example. By their metric 2-dimensionality various causal determinants indeed allow conformal field theory in an effectively 2-dimensional sense.

3. The formula for the black hole entropy generalizes to elementary particle level and involves p-adic length scale hypothesis and p-adic mass calculations [K74].

4. The new element is the hierarchy of Planck constants [K97, K78, K38] inspired by the findings that gravitational Planck constant might have gigantic value [E1]. This leads to a vision about dark matter as phases of matter with large Planck constant and hence macroscopically quantum coherent since all quantum scales are scaled up. The space-time sheets mediating gravitational interaction would have gigantic value of Planck constant: $\hbar_{gr} = GM_1M_2/v_0$, $v_0 = 2^{-11}$ gives a good example about the situation. The implication is that black hole entropy proportional to $1/\hbar$ is of order unity if $\hbar_{gr} = GM^2/v_0$, $v_0 = 1/4$ holds true for black holes. This would change completely the view about black holes as highly entropic objects. In particular, Planck length scales as $\sqrt{\hbar}$ so that Schwartschild radius represents Planck length for this kind of black hole and defines naturally kind of minimum length scales below which the signature of induced metric becomes Euclidian in TGD Universe.

5. The progress in the understanding of the realization of the hierarchy of Planck constants in terms of book like structure of embedding space with the pages of book representing Cartesian products of singular coverings and factor spaces of causal diamond CD and CP_2 led to a detailed picture about identification of anyonic systems as macroscopic light-like 3-surfaces containing dark matter in anyonic form possessing fractional quantum numbers. Anyonicity means that the "partonic" 2-surface of macroscopic size system surrounds the tip of CD so that homologically non-trivial 2-surface is in question. Anyonic phase could be even responsible for the properties of living matter [K80, K34]. This also inspired the proposal that dark matter resides at light-like 3-surfaces of astrophysical and even cosmological size scale possessing very complex topology: typically spherical topologies glued together by flux tubes. Black holes in standard sense would result in gravitational collapse of this kind of systems. An open question is whether the topology actually transforms to simple spherical topology in this process or whether it is more or less conserved so that huge information about the topology of orbits of dark matter particles surrounding the object would be preserved.

More concrete ideas about black hole like structures emerged from the attempts to understand the strange events reported by RHIC (Relativistic Heavy Ion Collider) [C3, C8] during last years. This work led to a dramatic increase of understanding of TGD and allowed to fuse together separate threads of TGD [K98].

1. The scaled down TGD inspired cosmology involving (not so) big crunch followed by (not so) big bang serves as a model for the events, and predicts a new phase identifiable as color glass condensate identifiable as tightly tangled color magnetic flux tube modellable as a hadronic string in Hagedorn temperature.

   This state makes a phase transition to quark gluon plasma during a period of critical cosmology analogous to inflationary cosmology characterized completely by its duration and quark gluon plasma analogous to radiation dominated cosmology in turn hadronizes giving rise to the analog of matter dominated cosmology.

   The assumption that anyonicity is responsible for the formation of the gluonic Bose-Einstein condensate explains the liquid like character of color glass condensate. Anyonicity forces the system to behave like a single particle like unit since fractionally charged particles cannot leave the light-like 2-surface surrounding the tip of CD.

2. RHIC events suggest processes analogous to the formation and evaporation of black hole. The TGD inspired description in terms of the formation of hadronic black hole and its evaporation and essentially identical with the description as a mini bang. The hadronic black hole is the same tightly tangled color magnetic flux tube that defines the initial state of the hadronic mini bang. The attribute "hadronic" means that Planck length is replaced with hadronic length so that strong gravitation is in question. Black hole temperature is identifiable as Hagedorn temperature and predicted to be 195 MeV for bosonic strings in 4-D space-time and slightly higher than the hadronization temperature measured to be about 176 MeV [K98].

3. As also the small value of black hole entropy suggests, black holes and their scaled counterparts would not be merciless information destroyers in TGD Universe. The entanglement of particles possessing different conformal weights to give states with a vanishing net conformal weight and having particle like integrity would make black hole like states ideal candidates for quantum computer like systems [K5]. One could even imagine that the galactic black hole is a highly tangled cosmic string in Hagedorn temperature performing quantum computations the complexity of which is totally out of reach of human intellect! Indeed, TGD inspired consciousness predicts that evolution leads to the increase of information and intelligence, and the evolution of stars should not form exception to this. Also the interpretation of black hole as consisting of dark matter follows from this picture [K34].

Concerning the mathematical description of dark matter - and of matter quite generally- TGD has led to amazingly simple mathematical framework, which might have something to with Matrix theory approach. The characteristic aspects of the classical dynamic determined by Kähler action is its vacuum degeneracy and this not only allows but even forces the notion of finite measurement resolution originally inspired by the inclusions of hyper-finite factors of type II_1 (HFFs) having WCW Clifford algebra as a canonical representative. The notion of finite measurement resolution leads to a discretization of physics in terms of string like objects carrying the modes of the spinor fields. If conformal symmetry for spinor modes is realized as gauge symmetry, there is effectively only a finite number of fermionic oscillator operators characterizing any subsystem [K121]. Even the infinite-dimensional world of classical worlds can be described with arbitrary accuracy as a finite-dimensional space and these descriptions define a hierarchy of inclusions of HFFs associated with WCW Clifford algebra.

How the TGD analogs of black holes could relate to GRT black holes?

1. GRT space-time is obtained from many-sheeted space-time of TGD by a rather violent operation: the sheets of the many-sheeted space-time are lumped together to form a region of M^4 with the deviation of metric from M^4 metric given by the sum of corresponding deviations for the sheets and gauge potentials identified as sums of the induced gauge potentials for sheets. What remains visible about many-sheeted physics are anomalies of GRT [K102].

2. This description should be good in regions, where the gravitational field is weak. Since the coordinates for the Schwarthild metric in TGD framework correspond naturally to Minkowski coordinates, one must interpret the diverging deviation of the metric from Minkowski metric at horizon as a failure of this approximation (usually one would argue that curvature is small at horizon so that there is no reason to worry: firewall debate has forced to question this assumption).

By this argument GRT space-time in black hole length scales can be seen as a continuation of physics from regions, where TGD-GRT correspondence is a good approximation to regions where fails to be so. TGD physics could become visible at Schwartschild radius and at even longer distances. The description of the formation of gravitational bound states in terms of strings connecting partonic 2-surfaces assumes macroscopic quantum coherence and this is of course something completely new.

3. The space-time regions with Euclidian signature of the induced metric are not included at all in TGD description. One could of course consider also this kind of solutions and they have been used as a trick to make path integral well-defined. Einstein-Maxwell action with cosmological constant defined by CP_2 scale allows CP_2 as a gravitational instanton, and one might consider the possibility of an improved GRT limit with particles identified as deformations of CP_2 are glued along light-like 3-surfaces to Reissner-Nordström type metric. One might hope this to give an improved description of elementary particles. There is however a problem. In TGD framework work the existence of embeddings of CP_2 to embedding space with light-like curve as M^4 projection are essential for particle interpretation. It is difficult to see how particle interpretation could be possible in GRT framework.

5.4.6 WCW Gamma Matrices As Hyper-Octonionic Conformal Fields?

The fact that the Clifford algebra generated by WCW gamma matrices forms a canonical representation for hyper-finite factor of type II_1 (HFFs) and led to a breakthrough in the understanding of quantum TGD. The inclusions of hyper-finite factors of type II_1 led to a realization of finite quantum measurement resolution as a basic principle govering dynamics and together with zero energy ontology this approach led to the generalization of S-matrix to M-matrix identified as time like entanglement coefficients between positive and negative energy parts of zero energy state and its identification as Connes tensor product. HFFs generated also ideas about how quantum TGD might be reducible to a generalization of HFFs to its local variant which is necessarily complex-octonionic as also to a construction of quantum variant of gamma matrix algebra leading to identification of quantum counterparts of hyper-octonions and hyper-quaternions as unique structures.

Only the quantum variants of M^4 and M^8 emerge from local hyper-finite II_1 factors

The fantastic properties of hyperfinite factors of type II_1 (HFFs) inspire the idea that a localized hyper-octonionic version of Clifford algebra of WCW might allow to see space-time, embedding space, and WCW as structures emerging from a hyper-octonionic version of HFF. Surprisingly, commutativity and associativity imply most of the speculative "must-be-true's" of quantum TGD.

WCW gamma matrices act only in vibrational degrees of freedom of 3-surface. One must also include center of mass degrees of freedom which appear as zero modes. The natural idea is that the resulting local gamma matrices define a local version of HFF of type II_1 as a generalization of conformal field of gamma matrices appearing super string models obtained by replacing complex numbers with hyper-octonions identified as a subspace of complexified octonions.

As a matter fact, one can generalize octonions to quantum octonions for which quantum commutativity means restriction to a hyper-octonionic subspace of quantum octonions. Non-associativity is essential for obtaining something non-trivial: otherwise this algebra reduces to HFF of type II_1 since matrix algebra as a tensor factor would give an algebra isomorphic with the original one. The octonionic variant of conformal invariance fixes the dependence of local gamma matrix field on the coordinate of HO. The coefficients of Laurent expansion of this field must commute with octonions!

Super-symmetry suggests that the representations of CH Clifford algebra $\mathcal{M}$ as $\mathcal{N}$ module $\mathcal{M}/\mathcal{N}$ should have bosonic counterpart in the sense that the coordinate for M^8 representable as a particular $M^2(Q)$ element should have quantum counterpart. Same would apply to M^4 coordinate representable as $M^2(C)$ element. Quantum matrix representation of $\mathcal{M}/\mathcal{N}$ as $SL_q(2,F)$ matrix, $F = C, H$ is the natural candidate for this representation. As a matter fact, this guess is not quite correct. It is the interpretation of $M_2(C)$ as a quaternionic quantum algebra whose generalization to the octonionic quantum algebra works.

Quantum variants of M^D exist for all dimensions but only spaces M^4 and M^8 and their linear sub-spaces emerge from hyper-finite factors of type II_1. This is due to the non-associativity of the octonionic representation of the gamma matrices making it impossible to absorb the powers of the octonionic coordinate to the Clifford algebra element so that the local algebra character would disappear. Even more: quantum coordinates for these spaces are commutative operators so that their spectra define ordinary M^4 and M^8 which are thus already quantal concepts.

Consider first hyper-quaternions and the emergence of M^4.

1. The commutation relations for $M_{2,q}(C)$ matrices

$$\left(\begin{array}{cc} a & b \\ c & d \end{array}\right) \ , \tag{5.4.3}$$

read as

$$\begin{array}{ll} ab = qba \ , & ac = qac \ , \quad bd = qdb \ , cd = qdc \ , \\ [a,d] = (q-q^{-1})bc \ , & bc = cb \ . \end{array} \tag{5.4.4}$$

2. These relations could be extended by postulating complex conjugates of these relations for complex conjugates $a^\dagger, b^\dagger, c^\dagger, d^\dagger$ plus the following non-vanishing commutators of type $[x, y^\dagger]$:

$$[a,a^\dagger] = [b,b^\dagger] = [c,c^\dagger] = [d,d^\dagger] = 1 \ . \tag{5.4.5}$$

This extension is not necessary for what comes.

3. The matrices representing M^4 point must be expressible as sums of Pauli spin matrices. This can be represented as following conditions on physical states

$$\begin{array}{rcl} O|phys\rangle & = & 0 \ , \\ O & \in & \{a-a^\dagger, d-d^\dagger, b-c^\dagger, c-b^\dagger\} \ . \end{array} \tag{5.4.6}$$

For instance, the first two conditions follow from the reality of Pauli sigma matrices $\sigma_x, \sigma_y, \sigma_z$. These conditions are compatible only if the operators O commute. These conditions need not be consistent with the commutation relations between a, b, c, d and their Hermitian conjugates. This is easy to see by noticing that the difference of $J_+ - J_-$ acts apart from imaginary unit like J_y and annihilates $j_y = 0$ state for every representation of rotation group diagonalized with respect to J_y.

4. What is essential is that the operators of O are of form $A - A^\dagger$ and their commutators are also of the same form that the commutativity conditions reduce the condition that the Lie-algebra like structure generated by these operators annihilates the physical state. Hence it is possible to define quantum states for which M^4 coordinates have well-defined eigenvalues so that ordinary M^4 emerges purely quantally from quaternions whose real coefficients are made non-Hermitian operators to obtain operator complexificiation of quaternions. Also the quantum states in which M^4 coordinates are emerge naturally.

5. $M_{2,q}(C)$ matrices define the quantum analog of C^4 and one can wonder whether also other linear sub-spaces can be defined consistently or whether M^4_q and thus Minkowski signature is unique. This seems to be not the case. For instance, the replacement $a - a^\dagger \to a + a^\dagger$ making also time variable Euclidian is impossible since $[a + a^\dagger, d - d^\dagger] = 2(q - q^{-1})(bc + b^\dagger c^\dagger$ is not proportional to a difference of operator and its hermitian conjugate and one does not obtain closed algebra.

What about M^8: does it have analogous description in terms of physical states annihilated by the Lie algebra generated by the differences $a_i - a_i^\dagger$, $i = 0, ..7$?

1. The representation of M^4 point as $M_2(C)$ matrix can be interpreted a combination of 4-D gamma matrices defining hyper-quaternionic units. Hyper-octonionic units indeed have anti-commutation relations of gamma matrices of M^8 and would give classical representation of M^8. The counterpart of $M_{2,q}(C)$ would thus be obtained by replacing the coefficients of hyper-octonionic units with operators satisfying the generalization of $M_{2,q}(C)$ commutation relations. One should identify the reality conditions and find whether they are mutually consistent.

2. In quaternionic case basis for matrix algebra is formed by the sigma matrices and M^4 point is represented by a hermitian matrix expressible as linear combination of hermitian sigma matrices with coefficients which act on physical states like hermitian operators. In the hyper-octonionic case would expect that real octonion unit and octonionic imaginary units multiplied by commuting imaginary unit to define the counterparts of sigma matrices and that the physically representable sub-space of complex quantum octonions corresponds to operator valued coordinates which act like hermitian matrices. The restriction to complex quaternionic sub-space must give hyper-quaternions and M^4 so that the only sensible generalization is that M^8 holds quite generally. This is also required by SO^7 invariance allowing to choose the sub-space M^4 freely. Again the key point should be that the conditions giving rise to real eigenvalues give rise to a Lie-algebra which must annihilate the physical state. For other signatures one would not obtain Lie algebra.

3. One can also make guess for the concrete realization of the algebra. Introduce the coefficients of E^4 gamma matrices having interpretation as quaternionic units as

$$
\begin{array}{ll}
a_0 = ix(a+d) \ , & a_3 = x(a-d) \ , \\
a_1 = x(ib+c) \ , & a_2 = x(ib-c) \ , \\
x = \frac{1}{\sqrt{2}} \ , &
\end{array}
$$

and write the commutations relations for them to see how the generalization should be performed.

4. The selections of complex and quaternionic sub-algebras of octonions are fundamental for TGD and quantum octonionic algebra should reflect these selections in its structure. In the case of hyper-quaternions the selection of commutative sub-algebra implies the breaking of 4-D Lorentz symmetry. In the case of hyper-octonions the selection of hyper-quaternion sub-algebra should induce the breaking of 8-D Lorentz symmetry. Hyper-quaternionic sub-algebra obeys the commutations of $M_q(2,C)$ whereas the coefficients in the complement commute mutually and quantum commute with the complex sub-algebra. This nails down the commutation relations completely:

$$
\begin{array}{rcl}
[a_0,a_3] & = & \frac{i}{2}(q-q^{-1})(a_1^2-a_2^2) \ , \\
[a_i,a_j] & = & 0 \ , \ i,j \neq 0,3 \ , \\
a_0a_i & = & qa_ia_0 \ , \ i\neq 0,3 \ , \\
a_3a_i & = & qa_ia_3 \ , \ i\neq 0,3 \ .
\end{array} \tag{5.4.7}
$$

Note that there is symmetry breaking in the sense that the commutation relations for sub-algebras relating to both M^4 and M^2 are in distinguished role.

Dimensions $D=4$ and $D=8$ are indeed unique if one takes this argument seriously.

1. For dimensions other than $D=4$ and $D=8$ a representation of the point of M^D as element of Clifford algebra of M^D is needed. The coefficients should be real for the signatures and this requires that the elements of Clifford algebra are Hermitian. Gamma matrices are the only natural candidates and when Majorana conditions can be satisfied one obtains quantum representation of M^D. 10-D Minkowski space of super-string models would represent one example of this kind of situation.

2. For other dimensions $D \geq 8$ but now octonionic units must be replaced by gamma matrices and an explicit matrix representation can be introduced. These gamma matrices can be included as a tensor factor to the infinite-dimensional Clifford algebra so that the local Clifford algebra reduces to a mere Clifford algebra. The units of quantum octonions which are just ordinary octonion units do not however allow matrix representation so that this reduction is not possible and embedding space and space-time indeed emerge genuinely. The non-associativity of octonions would determine the laws of physics in TGD Universe!

WCW spinor fields as hyper-octonionic conformal fields

A further proposed application of this picture is to the construction of WCW spinor fields as generalizations of conformal fields. The basic problem is to treat center of mass degrees of freedom properly, and the idea that conformal invariance generalizes to hyper-octonionic - or at least hyper-quaternionic - conformal invariance is attractive. If so, the usual expansion in powers of complex coordinate z would be replaced in powers of hyper-octonionic coordinate h and the coefficients would be elements of Clifford algebra for sub- WCW consisting of light-like 3-surfaces with frozen center of mass degrees of freedom. This is possible if one can map the points of H to those of M^8 and $M^8 - H$ duality allows to achieve this.

One could use Laurent expansions with coefficients multiplying powers of h from right so that one could defined the notion of octonion analyticity in terms of a generalization of Riemann conditions as shown in [A83]. In the case of quaternionic analyticity one obtains also analyticity in two complex variables for one particular form of Riemann conditions and something similar might happen now.

Hyper-octonions do not define a number field but only linear sub-space of complexified octonions. This does not however matter in this case. Also the notions of quaternionic and complex sub-manifold are independent of signature.

The natural condition would be that N-point functions defined by WCW spinor fields for which M^8 coordinate labels the position of the tip of the causal diamond containing the zero energy state involve only those points which are mutually associative and would thus belong to a hyper-quaternionic sub-space $M^4 \subset M^8$ would be in question and the outcome would be the analog of M^4 quantum field theory.

Commutativity would restrict the points to $M^2 \subset M^4 \subset M^8$ and hyper-complex variant conformal field theory would result: this theory would be analogous with integrable models known as factorizing quantum field theories in M^2 in which particle scattering is almost trivial (interactions generate only phase lag).

5.4.7 Zero Energy Ontology And Witten's Approach To 3-D Quantum Gravitation

There is an interesting relationship of quantum TGD to the recent yet unpublished work of Witten related to 3-D quantum blackholes [B43], which - despite that it does not directly relate to M-theory - provides additional perspective.

1. The motivation of Witten is to find an exact quantum theory for blackholes in 3-D case. Witten proposes that the quantum theory for 3-D AdS_3 blackhole with a negative cosmological constant can be reduced by AdS_3/CFT_2 correspondence to a 2-D conformal field theory at the 2-D boundary of AdS_3 analogous to blackhole horizon. This conformal field theory would be a Chern-Simons theory associated with the isometry group $SO(1,2) \times SO(1,2)$ of AdS_3. Witten restricts the consideration to $\Lambda < 0$ solutions because $\Lambda = 0$ does not allow black-hole solutions and Witten believes that $\Lambda > 0$ solutions are non-perturbatively unstable.

2. This conformal theory would have the so called monster group [B43, B2] as the group of its discrete hidden symmetries. The primary fields of the corresponding conformal field theory would form representations of this group. The existence of this kind of conformal theory has been demonstrated already [B16]. In particular, it has been shown that this theory does not allow massless states. On the other hand, for the 3-D vacuum Einstein equations the vanishing of the Einstein tensor requires the vanishing of curvature tensor, which means that

gravitational radiation is not possible. HenceAdS_3 theory in Witten's sense might define this conformal field theory.

Witten's construction has obviously a strong structural similarity to TGD.

1. Chern-Simons action for the induced Kähler form - or equivalently, for the induced classical color gauge field proportional to Kähler form and having Abelian holonomy - corresponds to the Chern-Simons action in Witten's theory.
2. Light-like 3-surfaces can be regard as 3-D solutions of vacuum Einstein equations. Due to the effective 2-dimensionality of the induced metric Einstein tensor vanishes identically and vacuum Einstein equations are satisfied for $\Lambda = 0$. One can say that light-like partonic 3-surfaces correspond to empty space solutions of Einstein equations. Even more, partonic 3-surfaces are very much analogous to 3-D black-holes if one identifies the counterpart of black-hole horizon with the intersection of $\delta M^4_\pm \times CP_2$ with the partonic 2-surface.
3. For light-like 3-surfaces curvature tensor is non-vanishing which raises the question whether one obtains gravitons in this case. The fact that time direction does not contribute to the metric means that propagating waves are not possible so that no 3-D gravitational radiation is obtained. There is analog for this result at quantum level. If partonic fermions are assumed to be free fields as is done in the recent formulation of quantum TGD, gravitons can be obtained only as parton-antiparton bound states connected by flux tubes and are therefore genuinely stringy objects. Hence it is not possible to speak about 3-D gravitons as single parton states.
4. Vacuum Einstein equations can be regarded as gauge fixing allowing to eliminate partially the gauge degeneracy due to the general coordinate invariance. Additional super conformal symmetries are however present and have an identification in terms of additional symmetries related to the fact that space-time surfaces correspond to preferred extremals of Kähler action whose existence was concluded before the discovery of the formulation in terms of light-like 3-surfaces.

There are also interesting differences.

1. According to Witten, his theory has no obvious generalization to 4-D black-holes whereas 3-D light-like determinants define the generalization of blackhole horizons which are also light-like 3-surfaces in the induced metric. In particular, light-like 3-surfaces define a 4-D quantum holography.
2. Also the fermionic counterpart of Chern-Simons action for the induced spinors whose form is dictated by the super-conformal symmetry is present. Furthermore, partonic 3-surfaces are dynamical unlike AdS_3 and the analog of Witten's theory results by freezing the vibrational degrees of freedom in TGD framework.
3. The very notion of light-likeness involves the induced metric implying that the theory is almost-topological but not quite. This small but important distinction indeed guarantees that the theory is physically interesting.
4. In Witten's theory the gauge group corresponds to the isometry group $SO(1,2) \times SO(1,2)$ of AdS_3. The group of isometries of light-like 3-surface is something much much mightier. It corresponds to the conformal transformations of 2-dimensional section of the 3-surfaces made local with respect to the radial light-like coordinate in such a manner that radial scaling compensates the conformal scaling of the metric produced by the conformal transformation.

   The direct TGD counterpart of the Witten's gauge group would be thus infinite-dimensional and essentially same as the group of 2-D conformal transformations. Presumably this can be interpreted in terms of the extension of conformal invariance implied by the presence of ordinary conformal symmetries associated with 2-D cross section plus "conformal" symmetries with respect to the radial light-like coordinate. This raises the question about the possibility to formulate quantum TGD as something analogous to string field theory using using Chern-Simons action for this infinite-dimensional group.
5. Monster group does not have any special role in TGD framework. However, all finite groups and - as it seems - also compact groups can appear as groups of dynamical symmetries at the partonic level in the general framework provided by the inclusions of hyper-finite factors of type II_1 [K38]. Compact groups and their quantum counterparts would closely relate to a

hierarchy of Jones inclusions associated with the TGD based quantum measurement theory with finite measurement resolution defined by inclusion as well as to the generalization of the embedding space related to the hierarchy of Planck constants [K38]. Discrete groups would correspond to the number theoretical braids providing representations of Galois groups for extensions of rationals realized as braidings [K52].

6. To make it clear, I am not suggesting that AdS_3/CFT_2 correspondence should have a TGD counterpart. If it had, a reduction of TGD to a closed string theory would take place. The almost-topological QFT character of TGD excludes this on general grounds. More concretely, the dynamics would be effectively 2-dimensional if the radial superconformal algebras associated with the light-like coordinate would act as pure gauge symmetries. Concrete manifestations of the genuine 3-D character are following.

   (a) Generalized super-conformal representations decompose into infinite direct sums of stringy super-conformal representations.

   (b) In p-adic thermodynamics explaining successfully particle massivation radial conformal symmetries act as dynamical symmetries crucial for the particle massivation interpreted as a generation of a thermal conformal weight.

   (c) The maxima of Kähler function defining Kähler geometry in the world of classical worlds correspond to special light-like 3-surfaces analogous to bottoms of valleys in spin glass energy landscape meaning that there is infinite number of different 3-D light-like surfaces associated with given 2-D partonic configuration each giving rise to different background affecting the dynamics in quantum fluctuating degrees of freedom. This is the analogy of landscape in TGD framework but with a direct physical interpretation in say living matter.

As noticed, Witten's theory is essentially for 2-D fundamental objects. It is good to sum up what is needed to get a theory for 3-D fundamental objects in TGD framework from an approach similar to Witten's in many respects. This connection is obtained if one brings in 4-D holography, replaces 3-metrics with light-like 3-surfaces (light-likeness constraint is possible by 4-D general coordinate invariance), and accepts the new view about M-matrix implied by the zero energy ontology.

1. Light-like 3-surfaces can be regarded as solutions vacuum Einstein equations with vanishing cosmological constant (Witten considers solutions with non-vanishing cosmological constant). The effective 2-D character of the induced metric is what makes this possible.

2. Zero energy ontology is also an essential element: quantum states of 3-D theory in zero energy ontology correspond to generalized S-matrices: **Matrix** or M-matrix might be a proper term. **Matrix** is a "complex square root" of density matrix -matrix valued generalization of Schrodinger amplitude - defining time like entanglement coefficients. Its "phase" is unitary matrix and might be rather universal. **Matrix** is a functor from the category of Feyman cobordisms and matrices have groupoid like structure (see discussion below). Without this generalization theory would reduce to a theory for 2-D fundamental objects.

3. Theory becomes genuinely 4-D because M-matrix is not universal anymore but characterizes zero energy states.

4. 4-D holography is obtained via the Kähler metric of the world of classical worlds assigning to light-like 3-surface a preferred extremal of Kähler action as the analog of Bohr orbit containing 3-D light-like surfaces as sub-manifolds (analogs of blackhole horizons and light-like boundaries) [K28]. Interiors of 4-D space-time sheets corresponds to zero modes of the metric and to the classical variables of quantum measurement theory (quantum classical correspondence). The conjecture is that Dirac determinant for the Kähler-Dirac action associated with partonic 3-surfaces defines the vacuum functional as the exponent of Kähler function with Kähler coupling strength fixed completely as the analog of critical temperature so that everything reduces to almost topological QFT [K121].

5.5 What Went Wrong With String Models?

As will be found, the few physical predictions of M-theory are wrong. It is instructive to try to understand what went wrong with M-theories and string models by comparing it with earlier successful theories and with TGD.

5.5.1 Problems Of M-Theory

At the physical side the situation in M-theory can be regarded as a catastrophe and without the association of the attribute "the only known candidate for the quantum theory of gravitation..." to the letter M bringing in mind Pavlov dogs, no-one could take it seriously. The various problems of M-theory have been discussed in the article of Smolin [B44] as also by Penrose in his lecture series "Fashion, Faith and Fantasy in Theoretical Physics" [B55]. The discussions of "Not Even Wrong" [B4] group provide a vivid critical view about the situation.

1. M-theory has not been able to explain why the dimension of the space-time is four and has even failed to reproduce the standard model. Unless one assumes that the small dimensions form a singular manifold (something so ugly that it turns my stomach around), M-theory predicts chiral symmetry just like Kaluza-Klein theories: the symmetry is inconsistent with the standard model. Ironically, just this was the reason why superstrings replaced Kaluza-Klein theories in the first superstring revolution. This full π twist represents a good example of M-logic.

   The predicted massless scalar fields have not been observed. The predicted low energy super-symmetry is experimentally absent, and now papers have begun to appear suggesting that M-theory after all might predict only high energy super-symmetry. One of the first findings after the second superstring revolution was that the prediction for the unification scale was wrong. I remember that Witten proposed at that time a suitable compactification of the 11^{th} dimension to a circle to circumvent this problem.

2. Cosmological constant is now believed to be non-vanishing and positive [E2] whereas the cosmological constant predicted by M-theory is negative. M-theories provide no explanation for the accelerated expansion [E2]. There is a plethora of cosmological observations which M-theory cannot even address.

This sad state of affairs has led to the introduction of the anthropic principle [B45] but not in the sense that it would really predict something but as an M-logic proof that M-theory after all predicts among other things also the cosmological constant correctly. The premise is that M-theory is correct and the conclusion is that the observed universe must represent some distant corner of the M-landscape, and we must be ready to accept as a fact, that we will never be able to find our way to this distant part of the Theory Universe, and be happy with learning new dualities.

5.5.2 Mouse As A Tailor

The history of string models differs dramatically from that for theories which has been successful as physical theories. As a rule, new theories have started from a precise problem which earlier theories have not been able to solve, and have led to a new ontology and inspired new mathematics.

String model was born as a model of hadrons. It however became gradually clear that the constraints on space-time dimensions make it unrealistic for this purpose. The conclusion of the mouse was not so humble as in the tale: admittedly string models fail for hadrons but who knows, they might describe everything.

After a decade of tailoring the cat was told that superstrings do not seem to make a TOE after all. The mouse said that he could tailor even something more grandiose just by sewing together all the previous failures. Now it has become clear that the result is an enormous bundle of solutions of the possibly existing M-theory, which at practical level is reduced after few heuristic arguments to compactifications of 11-D super gravity. There is still however a little problem: not a single one of these solutions seems to describe the Universe we live in. Now the mouse suggests that we should give up the dream about a theory of the observable universe as unrealistic, stop complaining and be happy with all these beautiful dualities.

Is the time ripe for the story to end as its original version did or shall the cat provide still another decade of financial support for the expensive tailor?

5.5.3 The Dogma Of Reductionism

M-theory as an outcome of hard-nosed reductionism

The philosophical background of string models is hard-nosed reductionism taken down to Planck length: something taken to be so self-evident that it has not been even mentioned. Hence the theory cannot make any predictions about or utilize the rich experimental input coming from the known physics.

This means that string theorists do not pay any attention to the pressing problems of quantum measurement theory, to the problems related to the relationship between experienced and geometric time, and to the problems surrounding to the poor understanding of second law. Not to even mention the questions about the difference between animate and in-animate matter, and about what it means to be a conscious system.

The belief that the action defining functional integral summarizes the physics leads to an approach which is extremely pragmatic: start from the existing formulas of perturbative field theories and try to combine them in order to cook up a more general theory. The danger that theoreticians fall into a kind of mathematical insanity in this kind of situation is obvious, and the possible failure of reductionism means a tragic failure of the entire approach.

Giving up reductionism

TGD cannot be regarded as a success from the point of view of sociology of science but the success of TGD as a physical theory is undeniable and basically due to the facts that TGD emerged as a solution to a well-defined problem, and that the notion of many-sheeted space-time plus p-adic length scale hypothesis [K74] provide a precise quantitative formulation for how reductionism fails.

1. I ended up with TGD by starting from a very real problem of general relativity and soon found that I could end up to TGD also from string models. From the beginning the contact of TGD with experimental physics was very intimate. Later the quantum classical correspondence has become a basic guide line in the construction of the theory.

2. One cannot deny that string theories partially solved the divergence problem of perturbative quantum field theories. Unfortunately, is is highly implausible that the sum of the perturbation series would converge so that as such it is useless. This has in fact been seen as a victory of the theory since one can hope that a genuinely non-perturbative approach could lead to a unique theory.

   In TGD framework the absence of the basic divergences is highly plausible already from the basic construction involving new ontology of space-time. Vacuum functional identified as an exponent of Kähler function is not anymore a local functional of 3-surface so that basic perturbative divergences resulting from the micro-locality are absent. Also Gaussian and metric determinants cancel and the definition of Kähler function in terms of Dirac determinant is free of divergencies [K121].

3. The construction of quantum TGD was not possible without the theory of consciousness. Key element is the replacement of space-time micro-locality with classical locality in the "world of classical worlds" making possible to understand how macroscopic and macro-temporal quantum coherence are possible [K55, K17, K56]. Thanks to the notion of self [K94, **?**, K21], observer ceases to be an outsider and quantum measurement theory is becomes an essential part of the theory. Completely un-expected outcomes were the already mentioned generalizations of the number concept and the identification of the space-time correlates of cognition and intentionality.

4. TGD generalizes in a dramatic manner the ontology of space-time in terms of the notion of the many-sheeted space-time involving also the new view about numbers. The identification of space-time sheets as space-time counterparts of physical objects resolves the question about the generation of structures. The ontology of quantum TGD is discussed in [K21] from the point of view of category theory. One important implication is that even quantum

superposition and quantum logic can have space-time correlates at the level of many-sheeted space-time.

5. TGD resolves the paradoxes due to the conflict between the non-determinism of quantum jump and determinism of Schrödinger equation and, by the classical non-determinism, quantum-classical correspondence can be realized at the space-time level even for quantum jump sequences. TGD leads to a new view about the relationship between geometric and subjectively experienced time rather than just identifying them [?].

6. Zero energy ontology replaces positive energy ontology. Zero energy states are superpositions of pairs of positive and negative energy states with opposite energies and other conserved quantum numbers assignable to the boundaries of causal diamond (CD). In ordinary ontology they corresponds to events consisting of initial and final state.

   Negative energies make possible what I call remote metabolism playing in key role in TGD inspired theory of consciousness and of quantum biology: the system can gain energy by sending negative energy to geometric past [?, K55, K56]. Time mirror mechanism (see **Fig.** `http://tgdtheory.fi/appfigures/timemirror.jpg` or **Fig. ??** in the appendix of this book) makes possible communications with geometric past and future and communications with an effectively super-luminal velocity become possible.

7. The duality between theory and reality is resolved. TGD based ontology postulates only three levels of existence corresponding to existences in these sense of classical and quantum physics, and conscious existence which corresponds to the quantum jumps between the quantum states [K21]. The possibility that space-time points are infinitely structured in p-adic sense although this structure is not visible in real sense [K103], would resolve the challenge posed by the question why all those structures that we can imagine mathematically, are not realized physically. Obviously, a reincarnation of the monad idea of Leibniz is in question.

5.5.4 The Loosely Defined M

In a sharp contrast with M-theory [B19], Newton's mechanics and gravitational theory, Maxwell's electrodynamics, Special and General Relativities, and even Bohr's rules were from the beginning relatively precisely defined theories able to make testable predictions. The lack of a precise definition of what "M" means has led to a flood of speculations based on speculations based on...

"M" as "membrane" would be a rather precise definition but does not really make sense since the huge conformal invariance of string models is lost as objects become 2-dimensional. For this reason one prefers to replace "M" with Mystery, Mother, or perhaps Matrix, but still think in terms of membranes which behave like strings. It became however clear that also branes of various dimensions are needed as discovered by Polchinski [B38] and identified as non-perturbative objects at which string ends are attached to: this interpretation is the only possible one since otherwise momentum conservation would be lost for D-branes.

Needless to say, a theory using geometric structures consisting of parts possessing different dimensions does not satisfy the standards of the conventional mathematical aesthetics. An outsider could argue that the non-uniqueness of the boundary conditions (Neumann, Dirichlet and mixtures of them) is the fundamental failure of the string theory, and that a viable theory should predict the dynamics of boundaries. This is indeed the case in TGD where the criticality of the Kähler action guaranteeing general coordinate invariance in 4-D sense does this and implies that the space-time surface is a field theory counterpart of Bohr orbit.

A good example of brave new M-logic is provided by the construction of what is called Matrix Theory [B26]. One starts from M-theory "known" to have 11-D supergravity as a low energy limit, replaces it with a 11-D supergravity, restricts the consideration to N 0-branes (point particles) living in an effectively 10-D space, in an ad hoc manner replaces their position coordinates in 10-D space with non-commuting $N\times N$-matrix valued coordinates assuming that eigenvalues correspond to N space-time points, postulates a non-relativistic Schrödinger equation for this matrix, and by generalizing bravely the notion of holography, concludes that the original theory and even more follows from this very-very special theory at $N\rightarrow\infty$ limit. From Matrix Theory one then deduces all superstring dualitiesandblack hole physics using an argumentation with a comparable rigor.

It must be added that TGD predicts a rich variety of objects resulting as asymptotic self-organization patterns for which Kähler-Lorentz 4-force vanishes by quantum classical correspondence. The solutions are classified by the dimension of either their M^4 or CP_2 projection [K14]. This variety includes cosmic strings and magnetic flux tubes besides space-time sheets. Magnetic flux tubes and string like objects can indeed attach to the boundaries of space-time sheets and there are obvious correspondences with branes with dimensions of branes restricted to run from 0 to 4 ($p = -1, .., 3$) but only as objects obtained by idealizing 4-dimensional object with a lower-dimensional object.

Even the possibility of single space-time point or space-time curve to mimic the quantum dynamics of the quantum state of Universe is predicted but only at the level of cognition and relying on the new notion about what mathematical point is [K103]. I however do not think that this has much to do with Matrix Theory.

5.5.5 What Went Wrong With Symmetries?

Theoretical physics is in deep crisis. This is not bad at all. Crisis forces eventually to challenge the existing beliefs. Crisis gives also hopes about profound changes. In physical systems criticality means sensitivity, long range fluctuations and long range correlations, and this makes phase transition possible. In TGD framework life emerges at criticality!

The crisis of theoretical physics has many aspects. The crisis relates closely to the sociology of science and to the only game in the town attitude. The prevailing materialistic philosophy of science combined with the naïve length scale reductionism form part of the sad story. The seeds of the crisis were sown in birthdays of quantum mechanics. The fathers of quantum theory were well aware that quantum measurement theory is the Achilles heel of the newborn quantum theory but later the pragmatically thinking theoreticians labelled questioning of the basic concepts as "philosophy" not meant for a respectable physicist.

The recent quantum measurement theory is just a collection of rules and observer still remains an outsider. To my view the proper formulation of quantum measurement theory requires making observer a part of systems. This means that physics must be extended to a theory of consciousness.

This raises several fundamental challenges and questions. How to define "self" as conscious entity? How to resolve the conflict between two causalities: that of field equations and that of "free will"? What is the relationship between the geometric time of physicist and the experienced time? How is the arrow of time determined and is it always the same? The evidence that living matter is macroscopic quantum system is accumulating: is a generalization of quantum theory required to describe quantum systems? What about dark matter: can we understand it in the framework of existing quantum theory? This list could be continued.

In the following I will not consider this aspect more but restrict the consideration to an important key notion of recent day theoretical physics, namely symmetries. Physical theories rely nowadays on postulates about symmetries and there are many who say that quantum theory reduces almost totally group representation theory. There are refined mathematical tools making possible to derive the implications of symmetries in quantum theory such as Noether's theorem. These technical tools are extremely useful but it seems that methodology has replaced critical thought.

By this I mean that the real nature of various symmetries has not been considered seriously enough and that this is one of the basic reasons for the recent dead end. In the following I describe what I see as the mistakes due to sloppy thinking (maybe "sloppying" might be shorthand for it) and discuss briefly the TGD based solution of the problems involved.

This sloppiness manifests itself already in general relativity, in standard model there is no unification of color and electroweak symmetries and their different character is not understood, GUT approach is based on naïve extension of gauge group and makes problematic predictions, supersymmetry in its standard form predicted to become visible at LHC energies is now strongly dis-favoured experimentally, and superstring model led to landscape catastrophe what has left is AdS/CFT correspondence which has not led to victories. Could it be that also conformal invariance should be re-considered seriously: a non-trivial generalization to 4-D context is highly desirable so that 10-D bulk would be replaced by 4-D space-time in the counterpart of AdS/CFT duality.

Energy problem of GRT

Energy and momentum are not well-defined notions in General Relativity. The Poincare symmetry of flat Minkowski space is lost and one cannot apply Noether's theorem so that the identification of classical conserved charges is lost and one can talk only about local conservation guaranteed by Einstein's equations realizing Equivalence Principle in weak form.

In quantum theory this kind of situation is highly unsatisfactory since Uncertainty Principle means that momentum eigenstates are delocalized. This is sloppy thinking and the fact that quantization is to high extend representation theory for symmetry groups might well explain the failure of the attempts to quantize general relativity.

TGD was born as a reaction to the challenge of constructing Poincare invariant theory of gravitation. The identification of space-times as 4-surfaces of some higher-dimensional space of form $H = M^4 \times S$ lifts Poincare symmetries from space-time level to the level of embedding space H.

In this framework GRT space-time is an approximate macroscopic description obtained by replacing the space-time sheets of many-sheeted space-time with single piece of M^4 which is slightly curved. Gravitational fields -deviations of induced metric from Minkowski metric- are replaced with their sum for various sheets. Same applies to gauge potentials. Einstein's equations express the remnants of Poincare symmetry for the GRT space-time obtained in this manner.

In superstring models one actually considers 10-D Minkowski space so that the lifting of symmetries is possible. Also the compactification (say Calabi-Yau) to $M^4 \times C$ still have Poincare symmetries. But after that one has 10-D gravitation and the same problems that one wanted to solve by introducing strings! School example about sloppying!

Is color symmetry really understood?

May colleagues use to think that standard model is a closed chapter of theoretical physics. This is a further example of sloppy thinking.

1. Standard model gauge group is product of color and electro-weak groups which are totally independent. The analogy with Maxwell's equations is obvious. Only after Maxwell and Einstein they could be seen as parts of single tensor representing gauge field.

2. QCD and electroweak interactions differ in crucial manner. Color symmetry is exact (no Higgs fields in QCD) whereas electroweak symmetry is broken, and QCD is asymptotically free unlike electroweak interactions. In QCD color confinement takes place at low energies and remains still poorly understood.

Again TGD approach suggests a solution to these problems in terms of induced gauge field concept and a more refined view about QCD color.

1. $S = CP_2$ has color group SU(3) as isometries and electroweak gauge group as holonomies: hence CP_2 unifies these symmetries just like Maxwell's theory unified electric and magnetic fields. Note that the choice of $H = M^4 \times CP_2$ is not adhoc: its factors are the only 4-D spaces allowing twistor spaces with Kähler structure.

2. One can understand also the different nature of these symmetries. Color group represents exact symmetries so that symmetry breaking should not take place. Holonomies are tangent space symmetries and broken already at the level of CP_2 geometry and does not therefore give rise to genuine Noether symmetries. One can however assign broken electroweak gauge symmetries to the holonomies.

   The isometry group defines Kac-Moody algebra in quantum TGD and color group acts as Kac-Moody group rather than gauge group. The differences is very delicate since only the central extension of Kac-Moody algebra distinguishes it from gauge algebra.

3. Color is not spin-like quantum number as in QCD but colored states correspond to color partial waves in CP_2 rather. Both leptons and quarks allow colored excitations which are however expected to be very heavy.

Is Higgs mechanism only a parameterization of particle masses?

The discovery of Higgs at LHC was very important step of progress but did not prove Higgs mechanism as a mechanism of massivation as sloppy thinkers believe. Fermion masses are not a prediction of the theory: they are put in by hand by assuming that Higgs couplings are proportional to the Higgs mass. It might well be that Higgs vacuum expectation value is the unique quantum field theoretic representation of particle massivation but that QFT approach cannot predict the masses and that the understanding of the massivation requires transcending QFT so that one describing particles as extended objects. String models were the first step to this direction but one step was not enough.

In TGD framework more radical generalization is performed. Point-like particle is replaced with a 3-surface and particle massivation is described in terms of p-adic thermodynamics, which relies on very general assumptions such as a non-trivial generalization of 2-D conformal invariance to 4-D context to be discussed later, p-adic thermodynamics, p-adic length scale hypothesis, and mapping of the predictions for p-adic mass squared to real mass squared by what I call anonical identification. In this framework Higgs vacuum expectation value parametrizes the QFT limit already described and is calculable from generalized Feynman diagrammatics.

GUT approach as more sloppy thoughts

After the successes of standard model the naïve guess was that theory of everything could be constructed by a simple trick: extend the gauge group to a larger group containing standard model gauge group as sub-group. One can do this and there is a refined machinery allowing to deduce particle multiplets, effective actions, beta functions, etc.. There exists of course an infinite variety of Lie groups and endless variety of GUTs have been proposed.

The view about the Universe provided by GUTs is rather weird looking.

1. Above weak mass scale there should be a huge desert of 14 orders of magnitudes containing no new physics! This is like claiming that the world ends at my backyard.
2. Only the sum of baryon and lepton numbers would be conserved and proton would be unstable. The experimental lower limit for proton lifetime has been however steadily increasing and all GUTs derived from superstring models share a fine tuning to keep proton alive.
3. Standard model gauge group seems to be all that is needed: there are no indications for larger gauge group. Fermion families seem to be copies of each other with different mass scales. Also the mass scales of these fermions differ dramatically and forcing them to multiplets of single gauge group could also be sloppy thinking. One would expect that the masses differ by simple numerical factors but they do not.

From TGD viewpoint the GUT approach is un-necessary.

1. In TGD quarks and leptons correspond to different chiralities of embedding space spinors. 8-D chiral invariance implies that quark and lepton numbers are separately conserved so that proton does not decay - at least in the manner predicted by GUTs. CP_2 mass scale is of same order of magnitude as the mass scale assigned to the super heavy additional gauge bosons mediating proton decay.
2. Family replication phenomenon does not require extension of gauge group since fermion families correspond to different topologies for partonic 2-surfaces representing fundamental particles (genus-generation correspondence) [K25]). Note that the orbits of partonic 2-surfaces correspond to light-like 3-surface at which the induced metric changes its signature from Euclidian to Minkowskian: these surfaces or equivalently the 4-surfaces with Euclidian signature can be regarded as lines of generalized Feynman diagrams. The three lowest genera are special in the sense that they always allow Z_2 as global conformal symmetry whereas higher genera allow this symmetry only in case of hyper-elliptic surfaces: this leads to an explanation for the experimental absence of higher genera. Higher genera could be more naturally many particle states with continuum mass spectrum with handles taking the role of particles.
3. p-Adic length scale hypothesis emerging naturally in TGD framework allows to understand the mass ratios of fermions which are very un-natural if different fermion families are assumed to be related by gauge symmetries.

Supersymmetry in crisis

Supersymmetry is very beautiful generalization of the ordinary symmetry concept by generalizing Lie-algebra by allowing grading such that ordinary Lie algebra generators are accompanied by super-generators transforming in some representation of the Lie algebra for which Lie-algebra commutators are replaced with anti-commutators. In the case of Poincare group the super-generators would transform like spinors. Clifford algebras are actually super-algebras. Gamma matrices anti-commute to metric tensor and transform like vectors under the vielbein group (SO(n) in Euclidian signature). In supersymmetric gauge theories one introduced super translations anti-commuting to ordinary translations.

Supersymmetry algebras defined in this manner are characterized by the number of super-generators and in the simplest situation their number is one: one speaks about $\mathcal{N} = 1$ SUSY and minimal super-symmetric extension of standard model (MSSM) in this case. These models are most studied because they are the simplest ones. They have however the strange property that the spinors generating SUSY are Majorana spinors- real in well-defined sense unlike Dirac spinors. This implies that fermion number is conserved only modulo two: this has not been observed experimentally. A second problem is that the proposed mechanisms for the breaking of SUSY do not look feasible.

LHC results suggest MSSM does not become visible at LHC energies. This does not exclude more complex scenarios hiding simplest $\mathcal{N} = 1$ to higher energies but the number of real believers is decreasing. Something is definitely wrong and one must be ready to consider more complex options or totally new view abot SUSY.

What is the situation in TGD? Here I must admit that I am still fighting to gain understanding of SUSY in TGD framework [K96]. That I can still imagine several scenarios shows that I have not yet completely understood the problem but I am working hardly to avoid falling to the sin of sloppying myself. In the following I summarize the situation as it seems just now.

1. In TGD framework $\mathcal{N} = 1$ SUSY is excluded since B and L and conserved separately and embedding space spinors are not Majorana spinors. The possible analog of space-time SUSY should be a remnant of a much larger super-conformal symmetry in which the Clifford algebra generated by fermionic oscillator operators giving also rise to the Clifford algebra generated by the gamma matrices of the "world of classical worlds" (WCW) and assignable with string world sheets. This algebra is indeed part of infinite-D super-conformal algebra behind quantum TGD. One can construct explicitly the conserved super conformal charges accompanying ordinary charges and one obtains something analogous to $\mathcal{N} = \infty$ super algebra. This SUSY is however badly broken by electroweak interactions.
2. The localization of induced spinors to string world sheets emerges from the condition that electromagnetic charge is well-defined for the modes of induced spinor fields. There is however an exception: covariantly constant right handed neutrino spinor ν_R: it can be de-localized along entire space-time surface. Right-handed neutrino has no couplings to electroweak fields. It couples however to left handed neutrino by induced gamma matrices except when it is covariantly constant. Note that standard model does not predict ν_R but its existence is necessary if neutrinos develop Dirac mass. ν_R is indeed something which must be considered carefully in any generalization of standard model.

Could covariantly constant right-handed spinors generate exact $\mathcal{N} = 2$ SUSY? There are two spin directions for them meaning the analog $\mathcal{N} = 2$ Poincare SUSY. Could these spin directions correspond to right-handed neutrino and antineutrino. This SUSY would not look like Poincare SUSY for which anti-commutator of super generators would be proportional to four-momentum. The problem is that four-momentum vanishes for covariantly constant spinors! Does this mean that the sparticles generated by covariantly constant ν_R are zero norm states and represent super gauge degrees of freedom? This might well be the case although I have considered also alternative scenarios.

Both embedding space spinor harmonics and the Kähler-Dirac equation have also right-handed neutrino spinor modes not constant in M^4. If these are responsible for SUSY then SUSY is broken.

1. Consider first the situation at space-time level. Both induced gamma matrices and their generalizations to Kähler-Dirac gamma matrices defined as contractions of embedding space

gamma matrices with the canonical momentum currents for Kähler action are superpositions of M^4 and CP_2 parts. This gives rise to the mixing of right-handed and left-handed neutrinos. Note that non-covariantly constant right-handed neutrinos must be localized at string world sheets.

This in turn leads neutrino massivation and SUSY breaking. Given particle would be accompanied by sparticles containing varying number of right-handed neutrinos and antineutrinos localized at partonic 2-surfaces.

2. One an consider also the SUSY breaking at embedding space level. The ground states of the representations of extended conformal algebras are constructed in terms of spinor harmonics of the embedding space and form the addition of right-handed neutrino with non-vanishing four-momentum would make sense. But the non-vanishing four-momentum means that the members of the super-multiplet cannot have same masses. This is one manner to state what SUSY breaking is.

3. The simplest form of massivation would be that all members of the super-multiplet obey the same mass formula but that the p-adic length scales associated with them are different. This could allow very heavy sparticles. What fixes the p-adic mass scales of sparticles? If this scale is CP_2 mass scale SUSY would be experimentally unreachable.

4. One can even consider the possibility that SUSY breaking makes sparticles unstable against phase transition to their dark variants with $h_{eff} = n \times h$. Sparticles could have same mass but be non-observable as dark matter not appearing in same vertices as ordinary matter! Geometrically the addition of right-handed neutrino to the state would induce many-sheeted covering in this case with right handed neutrino perhaps associated with different space-time sheet of the covering.

   This idea need not be so outlandish at it looks first. The generation of many-sheeted covering has interpretation in terms of breaking of conformal invariance. The sub-algebra for which conformal weights are n-tuples of integers becomes the algebra of conformal transformations and the remaining conformal generators do note represent gauge degrees of freedom anymore. They could however represent conserved conformal charges still.

   This generalization of conformal symmetry breaking gives rise to infinite number of fractal hierarchies formed by sub-algebras of conformal algebra and is also something new and a fruit of an attempt to avoid sloppy thinking. The breaking of conformal symmetry is indeed expected in massivation related to the SUSY breaking.

Have we been thinking sloppily also about super-conformal symmetries?

Super string models were once seen as the only possible candidate for the TOE. By looking at the proceedings of string theory conferences one sees that the age of of super strings is over. Landscape problem and multiverse do not give much hopes about predictive theory and the only defence for super string models is as the only game in the town. Super string gurus do not know about competing scenarion but this is not a wonder given the fact that publishing of competing scenarios has been impossible since superstrings have indeed been the only game in the town! One of the very few almost-predictions of superstring theory was $\mathcal{N} = 1$ SUSY at LHC and it seems that it is already now excluded at LHC energies.

AdS/CFT correspondence (`http://tinyurl.com/ye73k6u`) is a deep mathematical discovery inspired by super-string models. One of its variants states that there is duality between conformal theory in M^4 appearing as boundary of 5-D AdS and string theory in 10-D space $AdS_5 \times S^5$. A more general duality would be between conformal theory in M^n and 10-D space $AdS_{n+1} \times S^{10-n-1}$. For $n = 2$ the CFT would give conformal theory at 2-D Minkowski space for which conformal symmetries (actually their hypercomplex variant) form an infinite-D group. Duality has interpretation in terms of holography but the notion of holography is much more general than AdS/CFT.

AdS/CFT have been applied to nuclear physics but nothing sensational have been discovered. AdS/CFT have been tried also to explain the finding that what was expected to be QCD plasma behaves very differently. The first findings came from RHIC for heavy ion collisions and LHC has found that the strange effects appear already for proton heavy ion collisions. Essentially a deviation from QCD predictions is in question and in the regime where QCD should be a good description. AdS/CFT has not been a success (http://tinyurl.com/65gkpkj). AdS/CFT is now applied also to condensed matter physics. At least hitherto no dramatic successes have been reported.

This leads to ask whether sloppy thinking should be blamed again. AdS/CFT is mathematically rather sound and well-tested but is the notion of conformal invariance behind it really the one that applies to real world physics?

1. In TGD framework the ordinary conformal invariance is generalized so that it becomes 4-D one [K27, K121]: of course, the ordinary finite-dimensional conformal group in M^4 is not in question. The basic observation is that light-like 3-surfaces are metrically 2-dimensional and that this leads to a generalization of conformal transformations. One can locally express light-like 3-surfaces as $X^2 \times R$ and what happens is that the conformal transformations of X^2 are localized with respect to the light-like coordinate of R. Light-like orbits of partonic 2-surfaces carrying elementary particle quantum numbers would have this extended conformal invariance.

2. This is not all. In zero energy ontology (ZEO) the diamond like intersections of future and past directed light-cones - causal diamonds (CDs) are the basic objects. The space-time surfaces having 3-D ends at the boundaries of CD are the basic dynamical units. The boundaries of CD are pieces of $\delta M^4_\pm \times CP_2$. The boundary $\delta M^4_\pm = S^2 \times R_+$ is light-like 3-surface and thus allows a huge extension of conformal symmetries: with complex coordinate of S^2 and light-like radial coordinate playing the roles of complex coordinate for ordinary conformal symmetry.

   One can assign superconformal symmetry also the modes of the Kähler-Dirac operator localized at the string world sheets as the analog of super-conformal symmetry of superstring models.

   Besides this there is a further analog of conformal symmetry. The symplectic transformations of $\delta M^4_\pm \times CP_2$ can be regarded as symplectic transformations of $S^2 \times CP_2$ localized with respect to the light-like coordinate of R_+ defining the analog of the complex coordinate z. In TGD Universe a gigantic extension of the conformal symmetry of superstring models experiences applies.

3. Even these extended symmetries extend to a multi-local (loci correspond to partonic 2-surfaces at boundaries of CD) Yangian variant [L8]. Yangian symmetry is very closely related to quantum groups studied for decades but again without serious consideration of the question "Why quantum groups?". The hazy belief has been that they somehow emerge at Planck length scale, which itself is a hazy notion based solely on dimensional analysis and involving Planck constant and Newton's constant characterizing macroscopic gravitation.

   In TGD framework hyper-finite factors of type II_1 [K120] emerge naturally at the level of WCW since fermionic Fock space provides a canonical representation for them and their inclusions provide an elegant description for finite measurement resolution: the included algebra generates states which are not experimentally distinguishable from the original state.

4. Against this it is astonishing that AdS/CFT duality has very simple generalization in TGD framework and emerge from a generalization of General Coordinate Invariance (GCI) [K27] implying holography. Strong form of GCI postulates that either the space-like 3-surfaces at the ends of causal diamonds or the light-like orbits of partonic 2-surfaces can be taken as 3-surfaces defining the WCW : this is just gauge fixing for general coordinate invariance. If this is true then partonic 2-surfaces and their 4-D tangent space data at the boundaries of CD must code for physics. One would have strong form of holography. This might be too much to require: string world sheets carrying induced spinor fields are present and it might be that they cannot be reduced to data at partonic 2-surfaces.

   In any case, for this duality the 10-D space of AdS/CFT duality would be replaced with space-time surface. M^n would be replaced with the light-like parton orbits and/or space-like ends of CD. Surprisingly, this holography would be very much like holography in its original form!

5.5.6 Los Alamos, M-Theory, And TGD

String models have been seen not only as a kind of holy grail of modern physics but also as an ideology promising an Utopia. As a rule, ideologies have tried to establish the new world order using censorship. String model hegemony has followed the tradition.

For about decade ago it became impossible for me to get anything to hep-th and other physics related archives. Interestingly, for few years ago my article about Riemann hypothesis was accepted to the math archives of Los Alamos and is also published [L1]: it was however not possible to get it cross-listed to hep-th. For a few years American Mathematical Society has had a link to my homepage [A1] as one of the few examples about new mathematics related to quantum physics.

I have learned that I am not the only victim of the string revolution (see the comments in "Not Even Wrong" discussion group [B4]). Despite the official statement that anyone can contribute to LANL, an invisible peer system is acting. After 20 years of string revolutions it seems that physics itself has become the victim which has suffered the most severe injuries.

5.6 K-Theory, Branes, And TGD

K-theory has played important role in brane classification in super string models and M-theory. The excellent lectures by Harah Evslin with title *What doesn't K-theory classify?* (see `http://tinyurl.com/y9og83ut`) [B36] make it possible to learn the basic motivations for the classification, what kind of classifications are possible, and what are the failures. Also the Wikipedia article (see `http://tinyurl.com/ycuuh7j4`) [B1] gives a bird's eye of view about problems. As a by-product one learns something about the basic ideas of K-theory - at least I hope so - and about possible mathematical and physical problems of string theories and M-theory.

In the sequel I will discuss critically the basic assumptions of brane world scenario, sum up my meager understanding about the problems related to the topological classification of branes and also to the notion itself, ask what could go wrong with branes and demonstrate how the problems could be avoided in TGD framework, and just to irritate colleauges conclude with a proposal for a natural generalization of K-theory to include also the division of bundles inspired by the generalization of Feynman diagrammatics in quantum TGD, by zero energy ontology, and by the notion of finite measurement resolution.

5.6.1 Brane World Scenario

The brane world scenario looks attractive from the mathematical point of view one is able to get accustomed with the idea that basic geometric objects have varying dimensions. Even accepting the varying dimensions, the basic physical assumptions behind this scenario are vulnerable to criticism.

1. Branes (see `http://tinyurl.com/665osee`) are geometric objects of varying dimension in the 10-/11-dimensional space-time -call it M- of superstring theory/M-theory. In M-theory the fundamental strings are replaced with M-branes, which are 2-D membranes with 3-dimensional orbit having as its magnetic dual 6-D M5-brane. Branes are thought to emerge non-perturbatively from fundamental 2-branes but what this really means is not understood. One has D-p-branes (see `http://tinyurl.com/y7tdcmbp`) with Dirichlet boundary conditions fixing a $p+1$-dimensional surface of M as brane orbit: one of the dimensions corresponds to time. Also S-branes localized in time have been proposed.

2. In the description of the classical limit branes interact with the classical fields of the target space by the generalization of the minimal coupling of charged point-like particle to electromagnetic gauge potential. The coupling is simply the integral of the gauge potential over the world-line - the value of 1-form for the word-line. Point like particle represents 0-brane and in the case of p-brane the generalization is obtained by replacing the gauge potential represented by a 1-from with $p+1$-form. The exterior derivative of this $p+1$-form is $p+2$-form representing the analog of electromagnetic field. Complete dimensional democracy strongly suggests that string world sheets should be regarded as 1-branes.

3. From TGD point of view the introduction of branes looks a rather ad hoc trick. By generalizing the coupling of electromagnetic gauge potential to the word line of point like particle one could introduce extended objects of various dimensions also in the ordinary 4-D Maxwell theory but they would be always interpreted as idealizations for the carriers of 4- currents. Therefore

the crucial step leading to branes involves classical idealization in conflict with Uncertainty Principle and the genuine quantal description in terms of fields coupled to gauge potentials.

My view is that the most natural interpretation for what is behind branes is in terms of currents in D=10 or D= 11 space-time. In this scheme branes have role only as semi-classical idealizations making sense only above some scale. Both the reduction of string theories to quantum field theories by holography and the dynamical character of the metric of the target space conforms with super-gravity interpretation. Internal consistency requires also the identification of strings as branes so that superstring theories and M-theory would reduce to an idealization to 10-/11-dimensional quantum gravity.

In this framework the brave brane world episode would have been a very useful Odysseia. The possibility to interpret various geometric objects physically has proved to be an extremely powerful tool for building provable mathematical conjectures and has produced lots of immensely beautiful mathematics. As a fundamental theory this kind of approach does not look convincing to me.

5.6.2 The Basic Challenge: Classify The Conserved Brane Charges Associated With Branes

One can of course forget these critical arguments and look whether this general picture works. The first thing that one can do is to classify the branes topologically. I made the same question about 32 years ago in TGD framework: I thought that cobordism for 3-manifolds might give highly interesting topological conservation laws. I was disappointed. The results of Thom's classical article about manifold cobordism demonstrated that there is no hope for really interesting conservation laws. The assumption of Lorentz cobordism meaning the existence of global time-like vector field would make the situation more interesting but this condition looked too strong and I could not see a real justification for it. In generalized Feynman diagrammatics there is no need for this kind of condition.

There are many alternative approaches to the classification problem. One can use homotopy, homology, cohomology and their relative and other variants, topological or algebraic K-theory, twisted K-theory, and variants of K-theory not yet existing but to be proposed within next years. The list is probably endless unless something like motivic cohomology brings in enlightenment.

1. First of all one must decide whether one classifies p-dimensional time=constant sections of p-branes or their $p+1$-dimensional orbits. Both approaches have been applied although the first one is natural in the standard view about spontaneous compactification. For the first option topological invariants could be seen as conserved charges: homotopy invariants and homological and cohomological characteristics of branes provide this kind of invariants. For the latter option the invariants would be analogous to instanton number characterizing the change of magnetic charge.

2. Purely topological invariants come first in mind. Homotopy groups of the brane are invariants inherent to the brane (the brane topology can however change). Homological and cohomological characteristics of branes in singular homology characterize the embedding to the target space. There are also more delicate differential topological invariants such as de Rham cohomology defining invariants analogous to magnetic charges. Dolbeault cohomology emerges naturally for even-dimensional branes with complex structure.

3. Gauge theories - both abelian and non-Abelian - define a standard approach to the construction of brane charges for the bundle structures assigned with branes. Chern-Simons classes are fundamental invariants of this kind. Also more delicate invariants associated with gauge potentials can be considered. Chern-Simons theory with vanishing field strengths for solutions of field equations provides a basic example about this. For instance, SU(2) Chern-Simons theory provides 3-D topological invariants and knot invariants.

4. More refined approaches involve K-theory -closely related to motivic cohomology - and its twisted version. The idea is to reduce the classification of branes to the classification of the bundle structures associated with them. This approach has had remarkable successes but has also its short-comings.

The challenge is to find the mathematical classification which suits best the physical intuitions (, which might be fatally wrong as already proposed) but is universal at the same time. This challenge has turned out to be tough. The Ramond-Ramond (RR) p-form fields (see http://tinyurl.com/y9kmbxoy) of type II superstring theory are rather delicate objects and a source of most of the problems. The difficulties emerge also by the presence of Neveu-Schwartz 3-form $H = dB$ defining classical background field.

K-theory has emerged as a good candidate for the classification of branes. It leaves the confines of homology and uses bundle structures associated with branes and classifies these. There are many K-theories. In topological K-theory bundles form an algebraic structure with sum, difference, and multiplication. Sum is simply the direct sum for the fibers of the bundle with common base space. Product reduces to a tensor product for the fibers. The difference of bundles represents a more abstract notion. It is obtained by replacing bundles with pairs in much the same way as rationals can be thought of as pairs of integers with equivalence $(m, n) = (km, kn)$, k integer. Pairs $(n, 1)$ representing integers and pairs $(1, n)$ their inverses. In the recent case one replaces multiplication with sum and regards bundle pairs and (E, F) and $(E + G, F + G)$ equivalent. Although the pair as such remains a formal notion, each pair must have also a real world representativs. Therefore the sign for the bundle must have meaning and corresponds to the sign of the charges assigned to the bundle. The charges are analogous to winding of the brane and one can call brane with negative winding antibrane. The interpretation in terms of orientation looks rather natural. Later a TGD inspired concrete interpretation for the bundle sum, difference, product and also division will be proposed.

5.6.3 Problems

The classification of brane structures has some problems and some of them could be argued to be not only technical but reflect the fact that the physical picture is wrong.

Problems related to the existence of spinor structure

Many problems in the classification of brane charges relate to the existence of spinor structure. The existence of spinor structure is a problem already in general general relativity since ordinary spinor structure exists only if the second Stiefel- Whitney class (see http://tinyurl.com/y7m9ksq7) [A20] of the manifold is non-vanishing: if the third Stiefel-Whitney class vanishes one can introduce so called spin^c structure. This kind of problems are encountered already in lattice QCD, where periodic boundary conditions imply non-uniqueness having interpretation in terms of 16 different spinor structures with no obvious physical interpretation. One the strengths of TGD is that the notion of induced spinor structure eliminates all problems of this kind completely. One can therefore find direct support for TGD based notion of spinor structure from the basic inconsistency of QCD lattice calculations!

1. Freed-Witten anomaly (see http://tinyurl.com/y77znbqr) [B29] appearing in type II string theories represents one of the problems. Freed and Witten show that in the case of 2-branes for which the generalized gauge potential is 3-form so called spin^c structure is needed and exists if the third Stiefel-Whitney class w_3 related to second Stiefel Whitney class whose vanishing guarantees the existence of ordinary spin structure (in TGD framework spin^c structure for CP_2 is absolutely essential for obtaining standard model symmetries).

   It can however happen that w_3 is non-vanishing. In this case it is possible to modify the spin^c structure if the condition $w_3 + [H] = 0$ holds true. It can however happen that there is an obstruction for having this structure - in other words $w_3 + [H]$ does not vanish - known as Freed-Witten anomaly. In this case K-theory classification fails. Witten and Freed argue that physically the wrapping of cycle with non-vanishing $w_3 + [H]$ by a Dp-brane requires the presence of $D(p-2)$ brane cancelling the anomaly. If $D(p-2)$ brane ends to anti-Dp in which case charge conservation is lost. If there is not place for it to end one has semi-infinite brane with infinite mass, which is also problematic physically. Witten calls these branes baryons: these physically very dubious objects are not classified by K-theory.

2. The non-vanishing of $w_3 + [H] = 0$ forces to generalize K-theory to twisted K-theory (see http://tinyurl.com/ya2awfuk) [A25]. This means a modification of the exterior derivative

to get twisted de Rham cohomology and twisted K-theory and the condition of closedness in this cohomology for certain form becomes the condition guaranteeing the existence of the modified spin^c structure. D-branes act as sources of these fields and the coupling is completely analogous to that in electrodynamics. In the presence of classical Neveu-Schwartz (NS-NS) 3-form field H associated with the back-ground geometry the field strength $G^{p+1} = dC_p$ is not gauge invariant anymore. One must replace the exterior derivative with its twisted version to get twisted de Rham cohomology:

$$d \rightarrow d + H \wedge \quad .$$

There is a coupling between p- and p+2-forms together and gauge symmetries must be modified accordingly. The fluxes of twisted field strengths are not quantized but one can return to original p-forms which are quantized. The coupling to external sources also becomes more complicated and in the case of magnetic charges one obtains magnetically charged Dp-branes. Dp-brane serves as a source for $D(p-2)$- branes.

This kind of twisted cohomology is known by mathematicians as Deligne cohomology. At the level of homology this means that if branes with dimension of p are presented then also branes with dimension $p+2$ are there and serve as source of Dp-branes emanating from them or perhaps identifiable as their sub-manifolds. Ordinary homology fails in this kind of situation and the proposal is that so called twisted K-theory could allow to classify the brane charges.

3. A Lagrangian formulation of brane dynamics based on the notion of p-brane democracy (see `http://tinyurl.com/yb462wn9`) [B51] due to Peter Townsend has been developed by various authors.

Ashoke Sen (see `http://tinyurl.com/yannv4q2`) has proposed a grand vision for understanding the brane classification in terms of tachyon condensation in absence of NS-NS field H [B6]. The basic observation is that stacks of space-filling D- and anti D-branes are unstable against process called tachyon condensation which however means fusion of $p+1$-D brane orbits rather than p-dimensional time slices of branes. These branes are however accompanied by lower-dimensional branes and the decay process cannot destroy these. Therefore the idea arises that suitable stacks of D9 branes and anti-D9-branes could code for all lower-dimensional brane configurations as the end products of the decay process.

This leads to a creation of lower-dimensional branes. All decay products of branes resulting in the decay cascade would be by definition equivalent. The basic step of the decay process is the fusion of D-branes in stack to single brane. In bundle theoretic language one can say that the D-branes and anti-D branes in the stack fuse together to single brane with bundle fiber which is direct sum of the fibers on the stack. This fusion process for the branes of stack would correspond in topological K-theory. The fusion of D-branes and anti-D branes would give rise to nothing since the fibers would have opposite sign. The classification would reduce to that for stacks of D9-branes and anti D9-branes.

Problems with Hodge duality and S-duality

The K-theory classification is plagued by problems all of which need not be only technical.

1. R-R fields are self dual and since metric is involved with the mapping taking forms to their duals one encounters a problem. Chern characters appearing in K-theory are rational valued but the presence of metric implies that the Chern characters for the duals need not be rational valued. Hence K-theory must be replaced with something less demanding.

   The geometric quantization inspired proposal of Diaconescu, Moore and Witten [B15] is based on the polarization using only one half of the forms to get rid of the proboem. This is like thinking the 10-D space-time as phase space and reducing it effectively to 5-D space: this brings strongly in mind the identification of space-time surfaces as hyper-quaternionic (associative) sub-manifolds of embedding space with octonionic structure and one can ask whether the basic objects also in M-theory should be taken 5-dimensional if this line of thought is taken seriously. An alternative approach uses K-theory to classify the intersections of branes with 9-D space-time slice as has been porposed by Maldacena, Moore and Seiberg (see `http://tinyurl.com/ycm3l9nt`) [B46].

2. There another problem related to classification of the brane charges. Witten, Moore and Diaconescu (see http://tinyurl.com/y8kdz6wm) [B15] have shown that there are also homology cycles which are unstable against decay and this means that twisted K-theory is inconsistent with the S-duality of type IIB string theory. Also these cycles should be eliminated in an improved classification if one takes charge conservation as the basic condition and an hitherto un-known modification of cohomology theory is needed.
3. There is also the problem that K-theory for time slices classifies only the R-R field strengths. Also R-R gauge potentials carry information just as ordinary gauge potentials and this information is crucial in Chern-Simons type topological QFTs. K-theory for entire target space classifies D-branes as $p+1$-dimensional objects but in this case the classification of R-R field strengths is lost.

The existence of non-representable 7-D homology classes for targent space dimension $D > 9$

There is a further nasty problem which destroys the hopes that twisted K-theory could provide a satisfactory classification. Even worse, something might be wrong with the superstring theory itself. The problem is that not all homology classes allow a representation as non-singular manifolds. The first dimension in which this happens is $D = 10$, the dimension of super-string models! Situation is of course the same in M-theory. The existence of the non-representables was demonstrated by Thom - the creator of catastrophe theory and of cobordism theory for manifolds- for a long time ago.

What happens is that there can exist 7-D cycles which allow only singular embeddings. A good example would be the embedding of twistor space CP_3, whose orbit would have conical singularity for which CP_3 would contract to a point at the "moment of big bang". Therefore homological classification not only allows but demands branes which are orbifolds. Should orbifolds be excluded as unphysical? If so then homology gives too many branes and the singular branes must be excluded by replacing the homology with something else. Could twisted K-theory exclude non-representable branes as unstable ones by having non-vanishing $w_3 + [H]$? The answer to the question is negative: D6-branes with $w_3 + [H] = 0$ exist for which K-theory charges can be both vanishing or non-vanishing.

One can argue that non-representability is not a problem in superstring models (M-theory) since spontaneous compactification leads to $M \times X_6$ ($M \times X_7$). On the other hand, Cartesian product topology is an approximation which is expected to fail in high enough length scale resolution and near big bang so that one could encounter the problem. Most importantly, if M-theory is theory of everything it cannot contain this kind of beauty spots.

5.6.4 What Could Go Wrong With Super String Theory And How TGD Circumvents The Problems?

As a proponent of TGD I cannot avoid the temptation to suggest that at least two things could go wrong in the fundamental physical assumptions of superstrings and M-theory.

1. The basic failure would be the construction of quantum theory starting from semiclassical approximation assuming localization of currents of 10 - or 11-dimensional theory to lower-dimensional sub-manifolds. What should have been a generalization of QFT by replacing point-like particles with higher-dimensional objects would reduce to an approximation of 10- or 11-dimensional supergravity.

   This argument does not bite in TGD. 4-D space-time surfaces are indeed fundamental objects in TGD as also partonic 2-surfaces and braids. This role emerges purely number theoretically inspiring the conjecture that space-time surfaces are associative sub-manifolds of octonionic embedding spaces, from the requirement of extended conformal invariance, and from the non-dynamical character of the embedding space.
2. The condition that all homology equivalence classes are representable as manifolds excludes all dimensions $D > 9$ and thus super-strings and M-theory as a physical theory. This would be the case since branes are unavoidable in M-theory as is also the landscape of compactifications. In semiclassical supergravity interpretation this would not be catastrophe but if

branes are fundamental objects this shortcoming is serious. If the condition of homological representability is accepted then target space must have dimension $D < 10$ and the arguments sequence leading to D=8 and TGD is rather short. The number theoretical vision provides the mathematical justification for TGD as the unique outcome.

3. The existence of spin structure is clearly the source of many problems related to R-R form. In TGD framework the induction of spin^c structure of the embedding space resolves all problems associated with sub-manifold spin structures. For some reason the notion of induced spinor structure has not gained attention in super string approach.

4. Conservative experimental physicist might criticize the emergence of branes of various dimensions as something rather weird. In TGD framework electric-magnetic duality can be understood in terms of general coordinate invariance and holography and branes and their duals have dimension 2, 3, and 4 organize to sub-manifolds of space-time sheets.

   The TGD counterpart for the fundamental D-2-brane is light-like 3-surface. Its magnetic dual has dimension given by the general formula (see `http://tinyurl.com/y9aueyup`) $p_{dual} = D-p-4$, where D is the dimension of the target space [B23]. In TGD one has $D = 8$ giving $p_{dual} = 2$. The first interpretation is in terms of self-duality. A more plausible interpretation relies on the identification of the duals of light-like 3-surfaces as space-like 3-surfaces at the light-like boundaries of CD. General Coordinate Invariance in strong sense implies this duality. For partonic 2-surface and string world sheets carrying spinor modes one would have $p = 1$ and $p_{dual} = 3$. The identification of the dual would be as 4-D space-time surface: does this correspond to strong form of holography?. The crucial distinction to M-theory would be that branes of different dimension would be sub-manifolds of space-time surface.

5. For $p = 0$ one would have $p_{dual} = 4$ assigning five-dimensional surface to orbits of point-like particles identifiable most naturally as braid strands. One cannot assign to it any direct physical meaning in TGD framework and gauge invariance for the analogs of brane gauge potentials indeed excludes even-dimensional branes in TGD since corresponding forms are proportional to Kähler gauge potential (so that they would be analogous to odd-dimensional branes allowed by type II_B superstrings).

   4-branes might be however mathematically useful by allowing to define Morse theory for the critical points of the Minkowskian part of Kähler action. While writing this I learned that Witten (see `http://tinyurl.com/y8ganhrz`) has proposed a 4-D gauge theory approach with $\mathcal{N} = 4$ SUSY to the classification of knots. Witten also ends up with a Morse theory using 5-D space-times in the category-theoretical formulation of the theory [A54]. For some time ago I also proposed that TGD as almost topological QFT defines a theory of knots, knot braidings, and of 2-knots in terms of string world sheets [K51]. Maybe the 4-branes could be useful for understanding of the extrema of TGD of the Minkowskian part of Kähler action which would take take the same role as Hamiltonian in Floer homology: the extrema of 5-D brane action would connect these extrema.

6. Light-like 3-surfaces could be seen as the analogs von Neuman branes for which the boundary conditions state that the ends of space-like 3-brane defined by the partonic 2-surfaces move with light-velocity. The interpretation of partonic 2-surfaces as space-like branes at the ends of CD would in turn make them D-branes so that one would have a duality between D-branes and N-brane interpretations. T-duality (see `http://tinyurl.com/ycvp7rnq`) exchanges von Neumann and Dirichlet boundary conditions so that strong from of general coordinate invariance would correspond to both electric-magnetic and T-duality in TGD framework. Note that T-duality exchanges type II_A and type II_B super-strings with each other.

7. What about causal diamonds and their 7-D light-like boundaries? Could one regard the light-like boundaries of CDs as analogs of 6-branes with light-like direction defining time-like direction so that space-time surfaces would be seen as 3-branes connecting them? This brane would not have magnetic dual since the formula for the dimensions of brane and its magnetic dual allows positive brane dimension p only in the range (1, 3).

5.6.5 Can One Identify The Counterparts Of R-R And NS-NS Fields In TGD?

R-R and NS-NS 3-forms are clearly in fundamental role in M-theory. Since in TGD partonic 2-surfaces define the analogs of fundamental D-2-branes, one can wonder whether these 3-forms could have TGD counterparts.

1. In TGD framework the 3-forms $G_{3,A} = dC_{2,A}$ defined as the exterior derivatives of the two-forms $C_{2,A}$ identified as products $C_{2,A} = H_A J$ of Hamiltonians H_A of $\delta M^4_\pm \times CP_2$ with Kähler forms of factors of $\delta M^4_\pm \times CP_2$ define an infinite family of closed 3-forms belonging to various irreducible representations of rotation group and color group. One can consider also the algebra generated by products $H_A A$, $H_A J$, $H_A A \wedge J$, $H_A J \wedge J$, where A *resp.* J denotes the Kähler gauge potential *resp.* Kähler form or either $\delta M^4_\pm$ or CP_2. A *resp.* Also the sum of Kähler potentials *resp.* forms of $\delta M^4_\pm$ and CP_2 can be considered.
2. One can define the counterparts of the fluxes $\int Adx$ as fluxes of $H_A A$ over braid strands, $H_A J$ over partonic 2-surfaces and string world sheets, $H_A A \wedge J$ over 3-surfaces, and $H_A J \wedge J$ over space-time sheets.Gauge invariance however suggests that for non-constant Hamiltonians one must exclude the fluxes assigned to odd dimensional surfaces so that only odd-dimensional branes would be allowed. This would exclude 0-branes and the problematic 4-branes. These fluxes should be quantized for the critical values of the Minkowskian contributions and for the maxima with respect to zero modes for the Euclidian contributions to Kähler action. The interpretation would be in terms of Morse function and Kähler function if the proposed conjecture holds true. One could even hope that the charges in Cartan algebra are quantized for all preferred extremals and define charges in these irreducible representations for the isometry algebra of WCW . The quantization of electric fluxes for string world sheets would give rise to the familiar quantization of the rotation $\int E \cdot dl$ of electric field over a loop in time direction taking place in superconductivity.
3. Should one interpret these fluxes as the analogs of NS-NS-fluxes or R-R fluxes? The exterior derivatives of the forms G_3 vanish which is the analog for the vanishing of magnetic charge densities (it is however possible to have the analogs of homological magnetic charge). The self-duality of Ramond p-forms could be posed formally ($G_p =^* G_{8-p}$) but does not have any implications for $p < 4$ since the space-time projections vanish in this case identically for $p > 3$. For $p = 4$ the dual of the instanton density $J \wedge J$ is proportional to volume form if M^4 and is not of topological interest. The approach of Witten eliminating one half of self dual R-R-fluxes would mean that only the above discussed series of fluxes need to be considered so that one would have no troubles with non-rational values of the fluxes nor with the lack of higher dimensional objects assignable to them. An interesting question is whether the fluxes could define some kind of K-theory invariants.
4. In TGD embedding space is non-dynamical and there seems to be no counterpart for the NS 3-form field $H = dB$. The only natural candidate would correspond to Hamiltonian $B = J$ giving $H = dB = 0$. At quantum level this might be understood in terms of bosonic emergence meaning that only Ramond representations for fermions are needed in the theory since bosons correspond to wormhole contacts with fermion and anti-fermions at opposite throats. Therefore twisted cohomology is not needed and there is no need to introduce the analogy of brane democracy and 4-D space-time surfaces containing the analogs of lower-dimensional brains as sub-manifolds are enough. The fluxes of these forms over partonic 2-surfaces and string world sheets defined non-abelian analogs of ordinary gauge fluxes reducing to rotations of vector potentials and suggested be crucial for understanding braidings of knots and 2-knots in TGD framework. [K51]. Note also that the unique dimension D=4 for space-time makes 4-D space-time surfaces homologically self-dual so that only they are needed.

5.6.6 What About Counterparts Of S And U Dualities In TGD Framework?

The natural question is what could be the TGD counterparts of $S-$, $T-$ and U-dualities. If one accepts the identification of U-duality as product $U = ST$ and the proposed counterpart of T duality as a strong form of general coordinate invariance, it remains to understand the TGD counterpart of S-duality - in other words electric-magnetic duality - relating the theories with gauge couplings g and $1/g$.

Quantum criticality selects the preferred value of g_K: Kähler coupling strength is very near to fine structure constant at electron length scale and can be equal to it. Note that the hierarchy of Planck constants (dark matters) could be understood in terms of a spectrumfor $\alpha_K = g_K^2/4\pi h_{eff}$, $h_{eff} = n \times h$: in thermodynamical analogy one would have accumulation of critical points at zero temperature.

If there is no coupling constant evolution associated with α_K, it does not make sense to say that g_K becomes strong and is replaced with its inverse at some point. One should be able to formulate the counterpart of S-duality as an identity following from the weak form of electric-magnetic duality and the reduction of TGD to almost topological QFT. This might be the case.

1. For preferred extremals the interior parts of Kähler action reduces to a boundary term if the term $j^\mu A_\mu$ from them vanishes. The weak form of electric-magnetic duality requires that Kähler electric charge is proportional to Kähler magnetic charge, which implies reduction to abelian Chern-Simons term: the Kähler coupling strength does not appear at all in Chern-Simons term. The proportionality constant beween the electric and magnetic parts J_E and J_B of Kähler form however enters into the dynamics through the boundary conditions stating the weak form of electric-magnetic duality. At the Minkowskian side the proportionality constant must be proportional to g_K^2 to guarantee a correct value for the unit of Kähler electric charge - equal to that for electric charge in electron length scale- from the assumption that electric charge is proportional to the topologically quantized magnetic charge. It has been assumed that

$$J_E = \alpha_K J_B$$

holds true at *both sides* of the wormhole throat but this is an un-necessarily strong assumption at the Euclidian side. In fact, the self-duality of CP_2 Kähler form stating

$$J_E = J_B$$

favours this boundary condition at the Euclidian side of the wormhole throat. Also the fact that one cannot distinguish between electric and magnetic charges in Euclidian region since all charges are magnetic can be used to argue in favor of this form. The same constraint arises from the condition that the action for CP_2 type vacuum extremal has the value required by the argument leading to a prediction for gravitational constant in terms of the square of CP_2 radius and α_K the effective replacement $g_K^2 \rightarrow 1$ would spoil the argument.

2. Minkowskian and Euclidian regions should correspond to a strongly/weakly interacting phase in which Kähler magnetic/electric charges provide the proper description. In Euclidian regions associated with CP_2 type extremals there is a natural interpretation of interactions between magnetic monopoles associated with the light-like throats: for CP_2 type vacuum extremal itself magnetic and electric charges are actually identical and cannot be distinguished from each other. Therefore the duality between strong and weak coupling phases seems to be trivially true in Euclidian regions if one has $J_B = J_E$ at Euclidian side of the wormhole throat. This is however an un-necessarily strong condition as the following argument shows.

3. In Minkowskian regions the interaction is via Kähler electric charges and elementary particles have vanishing total Kähler magnetic charge consisting of pairs of Kähler magnetic monopoles so that one has confinement characteristic for strongly interacting phase. Therefore Minkowskian regions naturally correspond to a weakly interacting phase for Kähler electric charges. One can write the action density at the Minkowskian side of the wormhole throat as

$$\frac{(J_E^2 - J_B^2)}{\alpha_K} = \alpha_K J_B^2 - \frac{J_B^2}{\alpha_K} \ .$$

The exchange $J_E \leftrightarrow J_B$ accompanied by $\alpha_K \rightarrow -1/\alpha_K$ leaves the action density invariant. Since only the behavior of the vacuum functional infinitesimally near to the wormhole throat matters by almost topological QFT property, the duality is realized. Note that the argument goes through also in Euclidian regions so that it does not allow to decide which is the correct form of weak form of electric-magnetic duality.

4. S-duality could correspond geometrically to the duality between partonic 2-surfaces responsible for magnetic fluxes and string worlds sheets responsible for electric fluxes as rotations of Kähler gauge potentials around them and would be very closely related with the counterpart of T-duality implied by the strong form of general coordinate invariance and saying that space-like 3-surfaces at the ends of space-time sheets are equivalent with light-like 3-surfaces connecting them.

The boundary condition $J_E = J_B$ at the Euclidian side of the wormhole throat inspires the question whether all Euclidian regions could be self-dual so that the density of Kähler action would be just the instanton density. Self-duality follows if the deformation of the metric induced by the deformation of the canonically imbedded CP_2 is such that in CP_2 coordinates for the Euclidian region the tensor $(g^{\alpha\beta}g^{\mu\nu} - g^{\alpha\nu}g^{\mu\beta})/\sqrt{g}$ remains invariant. This is certainly the case for CP_2 type vacuum extremals since by the light-likeness of M^4 projection the metric remains invariant. Also conformal scalings of the induced metric would satisfy this condition. Conformal scaling is not consistent with the degeneracy of the 4-metric at the wormhole throat. Self-duality is indeed an un-necessarily strong condition.

Comparison with standard view about dualities

One can compare the proposed realization of T, S and U to the more general dualities defined by the modular group $SL(2,Z)$, which in QFT framework can hold true for the path integral over all possible gauge field configurations. In the resent case the dualities hold true for every preferred extremal separately and the functional integral is only over the space-time projections of fixed Kähler form of CP_2. Modular invariance for Maxwell action was discussed by E. Verlinde for Maxwell action with θ term (see `http://tinyurl.com/ycx6lve3`) for a general 4-D compact manifold with Euclidian signature of metric in [B20]. In this case one has path integral giving sum over infinite number of extrema characterized by the cohomological equivalence class of the Maxwell field the action exponential to a high degree. Modular invariance is broken for CP_2: one obtains invariance only for $\tau \rightarrow \tau + 2$ whereas S induces a phase factor to the path integral.

1. In the recent case these homology equivalence classes would correspond to homology equivalence classes of holomorphic partonic 2-surfaces associated with the critical points of Kähler function with respect to zero modes.
2. In the case that the Euclidian contribution to the Kähler action is expressible solely in terms of wormhole throat Chern-Simons terms, and one can neglect the measurement interaction terms fixing the values of some classical conserved quantities to be equal with their quantal counterparts for the space-time surfaces allowed in quantum superposition, the exponent of Kähler action can be expressed in terms of Chern-Simons action density as

$$\begin{aligned} L &= \tau L_{C-S} \ , \\ L_{C-S} &= J \wedge A \ , \\ \tau &= \frac{1}{g_K^2} + i\frac{k}{4\pi} \ , \quad k=1 \ . \end{aligned} \tag{5.6.1}$$

Here the parameter τ transforms under full $SL(2,Z)$ group as

$$\tau \rightarrow \frac{a\tau + b}{c\tau + d} \ . \tag{5.6.2}$$

The generators of $SL(2,Z)$ transformations are $T: \tau \rightarrow \tau + 1$, $S: \tau \rightarrow -1/\tau$. The imaginary part in the exponents corresponds to Kac-Moody central extension $k = 1$.

This form corresponds also to the general form of Maxwell action with CP breaking θ term given by

$$L = \frac{1}{g_K^2} J \wedge^* J + i\frac{\theta}{8\pi^2} J \wedge J \ , \quad \theta = 2\pi \ . \tag{5.6.3}$$

Hence the Minkowskian part mimics the θ term but with a value of θ for which the term does not give rise to CP breaking in the case that the action is full action for CP_2 type vacuum extremal so that the phase equals to 2π and phase factor case is trivial. It would seem that the deviation from the full action for CP_2 due to the presence of wormhole throats reducing the value of the full Kähler action for CP_2 type vacuum extremal could give rise to CP breaking. One can visualize the excluded volume as homologically non-trivial geodesic spheres with some thickness in two transverse dimensions. At the limit of infinitely thin geodesic spheres CP breaking would vanish. The effect is exponentially sensitive to the volume deficit.

CP breaking and ground state degeneracy

Ground state degeneracy due to the possibility of having both signs for Minkowskian contribution to the exponent of vacuum functional provides a general view about the description of CP breaking in TGD framework.

1. In TGD framework path integral is replaced by inner product involving integral over WCV. The vacuum functional and its conjugate are associated with the states in the inner product so that the phases of vacuum functionals cancel if only one sign for the phase is allowed. Minkowskian contribution would have no physical significance. This of course cannot be the case. The ground state is actually degenerate corresponding to the phase factor and its complex conjugate since $\sqrt{g}$ can have two signs in Minkowskian regions. Therefore the inner products between states associated with the two ground states define 2×2 matrix and non-diagonal elements contain interference terms due to the presence of the phase factor. At the limit of full CP_2 type vacuum extremal the two ground states would reduce to each other and the determinant of the matrix would vanish.
2. A small mixing of the two ground states would give rise to CP breaking and the first principle description of CP breaking in systems like $K - \overline{K}$ and of CKM matrix should reduce to this mixing. K^0 mesons would be CP even and odd states in the first approximation and correspond to the sum and difference of the ground states. Small mixing would be present having exponential sensitivity to the actions of CP_2 type extremals representing wormhole throats. This might allow to understand qualitatively why the mixing is about 50 times larger than expected for B^0 mesons.
3. There is a strong temptation to assign the two ground states with two possible arrows of geometric time. At the level of M-matrix the two arrows would correspond to state preparation at either upper or lower boundary of CD. Do long- and short-lived neutral K mesons correspond to almost fifty-fifty orthogonal superpositions for the two arrow of geometric time or almost completely to a fixed arrow of time induced by environment? Is the dominant part of the arrow same for both or is it opposite for long and short-lived neutral mesons? Different lifetimes would suggest that the arrow must be the same and apart from small leakage that induced by environment. CP breaking would be induced by the fact that CP is performed only K^0 but not for the environment in the construction of states. One can probably imagine also alternative interpretations.

Remark: The proportionality of Minkowskian and Euclidian contributions to the same Chern-Simons term implies that the critical points with respect to zero modes appear for both the phase and modulus of vacuum functional. The Kähler function property does not allow extrema for vacuum functional as a function of complex coordinates of WCW since this would mean Kähler metric with non-Euclidian signature. If this were not the case. the stationary values of phase factor and extrema of modulus of the vacuum functional would correspond to different configurations.

5.6.7 Could One Divide Bundles?

TGD differs from string models in one important aspects: stringy diagrams do not have interpretation as analogs of vertices of Feynman diagrams: the stringy decay of partonic 2-surface to two pieces does not represent particle decay but a propagation along different paths for incoming particle. Particle reactions in turn are described by the vertices of generalized Feynman diagrams in which the ends of incoming and outgoing particles meet along partonic 2-surface. This suggests a generalization of K-theory for bundles assignable to the partonic 2-surfaces. It is good to start with a guess for the concrete geometric realization of the sum and product of bundles in TGD framework.

1. The analogs of string diagrams could represent the analog for direct sum. Difference between bundles could be defined geometrically in terms of trouser vertex $A + B \to C$. B would by definition represent $C - A$. Direct sum could make sense for single particle states and have as space-time correlate the conservation of braid strands.
2. A possible concretization in TGD framework for the tensor product is in terms of the vertices of generalized Feynman diagrams at which incoming light-like 3-D orbits of partons meet along their ends. The tensor product of incoming state spaces defined by fermionic oscillator algebras is naturally formed. Tensor product would have also now as a space-time correlate conservation of braid strands. This does not mean that the number of braid strands is conserved in reactions if also particular exchanges can carry the braid strands of particles coming to the vertex.

Why not define also division of bundles in terms of the division for tensor product? In terms of the 3-vertex for generalized Feynman diagrams $A \otimes B = C$ representing tensor product B would be by definition C/A. Therefore TGD would extend the K-theory algebra by introducing also division as a natural operation necessitated by the presence of the join along ends vertices not present in string theory. I would be surprised if some mathematician would not have published the idea in some exotic journal. Below I represent an argument that this notion could be also applied in the mathematical description of finite measurement resolution in TGD framework using inclusions of hyper-finite factor. Division could make possible a rigorous definition for for non-commutative quantum spaces.

Tensor division could have also other natural applications in TGD framework.

1. One could assign bundles M_+ and M_- to the upper and lower light-like boundaries of CD. The bundle M_+/M_- would be obtained by formally identifying the upper and lower light-like boundaries. More generally, one could assign to the boundaries of CD positive and negative energy parts of WCW spinor fields and corresponding bundle structures in "half WCW ". Zero energy states could be seen as sections of the unit bundle just like infinite rationals reducing to real units as real numbers would represent zero energy states.
2. Finite measurement resolution would encourage tensor division since finite measurement resolution means essentially the loss of information about everything below measurement resolution represented as a tensor product factor. The notion of coset space formed by hyper-finite factor and included factor could be understood in terms of tensor division and give rise to quantum group like space with fractional quantum dimension in the case of Jones inclusions [K120]. Finite measurement resolution would therefore define infinite hierarchy of finite dimensional non-commutative spaces characterized by fractional quantum dimension. In this case the notion of tensor product would be somewhat more delicate since complex numbers are effectively replaced by the included algebra whose action creates states not distinguishable from each other [K120]. The action of algebra elements to the state $|B\rangle$ in the inner product $\langle A|B\rangle$ must be equivalent with the action of its hermitian conjugate to the state $\langle A|$. Note that zero energy states are in question so that the included algebra generates always modifications of states which keep it as a zero energy state.

Chapter 6

Can one apply Occam's razor as a general purpose debunking argument to TGD?

Occarm's razor have been used to debunk TGD. The following arguments provide the information needed by the reader to decide himself. Considerations are at three levels.

The level of "world of classical worlds" (WCW) defined by the space of 3-surfaces endowed with Kähler structure and spinor structure and with the identification of WCW space spinor fields as quantum states of the Universe: this is nothing but Einstein's geometrization program applied to quantum theory. Second level is space-time level.

Space-time surfaces correspond to preferred extremals of Käction in $M^4 \times CP_2$. The number of field like variables is 4 corresponding to 4 dynamically independent embedding space coordinates. Classical gauge fields and gravitational field emerge from the dynamics of 4-surfaces. Strong form of holography reduces this dynamics to the data given at string world sheets and partonic 2-surfaces and preferred extremals are minimal surface extremals of Kähler action so that the classical dynamics in space-time interior does not depend on coupling constants at all which are visible via boundary conditions only. Continuous coupling constant evolution is replaced with a sequence of phase transitions between phases labelled by critical values of coupling constants: loop corrections vanish in given phase. Induced spinor fields are localized at string world sheets to guarantee well-definedness of em charge.

At embedding space level the modes of embedding space spinor fields define ground states of super-symplectic representations and appear in QFT-GRT limit. GRT involves post-Newtonian approximation involving the notion of gravitational force. In TGD framework the Newtonian force correspond to a genuine force at embedding space level.

I was also asked for a summary about what TGD is and what it predicts. I decided to add this summary to this chapter although it is goes slightly outside of its title.

6.1 Introduction

Occam's razor argument is one the standard general purpose arguments used in debunking: the debunked theory is claimed to be hopelessly complicated. This argument is more refined that mere "You are a crackpot!" but is highly subjective and often the arguments pro or con are not given. Combined with the claim that the theory does not predict anything Occam's razor is very powerful argument unless the audience includes people who have bothered to study the debunked theory.

Let us take a closer look on this argument and compare TGD superstring models and seriously ask which of these theories is simple.

In superstring models one has strings as basic dynamical objects. They live in target space M^{10}, which in some mysterious way (something "non-perturbative" it is) spontaneously compactifies to $M^4 \times C$, C is Calabi-Yau space. The number of them is something like 10^{500} or probably infinite: depends on the counting criterion. And this estimate leaves their metric open. This leads to landscape and multiverse catastrophe: theory cannot predict anything. As a matter fact $M^4 \times C$:s must be allowed to deform still in Kaluza-Klein paradigm in which space-time has Calabi-Yau as small additional dimensions. An alternative way to obtain space-time is as 3-brane. One obtains also higher-D objects. Again by some "nonperturbative" mechanisms. One does not even know what space-time is! Situation looks to me a totally hopeless mess. Reader can conclude whether to regard this as simple and elegant.

I will consider TGD at three levels. At the level of "world of classical worlds" (WCW), at space-time level, and at the level of embedding space $H = M^4 \times CP_2$. I hope that I can convince the reader about the simplicity of the approach. The simplicity is actually quite shocking and certainly an embarrassing experience for the unhappy super string theorists meandering around in the landscape and multiverse. Behind this simplicity are however principles - something, which colleagues usually regard as unpractical philosophizing: "shut-up-and-calculate!"!

I was also asked for a summary about what TGD is and what it predicts. I decided to add this summary to this chapter although it is goes slightly outside of its title.

6.2 Simplicity at various levels

6.2.1 WCW level: a generalization of Einstein's geometrization program to entire quantum physics

I hope that the reader would read the following arguments keeping in mind the question "Is TGD really hopelessly complicated mess of pieces picked up randomly from theoretical physics?" as one debunker who told that he does not have time to read TGD formulated it.

1. Einstein's geometrization program for gravitation has been extremely successful but has failed for other classical fields, which do not have natural geometrization in the case of abstract four-manifolds with metric. One should understand standard model quantum numbers and also family replication for fermions.

   However, if space-time can be regarded surface in $H = M^4 \times CP_2$ also the classical fields find a natural geometrization as induced fields obtained basically by projecting. Also spinor structure can be induced and one avoids the problems due the fact that generic space-time as abstract 4-manifold does not allow spinor structure. The dynamics of space-time surfaces incredibly simple: only 4 field-like variables corresponding to *four* embedding space coordinates and induced that of classical geometric fields. Nowadays one would speak of emergence. The complexity emerges from the topology of space-time surfaces giving rise to many-sheeted space-time.

2. Even this view about geometrization is generalized in TGD. Einstein's geometrization program is applied to the entire quantum physics in terms of the geometry of WCW consisting of 3-D surfaces of H. More precisely, in zero energy ontology (ZEO) it consists of pairs of 3-surfaces at opposite boundaries of causal diamond (CD) connected by a preferred extremals of a variational principle to be discussed.

   Quantum states of the Universe would correspond to the modes of formally classical WCW spinor field satisfying the analog of Dirac equation. No quantization: just the construction of WCW geometry and spinor structure. The only genuinely quantal element of quantum theory would be state function reduction and in ZEO its description leads to a quantum theory of consciousness.

To me this sounds not only simple but shockingly simple.

WCW geometry

Consider first the generalization of Einstein's program of at the level of WCW geometry [K93, K50, K28].

1. Since complex conjugation must be geometrized, WCW must allow a geometric representation of imaginary unit as an antisymmetric tensor, which is essentially square root of the negative of the metric tensor and thus allow Kähler structure coded by Kähler function. One must

have 4-D general coordinate invariance (GCI) but basic objects are 3-D surfaces. Therefore the definition of Kähler function must assign to 3-surface a unique 4-surface.

Kähler function should have physical meaning and the natural assumption is that it is Kähler action plus possibly also volume term (twistor lift implies it). Space-time surface would be a preferred extremal of this action. The interpretation is also as an analog of Bohr orbit so that Bohr orbitology would correspond exact rather than only approximate part of quantum theory in TGD framework. One could speak also of quantum classical correspondence.

2. The action principle involves coupling parameters analogous to thermodynamical parameters. Their value spectrum is fixed by the conditions that TGD is quantum critical. For instance Kähler couplings strength is analogous to critical temperature. Different values correspond to different phases. Coupling constant evolution correspond to phase transitions between these phases and loops vanish as in free field theory for $\mathcal{N} = 4$ SYM.

3. The infinite-dimensionality of WCW is a crucial element of simplicity. Already in the case of loop spaces the geometry is essentially unique: loop space is analogous to a symmetric space points of the loop space being geometrically equivalent. For loop spaces Riemann connection exists only of the metric has maximal isometries defined by Kac-Moody algebra.

   The generalization to 3-D case is compelling. In TGD Kac-Moody algebra is replaced by super-symplectic algebra, which is much larger but has same basic structure (conformal weights of two kinds) and a fractal hierarchy of isomorphic sub-algebas with conformal weights coming as multiples of those for the entire algebra is crucial. Physics is unique because of its mathematical existence. WCW decompose to a union of sectors, which are infinite-D variants of symmetric spaces labelled by zero modes whose differentials do not appear in the line element of WCW.

All this sounds to me shockingly simple.

WCW spinor structure

One must construct also spinor structure for WCW [K121, K93].

1. The modes of WCW spinor fields would correspond to the solutions of WCW Dirac equation and would define the quantum states of the Universe. WCW spinors (assignable to given 3-surface) would correspond to fermionic Fock states created by fermionic creation operators. In ZEO 3-surfaces are pairs of 3-surfaces assignable to the opposite boundaries of WCW connected by preferred extremal.

   The fermionic states are superpositions of pairs of fermion states with opposite net quantum numbers at the opposite ends of space-time surface at boundaries of CD. The entanglement coefficients define the analogs of S-matrix elements. The analog of Dirac equation is analog for super-Virasoro conditions in string models but assignable to the infinite-D supersymplectic algebra of WCW defining its isometries.

2. The construction of the geometry of WCW requires that the anticommuting gamma matrices of WCW are expressible in terms of fermionic oscillator operators assignable to the induced spinor fields at space-time surface. Fermionic anti-commutativity at space-time level is not assumed but is forced by the anticommutativity of gamma matrices to metric. Fermi statistics is geometrized.

3. The gamma matrices of WCW in the coordinates assignable to isometry generators can be regarded as generators of superconformal symmetries. They correspond to classical charges assignable to the preferred extremals and to fermionic generators. The fermionic isometry generators are fermionic bilinears and super-generators are obtained from them by replacing the second second quantized spinor field with its mode. Quantum classical correspondence between fermionic dynamics and classical dynamics (SH) requires that the eigenvalues of the fermionic Cartan charges are equal to corresponding bosonic Noether charges.

4. The outcome is that quantum TGD reduces to a theory of formally *classical* spinor fields at the level of WCW and by infinite symmetries the construction of quantum states reduces to the construction of representations of super-symplectic algebra which generalizes to Yangian algebra as twistorial picture suggests. In ZEO everything would reduce to group theory, even

the construction of scattering amplitudes! In ZEO the construction of zero energy states and thus scattering amplitudes would reduce to that for the representations of Yangian variant of super-symplectic algebra [A27] [B25, B17, B18].

5. One can go to the extreme and wonder whether the scattering amplitudes as entanglement coefficients for Yangian zero energy states are just constant scalars for given values of zero modes as group invariant for isometries. This would leave only integration over zero modes and if number theoretical universality is assumed this integral reduces to sum over points with algebraic coordinates in the preferred coordinates made possible by the symmetric space property. Certainly this is one of the lines of research to be followed in future.

   Personally I find it hard to imagine anything simpler!

6.2.2 Space-time level: many-sheeted space-time and emergence of classical fields and GRT space-time

At space-time level one must consider dynamics of space-time surface and spinorial dynamics.

Dynamics of space-time surfaces

Consider first simplicity at space-time level.

1. Space-time is identified as 4-D surface in certain embedding space required to have symmetries of special relativity - Poincare invariance. This resolves the energy problem and many other problems of GRT [K123].

   This allows also to see TGD as generalization of string models obtained by replacing strings with 3-surfaces and 2-D string world sheets with 4-D space-time surfaces. Small space-time surfaces are particles, large space-time surfaces the background space-time in which these particles "live". There are only 4 dynamical field like variables for 8-D $M^4 \times CP_2$ since GCI eliminates 4 embedding space coordinates (they can be taken as space-tme coordinates). This should be compared with the myriads of classical fields for 10-D Einstein's theory coupled to matter fields (do not forget landscape and multiverse!)

2. Classical fields are induced at the level of single space-time sheet from their geometric counterparts in embedding space. A more fashionable way to say the same is that they emerge. Classical gravitational field correspond to the induced metric, electroweak gauge potentials to induced spinor connection of CP_2 and color gauge potentials to projections of Killing vector fields for CP_2.

3. In TGD the space-time of GRT is replaced by many-sheeted space-time constructed from basic building bricks, which are preferred extremals of Kähler action + volume term. This action emerges in twistor lift of TGD existing only for $H = M^4 \times CP_2$: TGD is completely unique since only M^4 and CP_2 allows twistor space with Kähler structure. This also predicts Planck length as radius of twistor sphere associated with M^4. Cosmological constant appears as the coefficient of the volume term and obeys p-adic length scale evolution predicting automatically correct order of magnitude in the scale of recent cosmos. Besides this one has CP_2 size which is of same order of magnitude as GUT scale, and Kähler coupling strength. By quantum criticality the various parameters are quantized.

   Quantum criticality is basic dynamical principle [K50, K41] and discretizes coupling constant evolution: only coupling constants corresponding to quantum criticality are realized and discretized coupling constant evolution corresponds to phase transitions between these values of coupling constants. All radiative corrections vanish so that only tree diagram contribute.

4. Preferred extremals realize strong form of holography (SH) implied by strong form of GCI (SGCI) emerging naturally in TGD framework. That GCI implies SH meaning an enormous simplification at the conceptual level.

   One has two choices for fundamental 3-D objects. They could be light-like boundaries between regions of Minkowskian and Euclidian signatures of the induced metric or they could be pairs of space-time 3-surfaces at the ends of space-time surface at opposite boundaries of causal diamond (CD) (CDs for a scale hierarchy). Both options should be correct so that the intersections of these 3-surfaces consisting of partonic 2-surfaces at which light-like partonic orbits and space-like 3-surfaces intersect should carry the data making possible holography. Also data about normal space of partonic 2-surface is involved.

SH generalizes AdS/CFT correspondence by replacing holography with what is very much like the familiar holography. String world, sheets, which are minimal surfaces carrying fermion fields and partonic 2-surfaces intersecting string world sheets at discrete points determine by SH the entire 4-D dynamics. The boundaries of string world sheets are world lines with fermion number coupling to classical Kähler force. In the interior Kähler force vanishes so that one has "dynamics of avoidance" [L18] required also by number theoretic universality satisfied if the coupling constants do not appear in the field equations at all: they are however seen in the boundary values stating vanishing of the classical super-symplectic charges (Noether's theorem) so that one obtains dependence of coupling constants via boundary conditions and coupling constant evolutions makes it manifest also classically. Hence the preferred extremals from which the space-time surfaces are engineered are extremely simple objects.

5. In twistor formulation the assumption that the inverse of Kähler coupling strength has zeros of Riemann zeta [L14] as the spectrum of its quantum critical values gives excellent prediction for the coupling constant of U(1) coupling constant of electroweak interactions. Complexity means that extremals are extremals of both Kähler action and volume term: minimal surfaces extremals of Kähler action. This would be part of preferred extremal property.

   Why α_K should be complex? If α_K is real, both bosonic and fermionic degrees of freedom for Euclidian and Minkowskian regions decouple completely. This is not physically attractive. If α_K is complex there is coupling between the two regions and the simplest assumption is that there is no Chern-Simons term in the action and one has just continuity conditions for canonical momentum current and hits super counterpart. Note the analogy with the possibility of blackhole evaporation. The presence of momentum exchange is also natural since it gives classical space-time correlates for interactions as momentum exchange.

   The conditions state that sub-algebra of super-symplectic algebra isomorphic to itself and its commutator with the entire algebra annihilate the physical states (classical Noether charges vanish). The condition could follow from minimal surface extremality or provide additional conditions reducing the degrees of freedom. In any case, 3-surfaces would be almost 2-D objects.

6. GRT space-time emerges from many-sheeted space-time as one replaces the sheets of many-sheeted space-time (4-D M^4 projection) to single slightly curved region of M^4 defining GRT space-time. Since test particle regarded as 3-surface touching the space-time sheets of many-sheeted spacetime, test particle experiences the sum of forces associated with the classical fields at the space-time sheets. Hence the classical fields of GRT space-time are sums of these fields. Disjoint union for space-time sheets maps to the sum of the induced fields. This gives standard model and GRT as long range scale limit of TGD.

How to build TGD space-time from legos?

TGD predicts shocking simplicity of both quantal and classical dynamics at space-time level. Could one imagine a construction of more complex geometric objects from basic building bricks - space-time legos?

Let us list the basic ideas.

1. Physical objects correspond to space-time surfaces of finite size - we see directly the non-trivial topology of space-time in everyday length scales.
2. There is also a fractal scale hierarchy: 3-surfaces are topologically summed to larger surfaces by connecting them with wormhole contact, which can be also carry monopole magnetic flux in which one obtains particles as pairs of these: these contacts are stable and are ideal for nailing together pieces of the structure stably.
3. In long length scales in which space-time surface tend to have 4-D M^4 projection this gives rise to what I have called many-sheeted spacetime. Sheets are deformations of canonically imbedded M^4 extremely near to each other (the maximal distance is determined by CP_2 size scale about 10^4 Planck lengths. The sheets touch each other at topological sum contacts, which can be also identified as building bricks of elementary particles if they carry monopole flux and are thus stable. In $D = 2$ it is easy to visualize this hierarchy.

What could be the simplest surfaces of this kind - the legos?

1. Assume twistor lift [K41, K10] so that action contain volume term besides Kähler action: preferred extremals can be seen as non-linear massless fields coupling to self-gravitation. They also simultaneously extremals of Kähler action. Also hydrodynamical interpretation makes sense in the sense that field equations are conservation laws. What is remarkable is that the solutions have no dependence on coupling parameters: this is crucial for realizing number theoretical universality. Boundary conditions however bring in the dependence on the values of coupling parameters having discrete spectrum by quantum criticality.

2. The simplest solutions corresponds to Lagrangian sub-manifolds of CP_2: induced Kähler form vanishes identically and one has just minimal surfaces. The energy density defined by scale dependent cosmological constant is small in cosmological scales - so that only a template of physical system is in question. In shorter scales the situation changes if the cosmological constant is proportional the inverse of p-adic prime.

   The simplest minimal surfaces are constructed from pieces of geodesic manifolds for which not only the trace of second fundamental form but the form itself vanishes. Geodesic sub-manifolds correspond to points, pieces of lines, planes, and 3-D volumes in E^3. In CP_2 one has points, circles, geodesic spheres, and CP_2 itself.

3. CP_2 type extremals defining a model for wormhole contacts, which can be used to glue basic building bricks at different scales together stably: stability follows from magnetic monopole flux going through the throat so that it cannot be split like homologically trivial contact. Elementary particles are identified as pairs of wormhole contacts and would allow to nail the legos together to from stable structures.

Amazingly, what emerges is the elementary geometry. My apologies for those who hated school geometry.

1. Geodesic minimal surfaces with vanishing induced gauge fields

Consider first *static* objects with 1-D CP_2 projection having thus *vanishing* induced gauge fields. These objects are of form $M^1 \times X^3$, $X^3 \subset E^3 \times CP_2$. M^1 corresponds to time-like or possible light-like geodesic (for CP_2 type extremals). I will consider mostly Minkowskian space-time regions in the following.

1. Quite generally, the simplest legos consist of 3-D geodesic sub-manifolds of $E^3 \times CP_2$. For E^3 their dimensions are $D = 1, 2, 3$ and for CP_2, $D = 0, 1, 2$. CP_2 allows both homologically non-trivial resp. trivial geodesic sphere S^2_I resp. S^2_{II}. The geodesic sub-manifolds cen be products $G_3 = G_{D_1} \times G_{D_2}$, $D_2 = 3 - D_1$ of geodesic manifolds G_{D_1}, $D_1 = 1, 2, 3$ for E^3 and G_{D_2}, $D_2 = 0, 1, 2$ for CP_2.

2. It is also possible to have twisted geodesic sub-manifolds G_3 having geodesic circle S^1 as CP_2 projection corresponding to the geodesic lines of $S^1 \subset CP_2$, whose projections to E^3 and CP_2 are geodesic line and geodesic circle respectively. The geodesic is characterized by S^1 wave vector. One can have this kind of geodesic lines even in $M^1 \times E^3 \times S^1$ so that the solution is characterized also by frequency and is not static in CP_2 degrees of freedom anymore.

   These parameters define a four-D wave vector characterizing the warping of the space-time surface: the space-time surface remains flat but is warped. This effect distinguishes TGD from GRT. For instance, warping in time direction reduces the effective light-velocity in the sense that the time used to travel from A to B increases. One cannot exclude the possibility that the observed freezing of light in condensed matter could have this warping as space-time correlate in TGD framework.

   For instance, one can start from 3-D minimal surfaces $X^2 \times D$ as local structures (thin layer in E^3). One can perform twisting by replacing D with twisted closed geodesics in $D \times S^1$: this gives valued map from D to S^1 (subset CP_2) representing geodesic line of $D \times S^1$. This geodesic sub-manifold is trivially a minimal surface and defines a two-sheeted cover of $X^2 \times D$. Wormhole contact pairs (elementary particles) between the sheets can be used to stabilize this structure.

3. Structures of form $D^2 \times S^1$, where D^2 is polygon, are perhaps the simplest building bricks for more complex structures. There are continuity conditions at vertices and edges at which

polygons D_i^2 meet and one could think of assigning magnetic flux tubes with edes in the spirit of homology: edges as magnetic flux tubes, faces as 2-D geodesic sub-manifolds and interiors as 3-D geodesic sub-manifolds.

Platonic solids as 2-D surfaces can be build are one example of this and are abundant in biology and molecular physics. An attractive idea is that molecular physics utilizes this kind of simple basic structures. Various lattices appearing in condensed matter physics represent more complex structures but could also have geodesic minimal 3-surfaces as building bricks. In cosmology the honeycomb structures having large voids as basic building bricks could serve as cosmic legos.

4. This lego construction very probably generalizes to cosmology, where Euclidian 3-space is replaced with 3-D hyperbolic space $SO(3,1)/SO(3)$. Also now one has pieces of lines, planes and 3-D volumes associated with an arbitrarily chosen point of hyperbolic space. Hyperbolic space allows infinite number of tessellations serving as analogs of 3-D lattices and the characteristic feature is quantization of redshift along line of sight for which empirical evidence is found.

5. The structures as such are still too simple to represent condensed matter systems. These basic building bricks can glued together by wormhole contact pairs defining elementary particles so that matter emerges as stabilizer of the geometry: they are the nails allowing to fix planks together, one might say.

2. Geodesic minimal surfaces with non-vanishing gauge fields

What about minimal surfaces and geodesic sub-manifolds carrying non-vanishing gauge fields - in particular em field (Kähler form identifiable as U(1) gauge field for weak hypercharge vanishes and thus also its contribution to em field)? Now one must use 2-D geodesic spheres of CP_2 combined with 1-D geodesic lines of E^2. Actually both homologically non-trivial resp. trivial geodesic spheres S_I^2 resp. S_{II}^2 can be used so that also non-vanishing Kähler forms are obtained.

The basic legos are now $D \times S_i^2$, $i = I, II$ and they can be combined with the basic legos constructed above. These legos correspond to two kinds of magnetic flux tubes in the ideal infinitely thin limit. There are good reasons to expected that these infinitely thin flux tubes can be thickened by deforming them in E^3 directions orthogonal to D. These structures could be used as basic building bricks assignable to the edges of the tensor networks in TGD.

3. Static minimal surfaces, which are not geodesic sub-manifolds

One can consider also more complex static basic building bricks by allowing bricks which are not anymore geodesic sub-manifolds. The simplest static minimal surfaces are form $M^1 \times X^2 \times S^1$, $S^1 \subset CP_2$ a geodesic line and X^2 minimal surface in E^3.

Could these structures represent higher level of self-organization emerging in living systems? Could the flexible network formed by living cells correspond to a structure involving more general minimal surfaces - also non-static ones - as basic building bricks? The Wikipedia article about minimal surfaces in E^3 suggests the role of minimal surface for instance in bio-chemistry (see `http://tinyurl.com/zqlv322`).

The surfaces with constant positive curvature do not allow embedding as minimal surfaces in E^3. Corals provide an example of surface consisting of pieces of 2-D hyperbolic space H^2 immersed in E^3 (see `http://tinyurl.com/ho9uvcc`. Minimal surfaces have negative curvature as also H^2 but minimal surface immersions of H^2 do not exist. Note that pieces of H^2 have natural embedding to E^3 realized as light-one proper time constant surface but this is not a solution to the problem.

Does this mean that the proposal fails?

1. One can build approximately spherical surfaces from pieces of planes. Platonic solids represents the basic example. This picture conforms with the notion of monadic manifold having as a spine a discrete set of points with coordinates in algebraic extension of rationals (preferred coordinates allowed by symmetries are in question). This seems to be the realistic option.

2. The boundaries of wormhole throats at which the signature of the induced metric changes can have arbitrarily large M^4 projection and they take the role of blackhole horizon. All physical systems have such horizon and the approximately boundaries assignable to physical objects could be horizons of this kind. In TGD one has minimal surface in $E^3 \times S^1$ rather than E^3. If

3-surface have no space-like boundaries they must be multi-sheeted and the sheets co-incide at some 2-D surface analogous to boundary. Could this 3-surface give rise to an approximately spherical boundary.

3. Could one lift the immersions of H^2 and S^2 to E^3 to minimal surfaces in $E^3 \times S^1$? The constancy of scalar curvature, which is for the immersions in question quadratic in the second fundamental form would pose one additional condition to non-linear Laplace equations expressing the minimal surface property. The analyticity of the minimal surface should make possible to check whether the hypothesis can make sense. Simple calculations lead to conditions, which very probably do not allow solution.

4. Dynamical minimal surfaces: how space-time manages to engineer itself?

At even higher level of self-organization emerge dynamical minimal surfaces. Here string world sheets as minimal surfaces represent basic example about a building block of type $X^2 \times S^2_i$. As a matter fact, S^2 can be replaced with complex sub-manifold of CP_2.

One can also ask about how to perform this building process. Also massless extremals (MEs) representing TGD view about topologically quantized classical radiation fields are minimal surfaces but now the induced Kähler form is non-vanishing. MEs can be also Lagrangian surfaces and seem to play fundamental role in morphogenesis and morphostasis as a generalization of Chladni mechanism [K110, K10]. One might say that they represent the tools to assign material and magnetic flux tube structures at the nodal surfaces of MEs. MEs are the tools of space-time engineering. Here many-sheetedness is essential for having the TGD counterparts of standing waves.

Spherically symmetry metric as minimal surface

Physical intuition and the experience with the vacuum extremals as models for GRT space-times suggests that Kähler charge is not important in the case of astrophysical objects like stars so that it might be possible to model them as minimal surfaces, which in the simplest situation have spherically symmetric metric analogous to Schwartschild solution. The vanishing of the induced Kähler form does not of course exclude the presence of electromagnetic fields. It must be of course emphasized that the assumption that single-sheeted space-time surface can model GRT-QFT limit based on many-sheeted space-time could be un-realistic.

At 90's I studied the embeddings of Schwartschild-Nordström solution as vacuum extremals of Kähler action and found that the solution is necessarily electromagnetically charged [K114]. This property is unavoidable. The embedding in coordinates (t, r, θ, ϕ) for X^4, (m^0, r, θ, ϕ) for M^4 and (Θ, Φ) for the trivial geodesic sphere S^2_{II} of CP_2 was not stationary as the first guess might be. m^0 relates to Schwartschild time and radial coordinate r by a shift $m^0 = \Lambda t + h(r)$. Without this shift the perihelion shift would be negligibly small.

One has $(cos(\Theta) = f(r), \Phi = \omega t + k(r))$. Also the dependence of Φ is not the first possibility to come in mind. The shifts $h(r)$ and $k(r)$ are such that the non-diagonal contribution g_{tr} to the induced metric vanishes. The question is whether one obtains spherically symmetric metric as a minimal surface.

5. General form of minimal surface equations

Consider first the minimal surface equations generally.

1. The field equations are analogous to massless wave equations for scalar fields defined by CP_2 coordinates having gravitational self coupling and also covariant derivative coupling due to the non-flatness of CP_2. One might therefore expect that the Newtonian gravitation based on Laplace equation in empty space-time regions follows as an approximation. Therefore also something analogous to Schwartschild metric is to be expected. Note that also massless extremals (MEs) are obtained as minimal surfaces so that also the topologically quantized counterparts of em and gravitational radiation emerge.

2. The general field equations can be written as vanishing of the covariant divergence for canonical momentum current $T^{k\alpha}$

$$
\begin{aligned}
D_\alpha(T^{k\alpha}\sqrt{g}) &= \partial_\alpha\left[T^{k\alpha}\sqrt{g}\right] + \left\{ {k \atop \alpha\ m} \right\} T^{m\alpha}\sqrt{g} = 0 \ , \\
T^{k\alpha} &= g^{\alpha\beta}\partial_\beta h^k \ , \\
\left\{ {k \atop \alpha\ m} \right\} &= \left\{ {k \atop l\ m} \right\}\partial_\alpha h^l \ .
\end{aligned}
\tag{6.2.1}
$$

D_α is covariant derivative taking into account that gradient $\partial_\alpha h^k$ is embedding space vector.

3. For isometry currents $j^{A,k}$ (Killing vector fields)

$$
T^{A,\alpha} = T^{\alpha k}h_{kl}j^{A,l} \tag{6.2.2}
$$

the covariant divergence simplifies to ordinary divergence

$$
\partial_\alpha\left[T^{A,\alpha}\sqrt{g}\right] = 0 \ . \tag{6.2.3}
$$

This allows to simplify the equations considerably.

6. Spherically symmetric stationary minimal surface

Consider now the spherically symmetric stationary metric representable as minimal surface.

1. In the following we consider only the region exterior to the surface defining the TGD counterpart of Schwartschild horizon and the possible horizon at which the signature of the induced metric. The first possibility is $g_{tt} = 0$ at horizon. If g_{rr} remains non-vanishing, the signature changes to Euclidian. If also $g_{rr} = 0$, both g_{tt} and g_{rr} can change sign so that one has a smooth variant of Schwartschild horizon.

   Second possibility is $g_{rr} = 0$ at radius r_E in the region below Schwartschild radius. At r_E the determinant of 4-metric would vanish and the signature of the induced metric would change to Euclidian.

2. The reduction to the conservation of isometry currents can be used for isometry current corresponding to the rotation $\Phi \rightarrow \Phi + \epsilon$ and time translation $m^0 \rightarrow m^0 + \epsilon$.

3. With the experience coming from the embedding of Reissner-Nordström metric the ansatz is exactly the same and can be written as

$$
m^0 = \Lambda t + h(r) \ , \quad \Phi = \omega t + k(r) \ , \quad u \equiv cos(\Theta) = u(r) \ , \tag{6.2.4}
$$

4. The condition $g_{tr} = 0$ gives

$$
\Lambda\partial_r h = R^2\omega sin^2(\Theta)\partial_r k = 0 \ . \tag{6.2.5}
$$

   This allows to integrate $h(r)$ in terms of $k(r)$.

5. The interesting components of the induced metric are

$$
g_{tt} = \Lambda^2 - R^2\omega^2 sin^2(\Theta) \ , \quad g_{rr} = -1 - R^2(\partial_r\Theta)^2 + \Lambda^2(\partial_r h)^2 \ . \tag{6.2.6}
$$

6. The field equations reduce to conservation laws for various isometry currents. Consider energy current and the current related to the $SO(3) \subset SU(3)$ rotation acting on Φ as shift (call this current isospin current). The stationary character of the induced metric implies that the field equations reduce to the conservation of the radial current for energy current and isospin current. These two equations fix the solution together with diagonality condition. One obtains the following equations

$$\partial_r(\partial_r h \times g^{rr}\sqrt{g}) = 0 \ , \quad \partial_r(sin^2(\Theta)\partial_r k \times g^{rr}\sqrt{g}) = 0) \ . \tag{6.2.7}$$

These two equations can be satisfied simultaneously only if one has

$$\partial_r h \times g^{rr}r^2\sqrt{g_2} = Asin^2(\Theta)\partial_r k \times g^{rr}r^2\sqrt{g_2} + B \ , \quad g_2 \equiv -g_{tt}g_{rr} \ . \tag{6.2.8}$$

Note the presence of constant B.
Second implication is

$$g^{rr}\partial_r h\sqrt{g_2} = \frac{C}{r^2} \ , \quad g^{rr}sin^2(\Theta)\partial_r k\sqrt{g_2} = \frac{D}{r^2} \ , \quad C = AD + B \ . \tag{6.2.9}$$

By substituting the expressions for the metric one has

$$\partial_r h = \sqrt{-\frac{g_{rr}}{g_{tt}}} \times \frac{C}{r^2} \ , \quad sin^2(\Theta)\partial_r k = \sqrt{-\frac{g_{rr}}{g_{tt}}} \times \frac{D}{r^2} \ . \tag{6.2.10}$$

7. It is natural to look what one obtains in the approximation that the metric is flat expected to make sense at large distances. Putting $g_{tt} = -g_{rr} = 1$, one obtains

$$\partial_r h \simeq \frac{C}{r^2} \ , \quad sin^2(\Theta)\partial_r k \simeq \frac{D}{r^2} \ . \tag{6.2.11}$$

The time component of the induced metric is given by

$$g_{tt} = \Lambda^2 - R^2\omega^2 sin^2(\Theta) \simeq \Lambda^2 - \frac{D}{r^2\partial_r k} \ . \tag{6.2.12}$$

This gives $1/r$ gravitational potential of a mass point if one has $\partial_r k \simeq E/r$ giving for $\Lambda = 1$

$$g_{tt} = 1 - \frac{r_S}{r} \ , \quad r_S = 2GM = \frac{D}{E} \ . \tag{6.2.13}$$

with the identification $r_S = 2GM = D/E$ inspired by the behavior of the Scwartschild metric. It seems that one can take $\Lambda = 1$ without a loss of generality.

8. Using $g_{tr} = 0$ condition this gives for h the approximate expression

$$\partial_r h \simeq \frac{D}{r^2} \ , \quad D = \frac{R^2\omega^2}{\Lambda} \ . \tag{6.2.14}$$

so that the field equations are consistent with the $1/r$ behavior of gravitational potential. The solution carries necessarily a non-vanishing Abelian electroweak gauge field.

9. The asymptotic behaviors of k and h would be

$$k \simeq k_0 log(\frac{r}{r_0}) \ , \quad h \simeq h_0 - \frac{C}{r} \ . \tag{6.2.15}$$

7. Two horizons and layered structure as basic prediction

A very interesting question is whether $g_{tt} = 0$ defines Schwartschild type horizon at which the roles of the coordinates t and r change or whether one obtains horizon at which the signature of the induced metric becomes Euclidian. The most natural option turns out to be Schwartschild like horizon at which the roles of time and radial coordinate are changed and second inner horizon at which g_{rr} changes sign again so that the induced metric has Euclidian signature below this inner horizon.

1. Unless one has $g_{tt}g_{rr} = C \neq 0$ ($C = -1$ holds true in Scwhartschild-Nordström metric) the surface $g_{tt} = 0$ - if it exists - defines a light-like 3-surface identifiable as horizon at which the signature of the induced metric changes. The conditions $g_{tt} = 0$ gives

$$\Lambda^2 - R^2\omega^2(1-u^2) = 0 \ . \tag{6.2.16}$$

giving

$$0 < sin^2(\Theta) = 1 - u^2 \ = \ \frac{\Lambda^2}{R^2\omega^2} < 1 \ . \tag{6.2.17}$$

For $\Lambda = 1$ this condition implies that ω is a frequency of order of the inverse of CP_2 radius R. Note that $g_{tt} = 0$ need mean change of the metric signature to Euclidian if the analog of Schwarschild horizon is in question.

2. $g_{tt} = 0$ surface is light-like surface if g_{rr} has non-vanishing and finite value at it. g_{rr} could diverges at this surface guaranteeing $g_{tt}g_{rr} > 0$. The quantities $\partial_r h$ and $sin^2(\Theta)\partial_r k$ are proportional to $\sqrt{g_{rr}/g_{tt}}$, which diverges for $g_{tt} = 0$ unless also g_{rr} vanishes so that also these derivatives would diverge. The behavior of g_{rr} at this surface is

$$g_{rr} = -1 - R^2\tfrac{(\partial_r u)^2}{1-u^2} + \Lambda^2(\partial_r h)^2 \ \ , \quad u \equiv cos(\Theta) \ . \tag{6.2.18}$$

There are several options to consider.

(a) Option I: The divergence of $(\partial_r h)^2$ as cause for the divergence of g_{rr} is out of question. If this quantity increases for small values of r, g_{rr} can change sign for with finite value of $\partial_r h$ and $u^2 < 1$ at some larger radius r_S analogous to Schwartschild radius. Since it is impossible to have two time-like directions also the sign of g_{tt} must change so that one would have the analog of Schwartschild horizon at this radius - call it r_S: $r_S = 2GM$ need not hold true. The condition $g_{tt} = 0$ at this radius fixes the value of $sin^2(\Theta)$ at this radius

$$sin^2(\Theta_S) \ = \ \frac{\Lambda^2}{R^2\omega^2} \ . \tag{6.2.19}$$

If g_{rr} has finite value and is continuous, the metric has Euclidian signature in interior. If g_{rr} is discontinuous and changes sign as in the case of Schwartschild metric, one has counterpart of Scwartschild horizon without infinities. This option will be called Option I.

(b) Second possibility giving rise to would be that u becomes equal 1. This is not consistent with $sin^2(\Theta_S) = 0$.

(c) Option II: Voth g_{tt} and g_{rr} change their sign and vanish at r_S. This however requires both radial and time-like direction become null directions locally. Space-time surface would become locally metrically 2-dimensional at the horizon. This would conform with the idea of strong form of holography (SH) but it is not possible to have two different light-like directions simultaneously unless these directions are actually same. Mathematically it is certainly possible to have surfaces for which the dimension is locally reduced from the maximal one but it is difficult to visualize what this kind of metric reduction of local space-time dimension could mean. This option will be considered in what follows.

To sum up, g_{rr} changes sign at horizon. For Option I g_{rr} is finite and dis-continuous. For Option II g_{rr} vanishes and is continuous. Whether g_{rr} vanishes at horizon or not, remains open.

3. For Schwartschild-Nordström metric g_{rr} becomes infinite and changes sign at horizon. The change of the roles of g_{tt} and g_{rr} could for Option II take place smoothly so that both could become zero and change their sign at r_S. This would keep $\partial_r h$ and $sin^2(\Theta)\partial_r k$ finite. One would have the analog of the interior of Schwartschild metric.

   What happens at the smaller radii? The obvious constraint is that $sin^2(\Theta)$ remains below unity. If g_{rr}/g_{tt} remains bounded, the condition for $sin^2(\Theta)\partial_k$ however suggests that $sin^2(\Theta) = 1$ is eventually achieved. This is the case also for the embedding of Schwartschild metric. Could this horizon correspond to a surface at which the signature of the metric changes? g_{rr} should becomes zero in order to obtain light-like surface. g_{rr} contains indeed a term proportional to $1/sin^2(\Theta)$ which diverges at $u = 1$ so that g_{rr} must change sign for second time already above the radius for $sin^2(\Theta) = 1$ if h and k behaves smoothly enough. At this radius - call it r_E - g_{tt} would be finite and the signature would become Euclidian below this radius.

   One would therefore have two special radii r_S and r_E and a layer between these radii. $r_S = 2GM$ need not hold true but is expected to give a reasonable order of magnitude estimate.

Is there any empirical evidence for the existence of two horizons? There is evidence that the formation of the recently found LIGO blackhole (discussed from TGD view point in [L24]) is not fully consistent with the GRT based model (see `http://tinyurl.com/zbbz58w`). There are some indications that LIGO blackhole has a boundary layer such that the gravitational radiation is reflected forth and back between the inner and outer boundaries of the layer. In the proposed model the upper boundary would not be totally reflecting so that gravitational radiation leaks out and gave rise to echoes at times .1 sec, .2 sec, and .3 sec. It is perhaps worth of noticied that time scale .1 sec corresponds to the secondary p-adic time scale of electron (characterized by Mersenne prime $M_{127} = 2^{127} - 1$). If the minimal surface solution indeed has two horizons and a layer like structure between them, one might at least see the trouble of killing the idea that it could give rise to repeated reflections of gravitational radiation.

The proposed model (see `http://tinyurl.com/zbbz58w`) assumes that the inner horizon is Schwarstchild horizon. TGD would however suggests that the outer horizon is the TGD counterpart of Schwartschild horizon. It could have different radius since it would not be a singularity of g_{rr} (g_{tt}/g_{rr} would be finite at r_S which need not be $r_S = 2GM$ now). At r_S the tangent space of the space-time surface would become effectively 2-dimensional for $g_{rr} = 0$: the interpretation in terms of strong holography (SH) has been already mentioned.

The condition that the normal components of the canonical momentum currents for Kähler action and volume term are finite implies that $g^{nn}\sqrt{g_4}$ is finite at both sides of the horizon. Also the weak form of electric magnetic duality for Kähler form requires this. This condition can be satisfied if g_{tt} and g_{nn} approach to zero in the same manner at both sides of the horizon. Hence it seems that strong form of holography in the horizon is forced by finiteness.

One should understand why it takes rather long time $T = .1$ seconds for radiation to travel forth and back the distance $L = r_S - r_E$ between the horizons. The maximal signal velocity is reduced for the light-like geodesics of the space-time surface but the reduction should be rather large for $L \sim 20$ km (say). The effective light-velocity is measured by the coordinate time $\Delta t = \Delta m^0 + h(r_S) - h(r_E)$ needed to travel the distance from r_E to r_S. The Minkowski time Δm^0_{-+} would be the from null geodesic property and $m^0 = t + h(r)$

$$\Delta m^0_{-+} = \Delta t - h(r_S) + h(r_E) \ , \quad \Delta t = \int_{r_E}^{r_S} \sqrt{\frac{g_{rr}}{g_{tt}}} dr \equiv \int_{r_E}^{r_S} \frac{dr}{c_\#} \ . \tag{6.2.20}$$

Note that $c\#$ approaches zero at horizon if g_{rr} is non-vanishing at horizon.

The time needed to travel forth and back does not depend on h and would be given by

$$\Delta m^0 = 2\Delta t = 2 \int_{r_E}^{r_S} \frac{dr}{c_\#} \ . \tag{6.2.21}$$

This time cannot be shorter than the minimal time $(r_s - r_E)/c$ along light-like geodesic of M^4 since light-like geodesics at space-time surface are in general time-like curves in M^4. Since .1 sec corresponds to about 3×10^4 km, the average value of $c_\#$ should be for $L = 20$ km (just a rough guess) of order $c\# \sim 2^{-11}c$ in the interval $[r_E, r_S]$. As noticed, $T = .1$ sec is also the secondary p-adic time assignable to electron labelled by the Mersenne prime M_{127}. Since g_{rr} vanishes at r_E one has $c_\# \to \infty$. $c_\#$ is finite at r_S.

There is an intriguing connection with the notion of gravitational Planck constant. The formula for gravitational Planck constant given by $h_{gr} = GMm/v_0$ characterizing the magnetic bodies topologically for mass m topologically condensed at gravitational magnetic flux tube emanating from large mass M [K97, K78, **?**, K79]. The interpretation of the velocity parameter v_0 has remained open. Could v_0 correspond to the average value of $c_\#$? For the 4 inner planets one has $v_0 \simeq 2^{-11}$ so that the order of magnitude is same as for the estimate for $c_\#$.

Remark: More than year after after writing the above text I learned about additional evidence for blackhole echoes. Sabine Hossenfelder (see `http://tinyurl.com/ybd9gswm`) tells about the new evidence reported by Niayesh Afshordi, Professor of astrophysics at Perimeter Institute in the article "*Echoes from the Abyss: A highly spinning black hole remnant for the binary neutron star merger GW170817*" (see `https://arxiv.org/abs/1803.10454`). Now the earlier 2.5 sigma evidence has grown into 4.2 sigma evidence. 5 sigma is regarded as a criterion for discovery.

What about TGD inspired cosmology?

Before the discovery of the twistor lift TGD inspired cosmology has been based on the assumption that vacuum extremals provide a good estimate for the solutions of Einstein's equations at GRT limit of TGD [K114, K98] . One can find embeddings of Robertson-Walker type metrics as vacuum extremals and the general finding is that the cosmological with super-critical and critical mass density have finite duration after which the mass density becomes infinite: this period of course ends before this. The interpretation would be in terms of the emergence of new space-time sheet at which matter represented by smaller space-time sheets suffers topological condensation. The only parameter characterizing critical cosmologies is their duration. Critical (over-critical) cosmologies having $SO3 \times E^3$ ($SO(4)$) as isometry group is the duration and the CP_2 projection at homologically trivial geodesic sphere S^2: the condition that the contribution from S^2 to g_{rr} component transforms hyperbolic 3-metric to that of E^3 or S^3 metric fixes these cosmologies almost completely. Sub-critical cosmologies have one-dimensional CP_2 projection.

Do Robertson-Walker cosmologies have minimal surface representatives? Recall that minimal surface equations read as

$$
\begin{aligned}
D_\alpha(g^{\alpha\beta}\partial_\beta h^k\sqrt{g}) &= \partial_\alpha\left[g^{\alpha\beta}\partial_\beta h^k\sqrt{g}\right] + \left\{ \begin{matrix} k \\ \alpha\ m \end{matrix} \right\} g^{\alpha\beta}\partial_\beta h^m\sqrt{g} = 0 \ , \\
\left\{ \begin{matrix} k \\ \alpha\ m \end{matrix} \right\} &= \left\{ \begin{matrix} k \\ l\ m \end{matrix} \right\} \partial_\alpha h^l \ .
\end{aligned}
\tag{6.2.22}
$$

Sub-critical minimal surface cosmologies would correspond to $X^4 \subset M^4 \times S^1$. The natural coordinates are Robertson-Walker coordinates, which co-incide with light-cone coordinates $(a = \sqrt{(m^0)^2 - r_M^2}, r = r_M/a, \theta, \phi)$ for light-cone M^4_+. They are related to spherical Minkowski coordinates (m^0, r_M, θ, ϕ) by $(m^0 = a\sqrt{1+r^2}, r_M = ar)$. $\beta = r_M/m_0 = r/\sqrt{1+r^2}$ corresponds to the velocity along the line from origin (0,0) to (m^0, r_M). r corresponds to the Lorentz factor $\gamma\beta = \beta/\sqrt{1-\beta^2}$. The metric of M^4_+ is given by the diagonal form $[g_{aa} = 1, g_{rr} = a^2/(1+r^2), g_{\theta\theta} = a^2r^2, g_{\phi\phi} = a^2r^2 sin^2(\theta)]$. One can use the coordinates of M^4_+ also for X^4.

The ansatz for the minimal surface reads is $\Phi = f(a)$. For $f(a) = constant$ one obtains just the flat M^4_+. In non-trivial case one has $g_{aa} = 1 - R^2(df/da)^2$. The g^{aa} component of the metric becomes now $g^{aa} = 1/(1 - R^2(df/da)^2)$. Metric determinant is scaled by $\sqrt{g_{aa}} = 1 \to \sqrt{1 - R^2(df/da)^2}$. Otherwise the field equations are same as for M^4_+. Little calculation shows that they are not satisfied unless one as $g_{aa} = 1$.

Also the minimal surface embeddings of critical and over-critical cosmologies are impossible. The reason is that the criticality alone fixes these cosmologies almost uniquely and this is too much for allowing minimal surface property.

Thus one can have only the trivial cosmology M^4_+ carrying dark energy density as a minimal surface solution! This obviously raises several questions.

1. Could $\Lambda = 0$ case for which action reduces to Kähler action provide vacuum extremals provide single-sheeted model for Robertson-Walker cosmologies for the GRT limit of TGD for which many-sheeted space-time surface is replaced with a slightly curved region of M^4? Could $\Lambda = 0$ correspond to a genuine phase present in TGD as formal generalization of the view of mathematicians about reals as $p = \infty$ p-adic number suggest. p-Adic length scale would be strictly infinite implying that $\Lambda \propto 1/p$ vanishes.
2. Second possibility is that TGD is quantum critical in strong sense. Not only 3-space but the entire space-time surface is flat and thus M^4_+. Only the local gravitational fields created by topologically condensed space-time surfaces would make it curved but would not cause smooth expansion. The expansion would take as quantum phase transitions reducing the value of $\Lambda \propto 1/p$ as p-adic prime p increases. p-Adic length scale hypothesis suggests that the preferred primes are near but below powers of 2 $p \simeq 2^k$ for some integers k. This led for years ago to a model for Expanding Earth [K40].
3. This picture would explain why individual astrophysical objects have not been observed to expand smoothly (except possibly in these phase transitions) but participate cosmic expansion only in the sense that the distance to other objects increase. The smaller space-time sheets glued to a given space-time sheet preserving their size would emanate from the tip of M^4_+ for given sheet.
4. RW cosmology should emerge in the idealization that the jerk-wise expansion by quantum phase transitions and reducing the value of Λ (by scalings of 2 by p-adic length scale hypothesis) can be approximated by a smooth cosmological expansion.

One should understand why Robertson-Walker cosmology is such a good approximation to this picture. Consider first cosmic redshift.

1. The cosmic recession velocity is defined from the redshift by Doppler formula.

   $$z = \frac{1+\beta}{1-\beta} - 1 \simeq \beta = \frac{v}{c} \quad . \tag{6.2.23}$$

   In TGD framework this should correspond to the velocity defined in terms of the coordinate r of the object.

   Hubble law tells that the recession velocity is proportional to the proper distance D from the source. One has

   $$v = HD \quad , \quad H = (\frac{da/dt}{a}) = \frac{1}{\sqrt{g_{aa}}a} \quad . \tag{6.2.24}$$

   This brings in the dependence on the Robertson-Walker metric.

   For M^4_+ one has $a = t$ and one would have $g_{aa} = 1$ and $H = 1/a$. The experimental fact is however that the value of H is larger for non-empty RW cosmologies having $g_{aa} < 1$. How to overcome this problem?
2. To understand this one must first understand the interpretation of gravitational redshift. In TGD framework the gravitational redshift is property of observer rather than source. The point is that the tangent space of the 3-surface assignable to the observer is related by a Lorent boost to that associated with the source. This implies that the four-momentum of radiation from the source is boosted by this same boost. Redshift would mean that the Lorentz boost reduces the momentum from the real one. Therefore redshift would be consistent with momentum conservation implied by Poincare symmetry.

   g_{aa} for which a corresponds to the value of cosmic time for the observer should characterize the boost of observer relative to the source. The natural guess is that the boost is characterized by the value of g_{tt} in sufficiently large rest system assignable to observer with t is taken to be M^4 coordinate m^0. The value of g_{tt} fluctuates do to the presence of local gravitational fields. At the GRT limit g_{aa} would correspond to the average value of g_{tt}.

3. There is evidence that H is not same in short and long scales. This could be understood if the radiation arrives along different space-time sheets in these two situations.
4. If this picture is correct GRT description of cosmology is effective description taking into account the effect of local gravitation to the redshift, which without it would be just the M^4_+ redshift.

Einstein's equations for RW cosmology [K114, K98] should approximately code for the cosmic time dependence of mass density at given slightly deformed piece of M^4_+ representing particular sub-cosmology expanding in jerkwise manner.

1. Many-sheeted space-time implies a hierarchy of cosmologies in different p-adic length scales and with cosmological constant $\Lambda \propto 1/p$ so that vacuum energy density is smaller in long scale cosmologies and behaves on the average as $1/a^2$ where a characterizes the scale of the cosmology. In zero energy ontology given scale corresponds to causal diamond (CD) with size characterized by a defining the size scale for the distance between the tips of CD.
2. For the comoving volume with constant value of coordinate radius r the radius of the volume increases as a. The vacuum energy would increase as a^3 for comoving volume. This is in sharp conflict with the fact that the mass decreases as $1/a$ for radiation dominated cosmology, is constant for matter dominated cosmology, and is proportional to a for string dominated cosmology.

   The physical resolution of the problem is rather obvious. Space-time sheets representing topologically condensed matter have finite size. They do not expand except possibly in jerkwise manner but in this process Λ is reduced - in average manner like $1/a^2$.

   If the sheets are smaller than the cosmological space-time sheet in the scale considered and do not lose energy by radiation they represent matter dominated cosmology emanating from the vertex of M^4_+. The mass of the co-moving volume remains constant.

   If they are radiation dominated and in thermal equilibrium they lose energy by radiation and the energy of volume behaves like $1/a$.

   Cosmic strings and magnetic flux tubes have size larger than that the space-time sheet representing the cosmology. The string as linear structure has energy proportional to a for fixed value of Λ as in string dominated cosmology. The reduction of Λ decreasing on the average like $1/a^2$ implies that the contribution of given string is reduced like $1/a$ on the average as in radiation dominated cosmology.
3. GRT limit would code for these behaviours of mass density and pressure identified as scalars in GRT cosmology in terms of Einstein's equations. The time dependence of g_{aa} would code for the density of the topologically condensed matter and its pressure and for dark energy at given level of hierarchy. The vanishing of covariant divergence for energy momentum tensor would be a remnant of Poincare invariance and give Einstein's equations with cosmological term.
4. Why GRT limit would involve only the RW cosmologies allowing embedding as vacuum extremals of Kähler action? Can one demand continuity in the sense that TGD cosmology at $p \to \infty$ limit corresponds to GRT cosmology with cosmological solutions identifiable as vacuum extremals? If this is assumed the earlier results are obtained. In particular, one obtains the critical cosmology with 2-D CP_2 projection assumed to provide a GRT model for quantum phase transitions changing the value of Λ.

If this picture is correct, TGD inspired cosmology at the level of many-sheeted space-time would be extremely simple. The new element would be many-sheetedness which would lead to more complex description provided by GRT limit. This limit would however lose the information about many-sheetedness and lead to anomalies such as two Hubble constants.

Induced spinor structure

The notion of induced spinor field deserves a more detailed discussion. Consider first induced spinor structures [K121].

1. Induced spinor field are spinors of $M^4 \times CP_2$ for which modes are characterized by chirality (quark or lepton like) and em charge and weak isospin.
2. Induced spinor spinor structure involves the projection of gamma matrices defining induced gamma matrices. This gives rise to superconformal symmetry if the action contains only volume term.

   When Kähler action is present, superconformal symmetry requires that the modified gamma matrices are contractions of canonical momentum currents with embedding space gamma matrices. Modified gammas appear in the modified Dirac equation and action, whose solution at string world sheets trivializes by super-conformal invariance to same procedure as in the case of string models.
3. Induced spinor fields correspond to two chiralities carrying quark number and lepton number. Quark chirality does not carry color as spin-like quantum number but it corresponds to a color partial wave in CP_2 degrees of freedom: color is analogous to angular momentum. This reduces to spinor harmonics of CP_2 describing the ground states of the representations of super-symplectic algebra.

   The harmonics do not satisfy correct correlation between color and electroweak quantum numbers although the triality t=0 for leptonic waves and t=1 for quark waves. There are two ways to solve the problem.

   (a) Super-symplectic generators applied to the ground state to get vanishing ground states weight instead of the tachyonic one carry color and would give for the physical states correct correlation: leptons/quarks correspond to the same triality zero(one partial wave irrespective of charge state. This option is assumed in p-adic mass calculations [K59].

   (b) Since in TGD elementary particles correspond to pairs of wormhole contacts with weak isospin vanishing for the entire pair, one must have pair of left and right-handed neutrinos at the second wormhole throat. It is possible that the anomalous color quantum numbers for the entire state vanish and one obtains the experimental correlation between color and weak quantum numbers. This option is less plausible since the cancellation of anomalous color is not local as assume in p-adic mass calculations.

The understanding of the details of the fermionic and actually also geometric dynamics has taken a long time. Super-conformal symmetry assigning to the geometric action of an object with given dimension an analog of Dirac action allows however to fix the dynamics uniquely and there is indeed dimensional hierarchy resembling brane hierarchy.

1. The basic observation was following. The condition that the spinor modes have well-defined em charge implies that they are localized to 2-D string world sheets with vanishing W boson gauge fields which would mix different charge states. At string boundaries classical induced W boson gauge potentials guarantee this. Super-conformal symmetry requires that this 2-surface gives rise to 2-D action which is area term plus topological term defined by the flux of Kähler form.
2. The most plausible assumption is that induced spinor fields have also interior component but that the contribution from these 2-surfaces gives additional delta function like contribution: this would be analogous to the situation for branes. Fermionic action would be accompanied by an area term by supersymmetry fixing modified Dirac action completely once the bosonic actions for geometric object is known. This is nothing but super-conformal symmetry.

   One would actually have the analog of brane-hierarchy consisting of surfaces with dimension D= 4,3,2,1 carrying induced spinor fields which can be regarded as independent dynamical variables and characterized by geometric action which is D-dimensional analog of the action for Kähler charged point particle. This fermionic hierarchy would accompany the hierarchy of geometric objects with these dimensions and the modified Dirac action would be uniquely determined by the corresponding geometric action principle (Kähler charged point like particle, string world sheet with area term plus Kähler flux, light-like 3-surface with Chern-Simons term, 4-D space-time surface with Kähler action).
3. This hierarchy of dynamics is consistent with SH only if the dynamics for higher dimensional objects is induced from that for lower dimensional objects - string world sheets or maybe even their boundaries orbits of point like fermions. Number theoretic vision [K119] suggests

that this induction relies algebraic continuation for preferred extremals. Note that quaternion analyticity [K41] means that quaternion analytic function is determined by its values at 1-D curves.

4. Quantum-classical correspondences (QCI) requires that the classical Noether charges are equal to the eigenvalues of the fermionic charges for surfaces of dimension $D = 0, 1, 2, 3$ at the ends of the CDs. These charges would not be separately conserved. Charges could flow between objects of dimension $D + 1$ and D - from interior to boundary and vice versa. Four-momenta and also other charges would be complex as in twistor approach: could complex values relate somehow to the finite life-time of the state?

   If quantum theory is square root of thermodynamics as zero energy ontology suggests, the idea that particle state would carry information also about its life-time or the time scale of CD to which is associated could make sense. For complex values of α_K there would be also flow of canonical and super-canonical momentum currents between Euclidian and Minkowskian regions crucial for understand gravitational interaction as momentum exchange at embedding space level.

5. What could be the physical interpretation of the bosonic and fermionic charges associated with objects of given dimension? Condensed matter physicists assign routinely physical states to objects of various dimensions: is this assignment much more than a practical approximation or could condensed matter physics already be probing many-sheeted physics?

SUSY and TGD

From this one ends up to the possibility of identifying the counterpart of SUSY in TGD framework [K96].

1. In TGD the generalization of much larger super-conformal symmetry emerges from the super-symplectic symmetries of WCW. The mathematically questionable notion of super-space is not needed: only the realization of super-algebra in terms of WCW gamma matrices defining super-symplectic generators is necessary to construct quantum states. As a matter of fact, also in QFT approach one could use only the Clifford algebra structure for super-multiplets. No Majorana condition on fermions is needed as for $\mathcal{N} = 1$ space-time SUSY and one avoids problems with fermion number non-conservation.

2. In TGD the construction of sparticles means quite concretely adding fermions to the state. In QFT it corresponds to transformation of states of integer and half-odd integer spin to each other. This difference comes from the fact that in TGD particles are replaced with point like particles.

3. The analog of $\mathcal{N} = 2$ space-time SUSY could be generated by covariantly constant right handed neutrino and antineutrino. Quite generally the mixing of fermionic chiralities implied by the mixing of M^4 and CP_2 gamma matrices implies SUSY breaking at the level of particle masses (particles are massless in 8-D sense). This breaking is purely geometrical unlike the analog of Higgs mechanism proposed in standard SUSY.

There are several options to consider.

1. The analog of brane hierarchy is realized also in TGD. Geometric action has parts assignable to 4-surface, 3-D light like regions between Minkowskian and Euclidian regions, 2-D string world sheets, and their 1-D boundaries. They are fixed uniquely. Also their fermionic counterparts - analogs of Dirac action - are fixed by super-conformal symmetry. Elementary particles reduce so composites consisting of point-like fermions at boundaries of wormhole throats of a pair of wormhole contacts.

   This forces to consider 3 kinds of SUSYs! The SUSYs associated with string world sheets and space-time interiors would certainly be broken since there is a mixing between M^4 chiralities in the modified Dirac action. The mass scale of the broken SUSY would correspond to the length scale of these geometric objects and one might argue that the decoupling between the degrees of freedom considered occurs at high energies and explains why no evidence for SUSY has been observed at LHC. Also the fact that the addition of massive fermions at these dimensions can be interpreted differently. 3-D light-like 3-surfaces could be however an exception.

2. For 3-D light-like surfaces the modified Dirac action associated with the Chern-Simons term does not mix M^4 chiralities (signature of massivation) at all since modified gamma matrices have only CP_2 part in this case. All fermions can have well-defined chirality. Even more: the modified gamma matrices have no M^4 part in this case so that these modes carry no four-momentum - only electroweak quantum numbers and spin. Obviously, the excitation of these fermionic modes would be an ideal manner to create spartners of ordinary particles consting of fermion at the fermion lines. SUSY would be present if the spin of these excitations couples - to various interactions and would be exact in absence of coupling to interior spinor fields.

   What would be these excitations? Chern-Simons action and its fermionic counterpart are non-vanishing only if the CP_2 projection is 3-D so that one can use CP_2 coordinates. This strongly suggests that the modified Dirac equation demands that the spinor modes are covariantly constant and correspond to covariantly constant right-handed neutrino providing only spin.

   If the spin of the right-handed neutrino adds to the spin of the particle and the net spin couples to dynamics, $\mathcal{N}=2$ SUSY is in question. One would have just action with unbroken SUSY at QFT limit? But why also right-handed neutrino spin would couple to dynamics if only CP_2 gamma matrices appear in Chern-Simons-Dirac action? It would seem that it is independent degree of freedom having no electroweak and color nor even gravitational couplings by its covariant constancy. I have ended up with just the same SUSY-or-no-SUSY that I have had earlier.

3. Can the geometric action for light-like 3-surfaces contain Chern-Simons term?

   (a) Since the volume term vanishes identically in this case, one could indeed argue that also the counterpart of Kähler action is excluded. Moreover, for so called massless extremals of Kähler action reduces to Chern-Simons terms in Minkowskian regions and this could happen quite generally: TGD with only Kähler action would be almost topological QFT as I have proposed. Volume term however changes the situation via the cosmological constant. Kähler-Dirac action in the interior does not reduce to its Chern-Simons analog at light-like 3-surface.

   (b) The problem is that the Chern-Simons term at the two sides of the light-like 3-surface differs by factor $\sqrt{-1}$ coming from the ratio of $\sqrt{g_4}$ factors which themselves approach to zero: oOne would have the analog of dipole layer. This strongly suggests that one should not include Chern-Simons term at all.

   Suppose however that Chern-Simons terms are present at the two sides and α_K is real so that nothing goes through the horizon forming the analog of dipole layer. Both bosonic and fermionic degrees of freedom for Euclidian and Minkowskian regions would decouple completely but currents would flow to the analog of dipole layer. This is not physically attractive.

   The canonical momentum current and its super counterpart would give fermionic source term $\Gamma^n\Psi_{int,\pm}$ in the modified Dirac equation defined by Chern-Simons term at given side $\pm$: $\pm$ refers to Minkowskian/Euclidian part of the interior. The source term is proportional to $\Gamma^n\Psi_{int,\pm}$ and Γ^n is in principle mixture of M^4 and CP_2 gamma matrices and therefore induces mixing of M^4 chiralities and therefore also 3-D SUSY breaking. It must be however emphasized that Γ^n is singular and one must be consider the limit carefully also in the case that one has only continuity conditions. The limit is not completely understood.

   (c) If α_K is complex there is coupling between the two regions and the simplest assumption has been that there is no Chern-Simons term as action and one has just continuity conditions for canonical momentum current and hits super counterpart.

   The cautious conclusion is that 3-D Chern-Simons term and its fermionic counterpart are absent.

4. What about the addition of fermions at string world sheets and interior of space-time surface ($D=2$ and $D=4$). For instance, in the case of hadrons $D=2$ excitations could correspond to addition of quark in the interior of hadronic string implying additional states besides the states obtained assuming only quarks at string ends. Let us consider the interior ($D=4$). For instance, inn the case of hadrons $D=2$ excitations could correspond to addition of quark in the interior of hadronic string implying additional states besides the states obtained assuming

only quarks at string ends. The smallness of cosmological constant implies that the contribution to the four-momentum from interior should be rather small so that an interpretation in terms of broken SUSY might make sense. There would be mass $m \sim .03$ eV per volume with size defined by the Compton scale $\hbar/m$. Note however that cosmological constant has spectrum coming as inverse powers of prime so that also higher mass scales are possible.

This interpretation might allow to understand the failure to find SUSY at LHC. Sparticles could be obtained by adding interior right-handed neutrinos and antineutrinos to the particle state. They could be also associated with the magnetic body of the particle. Since they do not have color and weak interactions, SUSY is not badly broken. If the mass difference between particle and sparticle is of order $m = .03$ eV characterizing dark energy density ρ_{vac}, particle and sparticle could not be distinguished in higher energy physics at LHC since it probes much shorter scales and sees only the particle. I have already earlier proposed a variant of this mechanism but without SUSY breaking.

To discover SUSY one should do very low energy physics in the energy range $m \sim .03$ eV having same order of magnitude as thermal energy $kT = 2.6 \times 10^{-2}$ eV at room temperature 25 °C. One should be able to demonstrate experimentally the existence of sparticle with mass differing by about $m \sim .03$ eV from the mass of the particle (one cannot exclude higher mass scales since Λ is expected to have spectrum). An interesting question is whether the sparticles associated with standard fermions could give rise to Bose-Einstein condensates whose existence in the length scale of large neutron is strongly suggested by TGD view about living matter.

6.2.3 Embedding space level

In GRT the description of gravitation involve only space-time and gravitational force is eliminated. In TGD also embedding space level is involved with the description [K41].

1. The incoming and outgoing states of particle reaction are labelled by the quantum numbers associated with the isometries of the embedding space and by the contributions of super-symplectic generators and isometry generators to the quantum numbers. This follows from the fact that the ground states of super-symplectic representations correspond to the modes of embedding space spinors fields. These quantum numbers appear in the S-matrix of QFT limit too. In particular, color quantum numbers as angular momentum like quantum numbers at fundamental level are transformed to spin-like quantum numbers at QFT limit.

2. In GRT the applications rely on Post-Newtonian approximation (PNA). This means that the notion of gravitational force is brought to the theory although it has been eliminated from the basic GRT. This is not simple. One could argue that there is genuine physics behind this PNA and TGD suggests what this physics is.

   At the level of space-time surfaces particles move along geodesic lines and in TGD minimal surface equation states the generalization of the geodesic line property for 3-D particles. At the embedding space level gravitational interaction involves exchanges of four-momentum and in principle of color quantum numbers too. Indeed, there is an exchange of classical charges through the light-like 3-surfaces defining the boundaries of Euclidian regions defining Euclidian regions as "lines" of generalized scattering diagrams. This however requires that Kähler coupling strength is allowed to be complex (say correspond to zero of Riemann Zeta). Hence in TGD also Newtonian view would be correct and needed.

6.3 Some questions about TGD

In Face Book I was made a question about general aspects of TGD. It was impossible to answer the question with few lines and I decided to write a blog posting, which then gave rise to this section. This text talks from different perspective about same topics as the article *Can one apply Occam's razor as a general purpose debunking argument to TGD?* [L19] trying o emphasize the simplicity of the basic principles of TGD and of the resulting theory.

6.3.1 In what aspects TGD extends other theory/theories of physics?

I will replace "extends" with "modifies" since TGD also simplifies in many respects. I shall restrict the considerations to the ontological level which to my view is the really important level.

1. Space-time level is where TGD started from. Space-time as an abstract 4-geometry is replaced as space-time as 4-surface in $M^4 \times CP_2$. In GRT space-time is small deformation of Minkowski space.

   In TGD both Relativity Principle (RP) of Special Relativity (SRT) and General Coordinate Invariance (GCI) and Equivalence Principle (EP) of General Relativity hold true. In GRT RP is given up and leads to the loss of conservation laws since Noether theorem cannot be applied anymore: this is what led to the idea about space-time as surface in H. Strong form of holography (SH) is a further principle reducing to strong form of GCI (SGCI).

2. TGD as a physical theory extends to a theory of consciousness and cognition. Observer as something external to the Universe becomes part of physical system - the notion of self - and quantum measurement theory which is the black sheet of quantum theory extends to a theory of consciousness and also of cognition relying of p-adic physics as correlate for cognition. Also quantum biology becomes part of fundamental physics and consciousness and life are seen as basic elements of physical existence rather than something limited to brain.

   One important aspect is a new view about time: experienced time and geometric time are not one and same thing anymore although closely related. ZEO explains how the experienced flow and its direction emerges. The prediction is that both arrows of time are possible and that this plays central role in living matter.

3. p-Adic physics is a new element and an excellent candidate for a correlate of cognition. For instance, imagination could be understood in terms of non-determinism of p-adic partial differential equations for p-adic variants of space-time surfaces. p-Adic physics and fusion of real and various p-adic physics to adelic physics provides fusion of physics of matter with that of cognition in TGD inspired theory of cognition. This means a dramatic extension of ordinary physics. Number Theoretical Universality states that in certain sense various p-adic physics and real physics can be seen as extensions of physics based on algebraic extensions of rationals (and also those generated by roots of e inducing finite-D extensions of p-adics).

4. Zero energy ontology (ZEO) in which so called causal diamonds (CDs, analogs Penrose diagrams) can be seen as being forced by very simple condition: the volume action forced by twistor lift of TGD must be finite. CD would represent the perceptive field defined by finite volume of embedding space $H = M^4 \times CP_2$.

   ZEO implies that conservation laws formulated only in the scale of given CD do not anymore fix select just single solution of field equations as in classical theory. Theories are strictly speaking impossible to test in the old classical ontology. In ZEO testing is possible be sequence of state function reductions giving information about zero energy states.

   In principle transition between any two zero energy states - analogous to events specified by the initial and final states of event - is in principle possible but Negentropy Maximization Principle (NMP) as basic variational principle of state function reduction and of consciousness restricts the possibilities by forcing generation of negentropy: the notion of negentropy requires p-adic physics.

   Zero energy states are quantum superpositions of classical time evolutions for 3-surfaces and classical physics becomes exact part of quantum physics: in QFTs this is only the outcome of stationary phase approximation. Path integral is replaced with well-defined functional integral- not over all possible space-time surface but pairs of 3-surfaces at the ends of space-time at opposite boundaries of CD.

   ZEO leads to a theory of consciousness as quantum measurement theory in which observer ceases to be outsider to the physical world. One also gets rid of the basic problem caused by the conflict of the non-determinism of state function reduction with the determinism of the unitary evolution. This is obviously an extension of ordinary physics.

5. Hierarchy of Planck constants represents also an extension of quantum mechanics at QFT limit. At fundamental level one actually has the standard value of h but at QFT limit one has effective Planck constant $h_{eff}/h = n$, $n = 1, 2, ...$ This generalizes quantum theory.

This scaling of h has a simple topological interpretation: space-time surface becomes n-fold covering of itself and the action becomes n-multiple of the original which can be interpreted as $h_{eff}/h = n$.

The most important applications are to biology, where quantum coherence could be understood in terms of a large value of h_{eff}/h. The large n phases resembles the large N limit of gauge theories with gauge couplings behaving as $\alpha \propto 1/N$ used as a kind of mathematical trick. Also gravitation is involved: h_{eff} is associated with the flux tubes mediating various interactions (being analogs to wormholes in ER-EPR correspondence). In particular, one can speak about h_{gr}, which Nottale introduced originally and $h_{eff} = h_{gr}$ plays key role in quantum biology according to TGD.

6.3.2 In what sense TGD is simplification/extension of existing theory?

1. Classical level: Space-time as 4-surface of H means a huge reduction in degrees of freedom. There are only 4 field like variables - suitably chosen 4 coordinates of $H = M^4 \times CP_2$. All classical gauge fields and gravitational field are fixed by the surface dynamics. There are no primary gauge fields or gravitational fields nor any other fields in TGD Universe and they appear only at the QFT limit [K14, K8, K10].

   GRT limit would mean that many-sheeted space-time is replaced by single slightly curved region of M^4. The test particle - small particle like 3-surface - touching the sheets simultaneously experience sum of gravitational forces and gauge forces. It is natural to assume that this superposition corresponds at QFT limit to the sum for the deviations of induced metrics of space-time sheets from flat metric and sum of induce gauge potentials. These would define the fields in standard model + GRT. At fundamental level effects rather than fields would superpose. This is absolutely essential for the possibility of reducing huge number field like degrees of freedom. One can obviously speak of emergence of various fields.

   A further simplification is that only preferred extremals for which data coding for them are reduced by SH to 2-D string like world sheets and partonic 2-surfaces are allowed. TGD is almost like string model but space-time surfaces are necessary for understanding the fact that experiments must be analyzed using classical 4-D physics. Things are extremely simple at the level of single space-time sheet.

   Complexity emerges from many-sheetedness. From these simple basic building bricks - minimal surface extremals of Kähler action (not the extremal property with respect to Kähler action and volume term strongly suggested by the number theoretical vision plus analogs of Super Virasoro conditions in initial data) - one can engineer space-time surfaces with arbitrarily complex topology - in all length scales. An extension of existing space-time concept emerges. Extremely simple locally, extremely complex globally with topological information added to the Maxwellian notion of fields (topological field quantization allowing to talk about field identify of system/field body/magnetic body.

   Another new element is the possibility of space-time regions with Euclidian signature of the induced metric. These regions correspond to 4-D "lines" of general scattering diagrams. Scattering diagrams has interpretation in terms of space-time geometry and topology.

2. The construction of quantum TGD using canonical quantization or path integral formalism failed completely for Kähler action by its huge vacuum degeneracy. The presence of volume term still suffers from complete failure of perturbation theory and extreme non-linearity. This led to the notion of world of classical worlds (WCW) - roughly the space of 3-surfaces. Essentially pairs of 3-surfaces at the boundaries of given CD connected by preferred extremals of action realizing SH and SGCI.

   The key principle is geometrization of the entire quantum theory, not only of classical fields geometrized by space-time as surface vision. This requires geometrization of hermitian conjugation and representation of imaginary unit geometrically. Kähler geometry for WCW [K50, K28, K93] makes this possible and is fixed once Kähler function defining Kähler metric is known. Kähler action for a preferred extremal of Kähler action defining space-time surface as an analog of Bohr orbit was the first guess but twistor lift forced to add volume term having interpretation in terms of cosmological constant.

Already the geometrization of loop spaces demonstrated that the geometry - if it exists - must have maximal symmetries (isometries). There are excellent reasons to expect that this is true also in $D = 3$. Physics would be unique from its mathematical existence!

3. WCW has also spinor structure [K121, K93]. WCW spinors correspond to fermionic Fock states using oscillator operators assignable to the induced spinor fields - free spinor fiels. WCW gamma matrices are linear combinations of these oscillator operators and Fermi statistics reduces to spinor geometry.

4. There is **no quantization** in TGD framework at the level of WCW [K27, K41]. The construction of quantum states and S-matrix reduces to group theory by the huge symmetries of WCW. Therefore zero energy states of Universe (or CD) correspond formally to **classical** WCW spinor fields satisfying WCW Dirac equation analogous to Super Virasoro conditions and defining representations for the Yangian generalization of the isometries of WCW (so called super-symplectic group assignable to $\delta M^4_+ \times CP_2$. In ZEO stated are analogous to pairs of initial and final states and the entanglement coefficients between positive and negative energy parts of zero energy states expected to be fixed by Yangian symmetry define scattering matrix and have purely group theoretic interpretation. If this is true, entire dynamics would reduce to group theory in ZEO.

6.3.3 What is the hypothetical applicability of the extension - in energies, sizes, masses etc?

TGD is a unified theory and is meant to apply in all scales. Usually the unifications rely on reductionistic philosophy and try to reduce physics to Planck scale. Also super string models tried this and failed: what happens at long length scales was completely unpredictable (landscape catastrophe).

Many-sheeted space-time however forces to adopt fractal view. Universe would be analogous to Mandelbrot fractal down to CP_2 scale. This predicts scaled variants of say hadron physics and electroweak physics. p-Adic length scale hypothesis and hierarchy of phases of matter with $h_{eff}/h = n$ interpreted as dark matter gives a quantitative realization of this view.

1. p-Adic physics shows itself also at the level of real physics [K70]. One ends up to the vision that particle mass squared has thermal origin: the p-adic variant of particle mass square is given as thermal mass squared given by p-adic thermodynamics mappable to real mass squared by what I call canonical identification. p-Adic length scale hypothesis states that preferred p-adic primes characterizing elementary particles correspond to primes near to power of 2: $p \simeq 2^k$. p-Adic length scale is proportional to $p^{1/2}$.

   This hypothesis is testable and it turns out that one can predict particle mass rather accurately. This is highly non-trivial since the sensitivity to the integer k is exponential. So called Mersenne primes turn out to be especially favoured. This part of theory was originally inspired by the regularities of particle mass spectrum. I have developed arguments for why the crucial p-adic length scale hypothesis - actually its generalization - should hold true. A possible interpretation is that particles provide cognitive representations of themselves by p-adic thermodynamics.

2. p-Adic length scale hypothesis leads also to consider the idea that particles could appear as different p-adically scaled up variants. For instance, ordinary hadrons to which one can assign Mersenne prime $M_{107} = 2^{107} - 1$ could have fractally scaled variants. M_{89} and $M_{G,107}$ (Gaussian prime) would be two examples and there are indications at LHC for these scaled up variants of hadron physics [K66, K67]. These fractal copies of hadron physics and also of electroweak physics would correspond to extension of standard model.

3. Dark matter hierarchy predicts zoomed up copies of various particles. The simplest assumption is that masses are not changed in the zooming up. One can however consider that binding energy scale scales non-trivially. The dark phases would emerge are quantum criticality and give rise to the associated long range correlations (quantum lengths are typically scaled up by $h_{eff}/h = n$).

6.3.4 What is the leading correction/contribution to physical effects due to TGD onto particles, interactions, gravitation, cosmology?

1. Concerning particles I already mentioned the key predictions.
   (a) The existence of scaled variants of various particles and entire branches of physics. The fundamental quantum numbers are just standard model quantum numbers code by CP_2 geometry.
   (b) Particle families have topological description meaning that space-time topology would be an essential element of particle physics [K25]. The genus of partonic 2-surfaces (number of handles attached to sphere) is $g = 0, 1, 2, ...$ and would give rise to family replication. $g < 2$ partonic 2-surfaces have always global conformal symmetry Z_2 and this suggests that they give rise to elementary particles identifiable as bound states of g handles. For $g > 2$ this symmetry is absent in the generic case which suggests that they can be regarded as many-handle states with mass continuum rather than elementary particles. 2-D anyonic systems could represent an example of this.
   (c) A hierarchy of dynamical symmetries as remnants of super-symplectic symmetry however suggests itself [K27, K93]. The super-symplectic algebra possess infinite hierarchy of isomorphic sub-algebras with conformal weights being n-multiples of for those for the full algebra (fractal structure again possess also by ordinary conformal algebras). The hypothesis is that sub-algebra specified by n and its commutator with full algebra annihilate physical states and that corresponding classical Noether charges vanish. This would imply that super-symplectic algebra reduces to finite-D Kac-Moody algebra acting as dynamical symmetries. The connection with ADE hierarchy of Kac-Moody algebras suggests itself. This would predict new physics. Condensed matter physics comes in mind.
   (d) Number theoretic vision suggests that Galois groups for the algebraic extensions of rationals act as dynamical symmetry groups. They would act on algebraic discretizations of 3-surfaces and space-time surfaces necessary to realize number theoretical universality. This would be completely new physics.
2. Interactions would be mediated at QFT limit by standard model gauge fields and gravitons. QFT limit however loses all information about many-sheetedness and there would be anomalies reflecting this information loss. In many-sheeted space-time light can propagate along several paths and the time taken to travel along light-like geodesic from A to B depends on space-time sheet since the sheet is curved and warped. Neutrinos and gamma rays from SN1987A arriving at different times would represent a possible example of this. It is quite possible that the outer boundaries of even macroscopic objects correspond to boundaries between Euclidian and Minkowskian regions at the space-time sheet of the object.

   The failure of QFTs to describe bound states of say hydrogen atom could be second example: many-sheetedness and identification of bound states as single connected surface formed by proton and electron would be essential and taken into account in wave mechanical description but not in QFT description.
3. Concerning gravitation the basic outcome is that by number theoretical vision all preferred extremals are extremals of both Kähler action and volume term. This is true for all known extremals what happens if one introduces the analog of Kähler form in M^4 is an open question) [K10].

   Minimal surfaces carrying no Kähler field would be the basic model for gravitating system. Minimal surface equation are non-linear generalization of d'Alembert equation with gravitational self-coupling to induce gravitational metric. In static case one has analog for the Laplace equation of Newtonian gravity. One obtains analog of gravitational radiation as "massless extremals" and also the analog of spherically symmetric stationary metric.

   Blackholes would be modified. Besides Schwartschild horizon which would differ from its GRT version there would be horizon where signature changes. This would give rise to a layer structure at the surface of blackhole [K10].
4. Concerning cosmology the hypothesis has been that RW cosmologies at QFT limit can be modelled as vacuum extremals of Kä hler action. This is admittedly ad hoc assumption

inspired by the idea that one has infinitely long p-adic length scale so that cosmological constant behaving like $1/p$ as function of p-adic length scale assignable with volume term in action vanishes and leaves only Kähler action [K79]. This would predict that cosmology with critical is specified by a single parameter - its duration as also over-critical cosmology [K98]. Only sub-critical cosmologies have infinite duration.

One can look at the situation also at the fundamental level. The addition of volume term implies that the only RW cosmology realizable as minimal surface is future light-cone of M^4. Empty cosmology which predicts non-trivial slightly too small redshift just due to the fact that linear Minkowski time is replaced with light-cone proper time constant for the hyperboloids of M^4_+. Locally these space-time surfaces are however deformed by the addition of topologically condensed 3-surfaces representing matter. This gives rise to additional gravitational redshift and the net cosmological redshift. This also explains why astrophysical objects do not participate in cosmic expansion but only comove. They would have finite size and almost Minkowski metric.

The gravitational redshift would be basically a kinematical effect. The energy and momentum of photons arriving from source would be conserved but the tangent space of observer would be Lorentz-boosted with respect to source and this would course redshift.

The very early cosmology could be seen as gas of arbitrarily long cosmic strings in H (or M^4) with 2-D M^4 projection [K98, K63]. Horizon would be infinite and TGD suggests strongly that large values of h_{eff}/h makes possible long range quantum correlations. The phase transition leading to generation of space-time sheets with 4-D M^4 projection would generate many-sheeted space-time giving rise to GRT space-time at QFT limit. This phase transition would be the counterpart of the inflationary period and radiation would be generated in the decay of cosmic string energy to particles.

Part II

PHYSICS AS INFINITE-DIMENSIONAL SPINOR GEOMETRY

Chapter 7

The Geometry of the World of Classical Worlds

7.1 Introduction

The topics of this chapter are the purely geometric aspects of the vision about physics as an infinite-dimensional Kähler geometry of the "world of classical worlds", with " classical world" identified either as light-like 3-D surface of the unique Bohr orbit like 4-surface traversing through it. The non-determinism of Kähler action forces to generalize the notion of 3-surface so that unions of space-like surfaces with time like separations must be allowed. Zero energy ontology allows to formulate this picture elegantly in terms of causal diamonds defined as intersections of future and past directed light-cones. Also a a geometric realization of coupling constant evolution and finite measurement resolution emerges.

There are two separate but closely related tasks involved.

1. Provide WCW with Kähler geometry which is consistent with 4-dimensional general coordinate invariance so that the metric is Diff^4 degenerate. General coordinate invariance implies that the definition of the metric must assign to a given light-like 3-surface X^3 a 4-surface as a kind of Bohr orbit $X^4(X^3)$.
2. Provide WCW with a spinor structure. The great idea is to identify WCW gamma matrices in terms of super algebra generators expressible using second quantized fermionic oscillator operators for induced free spinor fields at the space-time surface assignable to a given 3-surface. The isometry generators and contractions of Killing vectors with gamma matrices would thus form a generalization of Super Kac-Moody algebra.

In this chapter a summary about basic ideas related to the construction of the Kähler geometry of infinite-dimensional configuration of 3-surfaces (more or less-equivalently, the corresponding 4-surfaces defining generalized Bohr orbits) or "world of classical worlds" (WCW).

7.1.1 The Quantum States Of Universe As Modes Of Classical Spinor Field In The "World Of Classical Worlds"

The vision behind the construction of WCW geometry is that physics reduces to the geometry of classical spinor fields in the infinite-dimensional WCW of 3-surfaces of $M^4_+ \times CP_2$ or $M^4 \times CP_2$, where M^4 and M^4_+ denote Minkowski space and its light cone respectively. This WCW might be called the "world of classical worlds".

Hermitian conjugation is the basic operation in quantum theory and its geometrization requires that WCW possesses Kähler geometry. One of the basic features of the Kähler geometry is that it is solely determined by the so called. which defines both the J and the components of the g in complex coordinates via the general formulas [A49]

$$\begin{aligned} J &= i\partial_k\partial_{\bar{l}}K dz^k \wedge d\bar{z}^l \ . \\ ds^2 &= 2\partial_k\partial_{\bar{l}}K dz^k d\bar{z}^l \ . \end{aligned} \tag{7.1.1}$$

Kähler form is covariantly constant two-form and can be regarded as a representation of imaginary unit in the tangent space of the WCW

$$J_{mr}J^{rn} = -g_m^{\ n} \ . \tag{7.1.2}$$

As a consequence Kähler form defines also symplectic structure in WCW.

7.1.2 WCW Kähler Metric From Kähler Function

The task of finding Kähler geometry for the WCW reduces to that of finding Kähler function and identifying the complexification. The main constraints on the Kähler function result from the requirement of Diff^4 symmetry and degeneracy. requires that the definition of the Kähler function assigns to a given 3-surface X^3, which in Zero Energy Ontology is union of 3-surfaces at the opposite boundaries of causal diamond CD, a unique space-time surface $X^4(X^3)$, the generalized Bohr orbit defining the classical physics associated with X^3. The natural guess is that Kähler function is defined by what might be called Kähler action, which is essentially Maxwell action with Maxwell field expressible in terms of CP_2 coordinates.

Absolute minimization was the first guess for how to fix $X^4(X^3)$ uniquely. It has however become clear that this option might well imply that Kähler is negative and infinite for the entire Universe so that the vacuum functional would be identically vanishing. This condition can make sense only inside wormhole contacts with Euclidian metric and positive definite Kähler action.

Quantum criticality of TGD Universe suggests the appropriate principle to be the criticality, that is vanishing of the second variation of Kähler action. This principle now follows from the conservation of Noether currents the Kähler-Dirac action. This formulation is still rather abstract and if spinors are localized to string world sheets, it it is not satisfactory. A further step in progress was the realization that preferred extremals could carry vanishing super-conformal Noether charges for sub-algebras whose generators have conformal weight vanishing modulo n with n identified in terms of effective Planck constant $h_{eff}/h = n$.

If Kähler action would define a strictly deterministic variational principle, Diff^4 degeneracy and general coordinate invariance would be achieved by restricting the consideration to 3-surfaces Y^3 at the boundary of M^4_+ and by defining Kähler function for 3-surfaces X^3 at $X^4(Y^3)$ and diffeo-related to Y^3 as $K(X^3) = K(Y^3)$. The classical non-determinism of the Kähler action however introduces complications. As a matter fact, the hierarchy of Planck constants has nice interpretation in terms of non-determinism: the space-time sheets connecting the 3-surface at the ends of CD form n conformal equivalence classes. This would correspond to the non-determinism of quantum criticality accompanied by generalized conformal invariance

7.1.3 WCW Kähler Metric From Symmetries

A complementary approach to the problem of constructing configuration space geometry is based on symmetries. The work of Dan [A43] [A43] has demonstrated that the Kähler geometry of loop spaces is unique from the existence of Riemann connection and fixed completely by the Kac Moody symmetries of the space. In 3-dimensional context one has even better reasons to expect uniqueness. The guess is that WCW is a union of symmetric spaces labelled by zero modes not appearing in the line element as differentials. The generalized conformal invariance of metrically 2-dimensional light like 3-surfaces acting as causal determinants is the corner stone of the construction. The construction works only for 4-dimensional space-time and embedding space which is a product of four-dimensional Minkowski space or its future light cone with CP_2.

The detailed formulas for the matrix elements of the Kähler metric however remain educated guesses so that this approach is not entirely satisfactory.

7.1.4 WCW Kähler Metric As Anti-commutators Of Super-Symplectic Super Noether Charges

The third approach identifies the Kähler metric of WCW as anti-commutators of WCW gamma matrices. This is not yet enough to get concrete expressions but the identification of WCW gamma matrices as Noether super-charges for super-symplectic algebra assignable to the boundary of WCW changes the situation. One also obtains a direct connection with elementary particle physics.

The super charges are linear in the mode of induced spinor field and second quantized spinor field itself, and involve the infinitesimal action of symplectic generator on the spinor field. One can fix fermionic anti-commutation relations by second quantization of the induced spinor fields (as a matter fact, here one can still consider two options). Hence one obtains explicit expressions for the matrix elements of WCW metric.

If the induced spinor fields are localized at string world sheets - as the well-definedness of em charge and number theoretic arguments suggest - one obtains an expression for the matrix elements of the metric in terms of 1-D integrals over strings connecting partonic 2-surfaces. If spinors are localized to string world sheets also in the interior of CP_2, the integral is over a closed circle and could have a representation analogous to a residue integral so that algebraic continuation to p-adic number fields might become straightforward.

The matrix elements of WCW metric are labelled by the conformal weights of spinor modes, those of symplectic vector fields for light-like CD boundaries and by labels for the irreducible representations of $SO(3)$ acting on light-cone boundary $\delta M^4_\pm = R_+ \times S^2$ and of $SU(3)$ acting in CP_2. The dependence on spinor modes and their conformal weights could not be guessed in the approach based on symmetries only. The presence of two rather than only one conformal weights distinguishes the metric from that for loop spaces [A43] and reflects the effective 2-dimensionality. The metric codes a rather scarce information about 3-surfaces. This is in accordance with the notion of finite measurement resolution. By increasing the number of partonic 2-surfaces and string world sheets the amount of information coded - measurement resolution - increases. Fermionic quantum state gives information about 3-geometry. The alternative expression for WCW metric in terms of Kähler function means analog of AdS/CFT duality: Kähler metric can be expressed either in terms of Kähler action associated with the Euclidian wormhole contacts defining Kähler function or in terms of the fermionic oscillator operators at string world sheets connecting partonic 2-surfaces.

In this chapter I will first consider the basic properties of the WCW, briefly discuss the various approaches to the geometrization of the WCW, and introduce the alternative strategies for the construction of Kähler metric based on a direct guess of Kähler function, on the group theoretical approach assuming that WCW can be regarded as a union of symmetric spaces, and on the identification of Kähler metric as anti-commutators of gamma matrices identified as Noether super charges for the symplectic algebra. After these preliminaries a definition of the Kähler function is proposed and various physical and mathematical motivations behind the proposed definition are discussed. The key feature of the Kähler action is classical non-determinism, and various implications of the classical non-determinism are discussed.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L9].

7.2 How To Generalize The Construction Of WCW Geometry To Take Into Account The Classical Non-Determinism?

If the embedding space were $H_+ = M^4_+ \times CP_2$ and if Kähler action were deterministic, the construction of WCW geometry reduces to $\delta M^4_+ \times CP_2$. Thus in this limit quantum holography principle [B14, B40] would be satisfied also in TGD framework and actually reduce to the general coordinate invariance. The classical non-determinism of Kähler action however means that this construction is not quite enough and the challenge is to generalize the construction.

7.2.1 Quantum Holography In The Sense Of Quantum GravityTheories

In string theory context quantum holography is more or less synonymous with Maldacena conjecture Maldacena which (very roughly) states that string theory in Anti-de-Sitter space AdS is equivalent with a conformal field theory at the boundary of AdS. In purely quantum gravitational context [B14] , quantum holography principle states that quantum gravitational interactions at high energy limit in AdS can be described using a topological field theory reducing to a conformal (and non-gravitational) field theory defined at the *time like* boundary of the AdS. Thus the time like boundary plays the role of a dynamical hologram containing all information about correlation functions of $d+1$ dimensional theory. This reduction also conforms with the fact that black hole entropy is proportional to the horizon area rather than the volume inside horizon.

Holography principle reduces to general coordinate invariance in TGD. If the action principle assigning space-time surface to a given 3-surface X^3 at light cone boundary were completely deterministic, four-dimensional general coordinate invariance would reduce the construction of the configuration geometry for the space of 3-surfaces in $M^4_+ \times CP_2$ to the construction of the geometry at the boundary of WCW consisting of 3-surfaces in $\delta M^4_+ \times CP_2$ (moment of big bang). Also the quantum theory would reduce to the boundary of the future light cone.

The classical non-determinism of Kähler action however implies that quantum holography in this strong form fails. This is very desirable from the point of view of both physics and consciousness theory. Classical determinism would also mean that time would be lost in TGD as it is lost in GRT. Classical non-determinism is also absolutely essential for quantum consciousness and makes possible conscious experiences with contents localized into finite time interval despite the fact that quantum jumps occur between WCW spinor fields defining what I have used to call quantum histories. Classical non-determinism makes it also possible to generalize quantum-classical correspondence in the sense that classical non-determinism at the space-time level provides correlate for quantum non-determinism. The failure of classical determinism is a difficult challenge for the construction of WCW geometry. One might however hope that the notion of quantum holography generalizes.

7.2.2 How Does The Classical Determinism Fail In TGD?

One might hope that determinism in a generalized sense might be achieved by generalizing the notion of 3-surface by allowing unions of space-like 3-surfaces with time like separations with very strong but not complete correlations between the space-like 3-surfaces. In this case the non-determinism would mean that the 3-surfaces Y^3 at light cone boundary correspond to at most enumerable number of preferred extremals $X^4(Y^3)$ of Kähler action so that one would get finite or at most enumerably infinite number of replicas of a given WCW region and the construction would still reduce to the light cone boundary.

1. This is probably quite too simplistic view. Any 4-surface which has CP_2 projection which belongs to so called Lagrange manifold of CP_2 having by definition vanishing induced Kähler form is vacuum extremal. Thus there is an infinite variety of 6-dimensional sub-manifolds of H for which all extremals of Kähler action are vacua.

2. CP_2 type vacuum extremals are different since they possess non-vanishing Kähler form and Kähler action. They are identifiable as classical counterparts of elementary particles have M^4_+ projection which is a random light like curve (this in fact gives rise to conformal invariance identifiable as counterpart of quaternion conformal invariance). Thus there are good reasons to suspect that classical non-determinism might destroy the dream about complete reduction to the light cone boundary.

3. The wormhole contacts connecting different space-time sheets together can be seen as pieces of CP_2 type extremals and one expects that the non-determinism is still there and that the metrically 2-dimensional elementary particle horizons (light like 3-surfaces of H surrounding wormhole contacts and having time-like M^4_+ projection) might be a crucial element in the understanding of quantum TGD. The non-determinism of CP_2 type extremals is absolutely crucial for the ordinary elementary particle physics. It seems that the conformal symmetries responsible for the ordinary elementary particle quantum numbers acting in these degrees of freedom do not contribute to the WCW metric line element.

The treatment of the non-determinism in a framework in which the prediction of time evolution is seen as initial value problem, seems to be difficult. Also the notion of WCW becomes a messy concept. ZEO changes the situation completely. Light-like 3-surfaces become representations of generalized Feynman diagrams and brings in the notion of finite time resolution. One obtains a direct connection with the concepts of quantum field theory with path integral with cutoff replaced with a sum over various preferred extremals with cutoff in time resolution.

7.2.3 The Notions Of Embedding Space, 3-Surface, And Configuration Space

The notions of embedding space, 3-surface (and 4-surface), and configuration space ("world of classical worlds", WCW) are central to quantum TGD. The original idea was that 3-surfaces are space-like 3-surfaces of $H = M^4 \times CP_2$ or $H = M^4_+ \times CP_2$, and WCW consists of all possible 3-surfaces in H. The basic idea was that the definition of Kähler metric of WCW assigns to each X^3 a unique space-time surface $X^4(X^3)$ allowing in this manner to realize general coordinate invariance. During years these notions have however evolved considerably. Therefore it seems better to begin directly from the recent picture.

The notion of embedding space

Two generalizations of the notion of embedding space were forced by number theoretical vision [K104, K105, K103] .

1. p-Adicization forced to generalize the notion of embedding space by gluing real and p-adic variants of embedding space together along rationals and common algebraic numbers. The generalized embedding space has a book like structure with reals and various p-adic number fields (including their algebraic extensions) representing the pages of the book.
2. With the discovery of ZEO [K121, K27] it became clear that the so called causal diamonds (CDs) interpreted as intersections $M^4_+ \cap M^4_-$ of future and past directed light-cones of $M^4 \times CP_2$ define correlates for the quantum states. The position of the "lower" tip of CD characterizes the position of CD in H. If the temporal distance between upper and lower tip of CD is quantized power of 2 multiples of CP_2 length, p-adic length scale hypothesis [K74] follows as a consequence. The upper *resp.* lower light-like boundary $\delta M^4_+ \times CP_2$ *resp.* $\delta M^4_- \times CP_2$ of CD can be regarded as the carrier of positive *resp.* negative energy part of the state. All net quantum numbers of states vanish so that everything is creatable from vacuum. Space-time surfaces assignable to zero energy states would would reside inside $CD \times CP_2$s and have their 3-D ends at the light-like boundaries of $CD \times CP_2$. Fractal structure is present in the sense that CDs can contains CDs within CDs, and measurement resolution dictates the length scale below which the sub-CDs are not visible.
3. The realization of the hierarchy of Planck constants [K38] led to a further generalization of the notion of embedding space - at least as a convenient auxiliary structure. Generalized embedding space is obtained by gluing together Cartesian products of singular coverings and factor spaces of CD and CP_2 to form a book like structure. The particles at different pages of this book behave like dark matter relative to each other. This generalization also brings in the geometric correlate for the selection of quantization axes in the sense that the geometry of the sectors of the generalized embedding space with non-standard value of Planck constant involves symmetry breaking reducing the isometries to Cartan subalgebra. Roughly speaking, each CD and CP_2 is replaced with a union of CDs and CP_2s corresponding to different choices of quantization axes so that no breaking of Poincare and color symmetries occurs at the level of entire WCW.

   It seems that the covering of embedding space is only a convenient auxiliary structure. The space-time surfaces in the n-fold covering correspond to the n conformal equivalence classes of space-time surfaces connecting fixed 3-surfaces at the ends of CD: the space-time surfaces are branched at their ends. The situation can be interpreted at the level of WCW in several ways. There is single 3-surface at both ends but by non-determinism there are n space-time branches of the space-time surface connecting them so that the Kähler action is multiplied by factor n. If one forgets the presence of the n branches completely, one can say that one has $h_{eff} = n \times h$ giving $1/\alpha_K = n/\alpha_K(n = 1)$ and scaling ofKähler action. One can also imagine that the 3-surfaces at the ends of CD are actually surfaces in the n-fold covering space consisting of n identical copies so that Kähler action is multiplied by n. One could also include the light-like partonic orbits to the 3-surface so that 3-surfaces would not have boundaries: in this case the n-fold degeneracy would come out very naturally.
4. The construction of quantum theory at partonic level brings in very important delicacies related to the Kähler gauge potential of CP_2. Kähler gauge potential must have what one

might call pure gauge parts in M^4 in order that the theory does not reduce to mere topological quantum field theory. Hence the strict Cartesian product structure $M^4 \times CP_2$ breaks down in a delicate manner. These additional gauge components -present also in CP_2- play key role in the model of anyons, charge fractionization, and quantum Hall effect [K80] .

The notion of 3-surface

The question what one exactly means with 3-surface turned out to be non-trivial.

1. The original identification of 3-surfaces was as arbitrary space-like 3-surfaces subject to Equivalence implied by General Coordinate Invariance. There was a problem related to the realization of General Coordinate Invariance since it was not at all obvious why the preferred extremal $X^4(Y^3)$ for Y^3 at $X^4(X^3)$ and Diff^4 related X^3 should satisfy $X^4(Y^3) = X^4(X^3)$.
2. Much later it became clear that light-like 3-surfaces have unique properties for serving as basic dynamical objects, in particular for realizing the General Coordinate Invariance in 4-D sense (obviously the identification resolves the above mentioned problem) and understanding the conformal symmetries of the theory. On basis of these symmetries light-like 3-surfaces can be regarded as orbits of partonic 2-surfaces so that the theory is locally 2-dimensional. It is however important to emphasize that this indeed holds true only locally. At the level of WCW metric this means that the components of the Kähler form and metric can be expressed in terms of data assignable to 2-D partonic surfaces and their 4-D tangent spaces. It is however essential that information about normal space of the 2-surface is needed.
3. At some stage came the realization that light-like 3-surfaces can have singular topology in the sense that they are analogous to Feynman diagrams. This means that the light-like 3-surfaces representing lines of Feynman diagram can be glued along their 2-D ends playing the role of vertices to form what I call generalized Feynman diagrams. The ends of lines are located at boundaries of sub-CDs. This brings in also a hierarchy of time scales: the increase of the measurement resolution means introduction of sub-CDs containing sub-Feynman diagrams. As the resolution is improved, new sub-Feynman diagrams emerge so that effective 2-D character holds true in discretized sense and in given resolution scale only.
4. A further complication relates to the hierarchy of Planck constants. At "microscopic" level this means that there number of conformal equivalence classes of space-time surfaces connecting the 3-surfaces at boundaries of CD matters and this information is coded by the value of $h_{eff} = n \times h$. One can divide WCW to sectors corresponding to different values of h_{eff} and conformal symmetry breakings connect these sectors: the transition $n_1 \rightarrow n_2$ such that n_1 divides n_2 occurs spontaneously since it reduces the quantum criticality by transforming super-generators acting as gauge symmetries to dynamical ones.

The notion of WCW

From the beginning there was a problem related to the precise definition of WCW ("world of classical worlds" (WCW)). Should one regard CH as the space of 3-surfaces of $M^4 \times CP_2$ or $M^4_+ \times CP_2$ or perhaps something more delicate.

1. For a long time I believed that the question "M^4_+ or M^4?" had been settled in favor of M^4_+ by the fact that M^4_+ has interpretation as empty Roberson-Walker cosmology. The huge conformal symmetries assignable to $\delta M^4_+ \times CP_2$ were interpreted as cosmological rather than laboratory symmetries. The work with the conceptual problems related to the notions of energy and time, and with the symmetries of quantum TGD, however led gradually to the realization that there are strong reasons for considering M^4 instead of M^4_+.
2. With the discovery of ZEO (with motivation coming from the non-determinism of Kähler action) it became clear that the so called causal diamonds (CDs) define excellent candidates for the fundamental building blocks of WCW or "world of classical worlds" (WCW). The spaces $CD \times CP_2$ regarded as subsets of H defined the sectors of WCW.
3. This framework allows to realize the huge symmetries of $\delta M^4_\pm \times CP_2$ as isometries of WCW. The gigantic symmetries associated with the $\delta M^4_\pm \times CP_2$ are also laboratory symmetries. Poincare invariance fits very elegantly with the two types of super-conformal symmetries of

TGD. The first conformal symmetry corresponds to the light-like surfaces $\delta M^4_\pm \times CP_2$ of the embedding space representing the upper and lower boundaries of CD. Second conformal symmetry corresponds to light-like 3-surface X^3_l, which can be boundaries of X^4 and light-like surfaces separating space-time regions with different signatures of the induced metric. This symmetry is identifiable as the counterpart of the Kac Moody symmetry of string models.

A rather plausible conclusion is that WCW (WCW) is a union of WCWs associated with the spaces $CD \times CP_2$. CDs can contain CDs within CDs so that a fractal like hierarchy having interpretation in terms of measurement resolution results. Since the complications due to p-adic sectors and hierarchy of Planck constants are not relevant for the basic construction, it reduces to a high degree to a study of a simple special case $\delta M^4_+ \times CP_2$.

A further piece of understanding emerged from the following observations.

1. The induced Kähler form at the partonic 2-surface X^2 - the basic dynamical object if holography is accepted- can be seen as a fundamental symplectic invariant so that the values of $\epsilon^{\alpha\beta}J_{\alpha\beta}$ at X^2 define local symplectic invariants not subject to quantum fluctuations in the sense that they would contribute to the WCW metric. Hence only induced metric corresponds to quantum fluctuating degrees of freedom at WCW level and TGD is a genuine theory of gravitation at this level.

2. WCW can be divided into slices for which the induced Kähler forms of CP_2 and $\delta M^4_\pm$ at the partonic 2-surfaces X^2 at the light-like boundaries of CDs are fixed. The symplectic group of $\delta M^4_\pm \times CP_2$ parameterizes quantum fluctuating degrees of freedom in given scale (recall the presence of hierarchy of CDs).

3. This leads to the identification of the coset space structure of the sub-WCW associated with given CD in terms of the generalized coset construction for super-symplectic and super Kac-Moody type algebras (symmetries respecting light-likeness of light-like 3-surfaces). WCW in quantum fluctuating degrees of freedom for given values of zero modes can be regarded as being obtained by dividing symplectic group with Kac-Moody group. Equivalently, the local coset space $S^2 \times CP_2$ is in question: this was one of the first ideas about WCW which I gave up as too naïve!

4. Generalized coset construction and coset space structure have very deep physical meaning since they realize Equivalence Principle at quantum level. Contrary to the original belief, this construction does not provide a realization of Equivalence Principle at quantum level. The proper realization of EP at quantum level seems to be based on the identification of classical Noether charges in Cartan algebra with the eigenvalues of their quantum counterparts assignable to Kähler-Dirac action. At classical level EP follows at GRT limit obtained by lumping many-sheeted space-time to M^4 with effective metric satisfying Einstein's equations as a reflection of the underlying Poincare invariance.

5. Now it has become clear that EP in the sense of quantum classical correspondence allows a concrete realization for the fermion lines defined by the light-like boundaries of string world sheets at light-like orbits of partonic 2-surfaces. Fermion lines are always light-like or space-like locally. Kähler-Dirac equation reducing to its algebraic counterpart with light-like 8-momentum defined by the tangent of the boundary curve. 8-D light-likeness means the possibility of massivation in M^4 sense and gravitational mass is defined in an obvious manner. The M^4-part of 8-momentum is by quantum classical correspondence equal to the 4-momentum assignable to the incoming fermion. EP generalizes also to CP_2 degrees of freedom and relates $SO(4)$ acting as symmetries of Eucldian part of 8-momentum to color $SU(3)$. $SO(4)$ can be assigned to hadrons and $SU(3)$ to quarks and gluons.

   The 8-momentum is light-like with respect to the effective metric defined by K-D gamma matrices. Is it also light-like with respect to the induced metric and proportional to the tangent vector of the fermion line? If this is not the case, the boundary curve is locally space-like in the induced metric. Could this relate to the still poorly understand question how the necessariy tachyonic ground state conformal weight of super-conformal representations needed in padic mass calculations [K59] emerges? Could it be that "empty" lines carrying no fermion number are tachyonic with respect to the induced metric?

7.2.4 The Treatment Of Non-Determinism Of Kähler Action In Zero Energy Ontology

The non-determinism of Kähler action means that the reduction of the construction of WCW geometry to the light cone boundary fails. Besides degeneracy of the preferred extrema of Kähler action, the non-determinism should manifest itself as a presence of causal determinants also other than light cone boundary.

One can imagine two kinds of causal determinants.

1. Elementary particle horizons and light-like boundaries $X_l^3 \subset X^4$ of 4-surfaces representing wormhole throats act as causal determinants for the space-time dynamics defined by Kähler action. The boundary values of this dynamics have been already considered.
2. At embedding space level causal determinants correspond to light like CD forming a fractal hierarchy of CDs within CDs. These causal determinants determine the dynamics of zero energy states having interpretation as pairs of initial and final states in standard quantum theory.

The manner to treat the classical non-determinism would be roughly following.

1. The replacement of space-like 3-surface X^3 with X_l^3 transforms initial value problem for X^3 to a boundary value problem for X_l^3. In principle one can also use the surfaces $X^3 \subset \delta CD \times CP_2$ if X_l^3 fixes $X^4(X_l^3)$ and thus X^3 uniquely. For years an important question was whether both X^3 and X_l^3 contribute separately to WCW geometry or whether they provide descriptions, which are in some sense dual.
2. Only Super-Kac-Moody type conformal algebra makes sense in the interior of X_l^3. In the 2-D intersections of X_l^3 with the boundary of causal diamond (CD) defined as intersection of future and past directed light-cones super-symplectic algebra makes sense. This implies effective two-dimensionality which is broken by the non-determinism represented using the hierarchy of CDs meaning that the data from these 2-D surfaces and their normal spaces at boundaries of CDs in various scales determine the WCW metric.
3. An important question has been whether Kac-Moody and super-symplectic algebras provide descriptions which are dual in some sense. At the level of Super-Virasoro algebras duality seems to be satisfied in the sense of generalized coset construction meaning that the differences of Super Virasoro generators of super-symplectic and super Kac-Moody algebras annihilate physical states. Among other things this means that four-momenta assignable to the two Super Virasoro representations are identical. T he interpretation is in terms of a generalization of Equivalence Principle [K121, K27] . This gives also a justification for p-adic thermodynamics applying only to Super Kac-Moody algebra.
4. Light-like 3-surfaces can be regarded also as generalized Feynman diagrams. The finite length resolution mean means also a cutoff in the number of generalized Feynman diagrams and this number remains always finite if the light-like 3-surfaces identifiable as maxima of Kähler function correspond to the diagrams. The finiteness of this number is also essential for number theoretic universality since it guarantees that the elements of M-matrix are algebraic numbers if momenta and other quantum numbers have this property. The introduction of new sub-CDs means also introduction of zero energy states in corresponding time scale.
5. The notion of finite measurement resolution expressed in terms of hierarchy of CDs within CDs is important for the treatment of classical non-determinism. In a given resolution the non-determinism of Kähler action remains invisible below the time scale assigned to the smallest CDs. One could also say that complete non-determinism characterized in terms path integral with cutoff is replaced in TGD framework with the partial failure of classical non-determinism leading to generalized Feynman diagrams. This gives rise to discrete coupling constant evolution and avoids the mathematical ill-definedness and infinities plaguing path integral formalism since the functional integral over 3-surfaces is well defined.

7.2.5 Category Theory And WCW Geometry

Due the effects caused by the classical non-determinism even classical TGD universes are very far from simple Cartesian clockworks, and the understanding of the general structure of WCW is a formidable challenge. Category theory is a branch of mathematics which is basically a theory about universal aspects of mathematical structures. Thus category theoretical thinking might help in disentangling the complexities of WCW geometry and the basic ideas of category theory are discussed in this spirit and as an innocent layman. It indeed turns out that the approach makes highly non-trivial predictions.

In ZEO the effects of non-determinism are taken into account in terms of causal diamonds forming a hierarchical fractal structure. One must allow also the unions of CDs, CDs within CDs, and probably also overlapping of CDs, and there are good reasons to expert that CDs and corresponding algebraic structures could define categories. If one does not allow overlapping CDs then set theoretic inclusion map defines a natural arrow. If one allows both unions and intersections then CDs would form a structure analogous to the set of open sets used in set theoretic topology. One could indeed see CDs (or rather their Cartesian products with CP_2) as analogs of open sets in Minkowskian signature.

So called ribbon categories seem to be tailor made for the formulation of quantum TGD and allow to build bridge to topological and conformal field theories. This discussion based on standard ontology. In [K20] rather detailed category theoretical constructions are discussed. Important role is played by the notion of operad operad,operads : this structure can be assigned with both generalized Feynman diagrams and with the hierarchy of symplectic fusion algebras realizing symplectic analogs of the fusion rules of conformal field theories.

7.3 Constraints On WCW Geometry

The constraints on WCW ("world of classical worlds") geometry result both from the infinite dimension of WCW and from physically motivated symmetry requirements. There are three basic physical requirements on the WCW geometry: namely four-dimensional Diff invariance, Kähler property and the decomposition of WCW into a union $\cup_i G/H_i$ of symmetric spaces G/H_i, each coset space allowing G-invariant metric such that G is subgroup of some "universal group" having natural action on 3-surfaces. Together with the infinite dimensionality of WCW these requirements pose extremely strong constraints on WCW geometry. In the following these requirements are considered in more detail.

7.3.1 WCW

The first naïve view about WCW of TGD was that it consists of all 3-surfaces of $M^4_+ \times CP_2$ containing sets of

1. surfaces with all possible manifold topologies and arbitrary numbers of components (N-particle sectors)
2. singular surfaces topologically intermediate between two manifold topologies (see **Fig. ??**).

The symbol $C(H)$ will be used to denote the set of 3-surfaces $X^3 \subset H$. It should be emphasized that surfaces related by $Diff^3$ transformations will be regarded as different surfaces in the sequel.

These surfaces form a connected(!) space since it is possible to glue various N-particle sectors to each other along their boundaries consisting of sets of singular surfaces topologically intermediate between corresponding manifold topologies. The connectedness of the WCW is a necessary prerequisite for the description of topology changing particle reactions as continuous paths in WCW (see **Fig. 7.2**).

7.3.2 Diff^4 Invariance And Diff^4 Degeneracy

Diff^4 plays fundamental role as the gauge group of General Relativity. In string models $Diff^2$ invariance ($Diff^2$ acts on the orbit of the string) plays central role in making possible the elimination of the time like and longitudinal vibrational degrees of freedom of string. Also in the present case the elimination of the tachyons (time like oscillatory modes of 3-surface) is a physical necessity and Diff^4 invariance provides an obvious manner to do the job.

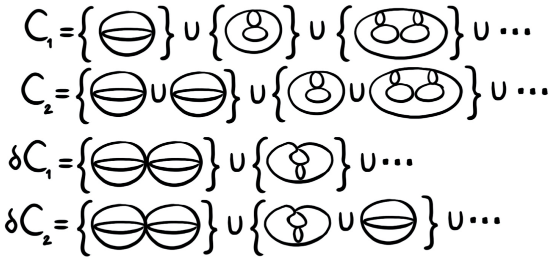

Figure 7.1: Structure of WCW: two-dimensional visualization

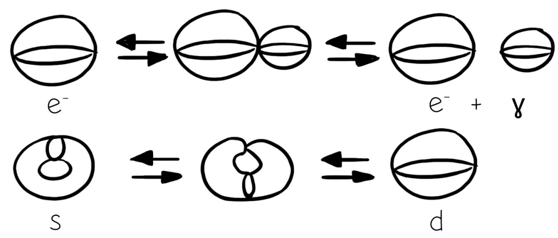

Figure 7.2: Two-dimensional visualization of topological description of particle reactions. a) Generalization of stringy diagram describing particle decay: 4-surface is smooth manifold and vertex a non-unique singular 3-manifold, b) Topological description of particle decay in terms of a singular 4-manifold but smooth and unique 3-manifold at vertex. c) Topological origin of Cabibbo mixing.

In the standard functional integral formulation the realization of Diff^4 invariance is an easy task at the formal level. The problem is however that the path integral over four-surfaces is plagued by divergences and doesn't make sense. In the present case the WCW consists of 3-surfaces and only $Diff^3$ emerges automatically as the group of re-parameterizations of 3-surface. Obviously one should somehow define the action of Diff^4 in the space of 3-surfaces. Whatever the action of Diff^4 is it must leave the WCW metric invariant. Furthermore, the elimination of tachyons is expected to be possible only provided the time like deformations of the 3-surface correspond to zero norm vector fields of WCW so that 3-surface and its Diff^4 image have zero distance. The conclusion is that WCW metric should be both Diff^4 invariant and Diff^4 degenerate.

The problem is how to define the action of Diff^4 in $C(H)$. Obviously the only manner to achieve Diff^4 invariance is to require that the very definition of the WCW metric somehow associates a unique space-time surface to a given 3-surface for Diff^4 to act on! The obvious physical interpretation of this space time surface is as "classical space time" so that "Classical Physics" would be contained in WCW geometry. It is this requirement, which has turned out to be decisive concerning the understanding of the configuration space geometry. Amusingly enough, the historical development was not this: the definition of Diff^4 degenerate Kähler metric was found by a guess and only later it was realized that Diff^4 invariance and degeneracy could have been postulated from beginning!

7.3.3 Decomposition Of WCW Into A Union Of Symmetric Spaces G/H

The extremely beautiful theory of finite-dimensional symmetric spaces constructed by Elie Cartan suggests that WCW should possess a decomposition into a union of coset spaces $CH = \cup_i G/H_i$

such that the metric inside each coset space G/H_i is left invariant under the infinite dimensional isometry group G. The metric equivalence of surfaces inside each coset space G/H_i does not mean that 3-surfaces inside G/H_i are physically equivalent. The reason is that the vacuum functional is exponent of Kähler action which is not isometry invariant so that the 3-surfaces, which correspond to maxima of Kähler function for a given orbit, are in a preferred position physically. For instance, one can calculate functional integral around this maximum perturbatively. The sum of over i means actually integration over the zero modes of the metric (zero modes correspond to coordinates not appearing as coordinate differentials in the metric tensor).

The coset space G/H is a symmetric space only under very special Lie-algebraic conditions. Denoting the Cartan decomposition of the Lie-algebra g of G to the direct sum of H Lie-algebra h and its complement t by $g = h \oplus t$, one has

$$[h,h] \subset h \ , \quad [h,t] \subset t \ , \quad [t,t] \subset h \ .$$

This decomposition turn out to play crucial role in guaranteeing that G indeed acts as isometries and that the metric is Ricci flat.

The four-dimensional $Diff$ invariance indeed suggests to a beautiful solution of the problem of identifying G. The point is that any 3-surface X^3 is $Diff^4$ equivalent to the intersection of $X^4(X^3)$ with the light cone boundary. This in turn implies that 3-surfaces in the space $\delta H = \delta M^4_+ \times CP_2$ should be all what is needed to construct WCW geometry. The group G can be identified as some subgroup of diffeomorphisms of δH and H_i diffeomorphisms of the 3-surface X^3. Since G preserves topology, WCW must decompose into union $\cup_i G/H_i$, where i labels 3-topologies and various zero modes of the metric. For instance, the elements of the Lie-algebra of G invariant under WCW complexification correspond to zero modes.

The reduction to the light cone boundary, identifiable as the moment of big bang, looks perhaps odd at first. In fact, it turns out that the classical non-determinism of Kähler action forces does not allow the complete reduction to the light cone boundary: physically this is a highly desirable implication but means a considerable mathematical challenge.

Kähler property implies that the tangent space of the configuration space allows complexification and that there exists a covariantly constant two-form J_{kl}, which can be regarded as a representation of the imaginary unit in the tangent space of the WCW:

$$J_k^{\ r} J_{rl} = -G_{kl} \ . \tag{7.3.1}$$

There are several physical and mathematical reasons suggesting that WCW metric should possess Kähler property in some generalized sense.

1. Kähler property turns out to be a necessary prerequisite for defining divergence free WCW integration. We will leave the demonstration of this fact later although the argument as such is completely general.
2. Kähler property very probably implies an infinite-dimensional isometry Freed shows that loop group allows only single Kähler metric with well Riemann connection and this metric allows local G as its isometries!

   To see this consider the construction of Riemannian connection for $Map(X^3, H)$. The defining formula for the connection is given by the expression

$$\begin{aligned} 2(\nabla_X Y, Z) &= X(Y,Z) + Y(Z,X) - Z(X,Y) \\ &+ ([X,Y],Z) + ([Z,X],Y) - ([Y,Z],X) \end{aligned} \tag{7.3.2}$$

   X, Y, Z are smooth vector fields in $Map(X^3, G)$. This formula defines $\nabla_X Y$ uniquely provided the tangent space of Map is complete with respect to Riemann metric. In the finite-dimensional case completeness means that the inverse of the covariant metric tensor exists so that one can solve the components of connection from the conditions stating the covariant constancy of the metric. In the case of the loop spaces with Kähler metric this is however not the case.

Now the symmetry comes into the game: if X, Y, Z are left (local gauge) invariant vector fields defined by the Lie-algebra of local G then the first three terms drop away since the scalar products of left invariant vector fields are constants. The expression for the covariant derivative is given by

$$\nabla_X Y = (Ad_X Y - Ad_X^* Y - Ad_Y^* X)/2 \quad (7.3.3)$$

where Ad_X^* is the adjoint of Ad_X with respect to the metric of the loop space.

At this point it is important to realize that Freed's argument does not force the isometry group of WCW to be $Map(X^3, M^4 \times SU(3))$! Any symmetry group, whose Lie algebra is complete with respect to the WCW metric (in the sense that any tangent space vector is expressible as superposition of isometry generators modulo a zero norm tangent vector) is an acceptable alternative.

The Kähler property of the metric is quite essential in one-dimensional case in that it leads to the requirement of left invariance as a mathematical consistency condition and we expect that dimension three makes no exception in this respect. In 3-dimensional case the degeneracy of the metric turns out to be even larger than in 1-dimensional case due to the four-dimensional Diff degeneracy. So we expect that the metric ought to possess some infinite-dimensional isometry group and that the above formula generalizes also to the 3-dimensional case and to the case of local coset space. Note that in M^4 degrees of freedom $Map(X^3, M^4)$ invariance would imply the flatness of the metric in M^4 degrees of freedom.

The physical implications of the above purely mathematical conjecture should not be underestimated. For example, one natural looking manner to construct physical theory would be based on the idea that WCW geometry is dynamical and this approach is followed in the attempts to construct string theories [B11] . Various physical considerations (in particular the need to obtain oscillator operator algebra) seem to imply that WCW geometry is necessarily Kähler. The above result however states that WCW Kähler geometry cannot be dynamical quantity and is dictated solely by the requirement of internal consistency. This result is extremely nice since it has been already found that the definition of the WCW metric must somehow associate a unique classical space time and "classical physics" to a given 3-surface: uniqueness of the geometry implies the uniqueness of the "classical physics".

3. The choice of the embedding space becomes highly unique. In fact, the requirement that WCW is not only symmetric space but also (contact) Kähler manifold inheriting its (degenerate) Kähler structure from the embedding space suggests that spaces, which are products of four-dimensional Minkowski space with complex projective spaces CP_n, are perhaps the only possible candidates for H. The reason for the unique position of the four-dimensional Minkowski space turns out to be that the boundary of the light cone of D-dimensional Minkowski space is metrically a sphere S^{D-2} despite its topological dimension $D-1$: for $D = 4$ one obtains two-sphere allowing Kähler structure and infinite parameter group of conformal symmetries!
4. It seems possible to understand the basic mathematical structures appearing in string model in terms of the Kähler geometry rather nicely.
   (a) The projective representations of the infinite-dimensional isometry group (not necessarily Map!) correspond to the ordinary representations of the corresponding centrally extended group [A57]. The representations of Kac Moody group Schwartz,Green and WCW approach would explain their occurrence, not as a result of some quantization procedure, but as a consequence of symmetry of the underlying geometric structure.
   (b) The bosonic oscillator operators of string models would correspond to centrally extended Lie-algebra generators of the isometry group acting on spinor fields of the WCW.
   (c) The "fermionic" fields (Ramond fields, Schwartz,Green) should correspond to gamma matrices of the WCW. Fermionic oscillator operators would correspond simply to contractions of isometry generators j_A^k with complexified gamma matrices of WCW

$$\begin{aligned} \Gamma_A^{\pm} &= j_A^k \Gamma_k^{\pm} \\ \Gamma_k^{\pm} &= (\Gamma^k \pm J^k_{\ l}\Gamma^l)/\sqrt{2} \end{aligned} \quad (7.3.4)$$

(J^k_l is the Kähler form of WCW) and would create various spin excitations of WCW spinor field. $\Gamma^{\pm}_k$ are the complexified gamma matrices, complexification made possible by the Kähler structure of the WCW.

This suggests that some generalization of the so called Super Kac Moody algebra of string models [B39, B35] should be regarded as a spectrum generating algebra for the solutions of field equations in configuration space.

Although the Kähler structure seems to be physically well motivated there is a rather heavy counter argument against the whole idea. Kähler structure necessitates complex structure in the tangent space of WCW. In CP_2 degrees of freedom no obvious problems of principle are expected: WCW should inherit in some sense the complex structure of CP_2.

In Minkowski degrees of freedom the signature of the Minkowski metric seems to pose a serious obstacle for complexification: somehow one should get rid of two degrees of freedom so that only two Euclidian degrees of freedom remain. An analogous difficulty is encountered in quantum field theories: only two of the four possible polarizations of gauge boson correspond to physical degrees of freedom: mathematically the wrong polarizations correspond to zero norm states and transverse Hilbert space with Euclidian metric. Also in string model analogous situation occurs: in case of D-dimensional Minkowski space only $D-2$ transversal degrees of freedom are physical. The solution to the problem seems therefore obvious: WCW metric must be degenerate so that each vibrational mode spans effectively a 2-dimensional Euclidian plane allowing complexification.

It will be found that the definition of Kähler function to be proposed indeed provides a solution to this problem and also to the problems listed before.

1. The definition of the metric doesn't differentiate between 1- and N-particle sectors, avoids spin statistics difficulty and has the physically appealing property that one can associate to each 3-surface a unique classical space time: classical physics is described by the geometry of WCW! And the geometry of WCW is determined uniquely by the requirement of mathematical consistency.

2. Complexification is possible only provided the dimension of the Minkowski space equals to four.

3. It is possible to identify a unique candidate for the necessary infinite-dimensional isometry group G. G is subgroup of the diffeomorphism group of $\delta M^4_\pm \times CP_2$. Essential role is played by the fact that the boundary of the four-dimensional light cone, which, despite being topologically 3-dimensional, is metrically two-dimensional(!) Euclidian sphere, and therefore allows infinite-parameter groups of isometries as well as conformal and symplectic symmetries and also Kähler structure unlike the higher-dimensional light cone boundaries. Therefore WCW metric is Kähler only in the case of four-dimensional Minkowski space and allows symplectic $U(1)$ central extension without conflict with the no-go theorems about higher dimensional central extensions.

   The study of the vacuum degeneracy of Kähler function defined by Kähler action forces to conclude that the isometry group must consist of the symplectic transformations of $\delta H = \delta M^4_\pm \times CP_2$. The corresponding Lie algebra can be regarded as a loop algebra associated with the symplectic group of $S^2 \times CP_2$, where S^2 is $r_M = constant$ sphere of light cone boundary. Thus the finite-dimensional group G defining loop group in case of string models extends to an infinite-dimensional group in TGD context. This group is a real monster! The radial Virasoro localized with respect to $S^2 \times CP_2$ defines naturally complexification for both G and H. The general form of the Kähler metric deduced on basis of this symmetry has same qualitative properties as that deduced from Kähler function identified as the absolute minimum of Kähler action. Also the zero modes, among them isometry invariants, can be identified.

4. The construction of the WCW spinor structure is based on the identification of the WCW gamma matrices as linear superpositions of the oscillator operators associated with the induced spinor fields. The extension of the symplectic invariance to super symplectic invariance fixes the anti-commutation relations of the induced spinor fields, and WCW gamma matrices correspond directly to the super generators. Physics as number theory vision suggests strongly that WCW geometry exists for 8-dimensional embedding space only and that the choice $M^4_+ \times CP_2$ for the embedding space is the only possible one.

7.4 Kähler Function

There are two approaches to the construction of WCW geometry: a direct physics based guess of the Kähler function and a group theoretic approach based on the hypothesis that CH can be regarded as a union of symmetric spaces. The rest of this chapter is devoted to the first approach.

7.4.1 Definition Of Kähler Function

Kähler metric in terms of Kähler function

Quite generally, Kähler function K defines Kähler metric in complex coordinates via the following formula

$$J_{k\bar{l}} = ig_{k\bar{l}} = i\partial_k\partial_{\bar{l}}K \ . \tag{7.4.1}$$

Kähler function is defined only modulo a real part of holomorphic function so that one has the gauge symmetry

$$K \rightarrow K + f + \overline{f} \ . \tag{7.4.2}$$

Let X^3 be a given 3-surface and let X^4 be any four-surface containing X^3 as a sub-manifold: $X^4 \supset X^3$. The 4-surface X^4 possesses in general boundary. If the 3-surface X^3 has nonempty boundary δX^3 then the boundary of X^3 belongs to the boundary of X^4: $\delta X^3 \subset \delta X^4$.

Induced Kähler form and its physical interpretation

Induced Kähler form defines a Maxwell field and it is important to characterize precisely its relationship to the gauge fields as they are defined in gauge theories. Kähler form J is related to the corresponding Maxwell field F via the formula

$$J = xF \ , \ x = \frac{g_K}{\hbar} \ . \tag{7.4.3}$$

Similar relationship holds true also for the other induced gauge fields. The inverse proportionality of J to $\hbar$ does not matter in the ordinary gauge theory context where one routinely choses units by putting $\hbar = 1$ but becomes very important when one considers a hierarchy of Planck constants [K38].

Unless one has $J = (g_K/\hbar_0)$, where $\hbar_0$ corresponds to the ordinary value of Planck constant, $\alpha_K = g_K^2/4\pi\hbar$ together the large Planck constant means weaker interactions and convergence of the functional integral defined by the exponent of Kähler function and one can argue that the convergence of the functional integral is what forces the hierarchy of Planck constants. This is in accordance with the vision that Mother Nature likes theoreticians and takes care that the perturbation theory works by making a phase transition increasing the value of the Planck constant in the situation when perturbation theory fails. This leads to a replacement of the M^4 (or more precisely, causal diamond CD) and CP_2 factors of the embedding space ($CD \times CP_2$) with its $r = h_{eff}/h$-fold singular covering (one can consider also singular factor spaces). If the components of the space-time surfaces at the sheets of the covering are identical, one can interpret r-fold value of Kähler action as a sum of r identical contributions from the sheets of the covering with ordinary value of Planck constant and forget the presence of the covering. Physical states are however different even in the case that one assumes that sheets carry identical quantum states and anyonic phase could correspond to this kind of phase [K80].

Kähler action

One can associate to Kähler form Maxwell action and also Chern-Simons anomaly term proportional to $\int_{X^4} J \wedge J$ in well known manner. Chern Simons term is purely topological term and well defined for orientable 4-manifolds, only. Since there is no deep reason for excluding non-orientable space-time surfaces it seems reasonable to drop Chern Simons term from consideration. Therefore Kähler action $S_K(X^4)$ can be defined as

$$S_K(X^4) = k_1 \int_{X^4;X^3\subset X^4} J\wedge(*J) \ . \tag{7.4.4}$$

The sign of the square root of the metric determinant, appearing implicitly in the formula, is defined in such a way that the action density is negative for the Euclidian signature of the induced metric and such that for a Minkowskian signature of the induced metric Kähler electric field gives a negative contribution to the action density.

The notational convention

$$k_1 \equiv \frac{1}{16\pi\alpha_K} \ , \tag{7.4.5}$$

where α_K will be referred as Kähler coupling strength will be used in the sequel. If the preferred extremals minimize/maximize [K105] the absolute value of the action in each region where action density has a definite sign, the value of α_K can depend on space-time sheet.

Kähler function

One can define the Kähler function in the following manner. Consider first the case $H = M^4_+\times CP_2$ and neglect for a moment the non-determinism of Kähler action. Let X^3 be a 3-surface at the light-cone boundary $\delta M^4_+\times CP_2$. Define the value $K(X^3)$ of Kähler function K as the value of the Kähler action for some preferred extremal in the set of four-surfaces containing X^3 as a sub-manifold:

$$K(X^3) = K(X^4_{pref}) \ , \ X^4_{pref}\subset \{X^4|X^3\subset X^4\} \ . \tag{7.4.6}$$

The most plausible identification of preferred extremals is in terms of quantum criticality in the sense that the preferred extremals allow an infinite number of deformations for which the second variation of Kähler action vanishes. Combined with the weak form of electric-magnetic duality forcing appearance of Kähler coupling strength in the boundary conditions at partonic 2-surfaces this condition might be enough to fix preferred extremals completely.

The precise formulation of Quantum TGD has developed rather slowly. Only quite recently-33 years after the birth of TGD - I have been forced to reconsider the question whether the precise identification of Kähler function. Should Kähler function actually correspond to the Kähler action for the space-time regions with Euclidian signature having interpretation as generalized Feynman graphs? If so what would be the interpretation for the Minkowskian contribution?

1. If one accepts just the formal definition for the square root of the metric determinant, Minkowskian regions would naturally give an imaginary contribution to the exponent defining the vacuum functional. The presence of the phase factor would give a close connection with the path integral approach of quantum field theories and the exponent of Kähler function would make the functional integral well-defined.
2. The weak form of electric magnetic duality would reduce the contributions to Chern-Simons terms from opposite sides of wormhole throats with degenerate four-metric with a constraint term guaranteeing the duality.

The motivation for this reconsideration came from the applications of ideas of Floer homology to TGD framework [K64]: the Minkowskian contribution to Kähler action for preferred extremals would define Morse function providing information about WCW homology. Both Kähler and Morse would find place in TGD based world order.

One of the nasty questions about the interpretation of Kähler action relates to the square root of the metric determinant. If one proceeds completely straightforwardly, the only reason conclusion is that the square root is imaginary in Minkowskian space-time regions so that Kähler action would be complex. The Euclidian contribution would have a natural interpretation as positive definite

Kähler function but how should one interpret the imaginary Minkowskian contribution? Certainly the path integral approach to quantum field theories supports its presence. For some mysterious reason I was able to forget this nasty question and serious consideration of the obvious answer to it. Only when I worked between possibile connections between TGD and Floer homology [K64] I realized that the Minkowskian contribution is an excellent candidate for Morse function whose critical points give information about WCW homology. This would fit nicely with the vision about TGD as almost topological QFT.

Euclidian regions would guarantee the convergence of the functional integral and one would have a mathematically well-defined theory. Minkowskian contribution would give the quantal interference effects and stationary phase approximation. The analog of Floer homology would represent quantum superpositions of critical points identifiable as ground states defined by the extrema of Kähler action for Minkowskian regions. Perturbative approach to quantum TGD would rely on functional integrals around the extrema of Kähler function. One would have maxima also for the Kähler function but only in the zero modes not contributing to the WCW metric.

There is a further question related to almost topological QFT character of TGD. Should one assume that the reduction to Chern-Simons terms occurs for the preferred extremals in *both* Minkowskian and Euclidian regions or only in Minkowskian regions?

1. All arguments for this have been represented for Minkowskian regions [K121] involve local light-like momentum direction which does not make sense in the Euclidian regions. This does not however kill the argument: one can have non-trivial solutions of Laplacian equation in the region of CP_2 bounded by wormhole throats: for CP_2 itself only covariantly constant right-handed neutrino represents this kind of solution and at the same time supersymmetry. In the general case solutions of Laplacian represent broken super-symmetries and should be in one-one correspondences with the solutions of the Kähler-Dirac equation. The interpretation for the counterparts of momentum and polarization would be in terms of classical representation of color quantum numbers.
2. If the reduction occurs in Euclidian regions, it gives in the case of CP_2 two 3-D terms corresponding to two 3-D gluing regions for three coordinate patches needed to define coordinates and spinor connection for CP_2 so that one would have two Chern-Simons terms. I have earlier claimed that without any other contributions the first term would be identical with that from Minkowskian region apart from imaginary unit and different coefficient. This statement is wrong since the space-like parts of the corresponding 3-surfaces are discjoint for Euclidian and Minkowskian regions.
3. There is also an argument stating that Dirac determinant for Chern-Simons Dirac action equals to Kähler function, which would be lost if Euclidian regions would not obey holography. The argument obviously generalizes and applies to both Morse and Kähler function which are definitely not proportional to each other.

CP breaking and ground state degeneracy

The Minkowskian contribution of Kähler action is imaginary due to the negativity of the metric determinant and gives a phase factor to vacuum functional reducing to Chern-Simons terms at wormhole throats. Ground state degeneracy due to the possibility of having both signs for Minkowskian contribution to the exponent of vacuum functional provides a general view about the description of CP breaking in TGD framework.

1. In TGD framework path integral is replaced by inner product involving integral over WCV. The vacuum functional and its conjugate are associated with the states in the inner product so that the phases of vacuum functionals cancel if only one sign for the phase is allowed. Minkowskian contribution would have no physical significance. This of course cannot be the case. The ground state is actually degenerate corresponding to the phase factor and its complex conjugate since $\sqrt{g}$ can have two signs in Minkowskian regions. Therefore the inner products between states associated with the two ground states define 2×2 matrix and non-diagonal elements contain interference terms due to the presence of the phase factor. At the limit of full CP_2 type vacuum extremal the two ground states would reduce to each other and the determinant of the matrix would vanish.

2. A small mixing of the two ground states would give rise to CP breaking and the first principle description of CP breaking in systems like $K - \overline{K}$ and of CKM matrix should reduce to this mixing. K^0 mesons would be CP even and odd states in the first approximation and correspond to the sum and difference of the ground states. Small mixing would be present having exponential sensitivity to the actions of CP_2 type extremals representing wormhole throats. This might allow to understand qualitatively why the mixing is about 50 times larger than expected for B^0 mesons.

3. There is a strong temptation to assign the two ground states with two possible arrows of geometric time. At the level of M-matrix the two arrows would correspond to state preparation at either upper or lower boundary of CD. Do long- and shortlived neutral K mesons correspond to almost fifty-fifty orthogonal superpositions for the two arrow of geometric time or almost completely to a fixed arrow of time induced by environment? Is the dominant part of the arrow same for both or is it opposite for long and short-lived neutral measons? Different lifetimes would suggest that the arrow must be the same and apart from small leakage that induced by environment. CP breaking would be induced by the fact that CP is performed only K^0 but not for the environment in the construction of states. One can probably imagine also alternative interpretations.

7.4.2 The Values Of The Kähler Coupling Strength?

Since the vacuum functional of the theory turns out to be essentially the exponent $exp(K)$ of the Kähler function, the dynamics depends on the normalization of the Kähler function. Since the Theory of Everything should be unique it would be highly desirable to find arguments fixing the normalization or equivalently the possible values of the Kähler coupling strength α_K.

Quantization of α_K follow from Dirac quantization in WCW?

The quantization of Kähler form of WCW could result in the following manner. It will be found that Abelian extension of the isometry group results by coupling spinors of WCW to a multiple of Kähler potential. This means that Kähler potential plays role of gauge connection so that Kähler form must be integer valued by Dirac quantization condition for magnetic charge. So, if Kähler form is co-homologically nontrivial the value of α_K is quantized.

Quantization from criticality of TGD Universe?

Mathematically α_K is analogous to temperature and this suggests that α_K is analogous to critical temperature and therefore quantized. This analogy suggests also a physical motivation for the unique value or value spectrum of α_K. Below the critical temperature critical systems suffer something analogous to spontaneous magnetization. At the critical point critical systems are characterized by long range correlations and arbitrarily large volumes of magnetized and non-magnetized phases are present. Spontaneous magnetization might correspond to the generation of Kähler magnetic fields: the most probable 3-surfaces are Kähler magnetized for subcritical values of α_K. At the critical values of α_K the most probable 3-surfaces contain regions dominated by either Kähler electric and or Kähler magnetic fields: by the compactness of CP_2 these regions have in general outer boundaries.

This suggests that 3-space has hierarchical, fractal like structure: 3-surfaces with all sizes (and with outer boundaries) are possible and they have suffered topological condensation on each other. Therefore the critical value of α_K allows the richest possible topological structure for the most probable 3-space. In fact, this hierarchical structure is in accordance with the basic ideas about renormalization group invariance. This hypothesis has highly nontrivial consequences even at the level of ordinary condensed matter physics.

Unfortunately, the exact definition of renormalization group concept is not at all obvious. There is however a much more general but more or less equivalent manner to formulate the condition fixing the value of α_K. Vacuum functional $exp(K)$ is analogous to the exponent $exp(-H/T)$ appearing in the definition of the partition function of a statistical system and S-matrix elements and other interesting physical quantities are integrals of type $\langle O\rangle = \int exp(K)O\sqrt{G}dV$ and therefore analogous to the thermal averages of various observables. α_K is completely analogous to temperature. The critical points of a statistical system correspond to critical temperatures T_c for which the partition function is non-analytic function of $T - T_c$ and according RGE hypothesis critical systems correspond to fixed points of renormalization group evolution. Therefore, a mathematically more precise manner to fix the value of α_K is to require that some integrals of type $\langle O\rangle$ (not necessary S-matrix elements) become non-analytic at $1/\alpha_K - 1/\alpha_K^c$.

Renormalization group invariance is closely related with criticality. The self duality of the Kähler form and Weyl tensor of CP_2 indeed suggest RG invariance. The point is that in $N = 1$ super-symmetric field theories duality transformation relates the strong coupling limit for ordinary particles with the weak coupling limit for magnetic monopoles and vice versa. If the theory is self dual these limits must be identical so that action and coupling strength must be RG invariant quantities. The geometric realization of the duality transformation is easy to guess in the standard complex coordinates ξ_1, ξ_2 of CP_2 (see Appendix of the book). In these coordinates the metric and Kähler form are invariant under the permutation $\xi_1 \leftrightarrow \xi_2$ having Jacobian -1.

Consistency requires that the fundamental particles of the theory are equivalent with magnetic monopoles. The deformations of so called CP_2 type vacuum extremals indeed serve as building bricks of a elementary particles. The vacuum extremals are are isometric embeddings of CP_2 and can be regarded as monopoles. Elementary particle corresponds to a pair of wormhole contacts and monopole flux runs between the throats of of the two contacts at the two space-time sheets and through the contacts between space-time sheets. The magnetic flux however flows in internal degrees of freedom (possible by nontrivial homology of CP_2) so that no long range $1/r^2$ magnetic field is created. The magnetic contribution to Kähler action is positive and this suggests that ordinary magnetic monopoles are not stable, since they do not minimize Kähler action: a cautious conclusion in accordance with the experimental evidence is that TGD does not predict magnetic monopoles. It must be emphasized that the prediction of monopoles of practically all gauge theories and string theories and follows from the existence of a conserved electromagnetic charge.

Does α_K have spectrum?

The assumption about single critical value of α_K is probably too strong.

1. The hierarchy of Planck constants which would result from non-determinism of Kähler action implying n conformal equivalences of space-time surface connecting 3-surfaces at the boundaries of causal diamond CD would predict effective spectrum of α_K as $\alpha_K = g_K^2/4\pi\hbar_{eff}$, $\hbar_{eff}/h = n$. The analogs of critical temperatures would have accumulation point at zero temperature.
2. p-Adic length scale hierarchy together with the immense vacuum degeneracy of the Kähler action leads to ask whether different p-adic length scales correspond to different critical values of α_K, and that ordinary coupling constant evolution is replaced by a piecewise constant evolution induced by that for α_K.

7.4.3 What Conditions Characterize The Preferred Extremals?

The basic vision forced by the generalization of General Coordinate Invariance has been that space-time surfaces correspond to preferred extremals $X^4(X^3)$ of Kähler action and are thus analogous to Bohr orbits. Kähler function $K(X^3)$ defining the Kähler geometry of the world of classical worlds would correspond to the Kähler action for the preferred extremal. The precise identification of the preferred extremals actually has however remained open.

In positive energy ontology space-time surfaces should be analogous to Bohr orbits in order to make possible possible realization of general coordinate invariance. The first guess was that absolute minimization of Kähler action might be the principle selecting preferred extremals. One can criticize the assumption that extremals correspond to the absolute minima of Kähler action for entire space-time surface as too strong since the Kähler action from Minkowskian regions is proportional to imaginary unit and corresponds to ordinary QFT action defining a phase factor of vacuum functional. Absolute minimization could however make sense for Euclidian space-time regions defining the lines of generalized Feynman diagras, where Kähler action has definite sign. Kähler function is indeed the Kähler action for these regions. Furthermore, the notion of absolute minimization does not make sense in p-adic context unless one manages to reduce it to purely algebraic conditions.

Is preferred extremal property needed at all in ZEO?

It is good to start with a critical question. Could it be that the notion of preferred extremal might be un-necessary in ZEO (ZEO)? The reason is that 3-surfaces are now pairs of 3-surfaces at boundaries of causal diamonds and for deterministic dynamics the space-time surface connecting them is unique.

Now the action principle is non-deterministic but the non-determinism would give rise to additional discrete dynamical degrees of freedom naturally assignable to the hierarchy of Planck constants $h_{eff} = n \times h$, n the number of space-time surface with same fixed ends at boundaries of CD and same Kähler action and same conserved quantities. One must be however cautious: this leaves the possibility that there is a gauge symmetry present so that the n sheets correspond to gauge equivalence classes of sheets. Conformal gauge invariance is associated with 2-D criticality and is expected to be present also now. and this is the recent view.

One can of course ask whether one can assume that the pairs of 3-surfaces at the ends of CD are totally un-correlated - this the starting point in ZEO. If this assumption is not made then preferred extremal property would make sense also in ZEO and imply additional correlation between the members of these pairs. This kind of correlations might be present and correspond to the Bohr orbit property, space-time correlate for quantum states. This kind of correlates are also expected as space-time counterpart for the correlations between initial and final state in quantum dynamics. This indeed seems to be the correct conclusion.

How to identify preferred extremals?

What is needed is the association of a unique space-time surface to a given 3-surface defined as union of 3-surfaces at opposite boundaries of CD. One can imagine many ways to achieve this. "Unique" is too much to demand: for the proposal unique space-time surface is replaced with finite number of conformal gauge equivalence classes of space-time surfaces. In any case, it is better to talk just about preferred extremals of Kähler action and accept as the fact that there are several proposals for what this notion could mean.

1. For instance, one can consider the identification of space-time surface as associative (co-associative) sub-manifold meaning that tangent space of space-time surface can be regarded as associative (co-associative) sub-manifold of complexified octonions defining tangent space of embedding space. One manner to define "associative sub-manifold" is by introducing octonionic representation of embedding space gamma matrices identified as tangent space vectors. It must be also assumed that the tangent space contains a preferred commutative (co-commutative) sub-space at each point and defining an integrable distribution having identification as string world sheet (also slicing of space-time sheet by string world sheets can be considered). Associativity and commutativity would define the basic dynamical principle. A closely related approach is based on so called Hamilton-Jacobi structure [K14] defining also this kind of slicing and the approaches could be equivalent.

2. In ZEO 3-surfaces become pairs of space-like 3-surfaces at the boundaries of causal diamond (CD). Even the light-like partonic orbits could be included to give the analog of Wilson loop. In absence of non-determinism of Kähler action this forces to ask whether the attribute "preferred" is un-necessary. There are however excellent reasons to expect that there is an infinite gauge degeneracy assignable to quantum criticality and represented in terms of Kac-Moody type transformations of partonic orbits respecting their light-likeness and giving rise to the degeneracy behind hierarchy of Planck constants $h_{eff} = n \times h$. n would give the number of conformal equivalence classes of space-time surfaces with same ends. In given measurement resolution one might however hope that the "preferred" could be dropped away.

   The vanishing of Noether charges for sub-algebras of conformal algebras with conformal weights coming as multiples of n at the ends of space-time surface would be a concrete realization of this picture and looks the most feasible option at this moment since it is direct classical correlated for broken super-conformal gauge invariance at quantum level.

3. The construction of quantum TGD in terms of the Kähler-Dirac action associated with Kähler action suggested a possible answer to the question about the principle selecting preferred extremals. The Noether currents associated with Kähler-Dirac action are conserved if second variations of Kähler action vanish. This is nothing but space-time correlate for quantum criticality and it is amusing that I failed to realize this for so long time. A further very important result is that in generic case the modes of induced spinor field are localized at 2-D surfaces from the condition that em charge is well-defined quantum number (W fields must vanish and also Z^0 field above weak scale in order to avoid large parity breaking effects).

   The localization at string world sheets means that quantum criticality as definition of "preferred" works only if there selection of string world sheets, partonic 2-surfaces, and their light-like orbits fixes the space-time surface completely. The generalization of AdS/CFT correspondence (or strong form of holography) suggests that this is indeed the case. The criticality conditions are however rather complicated and it seems that the vanishing of the symplectic Noether charges is the practical manner to formulate what "preferred" does mean.

7.5 Construction Of WCW Geometry From Symmetry Principles

Besides the direct guess of Kähler function one can also try to construct WCW geometry using symmetry principles. The mere existence of WCW geometry as a union of symmetric spaces requires maximal possible symmetries and means a reduction to single point of WCW with fixed values of zero modes. Therefore there are good hopes that the construction might work in practice.

7.5.1 General Coordinate Invariance And Generalized Quantum Gravitational Holography

The basic motivation for the construction of WCW geometry is the vision that physics reduces to the geometry of classical spinor fields in the infinite-dimensional WCW of 3-surfaces of $M^4_+ \times CP_2$ or of $M^4 \times CP_2$. Hermitian conjugation is the basic operation in quantum theory and its geometrization requires that WCW possesses Kähler geometry. Kähler geometry is coded into Kähler function.

The original belief was that the four-dimensional general coordinate invariance of Kähler function reduces the construction of the geometry to that for the boundary of configuration space consisting of 3-surfaces on $\delta M^4_+ \times CP_2$, the moment of big bang. The proposal was that Kähler function $K(Y^3)$ could be defined as a preferred extremal of so called Kähler action for the unique space-time surface $X^4(Y^3)$ going through given 3-surface Y^3 at $\delta M^4_+ \times CP_2$. For Diff^4 transforms of Y^3 at $X^4(Y^3)$ Kähler function would have the same value so that Diff^4 invariance and degeneracy would be the outcome. The proposal was that the preferred extremal is absolute minimum of Kähler action.

This picture turned out to be too simple.

1. Absolute minima had to be replaced by preferred extremals containing M^2 in the tangent space of X^4 at light-like 3-surfaces X^3_l. The reduction to the light-cone boundary which in fact corresponds to what has become known as quantum gravitational holography must be replaced with a construction involving light-like boundaries of causal diamonds CD already described.

2. It has also become obvious that the gigantic symmetries associated with $\delta M^4_\pm \times CP_2 \subset CD \times CP_2$ manifest themselves as the properties of propagators and vertices. Cosmological considerations, Poincare invariance, and the new view about energy favor the decomposition of WCW to a union of configuration spaces assignable to causal diamonds CDs defined as intersections of future and past directed light-cones. The minimum assumption is that CDs label the sectors of CH: the nice feature of this option is that the considerations of this chapter restricted to $\delta M^4_+ \times CP_2$ generalize almost trivially. This option is beautiful because the center of mass degrees of freedom associated with the different sectors of CH would correspond to M^4 itself and its Cartesian powers.

The definition of the Kähler function requires that the many-to-one correspondence $X^3 \rightarrow X^4(X^3)$ must be replaced by a bijective correspondence in the sense that X^3 as light-like 3-surface is unique among all its Diff^4 translates. This also allows physically preferred "gauge fixing" allowing to get rid of the mathematical complications due to Diff4 degeneracy. The internal geometry of the space-time sheet $X^4(X^3)$ must define the preferred 3-surface X^3.

The realization of this vision means a considerable mathematical challenge. The effective metric 2-dimensionality of 3-dimensional light-like surfaces X_l^3 of M^4 implies generalized conformal and symplectic invariances allowing to generalize quantum gravitational holography from light like boundary so that the complexities due to the non-determinism can be taken into account properly.

7.5.2 Light-Like 3-D Causal Determinants And Effective 2-Dimensionality

The light like 3-surfaces X_l^3 of space-time surface appear as 3-D causal determinants. Examples are boundaries and elementary particle horizons at which Minkowskian signature of the induced metric transforms to Euclidian one. This brings in a second conformal symmetry related to the metric 2-dimensionality of the 3-D light-like 3-surface. This symmetry is identifiable as TGD counterpart of the Kac Moody symmetry of string models. The challenge is to understand the relationship of this symmetry to WCW geometry and the interaction between the two conformal symmetries.

The analog of conformal invariance in the light-like direction of X_l^3 and in the light-like radial direction of $\delta M_\pm^4$ implies that the data at either X^3 or X_l^3 are enough to determine WCW geometry. This implies that the relevant data is contained to their intersection X^2 plus 4-D tangent space of X^2 at least for finite regions of X^3. This is the case if the deformations of X_l^3 not affecting X^2 and preserving light likeness corresponding to zero modes or gauge degrees of freedom and induce deformations of X^3 also acting as zero modes. The outcome is effective 2-dimensionality. One must be however cautious in order to not make over-statements. The reduction to 2-D theory in global sense would trivialize the theory to string model like theory and does not occur even locally. Moreover, the reduction to effectively 2-D theory must takes places for finite region of X^3 only so one has in well defined sense three-dimensionality in discrete sense. A more precise formulation of this vision is in terms of hierarchy of causal diamonds (CDs) containing CDs containing.... The introduction of sub-CD: s brings in improved measurement resolution and means also that effective 2-dimensionality is realized in the scale of sub-CD only.

One cannot over-emphasize the importance of the effective 2-dimensionality. It indeed simplifies dramatically the earlier formulas for WCW metric involving 3-dimensional integrals over $X^3 \subset M_+^4 \times CP_2$ reducing now to 2-dimensional integrals. Note that X^3 is determined by preferred extremal property of $X^4(X_l^3)$ once X_l^3 is fixed and one can hope that this mapping is one-to-one.

The reduction of data to that associated with 2-D surfaces and their 4-D tangent space distributions conforms with the number theoretic vision about embedding space as having hyper-octonionic structure [K105]: the commutative sub-manifolds of H have dimension not larger than two and for them tangent space is complex sub-space of complexified octonion tangent space. Number theoretic counterpart of quantum measurement theory forces the reduction of relevant data to 2-D commutative sub-manifolds of X^3. These points are discussed in more detail in the next chapter whereas in this chapter the consideration will be restricted to $X_l^3 = \delta M_+^4$ case which involves all essential aspects of the problem.

7.5.3 Magic Properties Of Light-Cone Boundary And Isometries Of WCW

The special conformal, metric and symplectic properties of the light cone of four-dimensional Minkowski space: δM_+^4, the boundary of four-dimensional light-cone is metrically 2-dimensional(!) sphere allowing infinite-dimensional group of conformal transformations and isometries(!) as well as Kähler structure. Kähler structure is not unique: possible Kähler structures of light-cone boundary are parameterized by Lobatchevski space $SO(3,1)/SO(3)$. The requirement that the isotropy group $SO(3)$ of S^2 corresponds to the isotropy group of the unique classical 3-momentum assigned to $X^4(Y^3)$ defined as absolute minimum of Kähler action, fixes the choice of the complex structure uniquely. Therefore group theoretical approach and the approach based on Kähler action complement each other.

The allowance of an infinite-dimensional group of isometries isomorphic to the group of conformal transformations of 2-sphere is completely unique feature of the 4-dimensional light-cone boundary. Even more, in case of $\delta M^4_+ \times CP_2$ the isometry group of δM^4_+ becomes localized with respect to CP_2! Furthermore, the Kähler structure of δM^4_+ defines also symplectic structure.

Hence any function of $\delta M^4_+ \times CP_2$ would serve as a Hamiltonian transformation acting in both CP_2 and δM^4_+ degrees of freedom. These transformations obviously differ from ordinary local gauge transformations. This group leaves the symplectic form of $\delta M^4_+ \times CP_2$, defined as the sum of light-cone and CP_2 symplectic forms, invariant. The group of symplectic transformations of $\delta M^4_+ \times CP_2$ is a good candidate for the isometry group of WCW.

The approximate symplectic invariance of Kähler action is broken only by gravitational effects and is exact for vacuum extremals. This suggests that Kähler function is in a good approximation invariant under the symplectic transformations of CP_2 would mean that CP_2 symplectic transformations correspond to zero modes having zero norm in the Kähler metric of WCW.

The groups G and H, and thus WCW itself, should inherit the complex structure of the light-cone boundary. The diffeomorphims of M^4 act as dynamical symmetries of vacuum extremals. The radial Virasoro localized with respect to $S^2 \times CP_2$ could in turn act in zero modes perhaps inducing conformal transformations: note that these transformations lead out from the symmetric space associated with given values of zero modes.

7.5.4 Symplectic Transformations Of $\Delta M^4_+ \times CP_2$ As Isometries Of WCW

The symplectic transformations of $\delta M^4_+ \times CP_2$ are excellent candidates for inducing symplectic transformations of the WCW acting as isometries. There are however deep differences with respect to the Kac Moody algebras.

1. The conformal algebra of WCW is gigantic when compared with the Virasoro + Kac Moody algebras of string models as is clear from the fact that the Lie-algebra generator of a symplectic transformation of $\delta M^4_+ \times CP_2$ corresponding to a Hamiltonian which is product of functions defined in δM^4_+ and CP_2 is sum of generator of δM^4_+-local symplectic transformation of CP_2 and CP_2-local symplectic transformations of δM^4_+. This means also that the notion of local gauge transformation generalizes.

2. The physical interpretation is also quite different: the relevant quantum numbers label the unitary representations of Lorentz group and color group, and the four-momentum labeling the states of Kac Moody representations is not present. Physical states carrying no energy and momentum at quantum level are predicted. The appearance of a new kind of angular momentum not assignable to elementary particles might shed some light to the longstanding problem of baryonic spin (quarks are not responsible for the entire spin of proton). The possibility of a new kind of color might have implications even in macroscopic length scales.

3. The central extension induced from the natural central extension associated with $\delta M^4_+ \times CP_2$ Poisson brackets is anti-symmetric with respect to the generators of the symplectic algebra rather than symmetric as in the case of Kac Moody algebras associated with loop spaces. At first this seems to mean a dramatic difference. For instance, in the case of CP_2 symplectic transformations localized with respect to δM^4_+ the central extension would vanish for Cartan algebra, which means a profound physical difference. For $\delta M^4_+ \times CP_2$ symplectic algebra a generalization of the Kac Moody type structure however emerges naturally.

   The point is that δM^4_+-local CP_2 symplectic transformations are accompanied by CP_2 local δM^4_+ symplectic transformations. Therefore the Poisson bracket of two δM^4_+ local CP_2 Hamiltonians involves a term analogous to a central extension term symmetric with respect to CP_2 Hamiltonians, and resulting from the δM^4_+ bracket of functions multiplying the Hamiltonians. This additional term could give the entire bracket of the WCW Hamiltonians at the maximum of the Kähler function where one expects that CP_2 Hamiltonians vanish and have a form essentially identical with Kac Moody central extension because it is indeed symmetric with respect to indices of the symplectic group.

7.5.5 Could The Zeros Of Riemann Zeta Define The Spectrum Of Super-Symplectic Conformal Weights?

The idea about symmetric space is extremely beautiful but the identification of the precise form of the Cartan decomposition is far from obvious. The basic problem concerns the spectrum of conformal weights of the generators of the super-symplectic algebra.

For the spinor modes at string world sheets the conformal weights are integers. The symplectic generators are characterized by the conformal weight associated with the light-like radial coordinate r_M of $\delta M^4_\pm = S^2 \times R_+$ plus quantum numbers associated with $SO(3)$ acting at S^2 in and with color group $SU(3)$. The simplest option would be that the conformal weights are simply integers also for the symplectic algebra implying that Hamiltonians are proportional to r^n. The complexification at WCW level would be induced from $n \to -n$.

There is however also an alternative option to consider. The inspiration came from the finding that quantum TGD leads naturally to an extension of Super Algebras by combining Ramond and Neveu-Schwartz algebras into single algebra. This led to the introduction Virasoro generators and generators of symplectic algebra of CP_2 localized with respect to the light-cone boundary and carrying conformal weights with a half integer valued real part.

1. The conformal weights $h = -1/2 - i\sum_i y_i$, where $z_i = 1/2 + y_i$ are non-trivial zeros of Riemann Zeta, are excellent candidates for the super-symplectic ground state conformal weights and for the generators of the symplectic algebra whose commutators generate the algebra. Also the negatives $h = 2n$ of the trivial zeros $z = -2n$, $n > 0$ can be included. Thus the conjecture inspired by the work with Riemann hypothesis stating that the zeros of Riemann Zeta appear at the level of basic quantum TGD gets some support. This raises interesting speculations. The possibility of negative real part of conformal weight $Re(h) = -1/2$ is intriguing since p-adic mass calculations demand that the ground state has negative conformal weight (is tachyonic).
2. If the conjecture holds true, the generators of algebra (in the standard sense now), whose commutators define the basis of the entire algebra, have conformal weights given by the negatives of the zeros of Riemann Zeta or Dirac Zeta. The algebra would be generated as commutators from the generators of g_1 and g_2 such that one has $h = 2n > 0$ for g_1 and $h = 1/2 + iy_i$ for g_2. The resulting super-symplectic algebra could be christened as Riemann algebra.
3. The spectrum of conformal weights would be of form $h = n + iy$, n integer and $y = \sum n_i y_i$. If mass squared is proportional to h, the value of h must be a real integer: $\sum n_i y_i = 0$. The interpretation would be in terms of conformal confinement generalizing color confinement.
4. The scenario for the hierarchy of conformal symmetry breakings in the sense that only a sub-algebra of full conformal algebra isomorphic with the original algebra (fractality) annihilates the physical states, makes sense also now since the algebra has a hierarchy of sub-algebras with the conformal weights of the full algebra scaled by integer n. This condition could be true also for the scalings of the real part of h but now the sub-algebra is not isomorphic with the original one. One can even consider the hierarchy of sub-algebras with imaginary parts of weights which are multiples of $y = \sum m_i n_i y_i$. Also these algebras fail to be isomorphic with the full algebra.
5. The requirement that ordinary Virasoro and Kac Moody generators annihilate physical states corresponds now to the fact that the generators of h vanish at the point of WCW, which remains invariant under the action of h. The maximum of Kähler function corresponds naturally to this point and plays also an essential role in the integration over WCW by generalizing the Gaussian integration of free quantum field theories.

7.5.6 Attempts To Identify WCW Hamiltonians

I have made several attempts to identify WCW Hamiltonians. The first two candidates referred to as magnetic and electric Hamiltonians, emerged in a relatively early stage. The third candidate is based on the formulation of quantum TGD using 3-D light-like surfaces identified as orbits of partons. The proposal is out-of-date but the most recent proposal is obtained by a very straightforward generalization from the proposal for magnetic Hamiltonians discussed below.

Magnetic Hamiltonians

Assuming that the elements of the radial Virasoro algebra of δM^4_+ have zero norm, one ends up with an explicit identification of the symplectic structures of WCW. There is almost unique identification for the symplectic structure. WCW counterparts of $\delta M^4 \times CP_2$ Hamiltonians are defined by the generalized signedandunsigned Kähler magnetic fluxes

$$
\begin{aligned}
& Q_m(H_A, X^2) = Z \int_{X^2} H_A J \sqrt{g_2} d^2x \ , \\
& Q^+_m(H_A, r_M) = Z \int_{X^2} H_A |J| \sqrt{g_2} d^2x \ , \\
& J \equiv \epsilon^{\alpha\beta} J_{\alpha\beta} \ .
\end{aligned}
\tag{7.5.1}
$$

H_A is CP_2 Hamiltonian multiplied by a function of coordinates of light cone boundary belonging to a unitary representation of the Lorentz group. Z is a conformal factor depending on symplectic invariants. The symplectic structure is induced by the symplectic structure of CP_2.

The most general flux is superposition of signed and unsigned fluxes Q_m and Q^+_m.

$$
Q^{\alpha,\beta}_m(H_A, X^2) = \alpha Q_m(H_A, X^2) + \beta Q^+_m(H_A, X^2) \ . \tag{7.5.2}
$$

Thus it seems that symmetry arguments fix the form of the WCW metric apart from the presence of a conformal factor Z multiplying the magnetic flux and the degeneracy related to the signed and unsigned fluxes.

Generalization

The generalization for definition WCW super-Hamiltonians defining WCW gamma matrices is discussed in detail in [K93] feeds in the wisdom gained about preferred extremals of Kähler action and solutions of the Kähler-Dirac action: in particular, about their localization at string worlds sheets (right handed neutrino could be an exception). Second quantized Noether charges in turn define representation of WCW Hamiltonians as operators.

The basic formulas generalize as such: the only modification is that the super-Hamiltonian of $\delta M^4_\pm \times CP_2$ at given point of partonic 2-surface is replaced with the Noether super charge associated with the Hamiltonian obtained by integrating the 1-D super current over string emanating from partonic 2-surface. Right handed neutrino spinor is replaced with any mode of the Kähler-Dirac operator localized at string world sheet in the case of Kac-Moody sub-algebra of super-symplectic algebra corresponding to symplectic isometries at light-cone boundary and CP_2. The original proposal involved only the contractions with covariantly constant right- handed neutrino spinor mode but now one can allow contractions with all spinor modes - both quark like and leptonic ones. One obtains entire super-symplectic algebra and the direct sum of these algebras is used to construct physical states. This step is analogous to the replacement of point like particle with string.

The resulting super Hamiltonians define WCW gamma matrices. They are labelled by two conformal weights. The first one is the conformal weight associated with the light-like coordinate of $\delta M^4_\pm \times CP_2$. Second conformal weight is associated with the spinor mode and the coordinate along stringy curve and corresponds to the usual stringy conformal weight. The symplectic conformal weight can be more general - I have proposed its spectrum to be generated by the zeros of Riemann zeta. The total conformal weight of a physical state would be non-negative integer meaning conformal confinement. Symplectic conformal symmetry can be assumed to be broken: an entire hierarchy of breakings is obtained corresponding to hierarchies of sub-algebra of the symplectic algebra isomorphic with it quantum criticalities, Planck constants, and dark matter.

The presence of two conformal weights is in accordance with the idea that a generalization of conformal invariance to 4-D situation is in question. If Yangian extension of conformal symmetries is possible and would bring an additional integer n telling the degree of multilocality of Yangian generators defined as the number of partonic 2-surfaces at which the generator acts. For conformal algebra degree of multilocality equals to $n = 1$.

7.5.7 General Expressions For The Symplectic And Kähler Forms

One can derive general expressions for symplectic and Kähler forms as well as Kähler metric of WCW in the basis provided by symplectic generators. These expressions as such do not tell much.

To obtain more information about WCW Hamiltonians one can use the hypothesis that the Hamiltonians of the boundary of CD can be lifted to the Hamiltonians of WCW isometries defining the tangent space basis of WCW. Symmetry considerations inspire the notion of flux Hamiltonian. Hamiltonians seem to be crucial for the realization of symmetries in WCW degrees of freedom using harmonics of WCW spinor fields. Also the construction of WCW Killing vector fields represents a technical problem.

The Poisson brackets of the WCW Hamiltonians can be calculated without the knowledge of the contravariant Kähler form by using the fact that the Poisson bracket of WCW Hamiltonians is WCW Hamiltonian associated with the Poisson bracket of embedding space Hamiltonians. The explicit calculation of Kähler form is difficult using only symmetry considerations and the attempts that I have made are not convincing.

The expression of Kähler metric in terms of anti-commutators of symplectic Noether charges and super-charges gives explicit formulas as integrals over a string connecting two partonic 2-surfaces. A natural guess for super Hamiltonian is that one integrates over the strings connecting partonic 2-surface to each other with the weighting coming from Kähler flux and embedding space Hamiltonian replaced with the fermionic super Hamiltonian of Hamiltonian of the string. It is not clear whether the vanishing of induced W fields at string world sheets allows all possible strings or only a discrete set of them as finite measurement resolution would suggest. If all points pairs can be connected by string one has effective 3-dimensionality.

Closedness requirement

The fluxes of Kähler magnetic and electric fields for the Hamiltonians of $\delta M^4_+ \times CP_2$ suggest a general representation for the components of the symplectic form of the WCW. The basic requirement is that Kähler form satisfies the defining condition

$$X \cdot J(Y,Z) + J([X,Y],Z) + J(X,[Y,Z]) = 0 \ , \tag{7.5.3}$$

where X, Y, Z are now vector fields associated with Hamiltonian functions defining WCW coordinates.

Matrix elements of the symplectic form as Poisson brackets

Quite generally, the matrix element of $J(X(H_A), X(H_B))$ between vector fields $X(H_A))$ and $X(H_B))$ defined by the Hamiltonians H_A and H_B of $\delta M^4_+ \times CP_2$ isometries is expressible as Poisson bracket

$$J^{AB} = J(X(H_A), X(H_B)) = \{H_A, H_B\} \ . \tag{7.5.4}$$

J^{AB} denotes contravariant components of the symplectic form in coordinates given by a subset of Hamiltonians. The proposal is that the magnetic flux Hamiltonians $Q^{\alpha,\beta}_m(H_{A,k})$ provide an explicit representation for the Hamiltonians at the level of WCW so that the components of the symplectic form of WCW are expressible as classical charges for the Poisson brackets of the Hamiltonians of the light-cone boundary:

$$J(X(H_A), X(H_B)) = Q^{\alpha,\beta}_m(\{H_A, H_B\}) \ . \tag{7.5.5}$$

Recall that the superscript α, β refers the coefficients of J and $|J|$ in the superposition of these Kähler magnetic fluxes. Note that $Q_m^{\alpha,\beta}$ contains unspecified conformal factor depending on symplectic invariants characterizing Y^3 and is unspecified superposition of signed and unsigned magnetic fluxes.

This representation does not carry information about the tangent space of space-time surface at the partonic 2-surface, which motivates the proposal that also electric fluxes are present and proportional to magnetic fluxes with a factor K, which is symplectic invariant so that commutators of flux Hamiltonians come out correctly. This would give

$$Q_m^{\alpha,\beta}(H_A)_{em} = Q_e^{\alpha,\beta}(H_A) + Q_m^{\alpha,\beta}(H_A) = (1+K)Q_m^{\alpha,\beta}(H_A) \ . \tag{7.5.6}$$

Since Kähler form relates to the standard field tensor by a factor $e/\hbar$, flux Hamiltonians are dimensionless so that commutators do not involve $\hbar$. The commutators would come as

$$Q_{em}^{\alpha,\beta}(\{H_A, H_B\}) \rightarrow (1+K)Q_m^{\alpha,\beta}(\{H_A, H_B\}) \ . \tag{7.5.7}$$

The factor $1+K$ plays the same role as Planck constant in the commutators.

WCW Hamiltonians vanish for the extrema of the Kähler function as variational derivatives of the Kähler action. Hence Hamiltonians are good candidates for the coordinates appearing as coordinates in the perturbative functional integral around extrema (with maxima giving dominating contribution). It is clear that WCW coordinates around a given extremum include only those Hamiltonians, which vanish at extremum (that is those Hamiltonians which span the tangent space of G/H). In Darboux coordinates the Poisson brackets reduce to the symplectic form

$$\begin{aligned} \{P^I, Q^J\} &= J^{IJ} = J_I \delta^{I,J} \ . \\ J_I &= 1 \ . \end{aligned} \tag{7.5.8}$$

It is not clear whether Darboux coordinates with $J_I = 1$ are possible in the recent case: probably the unit matrix on right hand side of the defining equation is replaced with a diagonal matrix depending on symplectic invariants so that one has $J_I \neq 1$. The integration measure is given by the symplectic volume element given by the determinant of the matrix defined by the Poisson brackets of the Hamiltonians appearing as coordinates. The value of the symplectic volume element is given by the matrix formed by the Poisson brackets of the Hamiltonians and reduces to the product

$$Vol = \prod_I J_I$$

in generalized Darboux coordinates.

Kähler potential (that is gauge potential associated with Kähler form) can be written in Darboux coordinates as

$$A = \sum_I J_I P_I dQ^I \ . \tag{7.5.9}$$

General expressions for Kähler form, Kähler metric and Kähler function

The expressions of Kähler form and Kähler metric in complex coordinates can obtained by transforming the contravariant form of the symplectic form from symplectic coordinates provided by Hamiltonians to complex coordinates:

$$J^{Z^i \bar{Z}^j} = iG^{Z^i \bar{Z}^j} = \partial_{H^A} Z^i \partial_{H^B} \bar{Z}^j J^{AB} \ , \tag{7.5.10}$$

where J^{AB} is given by the classical Kähler charge for the light-cone Hamiltonian $\{H^A, H^B\}$. Complex coordinates correspond to linear coordinates of the complexified Lie-algebra providing exponentiation of the isometry algebra via exponential mapping. What one must know is the precise relationship between allowed complex coordinates and Hamiltonian coordinates: this relationship is in principle calculable. In Darboux coordinates the expressions become even simpler:

$$J^{Z^i\bar{Z}^j} = iG^{Z^i\bar{Z}^j} \quad = \quad \sum_I J(I)(\partial_{P^i}Z^i\partial_{Q^I}\bar{Z}^j - \partial_{Q^I}Z^i\partial_{P^I}\bar{Z}^j) \ . \tag{7.5.11}$$

Kähler function can be formally integrated from the relationship

$$\begin{aligned} A_{Z^i} &= i\partial_{Z^i}K \ , \\ A_{\bar{Z}^i} &= -i\partial_{Z^i}K \ . \end{aligned} \tag{7.5.12}$$

holding true in complex coordinates. Kähler function is obtained formally as integral

$$K \quad = \quad \int_0^Z (A_{Z^i}dZ^i - A_{\bar{Z}^i}d\bar{Z}^i) \ . \tag{7.5.13}$$

$Diff(X^3)$ invariance and degeneracy and conformal invariances of the symplectic form

$J(X(H_A), X(H_B))$ defines symplectic form for the coset space G/H only if it is $Diff(X^3)$ degenerate. This means that the symplectic form $J(X(H_A), X(H_B))$ vanishes whenever Hamiltonian H_A or H_B is such that it generates diffeomorphism of the 3-surface X^3. If effective 2-dimensionality holds true, $J(X(H_A), X(H_B))$ vanishes if H_A or H_B generates two-dimensional diffeomorphism $d(H_A)$ at the surface X_i^2.

One can always write

$$J(X(H_A), X(H_B)) = X(H_A)Q(H_B|X_i^2) \ .$$

If H_A generates diffeomorphism, the action of $X(H_A)$ reduces to the action of the vector field X_A of some X_i^2-diffeomorphism. Since $Q(H_B|r_M)$ is manifestly invariant under the diffemorphisms of X^2, the result is vanishing:

$$X_AQ(H_B|X_i^2) = 0 \ ,$$

so that $Diff^2$ invariance is achieved.

The radial diffeomorphisms possibly generated by the radial Virasoro algebra do not produce trouble. The change of the flux integrand X under the infinitesimal transformation $r_M \rightarrow r_M + \epsilon r_M^n$ is given by $r_M^n dX/dr_M$. Replacing r_M with $r_M^{-n+1}/(-n+1)$ as variable, the integrand reduces to a total divergence dX/du the integral of which vanishes over the closed 2-surface X_i^2. Hence radial Virasoro generators having zero norm annihilate all matrix elements of the symplectic form. The induced metric of X_i^2 induces a unique conformal structure and since the conformal transformations of X_i^2 can be interpreted as a mere coordinate changes, they leave the flux integrals invariant.

Complexification and explicit form of the metric and Kähler form

The identification of the Kähler form and Kähler metric in symplectic degrees of freedom follows trivially from the identification of the symplectic form and definition of complexification. The requirement that Hamiltonians are eigen states of angular momentum (and possibly Lorentz boost generator), isospin and hypercharge implies physically natural complexification. In order to fix the complexification completely one must introduce some convention fixing which states correspond to "positive" frequencies and which to "negative frequencies" and which to zero frequencies that is to decompose the generators of the symplectic algebra to three sets Can_+, Can_- and Can_0. One must distinguish between Can_0 and zero modes, which are not considered here at all. For instance, CP_2 Hamiltonians correspond to zero modes.

The natural complexification relies on the imaginary part of the radial conformal weight whereas the real part defines the $g = t + h$ decomposition naturally. The wave vector associated with the radial logarithmic plane wave corresponds to the angular momentum quantum number associated with a wave in S^1 in the case of Kac Moody algebra. One can imagine three options.

1. It is quite possible that the spectrum of k_2 does not contain $k_2 = 0$ at all so that the sector Can_0 could be empty. This complexification is physically very natural since it is manifestly invariant under $SU(3)$ and $SO(3)$ defining the preferred spherical coordinates. The choice of $SO(3)$ is unique if the classical four-momentum associated with the 3-surface is time like so that there are no problems with Lorentz invariance.
2. If $k_2 = 0$ is possible one could have

$$\begin{aligned} Can_+ &= \{H^a_{m,n,k=k1_+ik_2}, k_2 > 0\} \ , \\ Can_- &= \{H^a_{m,n,k}, k_2 < 0\} \ , \\ Can_0 &= \{H^a_{m,n,k}, k_2 = 0\} \ . \end{aligned} \tag{7.5.14}$$

3. If it is possible to $n_2 \neq 0$ for $k_2 = 0$, one could define the decomposition as

$$\begin{aligned} Can_+ &= \{H^a_{m,n,k}, k_2 > 0 \ \text{ or } \ k_2 = 0, n_2 > 0\} \ , \\ Can_- &= \{H^a_{m,n,k}, k_2 < 0 \ \text{ or} k_2 = 0, n_2 < 0\} \ , \\ Can_0 &= \{H^a_{m,n,k}, k_2 = n_2 = 0\} \ . \end{aligned} \tag{7.5.15}$$

In this case the complexification is unique and Lorentz invariance guaranteed if one can fix the $SO(2)$ subgroup uniquely. The quantization axis of angular momentum could be chosen to be the direction of the classical angular momentum associated with the 3-surface in its rest system.

The only thing needed to get Kähler form and Kähler metric is to write the half Poisson bracket defined by Eq. 7.5.17

$$\begin{aligned} J_f(X(H_A), X(H_B)) &= 2Im\,(iQ_f(\{H_A, H_B\}_{-+})) \ , \\ G_f(X(H_A), X(H_B)) &= 2Re\,(iQ_f(\{H_A, H_B\}_{-+})) \ . \end{aligned} \tag{7.5.16}$$

Symplectic form, and thus also Kähler form and Kähler metric, could contain a conformal factor depending on the isometry invariants characterizing the size and shape of the 3-surface. At this stage one cannot say much about the functional form of this factor.

Comparison of CP_2 Kähler geometry with WCW geometry

The explicit discussion of the role of $g = t + h$ decomposition of the tangent space of WCW provides deep insights to the metric of the symmetric space. There are indeed many questions to be answered. To what point of WCW (that is 3-surface) the proposed $g = t + h$ decomposition corresponds to? Can one derive the components of the metric and Kähler form from the Poisson brackets of complexified Hamiltonians? Can one characterize the point in question in terms of the properties of WCW Hamiltonians? Does the central extension of WCW reduce to the symplectic central extension of the symplectic algebra or can one consider also other options?

1. Cartan decomposition for CP_2

A good manner to gain understanding is to consider the CP_2 metric and Kähler form at the origin of complex coordinates for which the sub-algebra $h = u(2)$ defines the Cartan decomposition.

1. $g = t + h$ decomposition depends on the point of the symmetric space in general. In case of CP_2 $u(2)$ sub-algebra transforms as $g \circ u(2) \circ g^{-1}$ when the point s is replaced by gsg^{-1}. This is expected to hold true also in case of WCW (unless it is flat) so that the task is to identify the point of WCW at which the proposed decomposition holds true.
2. The Killing vector fields of h sub-algebra vanish at the origin of CP_2 in complex coordinates. The corresponding Hamiltonians need not vanish but their Poisson brackets must vanish. It is possible to add suitable constants to the Hamiltonians in order to guarantee that they vanish at origin.

3. It is convenient to introduce complex coordinates and decompose isometry generators to holomorphic components $J_+^a = j^{ak}\partial_k$ and $j_-^a = j^{a\bar{k}}\partial_{\bar{k}}$. One can introduce what might be called half Poisson bracket and half inner product defined as

$$\begin{aligned} \{H^a, H^b\}_{-+} &\equiv \partial_{\bar{k}} H^a J^{\bar{k}l} \partial_l H^b \\ &= j^{ak} J_{k\bar{l}} j^{b\bar{l}} = -i(j_+^a, j_-^b) \ . \end{aligned} \quad (7.5.17)$$

If the half Poisson bracket of embedding space Hamiltonians can be calculated. If it lifts (this is assumption!) to a half Poisson bracket of corresponding WCW Hamiltonians, pne can express Poisson bracket of Hamiltonians and the inner product of the corresponding Killing vector fields in terms of real and imaginary parts of the half Poisson bracket:

$$\begin{aligned} \{H^a, H^b\} &= 2Im\left(i\{H^a, H^b\}_{-+}\right) \ , \\ (j^a, j^b) &= 2Re\left(i(j_+^a, j_-^b)\right) = 2Re\left(i\{H^a, H^b\}_{-+}\right) \ . \end{aligned} \quad (7.5.18)$$

What this means that Hamiltonians and their half brackets code all information about metric and Kähler form. Obviously this is of utmost importance in the case of the WCW metric whose symplectic structure and central extension are derived from those of CP_2.

4. The objection is that the WCW Kähler metric identified as the anticommutators of fermionic super charges have as an additional pair of labels the conformal weights of spinor modes involved with the matrix element so that the number of matrix elements of WCW metric would be larger than suggested by lifting. On the other hand, the standard conformal symmetry realized as gauge invariance for strings would suggest that the Noether super charges vanish for non-vanishing spinorial conformal weights and the two representations are equivalent. The vanishing of conformal charges would realize the effective 2-dimensionality which would be natural. This allows the breaking of conformal symmetry as gauge invariance only for the symplectic algebra whereas the conformal symmetry for spinor modes would be exact gauge symmetry as in string models. This conforms with the vision that symplectic algebra is the dynamical conformal algebra.

Consider now the properties of the metric and Kähler form at the origin of WCW.

1. The relations satisfied by the half Poisson brackets can be written symbolically as

$$\begin{aligned} &\{h, h\}_{-+} = 0 \ , \\ &Re\left(i\{h, t\}_{-+}\right) = 0 \ , \quad Im\left(i\{h, t\}_{-+}\right) = 0 \ , \\ &Re\left(i\{t, t\}_{-+}\right) \neq 0 \ , \quad Im\left(i\{t, t\}_{-+}\right) \neq 0 \ . \end{aligned} \quad (7.5.19)$$

2. The first two conditions state that h vector fields have vanishing inner products at the origin. The first condition states also that the Hamiltonians for the commutator algebra $[h, h] = SU(2)$ vanish at origin whereas the Hamiltonian for $U(1)$ algebra corresponding to the color hyper charge need not vanish although it can be made vanishing. The third condition implies that the Hamiltonians of t vanish at origin.

3. The last two conditions state that the Kähler metric and form are non-vanishing between the elements of t. Since the Poisson brackets of t Hamiltonians are Hamiltonians of h, the only possibility is that $\{t, t\}$ Poisson brackets reduce to a non-vanishing $U(1)$ Hamiltonian at the origin or that the bracket at the origin is due to the symplectic central extension. The requirement that all Hamiltonians vanish at origin is very attractive aesthetically and forces to interpret $\{t, t\}$ brackets at origin as being due to a symplectic central extension. For instance, for S^2 the requirement that Hamiltonians vanish at origin would mean the replacement of the Hamiltonian $H = cos(\theta)$ representing a rotation around z-axis with $H_3 = cos(\theta) - 1$ so that the Poisson bracket of the generators H_1 and H_2 can be interpreted as a central extension term.

4. The conditions for the Hamiltonians of $u(2)$ sub-algebra state that their variations with respect to g vanish at origin. Thus $u(2)$ Hamiltonians have extremum value at origin.
5. Also the Kähler function of CP_2 has extremum at the origin. This suggests that in the case of the WCW the counterpart of the origin corresponds to the maximum of the Kähler function.

2. Cartan algebra decomposition at the level of WCW

The discussion of the properties of CP_2 Kähler metric at origin provides valuable guide lines in an attempt to understand what happens at the level of WCW. The use of the half bracket for WCW Hamiltonians in turn allows to calculate the matrix elements of the WCW metric and Kähler form explicitly in terms of the magnetic or electric flux Hamiltonians.

The earlier construction was rather tricky and formula-rich and not very convincing physically. Cartan decomposition had to be assigned with something and in lack of anything better it was assigned with Super Virasoro algebra, which indeed allows this kind of decompositions but without any strong physical justification.

It must be however emphasized that holography implying effective 2-dimensionality of 3-surfaces in some length scale resolution is absolutely essential for this construction since it allows to effectively reduce Kac-Moody generators associated with X^3_l to $X^2 = X^3_l \cap \delta M^4_\pm \times CP_2$. In the similar manner super-symplectic generators can be dimensionally reduced to X^2. Number theoretical compactification forces the dimensional reduction and the known extremals are consistent with it [K14]. The construction of WCW spinor structure and metric in terms of the second quantized spinor fields [K121] relies to this picture as also the recent view about M-matrix [K26].

In this framework the coset space decomposition becomes trivial.

1. The algebra g is labeled by color quantum numbers of CP_2 Hamiltonians and by the label (m,n,k) labeling the function basis of the light-cone boundary. Also a localization with respect to X^2 is needed. This is a new element as compared to the original view.
2. Super Kac-Moody algebra is labeled by color octet Hamiltonians and function basis of X^2. Since Lie-algebra action does not lead out of irreps, this means that Cartan algebra decomposition is satisfied.

Comparison with loop groups

It is useful to compare the recent approach to the geometrization of the loop groups consisting of maps from circle to Lie group G [A43], which served as the inspirer of the WCW geometry approach but later turned out to not apply as such in TGD framework.

In the case of loop groups the tangent space T corresponds to the local Lie-algebra $T(k,A) = exp(ik\phi)T_A$, where T_A generates the finite-dimensional Lie-algebra g and ϕ denotes the angle variable of circle; k is integer. The complexification of the tangent space corresponds to the decomposition

$$T = \{X(k>0,A)\} \oplus \{X(k<0,A)\} \oplus \{X(k=0,A)\} = T_+ \oplus T_- \oplus T_0$$

of the tangent space. Metric corresponds to the central extension of the loop algebra to Kac Moody algebra and the Kähler form is given by

$$J(X(k_1<0,A), X(k_2>0,B)) = k_2\delta(k_1+k_2)\delta(A,B) \ .$$

In present case the finite dimensional Lie algebra g is replaced with the Lie-algebra of the symplectic transformations of $\delta M^4_+ \times CP_2$ centrally extended using symplectic extension. The scalar function basis on circle is replaced with the function basis on an interval of length Δr_M with periodic boundary conditions; effectively one has circle also now.

The basic difference is that one can consider two kinds of central extensions now.

1. Central extension is most naturally induced by the natural central extension ($\{p,q\} = 1$) defined by Poisson bracket. This extension is anti-symmetric with respect to the generators of the symplectic group: in the case of the Kac Moody central extension it is symmetric with respect to the group G. The symplectic transformations of CP_2 might correspond to non-zero modes also because they are not exact symmetries of Kähler action. The situation is however

rather delicate since $k = 0$ light-cone harmonic has a diverging norm due to the radial integration unless one poses both lower and upper radial cutoffs although the matrix elements would be still well defined for typical 3-surfaces. For Kac Moody group $U(1)$ transformations correspond to the zero modes. light-cone function algebra can be regarded as a local $U(1)$ algebra defining central extension in the case that only CP_2 symplectic transformations local with respect to δM^4_+ act as isometries: for Kac Moody algebra the central extension corresponds to an ordinary $U(1)$ algebra. In the case that entire light-cone symplectic algebra defines the isometries the central extension reduces to a $U(1)$ central extension.

Symmetric space property implies Ricci flatness and isometric action of symplectic transformations

The basic structure of symmetric spaces is summarized by the following structural equations

$$\begin{array}{l} g = h + t \ , \\ [h,h] \subset h \ , \quad [h,t] \subset t \ , \quad [t,t] \subset h \ . \end{array} \tag{7.5.20}$$

In present case the equations imply that all commutators of the Lie-algebra generators of $Can(\neq 0)$ having non-vanishing integer valued radial quantum number n_2, possess zero norm. This condition is extremely strong and guarantees isometric action of $Can(\delta M^4_+ \times CP_2)$ as well as Ricci flatness of the WCW metric.

The requirement $[t,t] \subset h$ and $[h,t] \subset t$ are satisfied if the generators of the isometry algebra possess generalized parity P such that the generators in t have parity $P = -1$ and the generators belonging to h have parity $P = +1$. Conformal weight n must somehow define this parity. The first possibility to come into mind is that odd values of n correspond to $P = -1$ and even values to $P = 1$. Since n is additive in commutation, this would automatically imply $h \oplus t$ decomposition with the required properties. This assumption looks however somewhat artificial. TGD however forces a generalization of Super Algebras and N-S and Ramond type algebras can be combined to a larger algebra containing also Virasoro and Kac Moody generators labeled by half-odd integers. This suggests strongly that isometry generators are labeled by half integer conformal weight and that half-odd integer conformal weight corresponds to parity $P = -1$ whereas integer conformal weight corresponds to parity $P = 1$. Coset space would structure would state conformal invariance of the theory since super-symplectic generators with integer weight would correspond to zero modes.

Quite generally, the requirement that the metric is invariant under the flow generated by vector field X leads together with the covariant constancy of the metric to the Killing conditions

$$X \cdot g(Y,Z) \quad = \quad 0 = g([X,Y],Z) + g(Y,[X,Z]) \ . \tag{7.5.21}$$

If the commutators of the complexified generators in $Can(\neq 0)$ have zero norm then the two terms on the right hand side of Eq. (7.5.21) vanish separately. This is true if the conditions

$$Q_m^{\alpha,\beta}(\{H^A,\{H^B,H^C\}\}) \quad = \quad 0 \ , \tag{7.5.22}$$

are satisfied for all triplets of Hamiltonians in $Can_{\neq 0}$. These conditions follow automatically from the $[t,t] \subset h$ property and guarantee also Ricci flatness as will be found later.

It must be emphasized that for Kähler metric defined by purely magnetic fluxes, one cannot pose the conditions of Eq. (7.5.22) as consistency conditions on the initial values of the time derivatives of embedding space coordinates whereas in general case this is possible. If the consistency conditions are satisfied for a single surface on the orbit of symplectic group then they are satisfied on the entire orbit. Clearly, isometry and Ricci flatness requirements and the requirement of time reversal invariance might well force Kähler electric alternative.

7.6 Representation Of WCW Metric As Anti-Commutators Of Gamma Matrices Identified As Symplectic Super-Charges

WCW gamma matrices identified as symplectic super Noether charges suggest an elegant representation of WCW metric and Kähler form, which seems to be more practical than the representations in terms of Kähler function or representations guessed by symmetry arguments.

This representation is equivalent with the somewhat dubious representation obtained using symmetry arguments - that is by assuming that the half Poisson brackets of embedding space Hamiltonians defining Kähler form and metric can be lifted to the level of WCW, if the conformal gauge conditions hold true for the spinorial conformal algebra, which is the TGD counterpart of the standard Kac-Moody type algebra of the ordinary strings models. For symplectic algebra the hierarchy of breakings of super-conformal gauge symmetry is possible but not for the standard conformal algebras associated with spinor modes at string world sheets.

7.6.1 Expression For WCW Kähler Metric As Anticommutators As Symplectic Super Charges

During years I have considered several variants for the representation of symplectic Hamiltonians and WCW gamma matrices and each of these proposals have had some weakness. The key question has been whether the Noether currents assignable to WCW Hamiltonians should play any role in the construction or whether one can use only the generalization of flux Hamiltonians.

The original approach based on flux Hamiltonians did not use Noether currents.

1. Magnetic flux Hamiltonians do not refer to the space-time dynamics and imply genuine rather than only effective 2-dimensionality, which is more than one wants. If the sum of the magnetic and electric flux Hamiltonians and the weak form of self duality is assumed, effective 2-dimensionality might be achieved.

   The challenge is to identify the super-partners of the flux Hamiltonians and postulate correct anti-commutation relations for the induced spinor fields to achieve anti-commutation to flux Hamiltonians. It seems that this challenge leads to ad hoc constructions.

2. For the purposes of generalization it is useful to give the expression of flux Hamiltonian. Apart from normalization factors one would have

$$Q(H_A) = \int_{X^2} H_A J_{\mu\nu} dx^\mu \wedge dx^\nu \quad .$$

   Here A is a label for the Hamiltonian of $\delta M^4_\pm \times CP_2$ decomposing to product of $\delta M^4_\pm$ and CP_2 Hamiltonians with the first one decomposing to a product of function of the radial light-like coordinate r_M and Hamiltonian depending on S^2 coordinates. It is natural to assume that Hamiltonians have well- defined $SO(3)$ and $SU(3)$ quantum numbers. This expressions serves as a natural starting point also in the new approach based on Noether charges.

The approach identifying the Hamiltonians as symplectic Noether charges is extremely natural from physics point of view but the fact that it leads to 3-D expressions involving the induced metric led to the conclusion that it cannot work. In hindsight this conclusion seems wrong: I had not yet realized how profound that basic formulas of physics really are. If the generalization of AdS/CFT duality works, Kähler action can be expressed as a sum of string area actions for string world sheets with string area in the effective metric given as the anti-commutator of the Kähler-Dirac gamma matrices for the string world sheet so that also now a reduction of dimension takes place. This is easy to understand if the classical Noether charges vanish for a sub-algebra of symplectic algebra for preferred extremals.

1. If all end points for strings are possible, the recipe for constructing super-conformal generators would be simple. The embedding space Hamiltonian H_A appearing in the expression of the flux Hamiltonian given above would be replaced by the corresponding symplectic quantum Noether charge $Q(H_A)$ associated with the string defined as 1-D integral along the string. By

replacing Ψ or its conjugate with a mode of the induced spinor field labeled by electroweak quantum numbers and conformal weight nm one would obtain corresponding super-charged identifiable as WCW gamma matrices. The anti-commutators of the super-charges would give rise to the elements of WCW metric labelled by conformal weights n_1, n_2 not present in the naïve guess for the metric. If one assumes that the fermionic super-conformal symmetries act as gauge symmetries only $n_i = 0$ gives a non-vanishing matrix element.

Clearly, one would have weaker form of effective 2-dimensionality in the sense that Hamiltonian would be functional of the string emanating from the partonic 2-surface. The quantum Hamiltonian would also carry information about the presence of other wormhole contacts- at least one- when wormhole throats carry Kähler magnetic monopole flux. If only discrete set for the end points for strings is possible one has discrete sum making possible easy p-adicization. It might happen that integrability conditions for the tangent spaces of string world sheets having vanishing W boson fields do not allow all possible strings.

2. The super charges obtained in this manner are not however entirely satisfactory. The problem is that they involve only single string emanating from the partonic 2-surface. The intuitive expectation is that there can be an arbitrarily large number of strings: as the number of strings is increased the resolution improves. Somehow the super-conformal algebra defined by Hamiltonians and super-Hamiltonians should generalize to allow tensor products of the strings providing more physical information about the 3-surface.

3. Here the idea of Yangian symmetry [L8] suggests itself strongly. The notion of Yangian emerges from twistor Grassmann approach and should have a natural place in TGD. In Yangian algebra one has besides product also co-product, which is in some sense "time-reversal" of the product. What is essential is that Yangian algebra is also multi-local.

   The Yangian extension of the super-conformal algebra would be multi-local with respect to the points of partonic surface (or multi-stringy) defining the end points of string. The basic formulas would be schematically

$$O_1^A = f^A_{BC} T^B \otimes T^B \quad ,$$

   where a summation of B, C occurs and f^A_{BC} are the structure constants of the algebra. The operation can be iterated and gives a hierarchy of n-local operators. In the recent case the operators are n-local symplectic super-charges with unit fermion number and symplectic Noether charges with a vanishing fermion number. It would be natural to assume that also the n-local gamma matrix like entities contribute via their anti-commutators to WCW metric and give multi-local information about the partonic 2-surface and 3-surface.

   The operation generating the algebra well-defined if one an assumes that the second quantization of induced spinor fields is carried out using the standard canonical quantization. One could even assume that the points involved belong to different partonic 2-surfaces belonging even at opposite boundaries of CD. The operation is also well-defined if one assumes that induced spinor fields at different space-time points at boundaries of CD always anti-commute. This could make sense at boundary of CD but lead to problems with embedding space-causality if assumed for the spinor modes at opposite boundaries of CD.

7.6.2 Handful Of Problems With A Common Resolution

Theory building could be compared to pattern recognition or to a solving a crossword puzzle. It is essential to make trials, even if one is aware that they are probably wrong. When stares long enough to the letters which do not quite fit, one suddenly realizes what one particular crossword must actually be and it is soon clear what those other crosswords are. In the following I describe an example in which this analogy is rather concrete.

I will first summarize the problems of ordinary Dirac action based on induced gamma matrices and propose Kähler-Dirac action as their solution.

Problems associated with the ordinary Dirac action

In the following the problems of the ordinary Dirac action are discussed and the notion of Kähler-Dirac action is introduced.

Minimal 2-surface represents a situation in which the representation of surface reduces to a complex-analytic map. This implies that induced metric is hermitian so that it has no diagonal components in complex coordinates $(z, \overline{z})$ and the second fundamental form has only diagonal components of type H^k_{zz}. This implies that minimal surface is in question since the trace of the second fundamental form vanishes. At first it seems that the same must happen also in the more general case with the consequence that the space-time surface is a minimal surface. Although many basic extremals of Kähler action are minimal surfaces, it seems difficult to believe that minimal surface property plus extremization of Kähler action could really boil down to the absolute minimization of Kähler action or some other general principle selecting preferred extremals as Bohr orbits [K28, K105].

This brings in mind a similar long-standing problem associated with the Dirac equation for the induced spinors. The problem is that right-handed neutrino generates super-symmetry only provided that space-time surface and its boundary are minimal surfaces. Although one could interpret this as a geometric symmetry breaking, there is a strong feeling that something goes wrong. Induced Dirac equation and super-symmetry fix the variational principle but this variational principle is not consistent with Kähler action.

One can also question the implicit assumption that Dirac equation for the induced spinors is consistent with the super-symmetry of the WCW geometry. Super-symmetry would obviously require that for vacuum extremals of Kähler action also induced spinor fields represent vacua. This is however not the case. This super-symmetry is however assumed in the construction of WCW geometry so that there is internal inconsistency.

Super-symmetry forces Kähler-Dirac equation

The above described three problems have a common solution. Nothing prevents from starting directly from the hypothesis of a super-symmetry generated by covariantly constant right-handed neutrino and finding a Dirac action which is consistent with this super-symmetry. Field equations can be written as

$$\begin{aligned} D_\alpha T^\alpha_k &= 0 \ , \\ T^\alpha_k &= \frac{\partial}{\partial h^k{}_\alpha} L_K \ . \end{aligned} \tag{7.6.1}$$

Here T^α_k is canonical momentum current of Kähler action. If super-symmetry is present one can assign to this current its super-symmetric counterpart

$$\begin{aligned} J^{\alpha k} &= \overline{\nu_R} \Gamma^k T^\alpha_l \Gamma^l \Psi \ , \\ D_\alpha J^{\alpha k} &= 0 \ . \end{aligned} \tag{7.6.2}$$

having a vanishing divergence. The isometry currents currents and super-currents are obtained by contracting T^{α_k} and J^{α_k} with the Killing vector fields of super-symmetries. Note also that the super current

$$J^\alpha = \overline{\nu_R} T^\alpha_l \Gamma^l \Psi \tag{7.6.3}$$

has a vanishing divergence.

By using the covariant constancy of the right-handed neutrino spinor, one finds that the divergence of the super current reduces to

$$D_\alpha J^{\alpha k} = \overline{\nu_R} \Gamma^k T^\alpha_l \Gamma^l D_\alpha \Psi \ . \tag{7.6.4}$$

The requirement that this current vanishes is guaranteed if one assumes that Kähler-Dirac equation

$$\hat{\Gamma}^\alpha D_\alpha \Psi = 0 \ ,$$
$$\hat{\Gamma}^\alpha = T^\alpha_l \Gamma^l \ . \quad (7.6.5)$$

This equation must be derivable from a Kähler-Dirac action. It indeed is. The action is given by

$$L = \overline{\Psi}\hat{\Gamma}^\alpha D_\alpha \Psi \ . \quad (7.6.6)$$

Thus the variational principle exists. For this variational principle induced gamma matrices are replaced with Kähler-Dirac gamma matrices and the requirement

$$D_\mu \hat{\Gamma}^\mu = 0 \quad (7.6.7)$$

guaranteeing that super-symmetry is identically satisfied if the bosonic field equations are satisfied. For the ordinary Dirac action this condition would lead to the minimal surface property. What sounds strange that the essentially hydrodynamical equations defined by Kähler action have fermionic counterpart: this is very far from intuitive expectations raised by ordinary Dirac equation and something which one might not guess without taking super-symmetry very seriously.

As a matter fact, any mode of Kähler-Dirac equation contracted with second quantized induced spinor field or its conjugate defines a conserved super charge. Also super-symplectic Noether charges and their super counterparts can be assigned to symplectic generators as Noether charges but they need not be conserved.

Second quantization of the K-D action

Second quantization of Kähler-Dirac action is crucial for the construction of the Kähler metric of world of classical worlds as anti-commutators of gamma matrices identified as super-symplectic Noether charges. To get a unique result, the anti-commutation relations must be fixed uniquely. This has turned out to be far from trivial.

1. *Canonical quantization works after all*

The canonical manner to second quantize fermions identifies spinorial canonical momentum densities and their conjugates as $\overline{\Pi} = \partial L_{K_D}/\partial_\Psi = \overline{\Psi}\Gamma^t$ and their conjugates. The vanishing of Γ^t at points, where the induced Kähler form J vanishes can cause problems since anti-commutation relations are not internally consistent anymore. This led me to give up the canonical quantization and to consider various alternatives consistent with the possibility that J vanishes. They were admittedly somewhat ad hoc. Correct (anti-)commutation relations for various fermionic Noether currents seem however to fix the anti-commutation relations to the standard ones. It seems that it is better to be conservative: the canonical method is heavily tested and turned out to work quite nicely.

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Consider first the 4-D situation without the localization to 2-D string world sheets. The canonical anti-commutation relations would state $\{\overline{\Pi}, \Psi\} = \delta^3(x, y)$ at the space-like boundaries of the string world sheet at either boundary of CD. At points where J and thus T^t vanishes, canonical momentum density vanishes identically and the equation seems to be inconsistent.

If fermions are localized at string world sheets assumed to always carry a non-vanishing J at their boundaries at the ends of space-time surfaces, the situation changes since Γ^t is non-vanishing. The localization to string world sheets, which are not vacua saves the situation. The problem is that the limit when string approaches vacuum could be very singular and discontinuous. In the case of elementary particle strings are associated with flux tubes carrying monopole fluxes so that the problem disappears.

It is better to formulate the anti-commutation relations for the modes of the induced spinor field. By starting from

$$\{\overline{\Pi}(x), \Psi(y)\} = \delta^1(x,y) \tag{7.6.8}$$

and contracting with $\Psi(x)$ and $\Pi(y)$ and integrating, one obtains using orthonormality of the modes of Ψ the result

$$\{b_m^\dagger, b_n\} = \gamma^0 \delta_{m,n} \tag{7.6.9}$$

holding for the nodes with non-vanishing norm. At the limit $J \rightarrow 0$ there are no modes with non-vanishing norm so that one avoids the conflict between the two sides of the equation.

The proposed anti-commutator would realize the idea that the fermions are massive. The following alternative starts from the assumption of 8-D light-likeness.

2. Does one obtain the analogy of SUSY algebra? In super Poincare algebra anti-commutators of super-generators give translation generator: anti-commutators are proportional to $p^k\sigma_k$. Could it be possible to have an anti-commutator proportional to the contraction of Dirac operator $p^k\sigma_k$ of 4-momentum with quaternionic sigma matrices having or 8-momentum with octonionic 8-matrices?

This would give good hopes that the GRT limit of TGD with many-sheeted space-time replaced with a slightly curved region of M^4 in long length scales has large $\mathcal{N}$ SUSY as an approximate symmetry: $\mathcal{N}$ would correspond to the maximal number of oscillator operators assignable to the partonic 2-surface. If conformal invariance is exact, it is just the number of fermion states for single generation in standard model.

1. The first promising sign is that the action principle indeed assigns a conserved light-like 8-momentum to each fermion line at partonic 2-surface. Therefore octonionic representation of sigma matrices makes sense and the generalization of standard twistorialization of four-momentum also. 8-momentum can be characterized by a pair of octonionic 2-spinors $(\lambda, \overline{\lambda})$ such that one has $\lambda\ \overline{\lambda}) = p^k\sigma_k$.

2. Since fermion line as string boundary is 1-D curve, the corresponding octonionic sub-spaces is just 1-D complex ray in octonion space and imaginary axes is defined by the associated imaginary octonion unit. Non-associativity and non-commutativity play no role and it is as if one had light like momentum in say z-direction.

3. One can select the ininitial values of spinor modes at the ends of fermion lines in such a way that they have well-defined spin and electroweak spin and one can also form linear superpositions of the spin states. One can also assume that the 8-D algebraic variant of Dirac equation correlating M^4 and CP_2 spins is satisfied.

   One can introduce oscillator operators $b^\dagger_{m,\alpha}$ and $b_{n,\alpha}$ with α denoting the spin. The motivation for why electroweak spin is not included as an index is due to the correlation between spin and electroweak spin. Dirac equation at fermion line implies a complete correlation between directions of spin and electroweak spin: if the directions are same for leptons (convention only), they are opposite for antileptons and for quarks since the product of them defines embedding space chirality which distinguishes between quarks and leptons. Instead of introducing electroweak isospin as an additional correlated index one can introduce 4 kinds of oscillator operators: leptonic and quark-like and fermionic and antifermionic.

4. For definiteness one can consider only fermions in leptonic sector. In hope of getting the analog of SUSY algebra one could modify the fermionic anti-commutation relations such that one has

$$\{b^{\dagger}_{m,\alpha}, b_{n,\beta}\} = \pm i\epsilon_{\alpha\beta}\delta_{m,n} \quad . \tag{7.6.10}$$

Here α is spin label and ϵ is the standard antisymmetric tensor assigned to twistors. The anti-commutator is clearly symmetric also now. The anti-commmutation relations with different signs $\pm$ at the right-hand side distinguish between quarks and leptons and also between fermions and anti-fermions. $\pm = 1$ could be the convention for fermions in lepton sector.

5. One wants combinations of oscillator operators for which one obtains anti-commutators having interpretation in terms of translation generators representing in terms of 8-momentum. The guess would be that the oscillator operators are given by

$$B^{\dagger}_n = b^{\dagger}_{m,\alpha}\lambda^{\alpha} \quad , \quad B_n = \overline{\lambda}^{\alpha} b_{m,\alpha} \quad . \tag{7.6.11}$$

The anti-commutator would in this case be given by

$$\begin{aligned} &\{B^{\dagger}_m, B_n\} = i\overline{\lambda}^{\alpha}\epsilon_{\alpha\beta}\lambda^{\beta}\delta_{m,n} \\ &= Tr(p^k\sigma_k)\delta_{m,n} = 2p^0\delta_{m,n} \quad . \end{aligned} \tag{7.6.12}$$

The inner product is positive for positive value of energy p^0. This form of anti-commutator obviously breaks Lorentz invariance and this us due the number theoretic selection of preferred time direction as that for real octonion unit. Lorentz invariance is saved by the fact that there is a moduli space for the choices of the quaternion units parameterized by Lorentz boosts for CD.

The anti-commutator vanishes for covariantly constant antineutrino so that it does not generate sparticle states. Only fermions with non-vanishing four-momentum do so and the resulting algebra is very much like that associated with a unitary representation of super Poincare algebra.

6. The recipe gives one helicity state for lepton in given mode m (conformal weight). One has also antilepton with opposite helicity with $\pm = -1$ in the formula defining the anti-commutator. In the similar manner one obtains quarks and antiquarks.

7. Contrary to the hopes, one did not obtain the anti-commutator $p^k\sigma_k$ but $Tr(p^0\sigma_0)$. $2p^0$ is however analogous to the action of Dirac operator $p^k\sigma_k$ to a massless spinor mode with "wrong" helicity giving $2p^0\sigma^0$. Massless modes with wrong helicity are expected to appear in the fermionic propagator lines in TGD variant of twistor approach. Hence one might hope that the resulting algebra is consistent with SUSY limit.

   The presence of 8-momentum at each fermion line would allow also to consider the introduction of anti-commutators of form $p^k(8)\sigma_k$ directly making $\mathcal{N} = 8$ SUSY at parton level manifest. This expression restricts for time-like M^4 momenta always to quaternion and one obtains just the standard picture.

8. Only the fermionic states with vanishing conformal weight seem to be realized if the conformal symmetries associated with the spinor modes are realized as gauge symmetries. Super-generators would correspond to the fermions of single generation standard model: 4+4 =8 states altogether. Interestingly, $\mathcal{N} = 8$ correspond to the maximal SUSY for super-gravity. Right-handed neutrino would obviously generate the least broken SUSY. Also now mixing of M^4 helicities induces massivation and symmetry breaking so that even this SUSY is broken. One must however distinguish this SUSY from the super-symplectic conformal symmetry. The space in which SUSY would be realized would be partonic 2-surfaces and this distinguishes it from the usual SUSY. Also the conservation of fermion number and absence of Majorana spinors is an important distinction.

3. What about quantum deformations of the fermionic oscillator algebra?

Quantum deformation introducing braid statistics is of considerable interest. Quantum deformations are essentially 2-D phenomenon, and the experimental fact that it indeed occurs gives a further strong support for the localization of spinors at string world sheets. If the existence of anyonic phases is taken completely seriously, it supports the existence of the hierarchy of Planck constants and TGD view about dark matter. Note that the localization also at partonic 2-surfaces cannot be excluded yet.

I have wondered whether quantum deformation could relate to the hierarchy of Planck constants in the sense that $n = h_{eff}/h$ corresponds to the value of deformation parameter $q = exp(i2\pi/n)$.

A q-deformation of Clifford algebra of WCW gamma matrices is required. Clifford algebra is characterized in terms of anti-commutators replaced now by q-anticommutators. The natural identification of gamma matrices is as complexified gamma matrices. For q-deformation q-anti-commutators would define WCW Kähler metric. The commutators of the supergenerators should still give anti-symmetric sigma matrices. The q-anticommutation relations should be same in the entire sector of WCW considered and be induced from the q-anticommutation relations for the oscillator operators of induced spinor fields at string world sheets, and reflect the fact that permutation group has braid group as covering group in 2-D case so that braid statistics becomes possible.

In [A53] (`http://tinyurl.com/y9e6pg4d`) the q-deformations of Clifford algebras are discussed, and this discussion seems to apply in TGD framework.

1. It is assumed that a Lie-algebra g has action in the Clifford algebra. The q-deformations of Clifford algebra is required to be consistent with the q-deformation of the universal enveloping algebra Ug.
2. The simplest situation corresponds to group $su(2)$ so that Clifford algebra elements are labelled by spin $\pm 1/2$. In this case the q-anticommutor for creation operators for spin up states reduces to an anti-commutator giving q-deformation I_q of unit matrix but for the spin down states one has genuine q-anti-commutator containing besides I_q also number operator for spin up states at the right hand side.
3. The undeformed anti-commutation relations can be witten as

$$P_{ij}^{+kl} a_k a_l = 0 \ , \quad P_{ij}^{+kl} a_k^\dagger a_l^\dagger = 0 \ , \quad a^i a_j^\dagger + P_{jk}^{ih} a_h^\dagger a^k = \delta_j^i 1 \ . \tag{7.6.13}$$

   Here $P_{ij}^{kl} = \delta_l^i \delta_k^j$ is the permutator and $P_{ij}^{+kl} = (1+P)/2$ is projector. The q-deformation reduces to a replacement of the permutator and projector with q-permutator P_q and q-projector and P_q^+, which are both fixed by the quantum group.
4. Also the condition that deformed algebra has same Poincare series as the original one is posed. This says that the representation content is not changed that is the dimensions of summands in a representation as direct sum of graded sub-spaces are same for algebra and its q-deformation. If one has quantum group in a strict sense of the word (quasi-triangularity (genuine braid group) rather that triangularity requiring that the square of the deformed permutator P_q is unit matrix, one can have two situations.
   (a) $g = sl(N)$ (special linear group such as $SL(2,F)$, $F = R, C$) or $g = Sp(N = 2n)$ (symplectic group such as $Sp(2) = SL(2,R)$), which is subgroup of $sl(N)$. Creation (annihilation-) operators must form the N-dimensional defining representation of g.
   (b) $g = sl(N)$ and one has direct sum of M N-dimensional defining representations of g. The M copies of representation are ordered so that they can be identified as strands of braid so that the deformation makes sense at the space-like ends of string world sheet naturally. q-projector is proportional to so called universal R-matrix.
5. It is also shown that q-deformed oscillator operators can be expressed as polynomials of the ordinary ones.

The following argument suggest that the g must correspond to the minimal choices $sl(2,R)$ (or $su(2)$) in TGD framework.

1. The q-Clifford algebra structure of WCW should be induced from that for the fermionic oscillator algebra. g cannot correspond to $su(2)_{spin} \times su(2)_{ew}$ since spin and weak isospin label fermionic oscillator operators beside conformal weights but must relate closely to this group. The physical reason is that the separate conservation of quark and lepton numbers and light-likeness in 8-D sense imply correlations between the components of the spinors and reduce g.
2. For a given H-chirality (quark/ lepton) 8-D light-likeness forced by massless Dirac equation at the light-like boundary of the string world sheet at parton orbit implies correlation between M^4 and CP_2 chiralities. Hence there are 4+4 spinor components corresponding to fermions and antifermions with physical (creation oeprators) and unphysical (annihilation operators) polarizations. This allows two creation operators with given H-chirality (quark or lepton) and fermion number. Same holds true for antifermions. By fermion number conservation one obtains a reduction to $SU(2)$ doublets and the quantum group would be $sl(2) = sp(2)$ for which "special linear" implies "symplectic".

7.7 Ricci Flatness And Divergence Cancelation

Divergence cancelation in WCW integration requires Ricci flatness and in this section the arguments in favor of Ricci flatness are discussed in detail.

7.7.1 Inner Product From Divergence Cancelation

Forgetting the delicacies related to the non-determinism of the Kähler action, the inner product is given by integrating the usual Fock space inner product defined at each point of WCW over the reduced WCW containing only the 3-surfaces Y^3 belonging to $\delta H = \delta M^4_+ \times CP_2$ ("light-cone boundary") using the exponent $exp(K)$ as a weight factor:

$$\begin{aligned}
\langle \Psi_1 | \Psi_2 \rangle &= \int \overline{\Psi}_1(Y^3)\Psi_2(Y^3) exp(K)\sqrt{G} dY^3 \quad , \\
\overline{\Psi_1}(Y^3)\Psi_2(Y^3) &\equiv \langle \Psi_1(Y^3)|\Psi_2(Y^3)\rangle_{Fock} \quad . \qquad (7.7.1)
\end{aligned}$$

The degeneracy for the preferred extremals of Kähler action implies additional summation over the degenerate extremals associated with Y^3. The restriction of the integration on light cone boundary is Diff^4 invariant procedure and resolves in elegant manner the problems related to the integration over Diff^4 degrees of freedom. A variant of the inner product is obtained dropping the bosonic vacuum functional $exp(K)$ from the definition of the inner product and by assuming that it is included into the spinor fields themselves. Probably it is just a matter of taste how the necessary bosonic vacuum functional is included into the inner product: what is essential that the vacuum functional $exp(K)$ is somehow present in the inner product.

The unitarity of the inner product follows from the unitary of the Fock space inner product and from the unitarity of the standard L^2 inner product defined by WCW integration in the set of the L^2 integrable scalar functions. It could well occur that $Diff^4$ invariance implies the reduction of WCW integration to $C(\delta H)$.

Consider next the bosonic integration in more detail. The exponent of the Kähler function appears in the inner product also in the context of the finite dimensional group representations. For the representations of the non-compact groups (say $SL(2,R)$) in coset spaces (now $SL(2,R)/U(1)$ endowed with Kähler metric) the exponent of Kähler function is necessary in order to get square integrable representations [B21]. The scalar product for two complex valued representation functions is defined as

$$(f,g) = \int \overline{f} g exp(nK)\sqrt{g} dV \quad . \qquad (7.7.2)$$

By unitarity, the exponent is an integer multiple of the Kähler function. In the present case only the possibility $n = 1$ is realized if one requires a complete cancelation of the determinants. In finite dimensional case this corresponds to the restriction to single unitary representation of the group in question.

The sign of the action appearing in the exponent is of decisive importance in order to make theory stable. The point is that the theory must be well defined at the limit of infinitely large system. Minimization of action is expected to imply that the action of infinitely large system is bound from above: the generation of electric Kähler fields gives negative contributions to the action. This implies that at the limit of the infinite system the average action per volume is non-positive. For systems having negative average density of action vacuum functional $exp(K)$ vanishes so that only configurations with vanishing average action per volume have significant probability. On the other hand, the choice $exp(-K)$ would make theory unstable: probability amplitude would be infinite for all configurations having negative average action per volume. In the fourth part of the book it will be shown that the requirement that average Kähler action per volume cancels has important cosmological consequences.

Consider now the divergence cancelation in the bosonic integration. One can develop the Kähler function as a Taylor series around maximum of Kähler function and use the contravariant Kähler metric as a propagator. Gaussian and metric determinants cancel each other for a unique vacuum functional. Ricci flatness guarantees that metric determinant is constant in complex coordinates so that one avoids divergences coming from it. The non-locality of the Kähler function as a functional of the 3-surface serves as an additional regulating mechanism: if $K(X^3)$ were a local functional of X^3 one would encounter divergences in the perturbative expansion.

The requirement that quantum jump corresponds to a quantum measurement in the sense of quantum field theories implies that quantum jump involves localization in zero modes. Localization in the zero modes implies automatically p-adic evolution since the decomposition of the WCW into sectors D_P labeled by the infinite primes P is determined by the corresponding decomposition in zero modes. Localization in zero modes would suggest that the calculation of the physical predictions does not involve integration over zero modes: this would dramatically simplify the calculational apparatus of the theory. Probably this simplification occurs at the level of practical calculations if U-matrix separates into a product of matrices associated with zero modes and fiber degrees of freedom.

One must also calculate the predictions for the ratios of the rates of quantum transitions to different values of zero modes and here one cannot actually avoid integrals over zero modes. To achieve this one is forced to define the transition probabilities for quantum jumps involving a localization in zero modes as

$$P(x, \alpha \to y, \beta) = \sum_{r,s} |S(r, \alpha \to s, \beta)|^2 |\Psi_r(x)|^2 |\Psi_s(y)|^2 \ ,$$

where x and y correspond to the zero mode coordinates and r and s label a complete state functional basis in zero modes and $S(r, m \to s, n)$ involves integration over zero modes. In fact, only in this manner the notion of the localization in the zero modes makes mathematically sense at the level of S-matrix. In this case also unitarity conditions are well-defined. In zero modes state function basis can be freely constructed so that divergence difficulties could be avoided. An open question is whether this construction is indeed possible.

Some comments about the actual evaluation of the bosonic functional integral are in order.

1. Since WCW metric is degenerate and the bosonic propagator is essentially the contravariant metric, bosonic integration is expected to reduce to an integration over the zero modes. For instance, isometry invariants are variables of this kind. These modes are analogous to the parameters describing the conformal equivalence class of the orbit of the string in string models.

2. α_K is a natural small expansion parameter in WCW integration. It should be noticed that α_K, when defined by the criticality condition, could also depend on the coordinates parameterizing the zero modes.

3. Semiclassical approximation, which means the expansion of the functional integral as a sum over the extrema of the Kähler function, is a natural approach to the calculation of the

bosonic integral. Symmetric space property suggests that for the given values of the zero modes there is only single extremum and corresponds to the maximum of the Kähler function. There are theorems (Duistermaat-Hecke theorem) stating that semiclassical approximation is exact for certain systems (for example for integrable systems [A44]). Symmetric space property suggests that Kähler function might possess the properties guaranteeing the exactness of the semiclassical approximation. This would mean that the calculation of the integral $\int exp(K)\sqrt{G}dY^3$ and even more complex integrals involving WCW spinor fields would be completely analogous to a Gaussian integration of free quantum field theory. This kind of reduction actually occurs in string models and is consistent with the criticality of the Kähler coupling constant suggesting that all loop integrals contributing to the renormalization of the Kähler action should vanish. Also the condition that WCW integrals are continuable to p-adic number fields requires this kind of reduction.

7.7.2 Why Ricci Flatness

It has been already found that the requirement of divergence cancelation poses extremely strong constraints on the metric of the WCW. The results obtained hitherto are the following.

1. If the vacuum functional is the exponent of Kähler function one gets rid of the divergences resulting from the Gaussian determinants and metric determinants: determinants cancel each other.
2. The non-locality of the Kähler action gives good hopes of obtaining divergence free perturbation theory.

The following arguments show that Ricci flatness of the metric is a highly desirable property.

1. Dirac operator should be a well defined operator. In particular its square should be well defined. The problem is that the square of Dirac operator contains curvature scalar, which need not be finite since it is obtained via two infinite-dimensional trace operations from the curvature tensor. In case of loop spaces [A43] the Kähler property implies that even Ricci tensor is only conditionally convergent. In fact, loop spaces with Kähler metric are Einstein spaces (Ricci tensor is proportional to metric) and Ricci scalar is infinite.

   In 3-dimensional case situation is even worse since the trace operation involves 3 summation indices instead of one! The conclusion is that Ricci tensor had better to vanish in vibrational degrees of freedom.
2. For Ricci flat metric the determinant of the metric is constant in geodesic complex coordinates as is seen from the expression for Ricci tensor [A49]

$$R_{k\bar{l}} = \partial_k \partial_{\bar{l}} ln(det(g)) \tag{7.7.3}$$

   in Kähler metric. This obviously simplifies considerably functional integration over WCW: one obtains just the standard perturbative field theory in the sense that metric determinant gives no contributions to the functional integration.
3. The constancy of the metric determinant results not only in calculational simplifications: it also eliminates divergences. This is seen by expanding the determinant as a functional Taylor series with respect to the coordinates of WCW. In local complex coordinates the first term in the expansion of the metric determinant is determined by Ricci tensor

$$\delta\sqrt{g} \propto R_{k\bar{l}} z^k \bar{z}^l \ . \tag{7.7.4}$$

   In WCW integration using standard rules of Gaussian integration this term gives a contribution proportional to the contraction of the propagator with Ricci tensor. But since the propagator is just the contravariant metric one obtains Ricci scalar as result. So, in order to avoid divergences, Ricci scalar must be finite: this is certainly guaranteed if Ricci tensor vanishes.

4. The following group theoretic argument suggests that Ricci tensor either vanishes or is divergent. The holonomy group of the WCW is a subgroup of $U(n = \infty)$ ($D = 2n$ is the dimension of the Kähler manifold) by Kähler property and Ricci flatness is guaranteed if the $U(1)$ factor is absent from the holonomy group. In fact Ricci tensor is proportional to the trace of the $U(1)$ generator and since this generator corresponds to an infinite dimensional unit matrix the trace diverges: therefore given element of the Ricci tensor is either infinite or vanishes. Therefore the vanishing of the Ricci tensor seems to be a mathematical necessity. This naïve argument doesn't hold true in the case of loop spaces, for which Kähler metric with finite non-vanishing Ricci tensor exists [A43] . Note however that also in this case the sum defining Ricci tensor is only conditionally convergent.

There are indeed good hopes that Ricci tensor vanishes. By the previous argument the vanishing of the Ricci tensor is equivalent with the absence of divergences in WCW integration. That divergences are absent is suggested by the non-locality of the Kähler function as a functional of 3-surface: the divergences of local field theories result from the locality of interaction vertices. Ricci flatness in vibrational degrees of freedom is not only necessary mathematically. It is also appealing physically: one can regard Ricci flat WCW as a vacuum solution of Einstein's equations $G^{\alpha\beta} = 0$.

7.7.3 Ricci Flatness And Hyper Kähler Property

Ricci flatness property is guaranteed if WCW geometry is Hyper Kähler [A76, A34] (there exists 3 covariantly constant antisymmetric tensor fields, which can be regarded as representations of quaternionic imaginary units). Hyper Kähler property guarantees Ricci flatness because the contractions of the curvature tensor appearing in the components of the Ricci tensor transform to traces over Lie algebra generators, which are $SU(n)$ generators instead of $U(n)$ generators so that the traces vanish. In the case of the loop spaces left invariance implies that Ricci tensor in the vibrational degrees is a multiple of the metric tensor so that Ricci scalar has an infinite value. This is basically due to the fact that Kac-Moody algebra has $U(1)$ central extension.

Consider now the arguments in favor of Ricci flatness of the WCW.

1. The symplectic algebra of δM^4_+ takes effectively the role of the $U(1)$ extension of the loop algebra. More concretely, the $SO(2)$ group of the rotation group $SO(3)$ takes the role of $U(1)$ algebra. Since volume preserving transformations are in question, the traces of the symplectic generators vanish identically and in finite-dimensional this should be enough for Ricci flatness even if Hyper Kähler property is not achieved.

2. The comparison with CP_2 allows to link Ricci flatness with conformal invariance. The elements of the Ricci tensor are expressible in terms of traces of the generators of the holonomy group $U(2)$ at the origin of CP_2, and since $U(1)$ generator is non-vanishing at origin, the Ricci tensor is non-vanishing. In recent case the origin of CP_2 is replaced with the maximum of Kähler function and holonomy group corresponds to super-symplectic generators labelled by integer valued real parts k_1 of the conformal weights $k = k_1 + i\rho$. If generators with $k_1 = n$ vanish at the maximum of the Kähler function, the curvature scalar should vanish at the maximum and by the symmetric space property everywhere. These conditions correspond to Virasoro conditions in super string models.

   A possible source of difficulties are the generators having $k_1 = 0$ and resulting as commutators of generators with opposite real parts of the conformal weights. It might be possible to assume that only the conformal weights $k = k_1 + i\rho$, $k_1 = 0, 1, ...$ are possible since it is the imaginary part of the conformal weight which defines the complexification in the recent case. This would mean that the commutators involve only positive values of k_1.

3. In the infinite-dimensional case the Ricci tensor involves also terms which are non-vanishing even when the holonomy algebra does not contain $U(1)$ factor. It will be found that symmetric space property guarantees Ricci flatness even in this case and the reason is essentially the vanishing of the generators having $k_1 = n$ at the maximum of Kähler function.

There are also arguments in favor of the Hyper Kähler property.

1. The dimensions of the embedding space and space-time are 8 and 4 respectively so that the dimension of WCW in vibrational modes is indeed multiple of four as required by Hyper

Kähler property. Hyper Kähler property requires a quaternionic structure in the tangent space of WCW. Since any direction on the sphere S^2 defined by the linear combinations of quaternionic imaginary units with unit norm defines a particular complexification physically, Hyper Kähler property means the possibility to perform complexification in S^2-fold ways.

2. S^2-fold degeneracy is indeed associated with the definition of the complex structure of WCW. First of all, the direction of the quantization axis for the spherical harmonics or for the eigen states of Lorentz Cartan algebra at δM^4_+ can be chosen in S^2-fold ways. Quaternion conformal invariance means Hyper Kähler property almost by definition and the S^2-fold degeneracy for the complexification is obvious in this case.

If these naïve arguments survive a more critical inspection, the conclusion would be that the effective 2-dimensionality of light like 3-surfaces implying generalized conformal and symplectic symmetries would also imply Hyper Kähler property of WCW and make the theory well-defined mathematically. This obviously fixes the dimension of space-time surfaces as well as the dimension of Minkowski space factor of the embedding space.

In the sequel we shall show that Ricci flatness is guaranteed provided that the holonomy group of WCW is isomorphic to some subgroup of $SU(n = \infty)$ instead of $U(n = \infty)$ (n is the complex dimension of WCW) implied by the Kähler property of the metric. We also derive an expression for the Ricci tensor in terms of the structure constants of the isometry algebra and WCW metric. The expression for the Ricci tensor is formally identical with that obtained by Freed for loop spaces: the only difference is that the structure constants of the finite-dimensional group are replaced with the group $Can(\delta H)$. Also the arguments in favor of Hyper Kähler property are discussed in more detail.

7.7.4 The Conditions Guaranteeing Ricci Flatness

In the case of Kähler geometry Ricci flatness condition can be characterized purely Lie-algebraically: the holonomy group of the Riemann connection, which in general is subgroup of $U(n)$ for Kähler manifold of complex dimension n, must be subgroup of $SU(n)$ so that the Lie-algebra of this group consists of traceless matrices. This condition is easy to derive using complex coordinates. Ricci tensor is given by the following expression in complex vielbein basis

$$R^{A\bar{B}} = R^{A\bar{C}B}_{\ \ \ \ \bar{C}} \ , \tag{7.7.5}$$

where the latter summation is only over the antiholomorphic indices $\bar{C}$. Using the cyclic identities

$$\sum_{cycl\ \bar{C}B\bar{D}} R^{A\bar{C}B\bar{D}} = 0 \ , \tag{7.7.6}$$

the expression for Ricci tensor reduces to the form

$$R^{A\bar{B}} = R^{A\bar{B}C}_{\ \ \ \ C} \ , \tag{7.7.7}$$

where the summation is only over the holomorphic indices C. This expression can be regarded as a trace of the curvature tensor in the holonomy algebra of the Riemann connection. The trace is taken over holomorphic indices only: the traces over holomorphic and anti-holomorphic indices cancel each other by the antisymmetry of the curvature tensor. For Kähler manifold holonomy algebra is subalgebra of $U(n)$, when the complex dimension of manifold is n and Ricci tensor vanishes if and only if the holonomy Lie-algebra consists of traceless matrices, or equivalently: holonomy group is subgroup of $SU(n)$. This condition is expected to generalize also to the infinite-dimensional case.

We shall now show that if WCW metric is Kähler and possesses infinite-dimensional isometry algebra with the property that its generators form a complete basis for the tangent space (every tangent vector is expressible as a superposition of the isometry generators plus zero norm vector) it is possible to derive a representation for the Ricci tensor in terms of the structure constants of the isometry algebra and of the components of the metric and its inverse in the basis formed by the isometry generators and that Ricci tensor vanishes identically for the proposed complexification of the WCW provided the generators $\{H_{A,m\neq 0}, H_{B,n\neq 0}\}$ correspond to zero norm vector fields of WCW.

The general definition of the curvature tensor as an operator acting on vector fields reads

$$R(X,Y)Z = [\nabla_X, \nabla_Y]Z - \nabla_{[X,Y]}Z \ . \tag{7.7.8}$$

If the vector fields considered are isometry generators the covariant derivative operator is given by the expression

$$\begin{aligned} \nabla_X Y &= (Ad_X Y - Ad^*_X Y - Ad^*_Y X)/2 \ , \\ (Ad^*_X Y, Z) &= (Y, Ad_X Z) \ , \end{aligned} \tag{7.7.9}$$

where $Ad_X Y = [X,Y]$ and Ad^*_X denotes the adjoint of Ad_X with respect to WCW metric.

In the sequel we shall assume that the vector fields in question belong to the basis formed by the isometry generators. The matrix representation of Ad_X in terms of the structure constants $C_{X,Y:Z}$ of the isometry algebra is given by the expression

$$\begin{aligned} Ad^m_{Xn} &= C_{X,Y:Z}\hat{Y}_n Z^m \ , \\ [X,Y] &= C_{X,Y:Z} Z \ , \\ \hat{Y} &= g^{-1}(Y,V)V \ , \end{aligned} \tag{7.7.10}$$

where the summation takes place over the repeated indices and $\hat{Y}$ denotes the dual vector field of Y with respect to the WCW metric. From its definition one obtains for Ad^*_X the matrix representation

$$\begin{aligned} Ad^{*m}_{Xn} &= C_{X,Y:Z}\hat{Y}^m Z_n \ , \\ Ad^*_X Y &= C_{X,U:V}\, g(Y,U) g^{-1}(V,W) W = g(Y,U) g^{-1}([X,U],W) W \ , \end{aligned} \tag{7.7.11}$$

where the summation takes place over the repeated indices.

Using the representations of ∇_X in terms of Ad_X and its adjoint and the representations of Ad_X and Ad^*_X in terms of the structure constants and some obvious identities (such as $C_{[X,Y],Z:V} = C_{X,Y:U} C_{U,Z:V}$) one can by a straightforward but tedious calculation derive a more detailed expression for the curvature tensor and Ricci tensor. Straightforward calculation of the Ricci tensor has however turned to be very tedious even in the case of the diagonal metric and in the following we shall use a more convenient representation [A43] of the curvature tensor applying in case of the Kähler geometry.

The expression of the curvature tensor is given in terms of the so called Toeplitz operators T_X defined as linear operators in the "positive energy part" G_+ of the isometry algebra spanned by the $(1,0)$ parts of the isometry generators. In present case the positive and negative energy parts and cm part of the algebra can be defined just as in the case of loop spaces:

$$\begin{aligned} G_+ &= \{H^{Ak} | k > 0\} \ , \\ G_- &= \{H^{Ak} | k < 0\} \ , \\ G_0 &= \{H^{Ak} | k = 0\} \ . \end{aligned} \tag{7.7.12}$$

Here H^{Ak} denote the Hamiltonians generating the symplectic transformations of δH. The positive energy generators with non-vanishing norm have positive radial scaling dimension: $k \geq 0$, which corresponds to the imaginary part of the scaling momentum $K = k_1 + i\rho$ associated with the factors $(r_M/r_0)^K$. A priori the spectrum of ρ is continuous but it is quite possible that the spectrum of ρ is discrete and $\rho = 0$ does not appear at all in the spectrum in the sense that the flux Hamiltonians associated with $\rho = 0$ elements vanish for the maximum of Kähler function which can be taken to be the point where the calculations are done.

T_X differs from Ad_X in that the negative energy part of $Ad_X Y = [X,Y]$ is dropped away:

$$\begin{aligned} T_X : G_+ &\rightarrow G_+ \ , \\ Y &\rightarrow [X,Y]_+ \ . \end{aligned} \tag{7.7.13}$$

Here "+" denotes the projection to "positive energy" part of the algebra. Using Toeplitz operators one can associate to various isometry generators linear operators $\Phi(X_0)$, $\Phi(X_-)$ and $\Phi(X_+)$ acting on G_+:

$$\begin{aligned} \Phi(X_0) &= T_{X_0} \ , \ X_0 \varepsilon G_0 \ , \\ \Phi(X_-) &= T_{X_-} \ , \ X_- \varepsilon G_- \ , \\ \Phi(X_+) &= -T^*_{X_-} \ , \ X_+ \varepsilon G_+ \ . \end{aligned} \tag{7.7.14}$$

Here "*" denotes hermitian conjugate in the diagonalized metric: the explicit representation $\Phi(X_+)$ is given by the expression [A43]

$$\begin{aligned} \Phi(X_+) &= D^{-1} T_{X_-} D \ , \\ D X_+ &= d(X) X_- \ , \\ d(X) &= g(X_-, X_+) \ . \end{aligned} \tag{7.7.15}$$

Here $d(X)$ is just the diagonal element of metric assumed to be diagonal in the basis used. denotes the conformal factor associated with the metric.

The representations for the action of ,$\Phi(X_0)$, $\Phi(X_-)$ and $\Phi(X_+)$ in terms of metric and structure constants of the isometry algebra are in the case of the diagonal metric given by the expressions

$$\begin{aligned} \Phi(X_0) Y_+ &= C_{X_0, Y_+ : U_+} U_+ \ , \\ \Phi(X_-) Y_+ &= C_{X_-, Y_+ : U_+} U_+ \ , \\ \Phi(X_+) Y_+ &= \frac{d(Y)}{d(U)} C_{X_-, Y_- : U_-} U_+ \ . \end{aligned} \tag{7.7.16}$$

The expression for the action of the curvature tensor in positive energy part G_+ of the isometry algebra in terms of the these operators is given as [A43] :

$$R(X,Y) Z_+ = \{[\Phi(X), \Phi(Y)] - \Phi([X,Y])\} Z_+ \ . \tag{7.7.17}$$

The calculation of the Ricci tensor is based on the observation that for Kähler manifolds Ricci tensor is a tensor of type $(1,1)$, and therefore it is possible to calculate Ricci tensor as the trace of the curvature tensor with respect to indices associated with G_+.

$$Ricci(X_+, Y_-) = (\hat{Z}_+, R(X_+, Y_-) Z_+) \equiv Trace(R(X_+, Y_-)) \ , \tag{7.7.18}$$

where the summation over Z_+ generators is performed.

Using the explicit representations of the operators Φ one obtains the following explicit expression for the Ricci tensor

$$\begin{aligned} Ricci(X_+, Y_-) &= Trace\{[D^{-1} T_{X_+} D, T_{Y_-}] - T_{[X_+, Y_-]_{|G_0 + G_-}} \\ &- D^{-1} T_{[X_+, Y_-]_{|G_+}} D\} \ . \end{aligned} \tag{7.7.19}$$

This expression is identical to that encountered in case of loop spaces and the following arguments are repetition of those applying in the case of loop spaces.

The second term in the Ricci tensor is the only term present in the finite-dimensional case. This term vanishes if the Lie-algebra in question consists of traceless matrices. Since symplectic transformations are volume-preserving the traces of Lie-algebra generators vanish so that this term is absent. The last term gives a non-vanishing contribution to the trace for the same reason.

The first term is quadratic in structure constants and does not vanish in case of loop spaces. It can be written explicitly using the explicit representations of the various operators appearing in the formula:

$$\begin{aligned} Trace\{[D^{-1}T_{X_-}D, T_{Y_-}]\} &= \sum_{Z_+,U_+} [C_{X_-,U_-:Z_-}C_{Y_-,Z_+:U_+}\frac{d(U)}{d(Z)} \\ &- C_{X_-,Z_-:U_-}C_{Y_-,U_+:Z_+}\frac{d(Z)}{d(U)}] \ . \end{aligned} \tag{7.7.20}$$

Each term is antisymmetric under the exchange of U and Z and one might fail to conclude that the sum vanishes identically. This is not the case. By the diagonality of the metric with respect to radial quantum number, one has $m(X_-) = m(Y_-)$ for the non-vanishing elements of the Ricci tensor. Furthermore, one has $m(U) = m(Z) - m(Y)$, which eliminates summation over $m(U)$ in the first term and summation over $m(Z)$ in the second term. Note however, that summation over other labels related to symplectic algebra are present.

By performing the change $U \rightarrow Z$ in the second term one can combine the sums together and as a result one has finite sum

$$\begin{aligned} \sum_{0<m(Z)<m(X)} [C_{X_-,U_-:Z_-}C_{Y_-,Z_+:U_+}\frac{d(U)}{d(Z)} &= C \sum_{0<m(Z)<m(X)} \frac{m(X)}{m(Z)-m(X)} \ , \\ C &= \sum_{Z,U} C_{X,U:Z}C_{Y,Z:U}\frac{d_0(U)}{d_0(Z)} \ . \end{aligned} \tag{7.7.21}$$

Here the dependence of $d(X) = |m(X)|d_0(X)$ on $m(X)$ is factored out; $d_0(X)$ does not depend on k_X. The dependence on $m(X)$ in the resulting expression factorizes out, and one obtains just the purely group theoretic term C, which should vanish for the space to be Ricci flat.

The sum is quadratic in structure constants and can be visualized as a loop sum. It is instructive to write the sum in terms of the metric in the symplectic degrees of freedom to see the geometry behind the Ricci flatness:

$$C = \sum_{Z,U} g([Y,Z],U)g^{-1}([X,U],Z) \ . \tag{7.7.22}$$

Each term of this sum involves a commutator of two generators with a non-vanishing norm. Since tangent space complexification is inherited from the local coset space, the non-vanishing commutators in complexified basis are always between generators in $Can_{\neq 0}$; that is they do not not belong to rigid $su(2) \times su(3)$.

The condition guaranteeing Ricci flatness at the maximum of Kähler function and thus everywhere is simple. All elements of type $[X_{\neq 0}, Y_{\neq 0}]$ vanish or have vanishing norm. In case of CP_2 Kähler geometry this would correspond to the vanishing of the $U(2)$ generators at the origin of CP_2 (note that the holonomy group is $U(2)$ in case of CP_2). At least formally stronger condition is that the algebra generated by elements of this type, the commutator algebra associated with $Can_{\neq 0}$, consist of elements of zero norm. Already the (possibly) weaker condition implies that adjoint map $Ad_{X\neq 0}$ and its hermitian adjoint $Ad^*_{X_{\neq 0}}$ create zero norm states. Since isometry conditions involve also adjoint action the condition also implies that $Can_{\neq 0}$ acts as isometries. More concrete form for the condition is that all flux factors involving double Poisson bracket and three generators in $Can_{\neq 0}$ vanish:

$$Q_e(\{H_A,\{H_B,H_C\}\}) \; = \; 0 \; , \; \text{for } H_A,H_B,H_C \text{ in } Can_{\neq 0} \; . \qquad (7.7.23)$$

The vanishing of fluxes involving two Poisson brackets and three Hamiltonians guarantees isometry invariance and Ricci flatness and, as found in [K28] , is implied by the $[t,t] \subset h$ property of the Lie-algebra of coset space G/H having symmetric space structure.

The conclusion is that the mere existence of the proposed isometry group (guaranteed by the symmetric space property) implies the vanishing of the Ricci tensor and vacuum Einstein equations. The existence of the infinite parameter isometry group in turn follows basically from the condition guaranteeing the existence of the Riemann connection. Therefore vacuum Einstein equations seem to arise, not only as a consequence of a physically motivated variational principle but as a mathematical consistency condition in infinite dimensional Kähler geometry. The flux representation seems to provide elegant manner to formulate and solve these conditions and isometry invariance implies Ricci flatness.

7.7.5 Is WCW Metric Hyper Kähler?

The requirement that WCW integral integration is divergence free implies that WCW metric is Ricci flat. The so called Hyper-Kähler metrics [A76, A34] , [B42] are particularly nice representatives of Ricci flat metrics. In the following the basic properties of Hyper-Kähler metrics are briefly described and the problem whether Hyper Kähler property could realized in case of $M^4_+ \times CP_2$ is considered.

Hyper-Kähler property

Hyper-Kähler metric is a generalization of the Kähler metric. For Kähler metric metric tensor and Kähler form correspond to the complex numbers 1 and i and therefore define complex structure in the tangent space of the manifold. For Hyper Kähler metric tangent space allows three closed Kähler forms I, J, K, which with respect to the multiplication obey the algebra of quaternionic imaginary units and have square equal to - 1, which corresponds to the metric of Hyper Kähler space.

$$I^2 \; = \; J^2 = K^2 = -1 \;\; IJ = -JI = K, \text{ etc. } . \qquad (7.7.24)$$

To define Kähler structure one must choose one of the Kähler forms or any linear combination of I, J and K with unit norm. The group $SO(3)$ rotates different Kähler structures to each other playing thus the role of quaternion automorphisms. This group acts also as coordinate transformations in Hyper Kähler manifold but in general fails to act as isometries.

If K is chosen to define complex structure then K is tensor of type $(1,1)$ in complex coordinates, I and J being tensors of type $(2,0)+(0,2)$. The forms $I+iJ$ and $I-iJ$ are holomorphic and anti-holomorphic forms of type $(2,0)$ and $(0,2)$ respectively and defined standard step operators I_+ and I_- of $SU(2)$ algebra. The holonomy group of Hyper-Kähler metric is always $Sp(k)$, $k \leq dimM/4$, the group of $k \times k$ unitary matrices with quaternionic entries. This group is indeed subgroup of SU(2k), so that its generators are traceless and Hyper Kähler metric is therefore Ricci flat.

Hyper Kähler metrics have been encountered in the context of 3-dimensional super symmetric sigma models: a necessary prerequisite for obtaining $N = 4$ super-symmetric sigma model is that target space allows Hyper Kähler metric [B42, B8] . In particular, it has been found that Hyper Kähler property is decisive for the divergence cancelation.

Hyper-Kähler metrics arise also in monopole and instanton physics [A34] . The moduli spaces for monopoles have Hyper Kähler property. This suggests that Hyper Kähler property is characteristic for the configuration (or moduli) spaces of 4-dimensional Yang Mills types systems. Since YM action appears in the definition of WCW metric there are hopes that also in present case the metric possesses Hyper-Kähler property.

CP_2 allows what might be called almost Hyper-Kähler structure known as quaternionion structure. This means that the Weil tensor of CP_2 consists of three components in one-one correspondence with components of iso-spin and only one of them- the one corresponding to Kähler form- is covariantly constant. The physical interpretation is in terms of electroweak symmetry breaking selecting one isospin direction as a favored direction.

Does the "almost" Hyper-Kähler structure of CP_2 lift to a genuine Hyper-Kähler structure in WCW?

The Hyper-Kähler property of WCW metric does not seem to be in conflict with the general structure of TGD.

1. In string models the dimension of the "space-time" is two and Weyl invariance and complex structures play a decisive role in the theory. In present case the dimension of the space-time is four and one therefore might hope that quaternions play a similar role. Indeed, Weyl invariance implies YM action in dimension 4 and as already mentioned moduli spaces of instantons and monopoles enjoy the Hyper Kähler property.

2. Also the dimension of the embedding space is important. The dimension of Hyper Kähler manifold must be multiple of 4. The dimension of WCW is indeed infinite multiple of 8: each vibrational mode giving one "8".

3. The complexification of the WCW in symplectic degrees of freedom is inherited from $S^2 \times CP_2$ and CP_2 Kähler form defines the symplectic form of WCW. The point is that CP_2 Weyl tensor has 3 covariantly constant components, having as their square metric apart from sign. One of them is Kähler form, which is closed whereas the other two are non-closed forms and therefore fail to define Kähler structure. The group $SU(2)$ of electro-weak isospin rotations rotate these forms to each other. It would not be too surprising if one could identify WCW counterparts of these forms as representations of quaternionic units at the level of WCW. The failure of the Hyper Kähler property at the level of CP_2 geometry is due to the electro-weak symmetry breaking and physical intuition (in particular, p-adic mass calculations [K70]) suggests that electro-weak symmetry might not be broken at the level of WCW geometry).

A possible topological obstruction for the Hyper Kähler property is related to the cohomology of WCW: the three Kähler forms must be co-homologically trivial as is clear from the following argument. If any of 3 quaternionic 2-form is cohomologically nontrivial then by $SO(3)$ symmetry rotating Kähler forms to each other all must be co-homologically nontrivial. On the other hand, electro-weak isospin rotation leads to a linear combination of 3 Kähler forms and the flux associated with this form is in general not integer valued. The point is however that Kähler form forms only the $(1,1)$ part of the symplectic form and must be co-homologically trivial whereas the zero mode part is same for all complexifications and can be co-homologically nontrivial. The co-homological non-triviality of the zero mode part of the symplectic form is indeed a nice feature since it fixes the normalization of the Kähler function apart from a multiplicative integer. On the other hand the hypothesis that Kähler coupling strength is analogous to critical temperature provides a dynamical (and perhaps equivalent) manner to fix the normalization of the Kähler function.

Since the properties of the WCW metric are inherited from $M^4_+ \times CP_2$ then also the Hyper Kähler property should be understandable in terms of the embedding space geometry. In particular, the complex structure in CP_2 vibrational degrees of freedom is inherited from CP_2. Hyper Kähler property implies the existence of a continuum (sphere S^2) of complex structures: any linear superposition of 3 independent Kähler forms defines a respectable complex structure. Therefore also CP_2 should have this continuum of complex structures and this is certainly not the case.

Indeed, if we had instead of CP_2 Hyper Kähler manifold with 3 covariantly constant 2-forms then it would be easy to understand the Hyper Kähler structure of WCW. Given the Kähler structure of WCW would be obtained by replacing induced Kähler electric and magnetic fields in the definition of flux factors $Q(H_{A,m})$ with the appropriate component of the induced Weyl tensor. CP_2 indeed manages to be very nearly Hyper Kähler manifold!

How CP_2 fails to be Hyper Kähler manifold can be seen in the following manner. The Weyl tensor of CP_2 allows three independent components, which are self dual as 2-forms and rotated to each other by vielbein rotations.

$$
\begin{aligned}
W_{03} &= W_{12} \equiv 2I_3 = 2(e^0 \wedge e^3 + e^1 \wedge e^2) \ , \\
W_{01} &= W_{23} \equiv I_1 = -e^0 \wedge e^1 - e^2 \wedge e^3 \ , \\
W_{02} &= W_{31} \equiv I_2 = -e^0 \wedge e^2 - e^3 \wedge e^1 \ . \qquad (7.7.25)
\end{aligned}
$$

The component I_3 is just the Kähler form of CP_2. Remaining components are covariantly constant only with respect to spinor connection and not closed forms so that they cannot be interpreted as Maxwell fields. Their squares equal however apart from sign with the metric of CP_2, when appropriate normalization factor is used. If these forms were covariantly constant Kähler action defined by any linear superposition of these forms would indeed define Kähler structure in WCW and the group $SO(3)$ would rotate these forms to each other. The projections of the components of the Weyl tensor on 3-surface define 3 vector fields as their duals and only one of these vector fields (Kähler magnetic field) is divergenceless. One might regard these 3 vector fields as counter parts of quaternion units associated with the broken Hyper Kähler structure, that is quaternion structure. The interpretation in terms of electro-weak symmetry breaking is obvious.

One cannot exclude the possibility that the symplectic invariance of the induced Kähler electric field implies that the electric parts of the other two components of induced Weyl tensor are symplectic invariants. This is the minimum requirement. What is however obvious is that the magnetic parts cannot be closed forms for arbitrary 3-surfaces at light cone boundary. One counter example is enough and CP_2 type extremals seem to provide this counter example: the components of the induced Weyl tensor are just the same as they are for CP_2 and clearly not symplectically invariant.

Thus it seems that WCW could allow Hyper Kähler structure broken by electro-weak interactions but it cannot be inherited from CP_2. An open question is whether it allows genuine quaternionic structure. Good prospects for obtaining quaternionic structure are provided by the quaternionic counterpart QP_2 of CP_2, which is 8-dimensional and has coset space structure $QP_2 = Sp(3)/Sp(2) \times Sp(1)$. This choice does not seem to be consistent with the symmetries of the standard model. Note however that the over all symmetry group is obtained by replacing complex numbers with quaternions on the matrix representation of the standard model group.

Could different complexifications for M^4_+ and light like surfaces induce Hyper Kähler structure for WCW?

Quaternionic structure means also the existence of a family of complex structures parameterized by a sphere S^2. The complex structure of the WCW is inherited from the complex structure of some light like surface.

In the case of the light cone boundary δM^4_+ the complex structure corresponds to the choice of quantization axis of angular momentum for the sphere $r_M = constant$ so that the coordinates orthogonal to the quantization axis define a complex coordinate: the sphere S^2 parameterizes these choices. Thus there is a temptation to identify the choice of quantization axis with a particular imaginary unit and Hyper Kähler structure would directly relate to the properties rotation group. This would bring an additional item to the list of miraculous properties of light like surfaces of 4-dimensional space-times.

This might relate to the fact that WCW geometry is not determined by the symplectic algebra of CP_2 localized with respect to the light cone boundary as one might first expect but consists of $M^4_+ \times CP_2$ Hamiltonians so that infinitesimal symplectic transformation of CP_2 involves always also M^4_+-symplectic transformation. M^4_+ Hamiltonians are defined by a function basis generated as products of the Hamiltonians H_3 and $H_1 \pm iH_2$ generating rotations with respect to three orthogonal axes, and two of these Hamiltonians are complexified.

Also the light like 3-surfaces X^3_l associated with quaternion conformal invariance are determined by some 2-surface X^2 and the choice of complex coordinates and if X^2 is sphere the choices are labelled by S^2. In this case, the presence of quaternion conformal structure would be almost obvious since it is possible to choose some complex coordinate in several ways and the choices are labelled by S^2. The choice of the complex coordinate in turn fixes 2-surface X^2 as a surface for which the remaining coordinates are constant. X^2 need not however be located at the elementary particle horizon unless one poses additional constraint. One might hope that different choices of X^2

resulting in this manner correspond to all possible different selections of the complex structure and that this choice could fix uniquely the conformal equivalence class of X^2 appearing as argument in elementary particle vacuum functionals. If X^2 has a more complex topology the identification is not so clear but since conformal algebra SL(2,C) containing algebra of rotation group is involved, one might argue that the choice of quantization axis also now involves S^2 degeneracy. If these arguments are correct one could conclude that Hyper Kähler structure is implicitly involved and guarantees Ricci flatness of the WCW metric.

Chapter 8

Classical TGD

8.1 Introduction

A brief summary of what might be called basic principles is in order to facilitate the reader to assimilate the basic tools and rules of intuitive thinking involved.

8.1.1 Quantum-Classical Correspondence

The fundamental meta level guiding principle is quantum-classical correspondence (classical physics is an exact part of quantum TGD). The principle states that all quantum aspects of the theory, which means also various aspects of consciousness such as volition, cognition, and intentionality, should have space-time correlates [K108] . Real space-time sheets provide kind of symbolic representations whereas p-adic space-time sheets provide correlates for cognition. All that we can symbolically communicate about conscious experience relies on quantal space-time engineering to build these representations.

The progress in the understanding of quantum TGD has demonstrated that quantum classical correspondence is more or less equivalent with holography, quantum criticality, and criticality as the principle selecting the preferred extremals of Kähler action. It also guarantees 1-1 correspondence between quantum states and classical states essential for quantum measurement theory.

8.1.2 Classical Physics As Exact Part Of Quantum Theory

Classical physics corresponds to the dynamics of space-time surfaces determined by the criticality in the sense that extremals allow an infinite number of deformations giving rise to a vanishing second variation of the Kähler action [K105] . This dynamics have several unconventional features basically due to the possibility to interpret the Kähler action as a Maxwell action expressible in terms of the induced metric defining classical gravitational field and induced Kähler form defining a non-linear Maxwell field not as such identifiable as electromagnetic field however.

Classical long ranged weak and color fields as signature for a fractal hierarchy of copies standard model physics

The geometrization of classical fields means that various classical fields are expressible in terms of embedding space-coordinates and are thus not primary dynamical variables. This predicts the presence of long range weak and color (gluon) fields not possible in standard physics context. It took 26 years to end up with a convincing interpretation for this puzzling prediction.

What seems to be the correct interpretation is in terms of an infinite fractal hierarchy of copies of standard models physics with appropriately scaled down mass spectra for quarks, leptons, and gauge bosons. Both p-adic length scales and the values of Planck constant predicted by TGD [K120] label various physics in this hierarchy. Also other quantum numbers are predicted as labels. This means that universe would be analogous to an inverted Mandelbrot fractal with each bird's eye of view revealing new long length scale structures serving also as correlates for higher levels of self hierarchy.

Exotic dark weak forces and their dark variants are consistent with the experimental widths for ordinary weak gauge bosons since the particles belonging to different levels of the hierarchy do not have direct couplings at Feynman diagram level although they have indirect classical interactions and also the de-coherence reducing the value of $\hbar$ is possible. Classical long ranged weak fields play a key role in quantum control and communications in living matter [?, K35] . Long ranged classical color force in turn is the backbone in the model of color vision [K44] : colors correspond to the increments of color quantum numbers in this model. The increments of weak isospin in turn could define the basic color like quale associated with hearing (black-white $\leftrightarrow$ to silence-sound [K44, K87, K89]).

Topological field quantization and the notion of many-sheeted space-time

The compactness of CP_2 implies the notions of many-sheeted space-time and topological field quantization. Topological field quantization means that various classical field configurations decompose into topological field quanta. One can see space-time as a gigantic Feynman diagram with lines thickened to 4-surfaces. Criticality of the preferred extremals implies that only selected field configurations analogous to Bohr's orbits are realized physically so that quantum-classical correspondence becomes very predictive. An interpretation as a 4-D quantum hologram is a further very useful picture [K55] but will not be discussed in this chapter in any detail.

Topological field quantization implies that the field patterns associated with material objects form extremely complex topological structures which can be said to belong to the material objects. The notion of field body , in particular magnetic body, typically much larger than the material system, differentiates between TGD and Maxwell's electrodynamics, and has turned out to be of fundamental importance in the TGD inspired theory of consciousness. One can say that field body provides an abstract representation of the material body.

One implication of many-sheetedness is the possibility of macroscopic quantum coherence. By quantum classical correspondence large space-time sheets as quantum coherence regions are macroscopic quantum systems and therefore ideal sites of the quantum control in living matter.

1. The original argument was that each space-time sheet carrying matter has a temperature determined by its size and the mass of the particles residing at it via de Broglie wave length $\lambda_{dB} = \sqrt{2mE}$ assumed to define the p-adic length scale by the condition $L(k) < \lambda_{dB} < L(k_>)$. This would give very low temperatures when the size of the space-time sheet becomes large enough. The original belief indeed was that the large space-time sheets can be very cold because they are not in thermal equilibrium with the smaller space-time sheets at higher temperature.

2. The assumption about thermal isolation is not needed if one accepts the possibility that Planck constant is dynamical and quantized and that dark matter corresponds to a hierarchy of phases characterized by increasing values of Planck constant [K120, K34] . From $E = hf$ relationship it is clear that arbitrarily low frequency dark photons (say EEG photons) can have energies above thermal energy which would explain the correlation of EEG with consciousness. This vision allows to formulate more precisely the basic notions of TGD inspired theory of consciousness and leads to a model of living matter giving precise quantitative predictions. Also the ability of this vision to generate new insights to quantum biology provides strong support for it [K35] .

Many-sheeted space-time predicts also fundamental mechanisms of metabolism based on the dropping of particles between space-time sheets with an ensuing liberation of the quantized zero point kinetic energy. Also the notion of many-sheeted laser follows naturally and population inverted many-sheeted lasers serve as storages of metabolic energy [K56] .

Space-time sheets topologically condense to larger space-time sheets by wormhole contacts which have Euclidian signature of metric. This implies causal horizon (or elementary particle horizon) at which the signature of the induced metric changes from Minkowskian to Euclidian. This forces to modify the notion of sub-system. What is new is that two systems represented by space-time sheets can be unentangled although their sub-systems bound state entangle with the mediation of the join along boundaries bonds connecting the boundaries of sub-system space-time sheets. This is not allowed by the notion of sub-system in ordinary quantum mechanics. This notion in turn implies the central concept of fusion and sharing of mental images by entanglement [K108] .

Zero energy ontology

The notion of zero energy ontology emerged implicitly in cosmological context from the observation that the imbeddings of Robertson-Walker metrics are always vacuum extremals. In fact, practically all solutions of Einstein's equations have this property very naturally. The explicit formulation emerged with the progress in the formulation of quantum TGD. In zero energy ontology physical states are creatable from vacuum and have vanishing net quantum numbers, in particular energy. Zero energy states can be decomposed to positive and negative energy parts with definite geometro-temporal separation, call it T, and having interpretation in terms of initial and final states of particle reactions. Zero energy ontology is consistent with ordinary positive energy ontology at the limit when the time scale of the perception of observer is much shorter than T. One of the implications is a new view about fermions and bosons allowing to understand Higgs mechanism among other things.

Zero energy ontology leads to the view about S-matrix as a characterizer of time-like entanglement associated with the zero energy state and a generalization of S-matrix to what might be called M-matrix emerges. M-matrix is complex square root of density matrix expressible as a product of real valued "modulus" and unitary matrix representing phase and can be seen as a matrix valued generalization of Schrödinger amplitude. Also thermodynamics becomes an inherent element of quantum theory in this approach.

TGD Universe is quantum spin glass

Since Kähler action is Maxwell action with Maxwell field and induced metric expressed in terms of $M^4_+ \times CP_2$ coordinates, the gauge invariance of Maxwell action as a symmetry of the vacuum extremals (this implies is a gigantic vacuum degeneracy) but not of non-vacuum extremals. Gauge symmetry related space-time surfaces are not physically equivalent and gauge degeneracy transforms to a huge spin glass degeneracy. Spin glass degeneracy provides a universal mechanism of macro-temporal quantum coherence and predicts degrees of freedom called zero modes not possible in quantum field theories describing particles as point-like objects. Zero modes not contributing to the configuration space line element are identifiable as effectively classical variables characterizing the size and shape of the 3-surface as well as the induced Kähler field. Spin glass degeneracy as mechanism of macroscopic quantum coherence should be equivalent with dark matter hierarchy as a source of the coherence [K55] .

Classical and p-adic non-determinism

The vacuum degeneracy of Kähler action implies classical non-determinism, which means that space-like 3-surface is not enough to fix the space-time surface associated with it uniquely as an absolute minimum of action, and one must generalize the notion of 3-surface by allowing sequences of 3-surfaces with time like separations to achieve determinism in a generalized sense. These "association sequences" can be seen as symbolic representations for the sequences of quantum jumps defining selves and thus for contents of consciousness. Not only speech and written language define symbolic representations but all real space-time sheets of the space-time surfaces can be seen in a very general sense as symbolic representations of not only quantum states but also of quantum jump sequences. An important implication of the classical non-determinism is the possibility to have conscious experiences with contents localized with respect to geometric time. Without this non-determinism conscious experience would have no correlates localized at space-time surface, and there would be no psychological time.

p-Adic non-determinism follows from the inherent non-determinism of p-adic differential equations for any action principle and is due to the fact that integration constants, which by definition are functions with vanishing derivatives, are not constants but functions of the pinary cutoffs x_N defined as $x = \sum_k x_k p^k \rightarrow x_N = \sum_{k<N} x_k p^k$ of the arguments of the function. In p-adic topology one can therefore fix the behavior of the space-time surface at discrete set of space-time points *above* some length scale defined by p-adic concept of nearness by fixing the integration constants. In the real context this corresponds to the fixing the behavior *below* some time/length

scales since points p-adically near to each other are in real sense faraway. This is a natural correlate for the possibility to plan the behavior and p-adic non-determinism is assumed to be a classical correlate for the non-determinism of intentionality, and perhaps also imagination and cognition.

These two non-determinisms allow to understand the self-referentiality of consciousness at a very general level. In a given quantum jump a space-time surface can be created with the property that it represents symbolically or cognitively something about the contents of consciousness before the quantum jump. Thus it becomes possible to become conscious about being conscious of something. This is very much like mathematician expressing her thoughts as symbol sequences which provides feedback to go the next abstraction level.

Classical and p-adic non-determinisms force also the generalization of the notion of quantum entanglement. Time-like entanglement, crucial for understanding long term memory and precognition becomes possible. The notion of many-sheeted space-time forces also to modify the notion of sub-system, which implies that unentangled systems can have entangled sub-systems. One can partially understand this in terms of length scale dependent notion of entanglement (the entanglement of sub-systems is not seen in the length scale resolution defined by the size of unentangled systems) but only partially. The formation of join along boundaries bonds between sub-system space-time sheets and the fact that topologically condensed space-time sheets are separated by elementary particle horizons from larger space-time sheets, provide the deeper topological motivation for the generalization of sub-system concept.

Dark matter hierarchy and hierarchy of Planck constants

Dark matter revolution with levels of the hierarchy labeled by values of Planck constant forces a further generalization of the notion of embedding space and thus of space-time. One can say, that imbedding space is a book like structure obtained by gluing together infinite number of copies of the embedding space like pages of a book: two copies characterized by singular discrete bundle structure are glued together along 4-dimensional set of common points. These points have physical interpretation in terms of quantum criticality. Particle states belonging to different sectors (pages of the book) can interact via field bodies representing space-time sheets which have parts belonging to two pages of this book.

The hierarchy of Planck constants can be reduced to the quantum criticality of Kähler action due to the non-determinism of Kähler action and the generalization of embedding space is only a useful auxiliary tool to describe the situation mathematically.

The hierarchy of Planck constants is necessary in order to understand the formation of gravitational and in fact all bound states if one assumes that this is due to the fermionic strings connecting partonic 2-surfaces as AdS/CFT correspondence suggests. The generalization of AdS/CFT duality to TGD framework suggests strongly that Kähler action can be expressed as string area action for string world sheets with effective covariant metric defined by Kähler-Dirac gamma matrices proportional to $\alpha_K^2 \propto 1/h_{eff}^2$. Macroscopic quantum coherence even in astrophysical scales is unavoidable prediction and it becomes also clear that super string models cannot describe the formation of gravitational bound states.

Of course, all this is a work in progress and there are many uncertainties involved. Despite this it seems that it is good to sum up the recent view in order to make easier to refer to the new developments in the existing chapters.

p-Adic fractality of life and consciousness

p-Adic fractality of biology and consciousness has become an increasingly important guide line in the construction of the theory. This notion allows to relate phenomena occurring in the molecular level to phenomena like remote viewing and psychokinesis and it leads also to the view that topological field quanta of various fields of astrophysical size are crucial for the functioning of biosystems. If one accepts p-adic fractality, the theory can be tested in unexpected way, in particular in molecular and cellular length scales where the systems are much simpler. Sensory perception, long term memory, remote mental interactions, metabolism: all these phenomena rely on the same basic mechanisms. p-Adic length scale hypothesis allows to quantify the hypothesis with testable quantitative predictions.

Double slit experiment and classical non-determinism

Bohr's complementarity principle is the basic element of Copenhagen interpretation and at the same time one of the most poorly defined aspects of this interpretation. If the possibility of macroscopic quantum entanglement between measurement instrument and quantum system is accepted, complementary principle becomes un-necessary. This is however not all that is needed. If classical non-determinism makes it possible to represent quantum jump sequences at space-time level, a revision of space-time description of quantum measurement is necessary. This sounds very logical but to be honest, I write these lines only after having learned about the remarkable experiment done by Shahriar Afshar [J6] .

The variant of double slit experiment by Shahriar Afshar seems to contradict the Copenhagen interpretation which states that the particle and field aspects are complementarity and thus mutually exclusive. In the case of double slit experiment complementarity predicts that the measurement of whether the photon came to the detector through slit 1 or 2 should destroy the interference pattern of electromagnetic fields in the region behind the screen.

The experimental arrangement of Afshar differs from the standard double slit experiment in that a lens was added behind the screen. The lens transmitted the photons coming from slits 1 and 2 via mirrors to detectors A and B so that in particle picture a photon detected by A (B) could be regarded as coming from slit 1 (2). In the first step both slits were open and the detectors represented interference patterns representing diffraction through single slit. The other slit was then closed and metal wires at the positions of dark interference rings were added. These wires degraded somewhat the image in the second detector. After this the second slit was opened again. Surprisingly, the resulting interference pattern was the original one.

The measurement certainly measures the particle aspect of photons. On the other hand, the preservation of the detected patterns means that no photons did enter in the regions containing the wires so that also interference pattern is there. Hence wave and particle aspects seem to be mutually consistent.

This finding is difficult to understand in Copenhagen interpretation and also in the many-worlds interpretation of quantum mechanics. Afshar himself suggest that the very notion of photon must be questioned. It is however difficult to accept this view since the photon absorption quite concretely corresponds to a click in the detector and also because the mathematical formalism of second quantization works so fantastically.

The conclusion can be criticized. What is primarily measured is not basically through which slit the photons came but whether the direction of the momentum of the photon emerging from the lens was in the angle range characterizing the detector or not. One can however argue that in deterministic physics for fields the two measurements are equivalent so that the problem remains.

In TGD framework the classical physics is not completely deterministic and this has led to a generalization of the notion of quantum classical correspondence. Space-time surface provides a classical (unfaithful) representation not only for quantum states but for quantum jump sequences or equivalently, for sequences of quantum states. The most obvious identification for the quantum states is as the maximal non-deterministic regions of a given space-time sheet.

In the recent context this would mean that the fields in the region between the screen and lens represent the state before the state function reduction and thus the interference pattern, whereas the fields in the region between lens and detectors represent the situation after the state function reduction. The interaction with lens involves classical non-determinism.

This picture conforms also with the notion of topological field quantization. The space-time decomposes into space-time sheets interpreted, topological field quanta (topological light rays containing photons, flux quanta of magnetic field, etc..). Topological field quanta correspond to the coherence regions for classical fields with spinor fields included. De-coherence corresponds to the splitting of space-time sheet to smaller, possibly parallel space-time sheets. Topological field quantum carries classical fields inside it but behaves as a whole like particle. Hence particle and wave aspects are consistent in the sense that below the size scale L of the topological field quantum (say the thickness of a magnetic flux tube or topological light ray) the description as a wave applies and above L particle description makes sense. In the recent case the coherence is lost at the lens space-time sheet where the space-time sheet representing interference pattern decomposes to two sheets representing photon beams going to the two detectors.

8.1.3 Some Basic Ideas Of TGD Inspired Theory Of Consciousness And Quantum Biology

The following ideas of TGD inspired theory of consciousness and of quantum biology are the most relevant ones for what will follow.

"Everything is conscious and consciousness can be only lost" is the briefest way to summarize TGD inspired theory of consciousness. Quantum jump as moment of consciousness and the notion of self are key concepts of the theory. Self is a system able to avoid bound state entanglement with environment and can be formally seen as an ensemble of quantum jumps. The contents of consciousness of self are defined by the averaged increments of quantum numbers and zero modes (sensory and geometric qualia). Moment of consciousness can be said to be the counterpart of elementary particle and self the counterpart of many-particle state, either bound and free. The selves formed by macro-temporal quantum coherence are in turn the counterparts of atoms, molecules and larger structures. Macro-temporal quantum coherence effectively binds a sequence of quantum jumps to a single quantum jump as far as conscious experience is considered. The idea that conscious experience is about changes amplified to macroscopic quantum phase transitions, is the key philosophical guideline in the construction of various models, such as the model of qualia, the capacitor model of sensory receptor, the model of cognitive representations, and declarative memories.

2. Macro-temporal quantum coherence is a second consequence of the spin glass degeneracy [K55] . It is essentially due to the formation of bound states and has as a topological correlate the formation of join along boundaries bonds connecting the boundaries of the component systems. During macro-temporal coherence quantum jumps integrate effectively to single long-lasting quantum jump and one can say that system is in a state of oneness, eternal now, outside time. Macro-temporal quantum coherence makes possible stable non-entropic mental images. Negative energy MEs are one particular mechanism making possible macro-temporal quantum coherence via the formation of bound states, and remote metabolism and sharing of mental images are other facets of this mechanism. The real understanding of the origin of macroscopic quantum coherence requires the generalization of quantum theory allowing dynamical and quantized Planck constant [K34, K35] .

3. p-Adic physics as physics of intentionality and of cognition is a further key idea of TGD inspired theory of consciousness. p-Adic space-time sheets as correlates for intentions and p-adic-to-real transformations of them as correlates for the transformation of intentions to actions allow deeper understanding of also psychological time as a front of p-adic-to-real transition propagating to the direction of the geometric future. Negative energy MEs are absolutely essential for the understanding of how precisely targeted intentionality is realized.

8.1.4 About Preferred Extremals

The understanding of preferred extremals of Kähler action is the basic challenge of classical TGD. The field equations are known locally but the key problem is to give a precise meaning to the "preferred". Various attempts in this direction are discussed in [K14, K119]. These options give different perspectives to the properties of preferred extremals but provide no magic formula.

Before continuing, it must be emphasized that the notion of preferred extremal originated in positive energy ontology. In ZEO 3-surfaces are pairs of space-like 3-surfaces at the boundaries of CD. Also the light-like partonic orbits at which the induced metric changes its signature could be included to get a closed 3-surface analogous to Wilson loop. In deterministic theory one would expect that the extremals are unique so that "preferred" would become obsolete. Kähler action is non-deterministic and quantum criticality suggests that the preferred extremals have Kac-Moody type symmetries are gauge symmetries deforming partonic orbits and preserving their light-likeness. The number of gauge equivalence classes would be finite and correspond to the integer n defining the value of effective Planck constant $h_{eff} = n \times h$. The conformal subalgebra with conformal weights coming as multiples of n would act as gauge symmetries. The most that one might expect that above measurement resolution the attribute "preferred" is un-necessary in ZEO.

The idea about Bohr orbitology would require that "preferred" is not an empty attribute even in ZEO. There would be strong correlations between the space-like 3-surfaces at the oppposite boundaries of CD: the pairs would be like point pairs at Bohr orbits connecting the boundaries of CD.

It is good to summarize the attempts to give meaning to "preferred". The properties assigned to preferred extremals characterize also the known extremals.

1. The original proposal was that preferred extremals correspond to absolute minima of Kähler action. This makes sense only for Euclidian regions of space-time surfaces representing lines of generalized Feynman diagrams. This option is not number theoretically attractive since the very notion of minimum is p-adically poorly defined unless one can reduce absolute minimization to purely algebraic conditions making sense also p-adically.
2. Later I ended up with the idea that preferred extremals are critical being analogous to saddle points of potential function: what this exactly means is not obvious [K14, K121]. This option is not consistent with absolute minimization. It took a long time to realize that Minkowskian and Euclidian regions give imaginary *resp.* real contributions to the exponent of the vacuum functional having interpretation as Morse function *resp.* Kähler function of WCW. One might ask whether absolute minimization works in Euclidian regions and criticality in Minkowskian regions.
3. I have proposed the characterization of preferred extremal property in terms of Hamilton-Jacobi structure generalizing the notion of complex structure and being motivated by the huge super-conformal symmetries of "world of classical worlds" [K14]. This picture is consistent with the identification of Minkowskian regions consisting of massless and interaction topological field quanta since one can speak about local light direction and local polarization directions. This picture is very quantal since linear superposition is not possible and the counterpart of it is set theoretic union of space-time sheets representing quanta. Number theoretic vision based on classical number fields supports similar picture.
4. Almost topological QFT property requires that Kähler action reduces to "boundary" terms transformable to Chern-Simons terms. This is guaranteed if the Kähler current is orthogonal to Kähler gauge potential ($j \cdot A = 0$) and the weak form of electric-magnetic duality holds true.
5. The recent view emerged from the realization that the extended super-conformal invariances of TGD define naturally a hierarchy of broken conformal gauge symmetries with sub-algebras with conformal weights coming as multiples of some integer n defining gauge symmetries. At classical level this means that the corresponding Noether charges associated with 3-surfaces at the ends of CD vanish. These conditions realize the strong form of holography implied by the strong form of General Coordinate Invariance stating that partonic 2-surfaces and their tangent space data (or more or less equivalently string world sheets) fix the quantum states.

   The vanishing of conformal charges is extremely powerful condition and the proposal is that the space-time sheets connecting given 3-surfaces at the ends of CD form $n = h_{eff}/n$ conformal equivalence classes. It seems that this provides the long sought for realization of preferred extremal property.

   The physical interpretation for the sequences of conformal symmetry breakings with $n_{i+1} = \sum_{k<i+1} m_k$ is in terms of hierarchy of criticalities. At each phase transition new gauge generators are transformed to generators of genuine physical charges. These hierarchies also correspond to the gradual transformation of degrees of freedom below measurement resolution to physical degrees of freedom as the measurement resolution increases. At the level of biology the increase of h_{eff} corresponds to evolution leading to improved sensory and cognitive resolutions.

8.1.5 TGD Space-Time Viz. Space-Time Of GRT

It took decades to realize that GRT space-time is only an effective space-time obtained by replacing the sheets of many-sheeted space-time with single piece of Minkowski space with effective metric defined by the sum of Minkowski metric and deviations of the induced metrics of space-time sheets from Minkowski metric. Equivalence Principle as it is expressed by Einstein's equations would follow from Poincare invariance in long length scales. Therefore it is not necessary that Einstein's equations are satisfy in TGD and the vanishing of the divergence of energy momentum tensor for Kähler action could be enough.

I have however considered also other alternatives before ending up this view.

1. Vacuum extremals of Kähler action seem to be excellent candidates for defining cosmological models. In light of what was said above this would mean that GRT space-times having imbedding as vacuum extremals are in favored position. This would not be surprising since the addition of string world sheets carrying non-vanishing Kähler form to vacuum extremals should in good approximation give non-vacuum extremals serving as building bricks of the many-sheeted space-time.
2. An obvious question is whether Einstein equations could be true for preferred extremals [K14]. The condition that Kähler 4-force vanishes implies vanishing of the divergence of the energy momentum tensor. In general relativity this leads to Einstein's equations with cosmological constant term since the linear combination of Einstein tensor and metric tensor is automatically divergenceless. In TGD framework Einstein equations would have powerful consequences: for instance, curvature scalar would be constant so that mathematically highly interesting constant curvature spaces allowing to classify manifolds topologically, could emerge naturally. A weaker condition would be that energy momentum tensor is divergenceless only asymptotically when dissipation characterize by Lorentz force classically is absent.
3. In TGD framework one can also consider a weaker form of Einstein equations with cosmological constant: several (at most two) cosmological "constants" would appear in the counterpart of Einstein equations (see `http://tinyurl.com/y5e6dvdv`).

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L9].

8.2 Many-Sheeted Space-Time, Magnetic Flux Quanta, Electrets And MEs

TGD inspired theory of consciousness and of living matter relies on space-time sheets carrying ordinary matter, topological light rays (massless extremals, MEs), and magnetic and electric flux quanta. There are some new results which motivate a separate discussion of them.

8.2.1 Dynamical Quantized Planck Constant And Dark Matter Hierarchy

By quantum classical correspondence space-time sheets can be identified as quantum coherence regions. Hence the fact that they have all possible size scales more or less unavoidably implies that Planck constant must be quantized and have arbitrarily large values. If one accepts this then also the idea about dark matter as a macroscopic quantum phase characterized by an arbitrarily large value of Planck constant emerges naturally as does also the interpretation for the long ranged classical electro-weak and color fields predicted by TGD. Rather seldom the evolution of ideas follows simple linear logic, and this was the case also now. In any case, this vision represents the fifth, relatively new thread in the evolution of TGD and the ideas involved are still evolving.

Dark matter as phase with non-standard value of h_{eff}?

D. Da Rocha and Laurent Nottale have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also sub-harmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive.

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests astrophysical systems are not only quantum systems at larger space-time sheets but correspond to a gigantic value of gravitational Planck constant. The gravitational (ordinary) Schrödinger equation would provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale [K97] .

It has been found in CERN (seehttp://tinyurl.com/jbvwmrp3) that matter and antimatter atoms have no differences in the energies of their excited states. This is predicted by CPT symmetry. Notice however that CP and T can be separately broken and that this is indeed the case. Kaon is classical example of this in particle physics. Neutral kaon and anti-kaon behave slightly differently.

This finding forces to repeat an old question. Where does the antimatter reside? Or does it exist at all?

In TGD framework one possibility is that antimatter corresponds to dark matter in TGD sense that is a phase with $h_{eff} = n \times h$, $n = 1, 2, 3, ...$ such that the value of n for antimatter is different from that for visible matter. Matter and antimatter would not have direct interactions and would interact only via classical fields or by emission of say photons by matter (antimatter) suffering a phase transition changing the value of h_{eff} before absorbtion by antimatter (matter). This could be rather rare process. Bio-photons could be produced from dark photons by this process and this is assumed in TGD based model of living matter.

What the value of n for ordinary visible matter is? The naïve guess is that it is $n = 1$, the smallest possible value. Randell Mills [D5] has however claimed the existence of scaled down hydrogen atoms - Mills calls them hydrinos - with ground state binding energy considerably higher than for hydrogen atom. The experimental support for the claim is published in respected journals and the company of Mills is developing a new energy technology based on the energy liberated in the transition to hydrino state (see http://tinyurl.com/hajyqo6).

These findings can be understood in TGD framework if one has actually $n = 6$ for visible atoms and $n = 1, 2$, or 3 for hydrinos. Hydrino states would be stabilized in the presence of some catalysts [L22] (see http://tinyurl.com/goruuzm). This also suggest a universal catalyst action. Among other things catalyst action requires that reacting molecule gets energy to overcome the potential barrier making reaction very slow. If an atom - say hydrogen - in catalyst suffers a phase transition to hydrogen like state, it liberates binding energy, and if one of the reactant molecules receives it it can overcome the barrier. After the reaction the energy can be sent back and catalyst hydrino returns to the ordinary hydrogen state.

So: could it be that one has n=6 for stable matter and n is different from this for stable antimatter? Could the small CP breaking cause this?

Dark matter as a source of long ranged weak and color fields

Long ranged classical electro-weak and color gauge fields are unavoidable in TGD framework. The smallness of the parity breaking effects in hadronic, nuclear, and atomic length scales does not however seem to allow long ranged electro-weak gauge fields. The problem disappears if long ranged classical electro-weak gauge fields are identified as space-time correlates for massless gauge fields created by dark matter. Also scaled up variants of ordinary electro-weak particle spectra are possible. The identification explains chiral selection in living matter and unbroken $U(2)_{ew}$ invariance and free color in bio length scales become characteristics of living matter and of biochemistry and bio-nuclear physics. An attractive solution of the matter antimatter asymmetry is based on the identification of also antimatter as dark matter.

Dark matter hierarchy and consciousness

The emergence of the vision about dark matter hierarchy has meant a revolution in TGD inspired theory of consciousness. Dark matter hierarchy means also a hierarchy of long term memories with the span of the memory identifiable as a typical geometric duration of moment of consciousness at the highest level of dark matter hierarchy associated with given self so that even human life cycle represents at this highest level single moment of consciousness.

Dark matter hierarchy leads to detailed quantitative view about quantum biology with several testable predictions [K35] . The applications to living matter suggests that there is a hierarchy a hierarchy of Planck constants h_{eff} coming as integer multiples of ordinary Planck constant. The original - too limited - proposal was that in living matter there would be preferred values for the integer coming as power of 2^{11} $\hbar(k) = \lambda^k \hbar_0$, $\lambda = 2^{11}$ for $k = 0, 1, 2, ...$ [K35]. Also integer valued sub-harmonics and integer valued sub-harmonics of λ were found to be possible. The hypothesis $h_{eff} = n \times h$ turned out to follow naturally from TGD. Each p-adic length scale corresponds to this kind of hierarchy [K38]. One can also ask whether number theoretically very simple integers could define the values of h_{eff}/h. One candidate for this class of integers characterize polygons constructible using only compass and ruler. The sine and cosine of the angle $2\pi/n$ characterizing the polygon reduces to expression involving only square root operations applied on rationals. These integers are products of power of two with a subset of distinct Fermat primes.

The general prediction is that Universe is a kind of inverted Mandelbrot fractal for which each bird's eye of view reveals new structures in long length and time scales representing scaled down copies of standard physics and their dark variants. These structures would correspond to higher levels in self hierarchy. This prediction is consistent with the belief that 75 per cent of matter in the universe is dark.

1. Living matter and dark matter

Living matter as ordinary matter quantum controlled by the dark matter hierarchy has turned out to be a particularly successful idea. The hypothesis has led to models for EEG predicting correctly the band structure and even individual resonance bands and also generalizing the notion of EEG [K35] . Also a generalization of the notion of genetic code emerges resolving the paradoxes related to the standard dogma [K58, K35] . A particularly fascinating implication is the possibility to identify great leaps in evolution as phase transitions in which new higher level of dark matter emerges [K35] .

It seems safe to conclude that the dark matter hierarchy with levels labelled by the values of Planck constants explains the macroscopic and macro-temporal quantum coherence naturally. That this explanation is consistent with the explanation based on spin glass degeneracy is suggested by following observations. First, the argument supporting spin glass degeneracy as an explanation of the macro-temporal quantum coherence does not involve the value of $\hbar$ at all. Secondly, the failure of the perturbation theory assumed to lead to the increase of Planck constant and formation of macroscopic quantum phases could be precisely due to the emergence of a large number of new degrees of freedom due to spin glass degeneracy. Thirdly, the phase transition increasing Planck constant has concrete topological interpretation in terms of many-sheeted space-time consistent with the spin glass degeneracy.

2. Dark matter hierarchy and the notion of self

The vision about dark matter hierarchy leads to a more refined view about self hierarchy and hierarchy of moments of consciousness [K34, K35] . The larger the value of Planck constant, the longer the subjectively experienced duration. The naïve guess for the average geometric duration of self was $T(n) \propto h_{eff}/h = n$. The identification of self as a sequence of state function reductions at the same boundary of CD at which state does not change gives an identification of the lifetime of self as the increase of the temporal distance between the tips of CD during this period [K116, K7]. The order of magnitude could be however proportional to h_{eff}/h.

Dark matter hierarchy suggests also a slight modification of the notion of self. Each self involves a hierarchy of dark matter levels, and one is led to ask whether the highest level in this hierarchy corresponds to a single quantum jump rather than a sequence of quantum jumps. The averaging of conscious experience over quantum jumps would occur only for sub-selves at lower levels of dark matter hierarchy and these mental images would be ordered, and single moment of consciousness would be experienced as a history of events. One can ask whether even entire life cycle could be regarded as a single quantum jump at the highest level so that consciousness would not be completely lost even during deep sleep. This would allow to understand why we seem to know directly that this biological body of mine existed yesterday.

The fact that we can remember phone numbers with 5 to 9 digits supports the view that self corresponds at the highest dark matter level to single moment of consciousness. Self would experience the average over the sequence of moments of consciousness associated with each sub-self but there would be no averaging over the separate mental images of this kind, be their parallel or serial. These mental images correspond to sub-selves having shorter wake-up periods than self and would be experienced as being time ordered. Hence the digits in the phone number are experienced as separate mental images and ordered with respect to experienced time.

8.2.2 P-Adic Length Scale Hypothesis And The Connection Between Thermal De Broglie Wave Length And Size Of The Space-Time Sheet

Also real space-time sheets are assumed to be characterized by p-adic prime p and assumed to have a size determined by primary p-adic length scale L_p or possibly n-ary p-adic length scale $L_p(n)$.

The possibility to assign a p-adic prime to the real space-time sheets is required by the success of the elementary particle mass calculations and various applications of the p-adic length scale hypothesis. The original idea was that the non-determinism of Kähler action corresponds to p-adic determinism for some primes. It has been however difficult to make this more concrete.

Rational numbers are common to reals and all p-adic number fields. One can actually assign to any algebraic extension of rationals extensions of p-adic numbers and construct corresponding adeles. These extensions can be arranged according to the complexity and I have already earlier proposed that this hierarchy defines an evolutionary hierarchy.

How the existence of preferred p-adic primes characterizing space-time surfaces emerge was solved only quite recently. The solution relies on p-adicization based on strong holography and the idea that string world sheets and partonic surfaces with parameters in algebraic extensions of rationals define the intersection of reality and various p-adicities. The algebraic extension possess preferred primes as primes, which are ramified meaning that their decomposition to a product of primes of the extension contains higher than first powers of its primes (prime ideals to be more rigorous). These primes are obviously natural candidates for primes characterizing string world sheets number theoretically.

In strong form of holography p-adic continuations of 2-surfaces to preferred extremals identifiable as imaginations would be easy due to the existence of p-adic pseudo-constants. The continuation could fail for most configurations of partonic 2-surfaces and string world sheets in the real sector: the interpretation would be that some space-time surfaces can be imagined but not realized [K73]. For certain extensions the number of realizable imaginations could be exceptionally large. These extensions would be winners in the number theoretic fight for survivalandcorresponding ramified primes would be preferred p-adic primes. Whether these primes correspond to p-adic lengths scale hypothesis or its generalization to small primes, is an open question.

Weak form of NMP indeed allows also to justify a generalization of p-adic length scale hypothesis [K65]. Primes near but below powers of primes are favoured since they allow exceptionally large negentropy gain so that state function reductions to tend to select them.

Parallel space-time sheets with distance about 10^4 Planck lengths form a hierarchy. Each material object (...,atom, molecule, ..., cell,...) would correspond to this kind of space-time sheet. The p-adic primes $p \simeq 2^k$, k prime or power of prime, characterize the size scales of the space-time sheets in the hierarchy. The p-adic length scale $L(k)$ can be expressed in terms of cell membrane thickness as

$$L(k) = 2^{(k-151)/2} \times L(151) \ , \tag{8.2.1}$$

$L(151) \simeq 10$ nm. These are so called primary p-adic length scales but there are also n-ary p-adic length scales related by a scaling of power of $\sqrt{p}$ to the primary p-adic length scale. Quite recent model for photosynthesis [K56] gives additional support for the importance of also n-ary p-adic length scales so that the relevant p-adic length scales would come as half-octaves in a good approximation but prime and power of prime values of k would be especially important.

8.2.3 Topological Light Rays (Massless Extremals, Mes)

I have described MEs, or "topological light rays", in detail in [L3] and in [K77] newphys, and describe here only very briefly the basic characteristics of MEs and concentrate on new idea about their possible role for consciousness and life.

What MEs are?

MEs (massless extremals, topological light rays) can be regarded as topological field quanta of classical radiation fields [K77, K9] . They are typically tubular space-time sheets inside which radiation fields propagate with light velocity in single direction without dispersion. The simplest case corresponds to a straight cylindrical ME but also curved MEs, kind of curved light rays, are possible. The initial values for a given moment of time are arbitrary by light likeness. Therefore MEs are ideal for precisely targeted communications. What distinguishes MEs from Maxwellian radiation fields in empty space is that light like vacuum 4-current is possible: ordinary Maxwell's equations would state that this current vanishes. Quite generally, purely geometric vacuum charge densities and 3-currents are purely TGD based prediction and could be seen as a classical correlate of the vacuum polarization predicted by quantum field theories.

MEs are fractal structures containing MEs within MEs. The so called scaling law of homeopathy predicts that the high frequency MEs inside low frequency MEs are in a ratio having discrete values [K48] . One can indeed justify this relationship. As ions drop from smaller space-time sheets to magnetic flux tubes, zero point kinetic energy is liberated as high frequency MEs, and the ions dropped to magnetic flux tubes generate cyclotron radiation, and the ratio of the fundamental frequencies is constant not depending on particle mass and being determined solely by p-adic length scale hypothesis. The model for the radio waves induced by the irradiation of DNA by laser light [I4] gives support for this picture [K55] .

Two basic types of MEs

MEs have 2-dimensional CP_2 projection which means that electro-weak holonomy group is Abelian (color holonomy is always Abelian which suggests that physical states in TGD Universe correspond to states of color multiplets with vanishing color hypercharge and isospin rather than color singlets). If CP_2 projection belongs to a homologically non-trivial geodesic sphere, only em and Z^0 fields and Abelian color gauge fields are present. In the homologically trivial case only classical W fields are non-vanishing.

1. Neutral MEs can be assigned to various kinds of communications from biological body to the magnetic body and fractal hierarchy of EEGs and ZEGs represent the basic example in this respect [K35] .
2. Dark W MEs serving as correlate for dark W exchanges induce an exotic ionization of atomic nuclei [K101, K36, K35] . This induces charge entanglement between magnetic body and biological body generating dark plasma oscillation patterns inducing nerve pulse patterns and ion waves at the space-time sheets occupied by the ordinary matter. The mechanism is based on many-sheeted Faraday law inducing electromagnetic fields at ordinary space-time sheet in turn giving rise to ohmic currents. State function reduction selects one of the exotically ionized configurations. This mechanism is the most plausible candidate for how magnetic body as an intentional agent controls biological body.

Negative energy MEs

MEs can have either positive or negative energy depending on the time orientation. The understanding of negative energy MEs has increased considerably. Phase conjugate laser beams [D3] are the most plausible standard physics counterparts of negative energy MEs since they can be interpreted as time reversed laser beams and do not possess direct Maxwellian analog. By quantum-classical correspondence one can interpret the frequencies associated with negative energy MEs as energies. One can also assume that the Bose-Einstein condensed photons associated with negative energy MEs and with the coherent light generated by the light like vacuum current have negative energies.

For frequencies for which energy is above the thermal energy there is no system which could interact with negative energy MEs or absorb negative energy photons. Therefore negative energy MEs and corresponding photons should propagate through matter practically without any interaction. Feinberg has demonstrated that phase conjugate laser beams behave similarly: for instance, one can see through chickens using these laser beams [D2] . This means that negative energy MEs do not respect Faraday cages and thus represent an attractive candidate for the hypothetical Psi field.

Negative energy MEs have many applications.

1. Negative energy MEs ideal for generating time like entanglement. Since negative energies are involved, this entanglement can be seen as a correlate for the bound state entanglement leading to a macro-temporal quantum coherence. Negative energy MEs make thus possible telepathic sharing of mental images. Negative energy MEs are involved with both sensory perception, long term memory, and motor action. In the model for living matter [K35] The charge entanglement generated by W MEs inducing exotic weak charge and electromagnetic charge is assumed to be responsible for bio-control whereas neutral MEs in general carrying both em and Z^0 fields are responsible for communications.
2. Negative energy MEs are ideal for a precisely targeted realization of intentions. p-Adic ME having a large number of common rational points with negative energy ME is generated and transformed to a real ME in quantum jump. The system receives positive energy and momentum as a recoil effect and the transition is not masked by ordinary spontaneously occurring quantum transitions since the energy of the system increases. One can say that negative energy ME represents the desires communicated to the geometric past and inducing as a reaction the desired action realized as say neuronal activity and generation of positive energy MEs.
3. The generation of negative energy MEs is also in a key role in remote metabolism and MEs serve as quantum credit cards implying an extreme flexibility of the metabolism. If the system receiving negative energy MEs is a population inverted laser or its many-sheeted counterpart, then quite a small field intensity associated with negative energy MEs (intensity of negative energy photons) can lead to the amplification of the time reflected positive energy signal. The reason is that the rate for the induced emission is proportional to the number of particles dropped to the ground state from the excited state. Therefore even negative energy bio-photons might serve as quantum controllers of metabolism and induce much more intense beams of positive energy photons, say when interacting with mitochondria.

8.2.4 Magnetic Flux Quanta And Electrets

Magnetic flux tubes and electrets are extremals of Kähler action dual to each other. Also layer like magnetic flux quanta and their electric counterparts are possible. The magnetic/electric field is in a good approximation of constant magnitude but has varying direction.

Magnetic fields and life

The magnetic field associated with any material system is topologically quantized, and one can assign to any system a magnetic body. An attractive idea is that the relationship of the magnetic body to the material system is to some degree that of the manual to an electronic instrument. Quantitative arguments related to the dark matter hierarchy assuming that magnetic bodies are dark suggest that cognitions and emotions are regarded as somatosensory qualia of the magnetic body [K44, K35] . Magnetic body would in this case serve as a kind of computer screen at which the data items processes in say brain are communicated either classically (positive energy MEs) or by sharing of mental images (negative energy MEs).

Magnetic body is also an active intentional agent: motor actions are controlled from magnetic body and proceed as cascade like processes from long to short length and time scales as quantum communications of desires at various levels of hierarchy of magnetic bodies. Communication occurs backwards in geometric time by negative energy MEs. Motor action as a response to these desires occurs by classical communications by positive energy MEs and as neural activities. This explains the coherence and synchrony of motor actions difficult to understand in neuroscience framework. The sizes of flux quanta are astrophysical: for instance, EEG frequency of 7.8 Hz corresponds to a wave length defined by Earth's circumference. The non-locality in the length scale of magnetosphere, and even in length scales up to light life, is forced by Uncertainty Principle alone, if taken seriously in macroscopic length scales.

The leakage of supra currents of ions and their Cooper pairs from magnetic flux tubes of the Earth's magnetic field to smaller space-time sheets and their dropping back involving liberation of the zero point kinetic energy defines one particular metabolic "Karma's cycle".

In many-sheeted space-time particles topologically condense at all space-time sheets having projection to given region of space-time so that this option makes sense only near the boundaries of space-time sheet of a given system. Also p-adic phase transition increasing the size of the space-time sheet could take place and the liberated energy would correspond to the reduction of zero point kinetic energy. Particles could be transferred from a portion of magnetic flux tube portion to another one with different value of magnetic field and possibly also of Planck constant h_{eff} so that cyclotron energy would be liberated.

The dropping of protons from $k = 137$ atomic space-time sheet involved with the utilization of ATP molecules is only a special instance of the general mechanism involving an entire hierarchy of zero point kinetic energies defining universal metabolic currencies. This leads to the idea that the topologically quantized magnetic field of Earth defines the analog of central nervous system and blood circulation present already during the pre-biotic evolution and making possible primitive metabolism. This has far reaching implications for the understanding of how pre-biotic evolution led to living matter as we understand it [?] .

For instance, it has recently become clear that the dropping of atoms and molecules from space-time sheets labelled by p-adic prime $p \simeq 2^k$, $k = 131$, liberates photons at visible and near infrared wave lengths. The hot $k = 131$ space-time sheets (with temperatures above 1000 K) could have served as a source of metabolic energy for life-forms at cool $k = 137$ sheets. Photosynthesis could have developed in the circumstances where solar radiation was replaced with these photons. The correct prediction is that chlorophylls should be especially sensitive to these wave lengths. In particular, it is predicted that also IR wave lengths 700-1000 nm should have been utilized. There indeed are bacteria using only this portion of solar radiation. This leads to a scenario making sense only in TGD universe. Pre-biotic life could have developed at the cool space-time sheets in the hot interior of Earth below crust, where $k = 131$ space-time sheets are possible and this life could still be there [?] . Also the life as we know it, could involve hot spots generated by the cavitation of water inside cell. The classical repulsive Z^0 force causes a strong acceleration during final stages of bubble collapse creating high temperatures, and could explain also sono-luminescence [D4] , [D4] as suggested in [K36] .

Magnetic Mother Gaia could also form sensory and other representations receiving input from several brains via negative energy EEG MEs entangling magnetosphere with brains. The multi-brained magnetospheric selves could be responsible for the third person aspect of consciousness and for the evolution of social structures. For instance, the successful healing by prayer and meditation groups [J4] , and the experiments of Mark Germine [J7] provide support for the notion of multi-brained magnetospheric selves are involved. Magnetic flux tubes could function as wave guides for MEs and this aspect is crucial in the model of long term memory.

Electrets and bio-systems

Bio-systems are known to be full of electrets and liquid crystals [I9] . Perhaps the most fundamental electret structure is cell membrane. In particular, the water inside cells tends to be in gel phase which is liquid crystal phase. There are many good reasons for why water should be in ordered phase. One very fundamental reason is that bio-polymers are stable in liquid crystal/ordered water phase since there are no free water molecules available for the de-polymerization by hydration. In fact, only a couple of years ago it was experimentally discovered that bio-polymers can be stabilized around ice.

The capacitor model for sensory receptor is one very important application of the electret concept [K44] , [L4] . Sensory qualia result in the flow of particles with given quantum numbers from the plate to another one in quantum discharge. This kind of amplification of quantum number *resp.* zero mode increments would give rise to both geometric *resp.* non-geometric qualia [K44] .

Also micro-tubuli are electrets. Sol-gel transition, as any phase transition, is an good candidate for the representation of a conscious bit and controlled local sol-gel transitions between ordinary and liquid crystal water could be a basic control tool making possible cellular locomotion, changes of protein conformations, etc... The tubulin dimers of micro-tubuli could induce sol-gel transformations by generating negative energy MEs, and micro-tubular surface could provide bit maps of their environment somewhat like sensory areas of brain provide maps of body. If gel→sol transition around tubulin inducing conformational change induces sol→gel transformation in some point of environment as would be the case for the seesaw mechanism to be discussed below, a one-one correspondence would result. By this one-one correspondence micro-tubules would automatically generate kind of conscious log files about the control activities which could have evolved to micro-tubular declarative memory representations about what happens inside cell [K56] .

8.3 General View About Field Equations

In this section field equations are deduced and discussed in general level. The fact that the divergence of the energy momentum tensor, Lorentz 4-force, does not vanish in general, in principle makes possible the mimicry of even dissipation and of the second law. For asymptotic self organization patterns for which dissipation is absent the Lorentz 4-force must vanish. This condition is guaranteed if Kähler current is proportional to the instanton current in the case that CP_2 projection of the space-time sheet is smaller than four and vanishes otherwise. An attractive identification for the vanishing of Lorentz 4-force is as a condition equivalent with the selection of preferred extremal of Kähler action. If preferred extremals correspond to absolute minima this principle would be essentially equivalent with the second law of thermodynamics.

8.3.1 Field Equations

The requirement that Kähler action is stationary leads to the following field equations in the interior of the four-surface

$$\begin{aligned} D_\beta(T^{\alpha\beta}h^k_\alpha) &- j^\alpha J^k_{\ l}\partial_\alpha h^l = 0 \ , \\ T^{\alpha\beta} &= J^{\nu\alpha}J_\nu^{\ \beta} - \frac{1}{4}g^{\alpha\beta}J^{\mu\nu}J_{\mu\nu} \ . \end{aligned} \tag{8.3.1}$$

Here $T^{\alpha\beta}$ denotes the traceless canonical energy momentum tensor associated with the Kähler action. An equivalent form for the first equation is

$$\begin{aligned} T^{\alpha\beta}H^k_{\alpha\beta} &- j^\alpha(J_\alpha^{\ \beta}h^k_{\ \beta} + J^k_{\ l}\partial_\alpha h^l) = 0 \ . \\ H^k_{\alpha\beta} &= D_\beta\partial_\alpha h^k \ . \end{aligned} \tag{8.3.2}$$

$H^k_{\alpha\beta}$ denotes the components of the second fundamental form and $j^\alpha = D_\beta J^{\alpha\beta}$ is the gauge current associated with the Kähler field.

On the boundaries of X^4 and at wormhole throats the field equations are given by the expression

$$\frac{\partial L_K}{\partial_n h^k} = T^{n\beta}\partial_\beta h^k - J^{n\alpha}(J_\alpha^{\ \beta}\partial_\beta h^k + J^k_{\ l})\partial_\alpha h^k) = 0 \ . \tag{8.3.3}$$

At wormhole throats problems are caused by the vanishing of metric determinant implying that contravariant metric is singular.

For M^4 coordinates boundary conditions are satisfied if one assumes

$$T^{n\beta} = 0 \tag{8.3.4}$$

stating that there is no flow of four-momentum through the boundary component or wormhole throat. This means that there is no energy exchange between Euclidian and Minkowskian regions so that Euclidian regions provide representations for particles as autonomous units. This is in accordance with the general picture [K43] . Note that momentum transfer with external world necessarily involves generalized Feynman diagrams also at classical level.

For CP_2 coordinates the boundary conditions are more delicate. The construction of WCW spinor structure [K121] led to the conditions

$$g_{ni} = 0 \ , \ J_{ni} = 0 \ . \tag{8.3.5}$$

$J^{ni} = 0$ does not and should not follow from this condition since contravariant metric is singular. It seems that limiting procedure is necessary in order to see what comes out.

The condition that Kähler electric charge defined as a gauge flux is non-vanishing would require that the quantity $J^{nr}\sqrt{g}$ is finite (here r refers to the light-like coordinate of X^3_l). Also $g^{nr}\sqrt{g_4}$ which is analogous to gravitational flux if n is interpreted as time coordinate could be non-vanishing. These conditions are consistent with the above condition if one has

$$\begin{array}{llll} J_{ni} = 0 \ , & g_{ni} = 0 \ , & J_{ir} = 0 \ , & g_{ir} = 0 \ , \\ J^{nk} = 0 \ \ k \neq r \ , & g^{nk} = 0 \ \ k \neq r \ , & J^{nr}\sqrt{g_4} \neq 0 \ , & g^{nr}\sqrt{g_4} \neq 0 \ . \end{array} \tag{8.3.6}$$

The interpretation of this conditions is rather transparent.

1. The first two conditions state that covariant form of the induced Kähler electric field is in direction normal to X^3_l and metric separate into direct sum of normal and tangential contributions. Fifth and sixth condition state the same in contravariant form for $k \neq n$.
2. Third and fourth condition state that the induced Kähler field at X^3_l is purely magnetic and that the metric of x^3_l reduces to a block diagonal form. The reduction to purely magnetic field is of obvious importance as far as the understanding of the generalized eigen modes of the Kähler-Dirac operator is considered [K121] .
3. The last two conditions must be understood as a limit and $\neq$ means only the possibility of non-vanishing Kähler gauge flux or analog of gravitational flux through X^3_l.
4. The vision inspired by number theoretical compactification allows to identify r and n in terms of the light-like coordinates assignable to an integrable distribution of planes $M^2(x)$ assumed to be assignable to M^4 projection of $X^4(X^3_l)$. Later it will be found that Hamilton-Jacobi structure assignable to the extremals indeed means the existence of this kind of distribution meaning slicing of $X^4(X^3_l)$ both by string world sheets and dual partonic 2-surfaces as well as by light-like 3-surfaces Y^3_l.
5. The physical analogy for the situation is the surface of an ideal conductor. It would not be surprising that these conditions are satisfied by all induced gauge fields.

8.3.2 Topologization And Light-Likeness Of The Kähler Current AsAlternative ways To Guarantee Vanishing Of Lorentz 4-Force

The general solution of 4-dimensional Einstein-Yang Mills equations in Euclidian 4-metric relies on self-duality of the gauge field, which topologizes gauge charge. This topologization can be achieved by a weaker condition, which can be regarded as a dynamical generalization of the Beltrami condition. An alternative manner to achieve vanishing of the Lorentz 4-force is light-likeness of the Kähler 4-current. This does not require topologization.

Topologization of the Kähler current for $D_{CP_2} = 3$: covariant formulation

The condition states that Kähler 4-current is proportional to the instanton current whose divergence is instanton density and vanishes when the dimension of CP_2 projection is smaller than four: $D_{CP_2} < 4$. For $D_{CP_2} = 2$ the instanton 4-current vanishes identically and topologization is equivalent with the vanishing of the Kähler current.

If the simplest vision about light-like 3-surfaces as basic dynamical objects is accepted $D_{CP_2} = 2$, corresponds to a non-physical situation and only the deformations of these surfaces - most naturally resulting by gluing of CP_2 type vacuum extremals on them - can represent preferred extremals of Kähler action. One can however speak about $D_{CP_2} = 2$ phase if 4-surfaces are obtained are obtained in this manner.

$$j^\alpha \equiv D_\beta J^{\alpha\beta} = \psi \times j_I^\alpha = \psi \times \epsilon^{\alpha\beta\gamma\delta} J_{\beta\gamma} A_\delta \ . \tag{8.3.7}$$

Here the function ψ is an arbitrary function $\psi(s^k)$ of CP_2 coordinates s^k regarded as functions of space-time coordinates. It is essential that ψ depends on the space-time coordinates through the CP_2 coordinates only. Hence the representation as an imbedded gauge field is crucial element of the solution ansatz.

The field equations state the vanishing of the divergence of the 4-current. This is trivially true for instanton current for $D_{CP_2} < 4$. Also the contraction of $\nabla\psi$ (depending on space-time coordinates through CP_2 coordinates only) with the instanton current is proportional to the winding number density and therefore vanishes for $D_{CP_2} < 4$.

The topologization of the Kähler current guarantees the vanishing of the Lorentz 4-force. Indeed, using the self-duality condition for the current, the expression for the Lorentz 4-force reduces to a term proportional to the instanton density:

$$\begin{aligned} j^\alpha J_{\alpha\beta} &= \psi \times j_I^\alpha J_{\alpha\beta} \\ &= \psi \times \epsilon^{\alpha\mu\nu\delta} J_{\mu\nu} A_\delta J_{\alpha\beta} \ . \end{aligned} \tag{8.3.8}$$

Since all vector quantities appearing in the contraction with the four-dimensional permutation tensor are proportional to the gradients of CP_2 coordinates, the expression is proportional to the instanton density, and thus winding number density, and vanishes for $D_{CP_2} < 4$.

Remarkably, the topologization of the Kähler current guarantees also the vanishing of the term $j^\alpha J^{k_l} \partial_\alpha s^k$ in the field equations for CP_2 coordinates. This means that field equations reduce in both M^4_+ and CP_2 degrees of freedom to

$$T^{\alpha\beta} H^k_{\alpha\beta} = 0 \ . \tag{8.3.9}$$

These equations differ from the equations of minimal surface only by the replacement of the metric tensor with energy momentum tensor. The earlier proposal that quaternion conformal invariance in a suitable sense might provide a general solution of the field equations could be seen as a generalization of the ordinary conformal invariance of string models. If the topologization of the Kähler current implying effective dimensional reduction in CP_2 degrees of freedom is consistent with quaternion conformal invariance, the quaternion conformal structures must differ for the different dimensions of CP_2 projection.

Topologization of the Kähler current for $D_{CP_2} = 3$: non-covariant formulation

In order to gain a concrete understanding about what is involved it is useful to repeat these arguments using the 3-dimensional notation. The components of the instanton 4-current read in three-dimensional notation as

$$\overline{j}_I = \overline{E} \times \overline{A} + \phi \overline{B} \ , \quad \rho_I = \overline{B} \cdot \overline{A} \ . \tag{8.3.10}$$

The self duality conditions for the current can be written explicitly using 3-dimensional notation and read

$$\begin{aligned} \nabla \times \overline{B} - \partial_t \overline{E} &= \overline{j} = \psi \overline{j}_I = \psi \left(\phi \overline{B} + \overline{E} \times \overline{A} \right) \ , \\ \nabla \cdot E &= \rho = \psi \rho_I \ . \end{aligned} \tag{8.3.11}$$

For a vanishing electric field the self-duality condition for Kähler current reduces to the Beltrami condition

$$\nabla \times \overline{B} = \alpha \overline{B} \ , \quad \alpha = \psi\phi \ . \tag{8.3.12}$$

The vanishing of the divergence of the magnetic field implies that α is constant along the field lines of the flow. When ϕ is constant and $\overline{A}$ is time independent, the condition reduces to the Beltrami condition with $\alpha = \phi = constant$, which allows an explicit solution [B5].

One can check also the vanishing of the Lorentz 4-force by using 3-dimensional notation. Lorentz 3-force can be written as

$$\rho_I \overline{E} + \overline{j} \times \overline{B} = \psi \overline{B} \cdot \overline{A} \overline{E} + \psi \left(\overline{E} \times \overline{A} + \phi \overline{B} \right) \times \overline{B} = 0 \ . \tag{8.3.13}$$

The fourth component of the Lorentz force reads as

$$\overline{j} \cdot \overline{E} = \psi \overline{B} \cdot \overline{E} + \psi \left(\overline{E} \times \overline{A} + \phi \overline{B} \right) \cdot \overline{E} = 0 \ . \tag{8.3.14}$$

The remaining conditions come from the induction law of Faraday and could be guaranteed by expressing $\overline{E}$ and $\overline{B}$ in terms of scalar and vector potentials.

The density of the Kähler electric charge of the vacuum is proportional to the the helicity density of the so called helicity charge $\rho = \psi \rho_I = \psi B \cdot A$. This charge is topological charge in the sense that it does not depend on the induced metric at all. Note the presence of arbitrary function ψ of CP_2 coordinates.

Further conditions on the functions appearing in the solution ansatz come from the 3 independent field equations for CP_2 coordinates. What is remarkable that the generalized self-duality condition for the Kähler current allows to understand the general features of the solution ansatz to very high degree without any detailed knowledge about the detailed solution. The question whether field equations allow solutions consistent with the self duality conditions of the current will be dealt later. The optimistic guess is that the field equations and topologization of the Kähler current relate to each other very intimately.

Vanishing or light likeness of the Kähler current guarantees vanishing of the Lorentz 4-force for $D_{CP_2} = 2$

For $D_{CP_2} = 2$ one can always take two CP_2 coordinates as space-time coordinates and from this it is clear that instanton current vanishes so that topologization gives a vanishing Kähler current. In particular, the Beltrami condition $\nabla \times \overline{B} = \alpha \overline{B}$ is not consistent with the topologization of the instanton current for $D_{CP_2} = 2$.

$D_{CP_2} = 2$ case can be treated in a coordinate invariant manner by using the two coordinates of CP_2 projection as space-time coordinates so that only a magnetic or electric field is present depending on whether the gauge current is time-like or space-like. Light-likeness of the gauge current provides a second manner to achieve the vanishing of the Lorentz force and is realized in case of massless extremals having $D_{CP_2} = 2$: this current is in the direction of propagation whereas magnetic and electric fields are orthogonal to it so that Beltrami conditions is certainly not satisfied.

Under what conditions topologization of Kähler current yields Beltrami conditions?

Topologization of the Kähler 4-current gives rise to magnetic Beltrami fields if either of the following conditions is satisfied.

1. The $\overline{E} \times \overline{A}$ term contributing besides $\phi \overline{B}$ term to the topological current vanishes. This requires that $\overline{E}$ and $\overline{A}$ are parallel to each other

$$\overline{E} = \nabla \Phi - \partial_t \overline{A} = \beta \overline{A} \tag{8.3.15}$$

   This condition is analogous to the Beltrami condition. Now only the 3-space has as its coordinates time coordinate and two spatial coordinatesand$\overline{B}$ is replaced with $\overline{A}$. Since E and B are orthogonal, this condition implies $\overline{B} \cdot \overline{A} = 0$ so that Kähler charge density is vanishing.

2. The vector $\overline{E}\times\overline{A}$ is parallel to $\overline{B}$.

$$\overline{E}\times\overline{A} \;=\; \beta\overline{B} \tag{8.3.16}$$

The condition is consistent with the orthogonality of $\overline{E}$ and $\overline{B}$ but implies the orthogonality of $\overline{A}$ and $\overline{B}$ so that electric charge density vanishes

In both cases vector potential fails to define a contact structure since $B\cdot A$ vanishes (contact structures are discussed briefly below), and there exists a global coordinate along the field lines of $\overline{A}$ and the full contact structure is lost again. Note however that the Beltrami condition for magnetic field means that magnetic field defines a contact structure irrespective of whether $\overline{B}\cdot\overline{A}$ vanishes or not. The transition from the general case to Beltrami field would thus involve the replacement

$$(\overline{A},\overline{B})\rightarrow_{\nabla\times}(\overline{B},\overline{j})$$

induced by the rotor.

One must of course take these considerations somewhat cautiously since the inner product depends on the induced 4-metric and it might be that induced metric could allow small vacuum charge density and make possible genuine contact structure.

Hydrodynamic analogy

The field equations of TGD are basically hydrodynamic equations stating the local conservation of the currents associated with the isometries of the embedding space. Therefore it is intriguing that Beltrami fields appear also as solutions of ideal magnetohydrodynamics equations and as steady solutions of non-viscous incompressible flow described by Euler equations [B49] .

In hydrodynamics the role of the magnetic field is taken by the velocity field. TGD based models for nuclei [?] and condensed matter [K36] involve in an essential manner valence quarks having large $\hbar$ and exotic quarks giving nucleons anomalous color and weak charges creating long ranged color and weak forces. Weak forces have a range of order atomic radius and could be responsible for the repulsive core in van der Waals potential.

This raises the idea that the incompressible flow could occur along the field lines of the Z^0 magnetic field so that the velocity field would be proportional to the Z^0 magnetic field and the Beltrami condition for the velocity field would reduce to that for Z^0 magnetic field. Thus the flow lines of hydrodynamic flow would directly correspond to those of Z^0 magnetic field. The generalized Beltrami flow based on the topologization of the Z^0 current would allow to model also the non-stationary incompressible non-viscous hydrodynamical flows.

It would seem that one cannot describe viscous flows using flows satisfying generalized Beltrami conditions since the vanishing of the Lorentz 4-force says that there is no local dissipation of the classical field energy. One might claim that this is not a problem since in TGD framework viscous flow could be seen as a practical description of a quantum jump sequence by replacing the corresponding sequence of space-time surfaces with a single space-time surface.

One the other hand, quantum classical correspondence requires that also dissipative effects have space-time correlates. Kähler fields, which are dissipative, and thus correspond to a non-vanishing Lorentz 4-force, represent one candidate for correlates of this kind. If this is the case, then the fields satisfying the generalized Beltrami condition provide space-time correlates only for the asymptotic self organization patterns for which the viscous effects are negligible, and also the solutions of field equations describing effects of viscosity should be possible.

One must however take this argument with a grain of salt. Dissipation, that is the transfer of conserved quantities to degrees of freedom corresponding to shorter scales, could correspond to a transfer of these quantities between different space-time sheets of the many-sheeted space-time. Here the opponent could however argue that larger space-time sheets mimic the dissipative dynamics in shorter scales and that classical currents represent "symbolically" averaged currents in shorter length scales, and that the local non-conservation of energy momentum tensor consistent with local conservation of isometry currents provides a unique manner to mimic the dissipative dynamics. This view will be developed in more detail below.

The stability of generalized Beltrami fields

The stability of generalized Beltrami fields is of high interest since unstable points of space-time sheets are those around which macroscopic changes induced by quantum jumps are expected to be localized.

1. Contact forms and contact structures

The stability of Beltrami flows has been studied using the theory of contact forms in three-dimensional Riemann manifolds [B28] . Contact form is a one-form A (that is covariant vector field A_α) with the property $A \wedge dA \neq 0$. In the recent case the induced Kähler gauge potential A_α and corresponding induced Kähler form $J_{\alpha\beta}$ for any 3-sub-manifold of space-time surface define a contact form so that the vector field $A^\alpha = g^{\alpha\beta}A_\beta$ is not orthogonal with the magnetic field $B^\alpha = \epsilon^{\alpha\beta\delta}J_{\beta\gamma}$. This requires that magnetic field has a helical structure. Induced metric in turn defines the Riemann structure.

If the vector potential defines a contact form, the charge density associated with the topologized Kähler current must be non-vanishing. This can be seen as follows.

1. The requirement that the flow lines of a one-form X_μ defined by the vector field X^μ as its dual allows to define a global coordinate x varying along the flow lines implies that there is an integrating factor ϕ such that $\phi X = dx$ and therefore $d(\phi X) = 0$. This implies $dlog(\phi) \wedge X = -dX$. From this the necessary condition for the existence of the coordinate x is $X \wedge dX = 0$. In the three-dimensional case this gives $\overline{X} \cdot (\nabla \times \overline{X}) = 0$.
2. This condition is by definition not satisfied by the vector potential defining a contact form so that one cannot identify a global coordinate varying along the flow lines of the vector potential. The condition $\overline{B} \cdot \overline{A} \neq 0$ states that the charge density for the topologized Kähler current is non-vanishing. The condition that the field lines of the magnetic field allow a global coordinate requires $\overline{B} \cdot \nabla \times \overline{B} = 0$. The condition is not satisfied by Beltrami fields with $\alpha \neq 0$. Note that in this case magnetic field defines a contact structure.

Contact structure requires the existence of a vector ξ satisfying the condition $A(\xi) = 0$. The vector field ξ defines a plane field, which is orthogonal to the vector field A^α. Reeb field in turn is a vector field for which $A(X) = 1$ and $dA(X;) = 0$ hold true. The latter condition states the vanishing of the cross product $X \times B$ so that X is parallel to the Kähler magnetic field B^α and has unit projection in the direction of the vector field A^α. Any Beltrami field defines a Reeb field irrespective of the Riemannian structure.

2. Stability of the Beltrami flow and contact structures

Contact structures are used in the study of the topology and stability of the hydrodynamical flows [B28] , and one might expect that the notion of contact structure and its proper generalization to the four-dimensional context could be useful in TGD framework also. An example giving some idea about the complexity of the flows defined by Beltrami fields is the Beltrami field in R^3 possessing closed orbits with all possible knot and link types simultaneously [B28] !

Beltrami flows associated with Euler equations are known to be unstable [B28] . Since the flow is volume preserving, the stationary points of the Beltrami flow are saddle points at which also vorticity vanishes and linear instabilities of Navier-Stokes equations can develop. From the point of view of biology it is interesting that the flow is stabilized by vorticity which implies also helical structures. The stationary points of the Beltrami flow correspond in TGD framework to points at which the induced Kähler magnetic field vanishes. They can be unstable by the vacuum degeneracy of Kähler action implying classical non-determinism. For generalized Beltrami fields velocity and vorticity (both divergence free) are replaced by Kähler current and instanton current.

More generally, the points at which the Kähler 4-current vanishes are expected to represent potential instabilities. The instanton current is linear in Kähler field and can vanish in a gauge invariant manner only if the induced Kähler field vanishes so that the instability would be due to the vacuum degeneracy also now. Note that the vanishing of the Kähler current allows also the generation of region with $D_{CP_2} = 4$. The instability of the points at which induce Kähler field vanish is manifested in quantum jumps replacing the generalized Beltrami field with a new one such that something new is generated around unstable points. Thus the regions in which induced

Kähler field becomes weak are the most interesting ones. For example, unwinding of DNA could be initiated by an instability of this kind.

8.3.3 How To Satisfy Field Equations?

The topologization of the Kähler current guarantees also the vanishing of the term $j^\alpha J^{k_l}\partial_\alpha s^k$ in the field equations for CP_2 coordinates. This means that field equations reduce in both M^4_+ and CP_2 degrees of freedom to

$$T^{\alpha\beta}H^k_{\alpha\beta} = 0 . \quad (8.3.17)$$

These equations differ from the equations of minimal surface only by the replacement of the metric tensor with energy momentum tensor. The following approach utilizes the properties of Hamilton Jacobi structures of M^4_+ introduced in the study of massless extremals and contact structures of CP_2 emerging naturally in the case of generalized Beltrami fields.

String model as a starting point

String model serves as a starting point.

1. In the case of Minkowskian minimal surfaces representing string orbit the field equations reduce to purely algebraic conditions in light cone coordinates (u, v) since the induced metric has only the component g_{uv}, whereas the second fundamental form has only diagonal components H^k_{uu} and H^k_{vv}.
2. For Euclidian minimal surfaces (u, v) is replaced by complex coordinates $(w, \overline{w})$ and field equations are satisfied because the metric has only the component $g^{w\overline{w}}$ and second fundamental form has only components of type H^k_{ww} and $H^{\overline{k}}_{\overline{w}\overline{w}}$. The mechanism should generalize to the recent case.

The general form of energy momentum tensor as a guideline for the choice of coordinates

Any 3-dimensional Riemann manifold allows always a orthogonal coordinate system for which the metric is diagonal. Any 4-dimensional Riemann manifold in turn allows a coordinate system for which 3-metric is diagonal and the only non-diagonal components of the metric are of form g^{ti}. This kind of coordinates might be natural also now. When $\overline{E}$ and $\overline{B}$ are orthogonal, energy momentum tensor has the form

$$T = \begin{pmatrix} \frac{E^2+B^2}{2} & 0 & 0 & EB \\ 0 & \frac{E^2+B^2}{2} & 0 & 0 \\ 0 & 0 & \frac{-E^2+B^2}{2} & 0 \\ EB & 0 & 0 & \frac{E^2-B^2}{2} \end{pmatrix} \quad (8.3.18)$$

in the tangent space basis defined by time direction and longitudinal direction $\overline{E}\times\overline{B}$, and transversal directions $\overline{E}$ and $\overline{B}$. Note that T is traceless.

The optimistic guess would be that the directions defined by these vectors integrate to three orthogonal coordinates of X^4 and together with time coordinate define a coordinate system containing only g^{ti} as non-diagonal components of the metric. This however requires that the fields in question allow an integrating factor and, as already found, this requires $\nabla \times X \cdot X = 0$ and this is not the case in general.

Physical intuition suggests however that X^4 coordinates allow a decomposition into longitudinal and transversal degrees freedom. This would mean the existence of a time coordinate t and longitudinal coordinate z the plane defined by time coordinate and vector $\overline{E} \times \overline{B}$ such that the coordinates $u = t - z$ and $v = t + z$ are light like coordinates so that the induced metric would have only the component g^{uv} whereas g^{vv} and g^{uu} would vanish in these coordinates. In the transversal space-time directions complex space-time coordinate coordinate w could be introduced. Metric could have also non-diagonal components besides the components $g^{w\overline{w}}$ and g^{uv}.

Hamilton Jacobi structures in M^4_+

Hamilton Jacobi structure in M^4_+ can understood as a generalized complex structure combing transversal complex structure and longitudinal hyper-complex structure so that notion of holomorphy and Kähler structure generalize.

1. Denote by m^i the linear Minkowski coordinates of M^4. Let (S^+, S^-, E^1, E^2) denote local coordinates of M^4_+ defining a *local* decomposition of the tangent space M^4 of M^4_+ into a direct, not necessarily orthogonal, sum $M^4 = M^2 \oplus E^2$ of spaces M^2 and E^2. This decomposition has an interpretation in terms of the longitudinal and transversal degrees of freedom defined by local light-like four-velocities $v_\pm = \nabla S_\pm$ and polarization vectors $\epsilon_i = \nabla E^i$ assignable to light ray. Assume that E^2 allows complex coordinates $w = E^1 + iE^2$ and $\overline{w} = E^1 - iE^2$. The simplest decomposition of this kind corresponds to the decomposition $(S^+ \equiv u = t + z, S^- \equiv v = t - z, w = x + iy, \overline{w} = x - iy)$.

2. In accordance with this physical picture, S^+ and S^- define light-like curves which are normals to light-like surfaces and thus satisfy the equation:

$$(\nabla S_\pm)^2 = 0 \quad .$$

   The gradients of $S_\pm$ are obviously analogous to local light like velocity vectors $v = (1, \overline{v})$ and $\tilde{v} = (1, -\overline{v})$. These equations are also obtained in geometric optics from Hamilton Jacobi equation by replacing photon's four-velocity with the gradient ∇S: this is consistent with the interpretation of massless extremals as Bohr orbits of em field. $S_\pm = constant$ surfaces can be interpreted as expanding light fronts. The interpretation of $S_\pm$ as Hamilton Jacobi functions justifies the term Hamilton Jacobi structure.

   The simplest surfaces of this kind correspond to $t = z$ and $t = -z$ light fronts which are planes. They are dual to each other by hyper complex conjugation $u = t - z \rightarrow v = t + z$. One should somehow generalize this conjugation operation. The simplest candidate for the conjugation $S^+ \rightarrow S^-$ is as a conjugation induced by the conjugation for the arguments: $S^+(t - z, t + z, x, y) \rightarrow S^-(t - z, t + z, x, y) = S^+(t + z, t - z, x, -y)$ so that a dual pair is mapped to a dual pair. In transversal degrees of freedom complex conjugation would be involved.

3. The coordinates $(S_\pm, w, \overline{w})$ define local light cone coordinates with the line element having the form

$$\begin{aligned} ds^2 &= g_{+-}dS^+dS^- + g_{w\overline{w}}dwd\overline{w} \\ &+ g_{+w}dS^+dw + g_{+\overline{w}}dS^+d\overline{w} \\ &+ g_{-w}dS^-dw + g_{-\overline{w}}dS^-d\overline{w} \ . \end{aligned} \tag{8.3.19}$$

   Conformal transformations of M^4_+ leave the general form of this decomposition invariant. Also the transformations which reduces to analytic transformations $w \rightarrow f(w)$ in transversal degrees of freedom and hyper-analytic transformations $S^+ \rightarrow f(S^+), S^- \rightarrow f(S^-)$ in longitudinal degrees of freedom preserve this structure.

4. The basic idea is that of generalized Kähler structure meaning that the notion of Kähler function generalizes so that the non-vanishing components of metric are expressible as

$$\begin{aligned} g_{w\overline{w}} &= \partial_w \partial_{\overline{w}} K \ , \quad g_{+-} = \partial_{S^+}\partial_{S^-} K \ , \\ g_{w\pm} &= \partial_w \partial_{S^\pm} K \ , \quad g_{\overline{w}\pm} = \partial_{\overline{w}}\partial_{S^\pm} K \ . \end{aligned} \tag{8.3.20}$$

   for the components of the metric. The expression in terms of Kähler function is coordinate invariant for the same reason as in case of ordinary Kähler metric. In the standard light-cone coordinates the Kähler function is given by

$$K = w_0\overline{w}_0 + uv \ , \quad w_0 = x + iy \ , \quad u = t - z \ , \quad v = t + z \ . \tag{8.3.21}$$

The Christoffel symbols satisfy the conditions

$$\{{}^{k}_{w\,\overline{w}}\} = 0 \ , \quad \{{}^{k}_{+-}\} = 0 \ . \tag{8.3.22}$$

If energy momentum tensor has only the components $T^{w\overline{w}}$ and T^{+-}, field equations are satisfied in M^4_+ degrees of freedom.

5. The Hamilton Jacobi structures related by these transformations can be regarded as being equivalent. Since light-like 3- surface is, as the dynamical evolution defined by the light front, fixed by the 2-surface serving as the light source, these structures should be in one-one correspondence with 2-dimensional surfaces with two surfaces regarded as equivalent if they correspond to different time=constant snapshots of the same light front, or are related by a conformal transformation of M^4_+. Obviously there should be quite large number of them. Note that the generating two-dimensional surfaces relate also naturally to quaternion conformal invariance and corresponding Kac Moody invariance for which deformations defined by the M^4 coordinates as functions of the light-cone coordinates of the light front evolution define Kac Moody algebra, which thus seems to appear naturally also at the level of solutions of field equations.

The task is to find all possible local light cone coordinates defining one-parameter families 2-surfaces defined by the condition $S_i = constant$, $i = +\ or\ = -$, dual to each other and expanding with light velocity. The basic open questions are whether the generalized Kähler function indeed makes sense and whether the physical intuition about 2-surfaces as light sources parameterizing the set of all possible Hamilton Jacobi structures makes sense.

Hamilton Jacobi structure means the existence of foliations of the M^4 projection of X^4 by 2-D surfaces analogous to string word sheets labeled by w and the dual of this foliation defined by partonic 2-surfaces labeled by the values of S_i. Also the foliation by light-like 3-surfaces Y^3_l labeled by $S_\pm$ with $S_\mp$ serving as light-like coordinate for Y^3_l is implied. This is what number theoretic compactification and $M^8 - H$ duality predict when space-time surface corresponds to hyper-quaternionic surface of M^8 [K43, K105] .

Contact structure and generalized Kähler structure of CP_2 projection

In the case of 3-dimensional CP_2 projection it is assumed that one can introduce complex coordinates $(\xi, \overline{\xi})$ and the third coordinate s. These coordinates would correspond to a contact structure in 3-dimensional CP_2 projection defining transversal symplectic and Kähler structures. In these coordinates the transversal parts of the induced CP_2 Kähler form and metric would contain only components of type $g_{w\overline{w}}$ and $J_{w\overline{w}}$. The transversal Kähler field $J_{w\overline{w}}$ would induce the Kähler magnetic field and the components J_{sw} and $J_{s\overline{w}}$ the Kähler electric field.

It must be emphasized that the non-integrability of the contact structure implies that J cannot be parallel to the tangent planes of $s = constant$ surfaces, s cannot be parallel to neither A nor the dual of J, and ξ cannot vary in the tangent plane defined by J. A further important conclusion is that for the solutions with 3-dimensional CP_2 projection topologized Kähler charge density is necessarily non-vanishing by $A \wedge J \neq 0$ whereas for the solutions with $D_{CP_2} = 2$ topologized Kähler current vanishes.

Also the CP_2 projection is assumed to possess a generalized Kähler structure in the sense that all components of the metric except s_{ss} are derivable from a Kähler function by formulas similar to M^4_+ case.

$$s_{w\overline{w}} = \partial_w \partial_{\overline{w}} K \ , \quad s_{ws} = \partial_w \partial_s K \ , \quad s_{\overline{w}s} = \partial_{\overline{w}} \partial_s K \ . \tag{8.3.23}$$

Generalized Kähler property guarantees that the vanishing of the Christoffel symbols of CP_2 (rather than those of 3-dimensional projection), which are of type $\{{}^{k}_{\xi\,\overline{\xi}}\}$.

$$\{{}^{k}_{\xi\,\overline{\xi}}\} \ = \ 0 \ . \tag{8.3.24}$$

Here the coordinates of CP_2 have been chosen in such a manner that three of them correspond to the coordinates of the projection and fourth coordinate is constant at the projection. The upper index k refers also to the CP_2 coordinate, which is constant for the CP_2 projection. If energy momentum tensor has only components of type T^{+-} and $T^{w\overline{w}}$, field equations are satisfied even when if non-diagonal Christoffel symbols of CP_2 are present. The challenge is to discover solution ansatz, which guarantees this property of the energy momentum tensor.

A stronger variant of Kähler property would be that also s_{ss} vanishes so that the coordinate lines defined by s would define light like curves in CP_2. The topologization of the Kähler current however implies that CP_2 projection is a projection of a 3-surface with strong Kähler property. Using $(s,\xi,\overline{\xi},S^-)$ as coordinates for the space-time surface defined by the ansatz $(w=w(\xi,s),S^+=S^+(s))$ one finds that g_{ss} must be vanishing so that stronger variant of the Kähler property holds true for $S^-=constant$ 3-surfaces.

The topologization condition for the Kähler current can be solved completely generally in terms of the induced metric using $(\xi,\overline{\xi},s)$ and some coordinate of M^4_+, call it x^4, as space-time coordinates. Topologization boils down to the conditions

$$\begin{aligned} \partial_\beta(J^{\alpha\beta}\sqrt{g}) &= 0 \text{ for } \alpha\in\{\xi,\overline{\xi},s\} \ , \\ g^{4i}\neq 0 \ . \end{aligned} \tag{8.3.25}$$

Thus 3-dimensional empty space Maxwell equations and the non-orthogonality of X^4 coordinate lines and the 3-surfaces defined by the lift of the CP_2 projection.

A solution ansatz yielding light-like current in $D_{CP_2}=3$ case

The basic idea is that of generalized Kähler structure and solutions of field equations as maps or deformations of canonically imbedded M^4_+ respecting this structure and guaranteeing that the only non-vanishing components of the energy momentum tensor are $T^{\xi\xi}$ and T^{s-} in the coordinates $(\xi,\overline{\xi},s,S^-)$.

1. The coordinates (w,S^+) are assumed to holomorphic functions of the CP_2 coordinates (s,ξ)

$$S^+=S^+(s) \ , \quad w=w(\xi,s) \ . \tag{8.3.26}$$

   Obviously S^+ could be replaced with S^-. The ansatz is completely symmetric with respect to the exchange of the roles of (s,w) and (S^+,ξ) since it maps longitudinal degrees of freedom to longitudinal ones and transverse degrees of freedom to transverse ones.

2. Field equations are satisfied if the only non-vanishing components of the energy momentum tensor are of type $T^{\xi\overline{\xi}}$ and T^{s-}. The reason is that the CP_2 Christoffel symbols for projection and projections of M^4_+ Christoffel symbols are vanishing for these lower index pairs.

3. By a straightforward calculation one can verify that the only manner to achieve the required structure of energy momentum tensor is to assume that the induced metric in the coordinates $(\xi,\overline{\xi},s,S^-)$ has as non-vanishing components only $g_{\xi\overline{\xi}}$ and g_{s-}

$$g_{ss}=0 \ , \quad g_{\xi s}=0 \ , \quad g_{\overline{\xi}s}=0 \ . \tag{8.3.27}$$

   Obviously the space-time surface must factorize into an orthogonal product of longitudinal and transversal spaces.

4. The condition guaranteeing the product structure of the metric is

$$\begin{aligned} s_{ss} &= m_{+w}\partial_s w(\xi,s)\partial_s S^+(s)+m_{+\overline{w}}\partial_s\overline{w}(\xi,s)\partial_s S^+(s) \ , \\ s_{s\xi} &= m_{+w}\partial_\xi w(\xi)\partial_s S^+(s) \ , \\ s_{s\overline{\xi}} &= m_{+w}\partial_{\overline{\xi}} w(\overline{\xi})\partial_s S^+(s) \ . \end{aligned} \tag{8.3.28}$$

   Thus the function of dynamics is to diagonalize the metric and provide it with strong Kähler property. Obviously the CP_2 projection corresponds to a light-like surface for all values of

S^- so that space-time surface is foliated by light-like surfaces and the notion of generalized conformal invariance makes sense for the entire space-time surface rather than only for its boundary or elementary particle horizons.

5. The requirement that the Kähler current is proportional to the instanton current means that only the j^- component of the current is non-vanishing. This gives the following conditions

$$\begin{aligned} j^{\xi}\sqrt{g} = \partial_\beta(J^{\xi\beta}\sqrt{g}) = 0 \ , \quad j^{\overline{\xi}}\sqrt{g} = \partial_\beta(J^{\overline{\xi}\beta}\sqrt{g}) = 0 \ , \\ j^{+}\sqrt{g} = \partial_\beta(J^{+\beta}\sqrt{g}) = 0 \ . \end{aligned} \quad (8.3.29)$$

Since $J^{+\beta}$ vanishes, the condition

$$\sqrt{g}j^{+} = \partial_\beta(J^{+\beta}\sqrt{g}) \quad = \quad 0 \quad (8.3.30)$$

is identically satisfied. Therefore the number of field equations reduces to three.

The physical interpretation of the solution ansatz deserves some comments.

1. The light-like character of the Kähler current brings in mind CP_2 type extremals for which CP_2 projection is light like. This suggests that the topological condensation of CP_2 type extremal occurs on $D_{CP_2} = 3$ helical space-time sheet representing zitterbewegung. In the case of many-body system light-likeness of the current does not require that particles are massless if particles of opposite charges can be present. Field tensor has the form $(J^{\xi\overline{\xi}}, J^{\xi-}, J^{\overline{\xi}-})$. Both helical magnetic field and electric field present as is clear when one replaces the coordinates (S^+, S^-) with time-like and space-like coordinate. Magnetic field dominates but the presence of electric field means that genuine Beltrami field is not in question.
2. Since the induced metric is product metric, 3-surface is metrically product of 2-dimensional surface X^2 and line or circle and obeys product topology. If absolute minima correspond to asymptotic self-organization patterns, the appearance of the product topology and even metric is not so surprising. Thus the solutions can be classified by the genus of X^2. An interesting question is how closely the explanation of family replication phenomenon in terms of the topology of the boundary component of elementary particle like 3-surface relates to this. The heaviness and instability of particles which correspond to genera $g > 2$ (sphere with more than two handles) might have simple explanation as absence of (stable) $D_{CP_2} = 3$ solutions of field equations with genus $g > 2$.
3. The solution ansatz need not be the most general. Kähler current is light-like and already this is enough to reduce the field equations to the form involving only energy momentum tensor. One might hope of finding also solution ansätze for which Kähler current is time-like or space-like. Space-likeness of the Kähler current might be achieved if the complex coordinates $(\xi, \overline{\xi})$ and hyper-complex coordinates (S^+, S^-) change the role. For this solution ansatz electric field would dominate. Note that the possibility that Kähler current is always light-like cannot be excluded.
4. Suppose that CP_2 projection quite generally defines a foliation of the space-time surface by light-like 3-surfaces, as is suggested by the conformal invariance. If the induced metric has Minkowskian signature, the fourth coordinate x^4 and thus also Kähler current must be time-like or light-like so that magnetic field dominates. Already the requirement that the metric is non-degenerate implies $g_{s4} \neq 0$ so that the metric for the $\xi = constant$ 2-surfaces has a Minkowskian signature. Thus space-like Kähler current does not allow the lift of the CP_2 projection to be light-like.

Are solutions with time-like or space-like Kähler current possible in $D_{CP_2} = 3$ case?

As noticed in the section about number theoretical compactification, the flow of gauge currents along slices Y^3_l of $X^4(X^3_l)$ "parallel" to X^3_l requires only that gauge currents are parallel to Y^3_l and can thus space-like. The following ansatz gives good hopes for obtaining solutions with space-like and perhaps also time-like Kähler currents.

1. Assign to light-like coordinates coordinates (T, Z) by the formula $T = S^+ + S^-$ and $Z = S^+ - S^-$. Space-time coordinates are taken to be $(\xi, \overline{\xi}, s)$ and coordinate Z. The solution ansatz with time-like Kähler current results when the roles of T and Z are changed. It will however found that same solution ansatz can give rise to both space-like and time-like Kähler current.
2. The solution ansatz giving rise to a space-like Kähler current is defined by the equations

$$T = T(Z,s) \ , \quad w = w(\xi,s) \ . \tag{8.3.31}$$

   If T depends strongly on Z, the g_{ZZ} component of the induced metric becomes positive and Kähler current time-like.
3. The components of the induced metric are

$$\begin{array}{ll} g_{ZZ} = m_{ZZ} + m_{TT}\partial_Z T \partial_s T \ , & g_{Zs} = m_{TT}\partial_Z T \partial_s T \ , \\ g_{ss} = s_{ss} + m_{TT}\partial_s T \partial_s T \ , & g_{w\overline{w}} = s_{w\overline{w}} + m_{w\overline{w}}\partial_\xi w \partial_{\overline{\xi}} \overline{w} \ , \\ g_{s\xi} = s_{s\xi} \ , & g_{s\overline{\xi}} = s_{s\overline{\xi}} \ . \end{array} \tag{8.3.32}$$

   Topologized Kähler current has only Z-component and 3-dimensional empty space Maxwell's equations guarantee the topologization.

In CP_2 degrees of freedom the contractions of the energy momentum tensor with Christoffel symbols vanish if T^{ss}, $T^{\xi s}$ and $T^{\xi\xi}$ vanish as required by internal consistency. This is guaranteed if the condition

$$J^{\xi s} \ = \ 0 \tag{8.3.33}$$

holds true. Note however that $J^{\xi Z}$ is non-vanishing. Therefore only the components $T^{\xi\overline{\xi}}$ and $T^{Z\xi}$, $T^{Z\overline{\xi}}$ of energy momentum tensor are non-vanishing, and field equations reduce to the conditions

$$\begin{aligned} \partial_{\overline{\xi}}(J^{\xi\overline{\xi}}\sqrt{g}) + \partial_Z(J^{\xi Z}\sqrt{g}) &= 0 \ , \\ \partial_{\xi}(J^{\overline{\xi}\xi}\sqrt{g}) + \partial_Z(J^{\overline{\xi}Z}\sqrt{g}) &= 0 \ . \end{aligned} \tag{8.3.34}$$

In the special case that the induced metric does not depend on z-coordinate equations reduce to holomorphicity conditions. This is achieve if T depends linearly on Z: $T = aZ$.

The contractions with M^4_+ Christoffel symbols come from the non-vanishing of $T^{Z\xi}$ and vanish if the Hamilton Jacobi structure satisfies the conditions

$$\begin{array}{ll} \left\{ {}_{T}{}^{k}{}_{w} \right\} = 0 \ , & \left\{ {}_{T}{}^{k}{}_{\overline{w}} \right\} = 0 \ , \\ \left\{ {}_{Z}{}^{k}{}_{w} \right\} = 0 \ , & \left\{ {}_{Z}{}^{k}{}_{\overline{w}} \right\} = 0 \end{array} \tag{8.3.35}$$

hold true. The conditions are equivalent with the conditions

$$\left\{ {}_{\pm}{}^{k}{}_{w} \right\} = 0 \ , \quad \left\{ {}_{\pm}{}^{k}{}_{\overline{w}} \right\} = 0 \ . \tag{8.3.36}$$

These conditions possess solutions (standard light cone coordinates are the simplest example). Also the second derivatives of $T(s, Z)$ contribute to the second fundamental form but they do not give rise to non-vanishing contractions with the energy momentum tensor. The cautious conclusion is that also solutions with time-like or space-like Kähler current are possible.

$D_{CP_2} = 4$ case

The preceding discussion was for $D_{CP_2} = 3$ and one should generalize the discussion to $D_{CP_2} = 4$ case.

1. Hamilton Jacobi structure for M^4_+ is expected to be crucial also now.
2. One might hope that for $D_{CP_2} = 4$ the Kähler structure of CP_2 defines a foliation of CP_2 by 3-dimensional contact structures. This requires that there is a coordinate varying along the field lines of the normal vector field X defined as the dual of the three-form $A \wedge dA = A \wedge J$. By the previous considerations the condition for this reads as $dX = d(log\phi) \wedge X$ and implies $X \wedge dX = 0$. Using the self duality of the Kähler form one can express X as $X^k = J^{kl}A_l$. By a brief calculation one finds that $X \wedge dX \propto X$ holds true so that (somewhat disappointingly) a foliation of CP_2 by contact structures does not exist.

For $D_{CP_2} = 4$ case Kähler current vanishes and this case corresponds to what I have called earlier Maxwellian phase since empty space Maxwell's equations would be indeed satisfied, provided this phase exists at all. It however seems that Maxwell phase is probably realized differently.

1. Solution ansatz with a 3-dimensional M^4_+ projection

The basic idea is that the complex structure of CP_2 is preserved so that one can use complex coordinates (ξ^1, ξ^2) for CP_2 in which CP_2 Christoffel symbols and energy momentum tensor have automatically the desired properties. This is achieved the second light like coordinate, say v, is non-dynamical so that the induced metric does not receive any contribution from the longitudinal degrees of freedom. In this case one has

$$S^+ = S^+(\xi^1, \xi^2) \ , \quad w = w(\xi^1, \xi^2) \ , \quad S^- = constant \ . \tag{8.3.37}$$

The induced metric does possesses only components of type $g_{i\overline{j}}$ if the conditions

$$g_{+w} = 0 \ , \quad g_{+\overline{w}} = 0 \ . \tag{8.3.38}$$

This guarantees that energy momentum tensor has only components of type $T^{i\overline{j}}$ in coordinates (ξ^1, ξ^2) and their contractions with the Christoffel symbols of CP_2 vanish identically. In M^4_+ degrees of freedom one must pose the conditions

$$\{ {}^{\ k}_{w+} \} = 0 \ , \quad \{ {}^{\ k}_{\overline{w}+} \} = 0 \ , \quad \{ {}^{\ k}_{++} \} = 0 \ . \tag{8.3.39}$$

on Christoffel symbols. These conditions are satisfied if the the M^4_+ metric does not depend on S^+:

$$\partial_+ m_{kl} \ = \ 0 \ . \tag{8.3.40}$$

This means that m_{-w} and $m_{-\overline{w}}$ can be non-vanishing but like m_{+-} they cannot depend on S^+.

The second derivatives of S^+ appearing in the second fundamental form are also a source of trouble unless they vanish. Hence S^+ must be a linear function of the coordinates ξ^k:

$$S^+ \ = \ a_k\xi^k + \overline{a}_k\overline{\xi}^k \ . \tag{8.3.41}$$

Field equations are the counterparts of empty space Maxwell equations $j^\alpha = 0$ but with M^4_+ coordinates (u, w) appearing as dynamical variables and entering only through the induced metric. By holomorphy the field equations can be written as

$$\partial_j(J^{j\overline{i}}\sqrt{g}) = 0 \ , \quad \partial_{\overline{j}}(J^{\overline{j}i}\sqrt{g}) = 0 \ , \tag{8.3.42}$$

and can be interpreted as conditions stating the holomorphy of the contravariant Kähler form.

What is remarkable is that the M^4_+ projection of the solution is 3-dimensional light like surface and that the induced metric has Euclidian signature. Light front would become a concrete geometric object with one compactified dimension rather than being a mere conceptualization. One could see this as topological quantization for the notion of light front or of electromagnetic shock wave, or perhaps even as the realization of the particle aspect of gauge fields at classical level.

If the latter interpretation is correct, quantum classical correspondence would be realized very concretely. Wave and particle aspects would both be present. One could understand the interactions of charged particles with electromagnetic fields both in terms of absorption and emission of topological field quanta and in terms of the interaction with a classical field as particle topologically condenses at the photonic light front.

For CP_2 type extremals for which M^4_+ projection is a light like curve correspond to a special case of this solution ansatz: transversal M^4_+ coordinates are constant and S^+ is now arbitrary function of CP_2 coordinates. This is possible since M^4_+ projection is 1-dimensional.

2. Are solutions with a 4-dimensional M^4_+ projection possible?

The most natural solution ansatz is the one for which CP_2 complex structure is preserved so that energy momentum tensor has desired properties. For four-dimensional M^4_+ projection this ansatz does not seem to make promising since the contribution of the longitudinal degrees of freedom implies that the induced metric is not anymore of desired form since the components $g_{ij} = m_{+-}(\partial_{\xi^i} S^+ \partial_{\xi^j} S^- + m_{+-}\partial_{\xi^i} S^- \partial_{\xi^j} S^+)$ are non-vanishing.

1. The natural dynamical variables are still Minkowski coordinates $(w, \overline{w}, S^+, S^-)$ for some Hamilton Jacobi structure. Since the complex structure of CP_2 must be given up, CP_2 coordinates can be written as (ξ, s, r) to stress the fact that only "one half" of the Kähler structure of CP_2 is respected by the solution ansatz.
2. The solution ansatz has the same general form as in $D_{CP_2} = 3$ case and must be symmetric with respect to the exchange of M^4_+ and CP_2 coordinates. Transverse coordinates are mapped to transverse ones and longitudinal coordinates to longitudinal ones:

$$(S^+, S^-) = (S^+(s,r), S^-(s,r)) \quad , \quad w = w(\xi) \ . \tag{8.3.43}$$

This ansatz would describe ordinary Maxwell field in M^4_+ since the roles of M^4_+ coordinates and CP_2 coordinates are interchangeable.

It is however far from obvious whether there are any solutions with a 4-dimensional M^4_+ projection. That empty space Maxwell's equations would allow only the topologically quantized light fronts as its solutions would realize quantum classical correspondence very concretely.

The recent view conforms with this intuition. The Maxwell phase is certainly physical notion but would correspond effective fields experience by particle in many-sheeted space-time (see **Fig. `http://tgdtheory.fi/appfigures/manysheeted.jpg`** or **Fig. 2.2** in the appendix of this book). Test particle topological condenses to all the space-time sheets with projection to a given region of Minkowski space and experiences essentially the sum of the effects caused by the induced gauge fields at different sheets. This applies also to gravitational fields interpreted as deviations from Minkowski metric.

The transition to GRT and QFT picture means the replacement of many-sheeted space-time with piece of Minkowski space with effective metric defined as the sum of Minkowski metric and deviations of the induced metrics of space-time sheets from Minkowski metric. Effective gauge potentials are sums of the induced gauge potentials. Hence the rather simple topologically quantized induced gauge fields associated with space-time sheets become the classical fields in the sense of Maxwell's theory and gauge theories.

$D_{CP_2} = 2$ case

Hamilton Jacobi structure for M^4_+ is assumed also for $D_{CP_2} = 2$, whereas the contact structure for CP_2 is in $D_{CP_2} = 2$ case replaced by the induced Kähler structure. Topologization yields vanishing

Kähler current. Light-likeness provides a second manner to achieve vanishing Lorentz force but one cannot exclude the possibility of time- and space-like Kähler current.

1. Solutions with vanishing Kähler current

1. String like objects, which are products $X^2 \times Y^2 \subset M^4_+ \times CP_2$ of minimal surfaces Y^2 of M^4_+ with geodesic spheres S^2 of CP_2 and carry vanishing gauge current. String like objects allow considerable generalization from simple Cartesian products of $X^2 \times Y^2 \subset M^4 \times S^2$. Let $(w, \overline{w}, S^+, S^-)$ define the Hamilton Jacobi structure for M^4_+. $w = constant$ surfaces define minimal surfaces X^2 of M^4_+. Let ξ denote complex coordinate for a sub-manifold of CP_2 such that the embedding to CP_2 is holomorphic: $(\xi^1, \xi^2) = (f^1(\xi), f^2(\xi))$. The resulting surface $Y^2 \subset CP_2$ is a minimal surface and field equations reduce to the requirement that the Kähler current vanishes: $\partial_{\overline{\xi}}(J^{\xi\overline{\xi}}\sqrt{g_2}) = 0$. One-dimensional strings are deformed to 3-dimensional cylinders representing magnetic flux tubes. The oscillations of string correspond to waves moving along string with light velocity, and for more general solutions they become TGD counterparts of Alfven waves associated with magnetic flux tubes regarded as oscillations of magnetic flux lines behaving effectively like strings. It must be emphasized that Alfven waves are a phenomenological notion not really justified by the properties of Maxwell's equations.
2. Also electret type solutions with the role of the magnetic field taken by the electric field are possible. $(\xi, \overline{\xi}, u, v)$ would provide the natural coordinates and the solution ansatz would be of the form

$$(s, r) = (s(u, v), r(u, v)) \quad , \quad \xi = constant \quad , \tag{8.3.44}$$

and corresponds to a vanishing Kähler current.
3. Both magnetic and electric fields are necessarily present only for the solutions carrying non-vanishing electric charge density (proportional to $\overline{B} \cdot \overline{A}$). Thus one can ask whether more general solutions carrying both magnetic and electric field are possible. As a matter fact, one must first answer the question what one really means with the magnetic field. By choosing the coordinates of 2-dimensional CP_2 projection as space-time coordinates one can define what one means with magnetic and electric field in a coordinate invariant manner. Since the CP_2 Kähler form for the CP_2 projection with $D_{CP_2} = 2$ can be regarded as a pure Kähler magnetic field, the induced Kähler field is either magnetic field or electric field.

The form of the ansatz would be

$$(s, r) = (s, r)(u, v, w, \overline{w}) \quad , \quad \xi = constant \quad . \tag{8.3.45}$$

As a matter fact, CP_2 coordinates depend on two properly chosen M^4 coordinates only.

1. Solutions with light-like Kähler current

There are large classes of solutions of field equations with a light-like Kähler current and 2-dimensional CP_2 projection.

1. Massless extremals for which CP_2 coordinates are arbitrary functions of one transversal coordinate $e = f(w, \overline{w})$ defining local polarization direction and light like coordinate u of M^4_+ and carrying in the general case a light like current. In this case the holomorphy does not play any role.
2. The string like solutions thickened to magnetic flux tubes carrying TGD counterparts of Alfven waves generalize to solutions allowing also light-like Kähler current. Also now Kähler metric is allowed to develop a component between longitudinal and transversal degrees of freedom so that Kähler current develops a light-like component. The ansatz is of the form

$$\xi^i = f^i(\xi) \quad , \quad w = w(\xi) \quad , \quad S^- = s^- \quad , \quad S^+ = s^+ + f(\xi, \overline{\xi}) \quad . \tag{8.3.46}$$

Only the components $g_{+\xi}$ and $g_{+\overline{\xi}}$ of the induced metric receive contributions from the modification of the solution ansatz. The contravariant metric receives contributions to $g^{-\xi}$ and $g^{-\overline{\xi}}$ whereas $g^{+\xi}$ and $g^{+\overline{\xi}}$ remain zero. Since the partial derivatives $\partial_\xi\partial_+ h^k$ and $\partial_{\overline{\xi}}\partial_+ h^k$ and corresponding projections of Christoffel symbols vanish, field equations are satisfied. Kähler current develops a non-vanishing component j^-. Apart from the presence of the electric field, these solutions are highly analogous to Beltrami fields.

Could $D_{CP_2} = 2 \rightarrow 3$ transition occur in rotating magnetic systems?

I have studied the embeddings of simple cylindrical and helical magnetic fields in various applications of TGD to condensed matter systems, in particular in attempts to understand the strange findings about rotating magnetic systems [K113] .

Let S^2 be the homologically non-trivial geodesic sphere of CP_2 with standard spherical coordinates $(U \equiv cos(\theta), \Phi)$ and let (t, ρ, ϕ, z) denote cylindrical coordinates for a cylindrical space-time sheet. The simplest possible space-time surfaces $X^4 \subset M^4_+ \times S^2$ carrying helical Kähler magnetic field depending on the radial cylindrical coordinate ρ, are given by:

$$\begin{array}{ll} U = U(\rho) \ , & \Phi = n\phi + kz \ , \\ J_{\rho\phi} = n\partial_\rho U \ , & J_{\rho z} = k\partial_\rho U \ . \end{array} \tag{8.3.47}$$

This helical field is not Beltrami field as one can easily find. A more general ansatz corresponding defined by

$$\Phi = \omega t + kz + n\phi$$

would in cylindrical coordinates give rise to both helical magnetic field and radial electric field depending on ρ only. This field can be obtained by simply replacing the vector potential with its rotated version and provides the natural first approximation for the fields associated with rotating magnetic systems.

A non-vanishing vacuum charge density is however generated when a constant magnetic field is put into rotation and is implied by the condition $\overline{E} = \overline{v} \times \overline{B}$ stating vanishing of the Lorentz force. This condition does not follow from the induction law of Faraday although Faraday observed this effect first. This is also clear from the fact that the sign of the charge density depends on the direction of rotation.

The non-vanishing charge density is not consistent with the vanishing of the Kähler 4-current and requires a 3-dimensional CP_2 projection and topologization of the Kähler current. Beltrami condition cannot hold true exactly for the rotating system. The conclusion is that rotation induces a phase transition $D_{CP_2} = 2 \rightarrow 3$. This could help to understand various strange effects related to the rotating magnetic systems [K113] . For instance, the increase of the dimension of CP_2 projection could generate join along boundaries contacts and wormhole contacts leading to the transfer of charge between different space-time sheets. The possibly resulting flow of gravitational flux to larger space-time sheets might help to explain the claimed antigravity effects.

8.3.4 $D_{CP_2} = 3$ Phase Allows Infinite Number Of Topological Charges Characterizing The Linking Of Magnetic Field Lines

When space-time sheet possesses a $D = 3$-dimensional CP_2 projection, one can assign to it a non-vanishing and conserved topological charge characterizing the linking of the magnetic field lines defined by Chern-Simons action density $A \wedge dA/4\pi$ for induced Kähler form. This charge can be seen as classical topological invariant of the linked structure formed by magnetic field lines.

The topological charge can also vanish for $D_{CP_2} = 3$ space-time sheets. In Darboux coordinates for which Kähler gauge potential reads as $A = P_k dQ^k$, the surfaces of this kind result if one has $Q^2 = f(Q^1)$ implying $A = f dQ^1$, $f = P_1 + P_2\partial_{Q_1}Q^2$, which implies the condition $A \wedge dA = 0$. For these space-time sheets one can introduce Q^1 as a global coordinate along field lines of A and define the phase factor $exp(i\int A_\mu dx^\mu)$ as a wave function defined for the entire space-time sheet. This function could be interpreted as a phase of an order order parameter of super-conductor like state and there is a high temptation to assume that quantum coherence in this sense is lost for more general $D_{CP_2} = 3$ solutions.

Chern-Simons action is known as helicity in electrodynamics [B32] . Helicity indeed describes the linking of magnetic flux lines as is easy to see by interpreting magnetic field as incompressible fluid flow having A as vector potential: $B = \nabla \times A$. One can write A using the inverse of $\nabla\times$ as $A = (1/\nabla\times)B$. The inverse is non-local operator expressible as

$$\frac{1}{\nabla\times}B(r) = \int dV' \frac{(r-r')}{|r-r'|^3} \times B(r') \ ,$$

as a little calculation shows. This allows to write $\int A \cdot B$ as

$$\int dV A \cdot B = \int dV dV' B(r) \cdot \left(\frac{(r-r')}{|r-r'|^3} \times B(r') \right) \ ,$$

which is completely analogous to the Gauss formula for linking number when linked curves are replaced by a distribution of linked curves and an average is taken.

For $D_{CP_2} = 3$ field equations imply that Kähler current is proportional to the helicity current by a factor which depends on CP_2 coordinates, which implies that the current is automatically divergence free and defines a conserved charge for $D = 3$-dimensional CP_2 projection for which the instanton density vanishes identically. Kähler charge is not equal to the helicity defined by the inner product of magnetic field and vector potential but to a more general topological charge.

The number of conserved topological charges is infinite since the product of any function of CP_2 coordinates with the helicity current has vanishing divergence and defines a topological charge. A very natural function basis is provided by the scalar spherical harmonics of $SU(3)$ defining Hamiltonians of CP_2 canonical transformations and possessing well defined color quantum numbers. These functions define and infinite number of conserved charges which are also classical knot invariants in the sense that they are not affected at all when the 3-surface interpreted as a map from CP_2 projection to M^4_+ is deformed in M^4_+ degrees of freedom. Also canonical transformations induced by Hamiltonians in irreducible representations of color group affect these invariants via Poisson bracket action when the $U(1)$ gauge transformation induced by the canonical transformation corresponds to a single valued scalar function. These link invariants are additive in union whereas the quantum invariants defined by topological quantum field theories are multiplicative.

Also non-Abelian topological charges are well-defined. One can generalize the topological current associated with the Kähler form to a corresponding current associated with the induced electro-weak gauge fields whereas for classical color gauge fields the Chern-Simons form vanishes identically. Also in this case one can multiply the current by CP_2 color harmonics to obtain an infinite number of invariants in $D_{CP_2} = 3$ case. The only difference is that $A \wedge dA$ is replaced by $Tr(A \wedge (dA + 2A \wedge A/3))$.

There is a strong temptation to assume that these conserved charges characterize colored quantum states of the conformally invariant quantum theory as a functional of the light-like 3-surface defining boundary of space-time sheet or elementary particle horizon surrounding wormhole contacts. They would be TGD analogs of the states of the topological quantum field theory defined by Chern-Simons action as highest weight states associated with corresponding Wess-Zumino-Witten theory. These charges could be interpreted as topological counterparts of the isometry charges of WCW defined by the algebra of canonical transformations of CP_2.

The interpretation of these charges as contributions of light-like boundaries to WCW Hamiltonians would be natural. The dynamics of the induced second quantized spinor fields relates to that of Kähler action by a super-symmetry, so that it should define super-symmetric counterparts of these knot invariants. The anti-commutators of these super charges cannot however contribute to WCW Kähler metric so that topological zero modes are in question. These Hamiltonians and their super-charge counterparts would be responsible for the topological sector of quantum TGD.

8.3.5 Preferred Extremal Property And The Topologization And Light-Likeness Of Kähler Current?

The basic question is under what conditions the Kähler current is either topologized or light-like so that the Lorentz force vanishes. Does this hold for all preferred extremals of Kähler action? Or only asymptotically as suggested by the fact that generalized Beltrami fields can be interpreted as asymptotic self-organization patterns, when dissipation has become insignificant. Or does topologization take place in regions of space-time surface having Minkowskian signature of the induced metric? And what asymptotia actually means? Do absolute minima of Kähler action correspond to preferred extremals?

One can challenge the interpretation in terms of asymptotic self organization patterns assigned to the Minkowskian regions of space-time surface.

1. ZEO challenges the notion of approach to asymptotia in Minkowskian sense since the dynamics of light-like 3-surfaces is restricted inside finite volume $CD \subset M^4$ since the partonic 2-surfaces representing their ends are at the light-like boundaries of causal diamond in a given p-adic time scale.
2. One can argue that generic non-asymptotic field configurations have $D_{CP_2} = 4$, and would thus carry a vanishing Kähler four-current if Beltrami conditions were satisfied universally rather than only asymptotically. $j^\alpha = 0$ would obviously hold true also for the asymptotic configurations, in particular those with $D_{CP_2} < 4$ so that empty space Maxwell's field equations would be universally satisfied for asymptotic field configurations with $D_{CP_2} < 4$. The weak point of this argument is that it is 3-D light-like 3-surfaces rather than space-time surfaces which are the basic dynamical objects so that the generic and only possible case corresponds to $D_{CP_2} = 3$ for X_l^3. It is quite possible that preferred extremal property implies that $D_{CP_2} = 3$ holds true in the Minkowskian regions since these regions indeed represent empty space. Geometrically this would mean that the CP_2 projection does not change as the light-like coordinate labeling Y_l^3 varies. This conforms nicely with the notion of quantum gravitational holography.
3. The failure of the generalized Beltrami conditions would mean that Kähler field is completely analogous to a dissipative Maxwell field for which also Lorentz force vanishes since $\overline{j} \cdot \overline{E}$ is non-vanishing (note that isometry currents are conserved although energy momentum tensor is not). Quantum classical correspondence states that classical space-time dynamics is by its classical non-determinism able to mimic the non-deterministic sequence of quantum jumps at space-time level, in particular dissipation in various length scales defined by the hierarchy of space-time sheets. Classical fields would represent "symbolically" the average dynamics, in particular dissipation, in shorter length scales. For instance, vacuum 4-current would be a symbolic representation for the average of the currents consisting of elementary particles. This would seem to support the view that $D_{CP_2} = 4$ Minkowskian regions are present. The weak point of this argument is that there is fractal hierarchy of length scales represented by the hierarchy of causal diamonds (CDs) and that the resulting hierarchy of generalized Feynman graphs might be enough to represent dissipation classically.
4. One objection to the idea is that second law realized as an asymptotic vanishing of Lorentz-Kähler force implies that all space-like 3-surfaces approaching same asymptotic state have the same value of Kähler function assuming that the Kähler function assignable to space-like 3-surface is same for all space-like sections of $X^4(X_l^3)$ (assuming that one can realize general coordinate invariance also in this sense). This need not be the case. In any case, this need not be a problem since it would mean an additional symmetry extending general coordinate invariance. The exponent of Kähler function would be highly analogous to a partition function defined as an exponent of Hamiltonian with Kähler coupling strength playing the role of temperature.

It seems that asymptotic self-organization pattern need not be correct interpretation for non-dissipating regions, and the identification of light-like 3-surfaces as generalized Feynman diagrams encourages an alternative interpretation.

1. $M^8 - H$ duality states that also the H counterparts of co-hyper-hyperquaternionic surfaces of M^8 are preferred extremals of Kähler action. CP_2 type vacuum extremals represent the basic example of these and a plausible conjecture is that the regions of space-time with Euclidian signature of the induced metric represent this kind of regions. If this conjecture is correct, dissipation could be assigned with regions having Euclidian signature of the induced metric. This makes sense since dissipation has quantum description in terms of Feynman graphs

and regions of Euclidian signature indeed correspond to generalized Feynman graphs. This argument would suggest that generalized Beltrami conditions or light-likeness hold true inside Minkowskian regions rather than only asymptotically.

2. One could of course play language games and argue that asymptotia is with respect to the Euclidian time coordinate inside generalized Feynman graps and is achieved exactly when the signature of the induced metric becomes Minkowskian. This is somewhat artificial attempt to save the notion of asymptotic self-organization pattern since the regions outside Feynman diagrams represent empty space providing a holographic representations for the matter at X^3_l so that the vanishing of $j^\alpha F_{\alpha\beta}$ is very natural.

3. What is then the correct identification of asymptotic self-organization pattern. Could correspond to the negative energy part of the zero energy state at the upper light-like boundary δM^4_- of CD? Or in the case of phase conjugate state to the positive energy part of the state at δM^4_+? An identification consistent with the fractal structure of ZEO and TGD inspired theory of consciousness is that the entire zero energy state reached by a sequence of quantum jumps represents asymptotic self-organization pattern represented by the asymptotic generalized Feynman diagram or their superposition. Biological systems represent basic examples about self-organization, and one cannot avoid the questions relating to the relationship between experience and geometric time. A detailed discussion of these points can be found in [L6] .

Absolute minimization of Kähler action was the first guess for the criterion selecting preferred extremals. Absolute minimization in a strict sense of the word does not make sense in the p-adic context since p-adic numbers are not well-ordered, and one cannot even define the action integral as a p-adic number. The generalized Beltrami conditions and the boundary conditions defining the preferred extremals are however local and purely algebraic and make sense also p-adically. If absolute minimization reduces to these algebraic conditions, it would make sense.

8.3.6 Generalized Beltrami Fields And Biological Systems

The following arguments support the view that generalized Beltrami fields play a key role in living systems, and that $D_{CP_2} = 2$ corresponds to ordered phase, $D_{CP_2} = 3$ to spin glass phase and $D_{CP_2} = 4$ to chaos, with $D_{CP_2} = 3$ defining life as a phenomenon at the boundary between order and chaos. If the criteria suggested by the number theoretic compactification are accepted, it is not clear whether D_{CP_2} extremals can define preferred extremals of Kähler action. For instance, cosmic strings are not preferred extremals and the Y^3_l associated with MEs allow only covariantly constant right handed neutrino eigenmode of $D_K(X^2)$. The topological condensation of CP_2 type vacuum extremals around $D_{CP_2} = 2$ type extremals is however expected to give preferred extremals and if the density of the condensate is low enough one can still speak about $D_{CP_2} = 2$ phase. A natural guess is also that the deformation of $D_{CP_2} = 2$ extremals transforms light-like gauge currents to space-like topological currents allowed by $D_{CP_2} = 3$ phase.

Why generalized Beltrami fields are important for living systems?

Chirality, complexity, and high level of organization make $D_{CP_2} = 3$ generalized Beltrami fields excellent candidates for the magnetic bodies of living systems.

1. Chirality selection is one of the basic signatures of living systems. Beltrami field is characterized by a chirality defined by the relative sign of the current and magnetic field, which means parity breaking. Chirality reduces to the sign of the function ψ appearing in the topologization condition and makes sense also for the generalized Beltrami fields.

2. Although Beltrami fields can be extremely complex, they are also extremely organized. The reason is that the function α is constant along flux lines so that flux lines must in the case of compact Riemann 3-manifold belong to 2-dimensional $\alpha = constant$ closed surfaces, in fact two-dimensional invariant tori [B49] .

For generalized Beltrami fields the function ψ is constant along the flow lines of the Kähler current. Space-time sheets with 3-dimensional CP_2 projection serve as an illustrative example.

One can use the coordinates for the CP_2 projection as space-time coordinates so that one space-time coordinate disappears totally from consideration. Hence the situation reduces to a flow in a 3-dimensional sub-manifold of CP_2. One can distinguish between three types of flow lines corresponding to space-like, light-like and time-like topological current. The 2-dimensional $\psi = constant$ invariant manifolds are sub-manifolds of CP_2. Ordinary Beltrami fields are a special case of space-like flow with flow lines belonging to the 2-dimensional invariant tori of CP_2. Time-like and light-like situations are more complex since the flow lines need not be closed so that the 2-dimensional $\psi = constant$ surfaces can have boundaries.

For periodic self-organization patterns flow lines are closed and $\psi = constant$ surfaces of CP_2 must be invariant tori. The dynamics of the periodic flow is obtained from that of a steady flow by replacing one spatial coordinate with effectively periodic time coordinate. Therefore topological notions like helix structure, linking, and knotting have a dynamical meaning at the level of CP_2 projection. The periodic generalized Beltrami fields are highly organized also in the temporal domain despite the potentiality for extreme topological complexity.

For these reasons topologically quantized generalized Beltrami fields provide an excellent candidate for a generic model for the dynamics of biological self-organization patterns. A natural guess is that many-sheeted magnetic and Z^0 magnetic fields and their generalizations serve as templates for the helical molecules populating living matter, and explain both chiral selection, the complex linking and knotting of DNA and protein molecules, and even the extremely complex and self-organized dynamics of biological systems at the molecular level.

The intricate topological structures of DNA, RNA, and protein molecules are known to have a deep significance besides their chemical structure, and they could even define something analogous to the genetic code. Usually the topology and geometry of bio-molecules is believed to reduce to chemistry. TGD suggests that space-like generalized Beltrami fields serve as templates for the formation of bio-molecules and bio-structures in general. The dynamics of bio-systems would in turn utilize the time-like Beltrami fields as templates. There could even exist a mapping from the topology of magnetic flux tube structures serving as templates for bio-molecules to the templates of self-organized dynamics. The helical structures, knotting, and linking of bio-molecules would thus define a symbolic representation, and even coding for the dynamics of the bio-system analogous to written language.

$D_{CP_2} = 3$ systems as boundary between $D_{CP_2} = 2$ order and $D_{CP_2} = 4$ chaos

The dimension of CP_2 projection is basic classifier for the asymptotic self-organization patterns.

1. $D_{CP_2} = 4$ phase, dead matter, and chaos

$D_{CP_2} = 4$ phase - if present at all- would correspond to the ordinary Maxwellian phase in which Kähler current and charge density vanish and there is no topologization of Kähler current. By its maximal dimension this phase would naturally correspond to disordered phase, ordinary "dead matter". If one assumes that Kähler charge corresponds to either em charge or Z^0 charge then the signature of this state of matter would be em neutrality or Z^0 neutrality. As already found, Maxwell phase is very probably not realized in this manner but is essentially outcome of many-sheeted space-time concept.

2. $D_{CP_2} = 2$ phase as ordered phase

By the low dimension of CP_2 projection $D_{CP_2} = 2$ phase is the least stable phase possible only at cold space-time sheets. Kähler current is either vanishing or light-like, and Beltrami fields are not possible. This phase is highly ordered and much like a topological quantized version of ferro-magnet. In particular, it is possible to have a global coordinate varying along the field lines of the vector potential also now. The magnetic and Z^0 magnetic body of any system is a candidate for this kind of system. Z^0 field is indeed always present for vacuum extremals having $D_{CP_2} = 2$ and the vanishing of em field requires that $sin^2(\theta_W)$ (θ_W is Weinberg angle) vanishes.

3. $D_{CP_2} = 3$ corresponds to living matter

$D_{CP_2} = 3$ corresponds to highly organized phase characterized in the case of space-like Kähler current by complex helical structures necessarily accompanied by topologized Kähler charge density $\propto \overline{A}\cdot\overline{B} \neq 0$ and Kähler current $\overline{E}\times\overline{A}+\phi\overline{B}$. For time like Kähler currents the helical structures are replaced by periodic oscillation patterns for the state of the system. By the non-maximal dimension of CP_2 projection this phase must be unstable against too strong external perturbations and cannot survive at too high temperatures. Living matter is thus excellent candidate for this phase and it might be that the interaction of the magnetic body with living matter makes possible the transition from $D_{CP_2} = 2$ phase to the self-organizing $D_{CP_2} = 3$ phase.

Living matter which is indeed populated by helical structures providing examples of space-like Kähler current. Strongly charged lipid layers of cell membrane might provide example of time-like Kähler current. Cell membrane, micro-tubuli, DNA, and proteins are known to be electrically charged and Z^0 charge plays key role in TGD based model of catalysis discussed in [?] . For instance, denaturing of DNA destroying its helical structure could be interpreted as a transition leading from $D_{CP_2} = 3$ phase to $D_{CP_2} = 4$ phase. The prediction is that the denatured phase should be electromagnetically (or Z^0) neutral.

Beltrami fields result when Kähler charge density vanishes. For these configurations magnetic field and current density take the role of the vector potential and magnetic field as far as the contact structure is considered. For Beltrami fields there exist a global coordinate along the field lines of the vector potential but not along those of the magnetic field. As a consequence, the covariant consistency condition $(\partial_s - qeA_s)\Psi = 0$ frequently appearing in the physics of super conducting systems would make sense along the flow lines of the vector potential for the order parameter of Bose-Einstein condensate. If Beltrami phase is super-conducting, then the state of the system must change in the transition to a more general phase. It is impossible to assign slicing of 4-surface by 3-D surfaces labeled by a coordinate t varying along the flow lines. This means that one cannot speak about a continuous evolution of Schrödinger amplitude with t playing the role of time coordinate. One could perhaps say that the entire space-time sheet represents single quantum event which cannot be decomposed to evolution. This would conform with the assignment of macroscopic and macro-temporal quantum coherence with living matter.

The existence of these three phases brings in mind systems allowing chaotic de-magnetized phase above critical temperature T_c, spin glass phase at the critical point, and ferromagnetic phase below T_c. Similar analogy is provided by liquid phase, liquid crystal phase possible in the vicinity of the critical point for liquid to solid transition, and solid phase. Perhaps one could regard $D_{CP_2} = 3$ phase and life as a boundary region between $D_{CP_2} = 2$ order and $D_{CP_2} = 4$ chaos. This would naturally explain why life as it is known is possible in relatively narrow temperature interval.

Can one assign a continuous Schrödinger time evolution to light-like 3-surfaces?

Alain Connes wrote [A28] about factors of various types using as an example Schrödinger equation for various kinds of foliations of space-time to time=constant slices. If this kind of foliation does not exist, one cannot speak about time evolution of Schrödinger equation at all. Depending on the character of the foliation one can have factor of type I, II, or III. For instance, torus with slicing $dx = ady$ in flat coordinates, gives a factor of type I for rational values of a and factor of type II for irrational values of a.

1. 3-D foliations and type III factors

Connes mentioned 3-D foliations V which give rise to type III factors. Foliation property requires a slicing of V by a one-form v to which slices are orthogonal (this requires metric).

1. The foliation property requires that v multiplied by suitable scalar is gradient. This gives the integrability conditions $dv = w \wedge v$, $w = -d\psi/\psi = -dlog(\psi)$. Something proportional to $log(\psi)$ can be taken as a third coordinate varying along flow lines of v: the flow defines a continuous sequence of maps of 2-dimensional slice to itself.
2. If the so called Godbillon-Vey invariant defined as the integral of $dw \wedge w$ over V is non-vanishing, factor of type III is obtained using Schrödinger amplitudes for which the flow lines of foliation define the time evolution. The operators of the algebra in question are transversal operators acting on Schrödinger amplitudes at each slice. Essentially Schrödinger equation in 3-D space-time would be in question with factor of type III resulting from the exotic choice of the time coordinate defining the slicing.

2. What happens in case of light-like 3-surfaces?

In TGD light-like 3-surfaces are natural candidates for V and it is interesting to look what happens in this case. Light-likeness is of course a disturbing complication since orthogonality condition and thus contravariant metric is involved with the definition of the slicing. Light-likeness is not however involved with the basic conditions.

1. The one-form v defined by the induced Kähler gauge potential A defining also a braiding is a unique identification for v. If foliation exists, the braiding flow defines a continuous sequence of maps of partonic 2-surface to itself.
2. Physically this means the possibility of a super-conducting phase with order parameter satisfying covariant constancy equation $D\psi = (d/dt - ieA)\psi = 0$. This would describe a supra current flowing along flow lines of A.
3. If the integrability fails to be true, one *cannot* assign Schrödinger time evolution with the flow lines of v. One might perhaps say that 3-surface behaves like single quantum event not allowing slicing into a continuous Schrödinger time evolution.
4. In TGD Schrödinger amplitudes are replaced by second quantized induced spinor fields. Hence one does not face the problem whether it makes sense to speak about Schrödinger time evolution of complex order parameter along the flow lines of a foliation or not. Also the fact that the "time evolution" for the Kähler-Dirac operator corresponds to single position dependent generalized eigenvalue identified as Higgs expectation same for all transversal modes (essentially z^n labeled by conformal weight) is crucial since it saves from the problems caused by the possible non-existence of Schrödinger evolution.

4. Extremals of Kähler action

Some comments relating to the interpretation of the classification of the extremals of Kähler action by the dimension of their CP_2 projection are in order. It has been already found that the extremals can be classified according to the dimension D of the CP_2 projection of space-time sheet in the case that $A_a = 0$ holds true.

1. For $D_{CP_2} = 2$ integrability conditions for the vector potential can be satisfied for $A_a = 0$ so that one has generalized Beltrami flow and one can speak about Schrödinger time evolution associated with the flow lines of vector potential defined by covariant constancy condition $D\psi = 0$ makes sense. Kähler current is vanishing or light-like. This phase is analogous to a super-conductor or a ferromagnetic phase. For non-vanishing A_a the Beltrami flow property is lost but the analogy with ferromagnetism makes sense still.
2. For $D_{CP_2} = 3$ foliations are lost. The phase is dominated by helical structures. This phase is analogous to spin glass phase around phase transition point from ferromagnetic to non-magnetized phase and expected to be important in living matter systems.
3. $D_{CP_2} = 4$ is analogous to a chaotic phase with vanishing Kähler current and to a phase without magnetization. The interpretation in terms of non-quantum coherent "dead" matter is suggestive.

An interesting question is whether the ordinary 8-D embedding space which defines one sector of the generalized embedding space could correspond to $A_a = 0$ phase. If so, then all states for this sector would be vacua with respect to M^4 quantum numbers. M^4-trivial zero energy states in this sector could be transformed to non-trivial zero energy states by a leakage to other sectors.

8.4 Basic Extremals Of Kähler Action

The solutions of field equations can be divided to vacuum extremals and non-vacuum extremal. Vacuum extremals come as two basic types: CP_2 type vacuum extremals for which the induced Kähler field and Kähler action are non-vanishing and the extremals for which the induced Kähler field vanishes. The deformations of both extremals are expected to be of fundamental importance in TGD universe.

8.4.1 CP_2 Type Vacuum Extremals

These extremals correspond to various isometric embeddings of CP_2 to $M^4_+ \times CP_2$. One can also drill holes to CP_2. Using the coordinates of CP_2 as coordinates for X^4 the embedding is given by the formula

$$\begin{aligned} m^k &= m^k(u) \ , \\ m_{kl}\dot{m}^k\dot{m}^l &= 0 \ , \end{aligned} \tag{8.4.1}$$

where $u(s^k)$ is an arbitrary function of CP_2 coordinates. The latter condition tells that the curve representing the projection of X^4 to M^4 is light like curve. One can choose the functions $m^i, i = 1,2,3$ freely and solve m^0 from the condition expressing light likeness so that the number of this kind of extremals is very large.

The induced metric and Kähler field are just those of CP_2 and energy momentum tensor $T^{\alpha\beta}$ vanishes identically by the self duality of the Kähler form of CP_2. Also the canonical current $j^\alpha = D_\beta J^{\alpha\beta}$ associated with the Kähler form vanishes identically. Therefore the field equations in the interior of X^4 are satisfied. The field equations are also satisfied on the boundary components of CP_2 type extremal because the non-vanishing boundary term is, besides the normal component of Kähler electric field, also proportional to the projection operator to the normal space and vanishes identically since the induced metric and Kähler form are identical with the metric and Kähler form of CP_2.

As a special case one obtains solutions for which M^4 projection is light like geodesic. The projection of $m^0 = constant$ surfaces to CP_2 is $u = constant$ 3-sub-manifold of CP_2. Geometrically these solutions correspond to a propagation of a massless particle. In a more general case the interpretation as an orbit of a massless particle is not the only possibility. For example, one can imagine a situation, where the center of mass of the particle is at rest and motion occurs along a circle at say (m^1, m^2) plane. The interpretation as a massive particle is natural. Amusingly, there is nice analogy with the classical theory of Dirac electron: massive Dirac fermion moves also with the velocity of light (zitterbewegung). The quantization of this random motion with light velocity leads to Virasoro conditions and this led to a breakthrough in the understanding of the symmetries of TGD. Super Virasoro invariance is a general symmetry of WCW geometry and quantum TGD.

The action for all extremals is same and given by the Kähler action for the embedding of CP_2. The value of the action is given by

$$S = -\frac{\pi}{8\alpha_K} \ . \tag{8.4.2}$$

To derive this expression we have used the result that the value of Lagrangian is constant: $L = 4/R^4$, the volume of CP_2 is $V(CP_2) = \pi^2 R^4/2$ and the definition of the Kähler coupling strength $k_1 = 1/16\pi\alpha_K$ (by definition, πR is the length of CP_2 geodesics). Four-momentum vanishes for these extremals so that they can be regarded as vacuum extremals. The value of the action is negative so that these vacuum extremals are indeed favored by the minimization of the Kähler action. The principle selecting preferred extremals of Kähler action suggests that ordinary vacuums with vanishing Kähler action density are unstable against the generation of CP_2 type extremals. There are even reasons to expect that CP_2 type extremals are for TGD what black holes are for GRT. Indeed, the nice generalization of the area law for the entropy of black hole [K74] supports this view.

In accordance with the basic ideas of TGD topologically condensed vacuum extremals should somehow correspond to massive particles. The properties of the CP_2 type vacuum extremals are in accordance with this interpretation. Although these objects move with a velocity of light, the motion can be transformed to a mere so that the center of mass motion is trivial. Even the generation of the rest mass could might be understood classically as a consequence of the minimization of action. Long range Kähler fields generate negative action for the topologically condensed vacuum extremal (momentum zero massless particle) and Kähler field energy in turn is identifiable as the rest mass of the topologically condensed particle.

An interesting feature of these objects is that they can be regarded as gravitational instantons [A56] . A further interesting feature of CP_2 type extremals is that they carry nontrivial classical color charges. The possible relationship of this feature to color confinement raises interesting questions. Could one model classically the formation of the color singlets to take place through the emission of "colorons": states with zero momentum but non-vanishing color? Could these peculiar states reflect the infrared properties of the color interactions?

8.4.2 Vacuum Extremals With Vanishing Kähler Field

Vacuum extremals correspond to 4-surfaces with vanishing Kähler field and therefore to gauge field zero configurations of gauge field theory. These surfaces have CP_2 projection, which is Lagrange manifold. The condition expressing Lagrange manifold property is obtained in the following manner. Kähler potential of CP_2 can be expressed in terms of the canonical coordinates (P_i, Q_i) for CP_2 as

$$A = \sum_k P_k dQ^k \ . \tag{8.4.3}$$

The conditions

$$P_k = \partial_{Q^k} f(Q^i) \ , \tag{8.4.4}$$

where $f(Q^i)$ is arbitrary function of its arguments, guarantee that Kähler potential is pure gauge. It is clear that canonical transformations, which act as local $U(1)$ gauge transformations, transform different vacuum configurations to each other so that vacuum degeneracy is enormous. Also M^4_+ diffeomorphisms act as the dynamical symmetries of the vacuum extremals. Some sub-group of these symmetries extends to the isometry group of the WCW in the proposed construction of the WCW metric. The vacuum degeneracy is still enhanced by the fact that the topology of the four-surface is practically free.

Vacuum extremals are certainly not absolute minima of the action. For the induced metric having Minkowski signature the generation of Kähler electric fields lowers the action. For Euclidian signature both electric and magnetic fields tend to reduce the action. Therefore the generation of Euclidian regions of space-time is expected to occur. CP_2 type extremals, identifiable as real (as contrast to virtual) elementary particles, can be indeed regarded as these Euclidian regions.

Particle like vacuum extremals can be classified roughly by the number of the compactified dimensions D having size given by CP_2 length. Thus one has $D = 3$ for CP_2 type extremals, $D = 2$ for string like objects, $D = 1$ for membranes and $D = 0$ for pieces of M^4. As already mentioned, the rule $h_{vac} = -D$ relating the vacuum weight of the Super Virasoro representation to the number of compactified dimensions of the vacuum extremal is very suggestive. $D < 3$ vacuum extremals would correspond in this picture to virtual particles, whose contribution to the generalized Feynman diagram is not suppressed by the exponential of Kähler action unlike that associated with the virtual CP_2 type lines.

M^4 type vacuum extremals (representable as maps $M^4_+ \rightarrow CP_2$ by definition) are also expected to be natural idealizations of the space-time at long length scales obtained by smoothing out small scale topological inhomogenuities (particles) and therefore they should correspond to space-time of GRT in a reasonable approximation.

The reason would be "Yin-Yang principle" discussed in [K14] .

1. Consider first the option for which Kähler function corresponds to an absolute minimum of Kähler action. Vacuum functional as an exponent of Kähler function is expected to concentrate on those 3-surfaces for which the Kähler action is non-negative. On the other hand, the requirement that Kähler action is absolute minimum for the space-time associated with a given 3-surface, tends to make the action negative. Therefore the vacuum functional is expected to differ considerably from zero only for 3-surfaces with a vanishing Kähler action per volume. It could also occur that the degeneracy of 3-surfaces with same large negative action compensates the exponent of Kähler function.

2. If preferred extremals correspond to Kähler calibrations or their duals [K105] , Yin-Yang principle is modified to a more local principle. For Kähler calibrations (their duals) the

absolute value of action in given region is minimized (maximized). A given region with a positive (negative sign) of action density favors Kähler electric (magnetic) fields. In long length scales the average density of Kähler action per four-volume tends to vanish so that Kähler function of the entire universe is expected to be very nearly zero. This regularizes the theory automatically and implies that average Kähler action per volume vanishes. Positive and finite values of Kähler function are of course favored.

In both cases the vanishing of Kähler action per volume in long length scales makes vacuum extremals excellent idealizations for the smoothed out space-time surface. Robertson-Walker cosmologies provide a good example in this respect. As a matter fact the smoothed out space-time is not a mere fictive concept since larger space-time sheets realize it as a essential part of the Universe.

Several absolute minima could be possible and the non-determinism of the vacuum extremals is not expected to be reduced completely. The remaining degeneracy could be even infinite. A good example is provided by the vacuum extremals representable as maps $M^4_+ \rightarrow D^1$, where D^1 is one-dimensional curve of CP_2. This degeneracy could be interpreted as a space-time correlate for the non-determinism of quantum jumps with maximal deterministic regions representing quantum states in a sequence of quantum jumps.

8.4.3 Cosmic Strings

Cosmic strings are extremals of type $X^2 \times S^2$, where X^2 is minimal surface in M^4_+ (analogous to the orbit of a bosonic string) and S^2 is the homologically non-trivial geodesic sphere of CP_2. The action of these extremals is positive and thus absolute minima are certainly not in question. One can however consider the possibility that these extremals are building blocks of the absolute minimum space-time surfaces since the principle selecting preferred extremals of the Kähler action is global rather than a local. Cosmic strings can contain also Kähler charged matter in the form of small holes containing elementary particle quantum numbers on their boundaries and the negative Kähler electric action for a topologically condensed cosmic string could cancel the Kähler magnetic action.

The string tension of the cosmic strings is given by

$$T = \frac{1}{8\alpha_K R^2} \simeq .2210^{-6}\frac{1}{G} , \quad (8.4.5)$$

where $\alpha_K \simeq \alpha_{em}$ has been used to get the numerical estimate. The string tension is of the same order of magnitude as the string tension of the cosmic strings of GUTs and this leads to the model of the galaxy formation providing a solution to the dark matter puzzle as well as to a model for large voids as caused by the presence of a strongly Kähler charged cosmic string. Cosmic strings play also fundamental role in the TGD inspired very early cosmology.

8.4.4 Massless Extremals

Massless extremals are characterized by massless wave vector p and polarization vector ε orthogonal to this wave vector. Using the coordinates of M^4 as coordinates for X^4 the solution is given as

$$\begin{array}{ll} s^k = f^k(u,v) \ , & \\ u = p\cdot m \ , & v = \varepsilon\cdot m \ , \\ p\cdot\varepsilon = 0 \ , & p^2 = 0 \ . \end{array} \quad (8.4.6)$$

CP_2 coordinates are arbitrary functions of $p \cdot m$ and $\varepsilon \cdot m$. Clearly these solutions correspond to plane wave solutions of gauge field theories. It is important to notice however that linear super position doesn't hold as it holds in Maxwell phase. Gauge current is proportional to wave vector and its divergence vanishes as a consequence. Also cylindrically symmetric solutions for which the transverse coordinate is replaced with the radial coordinate $\rho = \sqrt{m_1^2 + m_2^2}$ are possible. In fact, v can be *any* function of the coordinates m^1, m^2 transversal to the light like vector p.

Boundary conditions on the boundaries of the massless extremal are satisfied provided the normal component of the energy momentum tensor vanishes. Since energy momentum tensor is of the form $T^{\alpha\beta} \propto p^\alpha p^\beta$ the conditions $T^{n\beta} = 0$ are satisfied if the M^4 projection of the boundary is given by the equations of form

$$\begin{array}{ll} H(p\cdot m, \varepsilon\cdot m, \varepsilon_1\cdot m) = 0 \ , & \\ \varepsilon\cdot p = 0 \ , & \varepsilon_1\cdot p = 0 \ , \quad \varepsilon\cdot\varepsilon_1 = 0 \ . \end{array} \tag{8.4.7}$$

where H is arbitrary function of its arguments. Recall that for M^4 type extremals the boundary conditions are also satisfied if Kähler field vanishes identically on the boundary.

The following argument suggests that there are not very many ways to satisfy boundary conditions in case of M^4 type extremals. The boundary conditions, when applied to M^4 coordinates imply the vanishing of the normal component of energy momentum tensor. Using coordinates, where energy momentum tensor is diagonal, the requirement boils down to the condition that at least one of the eigen values of $T^{\alpha\beta}$ vanishes so that the determinant $det(T^{\alpha\beta})$ must vanish on the boundary: this condition defines 3-dimensional surface in X^4. In addition, the normal of this surface must have same direction as the eigen vector associated with the vanishing eigen value: this means that three additional conditions must be satisfied and this is in general true in single point only. The boundary conditions in CP_2 coordinates are satisfied provided that the conditions

$$J^{n\beta} J^k_{\ l} \partial_\beta s^l = 0$$

are satisfied. The identical vanishing of the normal components of Kähler electric and magnetic fields on the boundary of massless extremal property provides a way to satisfy all boundary conditions but it is not clear whether there are any other ways to satisfy them.

The characteristic feature of the massless extremals is that in general the Kähler gauge current is non-vanishing. In ordinary Maxwell electrodynamics this is not possible. This means that these extremals are accompanied by vacuum current, which contains in general case both weak and electromagnetic terms as well as color part.

A possible interpretation of the solution is as the exterior space-time to a topologically condensed particle with vanishing mass described by massless CP_2 type extremal, say photon or neutrino. In general the surfaces in question have boundaries since the coordinates s^k are are bounded: this is in accordance with the general ideas about topological condensation. The fact that massless plane wave is associated with CP_2 type extremal combines neatly the wave and particle aspects at geometrical level.

The fractal hierarchy of space-time sheets implies that massless extremals should interesting also in long length scales. The presence of a light like electromagnetic vacuum current implies the generation of coherent photons and also coherent gravitons are generated since the Einstein tensor is also non-vanishing and light like (proportional to $k^\alpha k^\beta$). Massless extremals play an important role in the TGD based model of bio-system as a macroscopic quantum system. The possibility of vacuum currents is what makes possible the generation of the highly desired coherent photon states.

8.4.5 Does GRT really allow gravitational radiation: could cosmological constant save the situation?

In Facebook discussion Nils Grebäck mentioned Weyl tensor and I learned something that I should have noticed long time ago. Wikipedia article (see `http://tinyurl.com/y7fsnzk8`) lists the basic properties of Weyl tensor as the traceless part of curvature tensor, call it R. Weyl tensor C is vanishing for conformally flat space-times. In dimensions D=2,3 Weyl tensor vanishes identically so that they are always conformally flat: this obviously makes the dimension $D = 3$ for space very special. Interestingly, one can have non-flat space-times with nonvanishing Weyl tensor but the vanishing Schouten/Ricci/Einstein tensor and thus also with vanishing energy momentum tensor.

The rest of curvature tensor R can be expressed in terms of so called Kulkarni-Nomizu product $P \cdot g$ of Schouten tensor P and metric tensor g: $R = C + P \cdot g$, which can be also transformed to a definition of Weyl tensor using the definition of curvature tensor in terms of

Christoffel symbols as the fundamental definition. Kulkarni-Nomizu product $\cdot$ is defined as tensor product of two 2-tensors with symmetrization with respect to first and second index pairs plus antisymmetrization with respect to second and fourth indices.

Schouten tensor P is expressible as a combination of Ricci tensor Ric defined by the trace of R with respect to the first two indices and metric tensor g multiplied by curvature scalar s (rather than R in order to use index free notation without confusion with the curvature tensor). The expression reads as

$$P = \frac{1}{D-2}\left[Ric - \frac{s}{2(D-1)}g\right] \quad .$$

Note that the coefficients of Ric and g differ from those for Einstein tensor. Ricci tensor and Einstein tensor are proportional to energy momentum tensor by Einstein equations relate to the part.

Weyl tensor is assigned with gravitational radiation in GRT. What I see as a serious interpretational problem is that by Einstein's equations gravitational radiation would carry no energy and momentum in absence of matter. One could argue that there are no free gravitons in GRT if this interpretation is adopted! This could be seen as a further argument against GRT besides the problems with the notions of energy and momentum: I had not realized this earlier.

Interestingly, in TGD framework so called massless extremals (MEs) [K14, K8, K77] are four-surfaces, which are extremals of Kähler action, have Weyl tensor equal to curvature tensor and therefore would have interpretation in terms of gravitons. Now these extremals are however non-vacuum extremals.

1. Massless extremals correspond to graphs of possibly multi-valued maps from M^4 to CP_2. CP_2 coordinates are arbitrary functions of variables $u = k \cot m$ and $w = \epsilon \cdot m$. k is light-like wave vector and ϵ space-like polarization vector orthogonal to k so that the interpretation in terms of massless particle with polarization is possible. ME describes in the most general case a wave packet preserving its shape and propagating with maximal signal velocity along a kind of tube analogous to wave guide so that they are ideal for precisely targeted communications and central in TGD inspired quantum biology. MEs do not have Maxwellian counterparts. For instance, MEs can carry light-like gauge currents parallel to them: this is not possible in Maxwell's theory.

2. I have discussed a generalization of this solution ansatz so that the directions defined by light-like vector k and polarization vector ϵ orthogonal to it are not constant anymore but define a slicing of M^4 by orthogonal curved surfaces (analogs of string world sheets and space-like surfaces orthogonal to them). MEs in their simplest form at least are minimal surfaces and actually extremals of practically any general coordinate invariance action principle. For instance, this is the case if the volume term suggested by the twistor lift of Kähler action [K41] and identifiable in terms of cosmological constant is added to Kähler action.

3. MEs carry non-trivial induced gauge fields and gravitational fields identified in terms of the induced metric. I have identified them as correlates for particles, which correspond to pairs of wormhole contacts between two space-times such that at least one of them is ME. MEs would accompany to both gravitational radiation and other forms or radiation classically and serve as their correlates. For massless extremals the metric tensor is of form

$$g = m + a\epsilon \otimes \epsilon + bk \otimes k + c(\epsilon \otimes kv + k \otimes \epsilon) \quad ,$$

   where m is the metric of empty Minkowski space. The curvature tensor is necessarily quadrilinear in polarization vector ϵ and light-like wave vector k (light-like ifor both M^4 and ME metric) and from the general expression of Weyl tensor C in terms of R and g it is equal to curvature tensor: $C = R$.

   Hence the interpretation as graviton solution conforms with the GRT interpretation. Now however the energy momentum tensor for the induced Kähler form is non-vanishing and bilinear in velocity vector k and the interpretational problem is avoided.

What is interesting that also at GRT limit cosmological constant saves gravitons from reducing to vacuum solutions. The deviation of the energy density given by cosmological term from that for Minkowski metric is identifiable as gravitonic energy density. The mysterious cosmological constant would be necessary for making gravitons non-vacuum solutions. The value of graviton amplitude would be determined by the continuity conditions for Einstein's equations with cosmological term. The p-adic evolution of cosmological term predicted by TGD is however difficult to understand in GRT framework.

8.4.6 Gravitational memory effect and quantum criticality of TGD

Gary Ehlenberg sent an interesting post about the gravitational memory effect (see this and this).

Classical gravitational waves would leave a memory of its propagation to the metric of space-time affecting distances between mass points. The computations are done by treating Einstein's theory as a field theory in the background defined by the energy momentum tensor of matter and calculations are carried out only in the lowest non-trivial order.

There are two kinds of effects: the linear memory effect occurs for instance when a planet moves along non-closed hyperbolic orbit around a star and involves only the energy momentum tensor of the system. The non-linear memory effect also involves the energy momentum tensor of gravitational radiation as a source added to the energy momentum tensor of matter.

The effect is accumulative and involves integration over the history of the matter source over the entire past. The reason why the memory effect is non-vanishing is basically that the source of the gravitational radiation is quadratic in metric. In Maxwellian electrodynamics the source does not have this property.

I have never thought of the memory effect. The formula used to estimate the effect is however highly interesting.

1. In the formula for the non-linear memory effect, that is for the action of d'Alembert operator acting on the radiation contribution to the metric, the source term is obtained by adding to the energy momentum tensor of the matter, the energy momentum tensor of the gravitational radiation.

2. This formula can be iterated and if the limit as a fixed point exists, the energy momentum tensor of the gravitational radiation produced by the total energy momentum tensor, including also the radiative contribution, should vanish. This brings in mind fractals and criticality.

   One of the basic facts about iteration for polynomials is that it need not always converge. Limit cycles typically emerge. In more complex situations also objects known as strange attractors can appear. Does the same problem occur now, when the situation is much much more complex?

3. What is interesting is that gravitational wave solutions indeed have vanishing energy momentum tensors. This is problematic if one considers them as radiation in empty space. In the presence of matter, this might be true only for very special background metrics as a sum of matter part and radiation part: just these gravitationally critical fixed point metrics.

   Could the fixed point property of these metrics (matter plus gravitational radiation) be used to gain information of the total metric as sum of matter and gravitational parts?

4. As a matter of fact, all solutions of non-linear field theories are constructed by similar iteration and the radiative contribution in a given order is determined by the contribution in lower orders.

   Under what conditions can one assume convergence of the perturbation series, that is fixed point property? Are limit cycles and chaotic attractors, and only a specialist knows what, unavoidable? Could this fixed point property have some physical relevance? Could the fixed points correspond in quantum field theory context to fixed points of the renormalization group and lead to quantization of coupling constants?

Does the fixed point property have a TGD counterpart?

1. In the TGD, framework Einstein's equations are expected only at the QFT limit at which space-time sheets are replaced with a single region of M^4 carrying gauge fields and gravitational fields, which are sums of the induced fields associated with space-time sheets. What happens at the level of the basic TGD?

What is intriguing, is that quantum criticality is the basic principle of TGD and fixes discrete coupling constant evolution: could the quantum criticality realize itself also as gravitational criticality in the above sense? And even the idea that perturbation series can converge only at critical points and becomes actually trivial?

2. What does the year 2023 version of classical TGD say about the situation? In TGD, space-time surfaces obey almost deterministic holography required by general coordinate invariance [L55, L46]. Holography follows from the general coordinate invariance and implies that path integral trivializes to sum over the analogs of Bohr orbits of particles represented as 3-D surfaces. This states quantum criticality and fixed point property: radiative contributions vanish. This also implies a number theoretic view of coupling constant evolution based on number theoretic vision about physics.

   M^8-H duality [L42, L43, L54] implies that the space-time regions defined by Bohr orbits are extremely simple and form an evolutionary hierarchy characterized by extensions of rationals associated with polynomials characterizing the counterparts of space-time surfaces as 4-surfaces in complexified M^8 mapped to space-time surfaces in $M^4\times CP_2$ by M^8-H duality.

   There is also universality: the Bohr orbits in H are minimal surfaces [L47], which satisfy a 4-D generalization of 2-D holomorphy and are independent of the action principle as long as it is general coordinate invariant and constructible in terms of the induced geometry. The only dependence on coupling constants comes from singularities at which minimal surface property fails. Also classical conserved quantities depend on coupling constants.

3. The so called "massless extremals" (MEs) represent radiation with very special properties such as precisely targeted propagation with light velocity, absence of dispersion of wave packed, and restricted linear superposition for massless modes propagating in the direction of ME. They are analogous to laser beams, Bohr orbits for radiation fields. The gauge currents associated with MEs are light-like and Lorentz 4-force vanishes.

4. Could the Einstein tensor of ME vanish? The energy momentum tensor expressed in terms of Einstein tensor involves a dimensional parameter and measures the breaking of scale invariance. MEs are conformally invariant objects: does this imply the vanishing of the Einstein tensor? Note however that the energy momentum tensor assignable to the induced gauge fields is non-vanishing: however, its scale covariance is an inherent property of gauge fields so that it need not vanish.

8.4.7 Generalization Of The Solution Ansatz Defining Massless Extremals (MEs)

The solution ansatz for MEs has developed gradually to an increasingly general form and the following formulation is the most general one achieved hitherto. Rather remarkably, it rather closely resembles the solution ansatz for the CP_2 type extremals and has direct interpretation in terms of geometric optics. Equally remarkable is that the latest generalization based on the introduction of the local light cone coordinates was inspired by quantum holography principle.

The solution ansatz for MEs has developed gradually to an increasingly general form and the following formulation is the most general one achieved hitherto. Rather remarkably, it rather closely resembles the solution ansatz for the CP_2 type extremals and has direct interpretation in terms of geometric optics. Equally remarkable is that the latest generalization based on the introduction of the local light cone coordinates was inspired by quantum holography principle.

Local light cone coordinates

The solution involves a decomposition of M^4_+ tangent space localizing the decomposition of Minkowski space to an orthogonal direct sum $M^2\oplus E^2$ defined by light-like wave vector and polarization vector orthogonal to it. This decomposition defines what might be called local light cone coordinates.

1. Denote by m^i the linear Minkowski coordinates of M^4. Let (S^+,S^-,E^1,E^2) denote local coordinates of M^4_+ defining a *local* decomposition of the tangent space M^4 of M^4_+ into a direct *orthogonal* sum $M^4=M^2\oplus E^2$ of spaces M^2 and E^2. This decomposition has interpretation in terms of the longitudinal and transversal degrees of freedom defined by local light-like four-velocities $v_\pm=\nabla S_\pm$ and polarization vectors $\epsilon_i=\nabla E^i$ assignable to light ray.

2. With these assumptions the coordinates $(S_{\pm}, E^i)$ define local light cone coordinates with the metric element having the form

$$ds^2 = 2g_{+-}dS^+dS^- + g_{11}(dE^1)^2 + g_{22}(dE^2)^2 \ . \tag{8.4.8}$$

   If complex coordinates are used in transversal degrees of freedom one has $g_{11} = g_{22}$.

3. This family of light cone coordinates is not the most general family since longitudinal and transversal spaces are orthogonal. One can also consider light-cone coordinates for which one non-diagonal component, say m_{1+}, is non-vanishing if the solution ansatz is such that longitudinal and transversal spaces are orthogonal for the induced metric.

A conformally invariant family of local light cone coordinates

The simplest solutions to the equations defining local light cone coordinates are of form $S_{\pm} = k \cdot m$ giving as a special case $S_{\pm} = m^0 \pm m^3$. For more general solutions of from

$$S_{\pm} = m^0 \pm f(m^1, m^2, m^3) \ \ , \ \ (\nabla_3 f)^2 = 1 \ \ , \tag{8.4.9}$$

where f is an otherwise arbitrary function, this relationship reads as

$$S^+ + S^- = 2m^0 \ . \tag{8.4.10}$$

This condition defines a natural rest frame. One can integrate f from its initial data at some two-dimensional $f = constant$ surface and solution describes curvilinear light rays emanating from this surface and orthogonal to it. The flow velocity field $\overline{v} = \nabla f$ is irrotational so that closed flow lines are not possible in a connected region of space and the condition $\overline{v}^2 = 1$ excludes also closed flow line configuration with singularity at origin such as $v = 1/\rho$ rotational flow around axis.

One can identify E^2 as a local tangent space spanned by polarization vectors and orthogonal to the flow lines of the velocity field $\overline{v} = \nabla f(m^1, m^2, m^3)$. Since the metric tensor of any 3-dimensional space allows always diagonalization in suitable coordinates, one can always find coordinates (E^1, E^2) such that (f, E^1, E^2) form orthogonal coordinates for $m^0 = constant$ hyperplane. Obviously one can select the coordinates E^1 and E^2 in infinitely many ways.

Closer inspection of the conditions defining local light cone coordinates

Whether the conformal transforms of the local light cone coordinates $\{S_{\pm} = m^0 \pm f(m^1, m^2, m^3), E^i\}$ define the only possible compositions $M^2 \oplus E^2$ with the required properties, remains an open question. The best that one might hope is that any function S^+ defining a family of light-like curves defines a local decomposition $M^4 = M^2 \oplus E^2$ with required properties.

1. Suppose that S^+ and S^- define light-like vector fields which are not orthogonal (proportional to each other). Suppose that the polarization vector fields $\epsilon_i = \nabla E^i$ tangential to local E^2 satisfy the conditions $\epsilon_i \cdot \nabla S^+ = 0$. One can formally integrate the functions E^i from these condition since the initial values of E^i are given at $m^0 = constant$ slice.

2. The solution to the condition $\nabla S_+ \cdot \epsilon_i = 0$ is determined only modulo the replacement

$$\epsilon_i \rightarrow \hat{\epsilon}_i = \epsilon_i + k\nabla S_+ \ \ , \tag{8.4.11}$$

   where k is any function. With the choice

$$k = -\frac{\nabla E^i \cdot \nabla S^-}{\nabla S^+ \cdot \nabla S^-}$$
(8.4.12)

one can satisfy also the condition $\hat{\epsilon}_i \cdot \nabla S^- = 0$.

3. The requirement that also $\hat{\epsilon}_i$ is gradient is satisfied if the integrability condition

$$k = k(S^+)$$
(8.4.13)

is satisfied: in this case $\hat{\epsilon}_i$ is obtained by a gauge transformation from ϵ_i. The integrability condition can be regarded as an additional, and obviously very strong, condition for S^- once S^+ and E^i are known.

4. The problem boils down to that of finding local momentum and polarization directions defined by the functions S^+, S^- and E^1 and E^2 satisfying the orthogonality and integrability conditions

$$(\nabla S^+)^2 = (\nabla S^-)^2 = 0 \ , \quad \nabla S^+ \cdot \nabla S^- \neq 0 \ ,$$
$$\nabla S^+ \cdot \nabla E^i = 0 \ , \qquad \frac{\nabla E^i \cdot \nabla S^-}{\nabla S^+ \cdot \nabla S^-} = k_i(S^+) \ .$$
(8.4.14)

The number of integrability conditions is 3+3 (all derivatives of k_i except the one with respect to S^+ vanish): thus it seems that there are not much hopes of finding a solution unless some discrete symmetry relating S^+ and S^- eliminates the integrability conditions altogether.

A generalization of the spatial reflection $f \rightarrow -f$ working for the separable Hamilton Jacobi function $S_\pm = m^0 \pm f$ ansatz could relate S^+ and S^- to each other and trivialize the integrability conditions. The symmetry transformation of M^4_+ must perform the permutation $S^+ \leftrightarrow S^-$, preserve the light-likeness property, map E^2 to E^2, and multiply the inner products between M^2 and E^2 vectors by a mere conformal factor. This encourages the conjecture that all solutions are obtained by conformal transformations from the solutions $S_\pm = m^0 \pm f$.

General solution ansatz for MEs for given choice of local light cone coordinates

Consider now the general solution ansatz assuming that a local wave-vector-polarization decomposition of M^4_+ tangent space has been found.

1. Let $E(S^+, E^1, E^2)$ be an arbitrary function of its arguments: the gradient ∇E defines at each point of E^2 an S^+-dependent (and thus time dependent) polarization direction orthogonal to the direction of local wave vector defined by ∇S^+. Polarization vector depends on E^2 position only.

2. Quite a general family of MEs corresponds to the solution family of the field equations having the general form

$$s^k = f^k(S^+, E) \ ,$$
(8.4.15)

where s^k denotes CP_2 coordinates and f^k is an arbitrary function of S^+ and E. The solution represents a wave propagating with light velocity and having definite S^+ dependent polarization in the direction of ∇E. By replacing S^+ with S^- one obtains a dual solution. Field equations are satisfied because energy momentum tensor and Kähler current are light-like so that all tensor contractions involved with the field equations vanish: the orthogonality of M^2 and E^2 is essential for the light-likeness of energy momentum tensor and Kähler current.

3. The simplest solutions of the form $S_\pm = m^0 \pm m^3$, $(E^1, E^2) = (m^1, m^2)$ and correspond to a cylindrical MEs representing waves propagating in the direction of the cylinder axis with light velocity and having polarization which depends on point (E^1, E^2) and S^+ (and thus time). For these solutions four-momentum is light-like: for more general solutions this cannot be the case. Polarization is in general case time dependent so that both linearly and circularly polarized waves are possible. If m^3 varies in a finite range of length L, then "free" solution represents geometrically a cylinder of length L moving with a light velocity. Of course, ends could be also anchored to the emitting or absorbing space-time surfaces.
4. For the general solution the cylinder is replaced by a three-dimensional family of light like curves and in this case the rectilinear motion of the ends of the cylinder is replaced with a curvilinear motion with light velocity unless the ends are anchored to emitting/absorbing space-time surfaces. The non-rotational character of the velocity flow suggests that the freely moving particle like 3-surface defined by ME cannot remain in a infinite spatial volume. The most general ansatz for MEs should be useful in the intermediate and nearby regions of a radiating object whereas in the far away region radiation solution is excepted to decompose to cylindrical ray like MEs for which the function $f(m^1, m^2, m^2)$ is a linear function of m^i.
5. One can try to generalize the solution ansatz further by allowing the metric of M^4_+ to have components of type g_{i+} or g_{i-} in the light cone coordinates used. The vanishing of T^{11}, T^{+1}, and T^{--} is achieved if $g_{i\pm} = 0$ holds true for the induced metric. For $s^k = s^k(S^+, E^1)$ ansatz neither $g_{2\pm}$ nor g_{1-} is affected by the embedding so that these components of the metric must vanish for the Hamilton Jacobi structure:

$$ds^2 = 2g_{+-}dS^+dS^- + 2g_{1+}dE^1dS^+ + g_{11}(dE^1)^2 + g_{22}(dE^2)^2 \ . \tag{8.4.16}$$

$g_{1+} = 0$ can be achieved by an additional condition

$$m_{1+} = s_{kl}\partial_1 s^k \partial_+ s^k \ . \tag{8.4.17}$$

The diagonalization of the metric seems to be a general aspect of absolute minima. The absence of metric correlations between space-time degrees of freedom for asymptotic self-organization patterns is somewhat analogous to the minimization of non-bound entanglement in the final state of the quantum jump.

Are the boundaries of space-time sheets quite generally light like surfaces with Hamilton Jacobi structure?

Quantum holography principle naturally generalizes to an approximate principle expected to hold true also in non-cosmological length and time scales.

1. The most general ansatz for topological light rays or massless extremals (MEs) inspired by the quantum holographic thinking relies on the introduction of the notion of local light cone coordinates S_+, S_-, E_1, E_2. The gradients ∇S_+ and ∇S_- define two light like directions just like Hamilton Jacobi functions define the direction of propagation of wave in geometric optics. The two polarization vector fields ∇E_1 and ∇E_2 are orthogonal to the direction of propagation defined by either S_+ or S_-. Since also E_1 and E_2 can be chosen to be orthogonal, the metric of M^4_+ can be written locally as $ds^2 = g_{+-}dS_+dS_- + g_{11}dE_1^2 + g_{22}dE_2^2$. In the earlier ansatz S_+ and S_- where restricted to the variables $k \cdot m$ and $\tilde{k} \cdot m$, where k and $\tilde{k}$ correspond to light like momentum and its mirror image and m denotes linear M^4 coordinates: these MEs describe cylindrical structures with constant direction of wave propagation expected to be most important in regions faraway from the source of radiation.
2. Boundary conditions are satisfied if the 3-dimensional boundaries of MEs have one light like direction (S_+ or S_- is constant). This means that the boundary of ME has metric dimension $d = 2$ and is characterized by an infinite-dimensional super-symplectic and super-conformal

symmetries just like the boundary of the embedding space $M^4_+ \times CP_2$: The boundaries are like moments for mini big bangs (in TGD based fractal cosmology big bang is replaced with a silent whisper amplified to not necessarily so big bang).

3. These observations inspire the conjecture that boundary conditions for M^4 like space-time sheets fixed by the variational principle selecting preferred extremals of Kähler action quite generally require that space-time boundaries correspond to light like 3-surfaces with metric dimension equal to $d = 2$. This does not yet imply that light like surfaces of embedding space would take the role of the light cone boundary: these light like surface could be seen only as a special case of causal determinants analogous to event horizons.

Part III

PHYSICS AS GENERALIZED NUMBER THEORY

Chapter 9

Physics as a Generalized Number Theory

9.1 Introduction

Physics as a generalized number theory program involves three threads: various p-adic physics and their fusion together with real number based physics to a larger structure [K104]. the attempt to understand basic physics in terms of classical number fields [K105]. and infinite primes [K103] whose construction is formally analogous to a repeated second quantization of an arithmetic quantum field theory. A common denominator of these approaches is a precise mathematical formulation for the notion of finite measurement resolution, which could be taken as one of the basic guiding principles of quantum TGD and is at quantum level realized in terms of inclusions of hyper-finite factors about which configuration space spinor fields provide an example [K120]. In the following these threads are described briefly. More detailed summaries will be given in separate articles.

9.1.1 P-Adic Physics And Unification Of Real And P-Adic Physics

p-Adic numbers [A51, A37, A38] became a part of TGD through the successes of p-adic thermodynamics in the description of elementary particle massivation [K70]. The p-adicization program attempts to construct physics in various number fields as an algebraic continuation of physics in the field of rationals (or appropriate extension of rationals). The program involves in an essential way the generalization of number concept obtained by fusing reals and p-adic number fields to a larger structure by gluing them together along common rationals.

The program involves in an essential way the generalization of number concept obtained by fusing reals and p-adic number fields to a larger structure by gluing them together along common rationals or their algebraic extension. The resulting structure is a generalization of adeles by fusing reals and various p-adic number fields to a book-like structure with pages defined by the number fields glued together along rationals or their algebraic extension in which case the extension induces the extension of p-adic number fields. This structure in turn induces similar structure for embedding spaces, space-time surfaces, and even WCW.

Real and p-adic regions of the space-time as geometric correlates of matter and mind

One could end up with p-adic space-time sheets via field equations. The solutions of the equations determining space-time surfaces are restricted by the requirement that the coordinates are real. When this is not the case, one might apply instead of a real completion with some p-adic completion. It however seems that p-adicity is present at deeper level and automatically present via the generalization of the number concept obtained by fusing reals and p-adics along rationals and common algebraics.

p-Adic non-determinism due to the presence of non-constant functions with a vanishing derivative implies extreme flexibility and therefore suggests the identification of the p-adic regions as seats of cognitive representations. Unlike the completion of reals to complex numbers, the completions of p-adic numbers preserve the information about the algebraic extension of rationals and algebraic coding of quantum numbers must be associated with "mind like" regions of space-time. p-Adics and reals would be in the same relationship as map and territory.

The implications are far-reaching and consistent with TGD inspired theory of consciousness: p-adic regions are present even at elementary particle level and provide some kind of model of "self" and of external world. In fact, p-adic physics would model the p-adic cognitive regions representing real elementary particle regions rather than elementary particles themselves! p-Adic mass calculations would be a model of a model!

The generalization of the notion of number

The unification of real physics of material work and p-adic physics of cognition and intentionality leads to the generalization of the notion of number field. Reals and various p-adic number fields are glued along their common rationals (and common algebraic numbers too) to form a fractal book like structure. Allowing all possible finite-dimensional extensions of p-adic numbers brings additional pages to this "Big Book".

At space-time level the book like structure corresponds to the decomposition of space-time surface to real and p-adic space-time sheets glued together along the common back. What this back means is however not what comes first in mind: a subset of space-time points for which preferred embedding space coordinates in an algebraic extension or rationals. This would lead to serious problems with GCI. One must define the intersection of realities and p-adicities at the level of WCW, and demand that the intersection corresponds to space-time surfaces with parameters (WCW coordinats) in the algebraic extension of rationals. The strong form of holography allows to construct space-time surface from string world sheets and partonic 2-surfaces serving as "space-time genes", and the parameters correspond by conformal invariance to general coordinate invariant conformal moduli for these 2-surfaces. The adelization of TGD reduces to an algebraic continuation of the moduli and various quantum numbers to various number fields.

This has deep implications for the view about cognition. For instance, two points infinitesimally near p-adically are infinitely distant in real sense so that cognition becomes a cosmic phenomenon.

Zero energy ontology, cognition, and intentionality

One could argue that conservation laws forbid p-adic-real phase transitions in practice so that cognitions (intentions) realized as real-to-padic (p-adic-to-real) transitions would not be possible. The situation changes if one accepts zero energy ontology [K27, K26].

1. Zero energy ontology classically

In TGD inspired cosmology [K98] the imbeddings of Robertson-Walker cosmologies are vacuum extremals. Same applies to the imbeddings of Reissner-Nordström solution [K114] and in practice to all solutions of Einstein's equations imbeddable as extremals of Kähler action. Since four-momentum currents define a collection of vector fields rather than a tensor in TGD, both positive and negative signs for energy corresponding to two possible assignments of the arrow of the geometric time to a given space-time surface are possible. This leads to the view that all physical states have vanishing net energy classically and that physically acceptable universes are creatable from vacuum.

The result is highly desirable since one can avoid unpleasant questions such as "What are the net values of conserved quantities like rest mass, baryon number, lepton number, and electric charge for the entire universe?", "What were the initial conditions in the big bang?", "If only single solution of field equations is selected, isn't the notion of physical theory meaningless since in principle it is not possible to compare solutions of the theory?". This picture fits also nicely with the view that entire universe understood as quantum counterpart 4-D space-time is recreated in each quantum jump and allows to understand evolution as a process of continual re-creation.

2. Zero energy ontology at quantum level

The construction of S-matrix [K69, K26] leads to the conclusion that all physical states identified as zero energy states in ZEO possess vanishing conserved quantum numbers but that for a given zero energy state one can identify opposite quantum numbers to the opposite boundaries of causal diamond (CD). Note that ZEO also superposition of states with different conserved quantum numbers at given boundary: this would allow a more natural understanding of Bose-Einstein condensate of Cooper pairs.

Furthermore, the entanglement coefficients between positive and negative energy components of the state have interpretation as M-matrix identifiable as a "complex square root" of density matrix expressible as a product of positive diagonal square root of the density matrix and of a unitary S-matrix. S-matrix thus becomes a property of the zero energy state and physical states code by their structure what is usually identified as quantum dynamics.

The collection of M-matrices defines an orthonormal state basis for zero energy states and together they define unitary U-matrix charactering transition amplitudes between zero energy states. This matrix would not be however the counterpart of the usual S-matrix. Rather the unitary matrix phase of a given M-matrix would define the S-matrix measured in laboratory.

At space-time level this would mean that positive energy component and negative energy component are at a temporal distance characterized by the time scale of the causal diamond (CD) and the rational (perhaps integer) characterizing the value of Planck constant for the state in question. The interpretation in terms of a mini bang followed by a mini crunch suggests itself also. CDs are indeed important also in TGD inspired cosmology [K98].

3. Hyper-finite factors of type II_1 and new view about S-matrix

The representation of S-matrix as unitary entanglement coefficients would not make sense in ordinary quantum theory but in TGD the von Neumann algebra in question is not a type I factor as for quantum mechanics or a type III factor as for quantum field theories, but what is called hyper-finite factor of type II_1 [K120]. This algebra is an infinite-dimensional algebra with the almost defining, and at the first look very strange, property that the infinite-dimensional unit matrix has unit trace. The infinite dimensional Clifford algebra spanned by the configuration space gamma matrices (configuration space understood as the space of 3-surfaces, the " of classical worlds") is indeed very naturally algebra of this kind since infinite-dimensional Clifford algebras provide a canonical representations for hyper-finite factors of type II_1.

It has turned out that the fractal structure of HFFs implying hierarchies of Jones inclusions has the hierarchy of quantum criticalities and associated hierarchy of Planck constants $h_{eff} = n \times h$ as counterparts. Also the hierarchy of algebraic extensions of rationals partially labelled by the integer n defined by the product of the ramified primes of the extension seems to be closely related to these hierarchies.

4. The new view about quantum measurement theory

This mathematical framework leads to a new kind of quantum measurement theory. The basic assumption is that only a finite number of degrees of freedom can be quantum measured in a given measurement and the rest remain untouched. What is known as Jones inclusions $\mathcal{N} \subset \mathcal{M}$ of von Neumann algebras allow to realize mathematically this idea [K120]. $\mathcal{N}$ characterizes measurement resolution and quantum measurement reduces the entanglement in the non-commutative quantum space $\mathcal{M}/\mathcal{N}$. The outcome of the quantum measurement is still represented by a unitary S-matrix but in the space characterized by $\mathcal{N}$. It is not possible to end up with a pure state with a finite sequence of quantum measurements.

The obvious objection is that the replacement of a universal S-matrix coding entire physics with a state dependent unitary entanglement matrix is too heavy a price to be paid for the resolution of the above mentioned paradoxes. Situation could be saved if the S-matrices have fractal structure. The quantum criticality of TGD Universe indeed implies fractality. The possibility of an infinite sequence of Jones inclusions for hyperfinite type II_1 factors isomorphic as von Neumann algebras expresses this fractal character algebraically. Thus one can hope that the S-matrix appearing as entanglement coefficients is more or less universal in the same way as Mandelbrot fractal looks more or less the same in all length scales and for all resolutions. Whether this kind of universality must be posed as an additional condition on entanglement coefficients or is an automatic consequence of unitarity in type II_1 sense is an open question.

What number theoretical universality might mean?

Number theoretic universality has been one of the basic guide lines in the construction of quantum TGD. There are two forms of the principle.

1. The strong form of number theoretical universality states that physics for any system should effectively reduce to a physics in algebraic extension of rational numbers at the level of M-matrix so that an interpretation in both real and p-adic sense (allowing a suitable algebraic extension of p-adics) is possible. One can however worry whether this principle only means that physics is algebraic so that there would be no need to talk about real and p-adic physics at the level of M-matrix elements. It is not possible to get rid of real and p-adic numbers at the level of classical physics since calculus is a prerequisite for the basic variational principles used to formulate the theory. For this option the possibility of completion is what poses conditions on M-matrix.
2. The weak form of principle requires only that both real and p-adic variants of physics make sense and that the intersection of these physics consist of physics associated with various algebraic extensions of rational numbers. In this rational physics would be like rational numbers allowing infinite number of algebraic extensions and real numbers and p-adic number fields as its completions. Real and p-adic physics would be completions of rational physics. In this framework criticality with respect to phase transitions changing number field becomes a viable concept. This form of principle allows also purely p-adic phenomena such as p-adic pseudo non-determinism assigned to imagination and cognition. Genuinely p-adic physics does not however allow definition of notions like conserved quantities since the notion of definite integral is lacking and only the purely local form of real physics allows p-adic counterpart.

   Strong form of holography suggests a rather elegant and concrete realization of this vision based on string world sheets and partonic 2-surfaces as "space-time genes" and having conformal moduli in an algebraic extension of rationals.

Experience has taught that it is better to avoid too strong statements and perhaps the weak form of the principle is enough. It is however clear that number theoretical criticality could provide important insights to quantum TGD. p-Adic thermodynamics [K70] is an excellent example of this. Needless to say, zero energy ontology is absolutely essential: otherwise this kind of transitions would not make sense.

p-Adicization by algebraic continuation

The basic challenges of the p-adicization program are following.

1. The first problem -the conceptual one- is the identification of preferred coordinates in which functions are algebraic and for which algebraic values of coordinates are in preferred position. This problem is encountered both at the level of space-time, embedding space, and configuration space. Here the group theoretical considerations play decisive role and the selection of preferred coordinates relates closely to the selection of quantization axes. This selection has direct physical correlates at the level of embedding space and the hierarchy of Planck constants has interpretation as a correlate for the selection of quantization axes [K38].

   Algebraization does not necessarily mean discretization at space-time level: for instance, the coordinates characterizing partonic 2-surface can be algebraic so that algebraic point of the configuration space results and surface is not discretized. If this kind of function spaces are finite-dimensional, it is possible to fix X^2 completely data for a finite number of points only.
2. Local physics generalizes as such to p-adic context (field equations, etc...). The basic stumbling block of this program is integration already at space-time (Kähler action, flux Hamiltonians, etc..). The problem becomes really horrible looking at configuration space level (functional integral). Algebraic continuation could allow to circumvent this difficulty. Needless to say, the requirement that the continuation exists must pose immensely tight constraints on the physics. Also the existence of the Kähler geometry does this and the solution to the constraint is that WCW is a union of symmetric spaces.

   In the case of symmetric spaces Fourier analysis generalizes to harmonics analysis and one can reduce integration to summation for functions allowing Fourier decomposition. In p-adic context the existence of plane waves requires an algebraic extension allowing roots of unity characterizing the measurement accuracy for angle like variables. This leads in the case of symmetric spaces to a general p-adicization recipe. One starts from a discrete variant of the symmetric space for which points correspond to roots of unity and replaces each discrete point with is p-adic completion representing the p-adic variant of the symmetric space so that kind of fractal variant of the symmetric space is obtained. There is an infinite hierarchy of p-adicizations corresponding to measurement resolutions and to the choice of preferred coordinates and the interpretation is in terms of cognitive representations. This requires a refined view about General Coordinate Invariance taking into account the fact that cognition is also part of the quantum state.

One general idea which results as an outcome of the generalized notion of number is the idea of a universal function continuable from a function mapping rationals to rationals or to a finite extension of rationals to a function in any number field. This algebraic continuation is analogous to the analytical continuation of a real analytic function to the complex plane.

1. Rational functions with rational coefficients are obviously functions satisfying this constraint. Algebraic functions with rational coefficients satisfy this requirement if appropriate finite-dimensional algebraic extensions of p-adic numbers are allowed. Exponent function is such a function.
2. For instance, residue calculus essential in the construction of N-point functions of conformal field theory might be generalized so that the value of an integral along the real axis could be calculated by continuing it instead of the complex plane to any number field via its values in the subset of rational numbers forming the rim of the book like structure having number fields as its pages. If the poles of the continued function in the finitely extended number field allow interpretation as real numbers it might be possible to generalize the residue formula. One can also imagine of extending residue calculus to any algebraic extension. An interesting situation arises when the poles correspond to extended p-adic rationals common to different pages of the "great book". Could this mean that the integral could be calculated at any page having the pole common. In particular, could a p-adic residue integral be calculated in the ordinary complex plane by utilizing the fact that in this case numerical approach makes sense.
3. Algebraic continuation is the basic tool of p-adicization program. Entire physics of the TGD Universe should be algebraically continuable to various number fields. Real number based physics would define the physics of matter and p-adic physics would describe correlates of cognition.
4. For instance, the idea that number theoretically critical partonic 2-surfaces are expressible in terms of rational functions with rational or algebraic coefficients so that also p-adic variants of these surfaces make sense, is very attractive.
5. Finite sums and products respect algebraic number property and the condition of finiteness is coded naturally by the notion of finite measurement resolution in terms of the notion of (number theoretic) braid. This simplifies dramatically the algebraic continuation since configuration space reduces to a finite-dimensional space and the space of configuration space spinor fields reduces to finite-dimensional function space.

The real configuration space can well contain sectors for which p-adicization does not make sense. For instance, if the exponent of Kähler function and Kähler are not expressible in terms of algebraic functions with rational or at most algebraic functions or more general functions making sense p-adically, the continuation is not possible. p-Adic non-determinism in p-adic sectors makes also impossible the continuation to real sector. All this is consistent with vision about rational and algebraic physics as as analog of rational and algebraic numbers allowing completion to various continuous number fields.

Due to the fact that real and p-adic topologies are fundamentally different, ultraviolet and infrared cutoffs in the set of rationals are unavoidable notions and correspond to a hierarchy of different physical phases on one hand and different levels of cognition on the other hand. For instance, most points p-adic space-time sheets reside at infinity in real sense and p-adically infinitesimal is infinite in real sense. Two types of cutoffs are predictedp-adic length scale cutoff and a cutoff due to phase resolution related to the hierarchy of Planck constants. Zero energy ontology provides natural realization for the p-adic length scale cutoff. The latter cutoff seems to correspond naturally to the hierarchy of algebraic extensions of p-adic numbers and quantum phases $exp(i2\pi/n)$, $n \geq 3$, coming as roots of unity and defining extensions of rationals and p-adics allowing to define p-adically sensible trigonometric functions These phases relate closely to the hierarchy of quantum groups, braid groups, and II_1 factors of von Neumann algebra.

9.1.2 TGD And Classical Number Fields

This chapter is second one in a multi-chapter devoted to the vision about TGD as a generalized number theory. The basic theme is the role of classical number fields in quantum TGD. A central notion is $M^8 - H$ duality which might be also called number theoretic compactification. This duality allows to identify embedding space equivalently either as M^8 or $M^4 \times CP_2$ and explains the symmetries of standard model number theoretically. These number theoretical symmetries induce also the symmetries dictaging the geometry of the "world of classical worlds" (WCW) as a union of symmetric spaces. This infinite-dimensional Kähler geometry is expected to be highly unique from the mere requirement of its existence requiring infinite-dimensional symmetries provided by the generalized conformal symmetries of the light-cone boundary $\delta M^4_+ \times S$ and of light-like 3-surfaces and the answer to the question what makes 8-D embedding space and $S = CP_2$ so unique would be the reduction of these symmetries to number theory.

Zero energy ontology has become the corner stone of both quantum TGD and number theoretical vision. In zero energy ontology either light-like or space-like 3-surfaces can be identified as the fundamental dynamical objects, and the extension of general coordinate invariance leads to effective 2-dimensionality (strong form of holography) in the sense that the data associated with partonic 2-surfaces and the distribution of 4-D tangent spaces at them located at the light-like boundaries of causal diamonds (CDs) defined as intersections of future and past directed light-cones code for quantum physics and the geometry of WCW.

The basic number theoretical structures are complex numbers, quaternions and octonions, and their complexifications obtained by introducing additional commuting imaginary unit $\sqrt{-1}$. Hyper-octonionic (-quaternionic,-complex) sub-spaces for which octonionic imaginary units are multiplied by commuting $\sqrt{-1}$ have naturally Minkowskian signature of metric. The question is whether and how the hyper-structures could allow to understand quantum TGD in terms of classical number fields. The answer which looks the most convincing one relies on the existence of octonionic representation of 8-D gamma matrix algebra.

1. The first guess is that associativity condition for the sub-algebras of the local Clifford algebra defined in this way could select 4-D surfaces as associative (hyper-quaternionic) sub-spaces of this algebra and define WCW purely number theoretically. The associative sub-spaces in question would be spanned by the modified gamma matrices defined by the Kähler-Dirac action fixed by the variational principle (Kähler action) selecting space-time surfaces as preferred extremals.

2. This condition is quite not enough: one must strengthen it with the condition that a preferred commutative and thus hyper-complex sub-space is contained in the tangent space of the space-time surface. This condition actually generalizes somewhat since one can introduce a family of so called Hamilton-Jacobi coordinates for M^4 allowing an integrable distribution of decompositions of tangent space to the space of non-physical and physical polarizations [K14]. The physical interpretation is as a number theoretic realization of gauge invariance selecting a preferred local commutative plane of non-physical polarizations.

3. Even this is not yet the whole story: one can define also the notions of co-associativity and co-commutativity applying in the regions of space-time surface with Euclidian signature of the induced metric. The basic unproven conjecture is that the decomposition of space-time surfaces to associative and co-associative regions containing preferred commutative *resp.* co-commutative 2-plane in the 4-D tangent plane is equivalent with the preferred extremal property of Kähler action and the hypothesis that space-time surface allows a slicing by string world sheets and by partonic 2-surfaces.

Hyper-octonions and hyper-quaternions

The discussions for years ago with Tony Smith [A79] stimulated very general ideas about space-time surface as an associative, quaternionic sub-manifold of octonionic 8-space (for octonions see [A15]. Also the observation that quaternionic and octonionic primes have norm squared equal to prime in complete accordance with p-adic length scale hypothesis, led to suspect that the notion of primeness for quaternions, and perhaps even for octonions, might be fundamental for the formulation of quantum TGD. The original idea was that space-time surfaces could be regarded as four-surfaces in 8-D embedding space with the property that the tangent spaces of these spaces can be locally regarded as 4- *resp.* 8-dimensional quaternions and octonions.

It took some years to realize that the difficulties related to the realization of Lorentz invariance might be overcome by replacing quaternions and octonions with hyper-quaternions and hyper-octonions. Hyper-quaternions *resp.* -octonions is obtained from the algebra of ordinary quaternions and octonions by multiplying the imaginary part with $\sqrt{-1}$ and can be regarded as a sub-space of complexified quaternions *resp.* octonions. The transition is the number theoretical counterpart of the transition from Riemannian to pseudo-Riemannin geometry performed already in Special Relativity. The loss of number field and even sub-algebra property is not fatal and has a clear physical meaning. The notion of primeness is inherited from that for complexified quaternions *resp.* octonions.

Note that hyper-variants of number fields make also sense p-adically unlike the notions of number fields themselves unless restricted to be algebraic extensions of rational variants of number fields. What deserves separate emphasis is that the basic structure of the standard model would reduce to number theory.

Number theoretical compactification and $M^8 - H$ duality

The notion of hyper-quaternionic and octonionic manifold makes sense but it not plausible that $H = M^4 \times CP_2$ could be endowed with a hyper-octonionic manifold structure. Situation changes if H is replaced with hyper-octonionic M^8. Suppose that $X^4 \subset M^8$ consists of hyper-quaternionic and co-hyper-quaternionic regions. The basic observation is that the hyper-quaternionic sub-spaces of M^8 with a fixed hyper-complex structure (containing in their tangent space a fixed hyper-complex subspace M^2 or at least one of the light-like lines of M^2) are labeled by points of CP_2. Hence each hyper-quaternionic and co-hyper-quaternionic four-surface of M^8 defines a 4-surface of $M^4 \times CP_2$. One can loosely say that the number-theoretic analog of spontaneous compactification occurs: this of course has nothing to do with dynamics.

This picture was still too naive and it became clear that not all known extremals of Kähler action contain fixed $M^2 \subset M^4$ or light-like line of M^2 in their tangent space.

1. The first option represents the minimal form of number theoretical compactification. M^8 is interpreted as the tangent space of H. Only the 4-D tangent spaces of light-like 3-surfaces X^3_l (wormhole throats or boundaries) are assumed to be hyper-quaternionic or co-hyper-quaternionic and contain fixed M^2 or its light-like line in their tangent space. Hyper-quaternionic regions would naturally correspond to space-time regions with Minkowskian signature of the induced metric and their co-counterparts to the regions for which the signature is Euclidian. What is of special importance is that this assumption solves the problem of identifying the boundary conditions fixing the preferred extremals of Kähler action since in the generic case the intersection of M^2 with the 3-D tangent space of X^3_l is 1-dimensional. The surfaces $X^4(X^3_l) \subset M^8$ would be hyper-quaternionic or co-hyper-quaternionic but would not allow a local mapping between the 4-surfaces of M^8 and H.
2. One can also consider a more local map of $X^4(X^3_l) \subset H$ to $X^4(X^3_l) \subset M^8$. The idea is to allow $M^2 \subset M^4 \subset M^8$ to vary from point to point so that $S^2 = SO(3)/SO(2)$ characterizes the local choice of M^2 in the interior of X^4. This leads to a quite nice view about strong geometric form of $M^8 - H$ duality in which M^8 is interpreted as tangent space of H and $X^4(X^3_l) \subset M^8$ has interpretation as tangent for a curve defined by light-like 3-surfaces at X^3_l and represented by $X^4(X^3_l) \subset H$. Space-time surfaces $X^4(X^3_l) \subset M^8$ consisting of hyper-quaternionic and co-hyper-quaternionic regions would naturally represent a preferred extremal of E^4 Kähler action. The value of the action would be same as CP_2 Kähler action. $M^8 - H$ duality would apply also at the induced spinor field and at the level of configuration space.

3. Strong form of $M^8 - H$ duality satisfies all the needed constraints if it represents Kähler isometry between $X^4(X_l^3) \subset M^8$ and $X^4(X_l^3) \subset H$. This implies that light-like 3-surface is mapped to light-like 3-surface and induced metrics and Kähler forms are identical so that also Kähler action and field equations are identical. The only differences appear at the level of induced spinor fields at the light-like boundaries since due to the fact that gauge potentials are not identical.

4. The map of $X_l^3 \subset H \rightarrow X_l^3 \subset M^8$ would be crucial for the realization of the number theoretical universality. $M^8 = M^4 \times E^4$ allows linear coordinates as those preferred coordinates in which the points of embedding space are rational/algebraic. Thus the point of $X^4 \subset H$ is algebraic if it is mapped to algebraic point of M^8 in number theoretic compactification. This of course restricts the symmetry groups to their rational/algebraic variants but this does not have practical meaning. Number theoretical compactification could thus be motivated by the number theoretical universality.

5. The possibility to use either M^8 or H picture might be extremely useful for calculational purposes. In particular, M^8 picture based on $SO(4)$ gluons rather than $SU(3)$ gluons could perturbative description of low energy hadron physics. The strong $SO(4)$ symmetry of low energy hadron physics can be indeed seen direct experimental support for the $M^8 - H$ duality.

9.1.3 Infinite Primes

The notion of prime seems to capture something very essential about what it is to be elementary building block of matter and has become a fundamental conceptual element of TGD. The notion of prime gains it generality from its reducibility to the notion of prime ideal of an algebra. Thus the notion of prime is well-defined, not only in case of quaternions and octonions, but also in the case of hyper-quaternions and -octonions, which are especially natural physically and for which numbers having zero norm correspond physically to light-like 8-vectors. Many interpretations for infinite primes have been competing for survival but it seems that the recent state of TGD allows to exclude some of them from consideration.

The notion of infinite prime

Simple arguments show that the p-adic prime characterizing the 3-surface representing the entire universe increases in a statistical sense in the sequence of quantum jumps: the reason is simply that the size of primes is bounded below. This leads to a peculiar paradox: if the number of quantum jumps already occurred is infinite, this prime is most naturally infinite. On the other hand, if one assumes that only finite number of quantum jumps have occurred, one encounters the problem of understanding why the initial quantum history was what it was. Furthermore, since the size of the 3-surface representing the entire Universe is infinite, p-adic length scale hypothesis suggest also that the p-adic prime associated with the entire universe is infinite.

These arguments motivate the attempt to construct a theory of infinite primes and to extend quantum TGD so that also infinite primes are possible. Rather surprisingly, one can construct infinite primes by repeating a procedure analogous to a quantization of a super symmetric arithmetic quantum field theory. At given level of hierarchy one can identify the decomposition of space-time surface to p-adic regions with the corresponding decomposition of the infinite prime to primes at lower level of infinity: at the basic level are finite primes for which one cannot find any formula.

This and other observations suggest that the Universe of quantum TGD might basically provide a physical representation of number theory allowing also infinite primes. The proposal suggests also a possible generalization of real numbers to a number system akin to hyper-reals introduced by Robinson in his non-standard calculus [A32] providing rigorous mathematical basis for calculus. In fact, some rather natural requirements lead to a unique generalization for the concepts of integer, rational and real. Somewhat surprisingly, infinite integers and reals can be regarded as infinite-dimensional vector spaces with integer and real valued coefficients respectively and this raises the question whether the tangent space for the configuration space of 3-surfaces could be regarded as the space of generalized 8-D hyper-octonionic numbers.

Infinite primes and physics in TGD Universe

Several different views about how infinite primes, integers, and rationals might be relevant in TGD Universe have emerged.

1. Infinite primes, cognition, and intentionality

The correlation of infinite primes with cognition is the first fascinating possibility and this possibility has stimulated several ideas.

1. The hierarchy of infinite primes associated with algebraic extensions of rationals leading gradually towards algebraic closure of rationals would in turn define cognitive hierarchy corresponding to algebraic extensions of p-adic numbers.
2. Infinite primes form an infinite hierarchy so that the points of space-time and embedding space can be seen as infinitely structured and able to represent all imaginable algebraic structures. Certainly counter-intuitively, single space-time point -or more generally wave functions in the space of the units associated with the point- might be even capable of representing the quantum state of the entire physical Universe in its structure. For instance, in the real sense surfaces in the space of units correspond to the same real number 1, and single point, which is structure-less in the real sense could represent arbitrarily high-dimensional spaces as unions of real units. For real physics this structure is completely invisible and is relevant only for the physics of cognition. One can say that Universe is an algebraic hologram, and there is an obvious connection both with Brahman=Atman identity of Eastern philosophies and Leibniz's notion of monad.
3. One can assign to infinite primes at n^{th} level of hierarchy rational functions of n rational arguments which form a natural hierarchical structure in that highest level corresponds to a polynomial with coefficients which are rational functions of the arguments at the lower level. One can solve one of the arguments in terms of lower ones to get a hierarchy of algebraic extensions. At the lowest level algebraic extensions of rationals emerge, at the next level algebraic extensions of space of rational functions of single variable, etc... This would suggest that infinite primes code for the correlation between quantum states and the algebraic extensions appearing in their their physical description and characterizing their cognitive correlates. The hierarchy of infinite primes would also correlate with a hierarchy of logics of various orders (hierarchy of statements about statements about...).

2. Infinite primes and super-symmetric quantum field theory

Consider next the physical interpretation.

1. The discovery of infinite primes suggested strongly the possibility to reduce physics to number theory. The construction of infinite primes can be regarded as a repeated second quantization of a super-symmetric arithmetic quantum field theory. This suggests that configuration space spinor fields or at least the ground states of associated super-conformal representations could be mapped to infinite primes in both bosonic and fermionic degrees of freedom. The process might generalize so that it applies in the case of quaternionic and octonionic primes and their hyper counterparts. This hierarchy of second quantizations means enormous generalization of physics to what might be regarded a physical counterpart for a hierarchy of abstractions about abstractions about.... The ordinary second quantized quantum physics corresponds only to the lowest level infinite primes.
2. The ordinary primes appearing as building blocks of infinite primes at the first level of the hierarchy could be identified as coding for p-adic primes assignable to fermionic and bosonic partons identified as 2-surfaces of a given space-time sheet. The hierarchy of infinite primes would correspond to hierarchy of space-time sheets defined by the topological condensate. This leads also to a precise identification of p-adic and real variants of bosonic partonic 2-surfaces as correlates of intention and action and pairs of p-adic and real fermionic partons as correlates for cognitive representations.
3. The idea that infinite primes characterize quantum states of the entire Universe, perhaps ground states of super-conformal representations, if not all states, could be taken further. It turns out that this idea makes sense when one considers discrete wave functions in the space

of infinite primes and that one can indeed represent standard model quantum numbers in this way.

4. The number theoretical supersymmetry suggests also space-time supersymmetry TGD framework. Space-time super-symmetry in its standard form is not possible in TGD Universe and this cheated me to believe that this supersymmetry is completely absent in TGD Universe. The progress in the understanding of the properties of the modified Dirac action however led to a generalization of the space-time super-symmetry as a dynamical and broken symmetry of quantum TGD [**?**].

Here however emerges the idea about the number theoretic analog of color confinement. Rational (infinite) primes allow not only a decomposition to (infinite) primes of algebraic extensions of rationals but also to algebraic extensions of quaternionic and octonionic (infinite) primes. The physical analog is the decomposition of a particle to its more elementary constituents. This fits nicely with the idea about number theoretic resolution represented as a hierarchy of Galois groups defined by the extensions of rationals and realized at the level of physics in terms of Jones inclusions [K120] defined by these groups having a natural action on space-time surfaces, induced spinor fields, and on configuration space spinor fields representing physical states [K27].

3. Infinite primes and physics as number theory

The hierarchy of algebraic extensions of rationals implying corresponding extensions of p-adic numbers suggests that Galois groups, which are the basic symmetry groups of number theory, should have concrete physical representations using induced spinor fields and configuration space spinor fields and also infinite primes and real units formed as infinite rationals. These groups permute zeros of polynomials and thus have a concrete physical interpretation both at the level of partonic 2-surfaces dictated by algebraic equations and at the level of braid hierarchy. The vision about the role of hyperfinite factors of II_1 and of Jones inclusions as descriptions of quantum measurements with finite measurement resolution leads to concrete ideas about how these groups are realized.

G_2 acts as automorphisms of hyper-octonions and $SU(3)$ as its subgroup respecting the choice of a preferred imaginary unit. The discrete subgroups of $SU(3)$ permuting to each other hyper-octonionic primes are analogous to Galois group and turned out to play a crucial role in the understanding of the correspondence between infinite hyper-octonionic primes and quantum states predicted by quantum TGD.

4. The notion of finite measurement resolution as the key concept

TGD predicts several hierarchies: the hierarchy of space-time sheets, the hierarchy of infinite primes, the hierarchy of Jones inclusions identifiable in terms of finite measurement resolution [K120]. the dark matter hierarchy characterized by increasing values of $\hbar$ [K38]. the hierarchy of extensions of a given p-adic number field. TGD inspired theory of consciousness predicts the hierarchy of selves and quantum jumps with increasing duration with respect to geometric time. These hierarchies should be closely related.

The notion of finite measurement resolution turns out to be the key concept: the p-adic norm of the rational defined by the infinite prime characterizes the angle measurement resolution for given p-adic prime p. It is essential that one has what might be called a state function reduction selecting a fixed p-adic prime which could be also infinite. This gives direct connections with cognition and with the p-adicization program relying also on angle measurement resolution. Also the value the integers characterizing the singular coverings of CD and CP_2 defining as their product Planck constant characterize the measurement resolution for a given p-adic prime in CD and CP_2 degrees of freedom. This conforms with the fact that elementary particles are characterized by two infinite primes. Hence finite measurement resolution ties tightly together the three threads of the number theoretic vision. Finite measurement resolution relates also closely to the inclusions of hyper-finite factors central for TGD inspired quantum measurement theory so that the characterization of the finite measurement resolution, which has been the ugly duckling of theoretical physics, transforms to a beautiful swan.

5. Space-time correlates of infinite primes

Infinite primes code naturally for Fock states in a hierarchy of super-symmetric arithmetic quantum field theories. Quantum classical correspondence leads to ask whether infinite primes could also code for the space-time surfaces serving as symbolic representations of quantum states. This would a generalization of algebraic geometry would emerge and could reduce the dynamics of Kähler action to algebraic geometry and organize 4-surfaces to a physical hierarchy according to their algebraic complexity. Note that this conjecture should be consistent with two other conjectures about the dynamics of space-time surfaces (space-time surfaces as preferred extrema of Kähler action and space-time surfaces as quaternionic or co-quaternionic (as associative or co-associative) 4-surfaces of hyper-octonion space M^8).

The representation of space-time surfaces as algebraic surfaces in M^8 is however too naive idea and the attempt to map hyper-octonionic infinite primes to algebraic surfaces has not led to any concrete progress.

The solution came from quantum classical correspondence, which requires the map of the quantum numbers of configuration space spinor fields to space-time geometry. The Kähler-Dirac equation with measurement interaction term realizes this requirement. Therefore, if one wants to map infinite rationals to space-time geometry it is enough to map infinite primes to quantum numbers. This map can be indeed achieved thanks to the detailed picture about the interpretation of the symmetries of infinite primes in terms of standard model symmetries.

Generalization of ordinary number fields: infinite primes and cognition

Both fermions and p-adic space-time sheets are identified as correlates of cognition in TGD Universe. The attempt to relate these two identifications leads to a rather concrete model for how bosonic generators of super-algebras correspond to either real or p-adic space-time sheets (actions and intentions) and fermionic generators to pairs of real space-time sheets and their p-adic variants obtained by algebraic continuation (note the analogy with fermion hole pairs).

The introduction of infinite primes, integers, and rationals leads also to a generalization of classical number fields since an infinite algebra of real (complex, etc...) units defined by finite ratios of infinite rationals multiplied by ordinary rationals which are their inverses becomes possible. These units are not units in the p-adic sense and have a finite p-adic norm which can be differ from one. This construction generalizes also to the case of hyper- quaternions and -octonions although non-commutativity and in case of octonions also non-associativity pose technical problems. Obviously this approach differs from the standard introduction of infinitesimals in the sense that sum is replaced by multiplication meaning that the set of real and also more general units becomes infinitely degenerate.

Infinite primes form an infinite hierarchy so that the points of space-time and imbedding space can be seen as infinitely structured and able to represent all imaginable algebraic structures. Certainly counter-intuitively, single space-time point is even capable of representing the quantum state of the entire physical Universe in its structure. For instance, in the real sense surfaces in the space of units correspond to the same real number 1, and single point, which is structure-less in the real sense could represent arbitrarily high-dimensional spaces as unions of real units.

One might argue that for the real physics this structure is invisible and is relevant only for the physics of cognition. On the other hand, one can consider the possibility of mapping the configuration space and configuration space spinor fields to the number theoretical anatomies of a single point of embedding space so that the structure of this point would code for the world of classical worlds and for the quantum states of the Universe. Quantum jumps would induce changes of configuration space spinor fields interpreted as wave functions in the set of number theoretical anatomies of single point of embedding space in the ordinary sense of the word, and evolution would reduce to the evolution of the structure of a typical space-time point in the system. Physics would reduce to space-time level but in a generalized sense. Universe would be an algebraic hologram, and there is an obvious connection both with Brahman=Atman identity of Eastern philosophies and Leibniz's notion of monad.

Infinite rationals are in one-one correspondence with quantum states and in zero energy ontology hyper-octonionic units identified as ratios of the infinite integers associated with the positive and negative energy parts of the zero energy state define a representation of WCW spinor fields. The action of subgroups of SU(3) and rotation group SU(2) preserving hyper-octonionic and hyper-quaternionic primeness and identification of momentum and electro-weak charges in terms of components of hyper-octonionic primes makes this representation unique. Hence Brahman-Atman identity has a completely concrete realization and fixes completely the quantum number spectrum including particle masses and correlations between various quantum numbers.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L9].

9.2 P-Adic Physics And The Fusion Of Real And P-Adic Physics To A Single Coherent Whole

In this section basic facts about p-adic numbers [A51, A37, A38] and the question about their relation to real numbers are discussed. Also the basic technicalities related to the notion of p-adic physics are discussed. Also included is a section about the physics in the intersection of real and p-adic worlds relevant to living systems in TGD Universe.

9.2.1 Background

It is good to start with a summary of the basic mathematical problems related to the p-adicization of physics and a rough formulation for how one might resolve these problems.

Problems

It is far from obvious what the p-adic counterpart of real physics could mean and how one could fuse together real and p-adic physics. Therefore it is good to list the basic problems and proposals for their solution.

The first problem concerns the correspondence between real and p-adic numbers.

1. The success of p-adic mass calculations involves the notions of p-adic probability, thermodynamics, and the mapping of p-adic probabilities to the real ones by a continuous correspondence $x = \sum x_n p^n \rightarrow Id(x) = \sum x_n p^{-n}$ that I have christened canonical identification.

   The naïve guess is that canonical identification in some form could relate also real and p-adic preferred extremals and define cognitive representations at space-time level. The problem is that I n does not respect symmetries defined by isometries and also general coordinate invariance is possible only if one can identify preferred embedding space coordinates. The reason is that I does not commute with the basic arithmetic operations. I allows several variants and it is possible to have correspondence which respects symmetries in arbitrary accuracy in preferred coordinates. Thus I can play a role at space-time level only if one defines symmetries modulo measurement resolution. I would make sense only in the interval defining the measurement resolution for a given coordinate variable and the p-adic effective topology would make sense just because the finite measurement resolution does not allow to well-order the points.

2. The identification of real and p-adic numbers via rationals common to all number fields - or more generally along algebraic extension of rationals- respects symmetries and algebra but is not continuous. At the embedding space level preferred coordinates are required also now. The maximal symmetries of the embedding space allow identification of this kind of coordinates. They are not unique. For instance, M^4 linear coordinates look very natural but for CP_2 trigonometric functions of angle like coordinates look more suitablel and Fourier analysis suggests strongly the introduction of algebraic extensions involving roots of unity. Partly the non-uniqueness has an interpretation as an embedding space correlate for the selection of the quantization axes. The symmetric space [A22] property of WCW gives hopes that general coordinate invariance in quantal sense can be realized. The existence of p-adic harmonic analysis suggests a discretization of the p-adic variant of embedding space and WCW based on roots of unity.

3. One can consider a compromise between the two correspondences. Discretization via common algebraic points can be completed to a p-adic continuum by assigning to each real discretization interval (say angle increment $2\pi/N$) p-adic numbers with norm smaller than one.

4. It however turned out that more imaginative approach is needed [K119]. Strong from of holography allows to identify string world sheets and partonic 2-surfaces as space-time genes. One can transcend the discretization in an algebraic extension of rationals from space-time level to the level of WCW by demanding that the parameters characterizing these surfaces are in an algebraic extension of rationals. Also cutoffs can be introduced at this level. The outcome is general coordinate invariant (GCI) and problems with symmetries and GCI are avoided. Besides this answers to the basic questions of p-adicization emerge. One can assign to string world sheets purely number theoretically preferred primes and even generalize the p-adic length scale hypothesis using Negentropy Maximization Principle (NMP) [K65].

Second problem relates to integration and Fourier analysis. Both these procedures are fundamental for physics - be it classical or quantum. The p-adic variant of definite integral does not exist in the sense required by the action principles of physics although classical partial differential equations assigned to a particular variational principle make perfect sense. Fourier analysis is also possible only if one allows algebraic extension of p-adic numbers allowing a sufficient number of roots of unity correlating with the measurement resolution of angle. The finite number of them has interpretation in terms of finite angle resolution. Fourier analysis provides also an algebraic realization of definite integral when one integrates over the entire manifold as one indeed does in the case of WCW. If the space in question allows maximal symmetries as WCW and embedding space do, there are excellent hopes of having p-adic variants of both integration and harmonic analysis and the above described procedure allows a precise completion of the discretized variant of real manifold to its continuous p-adic variant.

The third problem relates to the definitions of the p-adic variants of Riemannian, symplectic [A42, A24, A23], and Kähler [A12] geometries. It is possible to generalize formally the notion of Riemann metric although non-local quantities like areas and total curvatures do not make sense if defined in terms of integrals. If all relevant quantities assignable to the geometry (family of Hamiltonians defining isometries, Killing vector fields, components of metric and Kähler form, Kähler function, etc...) are expressible in terms of rational functions involving only rational numbers as coefficients of polynomials, they allow an algebraic continuation to the p-adic context and the p-adic variant of the geometry makes sense.

The fourth problem relates to the question what one means with p-adic quantum mechanics. In TGD framework p-adic quantum theory utilizes p-adic Hilbert space. The motivation is that the notions of p-adic probability and unitarity are well defined. From the beginning it was clear that the straightforward generalization of Schrödinger equation is not very interesting physically and gradually the conviction has developed that the most realistic approach must rely on the attempt to find the p-adic variant of the TGD inspired quantum physics in all its complexity. The recent approach starts from a rather concrete view about generalized Feynman diagrams defining the points of WCW and leads to a rather detailed view about what the p-adic variants of QM could be and how they could be fused with real QM to a larger structure. Even more, just the requirement that this p-adicization exists, gives very powerful constraints on the real variant of the quantum TGD. Very briefly, algebraic continuation of the scattering amplitudes expressible using data associated with string world sheets and partonic 2-surfaces to various number fields allows to achieve number theoretical universality.

The fifth problem relates to the notion of information in p-adic context. p-Adic thermodynamics leads naturally to the question what p-adic entropy might mean and this in turn leads to the realization that for rational or even algebraic probabilities p-adic variant of Shannon entropy can be negative and has minimum for a unique prime. One can say that the entanglement in the intersection of real and p-adic worlds is negentropic. This leads to rather fascinating vision about how negentropic entanglement (see **Fig. ??** in the Appendix) makes it possible for living systems to overcome the second law of thermodynamics. The formulation of quantum theory in the intersection of real and living worlds becomes the basic challenge.

The proposed solutions to the technical problems could be rephrased in terms of the notion of algebraic universality. Various p-adic physics are obtained as algebraic continuation of real physics through the common algebraic points of real and p-adic worlds and by performing completion in the sense that the interval corresponding to finite measurement resolution are replaced with their p-adic counterpart via canonical identification. This allows to have exact symmetries as their discrete variants and also a continuous correspondence if desired.

Program

These ideas lead to a reasonably well defined p-adicization program. Try to define precisely the concepts of the p-adic space-time and "world of classical worlds" (WCW), formulate the finite-p p-adic versions of quantum TGD. Try to fuse together real and various p-adic quantum TGDs are to form a full theory of physics and cognition.

The construction of the p-adic TGD necessitates the generalization of the basic tools of standard physics such as differential and integral calculus, the concept of Hilbert space, Riemannian geometry, group theory, action principles, and the notions of probability and unitarity to the p-adic context. Also new physical thinking and philosophy is needed. The notions of Zero Energy Ontology (ZEO) , hierarchy of Planck constants reducible to a hierarchy of quantum criticalities, Negentropy Maximization Principle (NMP), strong form of holography, etc.. are essential but not discussed in detail in the following.

Quite recently it has become clear that strong holography implied by strong form of general coordinate invariance (GCI) is the crux of the construction. WCW has a book-like adelic structure. String world sheets and partonic 2-surfaces serve as number theoretically universal "space-time genes" and induced by algebraic extensions of rationals shared by reals and appropriate extensions of p-adic numbers. This core structure could be called intersection of reality and various p-adicities, the back of the Big Book. What can be said about quantum physics utilizes information about this structure continued algebraically to various real and p-adic sectors.

In the following I try to describe the most central problems and ideas of the p-adicization program. Page number of a readable article must be finite and this has forced to leave away a lot of topics. p-Adic mass calculations [K70], which form the corner stone of the entire approach would require entire article series. The vision about how to define generalized Feynman diagrams and their p-adic variants by utilizing the assumption that WCW is symmetric space allowing algebraization of functional integral crucial for the entire approach is discussed [L8]. Here huge symmetries of WCW, which include super-symplectic symmetry and generalize the conformal symmetries of string models, are in key role [K50, K28]. Negentropy Maximization Principle [K65] relevant for understanding the profound implications of the negentropic entanglement, in particular how the preferred p-adic primes emerge [K119] is not discussed. The applications of p-adic length scale hypothesis to the physics of living matter [K107] and the model of cognition [K24, K73] would provide additional insights and motivations but have been also left out.

9.2.2 Summary Of The Basic Physical Ideas

In the following various ways to end up with p-adic physic sand with the idea about p-adic physics as physics of cognition are discussed. There is also the idea about p-adic topology as an effective topology of real space-time surfaces in finite measurement resolution implying discretization but this idea is not so compelling.

p-Adic mass calculations briefly

p-Adic mass calculations based on p-adic thermodynamics with energy replaced with the generator $L_0 = zd/dz$ of infinitesimal scaling are described in the first part of [K70] .

1. p-Adic thermodynamics could be justified by the randomness of the motion of partonic 2-surfaces restricted only by the light-likeness of the orbit.
2. It is essential that the conformal symmetries associated with the light-like coordinates of parton and light-cone boundary are not gauge symmetries but dynamical symmetries. The point is that there are two kinds of super-conformal symmetries [A19, A21] : the super-symplectic conformal symmetries assignable to the light-like boundaries of $CD \times CP_2$ and super Kac-Moody symmetries [A11] assignable to light-like 3-surfaces defining fundamental dynamical objects. In so called coset construction [A58] the differences of super-conformal generators of these algebras annihilate the physical states. This leads to a generalization of Equivalence Principle since one can assign four-momentum to the generators of both algebras identifiable as inertial *resp.* gravitational four-momentum. A second important consequence is that the generators of either algebra do not act like gauge transformations so that it makes sense to construct p-adic thermodynamics for them.

3. In p-adic thermodynamics scaling generator L_0 having conformal weights as its eigen values replaces energy and Boltzmann weight $exp(H/T)$ is replaced by p^{L_0/T_p}. The quantization $T_p = 1/n$ of conformal temperature and thus quantization of mass squared scale is implied by number theoretical existence of Boltzmann weights. p-Adic length scale hypothesis states that primes $p \simeq 2^k$, k integer. A stronger hypothesis is that k is prime (in particular Mersenne prime or Gaussian Mersenne) makes the model very predictive and fine tuning is not possible.

   Mersenne primes are very special number theoretically because bit as the unit of information unit corresponds to $log(2)$ and can be said to exists for M_n-adic topology. The reason is that $log(1+p)$ existing always p-adically corresponds for $M_n = 2^n - 1$ to $log(2^n) \equiv nlog(2)$ so that one has $log(2 \equiv log(1+M_n)/n$. Since the powers of 2 modulo p give all integers $n \in \{1, p-1\}$ by Fermat's theorem, one can say that the logarithms of all integers modulo M_n exist in this sense and therefore the logarithms of all p-adic integers not divisible by p exist. For other primes one must introduce a transcendental extension containing $log(a)$ where are is so called primitive root. One could criticize the identification since $log(1 + M_n)$ corresponding in the real sense to n bits corresponds in p-adic sense to a very small information content since the p-adic norm of the p-adic bit is $1/M_n$.

The basic mystery number of elementary particle physics defined by the ratio of Planck mass and proton mass follows thus from number theory once CP_2 radius is fixed to about 10^4 Planck lengths. Mass scale becomes additional discrete variable of particle physics so that there is not more need to force top quark and neutrinos with mass scales differing by 12 orders of magnitude to the same multiplet of gauge group. Electron, muon, and τ correspond to Mersenne prime $k = 127$ (the largest non-super-astrophysical Mersenne), and Mersenne primes $k = 113, 107$. Intermediate gauge bosons and photon correspond to Mersenne M_{89}, and graviton to M_{127}.

The value of k for quark can depend on hadronic environment [K72] and this would produce precise mass formulas for low energy hadrons. This kind of dependence conforms also with the indications that neutrino mass scale depends on environment [C7]. Amazingly, the biologically most relevant length scale range between 10 nm and 4 μm contains four Gaussian Mersennes $(1 + i)^n - 1$, $n = 151, 157, 163, 167$ and scaled copies of standard model physics in cell length scale could be an essential aspect of macroscopic quantum coherence prevailing in cell length scale.

p-Adic mass thermodynamics is not quite enough: also Higgs boson is needed and wormhole contact carrying fermion and anti-fermion quantum numbers at the light-like wormhole throats is excellent candidate for Higgs [K59] . The coupling of Higgs to fermions can be small and induce only a small shift of fermion mass: this could explain why Higgs has not been observed. Also the Higgs contribution to mass squared can be understood thermodynamically if identified as absolute value for the thermal expectation value of the eigenvalues of the Kähler-Dirac operator having interpretation as complex square root of conformal weight.

The original belief was that only Higgs corresponds to wormhole contact. The assumption that fermion fields are free in the conformal field theory applying at parton level forces to identify all gauge bosons as wormhole contacts connecting positive and negative energy space-time sheets [K59] . Fermions correspond to topologically condensed CP_2 type extremals with single light-like wormhole throat. Gravitons are identified as string like structures involving pair of fermions or gauge bosons connected by a flux tube. Partonic 2-surfaces are characterized by genus which explains family replication phenomenon and an explanation for why their number is three emerges [K25] . Gauge bosons are labeled by pairs (g_1, g_2) of handle numbers and can be arranged to octet and singlet representations of the resulting dynamical SU(3) symmetry. Ordinary gauge bosons are SU(3) singlets and the heaviness of octet bosons explains why higher boson families are effectively absent. The different character of bosons could also explain why the p-adic temperature for bosons is $T_p = 1/n < 1$ so that Higgs contribution to the mass dominates.

The basis challenge is to understand why elementary particles seem to be characterized by preferred p-adic primes and why these primes seem to obey p-adic length scale hyptohesis- that is be near but below powers of two.

p-Adic length scale hypothesis, ZEO, and hierarchy of Planck constants

ZEO and the hierarchy of Planck constants realized in terms of the generalization of the embedding space lead to a deeper understanding of the origin of the p-adic length scale hypothesis.

1. ZEO

In ZEO one replaces positive energy states with zero energy states with positive and negative energy parts of the state at the light-like boundaries of CD. All conserved quantum numbers of the positive and negative energy states are of opposite sign so that these states can be created from vacuum. "Any physical state is creatable from vacuum" becomes thus a basic principle of quantum TGD and together with the notion of quantum jump resolves several philosophical problems (What was the initial state of universe?, What are the values of conserved quantities for Universe?, Is theory building completely useless if only single solution of field equations is realized?). At the level of elementary particle physics positive and negative energy parts of zero energy state are interpreted as initial and final states of a particle reaction so that quantum states become physical events.

At the level of WCW ZEO means that pairs of 3-surfaces residing at opposite boundaries of CD become basis objects or equivalent preferred extremals of Kähler acting [K8] having these 3-surfaces at ends replaced space-like 3-surfaces as basic objects. Preferred extremal property means that these space-time surfaces become archetypal spatiotemporal patterns: biologist would talk about behaviors, functions, or self-organization patterns [K62]. Self-organization is however understood in 4-D sense.

2. Does the finiteness of measurement resolution dictate the laws of physics?

The hypothesis that the mere finiteness of measurement resolution could determine the laws of quantum physics [K26] completely belongs to the category of not at all obvious first principles. The basic observation is that the Clifford algebra [A5] spanned by the gamma matrices of the "world of classical worlds" represents a von Neumann algebra [A65] known as hyperfinite factor of type II_1 (HFF) [K26, K120, K38] . HFF [A41, A52] is an algebraic fractal having infinite hierarchy of included sub-algebras isomorphic to the algebra itself [A2] . The structure of HFF is closely related to several notions of modern theoretical physics such as integrable statistical physical systems [A80] , anyons [D7] , quantum groups and conformal field theories [A39] , and knots and topological quantum field theories [A77, A47] .

ZEO is second key element. In ZEO these inclusions allow an interpretation in terms of a finite measurement resolution: in the standard positive energy ontology this interpretation is not possible. Inclusion hierarchy defines in a natural manner the notion of coupling constant evolution and p-adic length scale hypothesis follows as a prediction. In this framework the extremely heavy machinery of renormalized quantum field theory involving the elimination of infinities is replaced by a precisely defined mathematical framework. More concretely, the included algebra creates states which are equivalent in the measurement resolution used. Zero energy state can be modified in a time scale shorter than the time scale of the zero energy state itself.

One can imagine two kinds of measurement resolutions. The element of the included algebra can leave the quantum numbers of the positive and negative energy parts of the state invariant, which means that the action of subalgebra leaves M-matrix invariant. The action of the included algebra can also modify the quantum numbers of the positive and negative energy parts of the state such that the zero energy property is respected. In this case the Hermitian operators subalgebra must commute with M-matrix.

The temporal distance between the tips of CD corresponds to the secondary p-adic time scale $T_{p,2} = \sqrt{p}T_p$ by a simple argument based on the observation that light-like randomness of light-like 3-surface is analogous to Brownian motion. This gives the relationship $T_p = L_p^2/Rc$, where R is CP_2 size. The action of the included algebra corresponds to an addition of zero energy parts to either positive or negative energy part of the state and is like addition of quantum fluctuation below the time scale of the measurement resolution. The natural hierarchy of time scales is obtained as $T_n = 2^{-n}T$ since these insertions must belong to either upper or lower half of the causal diamond. This implies that preferred p-adic primes are near powers of 2. For electron the time scale in question is .1 seconds defining the fundamental biorhythm of 10 Hz.

M-matrix representing a generalization of S-matrix and expressible as a product of a positive square root of the density matrix and unitary S-matrix would define the dynamics of quantum theory [K26] . The notion of thermodynamical state would cease to be a theoretical fiction and in a well-defined sense quantum theory could be regarded as a square root of thermodynamics. Connes tensor product [A41] provides a mathematical description of the finite measurement resolution but does not fix the M-matrix as was the original hope. The remaining challenge is the calculation of M-matrix and the progress induced by ZEO during last years has led to rather concrete proposal for the construction of M-matrix.

It turns out however that the mathematical representation for the notion of finite resolution for angle measurement serves as a common denominator for all basic approaches to quantum TGD: the Kähler geometry [A12] of WCW identified as a union of infinite-dimensional symmetric spaces, inclusions of hyper finite factors as representation of finite measurement resolution, p-adicization program, the role of classical number fields [A15, A7, A18] , and infinite primes so that it is fair to say that all approaches to TGD which originally seemed almost independent, converge to a coherent mathematical structure.

3. How do p-adic coupling constant evolution and p-adic length scale hypothesis emerge?

In zero energy ontology zero energy states have as embedding space correlates causal diamonds for which the distance between the tips of the intersecting future and past directed light-cones comes as integer multiples of a fundamental time scale: $T_n = n \times T_0$. p-Adic length scale hypothesis allows to consider a stronger hypothesis $T_n = 2^n T_0$ and its generalization a slightly more general hypothesis $T_n = p^n T_0$, p prime. It however seems that these scales are dynamically favored but that also other scales are possible.

Could the coupling constant evolution in powers of 2 implying time scale hierarchy $T_n = 2^n T_0$ (or $T_p = pT_0$) induce p-adic coupling constant evolution and explain why p-adic length scales correspond to $L_p \propto \sqrt{p}R$, $p \simeq 2^k$, R CP_2 length scale? This looks attractive but there is a problem. p-Adic length scales come as powers of $\sqrt{2}$ rather than 2 and the strongly favored values of k are primes and thus odd so that $n = k/2$ would be half odd integer. This problem can be solved.

1. The observation that the distance traveled by a Brownian particle during time t satisfies $r^2 = Dt$ suggests a solution to the problem. p-Adic thermodynamics applies because the partonic 3-surfaces X^2 are as 2-D dynamical systems random apart from light-likeness of their orbit. For CP_2 type vacuum extremals the situation reduces to that for a one-dimensional random light-like curve in M^4. The orbits of Brownian particle would now correspond to light-like geodesics γ_3 at X^3. The projection of γ_3 to a time=constant section $X^2 \subset X^3$ would define the 2-D path γ_2 of the Brownian particle. The M^4 distance r between the end points of γ_2 would be given $r^2 = Dt$. The favored values of t would correspond to $T_n = 2^n T_0$ (the full light-like geodesic). p-Adic length scales would result as $L^2(k) = DT(k) = D2^k T_0$ for $D = R^2/T_0$. Since only CP_2 scale is available as a fundamental scale, one would have $T_0 = R$ and $D = R$ and $L^2(k) = T(k)R$.
2. p-Adic primes near powers of 2 would be in preferred position. p-Adic time scale would not relate to the p-adic length scale via $T_p = L_p/c$ as assumed implicitly earlier but via $T_p = L_p^2/R_0 = \sqrt{p}L_p$, which corresponds to secondary p-adic length scale. For instance, in the case of electron with $p = M_{127}$ one would have $T_{127} = .1$ second which defines a fundamental biological rhythm. Neutrinos with mass around .1 eV would correspond to $L(169) \simeq 5$ μm (size of a small cell) and $T(169) \simeq 1. \times 10^4$ years. A deep connection between elementary particle physics and biology becomes highly suggestive.
3. In the proposed picture the p-adic prime $p \simeq 2^k$ would characterize the thermodynamics of the random motion of light-like geodesics of X^3 so that p-adic prime p would indeed be an inherent property of X^3. For $T_p = pT_0$ the above argument is not enough for p-adic length scale hypothesis and p-adic length scale hypothesis might be seen as an outcome of a process analogous to natural selection. Resonance like effect favoring octaves of a fundamental frequency might be in question. In this case, p would a property of CD and all light-like 3-surfaces inside it and also that corresponding sector of WCW.

The above proposal involves of course ad hoc elements and can be seen only as a first attempt to understand what is involved. Later a more refined approach will be discussed.

4. Mersenne primes and Gaussian Mersennes

The generalization of the embedding space required by the postulated hierarchy of Planck constants [K38] means a book like structure for which the pages are products of singular coverings or factor spaces of CD (causal diamond defined as intersection of future and past directed light-cones) and of CP_2 [K38] . This predicts that Planck constants are rationals and that a given value of Planck constant corresponds to an infinite number of different pages of the Big Book, which might be seen as a drawback. If only singular covering spaces are allowed the values of Planck constant are products of integers and given value of Planck constant corresponds to a finite number of pages given by the number of decompositions of the integer to two different integers. The definition of the book like structure assigns to a given CD preferred quantization axes and so that quantum measurement has direct correlate at the level of moduli space of CDs.

TGD inspired quantum biology and number theoretical considerations suggest preferred values $h_{eff}/h = n$, n as integer. Ruler and compass integers defined by the products of distinct Fermat primes and power of two are number theoretically favored values for these integers because the phases $exp(i2\pi/n)$ in this case are number theoretically very simple and should have emerged first in the number theoretical evolution via algebraic extensions of p-adics and of rationals. p-Adic length scale hypothesis favors powers of two as values of n.

One can however ask whether a more precise characterization of preferred Mersennes could exist and whether there could exists a stronger correlation between hierarchies of p-adic length scales and Planck constants. Mersenne primes $M_k = 2^k - 1$, $k \in \{89, 107, 127\}$, and Gaussian Mersennes $M_{G,k} = (1+i)k-1$, $k \in \{113, 151, 157, 163, 167, 239, 241..\}$ are expected to be physically highly interesting and up to $k = 127$ indeed correspond to elementary particles. The number theoretical miracle is that all the four p-adic length scales with $k \in \{151, 157, 163, 167\}$ are in the biologically highly interesting range 10 nm-2.5 μm). The question has been whether these define scaled up copies of electro-weak and QCD type physics with ordinary value of h_{eff}. The proposal that this is the case and that these physics are in a well-defined sense induced by the dark scaled up variants of corresponding lower level physics leads to a prediction for the preferred values of $r = 2^{k_d}$, $k_d = k_i - k_j$.

Dark variant of exotic nuclear physics implies exotic physics with ordinary value of Planck constant in the new scale in a resonant manner: dark gauge bosons transform to their ordinary variants with the same Compton length. This transformation is natural since in length scales below the Compton length the gauge bosons behave as massless and free particles. As a consequence, lighter variants of weak bosons emerge and QCD confinement scale becomes longer.

This proposal will be referred to as Mersenne hypothesis. It leads to strong predictions about EEG [K35] since it predicts a spectrum of preferred Josephson frequencies for a given value of membrane potential and also assigns to a given value of h_{eff} a fixed size scale having interpretation as the size scale of the body part or magnetic body. Also a vision about evolution of life emerges. Mersenne hypothesis is especially interesting as far as new physics in condensed matter length scales is considered: this includes exotic scaled up variants of the ordinary nuclear physics and their dark variants. Even dark nucleons are possible and this gives justification for the model of dark nucleons predicting the counterparts of DNA,RNA, tRNa, and amino-acids as well as realization of vertebrate genetic code [K115] .

These exotic nuclear physics with ordinary value of Planck constant could correspond to ground states that are almost vacuum extremals corresponding to homologically trivial geodesic sphere of CP_2 near criticality to a phase transition changing Planck constant. Ordinary nuclear physics would correspond to homologically non-trivial geodesic sphere and far from vacuum extremal property. For vacuum extremals of this kind classical Z^0 field proportional to electromagnetic field is present and this modifies dramatically the view about cell membrane as Josephson junction. The model for cell membrane as almost vacuum extremal indeed led to a quantitative breakthrough in TGD inspired model of EEG and is therefore something to be taken seriously. The safest option concerning empirical facts is that the copies of electro-weak and color physics with ordinary value of Planck constant are possible only for almost vacuum extremals - that is at criticality against phase transition changing Planck constant.

The origin of the preferred p-adic length scales

This question was posed already for two decades ago has but remained without a convincing answer. Quite recently however the number theoretical vision allowed to understand both the origin of preferred p-adic number fields and the emergence of p-adic length scale hypothesis in a generalized form. Preferred primes are near but below powers prime which can be also larger that

$p = 2$.

The preferred primes correspond to so called ramified rational primes, which split in to products of the primes of the extension. If some prime appears as higher than first power, one has ramification. The number of ramified primes is finite.

In strong form of holography p-adic continuations of 2-surfaces to preferred extrmals identifiable as imaginations would be easy due to the existence of p-adic pseudo-constants. The continuation could fail for most configurations of partonic 2-surfaces and string world sheets in the real sector: the interpretation would be that some space-time surfaces can be imagined but not realized [K73]. For certain extensions the number of realizable imaginations could be exceptionally large. These extensions would be winners in the number theoretic fight for survivalandcorresponding ramified primes would be preferred p-adic primes. Whether the preferred primes satisfy p-adic length scale hypothesis or its generalization from $p = 2$ to small primes remains an open question.

The value of effective Planck constant $h_{eff}/h = n$ corresponds to the number of sheets of some kind of covering space defined by the space-time surface. The discretization of the space-time surface identified as a monadic manifold [L25] with embedding space preferred coordinates in extension of rationals defining the adele has Galois group of extension as a group of symmetries permuting the sheets of the covering group. Therefore $n = h_{eff}/h$ would naturally correspond to the dimension of the extension dividing the order of its Galois group.

Weak form of NMP allows to understand the emergence of preferred p-adic length scales. NMP favors ramified primes, for which the integer n is power of single prime p. If n is a prime slightly below $n_{max} = p^n$ defining the dimension of the sub-space corresponding to maximal negentropy gain, weak form of NMP favors its selection since the p-adic topology is farthest from the discrete topology assignable to formal p-adic topology characterized by $p = 1$ [K119].

p-Adic physics and the notion of finite measurement resolution

Canonical identification mapping p-adic numbers to reals in a continuous manner plays a key role in some applications of TGD and together with the discretization necessary to define the p-adic variants of integration and harmonic analysis suggests that p-adic topology identified as an effective topology could provide an elegant manner to characterize finite measurement resolution.

1. Finite measurement resolution can be characterized as an interval of minimum length. Below this length scale one cannot distinguish points from each other. A natural definition for this inability could be as an inability to well-order the points. The real topology is too strong in the modelling in kind of situation since it brings in large amount of processing of pseudo information whereas p-adic topology which lacks the notion of well-ordering could be more appropriate as effective topology and together with a pinary cutoff could allow to get rid of the irrelevant information.
2. This suggest that canonical identification applies only inside the intervals defining finite measurement resolution in a given discretization of the space considered by say small cubes. The canonical identification is unique only modulo diffeomorphism applied on both real and p-adic side but this is not a problem since this would only reflect the absence of the well-ordering lost by finite measurement resolution. Also the fact that the map makes sense only at positive real axis would be natural if one accepts this identification.

This interpretation would suggest that there is an infinite hierarchy of measurement resolutions characterized by the value of the p-adic prime. This would mean quite interesting refinement of the notion of finite measurement resolution. At the level of quantum theory it could be interpreted as a maximization of p-adic entanglement negentropy as a function of the p-adic prime. Perhaps one might say that there is a unique p-adic effective topology allowing to maximize the information content of the theory relying on finite measurement resolution.

p-Adic numbers and the analogy of TGD with spin-glass

The vacuum degeneracy of the Kähler action leads to a precise spin glass analogy at the level of the WCW geometry and the generalization of the energy landscape concept to TGD context leads to the hypothesis about how p-adicity could be realized at the level of WCW. Also the concept of p-adic space-time surface emerges rather naturally.

1. Spin glass briefly

The basic characteristic of the spin glass phase [B12] is that the direction of the magnetization varies spatially, being constant inside a given spatial region, but does not depend on time. In the real context this usually leads to large surface energies on the surfaces at which the magnetization direction changes. Regions with different direction of magnetization clearly correspond non-vacuum regions separated by almost vacuum regions. Amusingly, if 3-space is effectively p-adic and if magnetization direction is p-adic pseudo constant, no surface energies are generated so that p-adics might be useful even in the context of the ordinary spin glasses.

Spin glass phase allows a great number of different ground states minimizing the free energy. For the ordinary spin glass, the partition function is the average over a probability distribution of the coupling constants for the partition function with Hamiltonian depending on the coupling constants. Free energy as a function of the coupling constants defines "energy landscape" and the set of free energy minima can be endowed with an ultra-metric distance function using a standard construction [A78] .

2. Vacuum degeneracy of Kähler action

The Kähler action defining WCW geometry allows enormous vacuum degeneracy: any four-surface for which the induced Kähler form vanishes, is an extremal of the Kähler action. Induced Kähler form vanishes if the CP_2 projection of the space-time surface is Lagrangian manifold [A13] of CP_2: these manifolds are at most two-dimensional and any canonical transformation of CP_2 creates a new Lagrangian sub-manifold [A13] . An explicit representation for Lagrangian sub-manifolds is obtained using some canonical coordinates P_i, Q_i for CP_2: by assuming

$$P_i = \partial_i f(Q_1, Q_2) \ \ , \ \ i = 1, 2 \ \ ,$$

where f arbitrary function of its arguments. One obtains a 2-dimensional sub-manifold of CP_2 for which the induced Kähler form proportional to $dP_i \wedge dQ^i$ vanishes. The roles of P_i and Q_i can obviously be interchanged. A familiar example of Lagrange manifolds are $p_i = constant$ surfaces of the ordinary (p_i, q_i) phase space.

Since vacuum degeneracy is removed only by the classical gravitational interaction there are good reasons to expect large ground state degeneracy, when the system corresponds to a small deformation of a vacuum extremal. This degeneracy is very much analogous to the ground state degeneracy of spin glass but is 4-dimensional.

3. Vacuum degeneracy of the Kähler action and physical spin glass analogy

Quite generally, the dynamical reason for the physical spin glass degeneracy is the fact that Kähler action has a huge vacuum degeneracy. Any 4-surface with CP_2 projection, which is a Lagrangian sub-manifold (generically two-dimensional), is vacuum extremal. This implies that space-time decomposes into non-vacuum regions characterized by non-vanishing Kähler magnetic and electric fields such that the (presumably thin) regions between the the non-vacuum regions are vacuum extremals. Therefore no surface energies are generated. Also the fact that various charges and momentum and energy can flow to larger space-time sheets via wormholes is an important factor making possible strong field gradients without introducing large surfaces energies. From a given preferred extremal of Kähler action one obtains a new one by adding arbitrary space-time surfaces which is vacuum extremal and deforming them.

The symplectic invariance of the Kähler action for vacuum extremals allows a further understanding of the vacuum degeneracy. The presence of the classical gravitational interaction spoils the canonical group $Can(CP_2)$ as gauge symmetries of the action and transforms it to the isometry group of CH. As a consequence, the $U(1)$ gauge degeneracy is transformed to a spin glass type degeneracy and several, perhaps even infinite number of maxima of Kähler function become possible. Given sheet has naturally as its boundary the 3-surfaces for which two maxima of the Kähler function coalesce or are created from single maximum by a cusp catastrophe [A48] . In catastrophe regions there are several sheets and the value of the maximum Kähler function determines which give a measure for the importance of various sheets. The quantum jumps selecting one of these sheets can be regarded as phase transitions.

In TGD framework classical non-determinism forces to generalize the notion of the 3-surface by replacing it with a sequence of space like 3-surfaces having time like separations such that the sequence characterizes uniquely one branch of multi-furcation. This characterization works when non-determinism has discrete nature. For CP_2 type extremals which are bosonic vacua, basic objects are essentially four-dimensional since M^4_+ projection of CP_2 type extremal is random light like curve. This effective four-dimensionality of the basic objects makes it possible to topologize Feynman diagrammatics of quantum field theories by replacing the lines of Feynman diagrams with CP_2 type extremals.

In TGD framework spin glass analogy holds true also in the time direction, which reflects the fact that the vacuum extremals are non-deterministic. For instance, by gluing vacuum extremals with a finite space-time extension (also in time direction!) to a non-vacuum extremal and deforming slightly, one obtains good candidates for the degenerate preferred extremals. This non-determinism is expected to make the preferred extremals of the Kähler action highly degenerate. The construction of S-matrix at the high energy limit suggests that since a localization selecting one degenerate maximum occurs, one must accept as a fact that each choice of the parameters corresponds to a particular S-matrix and one must average over these choices to get scattering rates. This averaging for scattering rates corresponds to the averaging over the thermodynamical partition functions for spin glass. A more general is that one allows final state wave functions to depend on the zero modes which affect S-matrix elements: in the limit that wave functions are completely localized, one ends up with the simpler scenario.

4. p-Adic non-determinism and spin glass analogy

One must carefully distinguish between cognitive and physical spin-glass analogy. Cognitive spin-glass analogy is due to the p-adic non-determinism. p-Adic pseudo constants induce a non-determinism which essentially means that p-adic extrema depend on the p-adic pseudo constants which depend on a finite number of positive pinary digits of their arguments only. Thus p-adic extremals are glued from pieces for which the values of the integration constants are genuine constants. Obviously, an optimal cognitive representation is achieved if pseudo constants reduce to ordinary constants.

More precisely, any function

$$\begin{aligned} f(x) &= f(x_N) \ , \\ x_N &= \sum_{k\leq N} x_k p^k \ , \end{aligned} \tag{9.2.1}$$

which does not depend on the pinary digits x_n, $n > N$ has a vanishing p-adic derivative and is thus a pseudo constant. These functions are piecewise constant below some length scale, which in principle can be arbitrary small but finite. The result means that the constants appearing in the solutions the p-adic field equations are constants functions only below some length scale. For instance, for linear differential equations integration constants are arbitrary pseudo constants. In particular, the p-adic counterparts of the preferred extremals are highly degenerate because of the presence of the pseudo constants. This in turn means a characteristic randomness of the spin glass also in the time direction since the surfaces at which the pseudo constants change their values do not give rise to infinite surface energy densities as they would do in the real context.

The basic character of cognition would be spin glass like nature making possible "engineering" at the level of thoughts (planning) whereas classical non-determinism of the Kähler action would make possible "engineering" at the level of the real world.

Life as islands of rational/algebraic numbers in the seas of real and p-adic continua?

The possibility to define entropy differently for rational/algebraic entanglement and the fact that number theoretic entanglement entropy can be negative raises the question about which kind of systems can possess this kind of entanglement. I have considered several identifications but the most elegant interpretation is based on the idea that living matter resides in the intersection of real and p-adic worlds, somewhat like rational numbers live in the intersection of real and p-adic number fields. This intersection would be number theoretically universal in the sense that algebraic extension of rationals would be the number field but in rather abstract sense: for the parameters defining the WCW coordinates characterizing space-time surface rather than points of space-time surface.

The observation that Shannon entropy allows an infinite number of number theoretic variants for which the entropy can be negative in the case that probabilities are algebraic numbers leads to the idea that living matter in a well-defined sense corresponds to the intersection of real and p-adic worlds. This would mean that the mathematical expressions for the space-time surfaces (or at least 3-surfaces or partonic 2-surfaces and their 4-D tangent planes) make sense in both real and p-adic sense for some primes p. Same would apply to the expressions defining quantum states. In particular, entanglement probabilities would be rationals or algebraic numbers so that entanglement can be negentropic and the formation of bound states in the intersection of real and p-adic worlds generates information and is thus favored by NMP.

This picture has also a direct connection with consciousness.

1. The generation of non-rational (non-algebraic) bound state entanglement between the system and external world means that the system loses consciousness during the state function reduction process following the U-process generating the entanglement. What happens that the Universe corresponding to given CD decomposes to two un-entangled subsystems, which in turn decompose, and the process continues until all subsystems have only entropic bound state entanglement or negentropic algebraic entanglement with the external world.
2. If the sub-system generates entropic bound state entanglement in the process, it loses consciousness. Note that the entanglement entropy of the sub-system is a sum over entanglement entropies over all subsystems involved. This hierarchy of subsystems corresponds to the hierarchy if sub-CDs so that the survival without a loss of consciousness depends on what happens at all levels below the highest level for a given self. In more concrete terms, ability to stay conscious depends on what happens at cellular level too. The stable evolution of systems having algebraic entanglement is expected to be a process proceeding from short to long length scales as the evolution of life indeed is.
3. U-process generates a superposition of states in which any sub-system can have both real and algebraic entanglement with the external world. This would suggest that the choice of the type of entanglement is a volitional selection. A possible interpretation is as a choice between good and evil. The hedonistic complete freedom resulting as the entanglement entropy is reduced to zero on one hand, and the algebraic bound state entanglement implying correlations with the external world and meaning giving up the maximal freedom on the other hand. The hedonistic option is risky since it can lead to non-algebraic bound state entanglement implying a loss of consciousness. The second option means expansion of consciousness - a fusion to the ocean of consciousness as described by spiritual practices.
4. This formulation means a sharpening of the earlier statement "Everything is conscious and consciousness can be only lost" with the additional statement "This happens when non-algebraic bound state entanglement is generated and the system does not remain in the intersection of real and p-adic worlds anymore". Clearly, the quantum criticality of TGD Universe seems has very many aspects and life as a critical phenomenon in the number theoretical sense is only one of them besides the criticality of the space-time dynamics and the criticality with respect to phase transitions changing the value of Planck constant and other more familiar criticalities. How closely these criticalities relate remains an open question.

A good guess is that algebraic entanglement is essential for quantum computation, which therefore might correspond to a conscious process. Hence cognition could be seen as a quantum computation like process, a more appropriate term being quantum problem solving. Living-dead dichotomy could correspond to rational-irrational or to algebraic-transcendental dichotomy: this at least when life is interpreted as intelligent life. Life would in a well defined sense correspond to islands of rationality/algebraicity in the seas of real and p-adic continua.

The view about the crucial role of rational and algebraic numbers as far as intelligent life is considered, could have been guessed on very general grounds from the analogy with the orbits of a dynamical system. Rational numbers allow a predictable periodic decimal/pinary expansion and are analogous to one-dimensional periodic orbits. Algebraic numbers are related to rationals by a finite number of algebraic operations and are intermediate between periodic and chaotic orbits allowing an interpretation as an element in an algebraic extension of any p-adic number field. The projections of the orbit to various coordinate directions of the algebraic extension represent now periodic orbits. The decimal/pinary expansions of transcendentals are un-predictable being analogous to chaotic orbits. The special role of rational and algebraic numbers was realized already by Pythagoras, and the fact that the ratios for the frequencies of the musical scale are rationals supports the special nature of rational and algebraic numbers. The special nature of the Golden Mean, which involves $\sqrt{5}$, conforms the view that algebraic numbers rather than only rationals are essential for life.

p-Adic physics as physics of cognition

The vision about p-adic physics as physics of cognition has gradually established itself as one of the key idea of TGD inspired theory of consciousness. There are several motivations for this idea.

The strongest motivation is the vision about living matter as something residing in the intersection of real and p-adic worlds. One of the earliest motivations was p-adic non-determinism identified tentatively as a space-time correlate for the non-determinism of imagination. p-Adic non-determinism follows from the fact that functions with vanishing derivatives are piecewise constant functions in the p-adic context. More precisely, p-adic pseudo constants depend on the pinary cutoff of their arguments and replace integration constants in p-adic differential equations. In the case of field equations this means roughly that the initial data are replaced with initial data given for a discrete set of time values chosen in such a way that unique solution of field equations results. Solution can be fixed also in a discrete subset of rational points of the embedding space. Presumably the uniqueness requirement implies some unique pinary cutoff. Thus the space-time surfaces representing solutions of p-adic field equations are analogous to space-time surfaces consisting of pieces of solutions of the real field equations. p-Adic reality is much like the dream reality consisting of rational fragments glued together in illogical manner or pieces of child's drawing of body containing body parts in more or less chaotic order.

The obvious looking interpretation for the solutions of the p-adic field equations is as a geometric correlate of imagination. Plans, intentions, expectations, dreams, and cognition in general are expected to have p-adic space-time sheets as their geometric correlates. This in the sense that p-adic space-time sheets somehow initiate the real neural processes providing symbolic counterparts for the cognitive representations provided by p-adic space-time sheets and p-adic fermions. A deep principle seems to be involved: incompleteness is characteristic feature of p-adic physics but the flexibility made possible by this incompleteness is absolutely essential for imagination and cognitive consciousness in general.

Although p-adic space-time sheets as such are not conscious, p-adic physics would provide beautiful mathematical realization for the intuitions of Descartes. The formidable challenge is to develop experimental tests for p-adic physics. The basic problem is that we can perceive p-adic reality only as "thoughts" unlike the "real" reality which represents itself to us as sensory experiences. Thus it would seem that we should be able generalize the physics of sensory experiences to physics of cognitive experiences.

9.2.3 What Is The Correspondence Between P-Adic And Real Numbers?

There must be some kind of correspondence between reals and p-adic numbers. This correspondence can depend on context. In p-adic mass calculations one must map p-adic mass squared values to real numbers in a continuous manner and canonical identification $x = \sum x_n p^n \to Id(x) = \sum x_n p^{-n}$ is a natural first guess. Also p-adic probabilities could be mapped to their real counterparts by a suitable normalization. The minimalistic interpretation is that real and p-adic mass calculations must give same results- physics must be consistent with the existence of cognitive representations of it. In this case p-adic thermodynamics would constrain the temperature and scale parameters of real thermodynamics.

The possible existence and the nature of the correspondence at the level of embedding space and space-time surfaces is much more questionable and it is far from clear whether it is needed as a naïve map of real space-time points to p-adic space-time points by - say - canonical identification: the problem would be that symmetries are not respected if one demands continuity. One would like to various symmetries in real and p-adic variants and the correspondence should respect symmetries.

One can wonder whether p-adic valued S-matrices have any physical meaning and whether they could be obtained as algebraic continuation from a number theoretically universal S-matrix whose matrix elements are algebraic numbers allowing an interpretation as real or p-adic numbers in suitable algebraic extension: this would pose extremely strong constraints on S-matrix. If one wants to introduce p-adic physics at space-time level one must be able to relate p-adic and real space-time regions to each other. The identification along common rational points of real and various p-adic variants of the embedding space produces however problems with symmetries.

In the following these questions are discussed as I did them before the recent steps of progress summarized in the last subsection. I hope that the reader can forgive certain naïvete of the discussion: pioneering work is in question.

Generalization of the number concept

The recent view about the unification of real and p-adic physics is based on the generalization of number concept obtained by fusing together real and p-adic number fields along common rationals (see **Fig. ??**in the Appendix.

1. Rational numbers as numbers common to all number fields

The unification of real physics of material work and p-adic physics of cognition leads to the generalization of the notion of number field. Reals and various p-adic number fields are glued along common algebraic numbers defining an extension of p-adic numbers to form a fractal book like structure. Allowing all possible finite-dimensional algebraic and perhaps even transcendental extensions of rationals inducing those of p-adic numbers adds additional pages to this "Big Book".

This suggests a generalization of the notion of manifold as real manifold and its p-adic variants glued together along common points. This generalization might make sense under very high symmetries and that it is safest to lean strongly on the physical picture provided by quantum TGD. This construction is discussed in [K122] and one must make clear that it is plagued difficulties with symmetries.

1. The most natural guess is that the coordinates of common points are rational or in some algebraic extension of rational numbers. General coordinate invariance and preservation of symmetries require preferred coordinates existing when the manifold has maximal number of isometries. This approach might make sense in the case of linear spaces- in particular Minkowski space M^4. The natural coordinates are in this case linear Minkowski coordinates. The choice of coordinates is however not completely unique and has interpretation as a geometric correlate for the choice of quantization axes for a given CD. Different choices are not equivalent.

2. As will be found, the need to have a well-defined integration based on Fourier analysis (or its generalization to harmonic analysis [A8] in symmetric spaces) poses very strong constraints and allows p-adicization only if the space has maximal symmetries. Fourier analysis requires the introduction of an algebraic extension of p-adic numbers containing sufficiently many roots of unity.

   (a) This approach is especially natural in the case of compact symmetric spaces such as CP_2 [A6] .

   (b) Also symmetric spaces such the 3-D proper time $a = constant$ hyperboloid of M^4-call it $H(a)$ -allowing Lorentz group as isometries allows a p-adic variant utilizing the hyperbolic counterparts for the roots of unity. $M^4 \times H(a = 2^n a_0)$ appears as a part of the moduli space of CDs.

   (c) For light-cone boundaries associated with CDs $SO(3)$ invariant radial coordinate r_M defining the radius of sphere S^2 defines the hyperbolic coordinate and angle coordinates of S^2 would correspond to phase angles and $M^4_\pm$ projections for the common points of real and p-adic variants of partonic 2-surfaces would be this kind of points. Same applies to CP_2 projections.
   In the "intersection of real and p-adic worlds" real and p-adic partonic 2-surfaces would obey same algebraic equations and would be obtained by an algebraic continuation from the corresponding equations making sense in the discrete variant of $M^4_\pm \times CP_2$. This

connection with discrete sub-manifold geometries means very powerful constraints on the partonic 2-surfaces in the intersection.

3. The common algebraic points of real and p-adic variant of the manifold form a discrete space but one could identify the p-adic counterpart of the real discretization intervals $(0, 2\pi/N)$ for angle like variables as p-adic numbers of norm smaller than 1 using canonical identification or some variant of it. Same applies to the the hyperbolic counterpart of this interval. The non-uniqueness of this map could be interpreted in terms of a finite measurement resolution. In particular, the condition that WCW allows Kähler geometry requires a decomposition to a union of symmetric spaces so that there are good hopes that p-adic counterpart is analogous to that assigned to CP_2.

This approach works for probabilities but has serious problems with symmetries. The only manner to circumvent the problems is based on strong form of holography and abstraction of the real-p-adic correspondence so that it is not anymore local but maps entire surfaces to each other. One must have also now discretization and co-dimension two rule holds true. For instance, space-time surfaces are replaced with a collection of 2-D objects and partonic 2-surfaces by a discrete set of points. This rule is equivalent with strong from of holography.

The correspondence would be at the level of parameters defining WCW coordinates and intersection of reality and p-adicities would consist of discrete set of 2-surfaces. As already explained, strong form of holography suggests that real and p-adic space-time sheets are obtained by continuation of the 2-surfaces to preferred extremals by assuming that the classical Noether charges associated with super-symplectic algebra vanish for the 3-surfaces at the ends of space-time surface. By conformal invariance the parameters would be naturally general coordinate invariant conformal moduli for the 2-surfaces involved, and belong to the algebraic extension of rationals in the intersection. Their continuation to various number fields would give real and p-adic space-time sheets. Also scattering amplitudes could be constructed using the data assigned with 2-surfaces in the intersection and continued algebraically to various number fields. This picture conforms also with the recipe for constructing scattering amplitudes in twistor approach [L8].

2. How large p-adic space-time sheets can be?

Space-time region having finite size in the real sense can have arbitrarily large size in p-adic sense and vice versa. This raises a rather thought provoking questions. Could the p-adic space-time sheets have cosmological or even infinite size with respect to the real metric but have be p-adically finite? How large space-time surface is responsible for the p-adic representation of my body? Could the large or even infinite size of the cognitive space-time sheets explain why creatures of a finite physical size can invent the notion of infinity and construct cosmological theories? Could it be that pinary cutoff $O(p^n)$ defining the resolution of a p-adic cognitive representation would define the size of the space-time region needed to realize the cognitive representation?

These questions make sense if the real-padic correspondence is local - that is defined by the intersection real and p-adic space-time surfaces. In the more abstract approach it does not make sense.

In fact, the mere requirement that the neighborhood of a point of the p-adic space-time sheet contains points, which are p-adically infinitesimally near to it can mean that points infinitely distant from this point in the real sense are involved. A good example is provided by an integer valued point $x = n < p$ and the point $y = x + p^m$, $m > 0$: the p-adic distance of these points is p^{-m} whereas at the limit $m \to \infty$ the real distance goes as p^m and becomes infinite for infinitesimally near points. The points $n + y$, $y = \sum_{k>0} x_k p^k$, $0 < n < p$, form a p-adically continuous set around $x = n$. In the real topology this point set is discrete set with a minimum distance $\Delta x = p$ between neighboring points whereas in the p-adic topology every point has arbitrary nearby points. There are also rationals, which are arbitrarily near to each other both p-adically and in the real sense. Consider points $x = m/n$, m and n not divisible by p, and $y = (m/n) \times (1 + p^k r)/(1 + p^k s)$, $s = r + 1$ such that neither r or s is divisible by p and $k >> 1$ and $r >> p$. The p-adic and real distances are $|x - y|_p = p^{-k}$ and $|x - y| \simeq (m/n)/(r + 1)$ respectively. By choosing k and r large enough the points can be made arbitrarily close to each other both in the real and p-adic senses.

The idea about astrophysical size of the p-adic cognitive space-time sheets providing representation of body and brain is consistent with TGD inspired theory of consciousness, which forces to take very seriously the idea that even human consciousness involves astrophysical length scales.

It must be however emphasized that this kind of concretization seems to be un-necessary if the correspondence is at the level of WCW.

3. Generalization of complex analysis

One general idea which results as an outcome of the generalized notion of number is the idea of a universal function continuable from a function mapping rationals to rationals or to a finite extension of rationals to a function in any number field. This algebraic continuation is analogous to the analytical continuation of a real analytic function to the complex plane. Rational functions for which polynomials have rational coefficients are obviously functions satisfying this constraint. Algebraic functions for which polynomials have rational coefficients satisfy this requirement if appropriate finite-dimensional algebraic extensions of p-adic numbers are allowed.

For instance, one can ask whether residue calculus might be generalized so that the value of an integral along the real axis could be calculated by continuing it instead of the complex plane to any number field via its values in the subset of rational numbers forming the back of the book like structure (in very metaphorical sense) having number fields as its pages. If the poles of the continued function in the finitely extended number field allow interpretation as real numbers it might be possible to generalize the residue formula. One can also imagine of extending residue calculus to any algebraic extension. An interesting situation arises when the poles correspond to extended p-adic rationals common to different pages of the "Big Book". Could this mean that the integral could be calculated at any page having the pole common. In particular, could a p-adic residue integral be calculated in the ordinary complex plane by utilizing the fact that in this case numerical approach makes sense. Contrary to the first expectations the algebraically continued residue calculus does not seem to have obvious applications in quantum TGD.

Canonical identification

Canonical There exists a natural continuous map $Id : R_p \rightarrow R_+$ from p-adic numbers to non-negative real numbers given by the "pinary" expansion of the real number for $x \in R$ and $y \in R_p$ this correspondence reads

$$\begin{aligned} y &= \sum_{k>N} y_k p^k \rightarrow x = \sum_{k<N} y_k p^{-k} \ , \\ y_k &\in \{0,1,..,p-1\} \ . \end{aligned} \tag{9.2.2}$$

This map is continuous as one easily finds out. There is however a little difficulty associated with the definition of the inverse map since the pinary expansion like also desimal expansion is not unique ($1 = 0.999...$) for the real numbers x, which allow pinary expansion with finite number of pinary digits

$$\begin{aligned} x &= \sum_{k=N_0}^{N} x_k p^{-k} \ , \\ x &= \sum_{k=N_0}^{N-1} x_k p^{-k} + (x_N - 1)p^{-N} + (p-1)p^{-N-1} \sum_{k=0,..} p^{-k} \ . \end{aligned} \tag{9.2.3}$$

The p-adic images associated with these expansions are different

$$\begin{aligned} y_1 &= \sum_{k=N_0}^{N} x_k p^k \ , \\ y_2 &= \sum_{k=N_0}^{N-1} x_k p^k + (x_N - 1)p^N + (p-1)p^{N+1} \sum_{k=0,..} p^k \\ &= y_1 + (x_N - 1)p^N - p^{N+1} \ , \end{aligned} \tag{9.2.4}$$

so that the inverse map is either two-valued for p-adic numbers having expansion with finite number of pinary digits or single valued and discontinuous and non-surjective if one makes pinary expansion unique by choosing the one with finite number of pinary digits. The finite number of pinary digits expansion is a natural choice since in the numerical work one always must use a pinary cutoff on the real axis.

1. Canonical identification is a continuous map of non-negative reals to p-adics

The topology induced by the inverse of the canonical identification map in the set of positive real numbers differs from the ordinary topology. The difference is easily understood by interpreting the p-adic norm as a norm in the set of the real numbers. The norm is constant in each interval $[p^k, p^{k+1})$ (see **Fig. A-6.1**) and is equal to the usual real norm at the points $x = p^k$: the usual linear norm is replaced with a piecewise constant norm. This means that p-adic topology is coarser than the usual real topology and the higher the value of p is, the coarser the resulting topology is above a given length scale. This hierarchical ordering of the p-adic topologies will be a central feature as far as the proposed applications of the p-adic numbers are considered.

Ordinary continuity implies p-adic continuity since the norm induced from the p-adic topology is rougher than the ordinary norm. This allows two alternative interpretations. Either p-adic image of a physical systems provides a good representation of the system above some pinary cutoff or the physical system can be genuinely p-adic below certain length scale L_p and become in good approximation real, when a length scale resolution L_p is used in its description. The first interpretation is correct if canonical identification is interpreted as a cognitive map. p-Adic continuity implies ordinary continuity from right as is clear already from the properties of the p-adic norm (the graph of the norm is indeed continuous from right, see **Fig. A-6.1** of Appendix). This feature is one clear signature of the p-adic topology.

If one considers seriously the application of canonical identification to basic quantum TGD one cannot avoid the question about the p-adic counterparts of the negative real numbers. There is no satisfactory manner to circumvent the fact that canonical images of p-adic numbers are naturally non-negative. This is not a problem if canonical identification applies only to the coordinate interval $(0, 2\pi/N)$ or its hyperbolic variant defining the finite measurement resolution. That p-adicization program works only for highly symmetric spaces is not a problem from the point of view of TGD.

2. Canonical identification relates p-adic and real statistical physics

p-Adic mass calculations based on p-adic thermodynamics were the first and rather successful application of the p-adic physics (see the four chapters in [K70] . The essential element of the approach was the replacement of the Boltzmann weight $e^{-E/T}$ with its p-adic generalization p^{L_0/T_p}, where L_0 is the Virasoro generator corresponding to scaling and representing essentially mass squared operator instead of energy. T_p is inverse integer valued p-adic temperature. The predicted mass squared averages were mapped to real numbers by canonical identification.

One could also construct a real variant of this approach by considering instead of the ordinary Boltzmann weights the weights p^{-L_0/T_p}. The quantization of temperature to $T_p = log(p)/n$ would be a completely ad hoc assumption. In the case of real thermodynamics all particles are predicted to be light whereas in case of p-adic thermodynamics particle is light only if the ratio for the degeneracy of the lowest massive state to the degeneracy of the ground state is integer. Immense number of particles disappear from the spectrum of light particles by this criterion. For light particles the predictions are same as of p-adic thermodynamics in the lowest non-trivial order but in the next order deviations are possible.

Also p-adic probabilities and the p-adic entropy can be mapped to real numbers by canonical identification. The general idea is that a faithful enough cognitive representation of the real physics can by the number theoretical constraints involved make predictions, which would be extremely difficult to deduce from real physics.

3. Variant of canonical identification commuting with division of integers

The basic problems of canonical identification is that it does not respect unitarity. For this reason it is not well suited for relating p-adic and real scattering amplitudes. The problem of the correspondence via direct rationals or roots of unity is that it does not respect continuity. The restriction of S-matrix to a discrete intersection of real and p-adic worlds is one manner to solve this difficulty.

One can also consider alternative approach to achieve a compromise between algebra and topology achieved by using a modification of canonical identification $I_{R_p \rightarrow R}$ defined as $I_1(r/s) = I(r)/I(s)$. If the conditions $r \ll p$ and $s \ll p$ hold true, the map respects algebraic operations and also unitarity and various symmetries. It seems that this option must be used to relate p-adic transition amplitudes to real ones and vice versa [K66] . In particular, real and p-adic coupling constants are related by this map. Also some problems related to p-adic mass calculations find a nice resolution when I_1 is used.

This variant of canonical identification is not equivalent with the original one using the infinite expansion of q in powers of p since canonical identification does not commute with product and division. The variant is however unique in the recent context when r and s in $q = r/s$ have no common factors. For integers $n < p$ it reduces to direct correspondence.

Generalized numbers would be regarded in this picture as a generalized manifold obtained by gluing different number fields together along rationals. Instead of a direct identification of real and p-adic rationals, the p-adic rationals in R_p are mapped to real rationals (or vice versa) using a variant of the canonical identification $I_{R \rightarrow R_p}$ in which the expansion of rational number $q = r/s = \sum r_n p^n / \sum s_n p^n$ is replaced with the rational number $q_1 = r_1/s_1 = \sum r_n p^{-n} / \sum s_n p^{-n}$ interpreted as a p-adic number:

$$q = \frac{r}{s} = \frac{\sum_n r_n p^n}{\sum_m s_n p^n} \rightarrow q_1 = \frac{\sum_n r_n p^{-n}}{\sum_m s_n p^{-n}} \quad . \qquad (9.2.5)$$

R_{p1} and R_{p_2} are glued together along common rationals by an the composite map $I_{R \rightarrow R_{p_2}} I_{R_{p_1} \rightarrow R}$.

This variant of canonical identification seems to be an excellent candidate for mapping the predictions of p-adic mass calculations to real numbers and also for relating p-adic and real scattering amplitudes to each other [K66] . The deviations of predictions from those for standard form of canonical identification are however small.

The cautious conclusion of this section is that symmetric space approach involving both the identification along common rationals of roots of unity in large and canonical identification below the measurement resolution provide the safest approach to the p-adicization of quantum TGD. The impossibility to well-order the points below measurement resolution explains why effective p-adic topology works in real context. The discussion of integration and Fourier analysis will provide further support for the conclusion.

9.2.4 P-Adic Variants Of The Basic Mathematical Structures Relevant To Physics

The basic existential questions worrying a person planning to become a p-adic quantum physicist are rather obvious. How to define p-adic probabilities, p-adic thermodynamics, and p-adic unitarity and perhaps even p-adic Hilbert space? Is it possible to define the p-adic variant of the manifold concept? As already noticed for symmetric spaces p-adic variants might exist but what about space-time surfaces: could it be enough to consider only the p-adic variants of the partonic 2-surfaces in the manner already discussed? Can one somehow circumvent the difficulties related to the definition of the p-adic variant of definite integral? Perhaps by using Fourier analysis? How can one circumvent the fact that the basic variational principle involves integral over space-time surface which is p-adically notoriously difficult to define? Is all this just a waste of time or could it be that the enormous constraints from p-adicization could provide information about real physics not achievable otherwise (as in the case of p-adic mass calculations)?

p-Adic probabilities

p-Adic super conformal representations necessitate p-adic QM based on the p-adic unitarity and p-adic probability concepts. The concept of a p-adic probability indeed makes sense as shown by [A35] . p-Adic probabilities can be defined as relative frequencies N_i/N in a long series consisting of total number N of observations and N_i outcomes of type i. Probability conservation corresponds to

$$\sum_i N_i = N , \qquad (9.2.6)$$

and the only difference as compared to the usual probability is that the frequencies are interpreted as p-adic numbers.

The interpretation as p-adic numbers means that the relative frequencies converge to probabilities in a p-adic rather than real sense in the limit of a large number N of observations. If one requires that probabilities are limiting values of the frequency ratios in p-adic sense one must pose restrictions on the possible numbers of the observations N if N is larger than p. For N smaller than p, the situation is similar to the real case. This means that for $p = M_{127} \simeq 10^{38}$, appropriate for the particle physics experiments, p-adic probability differs in no observable manner from the ordinary probability.

If the number of observations is larger than p, the situation changes. If N_1 and N_2 are two numbers of observations they are near to each other in the p-adic sense if they differ by a large power of p. A possible interpretation of this restriction is that the observer at the p:th level of the condensate cannot choose the number of the observations freely. The restrictions to this freedom come from the requirement that the sensible statistical questions in a p-adically conformally invariant world must respect p-adic conformal invariance [A19] .

The most important application of the p-adic probability is the description of the particle massivation based on p-adic thermodynamics. Instead of energy, Virasoro generator l is thermalized and in the low temperature phase temperature is quantized in the sense that the counterpart of the Boltzmann weight $exp(H/T)$ is $p^{L_0/T}$, where $T = 1/n$ from the requirement that Boltzmann weight exists (L_0 has integer spectrum). The surprising success of the mass calculations shows that p-adic probability theory is much more than a formal possibility.

In particle physics context coupling constant evolution is replaced with a discrete p-adic coupling constant evolution and the renormalization is related to the change of the reduction of the p-adic length scale L_p in the length scale hierarchy rather than p-adic fractality for a fixed value of p. In ZEO the evolution corresponds to the hierarchy of CDs with scales coming as powers of 2 in accordance with p-adic length scale hypothesis.

1. p-Adic probabilities and p-adic fractals

p-Adic probabilities are natural in the statistical description of the fractal structures, which can contain same structural detail with all possible sizes.

1. The concept of a structural detail in a fractal seems to be reasonably well defined concept. The structural detail is clearly fixed by its topology and p-adic conformal invariants associated with it. Clearly, a finite resolution defined by some power of p of the p-adic cutoff scale must be present in the definition. For example, p-adic angles are conformal invariants in the p-adic case, too. The overall size of the detail doesn't matter. Let us therefore assume that it is possible to make a list, possibly infinite, of the structural details appearing in the p-adic fractal.

2. What kind of questions related to the structural details of the p-adic fractal one can ask? The first thing one can ask is how many times i:th structural detail appears in a finite region of the fractal structure: although this number is infinite as a real number it might possess (and probably does so!) finite norm as a p-adic number and provides a useful p-adic invariant of the fractal. If a complete list about the structural details of the fractal is at use one can calculate also the total number of structural details defined as $N = \sum_i N_i$. This means that one can also define p-adic probability for the appearance of i:th structural detail as a relative frequency $p_i = N_i/N$.

3. One can consider conditional probabilities, too. It is natural to ask what is the probability for the occurrence of the structural detail subject to the condition that part of the structural detail is fixed (apart from the p-adic conformal transformations). In order to evaluate these probabilities as relative frequencies one needs to look only for those structural details containing the substructure in question.

4. The evaluation of the p-adic probabilities of occurrence can be done by evaluating the required numbers N_i and N in a given resolution. A better estimate is obtained by increasing the resolution and counting the numbers of the hitherto unobserved structural details. The increase in the resolution greatly increases the number of the observations in case of p-adic fractal and the fluctuations in the values of N_i and N increase with the resolution so that N_i/N has no well defined limit as a real number although one can define the probabilities of occurrence as a resolution dependent concept. In the p-adic sense the increase in the values of N_i and fluctuations are small and the procedure should converge rapidly so that reliable estimates should result with quite a reasonable resolution. Notice that the increase of the fluctuations in the real sense, when resolution is increased is in accordance with the criticality of the system.

5. p-Adic frequencies and probabilities define via the canonical correspondence real valued invariants of the fractal structure.

p-Adic fractality in this sense could have practical applications only for small values of p. They could be important in the macroscopic length scales if the hierarchy of Planck constants meaning scaling up $L_p \to \sqrt{r}L_p$, $r = h_{eff}/h$, of the p-adic length scales. In elementary particle physics L_p is of the order of the Compton length associated with the particle for $r = 1$ and already in the first downward step CP_2 length scale R is achieved whereas upward step gives astrophysical length scale in the case of electron ($p = M_{127} = 2^{127} - 1$) for instance. For large enough values of Planck constant and for small p-adic primes p the situation changes.

2. Relationship between p-adic and real probabilities

There are uniqueness problems related to the mapping of p-adic probabilities to real ones. These problems find a nice resolution from the requirement that the map respects probability conservation. The implied modification of the original mapping does not change measurably the predictions for the masses of light particles.

a) How unique the map of p-adic probabilities and mass squared values are mapped to real numbers is?

The mapping of p-adic thermodynamical probabilities and mass squared values to real numbers is not completely unique.

1. The canonical identification $Id : \sum x_n p^n \to \sum x_n p^{-n}$ takes care of this mapping but does not respect the sum of probabilities so that the real images $I(p_n)$ of the probabilities must be normalized. This is a somewhat alarming feature.

2. The modification of the canonical identification mapping rationals by the formula $I(r/s) = I(r)/I(s)$ has appeared naturally in various applications, in particular because it respects unitarity of unitary matrices with rational elements with $r < p, s < p$. In the case of p-adic thermodynamic the formula $I(g(n)p^n/Z) \to I(g(n)p^n)/I(Z)$ would be very natural although Z need not be rational anymore. For $g(n) < p$ the real counterparts of the p-adic probabilities would sum up to one automatically for this option. One cannot deny that this option is more convincing than the original one. The generalization of this formula to map p-adic mass squared to a real one is obvious.

3. Options 1) and 2) differ dramatically when the $n = 0$ massless ground state has ground state degeneracy $D > 1$. For option 1) the real mass is predicted to be of order CP_2 mass whereas for option 2) it would be by a factor $1/D$ smaller than the minimum mass predicted by the option 1). Thus option 2) would predict a large number of additional exotic states. For those states which are light for option 1), the two options make identical predictions as far as the significant two lowest order terms are considered. Hence this interpretation would not change the predictions of the p-adic mass calculations in this respect. Option 2) is definitely more in accord with the real physics based intuitions and the main role of p-adic thermodynamics would be to guarantee the quantization of the temperature and fix practically uniquely the spectrum of the "Hamiltonian".

b) Under what conditions the mapping of p-adic ensemble probabilities to real probabilities respects probability conservation?

One can consider also a more general situation. Assume that one has an ensemble consisting of independent elementary events such that the number of events of type i is N_i. The probabilities are given by $p_i = N_i/N$ and $N = \sum N_i$ is the total number of elementary events. Even in the case that N is infinite as a real number it is natural to map the p-adic probabilities to their real counterparts using the rational canonical identification $I(p_i) = I(N_i)/I(N)$. Of course, N_i and N exist as well defined p-adic numbers under very stringent conditions only.

The question is under what conditions this map respects probability conservation. The answer becomes obvious by looking at the pinary expansions of N_i and N. If the integers N_i (possibly infinite as real integers) have pinary expansions having no common pinary digits, the sum of probabilities is conserved in the map. Note that this condition can assign also to a finite ensemble with finite number of a unique value of p.

This means that the selection of a basis for independent events corresponds to a decomposition of the set of integers labelling pinary digits to disjoint sets and brings in mind the selection of orthonormalized basis of quantum states in quantum theory. What is physically highly non-trivial that this "orthogonalization" alone puts strong constraints on probabilities of the allowed elementary events. One can say that the probabilities define distributions of pinary digits analogous to non-negative probability amplitudes in the space of integers labelling pinary digits, and the probabilities of independent events must be orthogonal with respect to the inner product defined by point-wise multiplication in the space of pinary digits.

p-Adic thermodynamics for which Boltzmann weights $g(E)exp(-E/T)$ are replaced by $g(E)p^{E/T}$ such that one has $g(E) < p$ and E/T is integer valued, satisfies this constraint. The quantization of E/T to integer values implies quantization of both T and "energy" spectrum and forces so called super conformal invariance [A19, A21] in TGD applications, which is indeed a basic symmetry of the theory.

There are infinitely many ways to choose the elementary events and each choice corresponds to a decomposition of the infinite set of integers n labelling the powers of p to disjoint subsets. These subsets can be also infinite. One can assign to this kind of decomposition a resolution which is the poorer the larger the subsets involved are. p-Adic thermodynamics would represent the situation in which the resolution is maximal since each set contains only single pinary digit. Note the analogy with the basis of completely localized wave functions in a lattice.

c) How to map p-adic transition probabilities to real ones?

p-Adic variants of TGD, if they exist, give rise to S-matrices and transition probabilities P_{ij}, which are p-adic numbers.

1. The p-adic probabilities defined by rows of S-matrix mapped to real numbers using canonical identification respecting the $q = r/s$ decomposition of rational number or its appropriate generalization should define real probabilities.

2. The simplest example would simple renormalization for the real counterparts of the p-adic probabilities $(P_{ij})_R$ obtained by canonical identification (or more probably its appropriate modification).

$$\begin{aligned} P_{ij} &= \sum_{k\geq 0} P_{ij}^k p^k \ , \\ P_{ij} &\rightarrow \sum_{k\geq 0} P_{ij}^k p^{-k} \equiv (P_{ij})_R \ , \\ (P_{ij})_R &\rightarrow \frac{(P_{ij})_R}{\sum_j (P_{ij})_R} \equiv P_{ij}^R \ . \end{aligned} \tag{9.2.7}$$

The procedure converges rapidly in powers of p and resembles renormalization procedure of quantum field theories. The procedure automatically divides away one four-momentum delta function from the square of S-matrix element containing the square of delta function with no well defined mathematical meaning. Usually one gets rid of the delta function interpreting it as the inverse of the four-dimensional measurement volume so that transition rate instead of transition probability is obtained. Of course, also now same procedure should work either as a discrete or a continuous version.

3. Probability interpretation would suggest that the real counterparts of p-adic probabilities sum up to unity. This condition is rather strong since it would hold separately for each row and column of the S-matrix.
4. A further condition would be that the real counterparts of the p-adic probabilities for a given prime p are identical with the transition probabilities defined by the real S-matrix for real space-time sheets with effective p-adic topology characterized by p. This condition might allow to deduce all relevant phase information about real and corresponding p-adic S-matrices using as an input only the observable transition probabilities.

d) What it means that p-adically independent events are not independent in real sense?

A further condition would be that p-adic quantum transitions represent also in the real sense independent elementary events so that the real counterpart for a sum of the p-adic probabilities for a finite number of transitions equals to the sum of corresponding real probabilities. This condition is definitely too strong in the general case since only a single transition could correspond to a given p-adic norm of transition probability P_{ij} with i fixed. In p-adic thermodynamics it can be satisfied if the degeneracy for an energy eigenstate for a given eigen value $L_0 = n$ is not larger than p. This condition fails for large values of n for super Virasoro representations since the degeneracy grows exponentially. This has not practical implications for the large values of p considered.

The crucial question concerns the physical difference between the real counterpart for the sum of the p-adic transition probabilities and for the sum of the real counterparts of these probabilities, which are in general different:

$$(\sum_j P_{ij})_R \neq \sum_j (P_{ij})_R \ . \tag{9.2.8}$$

The suggestion is that p-adic sum of the transition probabilities corresponds to the experimental situation, when one does not monitor individual transitions but using some common experimental signature only looks whether the transition leads to this set of the final states or not. When one looks each transition separately or effectively performs different experiment by considering only one transition channel in each experiment one must use the sum of the real probabilities. More precisely, the choice of the experimental signatures divides the set U of the final states to a disjoint union $U = \cup_i U_i$ and one must define the real counterparts for the transition probabilities P_{iU_k} as

$$\begin{aligned} P_{iU_k} &= \sum_{j\in U_k} P_{ij} \ , \\ P_{iU_k} &\rightarrow (P_{iU_k})_R \ , \\ (P_{iU_k})_R &\rightarrow \frac{(P_{iU_k})_R}{\sum_l (P_{iU_l})_R} \equiv P^R_{iU_k} \ . \end{aligned} \tag{9.2.9}$$

The assumption means deep a departure from the ordinary probability theory. If p-adic physics is the physics of cognitive systems, there need not be anything mysterious in the dependence of the behavior of system on how it is monitored. At least half-jokingly one might argue that the behavior of an intelligent system indeed depends strongly on whether the boss is nearby or not. The precise definition for the monitoring could be based on the decomposition of the density matrix representing the entangled subsystem into a direct sum over the subspaces associated with the degenerate eigenvalues of the density matrix. This decomposition provides a natural definition for the notions of the monitoring and resolution.

The renormalization procedure is in fact familiar from standard physics. Assume that the labels j correspond to momenta. The division of momentum space to cells of a given size so that the individual momenta inside cells are not monitored separately means that momentum resolution is finite. Therefore one must perform p-adic summation over the cells and define the real probabilities in the proposed manner. p-Adic effects resulting from the difference between p-adic and real summations could be the counterpart of the renormalization effects in QFT. It should be added that similar resolution can be defined also for the initial states by decomposing them into a union of disjoint subsets.

An alternative interpretation for the degenerate eigenvalues has emerged years after writing this. The sub-spaces corresponding to given eigenvalue of density matrix represent entangled states resulting in state function reduction interpreted as measurement of density matrix. This entanglement would be negentropic and represent a rule/concept, whose instances the superposed state pairs are. The information measure would Shannon entropy based on the replacement of the probability appearing as argument of logarithm with its p-adic norm. This entropy would be negative and therefore measure the information associated with the entanglement. This number theoretic entropy characterizes two particle state rather than single particle state and has nothing to do with the ordinary Shannon entropy.

Maybe one could say that finite measurement resolution implies automatically conceptualization and rule building. Abstractions are indeed obtained by dropping out the details.

2. p-Adic thermodynamics

The p-adic field theory limit as such is not expected to give a realistic theory at elementary particle physics level. The point is that particles are expected to be either massless or possess mass of order 10^{-4} Planck mass. The p-adic description of particle massivation described in [K70] shows that p-adic thermodynamics provides the proper formulation of the problem. What is thermalized is Virasoro generator L_0 (mass squared contribution is not included to L_0 so that states do not have a fixed conformal weight). Temperature is quantized purely number theoretically in low temperature limit ($exp(H/kT) \rightarrow p^{L_0/T}$, $T = 1/n$): in fact, the partition function does not even exist in high temperature phase. The extremely small mixing of massless states with Planck mass states implies massivation and predictions of the p-adic thermodynamics for the fermionic masses are in excellent agreement with experimental masses. Thermodynamic approach also explains the emergence of the length scale L_p for a given p-adic condensation level and one can develop arguments explaining why primes near prime powers of two are favored.

It should be noticed that rational p-adic temperatures $1/T = k/n$ are possible, if one poses the restriction that thermal probabilities are non-vanishing only for some subalgebra of the Super Virasoro algebra isomorphic to the Super Virasoro algebra itself. The generators L_{kn},G_{kn}, where k is a positive integer, indeed span this kind of a subalgebra by the fractality of the Super Virasoro algebra and $p^{L_0/T}$ is integer valued with this restriction.

One might apply thermodynamics approach should also in the calculation of S-matrix. What is is needed is thermodynamical expectation value for the transition amplitudes squared over incoming and outgoing states. In this expectation value 3-momenta are fixed and only mass squared varies.

3. Generalization of the notion of information

TGD inspired theory of consciousness, in particular the formulation of Negentropy Maximization Principle (NMP) in p-adic context, has forced to rethink the notion of the information concept. In TGD state preparation process is realized as a sequence of self measurements. Each self measurement means a decomposition of the sub-system involved to two unentangled parts. The decomposition is fixed highly uniquely from the requirement that the reduction of the entanglement entropy is maximal.

The additional assumption is that bound state entanglement is stable against self measurement. This assumption is somewhat ad hoc and it would be nice to get rid of it. The only manner to achieve this seems to be a generalized definition of entanglement entropy allowing to assign a negative value of entanglement entropy to the bound state entanglement, so that bound state entanglement would actually carry information, in fact conscious information (experience of understanding). This would be very natural since macro-temporal quantum coherence corresponds to a generation of bound state entanglement, and is indeed crucial for ability to have long lasting non-entropic mental images.

The generalization of the notion of number concept leads immediately to the basic problem. How to generalize the notion of entanglement entropy that it makes sense for a genuinely p-adic entanglement? What about the number-theoretically universal entanglement with entanglement probabilities, which correspond to finite extension of rational numbers? One can also ask whether the generalized notion of information could make sense at the level of the space-time as suggested by quantum-classical correspondence.

In the real context Shannon entropy is defined for an ensemble with probabilities p_n as

$$S = -\sum_n p_n log(p_n) \ . \qquad (9.2.10)$$

As far as theory of consciousness is considered, the basic problem is that Shannon entropy is always non-negative so that as such it does not define a genuine information measure. One could define information as a change of Shannon entropy and this definition is indeed attractive in the sense that quantum jump is the basic element of conscious experience and involves a change. One can however argue that the mere ability to transfer entropy to environment (say by aggressive behavior) is not all that is involved with conscious information, and even less so with the experience of understanding or moment of eureka. One should somehow generalize the Shannon entropy without losing the fundamental additivity property.

a) p-Adic entropies

The key observation is that in the p-adic context the logarithm function $log(x)$ appearing in the Shannon entropy is not defined if the argument of logarithm has p-adic norm different from 1. Situation changes if one uses an extension of p-adic numbers containing $log(p)$: the conjecture is that this extension is finite-dimensional. One might however argue that Shannon entropy should be well defined even without the extension.

p-Adic thermodynamics inspires a way to achieve this. One can replace $log(x)$ with the logarithm $log_p(|x|_p)$ of the p-adic norm of x, where log_p denotes p-based logarithm. This logarithm is integer valued ($log_p(p^n) = n$), and is interpreted as a p-adic integer. The resulting p-adic entropy

$$\begin{aligned} S_p &= \sum_n p_n k(p_n) \ , \\ k(p_n) &= -log_p(|p_n|) \ . \end{aligned} \qquad (9.2.11)$$

is additive: that is the entropy for two non-interacting systems is the sum of the entropies of composites. Note that this definition differs from Shannon's entropy by the factor $log(p)$. This entropy vanishes identically in the case that the p-adic norms of the probabilities are equal to one. This means that it is possible to have non-entropic entanglement for this entropy.

One can consider a modification of S_p using p-adic logarithm if the extension of the p-adic numbers contains $log(p)$. In this case the entropy is formally identical with the Shannon entropy:

$$S_p = -\sum_n p_n log(p_n) = -\sum_n p_n \left[-k(p_n) log(p) + p^{k_n} log(p_n/p^{k_n}\right] \ . \qquad (9.2.12)$$

It seems that this entropy cannot vanish.

One must map the p-adic value entropy to a real number and here canonical identification can be used:

$$\begin{aligned} S_{p,R} &= (S_p)_R \times log(p)) \ , \\ (\sum_n x_n p^n)_R &= \sum_n x_n p^{-n} \ . \end{aligned} \qquad (9.2.13)$$

The real counterpart of the p-adic entropy is non-negative.

b) Number theoretic entropies and metabolic energy

In the case that the probabilities are rational or belong to a finite-dimensional extension of rationals, it is possible to regard them as real numbers or p-adic numbers in some extension of p-adic numbers for any p. The visions that rationals and their finite extensions correspond to islands of order in the seas of chaos of real and p-adic transcendentals suggests that states having entanglement coefficients in finite-dimensional extensions of rational numbers are somehow very special. This is indeed the case. The p-adic entropy entropy $S_p = -\sum_n p_n log_p(|p_n|) log(p)$ can be interpreted in this case as an ordinary rational number in an extension containing $log(p)$.

What makes this entropy so interesting is that it can have also negative values in which case the interpretation as an information measure is natural. In the real context one can fix the value of the value of the prime p by requiring that S_p is maximally negative, so that the information content of the ensemble could be defined as

$$I \equiv Max\{-S_p, \ p \ \text{prime}\} \ . \tag{9.2.14}$$

This information measure is positive when the entanglement probabilities belong to a finite-dimensional extension of rational numbers. Thus kind of entanglement is stable against NMP [K65] , and has a natural interpretation as a negentropic entanglement.

There is no need to interpret negentropic entanglement as bound state entanglement as was the original proposal. This together with the vision about life as something in the intersection of the real and p-adic worlds inspires the idea about a connection between information and metabolism in living matter. Metabolic energy could be carried by negentropic entanglement and the feed of metabolic energy would be also feed of negentropy. In particular, the poorly understood high energy phosphate bond could be identified as a bond involving negentropic entanglement [K4] . The prediction would be that the negentropic states of real systems form a number theoretical hierarchy according to the prime $p$anddimension of algebraic extension characterizing the entanglement.

Number theoretically state function reduction and state preparation could be seen as information generating processes in the intersection of real and p-adic worlds.

How to define integration and p-adic Fourier analysis, integral calculus, and p-adic counterparts of geometric objects?

p-Adic differential calculus exists and obeys essentially the same rules as ordinary differential calculus. The only difference from real context is the existence of p-adic pseudo constants: any function which depends on finite number of pinary digits has vanishing p-adic derivative. This implies non-determinism of p-adic differential equations. One can defined p-adic integral functions using the fact that indefinite integral is the inverse of differentiation. The basis problem with the definite integrals is that p-adic numbers are not well-ordered so that the crucial ordering of the points of real axis in definite integral is not unique. Also p-adic Fourier analysis is problematic since direct counterparts of ep(ix) and trigonometric functions are not periodic. Also exp(-x) fails to converse exponentially since it has p-adic norm equal to 1. Note also that these functions exists only when the p-adic norm of x is smaller than 1.

The following considerations support the view that the p-adic variant of a geometric objects, integration and p-adic Fourier analysis exists but only when one considers highly symmetric geometric objects such as symmetric spaces. This is wellcome news from the point of view of physics. At the level of space-time surfaces this is problematic. The field equations associated with Kähler action and Kähler-Dirac equation make sense. Kähler action defined as integral over p-adic space-time surface fails to exist. If however the Kähler function identified as Kähler for a preferred extremal of Kähler action is rational or algebraic function of preferred complex coordinates of WCW with ratonal coefficients, its p-adic continuation is expected to exist.

1. Circle with rotational symmetries and its hyperbolic counterparts

Consider first circle with emphasis on symmetries and Fourier analysis.

1. In this case angle coordinate ϕ is the natural coordinate. It however does not make sense as such p-adically and one must consider either trigonometric functions or the phase $exp(i\phi)$ instead. If one wants to do Fourier analysis on circle one must introduce roots $U_{n,N} = exp(in2\pi/N)$ of unity. This means discretization of the circle. Introducing all roots $U_{n,p} = exp(i2\pi n/p)$, such that p divides N, one can represent all $U_{k,n}$ up to $n = N$. Integration is naturally replaced with sum by using discrete Fourier analysis on circle. Note that the roots of unity can be expressed as products of powers of roots of unity $exp(in2\pi/p^k)$, where p^k divides N.

2. There is a number theoretical delicacy involved. By Fermat's theorem $a^{p-1}\ mod\ p = 1$ for $a = 1, ...p-1$ for a given p-adic prime so that for any integer M divisible by a factor of $p-1$ the M:th roots of unity exist as ordinary p-adic numbers. The problem disappears if these values of M are excluded from the discretization for a given value of the p-adic prime. The manner to achieve this is to assume that N contains no divisors of $p-1$ and is consistent with the notion of finite measurement resolution. For instance, $N = p^n$ is an especially natural choice guaranteeing this.
3. The p-adic integral defined as a Fourier sum does not reduce to a mere discretization of the real integral. In the real case the Fourier coefficients must approach to zero as the wave vector $k = n2\pi/N$ increases. In the p-adic case the condition consistent with the notion of finite measurement resolution for angles is that the p-adic valued Fourier coefficients approach to zero as n increases. This guarantees the p-adic convergence of the discrete approximation of the integral for large values of N as n increases. The map of p-adic Fourier coefficients to real ones by canonical identification could be used to relate p-adic and real variants of the function to each other.

This finding would suggests that p-adic geometries -in particular the p-adic counterpart of CP_2, are discrete. Variables which have the character of a radial coordinate are in natural manner p-adically continuous whereas phase angles are naturally discrete and described in terms of algebraic extensions. The conclusion is disappointing since one can quite well argue that the discrete structures can be regarded as real. Is there any manner to escape this conclusion?

1. Exponential function $exp(ix)$ exists p-adically for $|x|_p \leq 1/p$ but is not periodic. It provides representation of p-adic variant of circle as group $U(1)$. One obtains actually a hierarchy of groups $U(1)_{p,n}$ corresponding to $|x|_p \leq 1/p^n$. One could consider a generalization of phases as products $Exp_p(N, n2\pi/N + x) = exp(in2\pi n/N)exp(ix)$ of roots of unity and exponent functions with an imaginary exponent. This would assign to each root of unity p-adic continuum interpreted as the analog of the interval between two subsequent roots of unity at circle. The hierarchies of measurement resolutions coming as $2\pi/p^n$ would be naturally accompanied by increasingly smaller p-adic groups $U(1)_{p,n}$.
2. p-Adic integration would involve summation plus possibly also an integration over each p-adic variant of discretization interval. The summation over the roots of unity implies that the integral of $\int exp(inx)dx$ would appear for $n = 0$. Whatever the value of this integral is, it is compensated by a normalization factor guaranteeing orthonormality.
3. If one interprets the p-adic coordinate as p-adic integer without the identification of points differing by a multiple of n as different points the question whether one should require p-adic continuity arises. Continuity is obtained if $U_n(x + mp^m) = U_n(x)$ for large values of m. This is obtained if one has $n = p^k$. In the spherical geometry this condition is not needed and would mean quantization of angular momentum as $L = p^k$, which does not look natural. If representations of translation group are considered the condition is natural and conforms with the spirit of the p-adic length scale hypothesis.

The hyperbolic counterpart of circle corresponds to the orbit of point under Lorentz group in two 2-D Minkowski space. Plane waves are replaced with exponentially decaying functions of the coordinate η replacing phase angle. Ordinary exponent function $exp(x)$ has unit p-adic norm when it exists so that it is not a suitable choice. The powers p^n existing for p-adic integers however approach to zero for large values of $x = n$. This forces discretization of η or rather the hyperbolic phase as powers of p^x, $x = n$. Also now one could introduce products of $Exp_p(nlog(p) + z) = p^n exp(x)$ to achieve a p-adic continuum. Also now the integral over the discretization interval is compensated by orthonormalization and can be forgotten. The integral of exponential function would reduce to a sum $\int Exp_p dx = \sum_k p^k = 1/(1-p)$. One can also introduce finite-dimensional but non-algebraic extensions of p-adic numbers allowing e and its roots $e^{1/n}$ since e^p exists p-adically.

2. Plane with translational and rotational symmetries

Consider first the situation by taking translational symmetries as a starting point. In this case Cartesian coordinates are natural and Fourier analysis based on plane waves is what one wants to define. As in the previous case, this can be done using roots of unity and one can also introduce p-adic continuum by using the p-adic variant of the exponent function. This would effectively reduce the plane to a box. As already noticed, in this case the quantization of wave vectors as multiples of $1/p^k$ is required by continuity.

One can take also rotational symmetries as a starting point. In this case cylindrical coordinates (ρ, ϕ) are natural.

1. Radial coordinate can have arbitrary values. If one wants to keep the connection $\rho = \sqrt{x^2 + y^2}$ with the Cartesian picture square root allowing extension is natural. Also the values of radial coordinate proportional to odd power of p are problematic since one should introduce $\sqrt{p}$: is this extension internally consistent? Does this mean that the points $\rho \propto p^{2n+1}$ are excluded so that the plane decomposes to annuli?
2. As already found, angular momentum eigen states can be described in terms of roots of unity and one could obtain continuum by allowing also phases defined by p-adic exponent functions.
3. In radial direction one should define the p-adic variants for the integrals of Bessel functions and they indeed might make sense by algebraic continuation if one consistently defines all functions as Fourier expansions. Delta-function renormalization causes technical problems for a continuum of radial wave vectors. One could avoid the problem by using exponentially decaying variants of Bessel function in the regions far from origin, and here the already proposed description of the hyperbolic counterparts of plane waves is suggestive.
4. One could try to understand the situation also using Cartesian coordinates. In the case of sphere this is achieved by introducing two coordinate patches with Cartesian coordinates. Pythagorean phases are rational phases (orthogonal triangles for which all sides are integer valued) and form a dense set on circle. Complex rationals (orthogonal triangles with integer valued short sides) define a more general dense subset of circle. In both cases it is difficult to imagine a discretized version of integration over angles since discretization with constant angle incrcríment is not possible.

3. The case of sphere and more general symmetric space

In the case of sphere spherical coordinates are favored by symmetry considerations. For spherical coordinates $sin(\theta)$ is analogous to the radial coordinate of plane. Legendre polynomials expressible as polynomials of $sin(\theta)$ and $cos(\theta)$ are expressible in terms of phases and the integration measure $sin^2(\theta)d\theta d\phi$ reduces the integral of S^2 to summation. As before one can introduce also p-adic continuum. Algebraic cutoffs in both angular momentum l and m appear naturally. Similar cutoffs appear in the representations of quantum groups and there are good reasons to expect that these phenomena are correlated.

Exponent of Kähler function appears in the integration over WCW. From the expression of Kähler gauge potential given by $A_\alpha = J_\alpha{}^\theta \partial_\theta K$ one obtains using $A_\alpha = cos(\theta)\delta_{\alpha,\phi}$ and $J_{\theta\phi} = sin(\theta)$ the expression $exp(K) = sin(\theta)$. Hence the exponent of Kähler function is expressible in terms of spherical harmonics.

The completion of the discretized sphere to a p-adic continuum- and in fact any symmetric space- could be performed purely group theoretically.

1. Exponential map maps the elements of the Lie-algebra to elements of Lie-group. This recipe generalizes to arbitrary symmetric space G/H by using the Cartan decomposition $g = t + h$, $[h, h] \subset h$,$[h, t] \subset t$,$[t, t] \subset h$. The exponentiation of t maps t to G/H in this case. The exponential map has a p-adic generalization obtained by considering Lie algebra with coefficients with p-adic norm smaller than one so that the p-adic exponent function exists. As a matter fact, one obtains a hierarchy of Lie-algebras corresponding to the upper bounds of the p-adic norm coming as p^{-k} and this hierarchy naturally corresponds to the hierarchy of angle resolutions coming as $2\pi/p^k$. By introducing finite-dimensional transcendental extensions containing roots of e one obtains also a hierarchy of p-adic Lie-algebras associated with transcendental extensions.
2. In particular, one can exponentiate the complement of the $SO(2)$ sub-algebra of $SO(3)$ Lie-algebra in p-adic sense to obtain a p-adic completion of the discrete sphere. Each point of the discretized sphere would correspond to a p-adic continuous variant of sphere as a symmetric space. Similar construction applies in the case of CP_2. Quite generally, a kind of fractal or

holographic symmetric space is obtained from a discrete variant of the symmetric space by replacing its points with the p-adic symmetric space.

3. In the N-fold discretization of the coordinates of M-dimensional space t one $(N-1)^M$ discretization volumes which is the number of points with non-vanishing t-coordinates. It would be nice if one could map the p-adic discretization volumes with non-vanishing t-coordinates to their positive valued real counterparts by applying canonical identification. By group invariance it is enough to show that this works for a discretization volume assignable to the origin. Since the p-adic numbers with norm smaller than one are mapped to the real unit interval, the p-adic Lie algebra is mapped to the unit cell of the discretization lattice of the real variant of t. Hence by a proper normalization this mapping is possible.

The above considerations suggests that the hierarchies of measurement resolutions coming as $\Delta\phi = 2\pi/p^n$ are in a preferred role. One must be however cautious in order to avoid too strong assumptions. The above considerations suggest that the hierarchies of measurement resolutions coming as $\Delta\phi = 2\pi/p^n$ are in a preferred role. One must be however cautious in order to avoid too strong assumptions. The following arguments however support this identification.

1. The vision about p-adicization characterizes finite measurement resolution for angle measurement in the most general case as $\Delta\phi = 2\pi M/N$, where M and N are positive integers having no common factors. The powers of the phases $exp(i2\pi M/N)$ define identical Fourier basis irrespective of the value of M unless one allows only the powers $exp(i2\pi kM/N)$ for which $kM < N$ holds true: in the latter case the measurement resolutions with different values of M correspond to different numbers of Fourier components. Otherwise the measurement resolution is just $\Delta\phi = 2\pi/p^n$. If one regards N as an ordinary integer, one must have $N = p^n$ by the p-adic continuity requirement.

2. One can also interpret N as a p-adic integer and assume that state function reduction selects one particular prime (no superposition of quantum states with different p-adic topologies). For $N = p^n M$, where M is not divisible by p, one can express $1/M$ as a p-adic integer $1/M = \sum_{k\geq 0} M_k p^k$, which is infinite as a real integer but effectively reduces to a finite integer $K(p) = \sum_{k=0}^{N-1} M_k p^k$. As a root of unity the entire phase $exp(i2\pi M/N)$ is equivalent with $exp(i2\pi R/p^n)$, $R = K(p)M \mod p^n$. The phase would non-trivial only for p-adic primes appearing as factors in N. The corresponding measurement resolution would be $\Delta\phi = R2\pi/N$. One could assign to a given measurement resolution all the p-adic primes appearing as factors in N so that the notion of multi-p p-adicity would make sense. One can also consider the identification of the measurement resolution as $\Delta\phi = |N/M|_p = 2\pi/p^k$. This interpretation is supported by the approach based on infinite primes [K103] .

4. What about integrals over partonic 2-surfaces and space-time surface?

One can of course ask whether also the integrals over partonic 2-surfaces and space-time surface could be p-adicized by using the proposed method of discretization. Consider first the p-adic counterparts of the integrals over the partonic 2-surface X^2.

1. WCW Hamiltonians and Kähler form are expressible using flux Hamiltonians defined in terms of X^2 integrals of JH_A, where H_A is $\delta CD \times CP_2$ Hamiltonian, which is a rational function of the preferred coordinates defined by the exponentials of the coordinates of the sub-space t in the appropriate Cartan algebra decomposition. The flux factor $J = \epsilon^{\alpha\beta} J_{\alpha\beta}\sqrt{g_2}$ is scalar and does not actually depend on the induced metric.

2. The notion of finite measurement resolution would suggest that the discretization of X^2 is somehow induced by the discretization of $\delta CD \times CP_2$. The coordinates of X^2 could be taken to be the coordinates of the projection of X^2 to the sphere S^2 associated with $\delta M^4_\pm$ or to the homologically non-trivial geodesic sphere of CP_2 so that the discretization of the integral would reduce to that for S^2 and to a sum over points of S^2.

3. To obtain an algebraic number as an outcome of the summation, one must pose additional conditions guaranteeing that both H_A and J are algebraic numbers at the points of discretization (recall that roots of unity are involved). Assume for definiteness that S^2 is $r_M = constant$ sphere. If the remaining preferred coordinates are functions of the preferred S^2 coordinates

mapping phases to phases at discretization points, one obtains the desired outcome. These conditions are rather strong and mean that the various angles defining CP_2 coordinates -at least the two cyclic angle coordinates- are integer multiples of those assignable to S^2 at the points of discretization. This would be achieved if the preferred complex coordinates of CP_2 are powers of the preferred complex coordinate of S^2 at these points. One could say that X^2 is algebraically continued from a rational surface in the discretized variant of $\delta CD \times CP_2$. Furthermore, if the measurement resolutions come as $2\pi/p^n$ as p-adic continuity actually requires and if they correspond to the p-adic group $G_{p,n}$ for which group parameters satisfy $|t|_p \leq p^{-n}$, one can precisely characterize how a p-adic prime characterizes the real partonic 2-surface. This would be a fulfilment of one of the oldest dreams related to the p-adic vision.

A even more ambitious dream would be that even the integral of the Kähler action for preferred extremals could be defined using a similar procedure. The conjectured slicing of Minkowskian space-time sheets by string world sheets and partonic 2-surfaces encourages these hopes.

1. One could introduce local coordinates of H at both ends of CD by introducing a continuous slicing of $M^4 \times CP_2$ by the translates of $\delta M^4_\pm \times CP_2$ in the direction of the time-like vector connecting the tips of CD. As space-time coordinates one could select four of the eight coordinates defining this slicing. For instance, for the regions of the space-time sheet representable as maps $M^4 \rightarrow CP_2$ one could use the preferred M^4 time coordinate, the radial coordinate of δM^4_+, and the angle coordinates of $r_M = constant$ sphere.
2. Kähler action density should have algebraic values and this would require the strengthening of the proposed conditions for X^2 to apply to the entire slicing meaning that the discretized space-time surface is a rational surface in the discretized $CD \times CP_2$. If this condition applies to the entire space-time surface it would effectively mean the discretization of the classical physics to the level of finite geometries. This seems quite strong implication but is consistent with the preferred extremal property implying the generalized Bohr rules.

5. Tentative conclusions

These findings suggest following conclusions.

1. Exponent functions play a key role in the proposed p-adicization. This is not an accident since exponent functions play a fundamental role in group theory and p-adic variants of real geometries exist only under symmetries- possibly maximal possible symmetries- since otherwise the notion of Fourier analysis making possible integration does not exist. The inner product defined in terms of integration reduce for functions representable in Fourier basis to sums and can be carried out by using orthogonality conditions. Convolution involving integration reduces to a product for Fourier components. In the case of embedding space and WCW these conditions are satisfied but for space-time surfaces this is not possible.
2. There are several ways to choose the Cartan algebra already in the case of sphere. In the case of plane one can consider either translations or rotations and this leads to different p-adic variants of plane. Also the realization of the hierarchy of Planck constants leads to the conclusion that the extended embedding space and therefore also WCW contains sectors corresponding to different choices of quantization axes meaning that quantum measurement has a direct geometric correlate. One an imagine also other discretizations and choices of preferred coordinates and the interpretation is that they correspond to different cognitive representations and to different p-adic physics. This means a refinement of General Coordinate Invariance taking into account cognition.
3. The above described 2-D examples represent symplectic geometries for which one has natural decomposition of coordinates to canonical pairs of cyclic coordinate (phase angle) and corresponding canonical conjugate coordinate. p-Adicization depends on whether the conjugate corresponds to an angle or non-compact coordinate. In both cases it is however possible to define integration. For instance, in the case of CP_2 one would have two canonically conjugate pairs and one can define the p-adic counterparts of CP_2 partial waves by generalizing the procedure applied to spherical harmonics. Products of functions expressible using partial waves can be decomposed by tensor product decomposition to spherical harmonics and can be integrated. In particular inner products can be defined as integrals. The Hamiltonians

generating isometries are rational functions of phases: this inspires the hope that also WCW Hamiltonians also rational functions of preferred WCW coordinates and thus allow p-adic variants.

4. Discretization by introducing algebraic extensions seems unavoidable in the p-adicization of geometrical objects but one can have p-adic continuum as the analog of the discretization interval and in the function basis expressible in terms of phase factors and p-adic counterparts of exponent functions. As already described, the exponential map for Lie group provide an elegant manner to realize this. This would give a precise meaning for the p-adic counterparts of the embedding space and WCW if the latter is a symmetric space allowing coordinatization in terms of phase angles and conjugate coordinates. The intersection of p-adic and real worlds in a given measurement resolution would be unique and correspond to the points defining the discretization.

p-Adic embedding space

The construction of both quantum TGD and p-adic QFT limit requires p-adicization of the embedding space geometry. Also the fact that p-adic Poincare invariance throws considerable light to the p-adic length scale hypothesis suggests that p-adic geometry is really needed. The construction of the p-adic version of the embedding space geometry and spinor structure relies on the symmetry arguments and to the generalization of the analytic formulas of the real case almost. The essential element is the notion of finite measurement resolution leading to discretization in large and to p-adicization below the resolution scale. This approach leads to a highly nontrivial generalization of the symmetry concept and p-adic Poincare invariance throws light to the p-adic length scale hypothesis. An important delicacy is related to the identification of the fundamental p-adic length scale, which corresponds to the unit element of the p-adic number field and is mapped to the unit element of the real number field in the canonical identification mapping p-adic mass squared to its real counterpart.

1. p-Adic Riemannian geometry depends on cognitive representation

p-Adic Riemann geometry is a direct formal generalization of the ordinary Riemann geometry. In the minimal purely algebraic generalization one does not try to define concepts like arch length and volume involving definite integrals but simply defines the p-adic geometry via the metric identified as a quadratic form in the tangent space of the p-adic manifold. Canonical identification would make it possible to define p-adic variant of Riemann integral formally allowing to calculate arc lengths and similar quantities but looks like a trick. The realization that the p-adic variant of harmonic analysis makes it possible to define definite integrals in the case of symmetric space became possible only after a detailed vision about what quantum TGD is [K121] had emerged.

Symmetry considerations dictate the p-adic counterpart of the Riemann geometry for $M^4_+ \times CP_2$ to a high degree but not uniquely. This non-uniqueness might relate to the distinction between different cognitive representations. For instance, in the case of Euclidian plane one can introduce linear or cylindrical coordinates and the manifest symmetries dictating the preferred coordinates correspond to translational and rotational symmetries in these two cases and give rise to different p-adic variants of the plane. Both linear and cylindrical coordinates are fixed only modulo the action of group consisting of translations and rotations and the degeneracy of choices can be interpreted in terms of a choice of quantization axies of angular momentum and momenta.

The most natural looking manner to define the p-adic counterpart of M^4 is by using a p-adic completion for a subset of rational points in coordinates which are preferred on physical basis. In case of M^4 linear Minkowski coordinates are an obvious choice but also the counterparts of Robertson-Walker coordinates for M^4_+ defined as $[t,(z,x,y)] = a\times[cosh(\eta), sinh(\eta)(cos(\theta), sin(\theta)cos(\phi), sin(\theta)sin(\phi))]$ expressible in terms of phases and their hyperbolic counterparts and transforming nicely under the Cartan algebra of Lorentz group are possible. p-Adic variant is obtained by introducing finite measurement resolution for angle and replacing angle range by finite number of roots of unity. Same applies to hyperbolic angles.

Rational CP_2 could be defined as a coset space $SU(3,Q)/U(2,Q)$ associated with complex rational unitary 3×3-matrices. CP_2 could be defined as coset space of complex rational matrices by choosing one point in each coset $SU(3,Q)/U(2,Q)$ as a complex rational 3×3-matrix representable in terms of Pythagorean phases [A16] and performing a completion for the elements of this matrix by multiplying the elements with the p-adic exponentials $exp(iu)$, $|u|_p < 1$ such that one obtains p-adically unitary matrix.

This option is not very natural as far as integration is considered. CP_2 however allows the analog of spherical coordinates for S^2 expressible in terms of angle variables alone and this suggests the introduction of the variant of CP_2 for which the coordinate values correspond to roots of unity. Completion would be performed in the same manner as for rational CP_2. This non-uniqueness need not be a drawback but could reflect the fact that the p-adic cognitive representation of real geometry are geometrically non-equivalent. This means a refinement of the principle of General Coordinate Invariance taking into account the fact that the cognitive representation of the real world affects the world with cognition included in a delicate manner.

2. The identification of the fundamental p-adic length scale

The fundamental p-adic length scale corresponds to the p-adic unit $e = 1$ and is mapped to the unit of the real numbers in the canonical identification. The correct physical identification of the fundamental p-adic length scale is of crucial importance since the predictions of the theory for p-adic masses depend on the choice of this scale.

In TGD the "radius" R of CP_2 is the fundamental length scale ($2\pi R$ is by definition the length of the CP_2 geodesics). In accordance with the idea that p-adic QFT limit makes sense only above length scales larger than the radius of CP_2 R is of same order of magnitude as the p-adic length scale defined as $l = \pi/m_0$, where m_0 is the fundamental mass scale and related to the "cosmological constant" Λ ($R_{ij} = \Lambda s_{ij}$) of CP_2 by

$$m_0^2 = 2\Lambda \ . \tag{9.2.15}$$

The relationship between R and l is uniquely fixed:

$$R^2 = \frac{3}{m_0^3} = \frac{3}{2\Lambda} = \frac{3l^2}{\pi^2} \ . \tag{9.2.16}$$

Consider now the identification of the fundamental length scale.

1. One must use R^2 or its integer multiple, rather than l^2, as the fundamental p-adic length scale squared in order to avoid the appearance of the p-adically ill defined π:s in various formulas of CP_2 geometry.
2. The identification for the fundamental length scale as $1/m_0$ leads to difficulties.
   (a) The p-adic length for the CP_2 geodesic is proportional to $\sqrt{3}/m_0$. For the physically most interesting p-adic primes satisfying $p \ mod \ 4 = 3$ so that $\sqrt{-1}$ does not exist as an ordinary p-adic number, $\sqrt{3} = i\sqrt{-3}$ belongs to the complex extension of the p-adic numbers. Hence one has troubles in getting real length for the CP_2 geodesic.
   (b) If m_0^2 is the fundamental mass squared scale then general quark states have mass squared, which is integer multiple of $1/3$ rather than integer valued as in string models.
3. These arguments suggest that the correct choice for the fundamental length scale is as $1/R$ so that $M^2 = 3/R^2$ appearing in the mass squared formulas is p-adically real and all values of the mass squared are integer multiples of $1/R^2$. This does not affect the real counterparts of the thermal expectation values of the mass squared in the lowest p-adic order but the effects, which are due to the modulo arithmetics, are seen in the higher order contributions to the mass squared. As a consequence, one must identify the p-adic length scale l as

   $$l \equiv \pi R \ ,$$

   rather than $l = \pi/m_0$. This is indeed a very natural identification. What is especially nice is that this identification also leads to a solution of some longstanding problems related to the p-adic mass calculations. It would be highly desirable to have the same p-adic temperature $T_p = 1$ for both the bosons and fermions rather than $T_p = 1/2$ for bosons and $T_p = 1$ for fermions. For instance, black hole elementary particle analogy as well as the need to get rid of light boson exotics suggests this strongly. It indeed turns out possible to achieve this with the proposed identification of the fundamental mass squared scale.

3. p-Adic counterpart of M^4_+

The construction of the p-adic counterpart of M^4_+ seems a relatively straightforward task and should reduce to the construction of the p-adic counterpart of the real axis with the standard metric. As already noticed, linear Minkowski coordinates are physically and mathematically preferred coordinates and it is natural to construct the metric in these coordinates.

There are some quite interesting delicacies related to the p-adic version of the Poincare invariance. Consider first translations. In order to have imaginary unit needed in the construction of the ordinary representations of the Poincare group one must have $p \ mod \ 4 = 3$ to guarantee that $\sqrt{-1}$ does not exist as an ordinary p-adic number. It however seems that the construction of the representations is at least formally possible by replacing imaginary unit with the square root of some other p-adic number not existing as a p-adic number.

It seems that only the discrete group of translations allows representations consisting of orthogonal plane waves. p-Adic plane waves can be defined in the lattice consisting of the multiples of $x_0 = m/n$ consisting of points with p-adic norm not larger that $|x_0|_p$ and the points $p^n x_0$ define fractally scaled-down versions of this set. In canonical identification these sets corresponds to volumes scaled by factors p^{-n}.

A physically interesting question is whether the Lorentz group should contain only the elements obtained by exponentiating the Lie-algebra generators of the Lorentz group or whether also large Lorentz transformations, containing as a subgroup the group of the rational Lorentz transformations, should be allowed. If the group contains only small Lorentz transformations, the quantization volume of M^4_+ (say the points with coordinates m^k having p-adic norm not larger than one) is also invariant under Lorentz transformations. This means that the quantization of the theory in the p-adic cube $|m^k| < p^n$ is a Poincare invariant procedure unlike in the real case.

The appearance of the square root of p, rather than the naïvely expected p, in the expression of the p-adic length scale can be undertood if the p-adic version of M^4 metric contains p as a scaling factor:

$$\begin{aligned} ds^2 &= pR^2 m_{kl} dm^k dm^l \ , \\ R &\leftrightarrow 1 \ , \end{aligned} \tag{9.2.17}$$

where m_{kl} is the standard M^4 metric $(1, -1, -1, -1)$. The p-adic distance function is obtained by integrating the line element using p-adic integral calculus and this gives for the distance along the k:th coordinate axis the expression

$$s = R\sqrt{p} m^k \ . \tag{9.2.18}$$

The map from p-adic M^4 to real M^4 is canonical identification plus a scaling determined from the requirement that the real counterpart of an infinitesimal p-adic geodesic segment is same as the length of the corresponding real geodesic segment:

$$m^k \rightarrow \pi(m^k)_R \ . \tag{9.2.19}$$

The p-adic distance along the k:th coordinate axis from the origin to the point $m^k = (p-1)(1+p+p^2+...) = -1$ on the boundary of the set of the p-adic numbers with norm not larger than one, corresponds to the fundamental p-adic length scale $L_p = \sqrt{p}l = \sqrt{p}\pi R$:

$$\sqrt{p}((p-1)(1+p+...))R \rightarrow \pi R \frac{(p-1)(1+p^{-1}+p^{-2}+...)}{\sqrt{p}} = L_p \ . \tag{9.2.20}$$

What is remarkable is that the shortest distance in the range $m^k = 1, ..m-1$ is actually $L/\sqrt{p}$ rather than l so that p-adic numbers in range span the entire R_+ at the limit $p \rightarrow \infty$. Hence p-adic topology approaches real topology in the limit $p \rightarrow \infty$ in the sense that the length of the discretization step approaches to zero.

4. The two variants of CP_2

As noticed, CP_2 allows two variants based on rational discretization and on the discretization based on roots of unity. The root of unity option corresponds to the phases associated with $1/(1+r^2) = tan^2(u/2) = (1-cos(u))/(1+cos(u))$ and implies that integrals of spherical harmonics can be reduced to summations when angular resolution $\Delta u = 2\pi/N$ is introduced. In the p-adic context, one can replace distances with trigonometric functions of distances along zig zag curves connecting the points of the discretization. Physically this notion of distance is quite reasonable since distances are often measured using interferometer.

In the case of rtional variant of CP_2 one can proceed by defining the p-adic counterparts of $SU(3)$ and $U(2)$ and using the identification $CP_2 = SU(3)/U(2)$. The p-adic counterpart of $SU(3)$ consists of all 3×3 unitary matrices satisfying p-adic unitarity conditions (rows/colums are mutually orthogonal unit vectors) or its suitable subgroup: the minimal subgroup corresponds to the exponentials of the Lie-algebra generators. If one allows algebraic extensions of the p-adic numbers, one obtains several extensions of the group. The extension allowing the square root of a p-adically real number is the most interesting one in this respect since the general solution of the unitarity conditions involves square roots.

The subgroup of $SU(3)$ obtained by exponentiating the Lie-algebra generators of $SU(3)$ normalized so that their non-vanishing elements have unit p-adic norm, is of the form

$$SU(3)_0 = \{x = exp(\sum_k it_kX_k) \ ; \ |t_k|_p < 1\} = \{x = 1+iy \ ; \ |y|_p < 1\} \ . \tag{9.2.21}$$

The diagonal elements of the matrices in this group are of the form $1 + O(p)$. In order $O(p)$ these matrices reduce to unit matrices.

Rational $SU(3)$ matrices do not in general allow a representation as an exponential. In the real case all $SU(3)$ matrices can be obtained from diagonalized matrices of the form

$$h = diag\{exp(i\phi_1), exp(i\phi_2), exp(exp(-i(\phi_1+\phi_2)\} \ . \tag{9.2.22}$$

The exponentials are well defined provided that one has $|\phi_i|_p < 1$ and in this case the diagonal elements are of form $1 + O(p)$. For $p \ mod \ 4 = 3$ one can however consider much more general diagonal matrices

$$h = diag\{z_1, z_2, z_3\} \ ,$$

for which the diagonal elements are rational complex numbers

$$z_i = \frac{(m_i + in_i)}{\sqrt{m_i^2 + n_i^2}} \tag{9.2.23}$$

satisfying $z_1z_2z_3 = 1$ such that the components of z_i are integers in the range $(0, p-1)$ and the square roots appearing in the denominators exist as ordinary p-adic numbers. These matrices indeed form a group as is easy to see. By acting with $SU(3)_0$ to each element of this group and by applying all possible automorphisms $h \rightarrow ghg^{-1}$ using rational $SU(3)$ matrices one obtains entire $SU(3)$ as a union of an infinite number of disjoint components.

The simplest (unfortunately not physical) possibility is that the "physical" $SU(3)$ corresponds to the connected component of $SU(3)$ represented by the matrices, which are unit matrices in order $O(p)$. In this case the construction of CP_2 is relatively straightforward and the real formalism should generalize as such. In particular, for $p \ mod \ 4 = 3$ it is possible to introduce complex coordinates ξ_1, ξ_2 using the complexification for the Lie-algebra complement of $su(2) \times u(1)$. The real counterparts of these coordinates vary in the range $[0, 1)$ and the end points correspond to the values of t_i equal to $t_i = 0$ and $t_i = -p$. The p-adic sphere S^2 appearing in the definition of the p-adic light cone is obtained as a geodesic sub-manifold of CP_2 ($\xi_1 = \xi_2$ is one possibility). From the requirement that real CP_2 can be mapped to its p-adic counterpart it is clear that one must allow all connected components of CP_2 obtained by applying discrete unitary matrices having no exponential representation to the basic connected component. In practice this corresponds to the allowance of all possible values of the p-adic norm for the components of the complex coordinates ξ_i of CP_2.

The simplest approach to the definition of the CP_2 metric is to replace the expression of the Kähler function in the real context with its p-adic counterpart. In standard complex coordinates for which the action of $U(2)$ subgroup is linear, the expression of the Kähler function reads as

$$\begin{aligned} K &= log(1+r^2) \ , \\ r^2 &= \sum_i \bar{\xi}_i \xi_i \ . \end{aligned} \tag{9.2.24}$$

p-Adic logarithm exists provided r^2 is of order $O(p)$. This is the case when ξ_i is of order $O(p)$. The definition of the Kähler function in a more general case, when all possible values of the p-adic norm are allowed for r, is based on the introduction of a p-adic pseudo constant C to the argument of the Kähler function

$$K = log(\frac{1+r^2}{C}) \ . \tag{9.2.25}$$

C guarantees that the argument is of the form $\frac{1+r^2}{C} = 1 + O(p)$ allowing a well-defined p-adic logarithm. This modification of the Kähler function leaves the definition of Kähler metric, Kähler form and spinor connection invariant.

A more elegant manner to avoid the difficulty is to use the exponent $\Omega = exp(K) = 1+r^2$ of the Kähler function instead of Kähler function, which indeed well defined for all coordinate values. In terms of Ω one can express the Kähler metric as

$$g_{k\bar{l}} = \frac{\partial_k \partial_{\bar{l}} \Omega}{\Omega} - \frac{\partial_k \Omega \partial_{\bar{l}} \Omega}{\Omega^2} \ . \tag{9.2.26}$$

The p-adic metric can be defined as

$$s_{i\bar{j}} = R^2 \partial_i \partial_{\bar{j}} K = R^2 \frac{(\delta_{i\bar{j}} r^2 - \bar{\xi}_i \xi_j)}{(1+r^2)^2} \ . \tag{9.2.27}$$

The expression for the Kähler form is the same as in the real case and the components of the Kähler form in the complex coordinates are numerically equal to those of the metric apart from the factor of i. The components in arbitrary coordinates can be deduced from these by the standard transformation formulas.

9.2.5 What Could Be The Origin Of Preferred P-Adic Primes And P-Adic Length Scale Hypothesis?

p-Adic mass calculations [K70] allow to conclude that elementary particles correspond to one or possible several preferred primes assigning p-adic effective topology to the real space-time sheets in discretization in some length scale range. TGD inspired theory of consciousness leads to the identification of p-adic physics as physics of cognition. Quite recent progress (2015) leads to the proposal that quantum TGD is adelic: all p-adic number fields are involved and each gives one particular view about physics.

Adelic approach [K52, K76] plus the view about evolution as emergence of increasingly complex extensions of rationals leads to a possible answer to th question of the title. The algebraic extensions of rationals are characterized by preferred rational primes, namely those which are ramified when expressed in terms of the primes of the extensions. These primes would be natural candidates for preferred p-adic primes. An argument relying on what I call weak form of NMP in turn allows to understand why primes near powers of 2 are preferred: as a matter of fact, also primes near powers of other primes are predicted to be favoured.

Earlier attempts

How the preferred primes emerge in TGD framework? I have made several attempts to answer this question. As a matter fact, the question has been slightly different: what determines the p-adic prime assigned to elementary particle by p-adic mass calculations [K59]. The recent view assigns to particle entire adele but some p-adic number fields in it are different.

1. Classical non-determinism at space-time level for real space-time sheets could in some length scale range involving rational discretization for space-time surface itself or for parameters characterizing it as a preferred extremal correspond to the non-determinism of p-adic differential equations due to the presence of pseudo constants which have vanishing p-adic derivative. Pseudo- constants are functions depend on finite number of pinary digits of its arguments.
2. The quantum criticality of TGD [?] is suggested to be realized in in terms of infinite hierarchies of super-symplectic symmetry breakings in the sense that only a sub-algebra with conformal weights which are n-ples of those for the entire algebra act as conformal gauge symmetries [K93]. This might be true for all conformal algebras involved. One has fractal hierarchy since the sub-algebras in question are isomorphic: only the scale of conformal gauge symmetry increases in the phase transition increasing n. The hierarchies correspond to sequences of integers $n(i)$ such tht $n(i)$ divides $n(i+1)$. These hierarchies would very naturally correspond to hierarchies of inclusions of hyper-finite factors and $m(i) = n(i+1)/n(i)$ could correspond to the integer n characterizing the index of inclusion, which has value $n \geq 3$. Possible problem is that $m(i) = 2$ would not correspond to Jones inclusion. Why the scaling by power of two would be different? The natural question is whether the primes dividing $n(i)$ or $m(i)$ could define the preferred primes.
3. Negentropic entanglement corresponds to entanglement for which density matrix is projector [K65]. For n-dimensional projector any prime p dividing n gives rise to negentropic entanglement in the sense that the number theoretic entanglement entropy defined by Shannon formula by replacing p_i in $log(p_i) = log(1/n)$ by its p-adic norm $N_p(1/n)$ is negative if p divides n and maximal for the prime for which the dividing power of prime is largest power-of-prime factor of n. The identification of p-adic primes as factors of n is highly attractive idea. The obvious question is whether n corresponds to the integer characterizing a level in the hierarchy of conformal symmetry breakings.
4. The adelic picture about TGD led to the question whether the notion of unitarity could be generalized. S-matrix would be unitary in adelic sense in the sense that $P_m = (SS^\dagger)_{mm} = 1$ would generalize to adelic context so that one would have product of real norm and p-adic norms of P_m. In the intersection of the realities and p-adicities P_m for reals would be rational and if real and p-adic P_m correspond to the same rational, the condition would be satisfied. The condition that $P_m \leq 1$ seems however natural and forces separate unitary in each sector so that this options seems too tricky.

These are the basic ideas that I have discussed hitherto.

Could preferred primes characterize algebraic extensions of rationals?

The intuitive feeling is that the notion of preferred prime is something extremely deep and the deepest thing I know is number theory. Does one end up with preferred primes in number theory? This question brought to my mind the notion of ramification of primes (see `http://tinyurl.com/hddljlf`) (more precisely, of prime ideals of number field in its extension), which happens only for special primes in a given extension of number field, say rationals. Could this be the mechanism assigning preferred prime(s) to a given elementary system, such as elementary particle? I have not considered their role earlier also their hierarchy is highly relevant in the number theoretical vision about TGD.

1. Stating it very roughly (I hope that mathematicians tolerate this language): As one goes from number field K, say rationals Q, to its algebraic extension L, the original prime ideals in the so called integral closure (see `http://tinyurl.com/js6fpvr`) over integers of K decompose to products of prime ideals of L (prime is a more rigorous manner to express primeness).

   Integral closure for integers of number field K is defined as the set of elements of K, which are roots of some monic polynomial with coefficients, which are integers of K and having the form $x^n + a_{n-1}x^{n-1} + ... + a_0$. The integral closures of both K and L are considered. For instance, integral closure of algebraic extension of K over K is the extension itself. The integral closure of complex numbers over ordinary integers is the set of algebraic numbers.

2. There are two further basic notions related to ramification and characterizing it. Relative discriminant is the ideal divided by all ramified ideals in K and relative different is the ideal of L divided by all ramified P_i:s. Note that te general ideal is analog of integer and these ideas represent the analogous of product of preferred primes P of K and primes P_i of L dividing them.

3. A physical analogy is provided by decomposition of hadrons to valence quarks. Elementary particles becomes composite of more elementary particles in the extension. The decomposition to these more elementary primes is of form $P = \prod P_i^{e(i)}$, where e_i is the ramification index - the physical analog would be the number of elementary particles of type i in the state (see `http://tinyurl.com/h9528pl`). Could the ramified rational primes could define the physically preferred primes for a given elementary system?

In TGD framework the extensions of rationals (see `http://tinyurl.com/h9528pl`) and p-adic number fields (see `http://tinyurl.com/zq22tvb`) are unavoidable and interpreted as an evolutionary hierarchy physically and cosmological evolution would have gradually proceeded to more and more complex extensions. One can say that string world sheets and partonic 2-surfaces with parameters of defining functions in increasingly complex extensions of prime emerge during evolution. Therefore ramifications and the preferred primes defined by them are unavoidable. For p-adic number fields the number of extensions is much smaller for instance for $p > 2$ there are only 3 quadratic extensions.

1. In p-adic context a proper definition of counterparts of angle variables as phases allowing definition of the analogs of trigonometric functions requires the introduction of algebraic extension giving rise to some roots of unity. Their number depends on the angular resolution. These roots allow to define the counterparts of ordinary trigonometric functions - the naïve generalization based on Taylors series is not periodic - and also allows to defined the counterpart of definite integral in these degrees of freedom as discrete Fourier analysis. For the simplest algebraic extensions defined by $x^n - 1$ for which Galois group is abelian are are unramified so that something else is needed. One has decomposition $P = \prod P_i^{e(i)}$, $e(i) = 1$, analogous to n-fermion state so that simplest cyclic extension does not give rise to a ramification and there are no preferred primes.

2. What kind of polynomials could define preferred algebraic extensions of rationals? Irreducible polynomials are certainly an attractive candidate since any polynomial reduces to a product of them. One can say that they define the elementary particles of number theory. Irreducible polynomials have integer coefficients having the property that they do not decompose to products of polynomials with rational coefficients. IT would be wrong to say that only these algebraic extensions can appear but there is a temptation to say that one can reduce the study of extensions to their study. One can even consider the possibility that string world sheets associated with products of irreducible polynomials are unstable against decay to those characterize irreducible polynomials.

3. What can one say about irreducible polynomials? Eisenstein criterion (see `http://tinyurl.com/47kxjz` states following. If $Q(x) = \sum_{k=0,..,n} a_k x^k$ is n:th order polynomial with integer coefficients and with the property that there exists at least one prime dividing all coefficients a_i except a_n and that p^2 does not divide a_0, then Q is irreducible. Thus one can assign one or more preferred primes to the algebraic extension defined by an irreducible polynomial Q of this kind - in fact any polynomial allowing ramification. There are also other kinds of irreducible polynomials since Eisenstein's condition is only sufficient but not necessary.

4. Furthermore, in the algebraic extension defined by Q, the prime ideals P having the above mentioned characteristic property decompose to an n :th power of single prime ideal P_i: $P = P_i^n$. The primes are maximally/completely ramified. The physical analog $P = P_0^n$ is Bose-Einstein condensate of n bosons. There is a strong temptation to identify the preferred primes of irreducible polynomials as preferred p-adic primes.

   A good illustration is provided by equations $x^2 + 1 = 0$ allowing roots $x_\pm = \pm i$ and equation $x^2+2px+p = 0$ allowing roots $x_\pm = -p \pm \sqrt{p}p - 1$. In the first case the ideals associated with $\pm i$ are different. In the second case these ideals are one and the same since $x_+ == -x_- + p$: hence one indeed has ramification. Note that the first example represents also an example of irreducible polynomial, which does not satisfy Eisenstein criterion. In more general case the n conditions on defined by symmetric functions of roots imply that the ideals are one and same when Eisenstein conditions are satisfied.

5. What does this mean in p-adic context? The identity of the ideals can be stated by saying $P = P_0^n$ for the ideals defined by the primes satisfying the Eisenstein condition. Very loosely one can say that the algebraic extension defined by the root involves n:th root of p-adic prime p. This does not work! Extension would have a number whose n:th power is zero modulo p. On the other hand, the p-adic numbers of the extension modulo p should be finite field but this would not be field anymore since there would exist a number whose n:th power vanishes. The algebraic extension simply does not exist for preferred primes. The physical meaning of this will be considered later.

6. What is so nice that one could readily construct polynomials giving rise to given preferred primes. The complex roots of these polymials could correspond to the points of partonic 2-surfaces carrying fermions and defining the ends of boundaries of string world sheet. It must be however emphasized that the form of the polynomial depends on the choices of the complex coordinate. For instance, the shift $x \to x + 1$ transforms $(x^n - 1)/(x - 1)$ to a polynomial satisfying the Eisenstein criterion. One should be able to fix allowed coordinate changes in such a way that the extension remains irreducible for all allowed coordinate changes.

   Already the integral shift of the complex coordinate affects the situation. It would seem that only the action of the allowed coordinate changes must reduce to the action of Galois group permuting the roots of polynomials. A natural assumption is that the complex coordinate corresponds to a complex coordinate transforming linearly under subgroup of isometries of the embedding space.

In the general situation one has $P = \prod P_i^{e(i)}$, $e(i) \geq 1$ so that aso now there are prefered primes so that the appearance of preferred primes is completely general phenomenon.

A connection with Langlands program?

In Langlands program (see `http://tinyurl.com/ycej7s43`) [A46, A45] the great vision is that the n-dimensional representations of Galois groups G characterizing algebraic extensions of rationals or more general number fields define n-dimensional adelic representations of adelic Lie groups, in particular the adelic linear group $Gl(n, A)$. This would mean that it is possible to reduce these representations to a number theory for adeles. This would be highly relevant in the vision about TGD as a generalized number theory. I have speculated with this possibility earlier [K52] but the mathematics is so horribly abstract that it takes decade before one can have even hope of building a rough vision.

One can wonder whether the irreducible polynomials could define the preferred extensions K of rationals such that the maximal abelian extensions of the fields K would in turn define the adeles utilized in Langlands program. At least one might hope that everything reduces to the maximally ramified extensions.

At the level of TGD string world sheets with parameters in an extension defined by an irreducible polynomial would define an adele containing various p-adic number fields defined by the primes of the extension. This would define a hierarchy in which the prime ideals of previous level would decompose to those of the higher level. Each irreducible extension of rationals would correspond to some physically preferred p-adic primes.

It should be possible to tell what the preferred character means in terms of the adelic representations. What happens for these representations of Galois group in this case? This is known.

1. For Galois extensions ramification indices are constant: $e(i) = e$ and Galois group acts transitively on ideals P_i dividing P. One obtains an n-dimensional representation of Galois group. Same applies to the subgroup of Galois group G/I where I is subgroup of G leaving P_i invariant. This group is called inertia group. For the maximally ramified case G maps the ideal P_0 in $P = P_0^n$ to itself so that $G = I$ and the action of Galois group is trivial taking P_0 to itself, and one obtains singlet representations.
2. The trivial action of Galois group looks like a technical problem for Langlands program and also for TGD unless the singletness of P_i under G has some physical interpretation. One possibility is that Galois group acts as like a gauge group and here the hierarchy of sub-algebras of super-symplectic algebra labelled by integers n is highly suggestive. This raises obvious questions. Could the integer n characterizing the sub-algebra of super-symplectic algebra acting as conformal gauge transformations, define the integer defined by the product of ramified primes? P_0^n brings in mind the n conformal equivalence classes which remain invariant under the conformal transformations acting as gauge transformations. . Recalling that relative discriminant is an of K ideal divisible by ramified prime ideals of K, this means that n would correspond to the relative discriminant for $K = Q$. Are the preferred primes those which are "physical" in the sense that one can assign to the states satisfying conformal gauge conditions?

   If the Galois group corresponds to gauge symmetries for these primes, it is physically natural that the p-adic algebraic extension does not exists and that p-adic variant of the Galois group is absent. Nothing is lost from cognition since there is nothing to cognize!

What could be the origin of p-adic length scale hypothesis?

The argument would explain the existence of preferred p-adic primes. It does not yet explain p-adic length scale hypothesis [K74, K59] stating that p-adic primes near powers of 2 are favored. A possible generalization of this hypothesis is that primes near powers of prime are favored. There indeed exists evidence for the realization of 3-adic time scale hierarchies in living matter [I8] (see `http://tinyurl.com/jbh9m27`) and in music both 2-adicity and 3-adicity could be present, this is discussed in TGD inspired theory of music harmony and genetic code [K87].

The weak form of NMP might come in rescue here.

1. Entanglement negentropy for a negentropic entanglement [K65] characterized by n-dimensional projection operator is the $log(N_p(n)$ for some p whose power divides n. The maximum negentropy is obtained if the power of p is the largest power of prime divisor of p, and this can be taken as definition of number theoretic entanglement negentropy. If the largest divisor is p^k, one has $N = k \times log(p)$. The entanglement negentropy per entangled state is $N/n = klog(p)/n$ and is maximal for $n = p^k$. Hence powers of prime are favoured which means that p-adic length scale hierarchies with scales coming as powers of p are negentropically favored and should be generated by NMP. Note that $n = p^k$ would define a hierarchy of $h_{eff}/h = p^k$. During the first years of h_{eff} hypothesis I believe that the preferred values obey $h_{eff} = r^k$, r integer not far from $r = 2^{11}$. It seems that this belief was not totally wrong.
2. If one accepts this argument, the remaining challenge is to explain why primes near powers of two (or more generally p) are favoured. $n = 2^k$ gives large entanglement negentropy for the final state. Why primes $p = n_2 = 2^k - r$ would be favored? The reason could be following. $n = 2^k$ corresponds to $p = 2$, which corresponds to the lowest level in p-adic evolution since it is the simplest p-adic topology and farthest from the real topology and therefore gives the poorest cognitive representation of real preferred extremal as p-adic preferred externmal (Note that $p = 1$ makes formally sense but for it the topology is discrete).
3. Weak form of NMP [K65, K116] suggests a more convincing explanation. The density matrix of the state to be reduced is a direct sum over contributions proportional to projection operators. Suppose that the projection operator with largest dimension has dimension n. Strong form of NMP would say that final state is characterized by n-dimensional projection operator.

Weak form of NMP allows free will so that all dimensions $n-k$, $k=0,1,...n-1$ for final state projection operator are possible. 1-dimensional case corresponds to vanishing entanglement negentropy and ordinary state function reduction isolating the measured system from external world.

4. The negentropy of the final state per state depends on the value of k. It is maximal if $n-k$ is power of prime. For $n=2^k=M_k+1$, where M_k is Mersenne prime $n-1$ gives the maximum negentropy and also maximal p-adic prime available so that this reduction is favoured by NMP. Mersenne primes would be indeed special. Also the primes $n=2^k-r$ near 2^k produce large entanglement negentropy and would be favored by NMP.
5. This argument suggests a generalization of p-adic length scale hypothesis so that $p=2$ can be replaced by any prime.

This argument together with the hypothesis that preferred prime is ramified would correlate the character of the irreducible extension and character of super-conformal symmetry breaking. The integer n characterizing super-symplectic conformal sub-algebra acting as gauge algebra would depends on the irreducible algebraic extension of rational involved so that the hierarchy of quantum criticalities would have number theoretical characterization. Ramified primes could appear as divisors of n and n would be essentially a characteristic of ramification known as discriminant. An interesting question is whether only the ramified primes allow the continuation of string world sheet and partonic 2-surface to a 4-D space-time surface. If this is the case, the assumptions behind p-adic mass calculations would have full first principle justification.

A connection with infinite primes?

Infinite primes are one of the mathematical outcomes of TGD [K103]. There are two kinds of infinite primes. There are the analogs of free many particle states consisting of fermions and bosons labelled by primes of the previous level in the hierarchy. They correspond to states of a supersymmetric arithmetic quantum field theory or actually a hierarchy of them obtained by a repeated second quantization of this theory. A connection between infinite primes representing bound statesandirreducible polynomials is highly suggestive.

1. The infinite prime representing free many-particle state decomposes to a sum of infinite part and finite part having no common finite prime divisors so that prime is obtained. The infinite part is obtained from "fermionic vacuum" $X=\prod_k p_k$ by dividing away some fermionic primes p_i and adding their product so that one has $X \rightarrow X/m+m$, where m is square free integer. Also $m=1$ is allowed and is analogous to fermionic vacuum interpreted as Dirac sea without holes. X is infinite prime and pure many-fermion state physically. One can add bosons by multiplying X with any integers having no common denominators with m and its prime decomposition defines the bosonic contents of the state. One can also multiply m by any integers whose prime factors are prime factors of m.
2. There are also infinite primes, which are analogs of bound states and at the lowest level of the hierarchy they correspond to irreducible polynomials $P(x)$ with integer coefficients. At the second levels the bound states would naturally correspond to irreducible polynomials $P_n(x)$ with coefficients $Q_k(y)$, which are infinite integers at the previous level of the hierarchy.
3. What is remarkable that bound state infinite primes at given level of hierarchy would define maximally ramified algebraic extensions at previous level. One indeed has infinite hierarchy of infinite primes since the infinite primes at given level are infinite primes in the sense that they are not divisible by the primes of the previous level. The formal construction works as such. Infinite primes correspond to polynomials of single variable at the first level, polynomials of two variables at second level, and so on. Could the Langlands program could be generalized from the extensions of rationals to polynomials of complex argument and that one would obtain infinite hierarchy?
4. Infinite integers in turn could correspond to products of irreducible polynomials defining more general extensions. This raises the conjecture that infinite primes for an extension K of rationals could code for the algebraic extensions of K quite generally. If infinite primes correspond to real quantum states they would thus correspond the extensions of rationals to

which the parameters appearing in the functions defining partonic 2-surfaces and string world sheets.

This would support the view that partonic 2-surfaces associated with algebraic extensions defined by infinite integers and thus not irreducible are unstable against decay to partonic 2-surfaces which corresponds to extensions assignable to infinite primes. Infinite composite integer defining intermediate unstable state would decay to its composites. Basic particle physics phenomenology would have number theoretic analog and even more.

5. According to Wikipedia, Eisenstein's criterion (`http://tinyurl.com/47kxjz`) allows generalization and what comes in mind is that it applies in exactly the same form also at the higher levels of the hierarchy. Primes would be only replaced with prime polynomials and the there would be at least one prime polynomial $Q(y)$ dividing the coefficients of $P_n(x)$ except the highest one such that its square would not divide P_0. Infinite primes would give rise to an infinite hierarchy of functions of many complex variables. At first level zeros of function would give discrete points at partonic 2-surface. At second level one would obtain 2-D surface: partonic 2-surfaces or string world sheet. At the next level one would obtain 4-D surfaces. What about higher levels? Does one obtain higher dimensional objects or something else. The union of n 2-surfaces can be interpreted also as $2n$-dimensional surface and one could think that the hierarchy describes a hierarchy of unions of correlated partonic 2-surfaces. The correlation would be due to the preferred extremal property of Kähler action.

   One can ask whether this hierarchy could allow to generalize number theoretical Langlands to the case of function fields using the notion of prime function assignable to infinite prime. What this hierarchy of polynomials of arbitrary many complex arguments means physically is unclear. Do these polynomials describe many-particle states consisting of partonic 2-surface such that there is a correlation between them as sub-manifolds of the same space-time sheet representing a preferred extremals of Kähler action?

This would suggest strongly the generalization of the notion of p-adicity so that it applies to infinite primes.

1. This looks sensible and maybe even practical! Infinite primes can be mapped to prime polynomials so that the generalized p-adic numbers would be power series in prime polynomial - Taylor expansion in the coordinate variable defined by the infinite prime. Note that infinite primes (irreducible polynomials) would give rise to a hierarchy of preferred coordinate variables. In terms of infinite primes this expansion would require that coefficients are smaller than the infinite prime P used. Are the coefficients lower level primes? Or also infinite integers at the same level smaller than the infinite prime in question? This criterion makes sense since one can calculate the ratios of infinite primes as real numbers.

2. I would guess that the definition of infinite-P p-adicity is not a problem since mathematicians have generalized the number theoretical notions to such a level of abstraction much above of a layman like me. The basic question is how to define p-adic norm for the infinite primes (infinite only in real sense, p-adically they have unit norm for all lower level primes) so that it is finite.

3. There exists an extremely general definition of generalized p-adic number fields (see `http://tinyurl.com/y5zreeg`). One considers Dedekind domain D, which is a generalization of integers for ordinary number field having the property that ideals factorize uniquely to prime ideals. Now D would contain infinite integers. One introduces the field E of fractions consisting of infinite rationals.

   Consider element e of E and a general fractional ideal eD as counterpart of ordinary rational and decompose it to a ratio of products of powers of ideals defined by prime ideals, now those defined by infinite primes. The general expression for the p-adic norm of x is $x^{-ord(P)}$, where n defines the total number of ideals P appearing in the factorization of a fractional ideal in E: this number can be also negative for rationals. When the residue field is finite (finite field G(p,1) for p-adic numbers), one can take c to the number of its elements ($c = p$ for p-adic numbers.

   Now it seems that this number is not finite since the number of ordinary primes smaller than P is infinite! But this is not a problem since the topology for completion does not depend on the value of c. The simple infinite primes at the first level (free many-particle states) can be mapped to ordinary rationals and q-adic norm suggests itself: could it be that infinite-P p-adicity corresponds to q-adicity discussed by Khrennikov [A29]. Note however that q-adic numbers are not a field.

Finally a loosely related question. Could the transition from infinite primes of K to those of L takes place just by replacing the finite primes appearing in infinite prime with the decompositions? The resulting entity is infinite prime if the finite and infinite part contain no common prime divisors in L. This is not the case generally if one can have primes P_1 and P_2 of K having common divisors as primes of L: in this case one can include P_1 to the infinite part of infinite prime and P_2 to finite part.

9.3 TGD And Classical Number Fields

This section is devoted to the vision about TGD as a generalized number theory. The basic theme is the role of classical number fields [A15, A7, A18] in quantum TGD. A central notion is $M^8 - H$ duality which might be also called number theoretic compactification. This duality allows to identify embedding space equivalently either as M^8 or $M^4 \times CP_2$ and explains the symmetries of standard model number theoretically. These number theoretical symmetries induce also the symmetries dictaying the geometry of the "world of classical worlds" (WCW) as a union of symmetric spaces [A22] . This infinite-dimensional Kähler geometry is expected to be highly unique from the mere requirement of its existence requiring infinite-dimensional symmetries provided by the generalized conformal symmetries of the light-cone boundary $\delta M^4_+ \times S$ and of light-like 3-surfaces and the answer to the question what makes 8-D embedding space and $S = CP_2$ so unique would be the reduction of these symmetries to number theory.

ZEO has become the corner stone of also number theoretical vision. In ZEO either light-like or space-like 3-surfaces can be identified as the fundamental dynamical objects, and the extension of general coordinate invariance leads to effective 2-dimensionality (strong form of holography) in the sense that the data associated with partonic 2-surfaces and the distribution of 4-D tangent spaces at them located at the light-like boundaries of causal diamonds (CDs) defined as intersections of future and past directed light-cones code for quantum physics and the geometry of WCW. Also the hierarchy of Planck constants [K38] plays a role but not so important one.

The basic number theoretical structures are complex numbers, quaternions [A18] and octonions [A15] , and their complexifications obtained by introducing additional commuting imaginary unit $\sqrt{-1}$. Hyper-octonionic (-quaternionic,-complex) sub-spaces for which octonionic imaginary units are multiplied by commuting $\sqrt{-1}$ have naturally Minkowskian signature of metric. The question is whether and how the hyper-structures could allow to understand quantum TGD in terms of classical number fields. The answer which looks the most convincing one relies on the existence of octonionic representation of 8-D gamma matrix algebra.

1. The first guess is that associativity condition for the sub-algebras of the local Clifford algebra defined in this manner could select 4-D surfaces as associative (hyper-quaternionic) sub-spaces of this algebra and define WCW purely number theoretically. The associative sub-spaces in question would be spanned by the modified gamma matrices defined by the Kähler-Dirac action fixed by the variational principle (Kähler action) selecting space-time surfaces as preferred extremals [K121] .
2. This condition is quite not enough: one must strengthen it with the condition that a preferred commutative and thus hyper-complex sub-algebra is contained in the tangent space of the space-time surface. This condition actually generalizes somewhat since one can introduce a family of so called Hamilton-Jacobi coordinates for M^4 allowing an integrable distribution of decompositions of tangent space to the space of non-physical and physical polarizations [K14] . The physical interpretation is as a number theoretic realization of gauge invariance selecting a preferred local commutative plane of non-physical polarizations.
3. Even this is not yet the whole story: one can define also the notions of co-associativity and co-commutativity applying in the regions of space-time surface with Euclidian signature of the induced metric. The basic unproven conjecture is that the decomposition of space-time surfaces to associative and co-associative regions containing preferred commutative *resp.*

co-commutative 2-plane in the 4-D tangent plane is equivalent with the preferred extremal property of Kähler action and the hypothesis that space-time surface allows a slicing by string world sheets and by partonic 2-surfaces [K121] .

9.3.1 Notations

Some notational conventions are in order before continuing. The fields of quaternions *resp.* octonions having dimension 4 *resp.* 8 and will be denoted by Q and O. Their complexified variants will be denoted by Q_C and O_C. The sub-spaces of hyper-quaternions HQ and hyper-octonions HO are obtained by multiplying the quaternionic and octonionic imaginary units by $\sqrt{-1}$. These sub-spaces are very intimately related with the corresponding algebras, and can be seen as Euclidian and Minkowkian variants of the same basic structure. Also the Abelianized versions of the hyper-quaternionic and -octonionic sub-spaces can be considered: these algebras have a representation in the space of spinors of embedding space $H = M^4 \times CP_2$.

9.3.2 Quaternion And Octonion Structures And Their Hyper Counterparts

In this introductory section the notions of quaternion and octonion structures and their hyper counterparts are introduced with strong emphasis on the physical interpretation. Literature contains several variants of these structures (Hyper-Kähler structure [A9] and quaternion Kähler structure possed also by CP_2 [A63]). The notion introduced here is inspired by the physical motivations coming from TGD. As usual the first proposal based on the notions of (hyper-)quaternion and (hyper-)octonion analyticity was not the correct one. Much later a local variant of the notion based on tangent space emerged.

Octonions and quaternions

In the following only the basic definitions relating to octonions and quaterions are given (see **Fig. 9.1**). There is an excellent article by John Baez [A15] describing octonions and their relations to the rest of mathematics and physics.

Octonions can be expressed as real linear combinations $\sum_k x^k I_k$ of the octonionic real unit $I_0 = 1$ (counterpart of the unit matrix) and imaginary units I_a, $a = 1, ..., 7$ satisfying

$$\begin{aligned} I_0^2 &= I_0 \equiv 1 \ , \\ I_a^2 &= -I_0 = -1 \ , \\ I_0 I_a &= I_a \ . \end{aligned} \tag{9.3.1}$$

Octonions are closed with respect to the ordinary sum of the 8-dimensional vector space and with respect to the octonionic multiplication, which is neither commutative ($ab \neq ba$ in general) nor associative ($a(bc) \neq (ab)c$ in general).

A concise manner to summarize octonionic multiplication is by using octonionic triangle. Each line (6 altogether) containing 3 octonionic imaginary units forms an associative triple which together with $I_0 = 1$ generate a division algebra of quaternions. Also the circle spanned by the 3 imaginary units at the middle of the sides of the triangle is associative triple. The multiplication rules for each associative triple are simple:

$$I_a I_b = \epsilon_{abc} I_c \ , \tag{9.3.2}$$

where ϵ_{abc} is 3-dimensional permutation symbol. $\epsilon_{abc} = 1$ for the clockwise sequence of vertices (the direction of the arrow along the circumference of the triangle and circle). As a special case this rule gives the multiplication table of quaternions. A crucial observation for what follows is that any pair of imaginary units belongs to one associative triple.

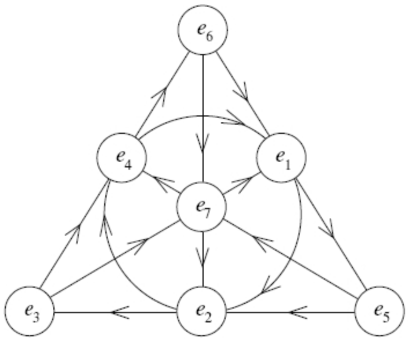

Figure 9.1: Octonionic triangle: the six lines and one circle containing three vertices define the seven associative triplets for which the multiplication rules of the ordinary quaternion imaginary units hold true. The arrow defines the orientation for each associative triplet. Note that the product for the units of each associative triplets equals to real unit apart from sign factor.

The non-vanishing structure constants ${d_{ab}}^c$ of the octonionic algebra can be read directly from the octonionic triangle. For a given pair I_a, I_b one has

$$\begin{aligned} I_aI_b &= {d_{ab}}^c I_c \ , \\ d_{ab} \quad c &= {\epsilon_{ab}}^c \ , \\ I_a^2 &= {d_{aa}}^0 I_0 = -I_0 \ , \\ I_0^2 &= {d_{00}}^0 I_0 \ , \\ I_0 I_a &= {d_{0a}}^a I_a = I_a \ . \end{aligned} \tag{9.3.3}$$

For ϵ_{abc} c belongs to the same associative triple as ab.

Non-associativity means that is not possible to represent octonions as matrices since matrix product is associative. Quaternions can be represented and the structure constants provide the defining representation as $I_a \rightarrow d_{abc}$, where b and c are regarded as matrix indices of 4×4 matrix. The algebra automorphisms of octonions form 14-dimensional group G_2, one of the so called exceptional Lie-groups. The isotropy group of imaginary octonion unit is the group $SU(3)$. The Euclidian inner product of the two octonions is defined as the real part of the product $\overline{x}y$

$$\begin{aligned} (x,y) &= Re(\overline{x}y) = \sum_{k=0,..7} x_k y_k \ , \\ \overline{x} &= x^0 I_0 - \sum_{i=1,..,7} x^k I_k \ , \end{aligned} \tag{9.3.4}$$

and is just the Euclidian norm of the 8-dimensional space.

Hyper-octonions and hyper-quaternions

The Euclidicity of the quaternion norm suggests that octonions are not a sensible concept in TGD context. One can imagine two ways to circumvent this conclusion.

1. Minkowskian metric for octonions and quaternions is obtained by identifying Minkowski inner product xy as the real counterpart of the product

$$x \cdot y \equiv Re(xy) = x^0 y^0 - \sum_k x^k y^k \ . \tag{9.3.5}$$

$SO(1,7)$ ($SO(1,3)$ in quaternionic case) Lorentz invariance appears completely naturally as the symmetry of the real part of the octonion (quaternion) product and hence of octonions/quaternions and there is no need to perform the complexification of the octonion algebra. Furthermore, only the signature $(1,7)$ ($(1,3)$ in the quaternionic case) is possible and this would raise $M^4_+ \times CP_2$ in a preferred position.

This norm does not give rise to a number theoretic norm defining a homomorphism to real numbers. Indeed, the number theoretic norm defined by the determinant of the linear mapping defined by the multiplication with quaternion or octonion, is inherently Euclidian. This is in conflict with the idea that quaternionic and octonionic primes and their infinite variants should have key role in TGD [K103] .

2. Hyper-octonions and hyper-quaternions provide a possible solution to these problems. These are obtained by multiplying imaginary units by commutative and associative $\sqrt{-1}$. These numbers form a sub-space of complexified octonions/quaternions and the cross product of imaginary parts leads out from this sub-space. In this case number theoretic norm induced from Q_C/O_C gives the fourth/eighth power of Minkowski length and Lorentz group acts as its symmetries. Light-like hyper-quaternions and -octonions causing the failure of the number field property have also a clear physical interpretation.

   A criticism against the notion of hyper-quaternionic and octonionic primeness is that the tangent space as an algebra property is lost and the notion of primeness is inherited from Q_C/O_C. Also non-commutativity and non-associativity could cause difficulties.

ZEO leads to a possible physical interpretation of complexified octonions. The moduli space for causal diamonds corresponds to a Cartesian product of $M^4 \times CP_2$ whose points label the position of either tip of $CD \times CP_2$ and space I whose points label the relative positive of the second tip with respect to the first one. p-Adic length scale hypothesis results if one assumes that the proper time distance between the tips comes in powers of two so that one has union of hyperboloids $H_n \times CP_2$, $H_n = \{m \in M^4_+ | a = 2^n a_0)\}$. A further quantization of hyperboloids H_n is obtained by replacing it with a lattice like structure is highly suggestive and would correspond to an orbit of a point of H_n under a subgroup of $SL(2,Q_C)$ or $SL(2,Z_C)$ acting as Lorentz transformations in standard manner. Also algebraic extensions of Q_C and Z_C can be considered. Also in the case of CP_2 discretization is highly suggestive so that one would have an orbit of a point of CP_2 under a discrete subgroup of $SU(3,Q)$.

The outcome could be interpreted by saying that the moduli space in question is $H \times I$ such that H corresponds to hyper-octonions and I to a discretized version of $\sqrt{-1}H$ and thus a subspace of complexified octonions. An open question whether the quantization has some deeper mathematical meaning.

Basic constraints

Before going to details it is useful to make clear the constraints on the concept of the hyper-octonionic structure implied by TGD view about physics.

$M^4 \times CP_2$ cannot certainly be regarded as having any global octonionic structure (for instance in the sense that it could be regarded as a coset space associated with some exceptional group). There are however clear indications for the importance of the hyper-quaternionic and -octonionic structures.

1. $SU(3)$ is the only simple 8-dimensional Lie-group and acts as the group of isometries of CP_2: if $SU(3)$ had some kind of octonionic structure, CP_2 would become unique candidate for the space S. The decomposition $SU(3) = h+t$ to $U(2)$ subalgebra and its complement corresponds rather closely to the decomposition of (hyper-)octonions to (hyper-)quaternionic sub-space and its complement. The electro-weak $U(2)$ algebra has a natural 1+3 decomposition and generators allow natural hyper-quaternionic structure. The components of the Weyl tensor of CP_2 behave with respect to multiplication like quaternionic imaginary units but only one of them is covariantly constant so that hyper Kähler structure [A9] with three covariantly constant quaternionic imaginary units represented by Kähler forms is not possible. These tensors and metric tensor however define quaternionic structure [A63] .
2. M^4_+ has a natural 1+3 decomposition and a unique cosmic time coordinate defined as the light cone proper time. Hyper-quaternionic structure is consistent with the Minkowskian signature

of the inner product and hyper quaternion units have a natural representation in terms of covariantly constant self-dual symplectic forms [A42, A24, A23] and their contractions with sigma matrices. It is not however clear whether this representation is physically interesting.

How to define hyper-quaternionic and hyper-octonionic structures?

I have considered several proposals for how to define quaternionic and octonionic structures and their hyper-counterparts.

1. (Hyper-)octonionic manifolds would obtained by gluing together coordinate patches using (hyper-)octonion analytic functions with real Laurent coefficients (this guarantees associativity and commutativity). This definition does not yet involve metric or any other structures (such as Kähler structure). This approach does not seem to be physically realistic.

2. Second option is based on the idea of representing quaternionic and octonionic imaginary units as antisymmetric tensors. This option makes sense for quaternionic manifolds [A17] and CP_2 indeed represents an example represents of this kind of manifold. The problem with the octonionic structure is that antisymmetric tensors cannot define non-associative product.

3. If the manifold is endowed with metric, octonionic structure should be defined as a local tangent space structure analogous to eight-bein structure and local gauge algebra structures. This can be achieved by contracting octo-bein vectors with the standard octonionic basis to get octonion form I_k. Each vector field a^k defines naturally octonion field $A = a^k I_k$. The product of two vector fields can be defined by the octonionic multiplication and this leads to the introduction of a tensor field d_{klm} of these structure constants obtained as the contraction of the octobein vectors with the octonionic structure constants d_{abc}. Hyper-octonion structure can defined in a completely analogous manner.

   It is possible to induce octonionic structure to any 4-dimensional space-time surface by forming the projection of I_k to the space-time surface and redefining the products of I_k:s by dropping away that part of the product, which is orthogonal to the space-time surface. This means that the structure constants of the new 4-dimensional algebra are the projections of d_{klm} to the space-time surface. One can also define similar induced algebra in the 4-dimensional normal space of the space-time surface. The hypothesis would be that the induced tangential is associative or hyper-quaternionic algebra. Also co-associativity defined as associativity of the normal space algebra is possible. This property would give for the 4-dimensionality of the space-time surface quite special algebraic meaning. The problem is now that there is no direct connection with quantum TGD proper- in particular the connection with the classical dynamics defined by Kähler action is lacking.

4. 8-dimensional gamma matrices allow a representation in terms of tensor products of octonions and 2×2 matrices. Genuine matrices are of course not in question since the product of the gamma matrices fails to be associative. An associative representation is obtained by restricting the matrices to a quaternionic plane of complex octonions. If the space-time surface is hyper-quaternionic in the sense that induced gamma matrices define a quaternionic plane of complexified octonions at each point of space-time surface the resulting local Clifford algebra is associative and structure constants define a matrix representation for the induced gamma matrices.

   A more general definition allows gamma matrices to be Kähler-Dirac gamma matrices defined by Kähler action appearing in the Kähler-Dirac action and forced both by internal consistency and super-conformal symmetry [K121] . The Kähler-Dirac gamma matrices associated with Kähler action do not in general define tangent space of the space-time surface as the induced gamma matrices do. Also co-associativity can be considered if one can identify a preferred imaginary unit such that the multiplication of the Kähler-Dirac gamma matrices with this unit gives a quaternionic basis. This condition makes sense only if the preferred extremals of the action are hyper-quaternionic surfaces in the sense defined by the action. That this is true for Kähler action at least is an is an unproven conjecture.

In the sequel only the fourh option will be considered.

How to end up to quantum TGD from number theory?

An interesting possibility is that quantum TGD could emerge from a condition that a local version of hyper-finite factor of type II_1 represented as a local version of infinite-dimensional Clifford algebra exists. The conditions are that "center or mass" degrees of freedom characterizing the position of CD separate uniquely from the "vibrational" degrees of freedom being represented in terms of octonions and that for physical states associativity holds true. The resulting local Clifford algebra would be identifiable as the local Clifford algebra of WCW (being an analog of local gauge groups and conformal fields [A19]).

The uniqueness of M^8 and $M^4 \times CP_2$ as well as the role of hyper-quaternionic space-time surfaces as fundamental dynamical objects indeed follow from rather weak conditions if one restricts the consideration to gamma matrices and spinors instead of assuming that M^8 coordinates are hyper-octonionic as was done in the first attempts.

1. The unique feature of M^8 and any 8-dimensional space with Minkowski signature of metric is that it is possible to have an octonionic representation of the complexified gamma matrices [K121, K27] and of spinors. This does not require octonionic coordinates for M^8. The restriction to a quaternionic plane for both gamma matrices and spinors guarantees the associativity.
2. One can also consider a local variant of the octonionic Clifford algebra in M^8. This algebra contains associative subalgebras for which one can assign to each point of M^8 a hyper-quaternionic plane. It is natural to assume that this plane is either a tangent plane of 4-D manifold defined naturally by the induced gamma matrices defining a basis of tangent space or more generally, by Kähler-Dirac gamma matrices defined by a variational principle (these gamma matrices do not define tangent space in general). Kähler action defines a unique candidate for the variational principle in question. Associativity condition would automatically select sub-algebras associated with 4-D hyper-quaternionic space-time surfaces.
3. This vision bears a very concrete connection to quantum TGD. In [K27] the octonionic formulation of the Kähler-Dirac equation is studied and shown to lead to a highly unique general solution ansatz for the equation working also for the matrix representation of the Clifford algebra. An open question is whether the resulting solution as such defined also solutions of the Kähler-Dirac equation for the matrix representation of gammas. Also a possible identification for 8-dimensional counterparts of twistors as octo-twistors follows: associativity implies that these twistors are very closely related to the ordinary twistors. In TGD framework octo-twistors provide an attractive manner to get rid of the difficulties posed by massive particles for the ordinary twistor formalism.
4. Associativity implies hyperquaternionic space-time surfaces (in a more general sense as usual) and this leads naturally to the notion of WCW and local Clifford algebra in this space. Number theoretic arguments imply $M^8 - H$ duality. The resulting infinite-dimensional Clifford algebra would differ from von Neumann algebras in that the Clifford algebra and spinors assignable to the center of mass degrees of freedom of causal diamond CD would be expressed in terms of octonionic units although they are associative at space-time surfaces. One can therefore say that quantum TGD follows by assuming that the tangent space of the embedding space corresponds to a classical number field with maximal dimension.
5. The slicing of the Minkowskian space-time surface inside CD by stringy world sheets and by partonic 2-surfaces inspires the question whether the Kähler-Dirac gamma matrices associated with the stringy world sheets *resp.* partonic 2-surfaces could be could commutative *resp.* co-commutative. Commutativity would also be seen as the justification for why the fundamental objects are effectively 2-dimensional.

This formulation is undeniably the most convincing one found hitherto since the notion of hyper-quaternionic structure is local and has elegant formulation in terms of Kähler-Dirac gamma matrices.

9.3.3 Number Theoretic Compactification And $M^8 - H$ Duality

This section summarizes the basic vision about number theoretic compactification reducing the classical dynamics to associativity or co-associativity. Originally $M^8 - H$ duality was introduced as a number theoretic explanation for $H = M^4 \times CP_2$. Much later it turned out that the completely exceptional twistorial properties of M^4 and CP_2 are enough to justify $X^4 \subset H$ hypothesis. Skeptic could therefore criticize the introduction of M^8 (or even its complexification) as an un-necessary mathematical complication producing only unproven conjectures and bundle of new statements to be formulated precisely.

The basic ideas in nutshell

The vision about the physical role of the classical number fields relies on certain speculative questions and ideas.

1. Could space-time surfaces X^4 be regarded as associative or co-associative ("quaternionic" is equivalent with "associative") surfaces of H endowed with octonionic structure in the sense that tangent space of space-time surface would be associative (co-associative) sub-space of octonions at each point of X^4 [K105]. This is certainly possible and an interesting conjecture is that the preferred extremals of Kähler action include associative and co-associative surfaces of H. Signature of M^8 could be a problem in M^8: M^8 can be regarded as linear sub-space of complexified octonions and the product of M^8 points does not belong to M^8. For tangent space this is not the case since one can complexify tangent space.
2. Could the notion of compactification generalize to that of number theoretic compactification in the sense that one can map associative (co-associative) surfaces of M^8 regarded as octonionic linear space to surfaces in $M^4 \times CP_2$ [K105]? This conjecture - $M^8 - H$ duality - would give for $M^4 \times CP_2$ deep number theoretic meaning. CP_2 would parametrize associative planes of octonion space containing fixed complex plane $M^2 \subset M^8$ and CP_2 point would thus characterize the tangent space of $X^4 \subset M^8$. The point of M^4 would be obtained by projecting the point of $X^4 \subset M^8$ to a point of M^4 identified as tangent space of X^4. This would guarantee that the dimension of space-time surface in H would be four. The conjecture is that the preferred extremals of Kähler action include these surfaces.
3. $M^8 - H$ duality can be generalized to a duality $H \rightarrow H$ if the images of the associative surface in M^8 is associative surface in H. One can start from associative surface of H and assume that it contains the preferred M^2 tangent plane in 8-D tangent space of H or integrable distribution $M^2(x)$ of them, and its points to H by mapping M^4 projection of H point to itself and associative tangent space to CP_2 point. This point need not be the original one! If the resulting surface is also associative, one can iterate the process indefinitely.
4. G_2 defines the automorphism group of octonions, and one might hope that the maps of octonions to octonions such that the action of Jacobian in the tangent space of associative or co-associative surface reduces to that of G_2 could produce new associative/co-associative surfaces. The action of G_2 would be analogous to that of gauge group.
5. One can also ask whether the notions of commutativity and co-commutavity could have physical meaning. The well-definedness of em charge as quantum number for the modes of the induced spinor field requires their localization to 2-D surfaces (right-handed neutrino is an exception) - string world sheets and partonic 2-surfaces. This can be possible only for Kähler action and could have commutativity and co-commutativity as a number theoretic counterpart. The basic vision would be that the dynamics of Kähler action realizes number theoretical geometrical notions like associativity and commutativity and their co-notions.

One can go even further and ask whether one could somehow construct the preferred extremals of Kähler action using real-octonion analytic functions, call them generically f. For some time I believed to this idea but it seems I was wrong. The fact that octonion real-analytic functions in M^8 section of M^8_c have values in the space of complexified octonions makes the complexification of octonions necessary. The simplest guess would be that quaternionic 4-surfaces correspond to the loci at which the values of function f are real quaternionic. One clearly obtains quaternionic planes as trivial solutions but it is not clear whether their inverse images in general case are quaternionic surfaces and whether non-trivial surfaces with physical properties are obtained. In complex case Riemann zeta serves as a discouraging much simpler analogy since real sug-manifolds of complex plane are just pieces of real axis. Quaternionicity would be replaced with reality and the loci of zeros of the imaginary part of function should be pieces of real axes. Zeta is real at real axis and also at the line $Im(s) = 1/2$ but the inverse image of this line is not real line. Therefore this approach does not look promising.

Is Kähler action needed also at the level of M^8

One can question the feasibility of $M^8 - H$ duality if the dynamics is purely number theoretic at the level of M^8 and determined by Kähler action at the level of H. Situation becomes more democratic if Kähler action defines the dynamics in both M^8 and H: this might mean that associativity could imply field equations for preferred extremals or vice versa or there might be equivalence between two. This means the introduction Kähler structure at the level of M^8, and motivates also the coupling of Kähler gauge potential to M^8 spinors characterized by Kähler charge or em charge. One could call this form of duality strong form of $M^8 - H$ duality.

The strong form $M^8 - H$ duality boils down to the assumption that space-time surfaces can be regarded either as 4-surfaces of H or as surfaces of M^8 composed of associative and co-associative regions identifiable as regions of space-time possessing Minkowskian *resp.* Euclidian signature of the induced metric.

Could they have the same induced metric and Kähler form and WCW associated with H should be essentially the same as that associated with M^8. Associativity corresponds to (hyper-)quaterniocity at the level of tangent space and co-associativity to co(-hyper)-quaternionicity - that is associativity/hyper-quaternionicity of the normal space. Both are needed to cope with known extremals. Since in Minkowskian context precise language would force to introduce clumsy terms like hyper-quaternionicity and co-hyper-quaternionicity, it is better to speak just about associativity or co-associativity.

For the octonionic spinor fields the octonionic analogs of electroweak couplings reduce to mere Kḧler or electromagnetic coupling and the solutions reduce to those for spinor d'Alembertian in 4-D harmonic potential breaking $SO(4)$ symmetry. Due to the enhanced symmetry of harmonic oscillator, one expects that partial waves are classified by $SU(4)$ and by reduction to $SU(3)\times U(1)$ by em charge and color quantum numbers just as for CP_2 - at least formally.

Harmonic oscillator potential defined by self-dual em field splits M^8 to $M^4 \times E^4$ and implies Gaussian localization of the spinor modes near origin so that E^4 effectively compactifies. The resulting physics brings strongly in mind low energy physics, where only electromagnetic interaction is visible directly, and one cannot avoid associations with low energy hadron physics. These are some of the reasons for considering $M^8 - H$ duality as something more than a mere mathematical curiosity.

Kähler form for M^8 non-trivial only in $E^4 \subset M^8$ implies unique decomposition $M^8 = M^4 \times E^4$ making possible to identify M^4 point in $M^8 - H$ duality uniquely. It however turns out that M^4 point corresponds naturally to a projection of M^8 point to the quaternionic tangent space.

Definition of complexified octonions and quaternions

The Minkowskian signatures of M^8 and M^4 produce technical nuisance if one tries to define octonion-real- analyticity. One might try to overcome it by Wick rotation, which is however somewhat questionable trick. $M^8_c = O_c$ provides another approach giving hopes. Complexified tangent space must be introduced in any case so that its detailed definition deserves to discussed.

1. The proper formulation for tangent space is in terms of complexified octonions and quaternions involving the introduction of commuting imaginary unit j. If complexified quaternions are used for H, Minkowskian signature requires the introduction of two commuting imaginary units j and i meaning double complexification.

2. Hyper-quaternions/octonions define as subspace of complexified quaternions/octonions spanned by real unit and jI_k, where I_k are quaternionic units. These spaces are obviously not closed under multiplication. One can however define the notion of associativity for the sub-space of M^8 by requiring that the products and sums of the tangent space vectors generate complexified quaternions.

3. Ordinary quaternions Q are expressible as $q = q_0 + q^k I_k$. Hyper-quaternions are expressible as $q = q_0 + jq^k I_k$ and form a subspace of complexified quaternions $Q_c = Q \oplus jQ$. Similar

formula applies to octonions and their hyper counterparts which can be regarded as subspaces of complexified octonions $O \oplus jO$.

4. One can consider two ways to identify the tangent space of H. Either as 8-D manifold for which tangent space is hyper-octonionic linear sub-space of complexified octonions O_c generated by sums and products of tangent vectors. Tangent space vectors of H could be also identified as hyper-quaternions $q_H = q_0 + jq^k I_k + jiq_2$ defining a subspace of doubly complexified quaternions: note the appearance of two imaginary units. This would imply an asymmetry between M^8 and H. The first option looks more elegant also because the composition of the duality maps can be iterated as maps of surfaces of H to those of H.

1. Are gamma matrices needed at all?

The recent definitions of associativity and $M^8 - H$- duality has evolved slowly from inaccurate characterizations and there are still open questions.

1. The standard spinor structure of H can be regarded as quaternionic in the sense that gamma matrices are essentially tensor products of quaternionic gamma matrices and reduce in matrix representation for quaternions to ordinary gamma matrices. Therefore the idea that one should introduce octonionic gamma matrices in H or even M^8 would mean doubling of the spinor structure: not an attractive idea.

   It is however important to notice that the introduction of octonionic gamma matrices is not necessary. Simplest option is just the interpretation of tangent basis vectors are octonions: octonion basis is obtained as contractions of vielbein vectors with "flat space" octonions.

2. The earlier formulation was in terms of octonionic flat space gamma matrices replacing the ordinary gamma matrices so that the formulation reduces to that in M^8 tangent space. This formulation is enough to define what associativity means although one can protest.

3. The known extremals provide a test for the associativity (co-associativity) hypothesis. I have not demonstrated that the associativity works for massless extremals (MEs) and vacuum extremals with the dimension of CP_2 projection not larger than 2.

4. Could one define associativity in H also in terms of modified gamma matrices defined by Kähler action (certainly not M^8)? The basic problem is that the space spanned by the Kähler-Dirac gamma matrices can have dimension smaller than that of 4 (so that co-basis would hve dimension larger than 4 if identified in terms of orthogonal complement). Second problem is that Kähler-Dirac gammas are in general not in the tangent space of space-time surface as vectors of the embedding space. Therefore the notions of associativity (co-associativity) defined in terms of tangent space (normal space) become problematic.

Basic formulation of $M^8 - H$ duality

If four-surfaces $X^4 \subset M^8$ under some conditions define 4-surfaces in $M^4 \times CP_2$ indirectly, the spontaneous compactification of super string models would correspond in TGD to two different ways to interpret the space-time surface. This correspondence could be called number theoretical compactification or $M^8 - H$ duality.

Basic mathematical facts

The hard mathematical facts behind the notion of number theoretical compactification are following.

1. One manner to define M^4 image of M^8 point uniquely would be to assume that M^8 has unique decomposition $M^8 = M^4 \times E^4$ (it turns out that this is not the correct manner!). This would be most naturally due to Kähler structure in E^4 defined by a self-dual Kähler form defining parallel constant electric and magnetic fields in Euclidian sense. Besides Kähler form there is vector field coupling to sigma matrix representing the analog of strong isospin: the corresponding octonionic sigma matrix however is imaginary unit times gamma matrix - say ie_1 in M^4 - defining a preferred plane M^2 in M^4. Here it is essential that the gamma matrices of E^4 defined in terms of octonion units commute to gamma matrices in M^4. What

is involved becomes clear from the Fano triangle illustrating octonionic multiplication table. One can however do also without the introduction of this structure and use only the octonionic structure.

2. The space of hyper-complex structures of the hyper-octonion space - they correspond to the choices of plane $M^2 \subset M^8$ - is parameterized by 6-sphere $S^6 = G^2/SU(3)$. The subgroup $SU(3)$ of the full automorphism group G_2 respects the a priori selected complex structure and thus leaves invariant one octonionic imaginary unit, call it e_1. Fixed complex structure therefore corresponds to a point of S^6.

3. Quaternionic sub-algebras of M^8 are parametrized by $G_2/U(2)$. The quaternionic sub-algebras of octonions with fixed complex structure (that is complex sub-space defined by real and preferred imaginary unit and parametrized by a point of S^6) are parameterized by $SU(3)/U(2) = CP_2$ just as the complex planes of quaternion space are parameterized by $CP_1 = S^2$. Same applies to hyper-quaternionic sub-spaces of hyper-octonions. $SU(3)$ would thus have an interpretation as the isometry group of CP_2, as the automorphism sub-group of octonions, and as color group. Thus the space of quaternionic structures can be parametrized by the 10-dimensional space $G_2/U(2)$ decomposing as $S^6 \times CP_2$ locally.

4. The basic result behind number theoretic compactification and $M^8 - H$ duality is that associative sub-spaces $M^4 \subset M^8$ containing a fixed commutative sub-space $M^2 \subset M^8$ are parameterized by CP_2. The choices of a fixed hyper-quaternionic basis $1, e_1, e_2, e_3$ with a fixed complex sub-space (choice of e_1) are labeled by $U(2) \subset SU(3)$. The choice of e_2 and e_3 amounts to fixing $e_2 \pm \sqrt{-1}e_3$, which selects the $U(2) = SU(2) \times U(1)$ subgroup of $SU(3)$. $U(1)$ leaves 1 invariant and induced a phase multiplication of e_1 and $e_2 \pm e_3$. $SU(2)$ induces rotations of the spinor having e_2 and e_3 components. Hence all possible completions of $1, e_1$ by adding e_2, e_3 doublet are labeled by $SU(3)/U(2) = CP_2$.

1. Formulation of $M^8 - H$ duality

Consider now the formulation of $M^8 - H$ duality.

1. The idea of the standard formulation is that associative manifold $X^4 \subset M^8$ has at its each point associative tangent plane. That is X^4 corresponds to an integrable distribution of $M^2(x) \subset M^8$ parametrized 4-D coordinate x that is map $x \rightarrow S^6$ such that the 4-D tangent plane is hyper-quaternionic for each x.

2. One should be able to assign a unique point of M^4 to a given point of $X^4 \subset M^8$.

   (a) The associative tangent space of space-time surface shifted to go through the origin of M^8 defines the preferred $M^4 \subset M^8$ uniquely, and one can projects the point of M^8 to this M^4 to get M^4 point. This identification implies that the dimension of tangent space projection to M^4 is maximum, and one avoids the situations in which the image surface of H has dimension smaller than 4.

   (b) One can imagine also second option which however fails. Since the Kähler structure of M^8 implies a unique decomposition $M^8 = M^4 \times E^4$, this surface in turn defines a surface in $M^4 \times CP_2$ obtained by assigning to the point of 4-surface point $(m, s) \in H = M^4 \times CP_2$: $m \in M^4$ is obtained as *projection* $M^8 \rightarrow M^4$ (this is modification to the original definition) and $s \in CP_2$ parametrizes the quaternionic tangent plane as point of CP_2. Here the local decomposition $G_2/U(2) = S^6 \times CP_2$ is essential for achieving uniqueness.
   One can however represent objection to this identification. The dimension of image in H is smaller than 4. For instance, hyperquaternionic plane M^4_1 which has M^2 the intersection with preferred M^4 corresponds to constant CP_2 point so that its H image is M^2.

2. Generalization to $H - H$ duality

As a matter fact, $M^8 - H$ duality might generalize to $H - H$ duality allowing to integrate space-time surfaces and thus WCW to a category.

1. The map of space-time surfaces of M^8 to those of $H = M^4 \times CP_2$ need not imply that the image surfaces in H are quaternionic in H. If they are, then the construction can be iterated.

It seems that one continue this series ad infinitum and could generate new solutions of field equations! If this is the case, one could iterate duality as a sequence $M^8 \rightarrow H \rightarrow H...$ by mapping the space-time surface to $M^4 \times CP_2$ by the same recipe as in case of M^8. One would obtain basically a category of space-time surfaces with arrows defined by the duality. Same probably applies to co-associative surfaces. This certainly makes the heart of mathematician beat.

2. It is not proven that associativity/co-associativity implies preferred extremal property for Kähler action. One thing to understand is why Kähler action. An argument in favor of preferred role of Kähler action is that only Kähler action allows localization of spinor modes to 2-D surfaces essential for the well-definedness of em charge [K121]. These surface would be string world sheets and possibly also partonic 2-surfaces and their could correspond to commutative and co-commutative 2-surfaces in number theoretic vision and be well-defined also for M^8. If so, Kähler action would provide a physical representation for the number theoretic notions like associativity and commutativity and their co-notions.
3. If all goes as in dreams, the mere associativity or co-associativity in M^8 would code for the preferred extremal property of Kähler action in H and would imply this property in H. The surfaces with this property would form category with arrow defined by the duality.
4. One could also map the associative surface in M^8 to surface in 10-dimensional $S^6 \times CP_2$. In this case the metric of the image surface cannot have Minkowskian signature and one cannot assume that the induced metrics are identical. It is not known whether S^6 allows genuine complex structure and Kähler structure which is essential for TGD formulation.

3. Some comments

A couple of comments are in order.

1. This definition differs from the first proposal for years ago stating that each point of X^4 contains a *fixed* $M^2 \subset M^4$ rather than $M_2(x) \subset M^8$ and also from the proposal assuming integrable distribution of $M^2(x) \subset M^4$. The older proposals are not consistent with the properties of massless extremals and string like objects for which the counterpart of M^2 depends on space-time point and is not restricted to M^4. The earlier definition $M^2(x) \subset M^4$ was problematic in the co-associative case since for the Euclidian signature is is not clear what the counterpart of $M^2(x)$ could be.
2. The new definition is consistent with the existence of Hamilton-Jacobi structure meaning slicing of space-time surface by string world sheets and partonic 2-surfaces with points of partonic 2-surfaces labeling the string world sheets [K14]. This structure has been proposed to characterize preferred extremals in Minkowskian space-time regions at least.
3. Co-associative Euclidian 4-surfaces, say CP_2 type vacuum extremal do not contain integrable distribution of $M^2(x)$. It is normal space which contains $M^2(x)$. Does this have some physical meaning? Or does the surface defined by $M^2(x)$ have Euclidian analog?

   A possible identification of the analog would be as string world sheet at which W boson field is pure gauge so that the modes of the modified Dirac operator [K121] restricted to the string world sheet have well-defined em charge. This condition appears in the construction of solutions of Kähler-Dirac operator.

   For octonionic spinor structure the W coupling is however absent so that the condition does not make sense in M^8. The number theoretic condition would be as commutative or co-commutative surface for which imaginary units in tangent space transform to real and imaginary unit by a multiplication with a fixed imaginary unit! One can also formulate co-associativity as a condition that tangent space becomes associative by a multiplication with a fixed imaginary unit.

   There is also another justification for the distribution of Euclidian tangent planes. The idea about associativity as a fundamental dynamical principle can be strengthened to the statement that space-time surface allows slicing by hyper-complex or complex 2-surfaces, which are commutative or co-commutative inside space-time surface. The physical interpretation would be as Minkowskian or Euclidian string world sheets carrying spinor modes. This would give a connection with string model and also with the conjecture about the general structure of preferred extremals.

4. Minimalist could argue that the minimal definition requires octonionic structure and associativity *only* in M^8. There is no need to introduce the counterpart of Kähler action in M^8 since the dynamics would be based on associativity or co-associativity alone. Not that the decomposition $M^8 = M^4 \times E^4$ is not necessary if M^4 projection is defined to the M^4 defined by hyper-quaternionic tangent place.

Hyper-octonionic Pauli "matrices" and the definition of associativity

Octonionic Pauli matrices suggest an interesting possibility to define precisely what associativity means at the level of M^8 using gamma matrices (for background see [L8]).

1. According to the standard definition space-time surface $X^4 \subset M^8$ is associative if the tangent space at each point of X^4 in $X^4 \subset M^8$ picture is associative. The definition can be given also in terms of octonionic gamma matrices whose definition is completely straightforward.
2. Could/should one define the analog of associativity at the level of H? One can identify the tangent space of H as M^8 and can define octonionic structure in the tangent space and this allows to define associativity locally. One can replace gamma matrices with their octonionic variants and formulate associativity in terms of them locally and this should be enough.

   Skeptic however reminds M^4 allows hyper-quaternionic structure and CP_2 quaternionic structure so that complexified quaternionic structure would look more natural for H. The tangent space would decompose as $M^8 = HQ + ijQ$, weher j is commuting imaginary unit and HQ is spanned by real unit and by units iI_k, where i second commutating imaginary unit and I_k denotes quaternionic imaginary units. There is no need to make anything associative.

   There is however far from obvious that octonionic spinor structure can be (or need to be!) defined globally. The lift of the CP_2 spinor connection to its octonionic variant has questionable features: in particular vanishing of the charged part and reduction of neutral part to photon. Therefore is is unclear whether associativity condition makes sense for $X^4 \subset M^4 \times CP_2$. What makes it so fascinating is that it would allow to iterate duality as a sequences $M^8 \to H \to H$.... This brings in mind the functional composition of octonion real-analytic functions suggested to produced associative or co-associative surfaces.

I have not been able to settle the situation. What seems the working option is associativity in both M^8 and H and Kähler-Dirac gamma matrices defined by appropriate Kähler action and correlation between associativity and preferred extremal property.

Are Kähler and spinor structures necessary in M^8?

If one introduces M^8 as dual of H, one cannot avoid the idea that hyper-quaternionic surfaces obtained as images of the preferred extremals of Kähler action in H are also extremals of M^8 Kähler action with same value of Kähler action defining Kähler function. As found, this leads to the conclusion that the $M^8 - H$ duality is Kähler isometry. Coupling of spinors to Kähler potential is the next step and this in turn leads to the introduction of spinor structure so that quantum TGD in H should have full M^8 dual.

1. Are also the 4-surfaces in M^8 preferred extremals of Kähler action?

It would be a mathematical miracle if associative and co-associative surfaces in M^8 would be in 1-1 correspondence with preferred extremals of Kähler action. This motivates the question whether Kähler action make sense also in M^8. This does not exclude the possibility that associativity implies or is equivalent with the preferred extremal property.

One expects a close correspondence between preferred extremals: also now vacuum degeneracy is obtained, one obtains massless extremals, string like objects, and counterparts of CP_2 type vacuum extremals. All known extremals would be associative or co-associative if modified gamma matrices define the notion (possible only in the case of H).

The strongest form of duality would be that the space-time surfaces in M^8 and H have same induced metric same induced Kähler form. The basic difference would be that the spinor connection for surfaces in M^8 would be however neutral and have no left handed components and only em gauge potential. A possible interpretation is that M^8 picture defines a theory in the phase in which electroweak symmetry breaking has happened and only photon belongs to the spectrum.

The question is whether one can define WCW also for M^8. Certainly it should be equivalent with WCW for H: otherwise an inflation of poorly defined notions follows. Certainly the general formulation of the WCW geometry generalizes from H to M^8. Since the matrix elements of symplectic super-Hamiltonians defining WCW gamma matrices are well defined as matrix elements involve spinor modes with Gaussian harmonic oscillator behavior, the non-compactness of E^4 does not pose any technical problems.

2. Spinor connection of M^8

There are strong physical constraints on M^8 dual and they could kill the hypothesis. The basic constraint to the spinor structure of M^8 is that it reproduces basic facts about electro-weak interactions. This includes neutral electro-weak couplings to quarks and leptons identified as different H-chiralities and parity breaking.

1. By the flatness of the metric of E^4 its spinor connection is trivial. E^4 however allows full S^2 of covariantly constant Kähler forms so that one can accommodate free independent Abelian gauge fields assuming that the independent gauge fields are orthogonal to each other when interpreted as realizations of quaternionic imaginary units. This is possible but perhaps a more natural option is the introduction of just single Kähler form as in the case of CP_2.
2. One should be able to distinguish between quarks and leptons also in M^8, which suggests that one introduce spinor structure and Kähler structure in E^4. The Kähler structure of E^4 is unique apart form $SO(3)$ rotation since all three quaternionic imaginary units and the unit vectors formed from them allow a representation as an antisymmetric tensor. Hence one must select one preferred Kähler structure, that is fix a point of S^2 representing the selected imaginary unit. It is natural to assume different couplings of the Kähler gauge potential to spinor chiralities representing quarks and leptons: these couplings can be assumed to be same as in case of H.
3. Electro-weak gauge potential has vectorial and axial parts. Em part is vectorial involving coupling to Kähler form and Z^0 contains both axial and vector parts. The naïve replacement of sigma matrices appearing in the coupling of electroweak gauge fields takes the left handed parts of these fields to zero so that only neutral part remains. Further, gauge fields correspond to curvature of CP_2 which vanishes for E^4 so that only Kähler form form remains. Kähler form couples to 3L and q so that the basic asymmetry between leptons and quarks remains. The resulting field could be seen as analog of photon.
4. The absence of weak parts of classical electro-weak gauge fields would conform with the standard thinking that classical weak fields are not important in long scales. A further prediction is that this distinction becomes visible only in situations, where H picture is necessary. This is the case at high energies, where the description of quarks in terms of $SU(3)$ color is convenient whereas $SO(4)$ QCD would require large number of E^4 partial waves. At low energies large number of $SU(3)$ color partial waves are needed and the convenient description would be in terms of $SO(4)$ QCD. Proton spin crisis might relate to this.

3. Dirac equation for leptons and quarks in M^8

Kähler gauge potential would also couple to octonionic spinors and explain the distinction between quarks and leptons.

1. The complexified octonions representing H spinors decompose to $1+1+3+\overline{3}$ under SU(3) representing color automorphisms but the interpretation in terms of QCD color does not make sense. Rather, the triplet and single combine to two weak isospin doublets and quarks and leptons corresponds to "spin" states of octonion valued 2-spinor. The conservation of quark and lepton numbers follows from the absence of coupling between these states.
2. One could modify the coupling so that coupling is on electric charge by coupling it to electromagnetic charge which as a combination of unit matrix and sigma matrix is proportional to $1+kI_1$, where I_1 is octonionic imaginary unit in $M^2 \subset M^4$. The complexified octonionic units can be chosen to be eigenstates of Q_{em} so that Laplace equation reduces to ordinary scalar Laplacian with coupling to self-dual em field.

3. One expects harmonic oscillator like behavior for the modes of the Dirac operator of M^8 since the gauge potential is linear in E^4 coordinates. One possibility is Cartesian coordinates is $A(A_x, A_y, A_z, A_t) = k(-y, x, t, -z)$. Thhe coupling would make E^4 effectively a compact space.
4. The square of Dirac operator gives potential term proportional to $r^2 = x^2 + y^2 + z^2 + t^2$ so that the spectrum of 4-D harmonic oscillator operator and $SO(4)$ harmonics localized near origin are expected. For harmonic oscillator the symmetry enhances to $SU(4)$.

   If one replaces Kähler coupling with em charge symmetry breaking of $SO(4)$ to vectorial $SO(3)$ is expected since the coupling is proportional to $1 + ike_1$ defining electromagnetic charge. Since the basis of complexified quaternions can be chosen to be eigenstates of e_1 under multiplication, octonionic spinors are eigenstates of em charge and one obtains two color singles $1 \pm e_1$ and color triplet and antitriplet. The color triplets cannot be however interpreted in terms of quark color.

   Harmonic oscillator potential is expected to enhance $SO(3)$ to SU(3). This suggests the reduction of the symmetry to $SU(3) \times U(1)$ corresponding to color symmetry and em charge so that one would have same basic quantum numbers as tof CP_2 harmonics. An interesting question is how the spectrum and mass squared eigenvalues of harmonics differ from those for CP_2.
5. In the square of Dirac equation $J^{kl}\Sigma_{kl}$ term distinguishes between different em charges ($\Sigma_k l$ reduces by self duality and by special properties of octonionic sigma matrices to a term proportional to iI_1 and complexified octonionic units can be chosen to be its eigenstates with eigen value ± 1. The vacuum mass squared analogous to the vacuum energy of harmonic oscillator is also present and this contribution are expected to cancel themselves for neutrinos so that they are massless whereas charged leptons and quarks are massive. It remains to be checked that quarks and leptons can be classified to triality $T = \pm 1$ and $t = 0$ representations of dynamical $SU(3)$ respectively.

4. What about the analog of Kähler-Dirac equation

Only the octonionic structure in $T(M^8)$ is needed to formulate quaternionicity of space-time surfaces: the reduction to O_c-real-analyticity would be extremely nice but not necessary (O_c denotes complexified octonions needed to cope with Minkowskian signature). Most importantly, there might be no need to introduce Kähler action (and Kähler form) in M^8. Even the octonionic representation of gamma matrices is un-necessary. Neither there is any absolute need to define octonionic Dirac equation and octonionic Kähler Dirac equation nor octonionic analog of its solutions nor the octonionic variants of embedding space harmonics.

It would be of course nice if the general formulas for solutions of the Kähler Dirac equation in H could have counterparts for octonionic spinors satisfying quaternionicity condition. One can indeed wonder whether the restriction of the modes of induced spinor field to string world sheets defined by integrable distributions of hyper-complex spaces $M^2(x)$ could be interpretated in terms of commutativity of fermionic physics in M^8. $M^8 - H$ correspondence could map the octonionic spinor fields at string world sheets to their quaternionic counterparts in H. The fact that only holomorphy is involved with the definition of modes could make this map possible.

How could one solve associativity/co-associativity conditions?

The natural question is whether and how one could solve the associativity/-co-associativity conditions explicitly. One can imagine two approaches besides $M^8 \rightarrow H \rightarrow H$... iteration generating new solutions from existing ones.

1.Could octonion-real analyticity be equivalent with associativity/co-associativity?

Analytic functions provide solutions to 2-D Laplace equations and one might hope that also the field equations could be solved in terms of octonion-real-analyticity at the level of M^8 perhaps also at the level of H. Signature however causes problems - at least technical. Also the compactness of CP_2 causes technical difficulties but they need not be insurmountable.

For E^8 the tangent space would be genuinely octonionic and one can define the notion octonion-real analytic map as a generalization of real-analytic function of complex variables (the coefficients of Laurent series are real to guarantee associativity of the series). The argument is complexified octonion in $O \oplus iO$ forming an algebra but not a field. The norm square is Minkowskian as difference of two Euclidian octonionic norms: $N(o_1 + io_2) = N(o_1) - N(o_2)$ and vanishes at 15-D light cone boundary. Obviously, differential calculus is possible outside the light-cone boundary. Rational analytic functions have however poles at the light-cone boundary. One can wonder whether the poles at M^4 light-cone boundary, which is subset of 15-D light-cone boundary could have physical significance and relevant for the role of causal diamonds in ZEO.

The candidates for associative surfaces defined by O_c-real-analytic functions (I use O_c for complexified octonions) have Minkowskian signature of metric and are 4-surfaces at which the projection of $f(o_1 + io_2)$ to $Im(O_1)$, $iIm(O_2)$, and $iRe(Q_2) \oplus Im(Q_1)$ vanish so that only the projection to hyper-quaternionic Minkowskian sub-space $M^4 = Re(Q_1) + iIm(Q_2)$ with signature $(1, -1, -, 1-, 1)$ is non-vanishing. The inverse image need not belong to M^8 and in general it belongs to M_c^8 but this is not a problem: all that is needed that the tangent space of inverse image is complexified quaternionic. If this is the case then $M^8 - H$ duality maps the tangent space of the inverse image to CP_2 point and image itself defines the point of M^4 so that a point of H is obtained. Co-associative surfaces would be surfaces for which the projections of image to $Re(O_1)$, $iRe(O_2)$, and to $Im(O_1)$ vanish so that only the projection to $iIm(O_2)$ with signature $(-1, -1, -1, -1)$ is non-vanishing.

The inverse images as 4-D sub-manifolds of M_c^8 (not M^8!) are excellent candidates for associative and co-associative 4-surfaces since $M^8 - H$ duality assignes to them a 4-surface in $M^4 \times CP_2$ if the tangent space at given point is complexified quaternionic. This is true if one believes on the analytic continuation of the intuition from complex analysis (the image of real axes under the map defined by O_c-real-analytic function is real axes in the new coordinates defined by the map: the intuition results by replacing "real" by "complexified quaternionic"). The possibility to solve field equations in this manner would be of enormous significance since besides basic arithmetic operations also the functional decomposition of O_c-real-analytic functions produces similar functions. One could speak of the algebra of space-time surfaces.

What is remarkable that the complexified octonion real analytic functions are obtained by analytic continuation from single real valued function of real argument. The real functions form naturally a hierarchy of polynomials (maybe also rational functions) and number theoretic vision suggests that there coefficients are rationals or algebraic numbers. Already for rational coefficients hierarchy of algebraic extensions of rationals results as one solves the vanishing conditions. There is a temptation to regard this hierarchy coding for space-time sheets as an analog of DNA.

Note that in the recent formulation there is no need to pose separately the condition about integrable distribution of $M^2(x) \subset M^4$.

2. Quaternionicity condition for space-time surfaces

Quaternionicity actually has a surprisingly simple formulation at the level of space-time surfaces. The following discussion applies to both M^8 and H with minor modifications if one accepts that also H can allow octonionic tangent space structure, which does not require gamma matrices.

1. Quaternionicity is equivalent with associativity guaranteed by the vanishing of the associator $A(a, b, c) = a(bc) - (ab)c$ for any triplet of imaginary tangent vectors in the tangent space of the space-time surface. The condition must hold true for purely imaginary combinations of tangent vectors.

2. If one is able to choose the coordinates in such a way that one of the tangent vectors corresponds to real unit (in the embedding map embedding space M^4 coordinate depends only on the time coordinate of space-time surface), the condition reduces to the vanishing of the octonionic product of remaining three induced gamma matrices interpreted as octonionic gamma matrices. This condition looks very simple - perhaps too simple!- since it involves only first derivatives of the embedding space vectors.

   One can of course whether quaternionicity conditions replace field equations or only select preferred extremals. In the latter case, one should be able to prove that quaternionicity conditions are consistent with the field equations.

3. Field equations would reduce to tri-linear equations in in the gradients of embedding space coordinates (rather than involving embedding space coordinates quadratically). Sum of analogs

of 3×3 determinants deriving from $a \times (b \times b)$ for different octonion units is involved.

4. Written explicitly field equations give in terms of vielbein projections e^A_α, vielbein vectors e^A_k, coordinate gradients $\partial_\alpha h^k$ and octonionic structure constants f_{ABC} the following conditions stating that the projections of the octonionic associator tensor to the space-time surface vanishes:

$$\begin{aligned} e^A_\alpha e^B_\beta e^C_\gamma A^E_{ABC} &= 0 \ , \\ A^E_{ABC} &= f_{AD}{}^E f_{BC}{}^D - f_{AB}{}^D f_{DC}{}^E \ , \\ e^A_\alpha &= \partial_\alpha h^k e^A_k \ , \\ \Gamma_k &= e^A_k \gamma_A \ . \end{aligned} \tag{9.3.6}$$

The very naïve idea would be that the field equations are indeed integrable in the sense that they reduce to these tri-linear equations. Tri-linearity in derivatives is highly non-trivial outcome simplifying the situation further. These equations can be formulated as the as purely algebraic equations written above plus integrability conditions

$$F^A_{\alpha\beta} = D_\alpha e^A_\beta - D_\beta e^A_\alpha = 0 \ . \tag{9.3.7}$$

One could say that vielbein projections define an analog of a trivial gauge potential. Note however that the covariant derivative is defined by spinor connection rather than this effective gauge potential which reduces to that in SU(2). Similar formulation holds true for field equations and one should be able to see whether the field equations formulated in terms of derivatives of vielbein projections commute with the associatitivity conditions.

5. The quaternionicity conditions can be formulated as vanishing of generalization of Cayley's hyperdeterminant for "hypermatrix" a_{ijk} with 2-valued indiced (see `http://tinyurl.com/ya7h3n9z`). Now one has 8 hyper-matrices with 3 8-valued indices associated with the vanishing $A^E_{BCD}x^B y^C z^D = 0$ of trilinear forms defined by the associators. The conditions say somethig only about the octonioni structure constants and since octonionic space allow quaternionic sub-spaces these conditions must be satisfied.

The inspection of the Fano triangle [A67] (see **Fig. 9.1**) expressing the multiplication table for octonionic imaginary units reveals that give any two imaginary octonion units e_1 and e_2 their product e_1e_2 (or equivalently commutator) is imaginary octonion unit (2 times octonion unit) and the three units span together with real unit quaternionic sub-algebra. There it seems that one can generate local quaternionic sub-space from two imaginary units plus real unit. This generalizes to the vielbein components of tangent vectors of space-time surface and one can build the solutions to the quaternionicity conditions from vielbein projections e_1, e_2, their product $e_3 = k(x)e_1e_2$ and real fourth "time-like" vielbein component which must be expressible as a combination of real unit and imaginary units:

$$e_0 = a \times 1 + b^i e_i$$

For static solutions this condition is trivial. Here summation over i is understood in the latter term. Besides these conditions one has integrability conditions and field equations for Kähler action. This formulation suggests that quaternionicity is additional - perhaps defining - property of preferred extremals.

Quaternionicity at the level of embedding space quantum numbers

From the multiplication table of octonions as illustrated by Fano triangle [A67] one finds that all edges of the triangle, the middle circle and the three the lines connecting vertices to the midpoints of opposite side define triplets of quaternionic units. This means that by taking real unit and any imaginary unit in quaternionic M^4 algebra spanning $M^2 \subset M^4$ and two imaginary units in the complement representing CP_2 tangent space one obtains quaternionic algebra. This suggests an explanation for the preferred M^2 contained in tangent space of space-time surface (the M^2:s could form an integrable distribution). Four-momentum restricted to M^2 and I_3 and Y interpreted as tangent vectors in CP_2 tangent space defined quaterionic sub-algebra. This could give content for the idea that quantum numbers are quaternionic.

I have indeed proposed that the four-momentum belongs to M^2. If $M^2(x)$ form a distribution as the proposal for the preferred extremals suggests this could reflect momentum exchanges between different points of the space-time surface such that total momentum is conserved or momentum exchange between two sheets connected by wormhole contacts.

Questions

In following some questions related to $M^8 - H$ duality are represented.

1. Could associativity condition be formulated using modified gamma matrices?

Skeptic can criticize the minimal form of $M^8 - H$ duality involving no Kähler action in M^8 is unrealistic. Why just Kähler action? What makes it so special? The only defense that I can imagine is that Kähler action is in many respects unique choice.

An alternative approach would replace induced gamma matrices with the modified ones to get the correlation In the case of M^8 this option cannot work. One cannot exclude it for H.

1. For Kähler action the Kähler-Dirac gamma matrices $\Gamma^\alpha = \frac{\partial L_K}{\partial h^k_\alpha}\Gamma^k$, $\Gamma_k = e^A_k\gamma_A$, assign to a given point of X^4 a 4-D space which need not be tangent space anymore or even its sub-space. The reason is that canonical momentum current contains besides the gravitational contribution coming from the induced metric also the "Maxwell contribution" from the induced Kähler form not parallel to space-time surface. In the case of M^8 the duality map to H is therefore lost.
2. The space spanned by the Kähler-Dirac gamma matrices need not be 4-dimensional. For vacuum extremals with at most 2-D CP_2 projection Kähler-Dirac gamma matrices vanish identically. For massless extremals they span 1- D light-like subspace. For CP_2 vacuum extremals the modified gamma matrices reduces to ordinary gamma matrices for CP_2 and the situation reduces to the quaternionicity of CP_2. Also for string like objects the conditions are satisfied since the gamma matrices define associative sub-space as tangent space of $M^2 \times S^2 \subset M^4 \times CP_2$. It seems that associativity is satisfied by all known extremals. Hence Kähler-Dirac gamma matrices are flexible enough to realize associativity in H.
3. Kähler-Dirac gamma matrices in Dirac equation are required by super conformal symmetry for the extremals of action and they also guarantee that vacuum extremals defined by surfaces in $M^4 \times Y^2$, Y^2 a Lagrange sub-manifold of CP_2, are trivially hyper-quaternionic surfaces. The modified definition of associativity in H does not affect in any manner $M^8 - H$ duality necessarily based on induced gamma matrices in M^8 allowing purely number theoretic interpretation of standard model symmetries. One can however argue that the most natural definition of associativity is in terms of induced gamma matrices in both M^8 and H.

Remark: A side comment not strictly related to associativity is in order. The anti-commutators of the Kähler-Dirac gamma matrices define an effective Riemann metric and one can assign to it the counterparts of Riemann connection, curvature tensor, geodesic line, volume, etc... One would have two different metrics associated with the space-time surface. Only if the action defining space-time surface is identified as the volume in the ordinary metric, these metrics are equivalent. The index raising for the effective metric could be defined also by the induced metric and it is not clear whether one can define Riemann connection also in this case. Could this effective metric have concrete physical significance and play a deeper role in quantum TGD? For instance, AdS-CFT duality leads to ask whether interactions be coded in terms of the gravitation associated with the effective metric.

Now skeptic can ask why should one demand $M^8 - H$ correspondence if one in any case is forced to introduced Kähler also at the level of M^8? Does $M^8 - H$ correspondence help to construct preferred extremals or does it only bring in a long list of conjectures? I can repeat the questions of the skeptic.

2. Minkowskian-Euclidian $\leftrightarrow$ associative–co-associative?

The 8-dimensionality of M^8 allows to consider both associativity of the tangent space and associativity of the normal space- let us call this co-associativity of tangent space- as alternative options. Both options are needed as has been already found. Since space-time surface decomposes into regions whose induced metric possesses either Minkowskian or Euclidian signature, there is a strong temptation to propose that Minkowskian regions correspond to associative and Euclidian regions to co-associative regions so that space-time itself would provide both the description and its dual.

The proposed interpretation of conjectured associative-co-associative duality relates in an interesting manner to p-adic length scale hypothesis selecting the primes $p \simeq 2^k$, k positive integer as preferred p-adic length scales. $L_p \propto \sqrt{p}$ corresponds to the p-adic length scale defining the size of the space-time sheet at which elementary particle represented as CP_2 type extremal is topologically condensed and is of order Compton length. $L_k \propto \sqrt{k}$ represents the p-adic length scale of the wormhole contacts associated with the CP_2 type extremal and CP_2 size is the natural length unit now. Obviously the quantitative formulation for associative-co-associative duality would be in terms $p \rightarrow k$ duality.

3. Can $M^8 - H$ duality be useful?

Skeptic could of course argue that $M^8 - H$ duality generates only an inflation of unproven conjectures. This might be the case. In the following I will however try to defend the conjecture. One can however find good motivations for $M^8 - H$ duality: both theoretical and physical.

1. If $M^8 - H$ duality makes sense for induced gamma matrices also in H, one obtains infinite sequence if dualities allowing to construct preferred extremals iteratively. This might relate to octonionic real-analyticity and composition of octonion-real-analytic functions.

2. $M^8 - H$ duality could provide much simpler description of preferred extremals of Kähler action as hyper-quaternionic surfaces. Unfortunately, it is not clear whether one should introduce the counterpart of Kähler action in M^8 and the coupling of M^8 spinors to Kähler form. Note that the Kähler form in E^4 would be self dual and have constant components: essentially parallel electric and magnetic field of same constant magnitude.

3. $M^8 - H$ duality provides insights to low energy physics, in particular low energy hadron physics. M^8 description might work when H-description fails. For instance, perturbative QCD which corresponds to H-description fails at low energies whereas M^8 description might become perturbative description at this limit. Strong $SO(4) = SU(2)_L \times SU(2)_R$ invariance is the basic symmetry of the phenomenological low energy hadron models based on conserved vector current hypothesis (CVC) and partially conserved axial current hypothesis (PCAC). Strong $SO(4) = SU(2)_L \times SU(2)_R$ relates closely also to electro-weak gauge group $SU(2)_L \times U(1)$ and this connection is not well understood in QCD description. $M^8 - H$ duality could provide this connection. Strong $SO(4)$ symmetry would emerge as a low energy dual of the color symmetry. Orbital $SO(4)$ would correspond to strong $SU(2)_L \times SU(2)_R$ and by flatness of E^4 spin like $SO(4)$ would correspond to electro-weak group $SU(2)_L \times U(1)_R \subset SO(4)$. Note that the inclusion of coupling to Kähler gauge potential is necessary to achieve respectable spinor structure in CP_2. One could say that the orbital angular momentum in $SO(4)$ corresponds to strong isospin and spin part of angular momentum to the weak isospin.

   This argument does not seem to be consistent with $SU(3) \times U(1) \subset SU(4)$ symmetry for Mx Dirac equation. One can however argue that $SU(4)$ symmetry combines $SO(4)$ multiplets together. Furthermore, $SO(4)$ represents the isometries leaving Kähler form invariant.

4. $M^8 - H$ duality in low energy physics and low energy hadron physics

$M^8 - H$ can be applied to gain a view about color confinement. The basic idea would be that $SO(4)$ and $SU(3)$ provide provide dual descriptions of quarks using E^4 and CP_2 partial waves and low energy hadron physics corresponds to a situation in which M^8 picture provides the perturbative approach whereas H picture works at high energies.

A possible interpretation is that the space-time surfaces vary so slowly in CP_2 degrees of freedom that can approximate CP_2 with a small region of its tangent space E^4. One could also say that color interactions mask completely electroweak interactions so that the spinor connection of CP_2 can be neglected and one has effectively E^4. The basic prediction is that $SO(4)$ should appear as dynamical symmetry group of low energy hadron physics and this is indeed the case.

Consider color confinement at the long length scale limit in terms of $M^8 - H$ duality.

1. At high energy limit only lowest color triplet color partial waves for quarks dominate so that QCD description becomes appropriate whereas very higher color partial waves for quarks and gluons are expected to appear at the confinement limit. Since WCW degrees of freedom begin to dominate, color confinement limit transcends the descriptive power of QCD.
2. The success of $SO(4)$ sigma model in the description of low lying hadrons would directly relate to the fact that this group labels also the E^4 Hamiltonians in M^8 picture. Strong $SO(4)$ quantum numbers can be identified as orbital counterparts of right and left handed electro-weak isospin coinciding with strong isospin for lowest quarks. In sigma model pion and sigma boson form the components of E^4 valued vector field or equivalently collection of four E^4 Hamiltonians corresponding to spherical E^4 coordinates. Pion corresponds to S^3 valued unit vector field with charge states of pion identifiable as three Hamiltonians defined by the coordinate components. Sigma is mapped to the Hamiltonian defined by the E^4 radial coordinate. Excited mesons corresponding to more complex Hamiltonians are predicted.
3. The generalization of sigma model would assign to quarks E^4 partial waves belonging to the representations of $SO(4)$. The model would involve also 6 $SO(4)$ gluons and their $SO(4)$ partial waves. At the low energy limit only lowest representations would be important whereas at higher energies higher partial waves would be excited and the description based on CP_2 partial waves would become more appropriate.
4. The low energy quark model would rely on quarks moving $SO(4)$ color partial waves. Left *resp.* right handed quarks could correspond to $SU(2)_L$ *resp.* $SU(2)_R$ triplets so that spin statistics problem would be solved in the same manner as in the standard quark model.
5. Family replication phenomenon is described in TGD framework the same manner in both cases so that quantum numbers like strangeness and charm are not fundamental. Indeed, p-adic mass calculations allowing fractally scaled up versions of various quarks allow to replace Gell-Mann mass formula with highly successful predictions for hadron masses [K72] .

To my opinion these observations are intriguing enough to motivate a concrete attempt to construct low energy hadron physics in terms of $SO(4)$ gauge theory.

Summary

The overall conclusion is that the most convincing scenario relies on the associativity/co-associativity of space-time surfaces define by induced gamma matrices and applying both for M^8 and H. The fact that the duality can be continued to an iterated sequence of duality maps $M^8 \rightarrow H \rightarrow H...$ is what makes the proposal so fascinating and suggests connection with fractality.

The introduction of Kähler action and coupling of spinors to Kähler gauge potentials is highly natural. One can also consider the idea that the space-time surfaces in M^8 and H have same induced metric and Kähler form: for iterated duality map this would mean that the steps in the map produce space-time surfaces which identical metric and Kähler form so that the sequence might stop. M^8_H duality might provide two descriptions of same underlying dynamics: M^8 description would apply in long length scales and H description in short length scales.

9.4 Infinite Primes

The notion of prime seems to capture something very essential about what it is to be elementary building block of matter and has become a fundamental conceptual element of TGD. The notion of prime gains it generality from its reducibility to the notion of prime ideal of an algebra. Thus the notion of prime is well-defined, not only in case of quaternions and octonions, but also for their complexifications and one can speak about infinite primes in the case of hyper-quaternions and -octonions, which are especially natural physically and for which numbers having zero norm correspond physically to light-like 8-vectors.

9.4.1 Basic Ideas

The notion of infinite prime

The original motivation for the notion of infinite prime came from the first attempts to construct TGD inspired theory of consciousness (around 1995) [K108] . Suppose very naïvely that the 4-surfaces in a given sector of the "world of classical worlds" (WCW) are labelled by a fixed p-adic prime. The natural expectation is that evolution by quantum jumps means dispersion in the space of these sectors and leads to the increase of the p-adic prime characterizing the Universe. As one moves backwards in subjective time (sequence of quantum jumps) one ends up to the situation in which the prime characterizing the universe was $p = 2$. Should one assume that there was the first quantum jump when everything began? If not, then it would seem that the p-adic prime characterizing the Universe must be infinite. Second problem is that the p-adic length scales are finite and if the size scale of Universe is given by p-adic length scale the Universe has finite sized: this does not make sense in TGD framework. The only way out of the problems is the assumption that the p-adic prime characterizing the entire Universe is literally infinite and that p-adic primes characterizing space-time sheets are finite.

These argument, which are by no means central for the recent view about p-adic primes, motivated the attempt to construct a theory of infinite primes and to extend quantum TGD accordingly. This turns out to be possible. The recipe for constructing infinite primes is structurally equivalent with a repeated second quantization of an arithmetic super-symmetric quantum field theory. At the lowest level one has fermionic and bosonic states labeled by finite primes and infinite primes correspond to many particle states of this theory. Also infinite primes analogous to bound states are predicted. This hierarchy of quantizations can be continued indefinitely by taking the many particle states of the previous level as elementary particles at the next level. It must be also emphasized that the notion of infinity is relativistic. With respect to the p-adic norm infinite primes have unit norm for all finite and infinite primes so that there is nothing to become scared of!

Construction could make sense also for hyper-quaternionic and hyper-octonionic primes although non-commutativity and non-associativity pose technical challenges. One can also construct infinite number of real units as ratios of infinite integers with a precise number theoretic anatomy. The fascinating finding is that the quantum states labeled by standard model quantum numbers allow a representation as wave functions in the discrete space of these units. Space-time point becomes infinitely richly structured in the sense that one can associate to it a wave function in the space of real (or octonionic) units allowing to represent the WCW spinor fields. One can speak about algebraic holography or number theoretic Brahman=Atman identity and one can also say that the points of embedding space and space-time surface are subject to a number theoretic evolution. In philosophical mood one can of course also ask whether there exists a hierarchy of embedding spaces in which the embedding space at the lower level represents something with infinitesimal size in the sense of real topology and whether this hierarchy is accompanied also by a hierarchy of conscious entities.

This picture suggest that the Universe of quantum TGD might basically provide a physical representation of number theory allowing also infinite primes. The proposal suggests also a possible generalization of real numbers to a number system akin to hyper-reals introduced by Robinson in his non-standard calculus [A32] providing a rigorous mathematical basis for calculus. In fact, some rather natural requirements lead to a unique generalization for the concepts of integer, rational and real. Infinite integers and reals can be regarded as infinite-dimensional vector spaces with integer and real valued coefficients respectively. Same generalization could make sense for all classical number fields [A15, A7, A18] .

Infinite primes and physics in TGD Universe

Several different views about how infinite primes, integers, and rationals might be relevant in TGD Universe have emerged.

1. *Infinite primes and super-symmetric quantum field theory*

Consider next the physical interpretation.

1. The discovery of infinite primes suggested strongly the possibility to reduce physics to number theory. The construction of infinite primes can be regarded as a repeated second quantization of a super-symmetric arithmetic quantum field theory. This suggests that WCW spinor fields or at least the ground states of associated super-conformal representations [A21] (for super-conformal invariance see [A21] could be mapped to infinite primes in both bosonic and fermionic degrees of freedom. The process might generalize so that it applies in the case of quaternionic and octonionic primes and their hyper counterparts. This hierarchy of second quantizations means enormous generalization of physics to what might be regarded a physical counterpart for a hierarchy of abstractions about abstractions about.... The ordinary second quantized quantum physics corresponds only to the lowest level infinite primes.
2. The ordinary primes appearing as building blocks of infinite primes at the first level of the hierarchy could be identified as coding for p-adic primes assignable to fermionic and bosonic partons identified as 2-surfaces of a given space-time sheet. The hierarchy of infinite primes would correspond to hierarchy of space-time sheets defined by the topological condensate. This leads also to a precise identification of p-adic and real variants of bosonic partonic 2-surfaces as correlates of intention and action and pairs of p-adic and real fermionic partons as correlates for cognitive representations.
3. The idea that infinite primes characterize quantum states of the entire Universe, perhaps ground states of super-conformal representations, if not all states, could be taken further. It turns out that this idea makes sense when one considers discrete wave functions in the space of infinite primes and that one can indeed represent standard model quantum numbers in this manner.
4. The number theoretical supersymmetry suggests also space-time supersymmetry TGD framework. Space-time super-symmetry in its standard form is not possible in TGD Universe and this cheated me to believe that this supersymmetry is completely absent in TGD Universe. The progress in the understanding of the properties of the modified Dirac action however led to a generalization of the space-time super-symmetry as a dynamical and broken symmetry of quantum TGD [**?**] .

Here however emerges the idea about the number theoretic analog of color confinement. Rational (infinite) primes allow not only a decomposition to (infinite) primes of algebraic extensions of rationals but also to algebraic extensions of quaternionic and octonionic (infinite) primes. The physical analog is the decomposition of a particle to its more elementary constituents. This fits nicely with the idea about number theoretic resolution represented as a hierarchy of Galois groups defined by the extensions of rationals and realized at the level of physics in terms of Jones inclusions [K120] defined by these groups having a natural action on space-time surfaces, induced spinor fields, and on WCW spinor fields representing physical states [K27] .

2. Infinite primes and physics as number theory

The hierarchy of algebraic extensions of rationals implying corresponding extensions of p-adic numbers [A51, A37, A38, A29] suggests that Galois groups, which are the basic symmetry groups of number theory, should have concrete physical representations using induced spinor fields and configuration space spinor fields and also infinite primes and real units formed as infinite rationals. These groups permute zeros of polynomials and thus have a concrete physical interpretation both at the level of partonic 2-surfaces dictated by algebraic equations and at the level of braid hierarchy. The vision about the role of hyperfinite factors of II_1 and of Jones inclusions as descriptions of quantum measurements with finite measurement resolution leads to concrete ideas about how these groups are realized.

G_2 acts as automorphisms of hyper-octonions and $SU(3)$ as its subgroup respecting the choice of a preferred imaginary unit. The discrete subgroups of $SU(3)$ permuting to each other hyper-octonionic primes are analogous to Galois group and turned out to play a crucial role in the understanding of the correspondence between infinite hyper-octonionic primes and quantum states predicted by quantum TGD.

3. The notion of finite measurement resolution as the key concept

TGD predicts several hierarchies: the hierarchy of space-time sheets, the hierarchy of infinite primes, the hierarchy of Jones inclusions identifiable in terms of finite measurement resolution

[K120] , the dark matter hierarchy characterized by increasing values of $\hbar$ [K38] , the hierarchy of extensions of a given p-adic number field. TGD inspired theory of consciousness predictes the hierarchy of selves and quantum jumps with increasing duration with respect to geometric time. These hierarchies should be closely related.

The notion of finite measurement resolution turns out to be the key concept: the p-adic norm of the rational defined by the infinite prime characterizes the angle measurement resolution for given p-adic prime p. It is essential that one has what might be called a state function reduction selecting a fixed p-adic prime which could be also infinite. This gives direct connections with cognition and with the p-adicization program relying also on angle measurement resolution. Also the value the integers characterizing the singular coverings of CD and CP_2 defining as their product Planck constant characterize the measurement resolution for a given p-adic prime in CD and CP_2 degrees of freedom. This conforms with the fact that elementary particles are characterized by two infinite primes. Hence finite measurement resolution ties tightly together the three threads of the number theoretic vision. Finite measurement resolution relates also closely to the inclusions of hyper-finite factors central for TGD inspired quantum measurement theory with finite measurement resolution.

4. Space-time correlates of infinite primes

Infinite primes code naturally for Fock states in a hierarchy of super-symmetric arithmetic quantum field theories. Quantum classical correspondence leads to ask whether infinite primes could also code for the space-time surfaces serving as symbolic representations of quantum states. This would a generalization of algebraic geometry would emerge and could reduce the dynamics of Kähler action to algebraic geometry and organize 4-surfaces to a physical hierarchy according to their algebraic complexity. This conjecture should be consistent with two other conjectures about the dynamics of space-time surfaces (space-time surfaces as preferred extrema of Kähler action and space-time surfaces as quaternionic or co-quaternionic (as associative or co-associative) 4-surfaces of hyper-octonion space M^8).

Quantum classical correspondence requires the map of the quantum numbers of WCW spinor fields to space-time geometry. The quantum numbers characterizing positive and negative energy parts of zero energy states couple directly to space-time geometry via the measurement interaction terms in Kähler action expressing the equality of classical conserved charges in Cartan algebra with their quantal counterparts for space-time surfaces in quantum superposition. This makes sense if classical charges parametrize zero modes. The localization in zero modes in state function reduction would be the WCW counterpart of state function collapse.

Therefore, if one wants to map infinite rationals to space-time geometry it is enough to map infinite primes to quantum numbers. This map might be i achieved thanks to the detailed picture about the interpretation of the symmetries of infinite primes in terms of standard model symmetries. The notion of finite measurement resolution allows to deduce much more detailed about this correspondence. In particular, the rational defined by the infinite prime classifies the finite sub-manifold geometry defined by the discretization of the partonic 2-surface implied by the finite measurement resolution. Also a direct correlation between integers defining Planck constant and the “fermionic” part of the infinite prime emerges.

Infinite primes and cognition

The correlation of infinite primes with cognition is the first fascinating possibility and this possibility has stimulated several ideas.

1. One can define the notion of prime also for the algebraic extensions of rationals. The hierarchy of infinite primes associated with algebraic extensions of rationals leading gradually towards algebraic closure of rationals would in turn define cognitive hierarchy corresponding to algebraic extensions of p-adic numbers.
2. The introduction of infinite primes, integers, and rationals leads also to a generalization of classical number fields since an infinite algebra of real (complex, etc...) units defined by finite ratios of infinite rationals multiplied by ordinary rationals which are their inverses becomes possible. These units are not units in the p-adic sense and have a finite p-adic norm which can be differ from one. This construction generalizes also to the case of hyper- quaternions and -octonions although non-commutativity and in case of octonions also non-associativity

pose technical problems. Obviously this approach differs from the standard introduction of infinitesimals in the sense that sum of infinitesimals (real zeros) is replaced by multiplication of real units meaning that the set of real and also more general units becomes infinitely degenerate.

3. Infinite primes form an infinite hierarchy so that the points of space-time and embedding space can be seen as infinitely structured and able to represent all imaginable algebraic structures. Certainly counter-intuitively, single space-time point -or more generally wave functions in the space of the units associated with the point- might be even capable of representing the quantum state of the entire physical Universe in its structure. For instance, in the real sense surfaces in the space of units correspond to the same real number 1, and single point, which is structure-less in the real sense could represent arbitrarily high-dimensional spaces as unions of real units. For real physics this structure is completely invisible and is relevant only for the physics of cognition. One can say that Universe is an algebraic hologram, and there is an obvious connection both with Brahman=Atman identity of Eastern philosophies and Leibniz's notion of monad.

4. In ZEO hyper-octonionic units identified as ratios of the infinite integers associated with the positive and negative energy parts of the zero energy state define a representation of WCW spinor fields. The action of subgroups of SU(3) and rotation group SU(2) preserving hyper-octonionic and hyper-quaternionic primeness and identification of momentum and electro-weak charges in terms of components of hyper-octonionic primes makes this representation unique. Hence Brahman-Atman identity has a completely concrete realization and fixes completely the quantum number spectrum including particle masses and correlations between various quantum numbers.

5. One can assign to infinite primes at n^{th} level of hierarchy rational functions of n rational arguments which form a natural hierarchical structure in that highest level corresponds to a polynomial with coefficients which are rational functions of the arguments at the lower level. One can solve one of the arguments in terms of lower ones to get a hierarchy of algebraic extensions. At the lowest level algebraic extensions of rationals emerge, at the next level algebraic extensions of space of rational functions of single variable, etc... This would suggest that infinite primes code for the correlation between quantum states and the algebraic extensions appearing in their their physical description and characterizing their cognitive correlates. The hierarchy of infinite primes would also correlate with a hierarchy of logics of various orders (hierarchy of statements about statements about...).

9.4.2 Infinite Primes, Integers, And Rationals

The definition of the infinite integers and rationals is a straightforward procedure and structurally similar to a repeated second quantization of a super-symmetric quantum field theory but including also the number theoretic counterparts of bound states.

The first level of hierarchy

In the following the concept of infinite prime is developed gradually by stepwise procedure rather than giving directly the basic definitions. The hope is that the development of the concept in the same manner as it actually occurred would make it easier to understand it.

Step 1

One could try to define infinite primes P by starting from the basic idea in the proof of Euclid for the existence of infinite number of primes. Take the product of all finite primes and add 1 to get a new prime:

$$\begin{aligned} P &= 1 + X \ , \\ X &= \prod_p p \ . \end{aligned} \tag{9.4.1}$$

If P were divisible by finite prime then $P - X = 1$ would be divisible by finite prime and one would encounter contradiction. One could of course worry about the possible existence of infinite primes smaller than P and possibly dividing P. The numbers $N = P - k$, $k > 1$, are certainly not primes since k can be taken as a factor. The number $P' = P - 2 = -1 + X$ could however be prime. P is certainly not divisible by $P - 2$. It seems that one cannot express P and $P - 2$ as product of infinite integer and finite integer. Neither it seems possible to express these numbers as products of more general numbers of form $\prod_{p \in U} p + q$, where U is infinite subset of finite primes and q is finite integer.

9.4.3 How To Interpret The Infinite Hierarchy Of Infinite Primes?

From the foregoing it should be clear that infinite primes might play key role in quantum physics. One can even consider the possibility that physics reduces to a generalized number theory, and that infinite primes are crucial for understanding mathematically consciousness and cognition. Of course, one must leave open the question whether infinite primes really provide really the mathematics of consciousness or whether they are only a beautiful but esoteric mathematical construct. In this spirit the following subsections give only different points of view to the problem with no attempt to a coherent overall view.

Infinite primes and hierarchy of super-symmetric arithmetic quantum field theories

Infinite primes are a generalization of the notion of prime. They turn out to provide number theoretic correlates of both free, interacting and bound states of a super-symmetric arithmetic quantum field theory. It turns also possible to assign to infinite prime space-time surface as a geometric correlate although the original proposal for how to achieve this failed. Hence infinite primes serve as a bridge between classical and quantum and realize quantum classical correspondence stating that quantum states have classical counterparts, and has served as a basic heuristic guideline of TGD. More precisely, the natural hypothesis is that infinite primes code for the ground states of super-symplectic representations (for instance, ordinary particles correspond to states of this kind).

1. Infinite primes and Fock states of a super-symmetric arithmetic QFT

The basic construction recipe for infinite primes is simple and generalizes to the quaternionic case.

1. Form the product of all primes and call it X:

$$X = \prod_p p \ .$$

2. Form the vacuum states

$$V_{\pm} = X \pm 1 \ .$$

3. From these vacua construct all *generating* infinite primes by the following process. Kick out from the Dirac sea some negative energy fermions: they correspond to a product s of first powers of primes: $V \to X/s \pm s$ (s is thus square-free integer). This state represents a state with some fermions represented as holes in Dirac sea but no bosons. Add bosons by multiplying by integer r, which decomposes into parts as $r = mn$: m corresponding to bosons in X/s is product of powers of primes dividing X/s and n corresponds to bosons in s and is product of powers of primes dividing s. This step can be described as $X/s \pm s \to mX/s \pm ns$.

Generating infinite primes are thus in one-one correspondence with the Fock states of a super-symmetric arithmetic quantum field theory and can be written as

$$P_{\pm}(m, n, s) = \frac{mX}{s} \pm ns \ ,$$

where X is product of all primes at previous level. s is square free integer. m and n have no common factors, and neither m and s nor n and X/s have common factors.

The physical analog of the process is the creation of Fock states of a super-symmetric arithmetic quantum field theory. The factorization of s to a product of first powers of primes corresponds to many-fermion state and the decomposition of m and n to products of powers of prime correspond to bosonic Fock states since p^k corresponds to k-particle state in arithmetic quantum field theory.

2. More complex infinite primes as counterparts of bound states

Generating infinite primes are not all that are possible. One can construct also polynomials of the generating primes and under certain conditions these polynomials are non-divisible by both finite primes and infinite primes already constructed. As found, the conjectured effective 2-dimensionality for hyper-octonionic primes allows the reduction of polynomial representation of hyper-octonionic primes to that for hyper-complex primes. This would be in accordance with the effective 2-dimensionality of the basic objects of quantum TGD.

The physical counterpart of n:th order irreducible polynomial is as a bound state of n particles whereas infinite integers constructed as products of infinite primes correspond to non-bound but interacting states. This process can be repeated at the higher levels by defining the vacuum state to be the product of all primes at previous levels and repeating the process. A repeated second quantization of a super-symmetric arithmetic quantum field theory is in question.

The infinite primes represented by irreducible polynomials correspond to quantum states obtained by mapping the superposition of the products of the generating infinite primes to a superposition of the corresponding Fock states. If complex rationals are the coefficient field for infinite integers, this gives rise to states in a complex Hilbert space and irreducibility corresponds to a superposition of states with varying particle number and the presence of entanglement. For instance, the superpositions of several products of type $\prod_{i=1,..,n} P_i$ of n generating infinite primes are possible and in general give rise to irreducible infinite primes decomposing into a product of infinite primes in algebraic extension of rationals.

3. Infinite rationals viz. quantum states and space-time surfaces

The most promising answer to the question how infinite rationals correspond to space-time surfaces is discussed in detail in the next section. Here it is enough to give only the basic idea.

1. In ZEO hyper-octonionic units (in real sense) defined by ratios of infinite integers have an interpretation as representations for pairs of positive and negative energy states. Suppose that the quantum number combinations characterizing positive and negative energy quantum states are representable as superpositions of real units defined by ratios of infinite integers at each point of the space-time surface. If this is true, the quantum classical correspondence coded by the measurement interaction term of the Kähler-Dirac action maps the quantum numbers also to space-time geometry and implies a correspondence between infinite rationals and space-time surfaces.

2. The space-time surface associated with the infinite rational is in general not a union of the space-time surfaces associated with the primes composing the integers defining the rational. There the classical description of interactions emerges automatically. The description of classical states in terms of infinite integers would be analogous to the description of many particle states as finite integers in arithmetic quantum field theory. This mapping could in principle make sense both in real and p-adic sectors of WCW.

The finite primes which correspond to particles of an arithmetic quantum field theory present in Fock state, correspond to the space-time sheets of finite size serving as the building blocks of the space-time sheet characterized by infinite prime.

4. What is the interpretation of the higher level infinite primes?

Infinite hierarchy of infinite primes codes for a hierarchy of Fock states such that many-particle Fock states of a given level serve as elementary particles at next level. The unavoidable conclusion is that higher levels represent totally new physics not described by the standard quantization procedures. In particular, the assignment of fermion/boson property to arbitrarily large system would be in some sense exact. Topologically these higher level particles could correspond to space-time sheets containing many-particle states and behaving as higher level elementary particles.

This view suggests that the generating quantum numbers are present already at the lowest level and somehow coded by the hyper-octonionic primes taking the role of momentum quantum number they have in arithmetic quantum field theories. The task is to understand whether and how hyper-octonionic primes can code for quantum numbers predicted by quantum TGD.

The quantum numbers coding higher level states are collections of quantum numbers of lower level states. At geometric level the replacement of the coefficients of polynomials with rational functions is the equivalent of replacing single particle states with new single particle states consisting of many-particle states.

Infinite primes, the structure of many-sheeted space-time, and the notion of finite measurement resolution

The mapping of infinite primes to space-time surfaces codes the structure of infinite prime to the structure of space-time surface in a rather non-implicit manner, and the question arises about the concrete correspondence between the structure of infinite prime and topological structure of the space-time surface. It turns out that the notion of finite measurement resolution is the key concept: infinite prime characterizes angle measurement resolution. This gives a direct connection with the p-adicization program relying also on angle measurement resolution as well as a connection with the hierarchy of Planck constants. Finite measurement resolution relates also closely to the inclusions of hyper-finite factors central for TGD inspired quantum measurement theory.

1. The first intuitions

The concrete prediction of the general vision is that the hierarchy of infinite primes should somehow correspond to the hierarchy of space-time sheets or partonic 2-surfaces if one accepts the effective 2-dimensionality. The challenge is to find space-time counterparts for infinite primes at the lowest level of the hierarchy.

One could hope that the Fock space structure of infinite prime would have a more concrete correspondence with the structure of the many-sheeted space-time. One might that the space-time sheets labeled by primes p would directly correspond to the primes appearing in the definition of infinite prime. This expectation seems to be too simplistic.

1. What seems to be a safe guess is that the simplest infinite primes at the lowest level of the hierarchy should correspond to elementary particles. If inverses of infinite primes correspond to negative energy space-time sheets, this would explain why negative energy particles are not encountered in elementary particle physics.
2. More complex infinite primes at the lowest level of the hierarchy could be interpreted in terms of structures formed by connecting these structures by join along boundaries bonds to get space-time correlates of bound states. Even simplest infinite primes must correspond to bound state structures if the condition that the corresponding polynomial has real-rational coefficients is taken seriously.

Infinite primes at the lowest level of hierarchy correspond to several finite primes rather than single finite prime. The number of finite primes is however finite.

1. A possible interpretation for multi-p property is in terms of multi-p p-adic fractality prevailing in the interior of space-time surface. The effective p-adic topology of these space-time sheets would depend on length scale. In the longest scale the topology would correspond to p_n, in some shorter length scale there would be smaller structures with $p_{n-1} < p_n$-adic topology, and so on... . A good metaphor would be a wave containing ripples, which in turn would contain still smaller ripples. The multi-p p-adic fractality would be assigned with the 4-D space-time sheets associated with elementary particles. The concrete realization of multi-p p-adicity would be in terms of infinite integers coming as power series $\sum x_n N^n$ and having interpretation as p-adic numbers for any prime dividing N.
2. Effective 2-dimensionality would suggest that the individual p-adic topologies could be assigned with the 2-dimensional partonic surfaces. Thus infinite prime would characterize at the lowest level space-time sheet and corresponding partonic 2-surfaces. There are however reasons to think that even single partonic 2-surface corresponds to a multi-p p-adic topology.

2. Do infinite primes code for the finite measurement resolution?

The above describe heuristic picture is not yet satisfactory. In order to proceed, it is good to ask what determines the finite prime or set of them associated with a given partonic 2-surface. It is good to recall first the recent view about the p-adicization program relying crucially on the notion of finite measurement resolution.

1. The vision about p-adicization characterizes finite measurement resolution for angle measurement in the most general case as $\Delta\phi = 2\pi M/N$, where M and N are positive integers having no common factors. The powers of the phases $exp(i2\pi M/N)$ define identical Fourier basis irrespective of the value of M and measurement resolution does not depend on on the value of M. Situation is different if one allows only the powers $exp(i2\pi kM/N)$ for which $kM < N$ holds true: in the latter case the measurement resolutions with different values of M correspond to different numbers of Fourier components. If one regards N as an ordinary integer, one must have $N = p^n$ by the p-adic continuity requirement.
2. One can also interpret N as a p-adic integer. For $N = p^n M$, where M is not divisible by p, one can express $1/M$ as a p-adic integer $1/M = \sum_{k\geq 0} M_k p^k$, which is infinite as a real integer but effectively reduces to a finite integer $K(p) = \sum_{k=0}^{N-1} M_k p^k$. As a root of unity the entire phase $exp(i2\pi M/N)$ is equivalent with $exp(i2\pi R/p^n)$, $R = K(p)M \mod p^n$. The phase would non-trivial only for p-adic primes appearing as factors in N. The corresponding measurement resolution would be $\Delta\phi = R2\pi/N$ if modular arithetics is used to define the measurement resolution. This works at the first level of the hierarchy but not at higher levels. The alternative manner to assign a finite measurement resolution to M/N for given p is as $\Delta\phi = 2\pi|N/M|_p = 2\pi/p^n$. In this case the small fermionic part of the infinite prime would fix the measurement resolution. The argument below shows that only this option works also at the higher levels of hierarchy and is therefore more plausible.
3. p-Adicization conditions in their strong form require that the notion of integration based on harmonic analysis [A8] in symmetric spaces [A22] makes sense even at the level of partonic 2-surfaces. These conditions are satisfied if the partonic 2-surfaces in a given measurement resolution can be regarded as algebraic continuations of discrete surfaces whose points belong to the discrete variant of the $\delta M^4_\pm \times CP_2$. This condition is extremely powerful since it effectively allows to code the geometry of partonic 2-surfaces by the geometry of finite submanifold geometries for a given measurement resolution. This condition assigns the integer N to a given partonic surface and all primes appearing as factors of N define possible effective p-adic topologies assignable to the partonic 2-surface.

How infinite primes could then code for the finite measurement resolution? Can one identify the measurement resolution for $M/N = M/(Rp^n)$ as $\Delta\phi = ((M/R)\ mod\ p^n) \times 2\pi/p^n$ or as $\Delta\phi = 2\pi/p^n$? The following argument allows only the latter option.

1. Suppose that p-adic topology makes sense also for infinite primes and that state function reduction selects power of infinite prime P from the product of lower level infinite primes defining the integer N in M/N. Suppose that the rational defined by infinite integer defines measurement resolution also at the higher levels of the hierarchy.
2. The infinite primes at the first level of hierarchy representing Fock states are in one-one correspondence with finite rationals M/N for which integers M and N can be chosen to characterize the infinite bosonic part and finite fermionic part of the infinite prime. This correspondence makes sense also at higher levels of the hierarchy but M and N are infinite integers. Also other option obtained by exchanging "bosonic" and "fermionic" but later it will be found that only the first identification makes sense.
3. The first guess is that the rational M/N characterizing the infinite prime characterizes the measurement resolution for angles and therefore partially classifies also the finite submanifold geometry assignable to the partonic 2-surface. One should define what $M/N = ((M/R)\ mod\ P^n) \times P^{-n}$ is for infinite primes. This would require expression of M/R in modular arithmetics modulo P^n. This does not make sense.
4. For the second option the measurement resolution defined as $\Delta\phi = 2\pi|N/M|_P = 2\pi/P^n$ makes sense. The Fourier basis obtained in this manner would be infinite but all states $exp(ik/P^n)$ would correspond in real sense to real unity unless one allows k to be infinite P-adic integer smaller than P^n and thus expressible as $k = \sum_{m<n} k_m P^m$, where k_m are

infinite integers smaller than P. In real sense one obtains all roots $exp(iq2\pi)$ of unity with $q < 1$ rational. For instance, for $n = 1$ one can have $0 < k/P < 1$ for a suitably chosen infinite prime k. Thus one would have essentially continuum theory at higher levels of the hierarchy. The purely fermionic part N of the infinite prime would code for both the number of Fourier components in discretization for each power of prime involved and the ratio characterize the angle resolution.

The proposed relation between infinite prime and finite measurement resolution implies very strong number theoretic selection rules on the reaction vertices.

1. The point is that the vertices of generalized Feyman diagrams correspond to partonic 2-surfaces at which the ends of light-like 3-surfaces describing the orbits of partonic 2-surfaces join together. Suppose that the partonic 2-surfaces appearing a both ends of the propagator lines correspond to same rational as finite sub-manifold geometries. If so, then for a given p-adic effective topology the integers assignable to all lines entering the vertex must contain this p-adic prime as a factor. Particles would correspond to integers and only the particles having common prime factors could appear in the same vertex.
2. In fact, already the work with modelling dark matter [K38] led to ask whether particle could be characterized by a collection of p-adic primes to which one can assign weak, color, em, gravitational interactions, and possibly also other interactions. It also seemed natural to assume that only the space-time sheets containing common primes in this collection can interact. This inspired the notions of relative and partial darkness. An entire hierarchy of weak and color physics such that weak bosons and gluons of given physics are characterized by a given p-adic prime p and also the fermions of this physics contain space-time sheet characterized by same p-adic prime, say M_{89} as in case of weak interactions. In this picture the decay widths of weak bosons do not pose limitations on the number of light particles if weak interactions for them are characterized by p-adic prime $p \neq M_{89}$. Same applies to color interactions.

The possibility of multi-p p-adicity raises the question about how to fix the p-adic prime characterizing the mass of the particle. The mass scale of the contribution of a given throat to the mass squared is given by $p^{-n/2}$, where $T = 1/n$ corresponds to the p-adic temperature of throat. Hence the dominating contribution to the mass squared corresponds to the smallest prime power p^n associated with the throats of the particle. This works if the integers characterizing other particles than graviton are divisible by the gravitonic p-adic prime or a product of p-adic primes assignable to graviton. If the smallest power p^n assignable to the graviton is large enough, the mass of graviton is consistent with the empirical bounds on it. The same consideration applies in the case of photons. Recall that the number theoretically very natural condition that in ZEO the number of generalized Feynman graphs contributing to a given process is finite is satisfied if all particles have a non-vanishing but arbitrarily small p-adic thermal mass [K121] .

3. Interpretational problem

The identification of infinite prime as a characterizer of finite measurement resolution looks nice but there is an interpretational problem.

1. The model characterizing the quantum numbers of WCW spinor fields to be discussed in the next section involves a pair of infinite primes P_+ and P_- corresponding to the two vacuum primes $X \pm 1$. Do they correspond to two different measurement resolutions perhaps assignable to CD and CP_2 degrees of freedom?
2. Different measurement resolutions in CD and CP_2 degrees of freedom need not be not a problem as long as one considers only the discrete variants of symmetric spaces involved. What might be a problem is that in the general case the p-adic primes associated with CD and CP_2 degrees of freedom would not be same unless the integers N_+ and N_- are assumed to have have same prime factors (they indeed do if $p^0 = 1$ is formally counted as prime power factors).
3. The idea of assigning different p-adic effective topologies to CD and CP_2 does not look attractive. Both CD and CP_2 and thus also partonic 2-surface could however possess simultaneously

both p-adic effective topologies. This kind of option might make sense since the integers representable as infinite powers series of integer N can be regarded as p-adic integers for all prime factors of N. As a matter fact, this kind of multi-p p-adicity could make sense also for the partonic 2-surfaces characterized by a measurement resolution $\Delta\phi = 2\pi M/N$. One would have what might be interpreted as N_+N_--adicity.

4. It will be found that quantum measurement means also the measurement of the p-adic prime selecting same p-adic prime from N_+ and N_-. If $N_\pm$ is divisible only by $p^0 = 1$, the corresponding angle measurement resolution is trivial. From the point of view of consciousness state function reduction selects also the p-adic prime characterizing the cognitive representation which is very natural since quantum superpositions of different p-adic topologies are not natural physically.

How the hierarchy of Planck constants could relate to infinite primes and p-adic hierarchy?

Besides the hierarchy of space-time sheets, TGD predicts, or at least suggests, several hierarchies such as the hierarchy of infinite primes, the hierarchy of Jones inclusions identifiable in terms of finite measurement resolution [K120], the hierarchy of super-symplectic gauge symmetry breakings closely related to the dark matter hierarchy characterized by increasing values of $h_{eff} = n \times h$ [K38] , the hierarchy of extensions of given p-adic number field associated with algebraic extensions of rationals, and the hierarchy of selves and quantum jumps with increasing duration with respect to geometric time. There are good reasons to expect that these hierarchies are closely related. Number theoretical considerations give hopes about developing a more quantitative vision about the relationship between these hierarchies, in particular between the hierarchy of infinite primes, p-adic length scale hierarchy, and the hierarchy if Planck constants.

This idea can be indeed made concrete.

1. The hierarchy of algebraic extensions of rationals corresponds to the hierarchy of quantum criticalities labelled by integer $n = h_{eff}/h$. There is a temptation to identify n as the product of ramified primes of the algebraic extension or its power. In accordance with the number theoretic vision number theoretic criticality would correspond to quantum criticality. The idea is that ramified primes are analogous to multiple roots of polynomial and criticality indeed corresponds to this kind of situation.

2. Infinite primes at the n:th level of hierarchy representing analogs of bound states correspond to irreducible polynomials of n-variables identifiable as polynomials of z_n with coefficients, which are polynomials of $z_1, .., z_{n-1}$. At the first level of hierarchy bound states correspond to irreducible polynomials of single variable and their roots define irreducible algebraic extensions of rationals. Infinite integers in turn correspond to products of reducible polynomials defining reducible extensions. The infinite integers at the first level of hierarchy would define the hierarchy of algebraic extensions of rationals in turn defining a hierarchy of quantum criticalities. This observation might generalize to the higher levels of hierarchy of infinite primes so that infinite primes would be part of quantum TGD although in much more abstract sense as thought originally.

Chapter 10

Unified Number Theoretical Vision

10.1 Introduction

Octonions, quaternions, quaternionic space-time surfaces, octonionic spinors and twistors and twistor spaces are highly relevant for quantum TGD. In the following some general observations distilled during years are summarized. This summary involves several corrections to the picture which has been developing for a decade or so.

A brief updated view about M^8-H duality and twistorialization is in order. There is a beautiful pattern present suggesting that M^8-H duality makes sense and that $H=M^4\times CP_2$ is completely unique on number theoretical grounds.

1. M^8-H duality allows to deduce $M^4\times CP_2$ via number theoretical compactification. For the option with minimal number of conjectures the associativity/co-associativity of the space-time surfaces in M^8 guarantees that the space-time surfaces in M^8 define space-time surfaces in H. The tangent/normal spaces of quaternionic/hyper-quaternionic surfaces in M^8 contain also an integrable distribution of hyper-complex tangent planes $M^2(x)$.

   An important correction is that associativity/co-associativity does not make sense at the level of H since the spinor structure of H is already complex quaternionic and reducible to the ordinary one by using matrix representations for quaternions. The associativity condition should however have some counterpart at level of H. One could require that the induced gamma matrices at each point could span a *real*-quaternionic sub-space of complexified quaternions for quaternionicity and a purely imaginary quaternionic sub-space for co-quaternionicity. One might hope that it is consistent with - or even better, implies - preferred extremal property. I have not however found a viable definition of quaternionic "reality". On the other hand, it is possible assigne the tangent space M^8 of H with octonion structure and define associativity as in case of M^8.

   M^8-H duality could generalize to $H-H$ duality in the sense that also the image of the space-time surface under duality map is not only preferred extremal but also associative (co-associative) surface. The duality map $H\rightarrow H$ could be iterated and would define the arrow for the category formed by preferred extremals.

2. M^4 and CP_2 are the unique 4-D spaces allowing twistor space with Kähler structure. M^8 allows twistor space for octonionic spinor structure obtained by direct generalization of the standard construction for M^4. $M^4\times CP_2$ spinors can be regarded as tensor products of quaternionic spinors associated with M^4 and CP_2: this trivial observation forces to challenge the earlier rough vision, which however seems to stand up the challenge.

3. Octotwistors generalise the twistorial construction from M^4 to M^8 and octonionic gamma matrices make sense also for H with quaternionicity condition reducing 12-D $T(M^8)=G_2/U(1)\times U(1)$ to the 12-D twistor space $T(H)=CP_3\times SU^3/U(1)\times U(1)$. The interpretation of the twistor space in the case of M^8 is as the space of choices of quantization axes

for the 2-D Cartan algebra of G_2 acting as octonionic automorphisms. For CP_2 one has space for the chocies of quantization axes for the 2-D $SU(3)$ Cartan algebra.

4. It is also possible that the dualities extend to a sequence $M^8 \rightarrow H \rightarrow H...$ by mapping the associative/co-associative tangent space to CP_2 and M^4 point to M^4 point at each step. One has good reasons to expect that this iteration generates fractal as the limiting space-time surface.

5. A fascinating structure related to octo-twistors is the non-associated analog of Lie group defined by automorphisms by octonionic imaginary units: this group is topologically 7-sphere. Second analogous structure is the 7-D Lie algebra like structure defined by octonionic analogs of sigma matrices.

The analogy of quaternionicity of M^8 pre-images of preferred extremals and quaternionicity of the tangent space of space-time surfaces in H with the Majorana condition central in super string models is very thought provoking. All this suggests that associativity at the level of M^8 indeed could define basic dynamical principle of TGD.

Number theoretical vision about quantum TGD involves both p-adic number fields and classical number fields and the challenge is to unify these approaches. The challenge is non-trivial since the p-adic variants of quaternions and octonions are not number fields without additional conditions. The key idea is that TGD reduces to the representations of Galois group of algebraic numbers realized in the spaces of octonionic and quaternionic adeles generalizing the ordinary adeles as Cartesian products of all number fields: this picture relates closely to Langlands program. Associativity would force sub-algebras of the octonionic adeles defining 4-D surfaces in the space of octonionic adeles so that 4-D space-time would emerge naturally. $M^8 - H$ correspondence in turn would map the space-time surface in M^8 to $M^4 \times CP_2$.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L9].

10.2 Number Theoretic Compactification And $M^8 - H$ Duality

This section summarizes the basic vision about number theoretic compactification reducing the classical dynamics to associativity or co-associativity. Originally $M^8 - H$ duality was introduced as a number theoretic explanation for $H = M^4 \times CP_2$. Much later it turned out that the completely exceptional twistorial properties of M^4 and CP_2 are enough to justify $X^4 \subset H$ hypothesis. Skeptic could therefore criticize the introduction of M^8 (actually its complexification) as an un-necessary mathematical complication producing only unproven conjectures and bundle of new statements to be formulated precisely. However, if quaternionicity can be realized in terms of M_c^8 using O_c-real analytic functions and if quaternionicity is equivalent with preferred extremal property, a huge simplification results and one can say that field equations are exactly solvable.

One can question the feasibility of $M^8 - H$ duality if the dynamics is purely number theoretic at the level of M^8 and determined by Kähler action at the level of H. Situation becomes more democratic if Kähler action defines the dynamics in both M^8 and H: this might mean that associativity could imply field equations for preferred extremals or vice versa or there might be equivalence between two. This means the introduction Kähler structure at the level of M^8, and motivates also the coupling of Kähler gauge potential to M^8 spinors characterized by Kähler charge or em charge. One could call this form of duality strong form of $M^8 - H$ duality.

The strong form $M^8 - H$ duality boils down to the assumption that space-time surfaces can be regarded either as 4-surfaces of H or as surfaces of M^8 or even M_c^8 composed of associative and co-associative regions identifiable as regions of space-time possessing Minkowskian *resp.* Euclidian signature of the induced metric. They have the same induced metric and Kähler form and WCW associated with H should be essentially the same as that associated with M^8. Associativity corresponds to hyper-quaterniocity at the level of tangent space and co-associativity to co-hyper-quaternionicity - that is associativity/hyper-quaternionicity of the normal space. Both are needed to cope with known extremals. Since in Minkowskian context precise language would force to introduce clumsy terms like hyper-quaternionicity and co-hyper-quaternionicity, it is better to speak just about associativity or co-associativity.

Remark: The original assumption was that space-times could be regarded as surfaces in M^8 rather than in its complexification M^8_c identifiable as complexified octonions. This assumption is un-necessarily strong and if one assumes that octonion-real analytic functions characterize these surfaces M^8_c must be assumed.

For the octonionic spinor fields the octonionic analogs of electroweak couplings reduce to mere K̈hler or electromagnetic coupling and the solutions reduce to those for spinor d'Alembertian in 4-D harmonic potential breaking $SO(4)$ symmetry. Due to the enhanced symmetry of harmonic oscillator, one expects that partial waves are classified by $SU(4)$ and by reduction to $SU(3)\times U(1)$ by em charge and color quantum numbers just as for CP_2 - at least formally.

Harmonic oscillator potential defined by self-dual em field splits M^8 to $M^4\times E^4$ and implies Gaussian localization of the spinor modes near origin so that E^4 effectively compactifies. The The resulting physics brings strongly in mind low energy physics, where only electromagnetic interaction is visible directly, and one cannot avoid associations with low energy hadron physics. These are some of the reasons for considering M^8-H duality as something more than a mere mathematical curiosity.

Remark: The Minkowskian signatures of M^8 and M^4 produce technical nuisance. One could overcome them by Wick rotation, which is however somewhat questionable trick. $M^8_c = O_c$ provides the proper formulation.

1. The proper formulation is in terms of complexified octonions and quaternions involving the introduction of commuting imaginary unit j.
2. Hyper-quaternions/octonions define as subspace of complexified quaternions/octonions spanned by real unit and jI_k, where I_k are quaternionic units. These spaces are obviously not closed under multiplication. One can however however define the notion of associativity for the subspace of M^8 by requiring that the products and sums of the tangent space vectors generate complexified quaternions.
3. Ordinary quaternions Q are expressible as $q = q_0 + q^k I_k$. Hyper-quaternions are expressible as $q = q_0 + jq^k I_k$ and form a subspace of complexified quaternions $Q_c = Q\oplus jQ$. Similar formula applies to octonions and their hyper counterparts which can be regarded as subspaces of complexified octonions $O\oplus jO$. Tangent space vectors of H correspond hyper-quaternions $q_H = q_0 + jq^k I_k + jiq_2$ defining a subspace of doubly complexified quaternions: note the appearance of two imaginary units.

The recent definitions of associativity and M^8 duality has evolved slowly from in-accurate characterizations and there are still open questions.

1. Kähler form for M^8 non-trivial only in $E^4\subset M^8$ implies unique decomposition $M^8 = M^4\times E^4$ needed to define M^8-H duality uniquely. This applies also to M^8_c. This forces to introduce also Kähler action, induced metric and induced Kähler form. Could strong form of duality meant that the space-time surfaces in M^8 and H have same induced metric and induced Kähler form? Could the WCW s associated with M^8 and H be identical with this assumption so that duality would provide different interpretations for the same physics?
2. One can formulate associativity in M^8 (or M^8_c) by introducing octonionic structure in tangent spaces or in terms of the octonionic representation for the induced gamma matrices. Does the notion have counterpart at the level of H as one might expect if Kähler action is involved in both cases? The analog of this formulation in H might be as quaternionic "reality" since tangent space of H corresponds to complexified quaternions: I have however found no acceptable definition for this notion.

   The earlier formulation is in terms of octonionic flat space gamma matrices replacing the ordinary gamma matrices so that the formulation reduces to that in M^8 tangent space. This formulation is enough to define what associativity means although one can protest. Somehow H is already complex quaternionic and thus associative. Perhaps this just what is needed since dynamics has two levels: *embedding space level* and *space-time level*. One must have embedding space spinor harmonics assignable to the ground states of super-conformal representations and quaternionicity and octonionicity of H tangent space would make sense at the level of space-time surfaces.

3. Whether the associativity using induced gamma matrices works is not clear for massless extremals (MEs) and vacuum extremals with the dimension of CP_2 projection not larger than 2.

4. What makes this notion of associativity so fascinating is that it would allow to iterate duality as a sequence $M^8 \rightarrow H \rightarrow H...$ by mapping the space-time surface to $M^4 \times CP_2$ by the same recipe as in case of M^8. This brings in mind the functional composition of O_c-real analytic functions (O_c denotes complexified octonions: complexification is forced by Minkowskian signature) suggested to produced associative or co-associative surfaces. The associative (co-associative) surfaces in M^8 would correspond to loci for vanishing of imaginary (real) part of octonion-real-analytic function.

It might be possible to define associativity in H also in terms of Kähler-Dirac gamma matrices defined by Kähler action (certainly not M^8).

1. All known extremals are associative or co-associative in H in this sense. This would also give direct correlation with the variational principle. For the known preferred extremals this variant is successful partially because the Kähler-Dirac gamma matrices need not span the entire tangent space. The space spanned by the Kähler-Dirac gammas is not necessarily tangent space. For instance for CP_2 type vacuum extremals the Kähler-Dirac gamma matrices are CP_2 gamma matrices plus an additional light-like component from M^4 gamma matrices.

   If the space spanned by Kähler-Dirac gammas has dimension D smaller than 3 co-associativity is automatic. If the dimension of this space is $D = 3$ it can happen that the triplet of gammas spans by multiplication entire octonionic algebra. For $D = 4$ the situation is of course non-trivial.

2. For Kähler-Dirac gamma matrices the notion of co-associativity can produce problems since Kähler-Dirac gamma matrices do not in general span the tangent space. What does co-associativity mean now? Should one replace normal space with orthogonal complement of the space spanned by Kähler-Dirac gamma matrices? Co-associativity option must be considered for $D = 4$ only. CP_2 type vacuum extremals provide a good example. In this case the Kähler-Dirac gamma matrices reduce to sums of ordinary CP_2 gamma matrices and ligt-like M^4 contribution. The orthogonal complement for the Kähler-Dirac gamma matrices consists of dual light-like gamma matrix and two gammas orthogonal to it: this space is subspace of M^4 and trivially associative.

10.2.1 Basic Idea Behind $M^8 - M^4 \times CP_2$ Duality

If four-surfaces $X^4 \subset M^8$ under some conditions define 4-surfaces in $M^4 \times CP_2$ indirectly, the spontaneous compactification of super string models would correspond in TGD to two different ways to interpret the space-time surface. This correspondence could be called number theoretical compactification or $M^8 - H$ duality.

The hard mathematical facts behind the notion of number theoretical compactification are following.

1. One must assume that M^8 has unique decomposition $M^8 = M^4 \times E^4$. This decomposition generalizes also to the case of M_c^8. This would be most naturally due to Kähler structure in E^4 defined by a self-dual Kähler form defining parallel constant electric and magnetic fields in Euclidian sense. Besides Kähler form there is vector field coupling to sigma matrix representing the analog of strong isospin: the corresponding octonionic sigma matrix however is imaginary unit times gamma matrix - say ie_1 in M^4 - defining a preferred plane M^2 in M^4. Here it is essential that the gamma matrices of E^4 defined in terms of octonion units commute to gamma matrices in M^4. What is involved becomes clear from the Fano triangle illustrating octonionic multiplication table.

2. The space of hyper-complex structures of the hyper-octonion space - they correspond to the choices of plane $M^2 \subset M^8$ - is parameterized by 6-sphere $S^6 = G^2/SU(3)$. The subgroup $SU(3)$ of the full automorphism group G_2 respects the a priori selected complex structure and thus leaves invariant one octonionic imaginary unit, call it e_1. Fixed complex structure therefore corresponds to a point of S^6.

3. Quaternionic sub-algebras of M^8 (and M^8_c) are parametrized by $G_2/U(2)$. The quaternionic sub-algebras of octonions with fixed complex structure (that is complex sub-space defined by real and preferred imaginary unit and parametrized by a point of S^6) are parameterized by $SU(3)/U(2) = CP_2$ just as the complex planes of quaternion space are parameterized by $CP_1 = S^2$. Same applies to hyper-quaternionic sub-spaces of hyper-octonions. $SU(3)$ would thus have an interpretation as the isometry group of CP_2, as the automorphism sub-group of octonions, and as color group. Thus the space of quaternionic structures can be parametrized by the 10-dimensional space $G_2/U(2)$ decomposing as $S^6 \times CP_2$ locally.

4. The basic result behind number theoretic compactification and $M^8 - H$ duality is that associative sub-spaces $M^4 \subset M^8$ containing a fixed commutative sub-space $M^2 \subset M^8$ are parameterized by CP_2. The choices of a fixed hyper-quaternionic basis $1, e_1, e_2, e_3$ with a fixed complex sub-space (choice of e_1) are labeled by $U(2) \subset SU(3)$. The choice of e_2 and e_3 amounts to fixing $e_2 \pm \sqrt{-1}e_3$, which selects the $U(2) = SU(2) \times U(1)$ subgroup of $SU(3)$. $U(1)$ leaves 1 invariant and induced a phase multiplication of e_1 and $e_2 \pm e_3$. $SU(2)$ induces rotations of the spinor having e_2 and e_3 components. Hence all possible completions of $1, e_1$ by adding e_2, e_3 doublet are labeled by $SU(3)/U(2) = CP_2$.

Consider now the formulation of $M^8 - H$ duality.

1. The idea of the standard formulation is that associative manifold $X^4 \subset M^8$ has at its each point associative tangent plane. That is X^4 corresponds to an integrable distribution of $M^2(x) \subset M^8$ parametrized 4-D coordinate x that is map $x \rightarrow S^6$ such that the 4-D tangent plane is hyper-quaternionic for each x.

2. Since the Kähler structure of M^8 implies unique decomposition $M^8 = M^4 \times E^4$, this surface in turn defines a surface in $M^4 \times CP_2$ obtained by assigning to the point of 4-surface point $(m, s) \in H = M^4 \times CP_2$: $m \in M^4$ is obtained as *projection* $M^8 \rightarrow M^4$ (this is modification to the earlier definition) and $s \in CP_2$ parametrizes the quaternionic tangent plane as point of CP_2. Here the local decomposition $G_2/U(2) = S^6 \times CP_2$ is essential for achieving uniqueness.

3. One could also map the associative surface in M^8 to surface in 10-dimensional $S^6 \times CP_2$. In this case the metric of the image surface cannot have Minkowskian signature and one cannot assume that the induced metrics are identical. It is not known whether S^6 allows genuine complex structure and Kähler structure which is essential for TGD formulation.

4. Does duality imply the analog of associativity for $X^4 \subset H$? The tangent space of H can be seen as a sub-space of doubly complexified quaternions. Could one think that quaternionic sub-space is replaced with sub-space analogous to that spanned by real parts of complexified quaternions? The attempts to define this notion do not however look promising. One can however define associativity and co-associativity for the tangent space M^8 of H using octonionization and can formulate it also terms of induced gamma matrices.

5. The associativity defined in terms of induced gamma matrices in both in M^8 and H has the interesting feature that one can assign to the associative surface in H a new associative surface in H by assigning to each point of the space-time surface its M^4 projection and point of CP_2 characterizing its associative tangent space or co-associative normal space. It seems that one continue this series ad infinitum and generate new solutions of field equations! This brings in mind iteration which is standard manner to generate fractals as limiting sets. This certainly makes the heart of mathematician beat.

6. Kähler structure in $E^4 \subset M^8$ guarantees natural $M^4 \times E^4$ decomposition. Does associativity imply preferred extremal property or vice versa, or are the two notions equivalent or only consistent with each other for preferred extremals?

A couple of comments are in order.

1. This definition generalizes to the case of M^8_c: all that matters is that tangent space-is is complexified quaternionic and there is a unique identification $M^4 \subset M^8_c$: this allows to assign the point of 4-surfaces a point of $M^4 \times CP_2$. The generalization is needed if one wants to formulate the hypothesis about O_c real-analyticity as a way to build quaternionic space-time surfaces properly.

2. This definition differs from the first proposal for years ago stating that each point of X^4 contains a *fixed* $M^2 \subset M^4$ rather than $M_2(x) \subset M^8$ and also from the proposal assuming integrable distribution of $M^2(x) \subset M^4$. The older proposals are not consistent with the properties of massless extremals and string like objects for which the counterpart of M^2 depends on space-time point and is not restricted to M^4. The earlier definition $M^2(x) \subset M^4$ was problematic in the co-associative case since for the Euclidian signature is is not clear what the counterpart of $M^2(x)$ could be.
3. The new definition is consistent with the existence of Hamilton-Jacobi structure meaning slicing of space-time surface by string world sheets and partonic 2-surfaces with points of partonic 2-surfaces labeling the string world sheets [K14]. This structure has been proposed to characterize preferred extremals in Minkowskian space-time regions at least.
4. Co-associative Euclidian 4-surfaces, say CP_2 type vacuum extremal do not contain integrable distribution of $M^2(x)$. It is normal space which contains $M^2(x)$. Does this have some physical meaning? Or does the surface defined by $M^2(x)$ have Euclidian analog?

   A possible identification of the analog would be as string world sheet at which W boson field is pure gauge so that the modes of the modified Dirac operator [K121] restricted to the string world sheet have well-defined em charge. This condition appears in the construction of solutions of Kähler-Dirac operator.

   For octonionic spinor structure the W coupling is however absent so that the condition does not make sense in M^8. The number theoretic condition would be as commutative or co-commutative surface for which imaginary units in tangent space transform to real and imaginary unit by a multiplication with a fixed imaginary unit! One can also formulate co-associativity as a condition that tangent space becomes associative by a multiplication with a fixed imaginary unit.

   There is also another justification for the distribution of Euclidian tangent planes. The idea about associativity as a fundamental dynamical principle can be strengthened to the statement that space-time surface allows slicing by hyper-complex or complex 2-surfaces, which are commutative or co-commutative inside space-time surface. The physical interpretation would be as Minkowskian or Euclidian string world sheets carrying spinor modes. This would give a connection with string model and also with the conjecture about the general structure of preferred extremals.
5. Minimalist could argue that the minimal definition requires octonionic structure and associativity *only* in M^8. There is no need to introduce the counterpart of Kähler action in M^8 since the dynamics would be based on associativity or co-associativity alone. The objection is that one must assumes the decomposition $M^8 = M^4 \times E^4$ without any justification.

   The map of space-time surfaces to those of $H = M^4 \times CP_2$ implies that the space-time surfaces in H are in well-defined sense quaternionic. As a matter of fact, the standard spinor structure of H can be regarded as quaternionic in the sense that gamma matrices are essentially tensor products of quaternionic gamma matrices and reduce in matrix representation for quaternions to ordinary gamma matrices. Therefore the idea that one should introduce octonionic gamma matrices in H is questionable. If all goes as in dreams, the mere associativity or co-associativity would code for the preferred extremal property of Kähler action in H. One could at least hope that associativity/co-associativity in H is consistent with the preferred extremal property.
6. One can also consider a variant of associativity based on modified gamma matrices - but only in H. This notion does not make sense in M^8 since the very existence of quaternionic tangent plane makes it possible to define $M^8 - H$ duality map. The associativity for modified gamma matrices is however consistent with what is known about extremals of Kähler action. The associativity based on induced gamma matrices would correspond to the use of the space-time volume as action. Note however that gamma matrices are *not* necessary in the definition.

10.2.2 Hyper-Octonionic Pauli "Matrices" And The Definition Of Associativity

Octonionic Pauli matrices suggest an interesting possibility to define precisely what associativity means at the level of M^8 using gamma matrices (for background see [L8, K10]).

1. According to the standard definition space-time surface $X^4 \subset M^8$ is associative if the tangent space at each point of X^4 in $X^4 \subset M^8$ picture is associative. The definition can be given also in terms of octonionic gamma matrices whose definition is completely straightforward.

2. Could/should one define the analog of associativity at the level of H? One can identify the tangent space of H as M^8 and can define octonionic structure in the tangent space and this allows to define associativity locally. One can replace gamma matrices with their octonionic variants and formulate associativity in terms of them locally and this should be enough.

   Skeptic however reminds M^4 allows hyper-quaternionic structure and CP_2 quaternionic structure so that complexified quaternionic structure would look more natural for H. The tangent space would decompose as $M^8 = HQ + ijQ$, weher j is commuting imaginary unit and HQ is spanned by real unit and by units iI_k, where i second commutating imaginary unit and I_k denotes quaternionic imaginary units. There is no need to make anything associative.

   There is however far from obvious that octonionic spinor structure can be (or need to be!) defined globally. The lift of the CP_2 spinor connection to its octonionic variant has questionable features: in particular vanishing of the charged part and reduction of neutral part to photon. Therefore is is unclear whether associativity condition makes sense for $X^4 \subset M^4 \times CP_2$. What makes it so fascinating is that it would allow to iterate duality as a sequences $M^8 \rightarrow H \rightarrow H....$ This brings in mind the functional composition of octonion real-analytic functions suggested to produced associative or co-associative surfaces.

I have not been able to settle the situation. What seems the working option is associativity in both M^8 and H and Kähler-Dirac gamma matrices defined by appropriate Kähler action and correlation between associativity and preferred extremal property.

10.2.3 Are Kähler And Spinor Structures Necessary In M^8?

If one introduces M^8 as dual of H, one cannot avoid the idea that hyper-quaternionic surfaces obtained as images of the preferred extremals of Kähler action in H are also extremals of M^8 Kähler action with same value of Kähler action defining Kähler function. As found, this leads to the conclusion that the $M^8 - H$ duality is Kähler isometry. Coupling of spinors to Kähler potential is the next step and this in turn leads to the introduction of spinor structure so that quantum TGD in H should have full M^8 dual.

Are also the 4-surfaces in M^8 preferred extremals of Kähler action?

It would be a mathematical miracle if associative and co-associative surfaces in M^8 would be in 1-1 correspondence with preferred extremals of Kähler action. This motivates the question whether Kähler action make sense also in M^8. This does not exclude the possibility that associativity implies or is equivalent with the preferred extremal property.

One expects a close correspondence between preferred extremals: also now vacuum degeneracy is obtained, one obtains massless extremals, string like objects, and counterparts of CP_2 type vacuum extremals. All known extremals would be associative or co-associative if modified gamma matrices define the notion (possible only in the case of H).

The strongest form of duality would be that the space-time surfaces in M^8 and H have same induced metric same induced Kähler form. The basic difference would be that the spinor connection for surfaces in M^8 would be however neutral and have no left handed components and only em gauge potential. A possible interpretation is that M^8 picture defines a theory in the phase in which electroweak symmetry breaking has happened and only photon belongs to the spectrum.

The question is whether one can define WCW also for M^8. Certainly it should be equivalent with WCW for H: otherwise an inflation of poorly defined notions follows. Certainly the general formulation of the WCW geometry generalizes from H to M^8. Since the matrix elements of symplectic super-Hamiltonians defining WCW gamma matrices are well defined as matrix elements involve spinor modes with Gaussian harmonic oscillator behavior, the non-compactness of E^4 does not pose any technical problems.

Spinor connection of M^8

There are strong physical constraints on M^8 dual and they could kill the hypothesis. The basic constraint to the spinor structure of M^8 is that it reproduces basic facts about electro-weak interactions. This includes neutral electro-weak couplings to quarks and leptons identified as different H-chiralities and parity breaking.

1. By the flatness of the metric of E^4 its spinor connection is trivial. E^4 however allows full S^2 of covariantly constant Kähler forms so that one can accommodate free independent Abelian gauge fields assuming that the independent gauge fields are orthogonal to each other when interpreted as realizations of quaternionic imaginary units. This is possible but perhaps a more natural option is the introduction of just single Kähler form as in the case of CP_2.
2. One should be able to distinguish between quarks and leptons also in M^8, which suggests that one introduce spinor structure and Kähler structure in E^4. The Kähler structure of E^4 is unique apart form $SO(3)$ rotation since all three quaternionic imaginary units and the unit vectors formed from them allow a representation as an antisymmetric tensor. Hence one must select one preferred Kähler structure, that is fix a point of S^2 representing the selected imaginary unit. It is natural to assume different couplings of the Kähler gauge potential to spinor chiralities representing quarks and leptons: these couplings can be assumed to be same as in case of H.
3. Electro-weak gauge potential has vectorial and axial parts. Em part is vectorial involving coupling to Kähler form and Z^0 contains both axial and vector parts. The naïve replacement of sigma matrices appearing in the coupling of electroweak gauge fields takes the left handed parts of these fields to zero so that only neutral part remains. Further, gauge fields correspond to curvature of CP_2 which vanishes for E^4 so that only Kähler form form remains. Kähler form couples to 3L and q so that the basic asymmetry between leptons and quarks remains. The resulting field could be seen as analog of photon.
4. The absence of weak parts of classical electro-weak gauge fields would conform with the standard thinking that classical weak fields are not important in long scales. A further prediction is that this distinction becomes visible only in situations, where H picture is necessary. This is the case at high energies, where the description of quarks in terms of $SU(3)$ color is convenient whereas $SO(4)$ QCD would require large number of E^4 partial waves. At low energies large number of $SU(3)$ color partial waves are needed and the convenient description would be in terms of $SO(4)$ QCD. Proton spin crisis might relate to this.

Dirac equation for leptons and quarks in M^8

Kähler gauge potential would also couple to octonionic spinors and explain the distinction between quarks and leptons.

1. The complexified octonions representing H spinors decompose to $1 + 1 + 3 + \overline{3}$ under SU(3) representing color automorphisms but the interpretation in terms of QCD color does not make sense. Rather, the triplet and single combine to two weak isospin doublets and quarks and leptons corresponds to "spin" states of octonion valued 2-spinor. The conservation of quark and lepton numbers follows from the absence of coupling between these states.
2. One could modify the coupling so that coupling is on electric charge by coupling it to electromagnetic charge which as a combination of unit matrix and sigma matrix is proportional to $1 + kI_1$, where I_1 is octonionic imaginary unit in $M^2 \subset M^4$. The complexified octonionic units can be chosen to be eigenstates of Q_{em} so that Laplace equation reduces to ordinary scalar Laplacian with coupling to self-dual em field.
3. One expects harmonic oscillator like behavior for the modes of the Dirac operator of M^8 since the gauge potential is linear in E^4 coordinates. One possibility is Cartesian coordinates is $A(A_x, A_y, A_z, A_t) = k(-y, x, t, -z)$. Thhe coupling would make E^4 effectively a compact space.
4. The square of Dirac operator gives potential term proportional to $r^2 = x^2 + y^2 + z^2 + t^2$ so that the spectrum of 4-D harmonic oscillator operator and $SO(4)$ harmonics localized near origin are expected. For harmonic oscillator the symmetry enhances to $SU(4)$.

If one replaces Kähler coupling with em charge symmetry breaking of $SO(4)$ to vectorial $SO(3)$ is expected since the coupling is proportional to $1 + ike_1$ defining electromagnetic charge. Since the basis of complexified quaternions can be chosen to be eigenstates of e_1 under multiplication, octonionic spinors are eigenstates of em charge and one obtains two color singles $1 \pm e_1$ and color triplet and antitriplet. The color triplets cannot be however interpreted in terms of quark color.

Harmonic oscillator potential is expected to enhance $SO(3)$ to SU(3). This suggests the reduction of the symmetry to $SU(3) \times U(1)$ corresponding to color symmetry and em charge so that one would have same basic quantum numbers as tof CP_2 harmonics. An interesting question is how the spectrum and mass squared eigenvalues of harmonics differ from those for CP_2.

5. In the square of Dirac equation $J^{kl}\Sigma_{kl}$ term distinguishes between different em charges ($\Sigma_k l$ reduces by self duality and by special properties of octonionic sigma matrices to a term proportional to iI_1 and complexified octonionic units can be chosen to be its eigenstates with eigen value ± 1. The vacuum mass squared analogous to the vacuum energy of harmonic oscillator is also present and this contribution are expected to cancel themselves for neutrinos so that they are massless whereas charged leptons and quarks are massive. It remains to be checked that quarks and leptons can be classified to triality $T = \pm 1$ and $t = 0$ representations of dynamical $SU(3)$ respectively.

What about the analog of Kähler Dirac equation

Only the octonionic structure in $T(M^8)$ is needed to formulate quaternionicity of space-time surfaces: the reduction to O_c-real-analyticity would be extremely nice but not necessary (O_c denotes complexified octonions needed to cope with Minkowskian signature). Most importantly, there might be no need to introduce Kähler action (and Kähler form) in M^8. Even the octonionic representation of gamma matrices is un-necessary. Neither there is any absolute need to define octonionic Dirac equation and octonionic Kähler Dirac equation nor octonionic analog of its solutions nor the octonionic variants of embedding space harmonics.

It would be of course nice if the general formulas for solutions of the Kähler Dirac equation in H could have counterparts for octonionic spinors satisfying quaternionicity condition. One can indeed wonder whether the restriction of the modes of induced spinor field to string world sheets defined by integrable distributions of hyper-complex spaces $M^2(x)$ could be interpretated in terms of commutativity of fermionic physics in M^8. $M^8 - H$ correspondence could map the octonionic spinor fields at string world sheets to their quaternionic counterparts in H. The fact that only holomorphy is involved with the definition of modes could make this map possible.

10.2.4 How Could One Solve Associativity/Co-Associativity Conditions?

The natural question is whether and how one could solve the associativity/-co-associativity conditions explicitly. One can imagine two approaches besides $M^8 \to H \to H...$ iteration generating new solutions from existing ones.

Could octonion-real analyticity be equivalent with associativity/co-associativity?

Analytic functions provide solutions to 2-D Laplace equations and one might hope that also the field equations could be solved in terms of octonion-real-analyticity at the level of M^8 perhaps also at the level of H. Signature however causes problems - at least technical. Also the compactness of CP_2 causes technical difficulties but they need not be insurmountable.

For E^8 the tangent space would be genuinely octonionic and one can define the notion octonion-real analytic map as a generalization of real-analytic function of complex variables (the coefficients of Laurent series are real to guarantee associativity of the series). The argument is complexified octonion in $O \oplus iO$ forming an algebra but not a field. The norm square is Minkowskian as difference of two Euclidian octonionic norms: $N(o_1 + io_2) = N(o_1) - N(o_2)$ and vanishes at 15-D light cone boundary. Obviously, differential calculus is possible outside the light-cone boundary. Rational analytic functions have however poles at the light-cone boundary. One can wonder whether the poles at M^4 light-cone boundary, which is subset of 15-D light-cone boundary could have physical significance and relevant for the role of causal diamonds in ZEO.

The candidates for associative surfaces defined by O_c-real-analytic functions (I use O_c for complexified octonions) have Minkowskian signature of metric and are 4-surfaces at which the projection of $f(o_1 + io_2)$ to $Im(O_1)$, $iIm(O_2)$, and $iRe(Q_2) \oplus Im(Q_1)$ vanish so that only the projection to hyper-quaternionic Minkowskian sub-space $M^4 = Re(Q_1) + iIm(Q_2)$ with signature $(1, -1, -, 1-, 1)$ is non-vanishing. The inverse image need not belong to M^8 and in general it belongs to M^8_c but this is not a problem: all that is needed that the tangent space of inverse image is complexified quaternionic. If this is the case then $M^8 - H$ duality maps the tangent space of the inverse image to CP_2 point and image itself defines the point of M^4 so that a point of H is obtained. Co-associative surfaces would be surfaces for which the projections of image to $Re(O_1)$, $iRe(O_2)$, and to $Im(O_1)$ vanish so that only the projection to $iIm(O_2)$ with signature $(-1, -1, -1, -1)$ is non-vanishing.

The inverse images as 4-D sub-manifolds of M^8_c (not M^8!) are excellent candidates for associative and co-associative 4-surfaces since $M^8 - H$ duality assignes to them a 4-surface in $M^4 \times CP_2$ if the tangent space at given point is complexified quaternionic. This is true if one believes on the analytic continuation of the intuition from complex analysis (the image of real axes under the map defined by O_c-real-analytic function is real axes in the new coordinates defined by the map: the intuition results by replacing "real" by "complexified quaternionic"). The possibility to solve field equations in this manner would be of enormous significance since besides basic arithmetic operations also the functional decomposition of O_c-real-analytic functions produces similar functions. One could speak of the algebra of space-time surfaces.

What is remarkable that the complexified octonion real analytic functions are obtained by analytic continuation from single real valued function of real argument. The real functions form naturally a hierarchy of polynomials (maybe also rational functions) and number theoretic vision suggests that there coefficients are rationals or algebraic numbers. Already for rational coefficients hierarchy of algebraic extensions of rationals results as one solves the vanishing conditions. There is a temptation to regard this hierarchy coding for space-time sheets as an analog of DNA.

Note that in the recent formulation there is no need to pose separately the condition about integrable distribution of $M^2(x) \subset M^4$.

Quaternionicity condition for space-time surfaces

Quaternionicity actually has a surprisingly simple formulation at the level of space-time surfaces. The following discussion applies to both M^8 and H with minor modifications if one accepts that also H can allow octonionic tangent space structure, which does not require gamma matrices.

1. Quaternionicity is equivalent with associativity guaranteed by the vanishing of the associator $A(a, b, c) = a(bc) - (ab)c$ for any triplet of imaginary tangent vectors in the tangent space of the space-time surface. The condition must hold true for purely imaginary combinations of tangent vectors.

2. If one is able to choose the coordinates in such a way that one of the tangent vectors corresponds to real unit (in the embedding map embedding space M^4 coordinate depends only on the time coordinate of space-time surface), the condition reduces to the vanishing of the octonionic product of remaining three induced gamma matrices interpreted as octonionic gamma matrices. This condition looks very simple - perhaps too simple!- since it involves only first derivatives of the embedding space vectors.

   One can of course whether quaternionicity conditions replace field equations or only select preferred extremals. In the latter case, one should be able to prove that quaternionicity conditions are consistent with the field equations.

3. Field equations would reduce to tri-linear equations in in the gradients of embedding space coordinates (rather than involving embedding space coordinates quadratically). Sum of analogs of 3×3 determinants deriving from $a \times (b \times b)$ for different octonion units is involved.

4. Written explicitly field equations give in terms of vielbein projections e^A_α, vielbein vectors e^A_k, coordinate gradients $\partial_\alpha h^k$ and octonionic structure constants f_{ABC} the following conditions stating that the projections of the octonionic associator tensor to the space-time surface vanishes:

$$\begin{aligned} e^A_\alpha e^B_\beta e^C_\gamma A^E_{ABC} &= 0 \ , \\ A^E_{ABC} &= f_{AD}{}^E f_{BC}{}^D - f_{AB}{}^D f_{DC}{}^E \ , \\ e^A_\alpha &= \partial_\alpha h^k e^A_k \ , \\ \Gamma_k &= e^A_k \gamma_A \ . \end{aligned} \tag{10.2.1}$$

The very naïve idea would be that the field equations are indeed integrable in the sense that they reduce to these tri-linear equations. Tri-linearity in derivatives is highly non-trivial outcome simplifying the situation further. These equations can be formulated as the as purely algebraic equations written above plus integrability conditions

$$F^A_{\alpha\beta} = D_\alpha e^A_\beta - D_\beta e^A_\alpha = 0 \ . \tag{10.2.2}$$

One could say that vielbein projections define an analog of a trivial gauge potential. Note however that the covariant derivative is defined by spinor connection rather than this effective gauge potential which reduces to that in SU(2). Similar formulation holds true for field equations and one should be able to see whether the field equations formulated in terms of derivatives of vielbein projections commute with the associatitivity conditions.

5. The quaternionicity conditions can be formulated as vanishing of generalization of Cayley's hyperdeterminant for "hypermatrix" a_{ijk} with 2-valued indiced (see `http://tinyurl.com/ya7h3n9z`). Now one has 8 hyper-matrices with 3 8-valued indices associated with the vanishing $A^E_{BCD}x^By^Cz^D = 0$ of trilinear forms defined by the associators. The conditions say somethig only about the octonioni structure constants and since octonionic space allow quaternionic sub-spaces these conditions must be satisfied.

The inspection of the Fano triangle [A67] (see **Fig. 10.1**) expressing the multiplication table for octonionic imaginary units reveals that give any two imaginary octonion units e_1 and e_2 their product e_1e_2 (or equivalently commutator) is imaginary octonion unit (2 times octonion unit) and the three units span together with real unit quaternionic sub-algebra. There it seems that one can generate local quaternionic sub-space from two imaginary units plus real unit. This generalizes to the vielbein components of tangent vectors of space-time surface and one can build the solutions to the quaternionicity conditions from vielbein projections e_1, e_2, their product $e_3 = k(x)e_1e_2$ and real fourth "time-like" vielbein component which must be expressible as a combination of real unit and imaginary units:

$$e_0 = a \times 1 + b^i e_i$$

For static solutions this condition is trivial. Here summation over i is understood in the latter term. Besides these conditions one has integrability conditions and field equations for Kähler action. This formulation suggests that quaternionicity is additional - perhaps defining - property of preferred extremals.

10.2.5 Quaternionicity At The Level Of Embedding Space Quantum Numbers

From the multiplication table of octonions as illustrated by Fano triangle [A67] one finds that all edges of the triangle, the middle circle and the three the lines connecting vertices to the midpoints of opposite side define triplets of quaternionic units. This means that by taking real unit and any imaginary unit in quaternionic M^4 algebra spanning $M^2 \subset M^4$ and two imaginary units in the complement representing CP_2 tangent space one obtains quaternionic algebra. This suggests an explanation for the preferred M^2 contained in tangent space of space-time surface (the M^2: s could form an integrable distribution). Four-momentum restricted to M^2 and I_3 and Y interpreted as

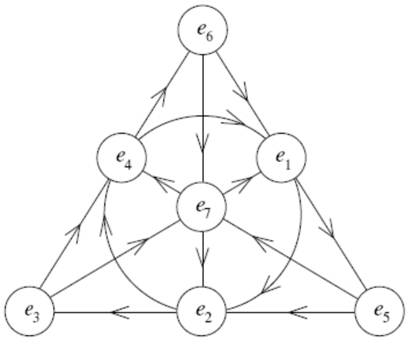

Figure 10.1: Octonionic triangle: the six lines and one circle containing three vertices define the seven associative triplets for which the multiplication rules of the ordinary quaternion imaginary units hold true. The arrow defines the orientation for each associative triplet. Note that the product for the units of each associative triplets equals to real unit apart from sign factor.

tangent vectors in CP_2 tangent space defined quaterionic sub-algebra. This could give content for the idea that quantum numbers are quaternionic.

I have indeed proposed that the four-momentum belongs to M^2. If $M^2(x)$ form a distribution as the proposal for the preferred extremals suggests this could reflect momentum exchanges between different points of the space-time surface such that total momentum is conserved or momentum exchange between two sheets connected by wormhole contacts.

10.2.6 Questions

In following some questions related to $M^8 - H$ duality are represented.

Could associativity condition be formulated using modified gamma matrices?

Skeptic can criticize the minimal form of $M^8 - H$ duality involving no Kähler action in M^8 is unrealistic. Why just Kähler action? What makes it so special? The only defense that I can imagine is that Kähler action is in many respects unique choice.

An alternative approach would replace induced gamma matrices with the modified ones to get the correlation In the case of M^8 this option cannot work. One cannot exclude it for H.

1. For Kähler action the Kähler-Dirac gamma matrices $\Gamma^\alpha = \frac{\partial L_K}{\partial h^k_\alpha}\Gamma^k$, $\Gamma_k = e^A_k \gamma_A$, assign to a given point of X^4 a 4-D space which need not be tangent space anymore or even its sub-space. The reason is that canonical momentum current contains besides the gravitational contribution coming from the induced metric also the "Maxwell contribution" from the induced Kähler form not parallel to space-time surface. In the case of M^8 the duality map to H is therefore lost.

2. The space spanned by the Kähler-Dirac gamma matrices need not be 4-dimensional. For vacuum extremals with at most 2-D CP_2 projection Kähler-Dirac gamma matrices vanish identically. For massless extremals they span 1- D light-like subspace. For CP_2 vacuum extremals the modified gamma matrices reduces to ordinary gamma matrices for CP_2 and the situation reduces to the quaternionicity of CP_2. Also for string like objects the conditions are satisfied since the gamma matrices define associative sub-space as tangent space of $M^2 \times S^2 \subset M^4 \times CP_2$. It seems that associativity is satisfied by all known extremals. Hence Kähler-Dirac gamma matrices are flexible enough to realize associativity in H.

3. Kähler-Dirac gamma matrices in Dirac equation are required by super conformal symmetry for the extremals of action and they also guarantee that vacuum extremals defined by surfaces in $M^4 \times Y^2$, Y^2 a Lagrange sub-manifold of CP_2, are trivially hyper-quaternionic surfaces.

The modified definition of associativity in H does not affect in any manner $M^8 - H$ duality necessarily based on induced gamma matrices in M^8 allowing purely number theoretic interpretation of standard model symmetries. One can however argue that the most natural definition of associativity is in terms of induced gamma matrices in both M^8 and H.

Remark: A side comment not strictly related to associativity is in order. The anti-commutators of the Kähler-Dirac gamma matrices define an effective Riemann metric and one can assign to it the counterparts of Riemann connection, curvature tensor, geodesic line, volume, etc... One would have two different metrics associated with the space-time surface. Only if the action defining space-time surface is identified as the volume in the ordinary metric, these metrics are equivalent. The index raising for the effective metric could be defined also by the induced metric and it is not clear whether one can define Riemann connection also in this case. Could this effective metric have concrete physical significance and play a deeper role in quantum TGD? For instance, AdS-CFT duality leads to ask whether interactions be coded in terms of the gravitation associated with the effective metric.

Now skeptic can ask why should one demand $M^8 - H$ correspondence if one in any case is forced to introduced Kähler also at the level of M^8? Does $M^8 - H$ correspondence help to construct preferred extremals or does it only bring in a long list of conjectures? I can repeat the questions of the skeptic.

Minkowskian-Euclidian $\leftrightarrow$ associative–co-associative?

The 8-dimensionality of M^8 allows to consider both associativity of the tangent space and associativity of the normal space- let us call this co-associativity of tangent space- as alternative options. Both options are needed as has been already found. Since space-time surface decomposes into regions whose induced metric possesses either Minkowskian or Euclidian signature, there is a strong temptation to propose that Minkowskian regions correspond to associative and Euclidian regions to co-associative regions so that space-time itself would provide both the description and its dual.

The proposed interpretation of conjectured associative-co-associative duality relates in an interesting manner to p-adic length scale hypothesis selecting the primes $p \simeq 2^k$, k positive integer as preferred p-adic length scales. $L_p \propto \sqrt{p}$ corresponds to the p-adic length scale defining the size of the space-time sheet at which elementary particle represented as CP_2 type extremal is topologically condensed and is of order Compton length. $L_k \propto \sqrt{k}$ represents the p-adic length scale of the wormhole contacts associated with the CP_2 type extremal and CP_2 size is the natural length unit now. Obviously the quantitative formulation for associative-co-associative duality would be in terms $p \to k$ duality.

Can $M^8 - H$ duality be useful?

Skeptic could of course argue that $M^8 - H$ duality generates only an inflation of unproven conjectures. This might be the case. In the following I will however try to defend the conjecture. One can however find good motivations for $M^8 - H$ duality: both theoretical and physical.

1. If $M^8 - H$ duality makes sense for induced gamma matrices also in H, one obtains infinite sequence if dualities allowing to construct preferred extremals iteratively. This might relate to octonionic real-analyticity and composition of octonion-real-analytic functions.

2. $M^8 - H$ duality could provide much simpler description of preferred extremals of Kähler action as hyper-quaternionic surfaces. Unfortunately, it is not clear whether one should introduce the counterpart of Kähler action in M^8 and the coupling of M^8 spinors to Kähler form. Note that the Kähler form in E^4 would be self dual and have constant components: essentially parallel electric and magnetic field of same constant magnitude.

3. $M^8 - H$ duality provides insights to low energy physics, in particular low energy hadron physics. M^8 description might work when H-description fails. For instance, perturbative QCD which corresponds to H-description fails at low energies whereas M^8 description might become perturbative description at this limit. Strong $SO(4) = SU(2)_L \times SU(2)_R$ invariance is the basic symmetry of the phenomenological low energy hadron models based on conserved vector current hypothesis (CVC) and partially conserved axial current hypothesis (PCAC).

Strong $SO(4) = SU(2)_L \times SU(2)_R$ relates closely also to electro-weak gauge group $SU(2)_L \times U(1)$ and this connection is not well understood in QCD description. $M^8 - H$ duality could provide this connection. Strong $SO(4)$ symmetry would emerge as a low energy dual of the color symmetry. Orbital $SO(4)$ would correspond to strong $SU(2)_L \times SU(2)_R$ and by flatness of E^4 spin like $SO(4)$ would correspond to electro-weak group $SU(2)_L \times U(1)_R \subset SO(4)$. Note that the inclusion of coupling to Kähler gauge potential is necessary to achieve respectable spinor structure in CP_2. One could say that the orbital angular momentum in $SO(4)$ corresponds to strong isospin and spin part of angular momentum to the weak isospin. This argument does not seem to be consistent with $SU(3) \times U(1) \subset SU(4)$ symmetry for Mx Dirac equation. One can however argue that $SU(4)$ symmetry combines $SO(4)$ multiplets together. Furthermore, $SO(4)$ represents the isometries leaving Kähler form invariant.

$M^8 - H$ duality in low energy physics and low energy hadron physics

$M^8 - H$ can be applied to gain a view about color confinement. The basic idea would be that $SO(4)$ and $SU(3)$ provide provide dual descriptions of quarks using E^4 and CP_2 partial waves and low energy hadron physics corresponds to a situation in which M^8 picture provides the perturbative approach whereas H picture works at high energies.

A possible interpretation is that the space-time surfaces vary so slowly in CP_2 degrees of freedom that can approximate CP_2 with a small region of its tangent space E^4. One could also say that color interactions mask completely electroweak interactions so that the spinor connection of CP_2 can be neglected and one has effectively E^4. The basic prediction is that $SO(4)$ should appear as dynamical symmetry group of low energy hadron physics and this is indeed the case.

Consider color confinement at the long length scale limit in terms of $M^8 - H$ duality.

1. At high energy limit only lowest color triplet color partial waves for quarks dominate so that QCD description becomes appropriate whereas very higher color partial waves for quarks and gluons are expected to appear at the confinement limit. Since WCW degrees of freedom begin to dominate, color confinement limit transcends the descriptive power of QCD.
2. The success of $SO(4)$ sigma model in the description of low lying hadrons would directly relate to the fact that this group labels also the E^4 Hamiltonians in M^8 picture. Strong $SO(4)$ quantum numbers can be identified as orbital counterparts of right and left handed electro-weak isospin coinciding with strong isospin for lowest quarks. In sigma model pion and sigma boson form the components of E^4 valued vector field or equivalently collection of four E^4 Hamiltonians corresponding to spherical E^4 coordinates. Pion corresponds to S^3 valued unit vector field with charge states of pion identifiable as three Hamiltonians defined by the coordinate components. Sigma is mapped to the Hamiltonian defined by the E^4 radial coordinate. Excited mesons corresponding to more complex Hamiltonians are predicted.
3. The generalization of sigma model would assign to quarks E^4 partial waves belonging to the representations of $SO(4)$. The model would involve also 6 $SO(4)$ gluons and their $SO(4)$ partial waves. At the low energy limit only lowest representations would be important whereas at higher energies higher partial waves would be excited and the description based on CP_2 partial waves would become more appropriate.
4. The low energy quark model would rely on quarks moving $SO(4)$ color partial waves. Left *resp.* right handed quarks could correspond to $SU(2)_L$ *resp.* $SU(2)_R$ triplets so that spin statistics problem would be solved in the same manner as in the standard quark model.
5. Family replication phenomenon is described in TGD framework the same manner in both cases so that quantum numbers like strangeness and charm are not fundamental. Indeed, p-adic mass calculations allowing fractally scaled up versions of various quarks allow to replace Gell-Mann mass formula with highly successful predictions for hadron masses [K72].

To my opinion these observations are intriguing enough to motivate a concrete attempt to construct low energy hadron physics in terms of $SO(4)$ gauge theory.

10.2.7 Summary

The overall conclusion is that the most convincing scenario relies on the associativity/co-associativity of space-time surfaces define by induced gamma matrices and applying both for M^8 and H. The fact that the duality can be continued to an iterated sequence of duality maps $M^8 \rightarrow H \rightarrow H...$ is what makes the proposal so fascinating and suggests connection with fractality.

The introduction of Kähler action and coupling of spinors to Kähler gauge potentials is highly natural. One can also consider the idea that the space-time surfaces in M^8 and H have same induced metric and Kähler form: for iterated duality map this would mean that the steps in the map produce space-time surfaces which identical metric and Kähler form so that the sequence might stop. M^8_H duality might provide two descriptions of same underlying dynamics: M^8 description would apply in long length scales and H description in short length scales.

10.3 Quaternions and TGD

10.3.1 Are Euclidian Regions Of Preferred Extremals Quaternion- Kähler Manifolds?

In blog comments Anonymous gave a link to an article (see `http://tinyurl.com/y7j9hxr8`) about construction of 4-D quaternion-Kähler metrics with an isometry: they are determined by so called $SU(\infty)$ Toda equation. I tried to see whether quaternion-Kähler manifolds could be relevant for TGD.

From Wikipedia (see `http://tinyurl.com/yd8feoev`) one can learn that QK is characterized by its holonomy, which is a subgroup of $Sp(n) \times Sp(1)$: $Sp(n)$ acts as linear symplectic transformations of $2n$-dimensional space (now real). In 4-D case tangent space contains 3-D submanifold identifiable as imaginary quaternions. CP_2 is one example of QK manifold for which the subgroup in question is $SU(2) \times U(1)$ and which has non-vanishing constant curvature: the components of Weyl tensor represent the quaternionic imaginary units. QKs are Einstein manifolds: Einstein tensor is proportional to metric.

What is really interesting from TGD point of view is that twistorial considerations show that one can assign to QK a special kind of twistor space (twistor space in the mildest sense requires only orientability). Wiki tells that if Ricci curvature is positive, this (6-D) twistor space is what is known as projective Fano manifold with a holomorphic contact structure. Fano variety has the nice property that as (complex) line bundle (the twistor space property) it has enough sections to define the embedding of its base space to a projective variety. Fano variety is also complete: this is algebraic geometric analogy of topological property known as compactness.

QK manifolds and twistorial formulation of TGD

How the QKs could relate to the twistorial formulation of TGD?

1. In the twistor formulation of TGD [L8] the space-time surfaces are 4-D base spaces of 6-D twistor spaces in the Cartesian product of 6-D twistor spaces of M^4 and CP_2 - the only twistor spaces with Kähler structure. In TGD framework space-time regions can have either Euclidian or Minkowskian signature of induced metric. The lines of generalized Feynman diagrams are Euclidian.
2. Could the twistor spaces associated with the lines of generalized Feynman diagrams be projective Fano manifolds? Could QK structure characterize Euclidian regions of preferred extremals of Kähler action? Could a generalization to Minkowskian regions exist.

   I have proposed that so called Hamilton-Jacobi structure [K121] characterizes preferred extremals in Minkowskian regions. It could be the natural Minkowskian counterpart for the quaternion Kähler structure, which involves only imaginary quaternions and could make sense also in Minkowski signature. Note that unit sphere of imaginary quaternions defines the sphere serving as fiber of the twistor bundle.

Why it would be natural to have QK that is corresponding twistor space, which is projective contact Fano manifold?

1. QK property looks very strong condition but might be true for the preferred extremals satisfying very strong conditions stating that the classical conformal charges associated with various conformal algebras extending the conformal algebras of string models [K121], [L16]. These conditions would be essentially classical gauge conditions stating that strong form of

holography implies by strong form of General Coordinate Invariance (GCI) is realized: that is partonic 2-surfaces and their 4-D tangent space data code for quantum physics.

2. Kähler property makes sense for space-time regions of Euclidian signature and would be natural is these regions can be regarded as small deformations of CP_2 type vacuum extremals with light-like M^4 projection and having the same metric and Kähler form as CP_2 itself.

3. Fano property implies that the 4-D Euclidian space-time region representing line of the Feynman diagram can be imbedded as a sub-manifold to complex projective space CP_n. This would allow to use the powerful machinery of projective geometry in TGD framework. This could also be a space-time correlate for the fact that CP_ns emerge in twistor Grassmann approach expected to generalize to TGD framework.

4. CP_2 allows both projective (trivially) and contact (even symplectic) structures. $\delta M^4_+ \times CP_2$ allows contact structure - I call it loosely symplectic structure. Also 3-D light-like orbits of partonic 2-surfaces allow contact structure. Therefore holomorphic contact structure for the twistor space is natural.

5. Both the holomorphic contact structure and projectivity of CP_2 would be inherited if QK property is true. Contact structures at orbits of partonic 2-surfaces would extend to holomorphic contact structures in the Euclidian regions of space-time surface representing lines of generalized Feynman diagrams. Projectivity of Fano space would be also inherited from CP_2 or its twistor space $SU(3)/U(1) \times U(1)$ (flag manifold identifiable as the space of choices for quantization axes of color isospin and hypercharge).

The article considers a situation in which the QK manifold allows an isometry. Could the isometry (or possibly isometries) for QK be seen as a remnant of color symmetry or rotational symmetries of M^4 factor of embedding space? The only remnant of color symmetry at the level of embedding space spinors is anomalous color hyper charge (color is like orbital angular momentum and associated with spinor harmonic in CP_2 center of mass degrees of freedom). Could the isometry correspond to anomalous hypercharge?

How to choose the quaternionic imaginary units for the space-time surface?

Parallellizability is a very special property of 3-manifolds allowing to choose quaternionic imaginary units: global choice of one of them gives rise to twistor structure.

1. The selection of time coordinate defines a slicing of space-time surface by 3-surfaces. GCI would suggest that a generic slicing gives rise to 3 quaternionic units at each point each 3-surface? The parallelizability of 3-manifolds - a unique property of 3-manifolds - means the possibility to select global coordinate frame as section of the frame bundle: one has 3 sections of tangent bundle whose inner products give rose to the components of the metric (now induced metric) guarantees this. The tri-bein or its dual defined by two-forms obtained by contracting tri-bein vectors with permutation tensor gives the quanternionic imaginary units. The construction depends on 3-metric only and could be carried out also in GRT context. Note however that topology change for 3-manifold might cause some non-trivialities. The metric 2-dimensionality at the light-like orbits of partonic 2-surfaces should not be a problem for a slicing by space-like 3-surfaces. The construction makes sense also for the regions of Minkowskian signature.

2. In fact, any 4-manifold (see `http://tinyurl.com/yb8l34b5`) [A83] allows almost quaternionic as the above slicing argument relying on parallelizibility of 3-manifolds strongly suggests.

3. In zero energy ntology (ZEO)- a purely TGD based feature - there are very natural special slicings. The first one is by linear time-like Minkowski coordinate defined by the direction of the line connecting the tips of the causal diamond (CD). Second one is defined by the light-cone proper time associated with either light-cone in the intersection of future and past directed light-cones defining CD. Neither slicing is global as it is easy to see.

The relationship to quaternionicity conjecture and $M^8 - H$ duality

One of the basic conjectures of TGD is that preferred extremals consist of quaternionic/ co-quaternionic (associative/co-associative) regions [K105]. Second closely related conjecture is $M^8 - H$ duality allowing to map quaternionic/co-quaternionic surfaces of M^8 to those of $M^4 \times CP_2$. Are these conjectures consistent with QK in Euclidian regions and Hamilton-Jacobi property in Minkowskian regions? Consider first the definition of quaternionic and co-quaternionic space-time regions.

1. Quaternionic/associative space-time region (with Minkowskian signature) is defined in terms of induced octonion structure obtained by projecting octonion units defined by vielbein of $H = M^4 \times CP_2$ to space-time surface and demanding that the 4 projections generate quaternionic sub-algebra at each point of space-time.

   If there is also unique complex sub-algebra associated with each point of space-time, one obtains one can assign to the tangent space-of space-time surface a point of CP_2. This allows to realize $M^8 - H$ duality [K105] as the number theoretic analog of spontaneous compactification (but involving no compactification) by assigning to a point of $M^4 = M^4 \times CP_2$ a point of $M^4 \times CP_2$. If the image surface is also quaternionic, this assignment makes sense also for space-time surfaces in H so that $M^8 - H$ duality generalizes to $H - H$ duality allowing to assign to given preferred extremal a hierarchy of extremals by iterating this assignment. One obtains a category with morphisms identifiable as these duality maps.

2. Co-quaternionic/co-associative structure is conjectured for space-time regions of Euclidian signature and 4-D CP_2 projection. In this case normal space of space-time surface is quaternionic/associative. A multiplication of the basis by preferred unit of basis gives rise to a quaternionic tangent space basis so that one can speak of quaternionic structure also in this case.

3. Quaternionicity in this sense requires unique identification of a preferred time coordinate as embedding space coordinate and corresponding slicing by 3-surfaces and is possible only in TGD context. The preferred time direction would correspond to real quaternionic unit. Preferred time coordinate implies that quaternionic structure in TGD sense is more specific than the QK structure in Euclidian regions.

4. The basis of induced octonionic imaginary unit allows to identify quaternionic imaginary units linearly related to the corresponding units defined by tri-bein vectors. Note that the multiplication of octonionic units is replaced with multiplication of antisymmetric tensors representing them when one assigns to the quaternionic structure potential QK structure. Quaternionic structure does not require Kähler structure and makes sense for both signatures of the induced metric. Hence a consistency with QK and its possible analog in Minkowskian regions is possible.

5. The selection of the preferred imaginary quaternion unit is necessary for $M^8 - H$ correspondence. This selection would also define the twistor structure. For quaternion-Kähler manifold this unit would be covariantly constant and define Kähler form - maybe as the induced Kähler form.

6. Also in Minkowskian regions twistor structure requires a selection of a preferred imaginary quaternion unit. Could the induced Kähler form define the preferred imaginary unit also now? Is the Hamilton-Jacobi structure consistent with this?

   Hamilton-Jacobi structure involves a selection of 2-D complex plane at each point of space-time surface. Could induced Kähler magnetic form for each 3-slice define this plane? It is not necessary to require that 3-D Kähler form is covariantly constant for Minkowskian regions. Indeed, massless extremals representing analogs of photons are characterized by local polarization and momentum direction and carry time-dependent Kähler-electric and -magnetic fields. One can however ask whether monopole flux tubes carry covariantly constant Kähler magnetic field: they are indeed deformations of what I call cosmic strings [K14, K29] for which this condition holds true?

10.3.2 The Notion of Quaternion Analyticity

The 4-D generalization of conformal invariance suggests strongly that the notion of analytic function generalizes somehow. The obvious ideas coming in mind are appropriately defined quaternionic and octonion analyticity. I have used a considerable amount of time to consider these possibilities but had to give up the idea about octonion analyticity could somehow allow to preferred extemals.

Basic idea

One can argue that quaternion analyticity is the more natural option in the sense that the local octonionic embedding space coordinate (or at least M^8 or E^8 coordinate, which is enough if M^8-H duality holds true) would for preferred extremals be expressible in the form

$$o(q) = u(q) + v(q) \times I \quad . \tag{10.3.1}$$

Here q is quaternion serving as a coordinate of a quaternionic sub-space of octonions, and I is octonion unit belonging to the complement of the quaternionic sub-space, and multiplies $v(q)$ from *right* so that quaternions and quaternionic differential operators acting from left do not notice these coefficients at all. A stronger condition would be that the coefficients are real. $u(q)$ and $v(q)$ would be quaternionic Taylor- of even Laurent series with coefficients multiplying powers of q from right for the same reason.

The signature of M^4 metric is a problem. I have proposed a complexification of M^8 and M^4 to get rid of the problem by assuming that the embedding space corresponds to surfaces in the space M^8 identified as octonions of form $o_8 = Re(o) + iIm(o)$, where o is imaginary part of ordinary octonion and i is commuting imaginary unit. M^4 would correspond to quaternions of form $q_4 = Re(q) + iIm(q)$. What is important is that powers of q_4 and o_8 belong to this sub-space (as follows from the vanishing of cross product term in the square of octonion/quaternion) so that powers of q_4 (o_8) has imaginary part proportional to $Im(q)$ ($Im(o)$)

I ended up to reconsider the idea of quaternion analyticity after having found two very interesting articles discussing the generalization of Cauchy-Riemann equations. The first article (see `http://tinyurl.com/yb8l34b5`) [A83] was about so called triholomorphic maps between 4-D almost quaternionic manifolds. The article gave as a reference an article (see `http://tinyurl.com/y7kww2o2`) [A69] about quaternionic analogs of Cauchy-Riemann conditions discussed by Fueter long ago (somehow I have managed to miss Fueter's work just like I missed Hitchin's work about twistorial uniqueness of M^4 and CP_2), and also a new linear variant of these conditions, which seems especially interesting from TGD point of view as will be found.

The first form of Cauchy-Riemann-Fueter conditions

Cauchy-Riemann-Fueter (CRF) conditions generalize Cauchy-Riemann conditions. These conditions are however not unique. Consider first the translationally invariant form of CRF conditions.

1. The translationally invariant form of CRF conditions is $\partial_{\overline{q}} f = 0$ or explicitly

$$\partial_{\overline{q}} f = d_1 f + d_2 f \equiv (\partial_t - \partial_x I) f - (\partial_y J + \partial_z K) f = 0 \quad . \tag{10.3.2}$$

   This form is not unique: one can perform SO(3) rotations of the quaternionic imaginary units acting as automorphisms of quaternions. This form does not allow quaternionic Taylor series as a solution. Note that the Taylor coefficients multiplying powers of the coordinate from right are arbitrary quaternions. What looks pathological is that even linear functions of q fail be solve this condition. What is however interesting that in flat space the equation is equivalent with Dirac equation for a pair of Majorana spinors [A83].

   Function $f = t + Ix - Jy - Kz$ is perhaps the simplest solution to the condition. One can define also other variants of $\overline{q}$, in particular the variant $\overline{q} = t + Ix - Jy - Kz$ giving $f = t + Ix + Jy + Kz$ as a solution.

2. The condition allows functions depending on complex coordinate z of some complex-plane only. It also allows functions satisfying two separate analyticity conditions, say $d_1 f = 0$ and $d_2 f = 0$, say

$$\partial_{\overline{u}} f = (\partial_t - \partial_x I) f = 0 \ ,$$

$$\partial_{\overline{v}} f = -(\partial_y J + \partial_z K) f = -J(\partial_y - \partial_z I) f = 0 \ . \tag{10.3.3}$$

   In the latter formula J multiplies from *left*! One has good hopes of obtaining holomorphic functions of two complex coordinates.

   The simplest solution to the conditions is complex value function $f(u = x + iy, v = y + iz)$ of two complex variables. The image of E^4 is 2-dimensional whereas for $f_0 = t + Ix - Jy - Kz$ it is 4-D.

   In Euclidian signature one obtains quaternion valued map if the Taylor coefficients a_{mn} in the series of $f(u, v)$ are quaternions and are taken to the right: $q = f(u, v) = \sum u^m v^n a_{mn}$ to avoid problems from non-commutativity. With this assumption the image would be 4-D in the generic case.

In TGD one must consider Minkowskian signature and it turns out that the situation changes dramatically, and the naïve view about quaternion analyticity must be given up. The experience about externals of Kähler action suggests a modification of the analyticity properties consistent with the signature but whether one should call this analyticity quaternion analyticity is a matter of taste.

Second form of CRF conditions

Second form of CRF conditions proposed in [A69] is tailored in order to realize the almost obvious manner to realize quaternion analyticity.

1. The ingenious idea is to replace preferred quaternionic imaginary unit by a imaginary unit which is in radial direction: $e_r = (xI + yJ + zK)/r$ and require analyticity with respect to the coordinate $t + e_r r$. The solution to the condition is power series in $t + e_r r = q$ so that one obtains quaternion analyticity.
2. The explicit form of the conditions is

$$(\partial_t - e_r \partial_r) f = (\partial_t - \tfrac{e_r}{r} r \partial_r) f = 0 \ . \tag{10.3.4}$$

   This form allows both the desired quaternionic Taylor series and ordinary holomorphic functions of complex variable in one of the 3 complex coordinate planes as general solutions.
3. This form of CRF is neither Lorentz invariant nor translationally invariant but remains invariant under simultaneous scalings of t and r and under time translations. Under rotations of either coordinates or of imaginary units the spatial part transforms like vector so that quaternionic automorphism group $SO(3)$ serves as a moduli space for these operators.
4. The interpretation of the latter solutions inspired by ZEO would be that in Minkowskian regions r corresponds to the light-like radial coordinate of the either boundary of CD, which is part of $\delta M^4_\pm$. The radial scaling operator is that assigned with the light-like radial coordinate of the light-cone boundary. A slicing of CD by surfaces parallel to the $\delta M^4_\pm$ is assumed and implies that the line $r = 0$ connecting the tips of CD is in a special role. The line connecting the tips of CD defines coordinate line of time coordinate. The breaking of rotational invariance corresponds to the selection of a preferred quaternion unit defining the twistor structure and preferred complex sub-space.

   In regions of Euclidian signature r could correspond to the radial Eguchi-Hanson coordinate of CP_2 and $r = 0$ corresponds to a fixed point of $U(2)$ subgroup under which CP_2 complex coordinates transform linearly.

5. Also in this case one can ask whether solutions depending on two complex local coordinates analogous to those for translationally invariant CRF condition are possible. The remain imaginary units would be associated with the surface of sphere allowing complex structure.

Generalization of CRF conditions?

Could the proposed forms of CRF conditions be special cases of much more general CRF conditions as CR conditions are?

1. Ordinary complex analysis suggests that there is an infinite number of choices of the quaternionic coordinates related by the above described quaternion-analytic maps with 4-D images. The form of of the CRF conditions would be different in each of these coordinate systems and would be obtained in a straightforward manner by chain rule.
2. One expects the existence of large number of different quaternion-conformal structures not related by quaternion-analytic transformations analogous to those allowed by higher genus Riemann surfaces and that these conformal equivalence classes of four-manifolds are characterized by a moduli space and the analogs of Teichmüller parameters depending on 3-topology. In TGD framework strong form of holography suggests that these conformal equivalence classes for preferred extremals could reduce to ordinary conformal classes for the partonic 2-surfaces. An attractive possibility is that by conformal gauge symmetries the functional integral over WCW reduces to the integral over the conformal equivalence classes.
3. The quaternion-conformal structures could be characterized by a standard choice of quaternionic coordinates reducing to the choice of a pair of complex coordinates. In these coordinates the general solution to quaternion-analyticity conditions would be of form described for the linear ansatz. The moduli space corresponds to that for complex or hyper-complex structures defined in the space-time region.

Geometric formulation of the CRF conditions

The previous naïve generalization of CRF conditions treats imaginary units without trying to understand their geometric content. This leads to difficulties when when tries to formulate these conditions for maps between quaternionic and hyper-quaternionic spaces using purely algebraic representation of imaginary units since it is not clear how these units relate to each other.

In [A83] the CRF conditions are formulated in terms of the antisymmetric (1, 1) type tensors representing the imaginary units: they exist for almost quaternionic structure. One might hope that this so also for the almost hyper-quaternionic structure needed in Minkowskian signature.

The generalization of CRF conditions is proposed in terms of the Jacobian J of the map mapping tangent space TM to TN and antisymmetric tensors J_u and j_u representing the quaternionic imaginary units of N and M respectively. The generalization of CRF conditions reads as

$$J - \sum_u J_u \circ J \circ j_u = 0 \quad . \tag{10.3.5}$$

For $N = M$ it reduces to the translationally invariant algebraic form of the conditions discussed above. These conditions reduce to CR conditions in 2-D case when one has only single J_u. In quaternionic case this form is only replaced with sum over all 3 antisymmetric forms representing quaternionic units.

These conditions are not unique. One can perform an SO(3) rotation (quaternion automorphism) of the imaginary units mediated by matrix Λ^{uv} to obtain

$$J - \Lambda^{uv} J_u \circ J \circ j_v = 0 \quad . \tag{10.3.6}$$

The matrix Λ can depend on point so that one has a kind of gauge symmetry. The most general triholomorphic map allows the presence of Λ. Note that these conditions make sense on any coordinates and complex analytic maps generate new forms of these conditions.

In Minkowskian signature one would have 3 forms iJ_u serving as counterparts for iI, iJ, iK. The most natural possibility is that i is represented as algebraic unit and I, J, K as antisymmetry self-dual em fields with E and B constant and parallel to each other and normalize to have unit lengths. Their directions would correspond to 3 orthogonal coordinate axis. The twistor lift forces to introduce the generalization of Kähler form of M^4 and one can introduce all these 3 independent forms as counterpart of hyperquaternionic units: they are introduced also for ordinary twistor structure but one of them is selected as a preferred one. The only change in the conditions is change of sign of the sign of the sum coming from $i^2 = -1$ so that one has

$$J + \sum_u J_u \circ J \circ j_u = 0 \ . \tag{10.3.7}$$

These conditions are therefore formally well-defined also when one maps quaternionic to hyper-quaternionic space or vice versa.

In 2-dimensional hypercomplex case the conditions allow to write hypercomplex map $X - Y = U = f(x-y)$ and $X + Y = V = f(x+y)$. In special case this solutions of massless d'Alembertian in M^2. Alternatively, one can express f as analytic function of $x + iIy$ and pick up $X - Y$ and $X + Y$. It is however not clear whether one can write a Taylor expansion in hyper-quaternionic coordinate in the similar manner.

Covariant forms of structure constant tensors reduce to octonionic structure constants and this allows to write the conditions explicitly. The index raising of the second index of the structure constants is however needed using the metrics of M and N. This complicates the situation and spoils linearity: in particular, for surfaces induced metric is needed. Whether local SO(3) rotation can eliminate the dependence on induced metric is an interesting question. Since M^4 imaginary units differ only by multiplication by i, Minkowskian structure constants differ only by sign from the Euclidian ones.

In the octonionic case the geometric generalization of CRF conditions does not seem to make sense. By non-associativity of octonion product it is not possible to have a matrix representation for the matrices so that a faithful representation of octonionic imaginary units as antisymmetric 1-1 forms does not make sense. If this representation exists it it must map octonionic associators to zero. Note however that CRF conditions do not involve products of three octonion units so that they make sense as algebraic conditions at least.

Does residue calculus generalize?

CRF conditions allow to generalize Cauchy formula allowing to express value of analytic function in terms of its boundary values [A83]. This would give a concrete realization of the holography in the sense that the physical variables in the interior could be expressed in terms of the data at the light-like partonic orbits and the ends of the space-time surface. Triholomorphic function satisfies d'Alembert/Laplace equations - in induced metric in TGD framework- so that the maximum modulus principle holds true. The general ansatz for a preferred extremals involving Hamilton-Jacobi structure leads to d'Alembert type equations for preferred extremals [K121].

Could the analog of residue calculus exist? Line integral would become 3-D integral reducing to a sum over poles and possible cuts inside the 3-D contour. The space-like 3-surfaces at the ends of CDs could define natural integration contours, and the freedom to choose contour rather freely would reflect General Coordinate Invariance. A possible choice for the integration contour would be the closed 3-surface defined by the union of space-like surfaces at the ends of CD and by the light-like partonic orbits.

Poles and cuts would be in the interior of the space-time surface. Poles have co-dimension 2 and cuts co-dimension 1. Strong form of holography suggests that partonic 2-surfaces and perhaps also string world sheets serve as candidates for poles. Light-like 3-surfaces (partonic orbits) defining the boundaries between Euclidian and Minkowskian regions are singular objects and could serve as cuts. The discontinuity would be due to the change of the signature of the induced metric. There are CDs inside CDs and one can also consider the possibility that sub-CDs define cuts, which in turn reduce to cuts associated with sub-CDs.

10.3.3 Are Preferred Extremals Quaternion-Analytic in Some Sense?

At what level quaternion analyticity could appear in TGD framework? Does it appear only in the formulation of conformal algebras and replace loop algebra with double loop algebra (roughly $z^m \rightarrow u^m v^n$)? Or does it appear in some form also at the level of preferred extremals for which geometric form of quaternionicity is expected to appear - at least at the level of M^8?

Minimalistic view

Before continuing it is good to bring in mind the minimal assumptions and general vision.

1. If $M^8 - H$ duality [K105] holds true, the space-time surface $X^4 \subset M^8 = M^4 \times E^4$ is quaternionic surface in the sense that it have quaternionic tangent space and contains preferred $M^2 \subset M^4$ as part of their tangent space or more generally the 2-D hyper-complex subspaces $M^2(x)$ define and integrable distribution defining 2-D surface.
2. Quaternionicity in geometric sense in M^8 alone *implies* the interpretation as a 4-D surface in $H = M^4 \times CP_2$. There is *no need* to assume quaternionicity in geometric sense in H although it cannot be excluded and would have strong implications [K105]. This one should remember in order to avoid drowning to an inflation of speculations.

It is not at all clear what quaternion analyticity in Minkowskian signature really means or whether it is even possible. The skeptic inside me has a temptation to conclude that the direct extrapolation of quaternion analyticity from Euclidian to Minkowskian signature for space-time surfaces in H is not necessary and might be even impossible. On the other hand, the properties of the known extremals strongly suggest its analog. Quaternion analyticity could however appear at the tangent space level for various generalized conformal algebras transformed to double loop algebras for the proposed realization of the quaternion analyticity.

The naïve generalization of quaternion analyticity to Minkowski signature fails

Quaternion analyticity works nicely in Euclidian signature for maps $E^4 \rightarrow E^4$. One can also consider quaternion analytic maps $E^4 \rightarrow E^8$ with E^8 regarded as octonionic space of form $E^4 \oplus E^4 J$, where E^4 is quaternionic space and J is octonion unit in the complement of $E^4 \subset E^8$. The maps decompose to sums $f_1 \oplus f_2 J$ where f_i are quaternion analytic maps $E^4 \rightarrow E^4$. Consider maps $f : E^4 \rightarrow E^8$, whose graph should define Euclidian space-time surface.

1. One can construct octonion valued maps $f(u, v) = f_0 + \sum u^m v^n a_{mn} : E^4 \rightarrow E^8$ with E^4 identified as quaternionic sub-space of E^8. Recall that one has $u = t + Iz$, $v = (x + Iy)J$. a_{mn} can be octonions with the proposed definition of the Taylor series. Since each power $u^m v^n$ is analytic function, one still has quaternion analyticity in the proposed sense. The image would be 4-D in the general case.
2. By linearity the solutions obey linear superposition. They can be also multiplied if the product is defined as ordered product in such a way that only the powers $t + ix$ and $y + iz$ are multiplied together at left and coefficients a_{mn} are multiplied together at right. The analogy with quantum non-commutativity is obvious.

Can one generalize this ansatz to Minkowskian signature? One can try to look the ansatz for the embedding $X^4 \subset M^8 = M^4 \times E^4 J$ as sum $f = (f_1, f_2)$ of quaternion analytic maps $f_1 : X^4 \rightarrow M^4$ and $f_2 : X^4 \rightarrow E^4$. The general quaternion analytic ansatz for $X^4 \subset E^8$ fails due to the non-commutativity of quaternions.

The comparison of 2-dimensional hypercomplex case with 4-D hyperquaternionic case reveals the basic problem.

1. The analogs CR conditions allow to write hypercomplex map $X - Y = U = f(x - y)$ and $X + Y = V = f(x + y)$. In special case this gives the solutions of massless d'Alembertian in M^2 as sum of these solutions. Alternatively, one can express f as analytic function of $x + iIy$ and pick up $X - Y$ and $X + Y$. The use of hypercomplex numbers and hypercomplex analyticity is equivalent with use of functions $f(x - y)$ or $f(x + y)$.

2. The essential point is that for M^2 regarded as a sub-space of "complexified" complex numbers $z_1 + iz_2$ consisting of points $x + iIy$, the multiplication of numbers of form $x + iIy$ does not lead out of M^2. For M^4 this is not anymore the case since $iI \times iJ = -K$ does not belong to the Minkowskian subspace of complexified quaternions. Hence there are no hopes about the existence of the analog of $f(z) = \sum a_n z^n$. For this reason also non-trivial powers $u^m v^n$ are excluded and one cannot build a Minkowskian generalization of quaternion analytic power series.

3. If one can allow the values of hyper-quaternion analytic functions to be in M^4_c rather than M^4, there are no problems but if one wants to represents space-time surfaces as graphs of hyper-quaternion analytic maps $f : M^4 \to M^8$ one must pose strong restrictions on allowed functions.

The restrictions on the allowed hyper-quaternion analytic functions look rather obvious for what might be called hyper-quaternion analytic maps $M^4 \to M^4$.

1. Assume a decomposition $M^4 = M^2 \times E^2$ such one has $f = (f_1, f_2)$, where $f_1 : M^2 \to M^2$ is analytic in hyper-complex sense and $f_2 : E^2 \to E^2$ is analytic in complex sense. Both these options are possible. One can write the map as $f(u, v) = f_1(u = t + iIz) + f_2(v = x + Iy)iJ$ and it satisfies the usual conditions $\partial_{\overline{u}} f = 0$ and $\partial_{\overline{v}} f = 0$. Note that iJ is taken to the right so that the differential operators acting from left in the analyticity conditions does not "notice" it.

   Linear superpositions of this kind of solutions with real coefficients are possible. One can multiply this kind of solutions if the multiplication is done separately in the Cartesian factors. Also functional composition is possible in the factors.

2. A generalization of the solution ansatz to integrable decompositions $M^4 = M^2(x) \oplus E^2(x)$ is rather plausible. This would mean a foliation of M^4 by pairs of 2-D surfaces. String world sheets and partonic 2-surfaces would be the physical counterpart for these foliations. I have called this kind of foliation Hamilton-Jacobi structure [K8] and it would serve as a generalization of the complex structure to 4-D Minkowskian case. In Euclidian signature it corresponds to ordinary complex structure in 4-D sense.

3. The analogy of double loop Lie algebra replacing powers z^m with $u^m v^n$ does does not however seem to be possible. Could this relate to SH forcing to code data using only functions of u or v and to select either string world sheet or partonic 2-surface (fixing the gauge)?

   On the other hand, the supersymplectic algebra (SSA) and extension of Kac-Moody algebras to light-like orbits of partonic 2-surfaces suggests strongly that functions of form $(t - z)^m v^n$ as basis associated with SSA and SKMAs must be allowed as basis at these 3-D light-like surfaces. These functions generate deformations of boundaries defining symmetries but the corresponding deformations in the interior of the preferred extremals are not expected to be of this form. Double loop algebra would not be lost but would have a nice separable form only at boundaries of CD and at light-like partonic orbits.

What can one conclude?

1. The general experience about the solutions of field equations conforms with this picture coded to the notion of Hamilton-Jacobi structure [K8]. Field equations and purely number theoretic conditions related to Minkowski signature force what might be called number theoretic spontaneous symmetry breaking. This symmetry breaking is analogous to a selection of single imaginary unit defining the analog of Kähler structure for M^4: this imaginary unit defines a new kind of $U(1)$ force in TGD explaining large scale breaking of CP,P, and T [L28]. This kind of selection occurs also for the quaternionic structure of CP_2 [L33].

2. The realistic form of analyticity condition abstractable from the properties known extremals seems to be following. For the Minkowskian space-time surfaces the complex coordinates of H are analytic functions of complex coordinates and of light-like coordinate assignable to space-time surface. These coordinates can be assigned to M^4 and define decomposition $M^4 = M^2 \times E^2$: this decomposition can be local but must be integrable (Hamilton-Jacobi structure). For Euclidian regions with 4-D CP_2 projection complex coordinate of E^2 is complex function of complex coordinates of CP_2 and M^2 light-like coordinate is function of real CP_2 coordinates and second light-like coordinate is constant.

3. The transition to Minkowskian signature by regarding M^4 as sub-space of complex-quaternionic M^4 does not respect the notion of quaternion analyticity in the naïvest sense. Both Euclidian and Minkowskian variants of quaternionic (associative) sub-manifold however makes sense as also co-quaternionic (co-associative) sub-manifold. An attractive hypothesis is that the geometric view about quaternionicity is consistent with the above view about analyticity. The known extremals are consistent with this form of analyticity. Analyticity in this sense should be consistent with the geometric quaternionicity of X^4 in Minkowskian signature and geometric co-quaternionicity in Euclidian signature.
4. The geometric form of quaternionicity (or associativity) requires that the associator $a(bc) - (ab)c$ for any 3 tangent space vectors vanishes. These conditions involve products of 3 partial derivates of embedding space coordinates. For co-associativity this holds true in the normal space. Again one must remember that these conditions might be needed only in M^8 but make sense also for H.

   One must be however cautious: quaternionicity (associativity) in M^8 in the geometric sense *need not* imply even the above realistic form of quaternion analyticity condition in M^8 and even less so in H: this however seems to be the case.

Can the known extremals satisfy the realistic form of quaternion-analyticity?

To test the consistency the realistic form of quaternion analyticity, at the level of M^8 or even H, the best thing to do is to look whether quaternion analyticity is possible for the known extremals for the twistor lift of Kähler action.

Twistor lift drops away most vacuum extremals from consideration and leaves only minimal surfaces. The surviving vacuum extremals include CP_2 type extremals with light-like geodesic rather than arbitrary light-like curve as M^4 projection. Vacuum extremals expressible as graph of map from M^4 to a Lagrangian sub-manifold of CP_2 remain in the spectrum only if they are also minimal surfaces: this kind of minimal surfaces are known to exist.

Massless extremals (MEs) with 2-D CP_2 projection remain in the spectrum. Cosmic strings of form $X^2 \times Y^2 \subset M^4 \times CP_2$ such that X^2 is string world sheet (minimal surface) and Y^2 complex sub-manifold of CP_2 are extremals of both Kähler action and volume term. One can also check whether Hamilton-Jacobi structure of M^4 and of Minkowskian space-time regions and complex structure of CP_2 have natural counterparts in the quaternion-analytic framework.

1. Consider first cosmic strings. In this case the quaternionic-analytic map from $X^4 = X^2 \times Y^2$ to $M^4 \times CP_2$ with octonion structure would be map X^4 to 2-D string world sheet in M^4 and Y^2 to 2-D complex manifold of CP_2. This could be achieved by using the linear variant of CRF condition. The map from X^4 to M^4 would reduce to ordinary hyper-analytic map from X^2 with hyper-complex coordinate to M^4 with hyper-complex coordinates just as in string models. The map from X^4 to CP_2 would reduce to an ordinary analytic map from X^2 with complex coordinates. One would not leave the realm of string models.
2. For the simplest massless extremals (MEs) CP_2 coordinates are arbitrary functions of light-like coordinate $u = k \cdot m$, k constant light-like vector, and of $v = \epsilon \cdot m$, ϵ - a polarization vector orthogonal to k. The interpretation as classical counterpart of photon or Bose-Einstein condensate of photons is obvious. There are good reasons to expect that this ansatz generalizes by replacing the variables u and v with coordinate along the light-like and space-like coordinate lines of Hamilton-Jacobi structure [K8]. The non-geodesic motion of photons with light-velocity and variation of the polarization direction would be due to the interactions with the space-time sheet to which it is topologically condensed.

   Now space-time surface would have naturally M^4 coordinates and the map $M^4 \to M^4$ would be just identity map satisfying the radial CRF condition. Can one understand CP_2 coordinates in terms of the realistic form of quaternion- analyticity? The dependence of CP_2 coordinates on $u = t - x$ only can be formulated as CFR condition $\partial_{\overline{u}} s^k = 0$ and this could is expected to generalize in the formulation using the geometric representation of quaternionic imaginary units at both sides. The dependence on light-light coordinate u follows from the translationally invariant CRF condition.

   The dependence on the real coordinate $v = t - z$ does not conform with the proposed ansatz since the dependence is naturally on complex coordinate w assignable to the polarization plane of form $z = f(w)$. This would give dependence on 2 transversal coordinates and CP_2 projection would be 3-D rather than 2-D. One can of course ask whether this dependence is actually present for preferred extremals? Could the polarization vector be complex local polarization vector orthogonal to the light-like vector? In quantum theory complex polarization vectors are used routinely and become oscillator operators in second quantization and in TGD Universe MEs indeed serve as space-time correlates for photons or their BE condensates.

If this picture makes sense, one must modify the ansatz for the preferred extremals with Minkowskian signature. The E^4 and coordinates and perhaps even real CP_2 coordinates can depend on light-like coordinate u.

3. Vacuum extremals with Lagrangian manifold as (in the generic case 2-D) CP_2 projection survive if they are minimal surfaces. This property should guarantee the realistic form of quaternion analyticity. Hyper-quaternionic structure for space-time surface using Hamilton-Jacobi structure is the first guess. CP_2 should allow a quaternionic coordinate decomposing to a pair of complex coordinates such that second complex coordinate is constant for 2-D Lagrangian manifold and second parameterizes it. Any 2-D surface allows complex structure defined by the induced metric so that there are good hopes that these coordinates exist. The quaternion-analytic map would map in the most general case is trivial for both hypercomplex and complex coordinate of M^4 but the quaternionic Taylor coefficients reduce to real numbers to that the image is 2-D.

4. For CP_2 type vacuum extremals surviving as extremals the M^4 projection is light-like geodesic with $t+z = 0$ with suitable choice of light-like coordinates in M^2. $t-z$ can arbitrary function of CP_2 coordinates. Associativity of the normal space is the only possible option now.

   One can fix the coordinates of X^4 to be complex coordinates of CP_2 so that one gets rid of the degeneracy due to the choice of coordinates. M^4 allows hyper-quaternionic coordinates and Hamilton-Jacobi structures define different choices of hyper-quaternionic coordinates. Now the second light- like coordinate would vary along random light-like geodesics providing slicing of M^4 by 3-D surfaces. Hamilton-Jacobi structure defines at each point a plane $M^2(x)$ fixed by the light-like vector at the point and the 2-D orthogonal plane. In fact 4-D coordinate grid is defined.

5. In the naïve generalization CRF conditions are linear. Whether this is the case in the formulation using the geometric representation of the imaginary units is not clear since the quaternionic imaginary units depend on the vielbein of the induced 3-metric (note however that the SO(3) gauge rotation appearing in the conditions could allow to compensate for the change of the tensors in small deformations of the spaced-time surface). If linearity is real and not true only for small perturbations, one could have linear superpositions of different types of solutions, which looks strange. Could the superpositions describe perturbations of say cosmic strings and massless extremals?

6. According to [A69] both forms of the algebraic CRF conditions generalize to the octonionic situation and right multiplication of powers of octonion by Taylor coefficients plus linearity imply that there are no problems with associativity. This inspires several questions.

   Could octonion analytic maps of embedding space allow to construct new solutions from the existing ones? Could quaternion analytic maps applied at space-time level act as analogs of holomorphic maps and generalize conformal gauge invariance to 4-D context?

Quaternion analyticity and generalized conformal algebras

The realistic quaternionic analyticity should apply at the level of conformal algebras for conformal algebra is replaced with a direct sum of 2-D conformal and hyper-conformal algebra assignable to string world sheets and partonic 2-surfaces. This would conform with SH and the considerations above.

It is however too early to exclude the possibility that the powers z^n of conformal algebras are replaced by $u^m z^n$ ($u = t - z$ and $w = x + iy$) for symmetries restricted to the light-like boundaries of CD and to the light-like orbits of partonic 2-surfaces. This preferred form at boundaries would be essential for reducing degrees of freedom implied by SSA and SKMA gauge conditions. In the interior of space-time surfaces this simple form would be lost.

This would realize the Minkowskian analog of double loop algebras suggested by 4-dimensionality. This option is encouraged by the structure of super-symplectic algebra and generalization of Kac-Moody algebras for light-like orbits of partonic 2-surfaces. Again one must however remember that these algebras should have a realization at the level of M^8 but might not be necessary at the level of H.

1. The basic vision of quantum TGD is that string world sheets are replaced with 4-D surfaces and this forces a generalization of the notion of conformal invariance and one indeed obtains generalized conformal invariances for both the light-like boundaries of CD and for the light-like 3-surfaces defining partonic orbits as boundaries between Minkowskian and Euclidian space-time regions. One can very roughly say that string string world sheets parameterized by complex coordinate are replaced by space-time surfaces parameterized by two complex coordinates. Quaternion analyticity in the sense discussed would roughly conform with this picture.
2. The recent work with the Yangians [K30] and so called n-structures related to the categorification of TGD [K13] suggest that double loop algebras for which string world sheets are replaced with 4-D complex spaces. Quantum groups and Yangians assignable to Kac-Moody algebras rather than Lie algebras should be also central. Also double quantum groups depending on 2 parameters with so called elliptic R-matrix seem to be important. This physical intuition agrees with the general vision of Russian mathematician Igor Frenkel, who is one of the pioneers of quantum groups. For the article summarizing the work of Frenkel see `http://tinyurl.com/y7eego8c`). This article tells also about the work of Frenkel related to quaternion analyticity, which he sees to be of physical relevance but as a mathematicians is well aware of the fact that the non-commutativity of quaternions poses strong interpretation problems and means the loss of many nice properties of the ordinary analyticity.
3. The twistor lift of TGD suggest similar picture [K41, K10, L33]. The 6-D twistor space of space-time surface would be 6-surface in the product $T(M^4) \times T(CP_2)$ of geometric twistor spaces of M^4 and CP_2 and have induced twistor structure. The detailed analysis of this statement strongly suggests that data given at surfaces with dimension not higher than $D = 2$ should fix the preferred extremals. For the twistor lift action contains besides Kähler action also volume term. Asymptotic solutions are extremals of both Kähler action and minimal surfaces and all non-vacuum extremals of Kähler action are minimal surfaces so that the only change is that vacuum extremals of Kähler action must be restricted to be minimal surfaces.

The article about the work of Igor Frenkel (see `http://tinyurl.com/y7eego8c`) explains the general mathematics inspired vision about 3-levelled hierarchy of symmetries.

1. At the lowest level are Lie algebras. Gauge theories are prime example about this level.
2. At the second level loop algebras and quantum groups (defined as deformations of enveloping algebra of Lie algebra) and also Yangians. Loop algebras correspond to string models and quantum groups to TQFTs formulated at 3-D spaces.
3. At the third level are double loop algebras, quantum variants of loop algebras (also Yangians), and double quantum quantum groups - deformations of Lie algebras for which the R-matrix is elliptic function and depends on 2 complex parameters.

The conjecture of Frenkel (see `http://tinyurl.com/y7eego8c`) based on mathematical intuition is that these levels are actually the only ones. The motivation for this claim is 2-dimensionality making possible braiding and various quantum algebras. The set of poles for the R-matrix forms Abelian group with respect to addition in complex plane and can have rank equal to 0, 1, or (single pole, poles along line, lattice of poles). Higher ranks are impossible in $D = 2$.

In TGD framework physical intuition leads to a similar vision.

1. The dimension $D = 4$ for space-time surface and the choice $H = M^4 \times CP_2$ have both number theoretical and twistorial motivations [K30]. The replacement of point like particle with partonic 2-surface implies that TGD corresponds to the third level since loop algebras are replaced with their double loop analogs. 4-dimensionality makes also possible 2-braids and reconnections giving rise to a new kind of topological physics.

The double loop group would represent the most dynamical level and its singly and doubly quantized variants correspond to a reduction in degrees of freedom, which one cannot exclude in TGD.

The interesting additional aspect relates to the adelic physics [L29] implying a hierarchy of physics labelled by extensions of rationals. For cognitive representations the dynamics is discretized [K13]. Light-like 3-surfaces as partonic orbits are part of the picture and Chern-Simons term is naturally associated with them. TGD as almost topological QFT has been one of the guiding ideas in the evolution of TGD.

2. Double loop algebras represent unknown territory of mathematical physics. Igor Frenkel has also considered a possible realization of double loop algebras (see `http://tinyurl.com/y7eego8c`). He starts from the work of Mickelson (by the way, my custos in my thesis defence in 1982!) giving a realization of loop algebras: the idea is clearly motivated by WZW model which is 2-D conformal field theory with action containing a term associated with a 3-ball having 2-sphere as boundary.

   Mickelson starts from a circle represented as a boundary of a disk at which the physical states of CFT are realized. CFT is obtained by gluing together two disks with the boundary circles identified. The sphere in turn can be regarded as a boundary of a ball. The proposal of Frenkel is to complexify all these structures: circle becomes a Riemann surface, disk becomes 4-D manifold possessing complex structure in some sense, and 3-ball becomes 6-D complex manifold in some sense conjectured to be Calabi-Yau manifold.

3. The twistor lift of TGD leads to an analogous proposal. Circle is replaced with partonic 2-surfaces and string world sheets. 2-D complex surface is replaced with space-time region with complex structure or Hamilton-Jacobi structure [K8] and possessing twistor structure. 6-D Calabi-Yau manifold is replaced with the 6-D twistor space of space-time surface (sphere bundle over space-time surface) represented as 6-surface in 12-D Cartesian product $T(H) = T(M^4) \times T(CP_2)$ of the geometric twistor spaces of M^4 and CP_2.

   Twistor structure is induced and this is conjectured to determine the dynamics to be that for the preferred extremals of Kähler action plus volume term. This vision would generalize Penrose's original vision by eliminating gauge fields as primary dynamical variables and replacing there dynamics with the geometrodynamics of space-time surface.

Do isometry currents of preferred extremals satisfy Frobenius integrability conditions?

During the preparation of the book I learned that Agostino Prastaro [A30, A31] has done highly interesting work with partial differential equations, also those assignable to geometric variational principles such as Kähler action of its twistor lift in TGD. I do not understand the mathematical details but the key idea is a simple and elegant generalization of Thom's cobordism theory, and it is difficult to avoid the idea that the application of Prastaro's idea might provide insights about the preferred extremals, whose identification is now on rather firm basis.

One could also consider a definition of what one might call dynamical homotopy groups as a genuine characteristics of WCW topology. The first prediction is that the values of conserved classical Noether charges correspond to disjoint components of WCW. Could the natural topology in the parameter space of Noether charges zero modes of WCW metric) be p-adic and realize adelic physics at the level of WCW? An analogous conjecture was made on basis of spin glass analogy long time ago. Second surprise is that the only the 6 lowest dynamical homotopy/homology groups of WCW would be non-trivial. The Kähler structure of WCW suggests that only Π_0, Π_2, and Π_4 are non-trivial.

The interpretation of the analog of Π_1 as deformations of generalized Feynman diagrams with elementary cobordism snipping away a loop as a move leaving scattering amplitude invariant conforms with the number theoretic vision about scattering amplitude as a representation for a sequence of algebraic operation can be always reduced to a tree diagram. TGD would be indeed topological QFT: only the dynamical topology would matter.

A further outcome is an ansatz for generalizing the earlier proposal for preferred extremals and stating that non-vanishing conserved isometry currents satisfy Frobenius integrability conditions so that one could assign global coordinate with their flow lines. This ansatz looks very similar to the CRF conditions stating quaternion analyticity [L12].

Conclusions

To sum up, connections between different conjectures related to the preferred extremals - $M^8 - H$ duality, Hamilton-Jacobi structure, induced twistor space structure, quaternion-Kähler property and its Minkowskian counterpart, and perhaps even quaternion analyticity - albeit not in the naïve form , are clearly emerging. The underlying reason is strong form of GCI forced by the construction of WCW geometry and implying strong from of holography posing extremely powerful quantization conditions on the extremals of Kähler action in ZEO. Without the conformal gauge conditions the mutual inconsistency of these conjectures looks rather infeasible.

10.4 Octo-Twistors And Twistor Space

The basic problem of the twistor approach is that one cannot represent massive momenta in terms of twistors in an elegant manner. One can also consider generalization of the notion of spinor and twistor. I have proposed a possible representation of massive states based on the existence of preferred plane of M^2 in the basic definition of theory allowing to express four-momentum as one of two light-like momenta allowing twistor description. One could however ask whether some more elegant representation of massive M^4 momenta might be possible by generalizing the notion of twistor -perhaps by starting from the number theoretic vision.

The basic idea is obvious: in quantum TGD massive states in M^4 can be regarded as massless states in M^8 and $M^4 \times CP_2$ (recall $M^8 - H$ duality). One can therefore map any massive M^4 momentum to a light-like M^8 momentum and hope that this association could be made in a unique manner. One should assign to a massless 8-momentum an 8-dimensional spinor of fixed chirality. The spinor assigned with the light-like four-momentum is not unique without additional conditions. The existence of covariantly constant right-handed neutrino in CP_2 degrees generating the super-conformal symmetries could allow to eliminate the non-uniqueness. 8-dimensional twistor in M^8 would be a pair of this kind of spinors fixing the momentum of massless particle and the point through which the corresponding light-geodesic goes through: the set of these points forms 8-D light-cone and one can assign to each point a spinor. In $M^4 \times CP_2$ definitions makes also in the case of $M^4 \times CP_2$ and twistor space would also now be a lifting of the space of light-like geodesics.

The possibility to interpret M^8 as hyperoctonionic space suggests also the possibility to define the 8-D counterparts of sigma matrices to hyperoctonions to obtain a representation of sigma matrix algebra which is not a matrix representation. The mapping of gamma matrices to this representation allows to define a notion of hyper-quaternionicity in terms of the Kähler-Dirac gamma matrices both in M^8 and H.

The basic challenge is to achieve twistorial description of four-momenta or even $M^4 \times CP_2$ quantum numbers: this applies both to the momenta of fundamental fermions at the lines of generalized Feynman diagrams and to the massive incoming and outcoming states identified as their composites.

1. A rather attractive way to overcome the problem at the level of fermions propagating along the braid strands at the light-like orbits of partonic 2-surfaces relies on the assumption that generalized Feynman diagrammatics effectively reduces to a form in which all fermions in the propagator lines are massless although they can have non-physical helicity [L8]. One can use ordinary M^4 twistors. This is consistent with the idea that space-time surfaces are quaternionic sub-manifolds of octonionic embedding space.
2. Incoming and outgoing states are composites of massless fermions and not massless. They are however massless in 8-D sense. This suggests that they could be described using generalization of twistor formalism from M^4 to M^8 and even betterm to $M^4 \times CP_2$.

In the following two possible twistorializations are considered.

10.4.1 Two ways To Twistorialize Embedding Space

In the following the generalization of twistor formalism for M^8 or $M^4 \times CP_2$ will be considered in more detail. There are two options to consider.

1. For the first option one assigns to $M^4 \times CP_2$ twistor space as a product of corresponding twistor spaces $T(M_4) = CP_3$ and the flag-manifold $T(CP_2) = SU(3)/U(1) \times U(1)$ parameterizing the choices of quantization axes for $SU(3)$: $T_H = T(M^4) \times T(CP_2)$. Quite remarkably, M^4 and CP_2 are the only 4-D manifolds allowing twistor space with Kähler structure. The twistor space is 12-dimensional. The choice of quantization axis is certainly a physically well-definec operation so that $T(CP_2)$ has physical interpretation. If all observable physical states are color singlets situation becomes more complex. If one assumes QCC for color quantum numbers Y and I_3, then also the choice of color quantization axis is fixed at the level of Kähler action from the condition that Y and I_3 have classically their quantal values.

2. For the second option one generalizes the usual construction for M^8 regarded as tangent space of $M^4 \times CP_2$ (unless one takes $M^8 - H$ duality seriously).

The tangent space option looks like follows.

1. One can map the points of M^8 to octonions. One can consider 2-component spinors with octonionic components and map points of M^8 light-cone to linear combinations of 2×2 Pauli sigma matrices but with octonionic components. By the same arguments as in the deduction of ordinary twistor space one finds that 7-D light-cone boundary is mapped to 7+8 D space since the octonionic 2-spinor/its conjugate can be multiplied/divided by arbitrary octonion without changing the light-like point. By standard argument this space extends to 8+8-D space. The points of M^8 can be identified as 8-D octonionic planes (analogs of complex sphere CP_1 in this space. An attractive identification is as octonionic projective space OP_2. Remarkably, octonions do not allow higher dimensional projective spaces.

2. If one assumes that the spinors are quaternionic the twistor space should have dimension 7+4+1=12. This dimension is same as for $M^4 \times CP_2$. Does this mean that quaternionicity assumption reduces $T(M^8) = OP_2$ to $T(H) = CP_3 \times SU(3)/U(1) \times U(1)$? Or does it yield 12-D space $G_2/U(1) \times U(1)$, which is also natural since G_2 has 2-D Cartan algebra? Number theoretical compactification would transform $T(M^8) = G_2/U(1) \times U(1)$ to $T(H) = CP_3 \times SU(3)/U(1) \times U(1)$. This would not be surprising since in $M^8 - H$-duality CP_2 parametrizes (hyper)quaternionic planes containing preferred plane M^2.

   Quaternionicity is certainly very natural in TGD framework. Quaternionicity for 8-momenta does not in general imply that they reduce to the observed M^4-momenta unless one identifies M^4 as one particular subspace of M^8. In $M^8 - H$ duality one in principle allows all choices of M^4: it is of course unclear whether this makes any physical difference. Color confinement could be interpreted as a reduction of M^8 momenta to M^4 momenta and would also allow the interpretational problems caused by the fact that CP_2 momenta are not possible.

3. Since octonions can be regarded as complexified quaternions with non-commuting imaginary unit, one can say that quaternionic spinors in M^8 are "real" and thus analogous to Majorana spinors. Similar interpretation applies at the level of H. Could one can interpret the quaternionicity condition for space-time surfaces and embedding space spinors as TGD analog of Majorana condition crucial in super string models? This would also be crucial for understanding supersymmetry in TGD sense.

10.4.2 Octotwistorialization Of M^8

Consider first the twistorialization in 4-D case. In M^4 one can map light-like momoment to spinors satisfying massless Dirac equation. General point m of M^4 can be mapped to a pair of massless spinors related by incidence relation defining the point m. The essential element of this association is that mass squared can be defined as determinant of the 2×2 matrix resulting in the assignment. Light-likeness is coded to the vanishing of the determinant implying that the spinors defining its rows are linearly independent. The reduction of M^4 inner product to determinant occurs because the 2×2 matrix can be regarded as a matrix representation of complexified quaternion. Massless means that the norm of a complexified quaternion defined as the product of q and its conjugate vanishes. Incidence relation $s_1 = xs_2$ relating point of M^4 and pair of spinors defining the corresponding twistor, can be interpreted in terms of product for complexified quaternions.

The generalization to the 8-D situation is straightforward: replace quaternions with octonions.

1. The transition to M^8 means the replacement of quaternions with octonions. Masslessness corresponds to the vanishing norm for complexified octonion (hyper-octonion).
2. One should assign to a massless 8-momentum an 8-dimensional spinor identifiable as octonion - or more precisely as hyper-octonion obtained by multiplying the imaginary part of ordinary octonion with commuting imaginary unit j and defining conjugation as a change of sign of j or that of octonionic imaginar units.
3. This leads to a generalization of the notion of twistor consisting of pair of massless octonion valued spinors (octonions) related by the incidence relation fixing the point of M^8. The incidence relation for Euclidian octonions says $s_1 = xs_2$ and can be interpreted in terms of triality for $SO(8)$ relating conjugate spinor octet to the product of vector octed and spinor octet. For Minkowskian subspace of complexified octonions light-like vectors and s_1 and s_2 can be taken light-like as octonions. Light like x can annihilate s_2.

The possibility to interpret M^8 as hyperoctonionic space suggests also the possibility to define the 8-D counterparts of sigma matrices to hyperoctonions to obtain a representation of sigma matrix algebra which is not a matrix representation. The mapping of gamma matrices to this representation allows to define a notion of hyper-quaternionicity in terms of the Kähler-Dirac gamma matrices both in M^8 and H.

10.4.3 Octonionicity, $SO(1,7)$, G_2, And Non-Associative Malcev Group

The symmetries assignable with octonions are rather intricate. First of all, octonions (their hyper-variants defining M^8) have SO(8) (SO(1, 7)) as isometries. $G_2 \subset SO(7)$ acts as automorphisms of octonions and $SO(1,7) \rightarrow G_2$ clearly means breaking of Lorentz invariance.

John Baez has described in a lucid manner G_2 geometrically (`http://tinyurl.com/ybd4lcpy`). The basic observation is that quaternionic sub-space is generated by two linearly independent imaginary units and by their product. By adding a fourth linearly independent imaginary unit, one can generated all octonions. From this and the fact that G_2 represents subgroup of $SO(7)$, one easily deduces that G_2 is 14-dimensional. The Lie algebra of G_2 corresponds to derivations of octonionic algebra as follows infinitesimally from the condition that the image of product is the product of images. The entire algebra $SO(8)$ is direct sum of G_2 and linear transformations generated by right and left multiplication by imaginary octonion: this gives $14 + 14 = 28 = D(SO(8))$. The subgroup $SO(7)$ acting on imaginary octonsions corresponds to the direct sum of derivations and adjoint transformations defined by commutation with imaginary octonions, and has indeed dimension $14 + 7 = 21$.

One can identify also a non-associative group-like structure.

1. In the case of octonionic spinors this group like structure is defined by the analog of phase multiplication of spinor generalizing to a multiplication with octonionic unit expressible as linear combinations of 8 octonionic imaginary units and defining 7-sphere plays appear as analog of automorphisms $o \rightarrow uou^{-1} = uou^*$.

   One can associate with these transformations a non-associative Lie group and Lie algebra like structures by defining the commutators just as in the case of matrices that is as $[a,b] = ab - ba$. One 7-D non-associative Lie group like structure with topology of 7-sphere S^7 whereas G_2 is 14-dimensional exceptional Lie group (having S^6 as coset space $S^6 = G_2/SU(3)$). This group like object might be useful in the treatment of octonionic twistors. In the case of quaternions one has genuine group acting as SO(3) rotations.
2. Octonionic gamma matrices allow to define as their commutators octonionic sigma matrices:

$$\Sigma_{kl} = \frac{i}{2}[\gamma_k, \gamma_l] \quad . \qquad (10.4.1)$$

   This algebra is 14-dimensional thanks to the fact that octonionic gamma matrices are of form $\gamma_0 = \sigma_1 \otimes 1$, $\gamma_i = \sigma_2 \otimes e_i$. Due to the non-associativity of octonions this algebra does not satisfy Jacobi identity - as is easy to verify using Fano triangle - and is therefore not a genuine Lie-algebra. Therefore these sigma matrices do not define a representation of G_2 as I thought first.

This algebra has decomposition $g = h + t$, $[h, t] \subset t$, $[t, t] \subset h$ characterizing for symmetric spaces. h is the 7-D algebra generated by Σ_{ij} and identical with the non-associative Malcev algebra generated by the commutators of octonionic units. The complement t corresponds to the generators Σ_{0i}. The algebra is clearly an octonionic non-associative analog fo $SO(1, 7)$.

10.4.4 Octonionic Spinors In M^8 And Real Complexified-Quaternionic Spinors In H?

This above observations about the octonionic sigma matrices raise the problem about the octonionic representation of spinor connection. In $M^8 = M^4 \times E^4$ the spinor connection is trivial but for $M^4 \times CP_2$ not. There are two options.

1. Assume that octonionic spinor structure makes sense for M^8 only and spinor connection is trivial.
2. An alternative option is to identify M^8 as tangent space of $M^4 \times CP_2$ possessing quaternionic structure defined in terms of octonionic variants of gamma matrices. Should one replace sigma matrices appearing in spinor connection with their octonionic analogs to get a sigma matrix algebra which is pseudo Lie algebra. Or should one map the holonomy algebra of CP_2 spinor connection to a sub-algebra of $G_2 \subset SO(7)$ and define the action of the sigma matrices as ordinary matrix multiplication of octonions rather than octonionic multiplication? This seems to be possible formally.

   The replacement of sigma matrices with their octonionic counterparts seems to lead to weird looking results. Octonionic multiplication table implies that the electroweak sigma matrices associated with CP_2 tangent space reduce to M^4 sigma matrices so that the spinor connection is quaternionic. Furthermore, left-handed sigma matrices are mapped to zero so that only the neutral part of spinor connection is non-vanishing. This supports the view that only M^8 gamma matrices make sense and that Dirac equation in M^8 is just free massless Dirac equation leading naturally also to the octonionic twistorialization.

   One might think that distinction between different H-chiralities is difficult to make but it turns out that quarks and leptons can be identified as different components of 2-component complexified octonionic spinors.

The natural question is what associativization of octonions gives. This amounts to a condition putting the associator $a(bc) - (ab)c$ to zero. It is enough to consider octonionic imaginary units which are different. By using the decomposition of the octonionic algebra to quaternionic sub-algebra and its complement and general structure of structure constants, one finds that quaternionic sub-algebra remains as such but the products of all imaginary units in the complement with different imaginary units vanish. This means that the complement behaves effectively as 4-D flat space-gamma matrix algebra annihilated by the quaternionic sub-algebra whose imaginary part acts like Lie algebra of $SO(3)$.

10.4.5 What The Replacement Of $SO(7, 1)$ Sigma Matrices With Octonionic Sigma Matrices Could Mean?

The basic implication of octonionization is the replacement of $SO(7, 1)$ sigma matrices with octonionic sigma matrices. For M^8 this has no consequences since since spinor connection is trivial.

For $M^4 \times CP_2$ situation would be different since CP_2 spinor connection would be replaced with its octonionic variant. This has some rather unexpected consequences and suggests that one should not try to octonionize at the level of $M^4 \times CP_2$ but interpret gamma matrices as tensor products of quaternionic gamma matrices, which can be replaced with their matrix representations. There are however some rather intriguing observations which force to keep mind open.

Octonionic representation of 8-D gamma matrices

Consider first the representation of 8-D gamma matrices in terms of tensor products of 7-D gamma matrices and 2-D Pauli sigma matrices.

1. The gamma matrices are given by

$$\gamma^0 = 1 \times \sigma_1 \ \ , \ \ \gamma^i = \gamma^i \otimes \sigma_2 \ \ , \ \ i = 1,..,7 \ \ . \tag{10.4.2}$$

 7-D gamma matrices in turn can be expressed in terms of 6-D gamma matrices by expressing γ^7 as

$$\gamma^{7)}_{i+1} = \gamma^{6)}_i , i = 1, ..., 6 \ \ , \ \ \gamma^{7)}_1 = \gamma^{6)}_7 = \prod_{i=1}^{6} \gamma^{6)}_i \ \ . \tag{10.4.3}$$

2. The octonionic representation is obtained as

$$\gamma_0 = 1 \otimes \sigma_1 \ \ , \ \ \gamma_i = e_i \otimes \sigma_2 \ \ . \tag{10.4.4}$$

 where e_i are the octonionic units. $e_i^2 = -1$ guarantees that the M^4 signature of the metric comes out correctly. Note that $\gamma_7 = \prod \gamma_i$ is the counterpart for choosing the preferred octonionic unit and plane M^2.

3. The octonionic sigma matrices are obtained as commutators of gamma matrices:

$$\Sigma_{0i} = je_i \times \sigma_3 \ \ , \ \ \Sigma_{ij} = jf_{ij}{}^{k} e_k \otimes 1 \ \ . \tag{10.4.5}$$

 Here j is commuting imaginary unit. These matrices span G_2 algebra having dimension 14 and rank 2 and having imaginary octonion units and their conjugates as the fundamental representation and its conjugate. The Cartan algebra for the sigma matrices can be chosen to be Σ_{01} and Σ_{23} and belong to a quaternionic sub-algebra.

4. The lower dimension $D = 14$ of the non-associative version of sigma matrix algebra algebra means that some combinations of sigma matrices vanish. All left or right handed generators of the algebra are mapped to zero: this explains why the dimension is halved from 28 to 14. From the octonionic triangle expressing the multiplication rules for octonion units [A15] one finds $e_4e_5 = e_1$ and $e_6e_7 = -e_1$ and analogous expressions for the cyclic permutations of e_4, e_5, e_6, e_7. From the expression of the left handed sigma matrix $I_L^3 = \sigma_{23} + \sigma^{30}$ representing left handed weak isospin (see the Appendix about the geometry of CP_2 [K15]) one can conclude that this particular sigma matrix and left handed sigma matrices in general are mapped to zero. The quaternionic sub-algebra $SU(2)_L \times SU(2)_R$ is mapped to that for the rotation group $SO(3)$ since in the case of Lorentz group one cannot speak of a decomposition to left and right handed subgroups. The elements of the complement of the quaternionic sub-algebra are expressible in terms of Σ_{ij} in the quaternionic sub-algebra.

Some physical implications of the reduction of $SO(7,1)$ to its octonionic counterpart

The octonization of spinor connection of CP_2 has some weird physical implications forcing to keep mind to the possibility that the octonionic description even at the level of H might have something to do with reality.

1. If $SU(2)_L$ is mapped to zero only the right-handed parts of electro-weak gauge field survive octonionization. The right handed part is neutral containing only photon and Z^0 so that the gauge field becomes Abelian. Z^0 and photon fields become proportional to each other ($Z^0 \rightarrow sin^2(\theta_W)\gamma$) so that classical Z^0 field disappears from the dynamics, and one would obtain just electrodynamics.

2. The gauge potentials and gauge fields defined by CP_2 spinor connection are mapped to fields in $SO(2) \subset SU(2) \times U(1)$ in quaternionic sub-algebra which in a well-defined sense corresponds to M^4 degrees of freedom and gauge group becomes $SO(2)$ subgroup of rotation group of $E^3 \subset M^4$. This looks like catastrophe. One might say that electroweak interactions are transformed to gravimagnetic interactions.

3. In very optimistic frame of mind one might ask whether this might be a deeper reason for why electrodynamics is an excellent description of low energy physics and of classical physics. This is consistent with the fact that CP_2 coordinates define 4 field degrees of freedom so that single Abelian gauge field should be enough to describe classical physics. This would remove also the interpretational problems caused by the transitions changing the charge state of fermion induced by the classical W boson fields.

4. Interestingly, the condition that electromagnetic charge is well-defined quantum number for the modes of the induced spinor field for $X^4 \subset H$ leads to the proposal that the solutions of the Kähler-Dirac equation are localized to string world sheets in Minkowskian regions of space-time surface at least. For CP_2 type vacuum extremals one has massless Dirac and this allows only covariantly constant right-handed neutrino as solution. One has however only a piece of CP_2 (wormhole contact) so that holomorphic solutions annihilated by two complexified gamma matrices are possible in accordance with the conformal symmetries.

Can one assume non-trivial spinor connection in M^8?

1. The simplest option encouraged by the requirement of maximal symmetries is that it is absent. Massless 8-momenta would characterize spinor modes in M^8 and this would give physical justification for the octotwistors.

2. If spinor connection is present at all, it reduces essentially to Kähler connection having different couplings to quarks and leptons identifiable as components of octonionic 2-spinors. It should be $SO(4)$ symmetric and since CP_2 is instant one might argue that now one has also instanton that is self-dual U(1) gauge field in $E^4 \subset M^4 \times E^4$ defining Kähler form. One can loosely say that one has of constant electric and magnetic fields which are parallel to each other. The rotational symmetry in E^4 would break down to $SO(2)$.

3. Without spinor connection quarks and leptons are in completely symmetric position at the level of M^8: this is somewhat disturbing. The difference between quarks and leptons in H is made possible by the fact that CP_2 does not allow standard spinor structure. Now this problem is absent. I have also consider the possibility that only leptonic spinor chirality is allowed and quarks result via a kind of anyonization process allowing them to have fractional em charges (see `http://tinyurl.com/y93aerea`).

4. If the solutions of the Kähler Dirac equation in Minkowskian regions are localized to two surfaces identifiable as integrable distributions of planes $M^2(x)$ and characterized by a local light-like direction defining the direction of massless momentum, they are holomorphic (in the sense of hyper-complex numbers) such that the second complexified Kähler-Dirac gamma matrix annihilates the solution. Same condition makes sense also at the level of M^8 for solutions restricted to string world sheets and the presence or absence of spinor connection does not affect the situation.

   Does this mean that the difference between quarks and leptons becomes visible only at the embedding space level where ground states of super-conformal representations correspond to embedding space spinor harmonics which in CP_2 cm degrees are different for quarks and leptons?

Octo-spinors and their relation to ordinary embedding space spinors

Octo-spinors are identified as octonion valued 2-spinors with basis

$$\Psi_{L,i} = e_i \begin{pmatrix} 1 \\ 0 \end{pmatrix} \ ,$$
$$\Psi_{q,i} = e_i \begin{pmatrix} 0 \\ 1 \end{pmatrix} \ . \tag{10.4.6}$$

One obtains quark and lepton spinors and conjugation for the spinors transforms quarks to leptons. Note that octospinors can be seen as 2-dimensional spinors with components which have values in the space of complexified octonions.

The leptonic spinor corresponding to real unit and preferred imaginary unit e_1 corresponds naturally to the two spin states of the right handed neutrino. In quark sector this would mean that right handed U quark corresponds to the real unit. The octonions decompose as $1+1+3+\overline{3}$ as representations of $SU(3) \subset G_2$. The concrete representations are given by

$$\begin{array}{ll} \{1 \pm ie_1\} \ , & e_R \text{ and } \nu_R \text{ with spin } 1/2 \ , \\ \{e_2 \pm ie_3\} \ , & e_R \text{ and } \nu_L \text{ with spin -}1/2 \ , \\ \{e_4 \pm ie_5\} & e_L \text{ and } \nu_L \text{ with spin } 1/2 \ , \\ \{e_6 \pm ie_7\} & e_L \text{ and } \nu_L \text{ with spin } 1/2 \ . \end{array} \quad (10.4.7)$$

Instead of spin one could consider helicity. All these spinors are eigenstates of e_1 (and thus of the corresponding sigma matrix) with opposite values for the sign factor $\epsilon = \pm$. The interpretation is in terms of vectorial isospin. States with $\epsilon = 1$ can be interpreted as charged leptons and D type quarks and those with $\epsilon = -1$ as neutrinos and U type quarks. The interpretation would be that the states with vanishing color isospin correspond to right handed fermions and the states with non-vanishing SU(3) isospin (to be not confused with QCD color isospin) and those with non-vanishing SU(3) isospin to left handed fermions.

The importance of this identification is that it allows a unique map of the candidates for the solutions of the octonionic Kähler-Dirac equation to those of ordinary one. There are some delicacies involved due to the possibility to chose the preferred unit e_1 so that the preferred subspace M^2 can corresponds to a sub-manifold $M^2 \subset M^4$.

10.4.6 About The Twistorial Description Of Light-Likeness In 8-D Sense Using Octonionic Spinors

The twistor approach to TGD [L8] require that the expression of light-likeness of M^4 momenta in terms of twistors generalizes to 8-D case. The light-likeness condition for twistors states that the 2×2 matrix representing M^4 momentum annihilates a 2-spinor defining the second half of the twistor. The determinant of the matrix reduces to momentum squared and its vanishing implies the light-likeness. This should be generalized to a situation in one has M^4 and CP_2 twistor which are not light-like separately but light-likeness in 8-D sense holds true.

The case of $M^8 = M^4 \times E^4$

$M^8 - H$ duality [K105] suggests that it might be useful to consider first the twistorialiation of 8-D light-likeness first the simpler case of M^8 for which CP_2 corresponds to E^4. It turns out that octonionic representation of gamma matrices provide the most promising formulation.

In order to obtain quadratic dispersion relation, one must have 2×2 matrix unless the determinant for the 4×4 matrix reduces to the square of the generalized light-likeness condition.

1. The first approach relies on the observation that the 2×2 matrices characterizing four-momenta can be regarded as hyper-quaternions with imaginary units multiplied by a commuting imaginary unit. Why not identify space-like sigma matrices with hyper-octonion units?
2. The square of hyper-octonionic norm would be defined as the determinant of 4×4 matrix and reduce to the square of hyper-octonionic momentum. The light-likeness for pairs formed by M^4 and E^4 momenta would make sense.
3. One can generalize the sigma matrices representing hyper-quaternion units so that they become the 8 hyper-octonion units. Hyper-octonionic representation of gamma matrices exists ($\gamma_0 = \sigma_z \times 1$, $\gamma_k = \sigma_y \times I_k$) but the octonionic sigma matrices represented by octonions span the Lie algebra of G_2 rather than that of $SO(1,7)$. This dramatically modifies the physical picture and brings in also an additional source of non-associativity. Fortunately, the flatness of M^8 saves the situation.
4. One obtains the square of $p^2 = 0$ condition from the massless octonionic Dirac equation as vanishing of the determinant much like in the 4-D case. Since the spinor connection is flat for M^8 the hyper-octonionic generalization indeed works.

This is not the only possibility that I have by-passingly considered [K27].

1. Is it enough to allow the four-momentum to be complex? One would still have 2×2 matrix and vanishing of complex momentum squared meaning that the squares of real and imaginary parts are same (light-likeness in 8-D sense) and that real and imaginary parts are orthogonal to each other. Could E^4 momentum correspond to the imaginary part of four-momentum?
2. The signature causes the first problem: M^8 must be replaced with complexified Minkowski space M^4_c for to make sense but this is not an attractive idea although M^4_c appears as subspace of complexified octonions.
3. For the extremals of Kähler action Euclidian regions (wormhole contacts identifiable as deformations of CP_2 type vacuum extremals) give imaginary contribution to the four-momentum. Massless complex momenta and also color quantum numbers appear also in the standard twistor approach. Also this suggest that complexification occurs also in 8-D situation and is not the solution of the problem.

The case of $M^8 = M^4 \times CP_2$

What about twistorialization in the case of $M^4 \times CP_2$? The introduction of wave functions in the twistor space of CP_2 seems to be enough to generalize Witten's construction to TGD framework and that algebraic variant of twistors might be needed only to realize quantum classical correspondence. It should correspond to tangent space counterpart of the induced twistor structure of space-time surface, which should reduce effectively to 4-D one by quaternionicity of the space-time surface.

1. For $H = M^4 \times CP_2$ the spinor connection of CP_2 is not trivial and the G_2 sigma matrices are proportional to M^4 sigma matrices and act in the normal space of CP_2 and to M^4 parts of octonionic embedding space spinors, which brings in mind co-associativity. The octonionic charge matrices are also an additional potential source of non-associativity even when one has associativity for gamma matrices.

   Therefore the octonionic representation of gamma matrices in entire H cannot be physical. It is however equivalent with ordinary one at the boundaries of string world sheets, where induced gauge fields vanish. Gauge potentials are in general non-vanishing but can be gauge transformed away. Here one must be of course cautious since it can happen that gauge fields vanish but gauge potentials cannot be gauge transformed to zero globally: topological quantum field theories represent basic example of this.
2. Clearly, the vanishing of the induced gauge fields is needed to obtain equivalence with ordinary induced Dirac equation. Therefore also string world sheets in Minkowskian regions should have 1-D CP_2 projection rather than only having vanishing W fields if one requires that octonionic representation is equivalent with the ordinary one. For CP_2 type vacuum extremals electroweak charge matrices correspond to quaternions, and one might hope that one can avoid problems due to non-associativity in the octonionic Dirac equation. Unless this is the case, one must assume that string world sheets are restricted to Minkowskian regions. Also embedding space spinors can be regarded as octonionic (possibly quaternionic or co-quaternionic at space-time surfaces): this might force vanishing 1-D CP_2 projection.
   (a) Induced spinor fields would be localized at 2-surfaces at which they have no interaction with weak gauge fields: of course, also this is an interaction albeit very implicit one! This would not prevent the construction of non-trivial electroweak scattering amplitudes since boson emission vertices are essentially due to re-groupings of fermions and based on topology change.
   (b) One could even consider the possibility that the projection of string world sheet to CP_2 corresponds to CP_2 geodesic circle so that one could assign light-like 8-momentum to entire string world sheet, which would be minimal surface in $M^4 \times S^1$. This would mean enormous technical simplification in the structure of the theory. Whether the spinor harmonics of embedding space with well-defined M^4 and color quantum numbers can coincide with the solutions of the induced Dirac operator at string world sheets defined by minimal surfaces remains an open problem.
   (c) String world sheets cannot be present inside wormhole contacts which have 4-D CP_2 projection so that string world sheets cannot carry vanishing induced gauge fields.

10.5 What Could Be The Origin Of Preferred P-Adic Primes And P-Adic Length Scale Hypothesis?

p-Adic mass calculations [K70] allow to conclude that elementary particles correspond to one or possible several preferred primes assigning p-adic effective topology to the real space-time sheets in discretization in some length scale range. TGD inspired theory of consciousness leads to the identification of p-adic physics as physics of cognition. The recent progress leads to the proposal that quantum TGD is adelic: all p-adic number fields are involved and each gives one particular view about physics. tgdquantum/tgdquantum Adelic approach [K52, K76] plus the view about evolution as emergence of increasingly complex extensions of rationals leads to a possible answer to th question of the title. The algebraic extensions of rationals are characterized by preferred rational primes, namely those which are ramified when expressed in terms of the primes of the extensions. These primes would be natural candidates for preferred p-adic primes. An argument relying on what I call weak form of NMP in turn allows to understand why primes near powers of 2 are preferred: as a matter of fact, also primes near powers of other primes are predicted to be favoured.

10.5.1 Earlier Attempts

How the preferred primes emerge in TGD framework? I have made several attempts to answer this question. As a matter fact, the question has been slightly different: what determines the p-adic prime assigned to elementary particle by p-adic mass calculations [K59]. The recent view assigns to particle entire adele but some p-adic number fields in it are different.

1. Classical non-determinism at space-time level for real space-time sheets could in some length scale range involving rational discretization for space-time surface itself or for parameters characterizing it as a preferred extremal correspond to the non-determinism of p-adic differential equations due to the presence of pseudo constants which have vanishing p-adic derivative. Pseudo- constants are functions depend on finite number of pinary digits of its arguments.
2. The quantum criticality of TGD [?] is suggested to be realized in in terms of infinite hierarchies of super-symplectic symmetry breakings in the sense that only a sub-algebra with conformal weights which are n-ples of those for the entire algebra act as conformal gauge symmetries [K93]. This might be true for all conformal algebras involved. One has fractal hierarchy since the sub-algebras in question are isomorphic: only the scale of conformal gauge symmetry increases in the phase transition increasing n. The hierarchies correspond to sequences of integers $n(i)$ such tht $n(i)$ divides $n(i+1)$. These hierarchies would very naturally correspond to hierarchies of inclusions of hyper-finite factors and $m(i) = n(i+1)/n(i)$ could correspond to the integer n characterizing the index of inclusion, which has value $n \geq 3$. Possible problem is that $m(i) = 2$ would not correspond to Jones inclusion. Why the scaling by power of two would be different? The natural question is whether the primes dividing $n(i)$ or $m(i)$ could define the preferred primes.
3. Negentropic entanglement corresponds to entanglement for which density matrix is projector [K65]. For n-dimensional projector any prime p dividing n gives rise to negentropic entanglement in the sense that the number theoretic entanglement entropy defined by Shannon formula by replacing p_i in $log(p_i) = log(1/n)$ by its p-adic norm $N_p(1/n)$ is negative if p divides n and maximal for the prime for which the dividing power of prime is largest power-of-prime factor of n. The identification of p-adic primes as factors of n is highly attractive idea. The obvious question is whether n corresponds to the integer characterizing a level in the hierarchy of conformal symmetry breakings.
4. The adelic picture about TGD led to the question whether the notion of unitarity could be generalized. S-matrix would be unitary in adelic sense in the sense that $P_m = (SS^\dagger)_{mm} = 1$ would generalize to adelic context so that one would have product of real norm and p-adic norms of P_m. In the intersection of the realities and p-adicities P_m for reals would be rational and if real and p-adic P_m correspond to the same rational, the condition would be satisfied. The condition that $P_m \leq 1$ seems however natural and forces separate unitary in each sector so that this options seems too tricky.

These are the basic ideas that I have discussed hitherto.

10.5.2 Could Preferred Primes Characterize Algebraic Extensions Of Rationals?

The intuitive feeling is that the notion of preferred prime is something extremely deep and the deepest thing I know is number theory. Does one end up with preferred primes in number theory? This question brought to my mind the notion of ramification of primes (see `http://tinyurl.com/hddljlf`) (more precisely, of prime ideals of number field in its extension), which happens only for special primes in a given extension of number field, say rationals. Could this be the mechanism assigning preferred prime(s) to a given elementary system, such as elementary particle? I have not considered their role earlier also their hierarchy is highly relevant in the number theoretical vision about TGD.

1. Stating it very roughly (I hope that mathematicians tolerate this language): As one goes from number field K, say rationals Q, to its algebraic extension L, the original prime ideals in the so called integral closure (see `http://tinyurl.com/js6fpvr`) over integers of K decompose to products of prime ideals of L (prime is a more rigorous manner to express primeness).

   Integral closure for integers of number field K is defined as the set of elements of K, which are roots of some monic polynomial with coefficients, which are integers of K and having the form $x^n + a_{n-1}x^{n-1} + ... + a_0$. The integral closures of both K and L are considered. For instance, integral closure of algebraic extension of K over K is the extension itself. The integral closure of complex numbers over ordinary integers is the set of algebraic numbers.

2. There are two further basic notions related to ramification and characterizing it. Relative discriminant is the ideal divided by all ramified ideals in K and relative different is the ideal of L divided by all ramified P_i:s. Note that te general ideal is analog of integer and these ideas represent the analogous of product of preferred primes P of K and primes P_i of L dividing them.

3. A physical analogy is provided by decomposition of hadrons to valence quarks. Elementary particles becomes composite of more elementary particles in the extension. The decomposition to these more elementary primes is of form $P = \prod P_i^{e(i)}$, where e_i is the ramification index - the physical analog would be the number of elementary particles of type i in the state (see `http://tinyurl.com/h9528pl`). Could the ramified rational primes could define the physically preferred primes for a given elementary system?

In TGD framework the extensions of rationals (see `http://tinyurl.com/h9528pl`) and p-adic number fields (see`http://tinyurl.com/zq22tvb`) are unavoidable and interpreted as an evolutionary hierarchy physically and cosmological evolution would have gradually proceeded to more and more complex extensions. One can say that string world sheets and partonic 2-surfaces with parameters of defining functions in increasingly complex extensions of prime emerge during evolution. Therefore ramifications and the preferred primes defined by them are unavoidable. For p-adic number fields the number of extensions is much smaller for instance for $p > 2$ there are only 3 quadratic extensions.

1. In p-adic context a proper definition of counterparts of angle variables as phases allowing definition of the analogs of trigonometric functions requires the introduction of algebraic extension giving rise to some roots of unity. Their number depends on the angular resolution. These roots allow to define the counterparts of ordinary trigonometric functions - the naïve generalization based on Taylors series is not periodic - and also allows to defined the counterpart of definite integral in these degrees of freedom as discrete Fourier analysis. For the simplest algebraic extensions defined by $x^n - 1$ for which Galois group is abelian are are unramified so that something else is needed. One has decomposition $P = \prod P_i^{e(i)}$, $e(i) = 1$, analogous to n-fermion state so that simplest cyclic extension does not give rise to a ramification and there are no preferred primes.

2. What kind of polynomials could define preferred algebraic extensions of rationals? Irreducible polynomials are certainly an attractive candidate since any polynomial reduces to a product of them. One can say that they define the elementary particles of number theory. Irreducible

polynomials have integer coefficients having the property that they do not decompose to products of polynomials with rational coefficients. IT would be wrong to say that only these algebraic extensions can appear but there is a temptation to say that one can reduce the study of extensions to their study. One can even consider the possibility that string world sheets associated with products of irreducible polynomials are unstable against decay to those characterize irreducible polynomials.

3. What can one say about irreducible polynomials? Eisenstein criterion (see `http://tinyurl.com/47kxjz` states following. If $Q(x) = \sum_{k=0,..,n} a_k x^k$ is n:th order polynomial with integer coefficients and with the property that there exists at least one prime dividing all coefficients a_i except a_n and that p^2 does not divide a_0, then Q is irreducible. Thus one can assign one or more preferred primes to the algebraic extension defined by an irreducible polynomial Q of this kind - in fact any polynomial allowing ramification. There are also other kinds of irreducible polynomials since Eisenstein's condition is only sufficient but not necessary.

4. Furthermore, in the algebraic extension defined by Q, the prime ideals P having the above mentioned characteristic property decompose to an n :th power of single prime ideal P_i: $P = P_i^n$. The primes are maximally/completely ramified. The physical analog $P = P_0^n$ is Bose-Einstein condensate of n bosons. There is a strong temptation to identify the preferred primes of irreducible polynomials as preferred p-adic primes.

   A good illustration is provided by equations $x^2+1=0$ allowing roots $x_\pm = \pm i$ and equation $x^2+2px+p=0$ allowing roots $x_\pm = -p \pm \sqrt{p}p-1$. In the first case the ideals associated with $\pm i$ are different. In the second case these ideals are one and the same since $x_+ == -x_- + p$: hence one indeed has ramification. Note that the first example represents also an example of irreducible polynomial, which does not satisfy Eisenstein criterion. In more general case the n conditions on defined by symmetric functions of roots imply that the ideals are one and same when Eisenstein conditions are satisfied.

5. What does this mean in p-adic context? The identity of the ideals can be stated by saying $P = P_0^n$ for the ideals defined by the primes satisfying the Eisenstein condition. Very loosely one can say that the algebraic extension defined by the root involves n:th root of p-adic prime p. This does not work! Extension would have a number whose n:th power is zero modulo p. On the other hand, the p-adic numbers of the extension modulo p should be finite field but this would not be field anymore since there would exist a number whose n:th power vanishes. The algebraic extension simply does not exist for preferred primes. The physical meaning of this will be considered later.

6. What is so nice that one could readily construct polynomials giving rise to given preferred primes. The complex roots of these polymials could correspond to the points of partonic 2-surfaces carrying fermions and defining the ends of boundaries of string world sheet. It must be however emphasized that the form of the polynomial depends on the choices of the complex coordinate. For instance, the shift $x \to x+1$ transforms $(x^n-1)/(x-1)$ to a polynomial satisfying the Eisenstein criterion. One should be able to fix allowed coordinate changes in such a way that the extension remains irreducible for all allowed coordinate changes.

   Already the integral shift of the complex coordinate affects the situation. It would seem that only the action of the allowed coordinate changes must reduce to the action of Galois group permuting the roots of polynomials. A natural assumption is that the complex coordinate corresponds to a complex coordinate transforming linearly under subgroup of isometries of the embedding space.

In the general situation one has $P = \prod P_i^{e(i)}$, $e(i) \geq 1$ so that aso now there are prefered primes so that the appearance of preferred primes is completely general phenomenon.

10.5.3 A Connection With Langlands Program?

In Langlands program (`http://tinyurl.com/ycej7s43`,`RecentAdvancesinLanglandsprogram`) [A46, A45] the great vision is that the n-dimensional representations of Galois groups G characterizing algebraic extensions of rationals or more general number fields define n-dimensional adelic representations of adelic Lie groups, in particular the adelic linear group $Gl(n, A)$. This would mean that it is possible to reduce these representations to a number theory for adeles. This would be highly relevant in the vision about TGD as a generalized number theory. I have speculated with this possibility earlier (http://tinyurl.com/y9ee3lk6) [K52] but the mathematics is so horribly abstract that it takes decade before one can have even hope of building a rough vision.

One can wonder whether the irreducible polynomials could define the preferred extensions K of rationals such that the maximal abelian extensions of the fields K would in turn define the adeles utilized in Langlands program. At least one might hope that everything reduces to the maximally ramified extensions.

At the level of TGD string world sheets with parameters in an extension defined by an irreducible polynomial would define an adele containing various p-adic number fields defined by the primes of the extension. This would define a hierarchy in which the prime ideals of previous level would decompose to those of the higher level. Each irreducible extension of rationals would correspond to some physically preferred p-adic primes.

It should be possible to tell what the preferred character means in terms of the adelic representations. What happens for these representations of Galois group in this case? This is known.

1. For Galois extensions ramification indices are constant: $e(i) = e$ and Galois group acts transitively on ideals P_i dividing P. One obtains an n-dimensional representation of Galois group. Same applies to the subgroup of Galois group G/I where I is subgroup of G leaving P_i invariant. This group is called inertia group. For the maximally ramified case G maps the ideal P_0 in $P = P_0^n$ to itself so that $G = I$ and the action of Galois group is trivial taking P_0 to itself, and one obtains singlet representations.
2. The trivial action of Galois group looks like a technical problem for Langlands program and also for TGD unless the singletness of P_i under G has some physical interpretation. One possibility is that Galois group acts as like a gauge group and here the hierarchy of sub-algebras of super-symplectic algebra labelled by integers n is highly suggestive. This raises obvious questions. Could the integer n characterizing the sub-algebra of super-symplectic algebra acting as conformal gauge transformations, define the integer defined by the product of ramified primes? P_0^n brings in mind the n conformal equivalence classes which remain invariant under the conformal transformations acting as gauge transformations. . Recalling that relative discriminant is an of K ideal divisible by ramified prime ideals of K, this means that n would correspond to the relative discriminant for $K = Q$. Are the preferred primes those which are "physical" in the sense that one can assign to the states satisfying conformal gauge conditions?

   If the Galois group corresponds to gauge symmetries for these primes, it is physically natural that the p-adic algebraic extension does not exists and that p-adic variant of the Galois group is absent. Nothing is lost from cognition since there is nothing to cognize!

10.5.4 What Could Be The Origin Of P-Adic Length Scale Hypothesis?

The argument would explain the existence of preferred p-adic primes. It does not yet explain p-adic length scale hypothesis [K74, K59] stating that p-adic primes near powers of 2 are favored. A possible generalization of this hypothesis is that primes near powers of prime are favored. There indeed exists evidence for the realization of 3-adic time scale hierarchies in living matter [I8] (http://tinyurl.com/jbh9m27) and in music both 2-adicity and 3-adicity could be present, this is discussed in TGD inspired theory of music harmony and genetic code [K87].

The weak form of NMP might come in rescue here.

1. Entanglement negentropy for a negentropic entanglement [K65] characterized by n-dimensional projection operator is the $log(N_p(n)$ for some p whose power divides n. The maximum negentropy is obtained if the power of p is the largest power of prime divisor of p, and this can be taken as definition of number theoretic entanglement negentropy. If the largest divisor is p^k, one has $N = k \times log(p)$. The entanglement negentropy per entangled state is $N/n = klog(p)/n$ and is maximal for $n = p^k$. Hence powers of prime are favoured which means that p-adic length scale hierarchies with scales coming as powers of p are negentropically favored and should be generated by NMP. Note that $n = p^k$ would define a hierarchy of $h_{eff}/h = p^k$. During the first years of h_{eff} hypothesis I believe that the preferred values obey $h_{eff} = r^k$, r integer not far from $r = 2^{11}$. It seems that this belief was not totally wrong.

2. If one accepts this argument, the remaining challenge is to explain why primes near powers of two (or more generally p) are favoured. $n = 2^k$ gives large entanglement negentropy for the final state. Why primes $p = n_2 = 2^k - r$ would be favored? The reason could be following. $n = 2^k$ corresponds to $p = 2$, which corresponds to the lowest level in p-adic evolution since it is the simplest p-adic topology and farthest from the real topology and therefore gives the poorest cognitive representation of real preferred extremal as p-adic preferred extermal (Note that $p = 1$ makes formally sense but for it the topology is discrete).
3. Weak form of NMP [K65, K116] suggests a more convincing explanation. The density matrix of the state to be reduced is a direct sum over contributions proportional to projection operators. Suppose that the projection operator with largest dimension has dimension n. Strong form of NMP would say that final state is characterized by n-dimensional projection operator. Weak form of NMP allows free will so that all dimensions $n-k$, $k = 0, 1, ...n-1$ for final state projection operator are possible. 1-dimensional case corresponds to vanishing entanglement negentropy and ordinary state function reduction isolating the measured system from external world.
4. The negentropy of the final state per state depends on the value of k. It is maximal if $n-k$ is power of prime. For $n = 2^k = M_k + 1$, where M_k is Mersenne prime $n-1$ gives the maximum negentropy and also maximal p-adic prime available so that this reduction is favoured by NMP. Mersenne primes would be indeed special. Also the primes $n = 2^k - r$ near 2^k produce large entanglement negentropy and would be favored by NMP.
5. This argument suggests a generalization of p-adic length scale hypothesis so that $p = 2$ can be replaced by any prime.

This argument together with the hypothesis that preferred prime is ramified would correlate the character of the irreducible extension and character of super-conformal symmetry breaking. The integer n characterizing super-symplectic conformal sub-algebra acting as gauge algebra would depends on the irreducible algebraic extension of rational involved so that the hierarchy of quantum criticalities would have number theoretical characterization. Ramified primes could appear as divisors of n and n would be essentially a characteristic of ramification known as discriminant. An interesting question is whether only the ramified primes allow the continuation of string world sheet and partonic 2-surface to a 4-D space-time surface. If this is the case, the assumptions behind p-adic mass calculations would have full first principle justification.

10.5.5 A Connection With Infinite Primes?

Infinite primes are one of the mathematical outcomes of TGD [K103]. There are two kinds of infinite primes. There are the analogs of free many particle states consisting of fermions and bosons labelled by primes of the previous level in the hierarchy. They correspond to states of a supersymmetric arithmetic quantum field theory or actually a hierarchy of them obtained by a repeated second quantization of this theory. A connection between infinite primes representing bound statesandirreducible polynomials is highly suggestive.

1. The infinite prime representing free many-particle state decomposes to a sum of infinite part and finite part having no common finite prime divisors so that prime is obtained. The infinite part is obtained from "fermionic vacuum" $X = \prod_k p_k$ by dividing away some fermionic primes p_i and adding their product so that one has $X \rightarrow X/m + m$, where m is square free integer. Also $m = 1$ is allowed and is analogous to fermionic vacuum interpreted as Dirac sea without holes. X is infinite prime and pure many-fermion state physically. One can add bosons by multiplying X with any integers having no common denominators with m and its prime decomposition defines the bosonic contents of the state. One can also multiply m by any integers whose prime factors are prime factors of m.
2. There are also infinite primes, which are analogs of bound states and at the lowest level of the hierarchy they correspond to irreducible polynomials $P(x)$ with integer coefficients. At the second levels the bound states would naturally correspond to irreducible polynomials $P_n(x)$ with coefficients $Q_k(y)$, which are infinite integers at the previous level of the hierarchy.
3. What is remarkable that bound state infinite primes at given level of hierarchy would define maximally ramified algebraic extensions at previous level. One indeed has infinite hierarchy of

infinite primes since the infinite primes at given level are infinite primes in the sense that they are not divisible by the primes of the previous level. The formal construction works as such. Infinite primes correspond to polynomials of single variable at the first level, polynomials of two variables at second level, and so on. Could the Langlands program could be generalized from the extensions of rationals to polynomials of complex argument and that one would obtain infinite hierarchy?

4. Infinite integers in turn could correspond to products of irreducible polynomials defining more general extensions. This raises the conjecture that infinite primes for an extension K of rationals could code for the algebraic extensions of K quite generally. If infinite primes correspond to real quantum states they would thus correspond the extensions of rationals to which the parameters appearing in the functions defining partonic 2-surfaces and string world sheets.

   This would support the view that partonic 2-surfaces associated with algebraic extensions defined by infinite integers and thus not irreducible are unstable against decay to partonic 2-surfaces which corresponds to extensions assignable to infinite primes. Infinite composite integer defining intermediate unstable state would decay to its composites. Basic particle physics phenomenology would have number theoretic analog and even more.

5. According to Wikipedia, Eisenstein's criterion (`http://tinyurl.com/47kxjz`) allows generalization and what comes in mind is that it applies in exactly the same form also at the higher levels of the hierarchy. Primes would be only replaced with prime polynomials and the there would be at least one prime polynomial $Q(y)$ dividing the coefficients of $P_n(x)$ except the highest one such that its square would not divide P_0. Infinite primes would give rise to an infinite hierarchy of functions of many complex variables. At first level zeros of function would give discrete points at partonic 2-surface. At second level one would obtain 2-D surface: partonic 2-surfaces or string world sheet. At the next level one would obtain 4-D surfaces. What about higher levels? Does one obtain higher dimensional objects or something else. The union of n 2-surfaces can be interpreted also as $2n$-dimensional surface and one could think that the hierarchy describes a hierarchy of unions of correlated partonic 2-surfaces. The correlation would be due to the preferred extremal property of Kähler action.

   One can ask whether this hierarchy could allow to generalize number theoretical Langlands to the case of function fields using the notion of prime function assignable to infinite prime. What this hierarchy of polynomials of arbitrary many complex arguments means physically is unclear. Do these polynomials describe many-particle states consisting of partonic 2-surface such that there is a correlation between them as sub-manifolds of the same space-time sheet representing a preferred extremals of Kähler action?

This would suggest strongly the generalization of the notion of p-adicity so that it applies to infinite primes.

1. This looks sensible and maybe even practical! Infinite primes can be mapped to prime polynomials so that the generalized p-adic numbers would be power series in prime polynomial - Taylor expansion in the coordinate variable defined by the infinite prime. Note that infinite primes (irreducible polynomials) would give rise to a hierarchy of preferred coordinate variables. In terms of infinite primes this expansion would require that coefficients are smaller than the infinite prime P used. Are the coefficients lower level primes? Or also infinite integers at the same level smaller than the infinite prime in question? This criterion makes sense since one can calculate the ratios of infinite primes as real numbers.

2. I would guess that the definition of infinite-P p-adicity is not a problem since mathematicians have generalized the number theoretical notions to such a level of abstraction much above of a layman like me. The basic question is how to define p-adic norm for the infinite primes (infinite only in real sense, p-adically they have unit norm for all lower level primes) so that it is finite.

3. There exists an extremely general definition of generalized p-adic number fields (see `http://tinyurl.com/y5zreeg`). One considers Dedekind domain D, which is a generalization of integers for ordinary number field having the property that ideals factorize uniquely to prime ideals. Now D would contain infinite integers. One introduces the field E of fractions consisting of infinite rationals.

Consider element e of E and a general fractional ideal eD as counterpart of ordinary rational and decompose it to a ratio of products of powers of ideals defined by prime ideals, now those defined by infinite primes. The general expression for the p-adic norm of x is $x^{-ord(P)}$, where n defines the total number of ideals P appearing in the factorization of a fractional ideal in E: this number can be also negative for rationals. When the residue field is finite (finite field G(p,1) for p-adic numbers), one can take c to the number of its elements ($c = p$ for p-adic numbers.

Now it seems that this number is not finite since the number of ordinary primes smaller than P is infinite! But this is not a problem since the topology for completion does not depend on the value of c. The simple infinite primes at the first level (free many-particle states) can be mapped to ordinary rationals and q-adic norm suggests itself: could it be that infinite-P p-adicity corresponds to q-adicity discussed by Khrennikov [A29]. Note however that q-adic numbers are not a field.

Finally a loosely related question. Could the transition from infinite primes of K to those of L takes place just by replacing the finite primes appearing in infinite prime with the decompositions? The resulting entity is infinite prime if the finite and infinite part contain no common prime divisors in L. This is not the case generally if one can have primes P_1 and P_2 of K having common divisors as primes of L: in this case one can include P_1 to the infinite part of infinite prime and P_2 to finite part.

10.6 More About Physical Interpretation Of Algebraic Extensions Of Rationals

The number theoretic vision has begun to show its power. The basic hierarchies of quantum TGD would reduce to a hierarchy of algebraic extensions of rationals and the parameters - such as the degrees of the irreducible polynomials characterizing the extension and the set of ramified primes (see `http://tinyurl.com/hddljlf`) - would characterize quantum criticality and the physics of dark matter as large h_{eff} phases. The value of $h_{eff}/h = n$ would naturally correspond to the order of the Galois group of the extension.

The conjecture is that preferred p-adic primes correspond to ramified primes for extensions of rationals for which especially many number theoretic discretizations of the space-time surfaces allow strong form of holography as an algebraic continuation of string world sheets to space-time surfaces. The generalization of the p-adic length scale hypothesis as a prediction of NMP is another conjecture. What remains to be shown that the primes predicted by generalization p-adic length scale hypothesis indeed are preferred primes in the proposed sense.

By strong form of holography the parameters characterizing string world sheets and partonic 2-surfaces serve as WCW coordinates. By various conformal invariances, one expects that the parameters correspond to conformal moduli, which means a huge simplification of quantum TGD since the mathematical apparatus of superstring theories becomes available and number theoretical vision can be realized. Scattering amplitudes can be constructed for a given algebraic extension and continued to various number fields by continuing the parameters which are conformal moduli and group invariants characterizing incoming particles.

There are many un-answered and even un-asked questions.

1. How the new degrees of freedom assigned to the n-fold covering defined by the space-time surface pop up in the number theoretic picture? How the connection with preferred primes emerges?
2. What are the precise physical correlates of the parameters characterizing the algebraic extension of rationals? Note that the most important extension parameters are the degree of the defining polynomial and ramified primes.

10.6.1 Some Basic Notions

Some basic information about extensions are in order. I emphasize that I am not a specialist.

Basic facts

The algebraic extensions of rationals are determined by roots of polynomials. Polynomials be decomposed to products of irreducible polynomials, which by definition do not contain factors which are polynomials with rational coefficients. These polynomials are characterized by their degree n, which is the most important parameter characterizing the algebraic extension.

One can assign to the extension primes and integers - or more precisely, prime and integer ideals. Integer ideals correspond to roots of monic polynomials $P_n(x) = x^n + ..a_0$ in the extension with integer coefficients. Clearly, for $n = 0$ (trivial extension) one obtains ordinary integers. Primes as such are not a useful concept since roots of unity are possible and primes which differ by a multiplication by a root of unity are equivalent. It is better to speak about prime ideals rather than primes.

Rational prime p can be decomposed to product of powers of primes of extension and if some power is higher than one, the prime is said to be ramified and the exponent is called ramification index. Eisenstein's criterion (see `http://tinyurl.com/47kxjz` states that any polynomial $P_n(x) = a_n x^n + a_{n-1}x^{n-1} + ...a_1 x + a_0$ for which the coefficients a_i, $i < n$ are divisible by p and a_0 is not divisible by p^2 allows p as a maximally ramified prime. The corresponding prime ideal is n:th power of the prime ideal of the extensions (roughly n:th root of p). This allows to construct endless variety of algebraic extensions having given primes as ramified primes.

Ramification is analogous to criticality. When the gradient potential function $V(x)$ depending on parameters has multiple roots, the potential function becomes proportional a higher power of $x - x_0$. The appearance of power is analogous to appearance of higher power of prime of extension in ramification. This gives rise to cusp catastrophe. In fact, ramification is expected to be number theoretical correlate for the quantum criticality in TGD framework. What this precisely means at the level of space-time surfaces, is the question.

Galois group as symmetry group of algebraic physics

I have proposed long time ago that Galois group (see `http://tinyurl.com/h9528pl`) acts as fundamental symmetry group of quantum TGD and even made clumsy attempt to make this idea more precise in terms of the notion of number theoretic braid. It seems that this notion is too primitive: the action of Galois group must be realized at more abstract level and WCW provides this level.

First some facts (I am not a number theory professional, as the professional reader might have already noticed!).

1. Galois group acting as automorphisms of the field extension (mapping products to products and sums to sums and preserves norm) characterizes the extension and its elements have maximal order equal to n by algebraic n-dimensionality. For instance, for complex numbers Galois group acs as complex conjugation. Galois group has natural action on prime ideals of extension mapping them to each other and preserving the norm determined by the determinant of the linear map defined by the multiplication with the prime of extension. For instance, for the quadratic extension $Q(\sqrt{5})$ the norm is $N(x + \sqrt{5}y) = x^2 - 5y^2$: not that number theory leads to Minkowkian metric signatures naturally. Prime ideals combine to form orbits of Galois group.
2. Since Galois group leaves the rational prime p invariant, the action must permute the primes of extension in the product representation of p. For ramified primes the points of the orbit of ideal degenerate to single ideal. This means that primes and quite generally, the numbers of extension, define orbits of the Galois group.

Galois group acts in the space of integers or prime ideals of the algebraic extension of rationals and it is also physically attractive to consider the orbits defined by ideals as preferred geometric structures. If the numbers of the extension serve as parameters characterizing string world sheets and partonic 2-surfaces, then the ideals would naturally define subsets of the parameter space in which Galois group would act.

The action of Galois group would leave the space-time surface invariant if the sheets co-incide at ends but permute the sheets. Of course, the space-time sheets permuted by Galois group need not co-incide at ends. In this case the action need not be gauge action and one could have non-trivial representations of the Galois group. In Langlands correspondence these representation relate to the representations of Lie group and something similar might take place in TGD as I have indeed proposed.

The value of effective Planck constant $h_{eff}/h = n$ corresponds to the number of sheets of some kind of covering space defined by the space-time surface. The discretization of the space-time surface identified as a monadic manifold [L25] with embedding space preferred coordinates in extension of rationals defining the adele has Galois group of extension as a group of symmetries permuting the sheets of the covering group. Therefore $n = h_{eff}/h$ would naturally correspond to the dimension of the extension dividing the order of its Galois group. Dark matter in TGD sense would correspond to number theoretic physics.

Remark: Strong form of holography supports also the vision about quaternionic generalization of conformal invariance implying that the adelic space-time surface can be constructed from the data associated with functions of two complex variables, which in turn reduce to functions of single variable.

If this picture is correct, it is possible to talk about quantum amplitudes in the space defined by the numbers of extension and restrict the consideration to prime ideals or more general integer ideals.

1. These number theoretical wave functions are physical if the parameters characterizing the 2-surface belong to this space. One could have purely number theoretical quantal degrees of freedom assignable to the hierarchy of algebraic extensions and these discrete degrees of freedom could be fundamental for living matter and understanding of consciousness.

2. The simplest assumption that Galois group acts as a gauge group when the ends of sheets co-incide at boundaries of CD seems however to destroy hopes about non-trivial number theoretical physics but this need not be the case. Physical intuition suggests that ramification somehow saves the situation and that the non-trivial number theoretic physics could be associated with ramified primes assumed to define preferred p-adic primes.

10.6.2 How New Degrees Of Freedom Emerge For Ramified Primes?

How the new discrete degrees of freedom appear for ramified primes?

1. The space-time surfaces defining singular coverings are n-sheeted in the interior. At the ends of the space-time surface at boundaries of CD however the ends co-incide. This looks very much like a critical phenomenon.

   Hence the idea would be that the end collapse can occur only for the ramified prime ideals of the parameter space - ramification is also a critical phenomenon - and means that some of the sheets or all of them co-incide. Thus the sheets would co-incide at ends only for the preferred p-adic primes and give rise to the singular covering and large h_{eff}. End-collapse would be the essence of criticality! This would occur, when the parameters defining the 2-surfaces are in a ramified prime ideal.

2. Even for the ramified primes there would be n distinct space-time sheets, which are regarded as physically distinct. This would support the view that besides the space-like 3-surfaces at the ends the full 3-surface must include also the light-like portions connecting them so that one obtains a closed 3-surface. The conformal gauge equivalence classes of the light-like portions would give rise to additional degrees of freedom. In space-time interior and for string world sheets they would become visible.

   For ramified primes n distint 3-surfaces would collapse to single one but the n discrete degrees of freedom would be present and particle would obtain them. I have indeed proposed number theoretical second quantization assigning fermionic Clifford algebra to the sheets with n oscillator operators. Note that this option does not require Galois group to act as gauge group in the general case. This number theoretical second quantization might relate to the realization of Boolean algebra suggested by weak form of NMP [K119].

10.6.3 About The Physical Interpretation Of The Parameters Characterizing Algebraic Extension Of Rationals In TGD Framework

It seems that Galois group is naturally associated with the hierarchy $h_{eff}/h = n$ of effective Planck constants defined by the hierarchy of quantum criticalities. n would naturally define the maximal order for the element of Galois group. The analog of singular covering with that of $z^{1/n}$ would suggest that Galois group is very closely related to the conformal symmetries and its action induces permutations of the sheets of the covering of space-time surface.

Without any additional assumptions the values of n and ramified primes are completely independent so that the conjecture that the magnetic flux tube connecting the wormhole contacts associated with elementary particles would not correspond to very large n having the p-adic prime p characterizing particle as factor ($p = M_{127} = 2^{127} - 1$ for electron). This would not induce any catastrophic changes.

TGD based physics could however change the situation and reduce number theoretical degrees of freedom: the intuitive hypothesis that p divides n might hold true after all.

1. The strong form of GCI implies strong form of holography. One implication is that the WCW Kähler metric can be expressed either in terms of Kähler function or as anti-commutators of super-symplectic Noether super-charges defining WCW gamma matrices. This realizes what can be seen as an analog of Ads/CFT correspondence. This duality is much more general. The following argument supports this view.
   (a) Since fermions are localized at string world sheets having ends at partonic 2-surfaces, one expects that also Kähler action can be expressed as an effective stringy action. It is natural to assume that string area action is replaced with the area defined by the effective metric of string world sheet expressible as anti-commutators of Kähler-Dirac gamma matrices defined by contractions of canonical momentum currents with embedding space gamma matrices. It string tension is proportional to h_{eff}^2, string length scales as h_{eff}.
   (b) AdS/CFT analogy inspires the view that strings connecting partonic 2-surfaces serve as correlates for the formation of - at least gravitational - bound states. The distances between string ends would be of the order of Planck length in string models and one can argue that gravitational bound states are not possible in string models and this is the basic reason why one has ended to landscape and multiverse non-sense.
2. In order to obtain reasonable sizes for astrophysical objects (that is sizes larger than Schwartschild radius $r_s = 2GM$) For $\hbar_{eff} = \hbar_{gr} = GMm/v_0$ one obtains reasonable sizes for astrophysical objects. Gravitation would mean quantum coherence in astrophysical length scales.
3. In elementary particle length scales the value of h_{eff} must be such that the geometric size of elementary particle identified as the Minkowski distance between the wormhole contacts defining the length of the magnetic flux tube is of order Compton length - that is p-adic length scale proportional to $\sqrt{p}$. Note that dark physics would be an essential element already at elementary particle level if one accepts this picture also in elementary particle mass scales. This requires more precise specification of what darkness in TGD sense really means.

   One must however distinguish between two options.
   (a) If one assumes $n \simeq \sqrt{p}$, one obtains a large contribution to classical string energy as $\Delta \sim m_{CP_2}^2 L_p/\hbar_{eff}^2 \sim m_{CP_2}/\sqrt{p}$, which is of order particle mass. Dark mass of this size looks un-feasible since p-adic mass calculations assign the mass with the ends wormhole contacts. One must be however very cautious since the interpretations can change.
   (b) Second option allows to understand why the minimal size scale associated with CD characterizing particle correspond to secondary p-adic length scale. The idea is that the string can be thought of as being obtained by a random walk so that the distance between its ends is proportional to the square root of the actual length of the string in the induced metric. This would give that the actual length of string is proportional to p and n is also proportional to p and defines minimal size scale of the CD associated with the particle. The dark contribution to the particle mass would be $\Delta m \sim m_{CP_2}^2 L_p/\hbar_{eff}^2 \sim m_{CP_2}/p$, and completely negligible suggesting that it is not easy to make the dark side of elementary visible.

4. If the latter interpretation is correct, elementary particles would have huge number of hidden degrees of freedom assignable to their CDs. For instance, electron would have $n = 2^{127} - 1 \simeq 10^{38}$ hidden discrete degrees of freedom and would be rather intelligent system - 127 bits is the estimate- and thus far from a point-like idiot of standard physics. Is it a mere accident that the secondary p-adic time scale of electron is .1 seconds - the fundamental biorhythm - and the size scale of the minimal CD is slightly large than the circumference of Earth?

   Note however, that the conservation option assuming that the magnetic flux tubes connecting the wormhole contacts representing elementary particle are in $h_{eff}/h = 1$ phase can be considered as conservative option.

10.7 p-Adicization and adelic physics

This section is devoted to the challenges related to p-adicization and adelization of physics in which the correspondence between real and p-adic numbers via canonical identification serves as the basic building brick. Also the problems associated with p-adic variants of integral, Fourier analysis, Hilbert space, and Riemann geometry should be solved in a way respecting fundamental symmetries and their p-adic variants must be met. The notion of number theoretical universality (NTU) plays a key role here. One should also answer to questions about the origin of preferred primes and p-adic length scale hypothesis.

10.7.1 Challenges

The basic challenges encountered are construction of the p-adic variants of real number based physics, understanding their relationship to real physics, and the fusion of various physics to single coherent whole.

The p-adicization of real physics is not just a straightforward formal generalization of scattering amplitudes of existing theories but requires a deeper understanding of the physics involved. The interpretation of p-adic physics as correlate for cognition and imagination is an important guideline and will be discussed l in more detail in separate section.

Definite integral and Fourier analysis are basic elements of standard physics and their generalization to the p-adic context defines a highly non-trivial challenge. Also the p-adic variants of Riemann geometry and Hilbert space are suggestive. There are however problems.

1. There are problems associated with p-adic definite integral. Riemann sum does not make sense since it approaches zero if the p-adic norm of discretization unit approaches zero. The problems are basically due to the absence of well-orderedness essential for the definition of definite integral and differential forms and their integrals.

   Residue integration might make sense in finite angle resolution. For algebraic extension containing $e^{i\pi/n}$ the number theoretically universal approximation $i\pi = n(e^{i\pi/n} - 1)$ could be used. In twistor approach integrations reduce to multiple residue integrations and since twistor approach generalizes in TGD framework, this approach to integration is very attractive.

   Positivity is a central notion in twistor Grassmannian approach [B24]. Since canonical identification maps p-adic numbers to non-negative real numbers, there is a strong temptation to think that positivity relates to NTU [L15].

2. There are problems with Fourier analysis. The naïve generalization of trigonometric functions by replacing e^{ix} with its p-adic counterpart is not physical. Same applies to e^x. Algebraic extensions are needed to get roots of unity ad e as counterparts of the phases and discretization is necessary and has interpretation in terms of finite resolution for angle/phase and its hyperbolic counterpart.

3. The notion of Hilbert space is problematic. The naïve generalization of Hilbert space norm square $|x|^2 = \sum x_n \overline{x}_n$ for state $(x_1, x_2, ...)$ can vanish p-adically. Also here NTU could help. State would contain as coefficients only roots of e and unity and only the overall factor could be p-adic number. Coefficients could be restricted to the algebraic numbers generating the algebraic extension of rational numbers and would not contain powers of p or even ordinary p-adic numbers expect in the overall normalization factor.

Second challenge relates to the relationship between real and p-adic physics. Canonical identification (CI) $\sum x_n p^n \rightarrow \sum x_n p^{-n}$ or some of its variants should play an important role. CI is expected to map the invariants appearing in scattering amplitudes to their real counterparts.

1. Real and p-adic variants of space-time surfaces should exist and relate to each other somehow. Is this relationship local and involve CI at space-time level or embedding space level? Or is it only a global and non-local assignment of preferred real extremals to their p-adic counterparts? Or is between these extreme options and involves algebraic discretization of the space-time surface weakening the strong form of SH as already proposed? How do real and p-adic embedding spaces relate to each other and can this relationship induce local correspondence between preferred extremals (PEs) [K8, K14, K10]?
2. NTU in some sense is a highly suggestive approach to these questions and would suggest that canonical identification applies to isometry invariants whereas angles and hyperbolic angles, or rather the corresponding "phases" belonging to an extension of p-adics containing roots of e and roots of unity are mapped to themselves. Note that the roots of e define extensions of rationals, which induce finite dimensional algebraic extensions of p-adic numbers. This would make possible to define embedding space in accordance with NTU. Also the Hilbert space could be defined by requiring that its points correspond to number theoretically universal angles expressible in terms of roots of unity.
3. What about real and p-adic variants of WCW? Are they needed at all? Or could their existence be used as a powerful constraint on real physics? The representability of WCW as a union of infinite-dimensional symmetric spaces labelled by zero modes suggests that the same description applies at the level of WCW and embedding space.

   One cannot circumvent the question about how to generalize functional integral from real WCW to p-adic WCWs. In particular, what is the p-adic variant of the action defining the dynamics of space-time surfaces. In the case of exponent of action the p-adic variant could be defined by assuming algebraic universality: again the roots of e and of unity would be in central role. Also the Kähler structure of WCW implying that Gaussian and metric determinants cancel each other in functional integral, would be absolutely crucial.

   One must remember that the exponents of action for scattering amplitudes for the stationary phase extremal cancel from the path integral representation of scattering amplitudes. Also now this mechanism would allow to get rid of the poorly defined exponent for single minimum. If there is sum over scattering amplitudes assignable to different maxima, normalization sould give ratios of these exponents for different extrema/maxima and only these ratios should belong to the extension of rationals.

   The zero modes of WCW metric are invariants of supersymplectic group so that canonical identification could relate their real and p-adic variants. Zero modes could break NTU and would be behind p-adic thermodynamics and dependence of mass scale on p-adic prime.

The third challenge relates to the fusion of p-adic physics and real physics to a larger structure. Here a generalization of number concept obtained by glueing reals and various p-adics together along an extension of rational numbers inducing the extensions of p-adic numbers is highly suggestive. Adeles associated with the extension of rationals are highly attractive and closely related notion. Real and various p-adic physics would be correlates for sensory and cognitive aspects of the same universal physics rather than separate physics in this framework. One important implication of this view is that real entropy and p-adic negentropies characterize the same entanglement with coefficients in an extension of rationals.

NTU for hyperbolic and ordinary phases is definetely the central idea. How the invariance of angles under conformal transformations does relate to this? Could one perhaps define a discretized version of conformal symmetry preserving the phases defined by the angles between vectors assignable with the tangent spaces of discretized geometric structures and thus respecting NTU? Of should one apply conformal symmetry at Lie algebra level only?

10.7.2 NTU and the correspondence between real and p-adic physics

p-Adic real correspondence is certainly the basic problem of p-adicization and adelization. One can make several general questions about p-adic real correspondence and canonical identification inspired by p-adic mass calculations.

How generally p-adic real correspondence does apply? Could canonical identification for group invariants combined with direct identification of ordinary and hyperbolic phases identified as roots of unity and e apply at WCW and embedding space level having maximally symmetric geometries? Could this make sense even at space-time level as a correspondence induced from embedding space level [L25]? Does canonical identification apply locally for the discretizations of space-time surface or only globally for the parameters characterizing PEs (string world sheets and partonic 2-surfaces by SH), which are general coordinate invariant and Poincare invariant quantities?

The following vision seems to be the most feasible one found hitherto.

1. Preservation of symmetries and continuity compete. Lorenz transformations do not commute with canonical identification. This suggests that canonical identification applies only to Lorentz invariants formed from quantum numbers. This is enough in the case of scattering amplitudes. Canonical identification applies only to isometry invariants at the level of WCW and the phases/exponents of ordinary/hyperbolic angles correspond to numbers in the algebraic extension common to extensions of rationals and various p-adics.
2. Canonical identification applies at the level of momentum space and maps p-adic Lorentz invariants of scattering amplitudes to their real counterparts. Phases of angles and their hyperbolic counterparts should correspond to parameters defining extension and should be mapped as such to their p-adic counterparts.
3. The constraints coming from GCI and symmetries do not allow local correspondence but allow to consider its discretized version at space-time leve induced by the correspondence at the level of embedding space.

   This requires the restriction of isometries and other symmetries to algebraic subgroups defined by the extension of rationals. This would imply reduction of symmetry due to finite cognitive/measurement resolution and should be acceptable. If one wants to realize the ideas about imagination, discretization must be applied also for the space-time interior meaning partial breaking of SH and giving rise to dark matter degrees freedom in TGD sense. SH could apply in real sector for realizable imaginations only. Note that the number of algebraic points of space-time surface is expected to be relatively small.

The correspondence must be considered at the level of embedding space, space-time, and WCW.

1. At the level of embedding space p-adic–real correspondence is induced by points in extension of rationals and is totally discontinuous. This requires that space-time dimension is smaller than embedding space dimension.
2. At space-time level the correspondence involves field equations derivable from a local variational principle make sense also p-adically although the action itself is ill-defined as 4-D integral. The notion of p-adic PE makes sense by strong form of holography applied to 2-surfaces in the intersection. p-Adically however only the vanishing of Noether currents for a sub-algebra of the super-symplectic algebra might make sense. This condition is stronger than the vanishing of Noether charges defined by 3-D integrals.
3. Correspondence at the level of WCW can make sense and reduces to that for string world sheets and partonic 2-surfaces by SH. Real and p-adic 4-surfaces would be obtained by algebraic continuation as PEs from 2-surfaces by assuming that the space-time surface contains subset of points of embedding space belonging to the extension of rationals. p-Adic pseudo constants make p-adic continuation easy. Real continuation need not exist always. p-Adic WCW would be considerably larger than real WCW and make possible a predictive quantum theory of imagination and cognition.

   What I have called intersection of realities and p-adicities can be identified as the set of 2-surfaces plus algebraic discretization of space-time interior. Also the values of induced spinor fields at the points of discretization must be given. The parameters characterizing the extremals (say coefficients of polynomials) - WCW coordinates - would be in extension of rationals inducing a finite-D extension of p-adic number fields.

   The hierarchy of algebraic extensions induces an evolutionary hierarchy of adeles. The interpretation could be as a mathematical correlate for cosmic evolution realized at the level of the core of WCW defined by the intersection? 2-surfaces could be called space-time genes.

4. Also the p-adic variant Kähler action or at least the exponent of Kähler action defining vacuum functional should be obtainable by algebraic continuation. The weakest condition states that the ratios of action exponents for the maxima of Kähler function to the sum of action exponents for maxima belong to the extension. Without this condition the hopes of satisfying NTU seem rather meager.

10.7.3 NTU at space-time level

What about NTU at space-time level? NTU requires a correspondence between real and p-adic numbers and the details of this corresponds have been a long standing problem.

1. The recent view about the correspondence between real PEs to their p-adic counterparts does not demand discrete local correspondence assumed in the earlier proposal [K122]. The most abstract approach would give up the local correspondence at space-time level altogether, and restrict the preferred coordinates of WCW (having maximal group of isometries) to numbers in the extension of rationals considered. WCW would be discretized.

   Intuitively a more realistic view is a correspondence at space-time level in the sense that real and p-adic space-time sheets intersect at points belonging to the extension of rationals and defining "cognitive representations". Only some p-adic space-time surfaces would have real counterpart.

2. The strongest form of NTU would require that the allowed points of embedding space belonging an extension of rationals are mapped as such to corresponding extensions of p-adic number fields (no canonical identification). At embedding space level this correspondence would be extremely discontinuous. The "spines" of space-time surfaces would however contain only a subset of points of extension, and a natural resolution length scale could emerge and prevent the fluctuation. This could be also seen as a reason for why space-times surfaces must be 4-D. The fact that the curve $x^n + y^n = z^n$ has no rational points for $n > 2$, raises the hope that the resolution scale could emerge spontaneously.

3. The notion of monadic geometry discussed in detail in [L25] would realize this idea. Define first a number theoretic discretization of embedding space in terms of points, whose coordinates in group theoretically preferred coordinate system belong to the extension of rationals considered. One can say that these algebraic points are in the intersection of reality and various p-adicities. Overlapping open sets assigned with this discretization define in the real sector a covering by open sets. In p-adic sector compact-open-topology allows to assign with each point 8^{th} Cartesian power of algebraic extension of p-adic numbers. These compact open sets define analogs for the monads of Leibniz and p-adic variants of field equations make sense inside them.

   The monadic manifold structure of H is induced to space-time surfaces containing discrete subset of points in the algebraic discretization with field equations defining a continuation to space-time surface in given number field, and unique only in finite measurement resolution. This approach would resolve the tension between continuity and symmetries in p-adic–real correspondence: isometry groups would be replaced by their sub-groups with parameters in extension of rationals considered and acting in the intersection of reality and p-adicities.

   The Galois group of extension acts non-trivially on the "spines" of space-time surfaces. Hence the number theoretical symmetries act as physical symmetries and define the orbit of given space-time surface as a kind of covering space. The coverings assigned to the hierarchy of Planck constants would naturally correspond to Galois coverings and dark matter would represent number theoretical physics.

   This would give rise to a kind of algebraic hierarchy of adelic 4-surfaces identifiable as evolutionary hierarchy: the higher the dimension of the extension, the higher the evolutionary level.

10.7.4 NTU and WCW

p-Adic–real correspondence at the level of WCW

It has not been obvious whether one should perform p-adicization and adelization at the level of WCW. Minimalist could argue that scattering amplitudes are all we want and that their p-adicization and adelization by algebraic continuation can be tolerated only if it can give powerful enough constraints on the amplitudes.

1. The anti-commutations for fermionic oscillator operators are number theoretically universal. Supersymmetry suggests that also WCW bosonic degrees of freedom satisfy NTU. This could mean that the coordinates of p-adic WCW consist of super-symplectic invariants mappable by canonical identification to their real counterparts plus phases and their hyperbolic counterparts expressible as genuinely algebraic numbers common to all number fields. This kind of coordinates are naturally assignable to symmetric spaces [L25].
2. Kähler structure should be mapped from p-adic to real sector and vice versa. Vacuum functional identified as exponent of action should be NTU. Algebraic continuation defined by SH involves p-adic pseudo constants. All p-adic continuations by SH should correspond to the same value of exponent of action obtained by algebraic continuation from its real value. The degeneracy associated with p-adic pseudo-constants would be analogous to gauge invariance - imagination in TGD inspired theory of consciousness.
3. Ist it possible have NTU for WCW functional integration? Or is it enough to realize NTU for scattering amplitudes only. What seems clear that functional integral must reduce to a discrete sum. Physical intuition suggests a sum over maxima of Kähler function forming a subset of PEs representing stationary points. One cannot even exclude the possibility that the set of PEs is discrete and that one can sum over all of them.

   Restriction to maximum/stationary phase approximation gives rise to sum over exponents multiplied with Gaussian determinants. The determinant of Kähler metric however cancels the Gaussian determinants, and one obtains only a sum over the exponents of action.

   The breaking of strong NTU could happen: consider only p-adic mass calculations. This breaking is however associated with the parts of quantum states assignable to the boundaries of CD, not with the vacuum functional.

NTU for functional integral

Number theoretical vision relies on NTU. In fermionic sector NTU is necessary: one cannot speak about real and p-adic fermions as separate entities and fermionic anti-commutation relations are indeed number theoretically universal.

What about NTU in case of functional integral? There are two opposite views.

1. One can define p-adic variants of field equations without difficulties if preferred extremals are minimal surface extremals of Kähler action so that coupling constants do not appear in the solutions. If the extremal property is determined solely by the analyticity properties as it is for various conjectures, it makes sense independent of number field. Therefore there would be no need to continue the functional integral to p-adic sectors. This in accordance with the philosophy that thought cannot be put in scale. This would be also the option favored by pragmatist.
2. Consciousness theorist might argue that also cognition and imagination allow quantum description. The supersymmetry NTU should apply also to functional integral over WCW (more precisely, its sector defined by CD) involved with the definition of scattering amplitudes.

1. Key observations

The general vision involves some crucial observations.

1. Only the expressions for the scatterings amplitudes should should satisfy NTU. This does not require that the functional integral satisfies NTU.

2. Since the Gaussian and metric determinants cancel in WCW Kähler metric the contributions form maxima are proportional to action exponentials $exp(S_k)$ divided by the $\sum_k exp(S_k)$. Loops vanish by quantum criticality.

3. Scattering amplitudes can be defined as sums over the contributions from the maxima, which would have also stationary phase by the double extremal property made possible by the complex value of α_K. These contributions are normalized by the vacuum amplitude.

   It is enough to require NTU for $X_i = exp(S_i)/\sum_k exp(S_k)$. This requires that $S_k - S_l$ has form $q_1 + q_2 i\pi + q_3 log(n)$. The condition brings in mind homology theory without boundary operation defined by the difference $S_k - S_l$. NTU for both S_k and $exp(S_k)$ would only values of general form $S_k = q_1 + q_2 i\pi + q_3 log(n)$ for S_k and this looks quite too strong a condition.

4. If it is possible to express the 4-D exponentials as single 2-D exponential associated with union of string world sheets, vacuum functional disappears completely from consideration! There is only a sum over discretization with the same effective action and one obtains purely combinatorial expression.

2. What does one mean with functional integral?

The definition of functional integral in WCW is one of the key technical problems of quantum TGD [K119]. NTU states that the integral should be defined simultaneously in all number fields in the intersection of real and p-adic worlds defined by string world sheets and partonic 2-surfaces with WCW coordinates in algebraic extension of rationals and allowing by strong holography continuation to 4-D space-time surface. NTU is powerful constraint and could help in this respect.

1. Path integral is not in question. Rather, the functional integral is analogous to Wiener integral and perhaps allows identification as a genuine integral in the real sector. In p-adic sectors algebraic continuation should give the integral and here number theoretical universality gives excellent hopes. The integral would have exactly the same form in real and p-adic sector and expressible solely in terms of algebraic numbers characterizing algebraic extension and finite roots of e and roots of unity $U_n = exp(i2\pi/n)$ in algebraic extension of p-adic numbers.

   Since vacuum functional $exp(S)$ is exponential of complex action S, the natural idea is that only rational powers e^q and roots of unity and phases $exp(i2\pi q)$ are involved and there is no dependence on p-adic prime p! This is true in the integer part of q is smaller than p so that one does not obtain e^{kp}, which is ordinary p-adic number and would spoil the number theoretic universality. This condition is not possible to satisfy for all values of p unless the value of Kähler function is smaller than 2. One might consider the possibility that the allow primes are above some minimum value.

   The minimal solution to NTU conditions is that the ratios of action exponentials for maxima of Kähler function to the sum of these exponentials belong to the extension of rationals considered.

2. What does one mean with functional integral? TGD is expected to be an integrable in some sense. In integrable QFTs functional integral reduces to a sum over stationary points of the action: typically only single point contributes - at least in good approximation.

   For real α_K and Λ vacuum functional decomposes to a product of exponents of real contribution from Euclidian regions ($\sqrt{g_4}$ real) and imaginary contribution Minkowskian regions ($\sqrt{g_4}$ imaginary). There would be no exchange of momentum between Minkowskian and Euclidian regions. For complex values of α_K [L14] situation changes and Kähler function as real part of action receives contributions from both Euclidian and Minkowskian regions. The imaginary part of action has interpretation as analog of Morse function and action as it appears in QFTs. Now saddle points must be considered.

   PEs satisfy extremely strong conditions [K8, K10]. All classical Noether charges for a sub-algebra associated with super-symplectic algebra and isomorphic to the algebra itself vanish at both ends of CD. The conformal weights of this algebra are $n > 0$-ples of those for the entire algebra. What is fascinating that the condition that the preferred extremals are minimal surface extremals of Kähler action could solve these conditions and guarantee also NTU at the level of space-time surfaces. Supersymplectic boundary conditions at the ends of CD would however pose number theoretic conditions on the coupling parameters. In p-adic case the conditions should reduce to purely local conditions since p-adic charges are not well-defined as integrals.

3. In TGD framework one is constructing zero energy states rather calculating the matrix elements of S-matrix in terms of path integral. This gives certain liberties but a natural expectation is that functional integral as a formal tool at least is involved.

   Could one *define* the functional integral as a discrete sum of contributions of standard form for the preferred extremals, which correspond to maxima in Euclidian regions and associated stationary phase points in Minkowskian regions? Could one assume that WCW spinor field is concentrated along single maximum/stationary point.

   Quantum classical correspondence suggests that in Cartan algebra isometry charges are equal to the quantal charges for quantum states expressible in number theoretically universal manner in terms of fermionic oscillator operators or WCW gamma matrices? Even stronger condition would be that classical correlation functions are identical with quantal ones for allowed space-time surfaces in the quantum superposition. Could the reduction to a discrete sum be interpreted in terms of a finite measurement resolution?

4. In QFT Gaussian determinants produce problems because they are often poorly defined. In the recent case they could also spoil the NTU based on the exceptional properties of e. In the recent case however Gaussian determinant and metric determinant for Kähler metric cancel each other and this problem disappears. One could obtain just a sum over products of roots of e and roots of unity. Thus also Kähler structure seems to be crucial for the dream about NTU.

10.7.5 Breaking of NTU at the level of scattering amplitudes

NTU in strong sense could be broken at the level of scattering amplitudes. At space-time level the breaking does not look natural in the recent framework. Consider only p-adic mass calculations predicting that mass scale depends on p-adic prime. Also for the action strong form of NTU might fail for small p-adic primes since the value of the real part of action would be larger than than p. Should one allow this? What does one actually mean with NTU in the case of action?

Canonical identification is an important element of p-adic mass calculations and might also be needed to map p-adic variants of scattering amplitudes to their real counterparts. The breaking of NTU would take place, when the canonical real valued image of the p-adic scattering amplitude differs from the real scattering amplitude. The interpretation would be in terms of finite measurement resolution. By the finite measurement/cognitive resolution characterized by p one cannot detect the difference.

The simplest form of the canonical identification is $x = \sum_n x_n p^n \rightarrow \sum x_n p^{-n}$. Product xy and sum $x + y$ do not in general map to product and sum in canonical identification. The interpretation would be in terms of a finite measurement resolution: $(xy)_R = x_R y_R$ and $(x+y)_R = x_R + y_R$ only modulo finite measurement resolution. p-Adic scattering amplitudes are obtained by algebraic continuation from the intersection by replacing algebraic number valued parameters (such as momenta) by general p-adic numbers. The real images of these amplitudes under canonical identification are in general not identical with real scattering amplitudes the interpretation being in terms of a finite measurement resolution.

In p-adic thermodynamics NTU in the strong sense fails since thermal masses depend on p-adic mass scale. NTU can be broken by the fermionic matrix elements in the functional integral so that the real scattering amplitudes differ from the canonical images of the p-adic scattering amplitudes. For instance, the elementary particle vacuum functionals in the space of Teichmueller parameters for the partonic 2-surfaces and string world sheets should break NTU [K25].

10.7.6 NTU and the spectrum of Kähler coupling strength

During years I have made several attempts to understand coupling evolution in TGD framework. The most convincing proposal has emerged rather recently and relates the spectrum of $1/\alpha_K$ to that for the zeros of Riemann zeta [L14] and to the evolution of the electroweak U(1) couplings strength.

1. The first idea dates back to the discovery of WCW Kähler geometry defined by Kähler function defined by Kähler action (this happened around 1990) [K50]. The only free parameter of the theory is Kähler coupling strength α_K analogous to temperature parameter α_K postulated to be is analogous to critical temperature. Whether only single value or entire spectrum of of values α_K is possible, remained an open question.

   About decade ago I realized that Kähler action is *complex* receiving a real contribution from space-time regions of Euclidian signature of metric and imaginary contribution from the Minkoswkian regions. Euclidian region would give Kähler function and Minkowskian regions analog of QFT action of path integral approach defining also Morse function. Zero energy ontology (ZEO) [K123] led to the interpretation of quantum TGD as complex square root of thermodynamics so that the vacuum functional as exponent of Kähler action could be identified as a complex square root of the ordinary partition function. Kähler function would correspond to the real contribution Kähler action from Euclidian space-time regions. This led to ask whether also Kähler coupling strength might be complex: in analogy with the complexification of gauge coupling strength in theories allowing magnetic monopoles. Complex α_K could allow to explain CP breaking. I proposed that instanton term also reducing to Chern-Simons term could be behind CP breaking.

   The problem is that the dynamics in Minkowskian and Euclidian regions decouple completely and if Euclidian regions serve as space-time correlates for physical objects, there would be no exchanges of classical charges between physical objects. Should one conclude that α_K must be complex?

2. p-Adic mass calculations for 2 decades ago [K59] inspired the idea that length scale evolution is discretized so that the real version of p-adic coupling constant would have discrete set of values labelled by p-adic primes. The simple working hypothesis was that Kähler coupling strength is renormalization group (RG) invariant and only the weak and color coupling strengths depend on the p-adic length scale. The alternative ad hoc hypothesis considered was that gravitational constant is RG invariant. I made several number theoretically motivated ad hoc guesses about coupling constant evolution, in particular a guess for the formula for gravitational coupling in terms of Kähler coupling strength, action for CP_2 type vacuum extremal, p-adic length scale as dimensional quantity [L36]. Needless to say these attempts were premature and a hoc.

3. The vision about hierarchy of Planck constants $h_{eff} = n \times h$ and the connection $h_{eff} = h_{gr} = GMm/v_0$, where $v_0 < c = 1$ has dimensions of velocity [**?**] forced to consider very seriously the hypothesis that Kähler coupling strength has a spectrum of values in one-one correspondence with p-adic length scales. A separate coupling constant evolution associated with h_{eff} induced by $\alpha_K \propto 1/\hbar_{eff} \propto 1/n$ looks natural and was motivated by the idea that Nature is theoretician friendly: when the situation becomes non-perturbative, Mother Nature comes in rescue and an h_{eff} increasing phase transition makes the situation perturbative again.

   Quite recently the number theoretic interpretation of coupling constant evolution [K119] [L13] in terms of a hierarchy of algebraic extensions of rational numbers inducing those of p-adic number fields encouraged to think that $1/\alpha_K$ has spectrum labelled by primes and values of h_{eff}. Two coupling constant evolutions suggest themselves: they could be assigned to length scales and angles which are in p-adic sectors necessarily discretized and describable using only algebraic extensions involve roots of unity replacing angles with discrete phases.

4. Few years ago the relationship of TGD and GRT was finally understood [K114]. GRT space-time is obtained as an approximation as the sheets of the many-sheeted space-time of TGD are replaced with single region of space-time. The gravitational and gauge potential of sheets add together so that linear superposition corresponds to set theoretic union geometrically. This forced to consider the possibility that gauge coupling evolution takes place only at the level of the QFT approximation and α_K has only single value. This is nice but if true, one does not have much to say about the evolution of gauge coupling strengths.

5. The analogy of Riemann zeta function with the partition function of complex square root of thermodynamics suggests that the zeros of zeta have interpretation as inverses of complex temperatures $s = 1/\beta$. Also $1/\alpha_K$ is analogous to temperature. This led to a radical idea to be discussed in detail in the sequel.

Could the spectrum of $1/\alpha_K$ reduce to that for the zeros of Riemann zeta or - more plausibly - to the spectrum of poles of fermionic zeta $\zeta_F(ks) = \zeta(ks)/\zeta(2ks)$ giving for $k = 1/2$ poles as zeros of zeta and as point $s = 2$? ζ_F is motivated by the fact that fermions are the only fundamental particles in TGD and by the fact that poles of the partition function are naturally associated with quantum criticality whereas the vanishing of ζ and varying sign allow no natural physical interpretation.

The poles of $\zeta_F(s/2)$ define the spectrum of $1/\alpha_K$ and correspond to zeros of $\zeta(s)$ and to the pole of $\zeta(s/2)$ at $s = 2$. The trivial poles for $s = 2n$, $n = 1, 2, ..$ correspond naturally to the values of $1/\alpha_K$ for different values of $h_{eff} = n \times h$ with n even integer. Complex poles would correspond to ordinary QFT coupling constant evolution. The zeros of zeta in increasing order would correspond to p-adic primes in increasing order and UV limit to smallest value of poles at critical line. One can distinguish the pole $s = 2$ as extreme UV limit at which QFT approximation fails totally. CP_2 length scale indeed corresponds to GUT scale.

6. One can test this hypothesis. $1/\alpha_K$ corresponds to the electroweak U(1) coupling strength so that the identification $1/\alpha_K = 1/\alpha_{U(1)}$ makes sense. One also knows a lot about the evolutions of $1/\alpha_{U(1)}$ and of electromagnetic coupling strength $1/\alpha_{em} = 1/[cos^2(\theta_W)\alpha_{U(1)}$. What does this predict?

   It turns out that at p-adic length scale $k = 131$ ($p \simeq 2^k$ by p-adic length scale hypothesis, which now can be understood number theoretically [K119]) fine structure constant is predicted with .7 per cent accuracy if Weinberg angle is assumed to have its value at atomic scale! It is difficult to believe that this could be a mere accident because also the prediction evolution of $\alpha_{U(1)}$ is correct qualitatively. Note however that for $k = 127$ labelling electron one can reproduce fine structure constant with Weinberg angle deviating about 10 per cent from the measured value of Weinberg angle. Both models will be considered.

7. What about the evolution of weak, color and gravitational coupling strengths? Quantum criticality suggests that the evolution of these couplings strengths is universal and independent of the details of the dynamics. Since one must be able to compare various evolutions and combine them together, the only possibility seems to be that the spectra of gauge coupling strengths are given by the poles of $\zeta_F(w)$ but with argument $w = w(s)$ obtained by a global conformal transformation of upper half plane - that is Möbius transformation (see `http://tinyurl.com/gwjs85b`) with real coefficients (element of $GL(2, R)$) so that one as $\zeta_F((as + b)/(cs + d))$. Rather general arguments force it to be and element of $GL(2, Q)$, $GL(2, Z)$ or maybe even $SL(2, Z)$ ($ad - bc = 1$) satisfying additional constraints. Since TGD predicts several scaled variants of weak and color interactions, these copies could be perhaps parameterized by some elements of $SL(2, Z)$ and by a scaling factor K.

   Could one understand the general qualitative features of color and weak coupling contant evolutions from the properties of corresponding Möbius transformation? At the critical line there can be no poles or zeros but could asymptotic freedom be assigned with a pole of $cs + d$ and color confinement with the zero of $as + b$ at real axes? Pole makes sense only if Kähler action for the preferred extremal vanishes. Vanishing can occur and does so for massless extremals characterizing conformally invariant phase. For zero of $as + b$ vacuum function would be equal to one unless Kähler action is allowed to be infinite: does this make sense?. One can however hope that the values of parameters allow to distinguish between weak and color interactions. It is certainly possible to get an idea about the values of the parameters of the transformation and one ends up with a general model predicting the entire electroweak coupling constant evolution successfully.

To sum up, the big idea is the identification of the spectra of coupling constant strengths as poles of $\zeta_F((as + b/)(cs + d))$ identified as a complex square root of partition function with motivation coming from ZEO, quantum criticality, and super-conformal symmetry; the discretization of the RG flow made possible by the p-adic length scale hypothesis $p \simeq k^k$, k prime; and the assignment of complex zeros of ζ with p-adic primes in increasing order. These assumptions reduce the coupling constant evolution to four real rational or integer valued parameters (a, b, c, d). In the sequel this vision is discussed in more detail.

10.7.7 Generalization of Riemann zeta to Dedekind zeta and adelic physics

10.7.8 Generalization of Riemann zeta to Dedekind zeta and adelic physics

A further insight to adelic physics comes from the possible physical interpretation of the L-functions appearing also in Langlands program [K53]. The most important L-function would be generalization of Riemann zeta to extension of rationals. I have proposed several roles for ζ, which would be the simplest L-function assignable to rational primes, and for its zeros.

1. Riemann zeta itself could be identifiable as an analog of partition function for a system with energies given by logarithms of prime. One can define also the fermionic counterpart of ζ as ζ_F. In ZEO this function could be regarded as complex square root of thermodynamical partition function in accordance with the interpretation of quantum theory as complex square root of thermodynamics.
2. The zeros of zeta could define the conformal weights for the generators of super-symplectic algebra so that the number of generators would be infinite. The rough idea - certainly not correct as such except at the limit of infinitely large CD - is that the scaling operator $L_0 = r_M d/dr_M$, where r_M is light-like coordinate of light-cone boundary (containing upper or lower boundary of the causal diamond (CD)), has as eigenfunctions the functions $(r_M/r_0)^{s_n}$ $s_n = 1/2 + iy_n$, where s_n is the radial conformal weight identified as complex zero of ζ. Periodic boundary conditions for CD do not allow all possible zeros as conformal weights so that for given CD only finite subset corresponds to generators of the supersymplectic algebra. Conformal confinement would hold true in the sense that the sum $\sum_n s_n$ for physical states would be integer. Roots and their conjugates should appear as pairs in physical states.
3. On basis of numerical evidence Dyson [A64] (`http://tinyurl.com/hjbfsuv`) has conjectured that the Fourier transform for the set formed by zeros of zeta consists of primes so that one could regard zeros as one-dimensional quasi-crystal. This hypothesis makes sense if the zeros of zeta decompose into disjoint sets such that each set corresponds to its own prime (and its powers) and one has $p^{iy} = U_{m/n} = exp(i2\pi m/n)$ (see the appendix of [L13]). This hypothesis is also motivated by number theoretical universality [K119, L30].
4. I have considered the possibility [L14] that the discrete values for the inverse of the electroweak $U(1)$ coupling constant for a gauge field assignable to the Kähler form of CP_2 assignable to p-adic coupling constant evolution corresponds to poles of the fermionic zeta $\zeta_F(s) = \zeta(s)/\zeta(2s)$ coming from $s_n/2$ (denominator) and pole at $s = 1$ (numerator) zeros of zeta assignable to rational primes. Note that also odd negative integers at real axis would be poles.

   It is also possible to consider scaling of the argument of $\zeta_F(s)$. More general coupling constant evolutions could correspond to $\zeta_F(m(s))$, where $m(s) = (as + b)/(cs + d)$ is Möbius transformation performed for the argument mapping upper complex plane to itself so that a, b, c, d are real and also rational by number theoretical universality.

Suppose for a moment that more precise formulations of these physics inspired conjectures make sense and even that their generalization for extensions K/Q of rationals holds true. This would solve a big part of adelic physics! Not surprisingly, the generalization of zeta function was proposed already by Dedekind (see `http://tinyurl.com/yarwbo6h`).

1. The definition of Dedekind zeta function ζ_K relies on the product representation and analytic continuation allows to deduce ζ_K elsewhere. One has a product over prime ideals of K/Q of rationals with the factors $1/(1 - p^{-s})$ associated with the ordinary primes in Riemann zeta replaced with the factors $X(P) = 1/(1 - N_{K/Q}(P)^{-s})$, where P is prime for the integers $O(K)$ of extension and $N_{K/Q}(P)$ is the norm of P in the extension. In the region $s > 1$ where the product converges, ζ_K is non-vanishing and $s = 1$ is a pole of ζ_K. The functional identifies of ζ hold true for ζ_K as well. Riemann hypothesis is generalized for ζ_K.
2. It is possible to understand ζ_K in terms of a physical picture. By the results of `http://tinyurl.com/yckfjgpk` one has $N_{K/Q}(P) = p^r$, $r > 0$ integer. This implies that one can

arrange in ζ_K all primes P for which the norm is power or given p in the same group. The prime ideals p of ordinary integers decompose to products of prime ideals P of the extension: one has $p = \prod_{r=1}^{g} P_r^{e_r}$, where e_r is so called ramification index. One can say that each factor of ζ decomposes to a product of factors associated with corresponding primes P with norm power of p. In the language of physics, the particle state represented by p decomposes in improved resolution to a product of many-particle states consisting of e_r particles in state P_r, very much like hadron decomposes to quarks.

The norms of $N_{K/Q}(P_r) = p^{d_r}$ satisfy the condition $\sum_{r=1}^{g} d_r e_r = n$. Mathematician would say that the prime ideals of Q modulo p decompose in n-dimensional extension K to products of prime power ideals $P_r^{e_r}$ and that P_r corresponds to a finite field $G(p, d_r)$ with algebraic dimension d_r. The formula $\sum_{r=1}^{g} d_r e_r = n$ reflects the fact the dimension n of extension is same independent of p even when one has $g < n$ and ramification occurs.

Physicist would say that the number of degrees of freedom is n and is preserved although one has only $g < n$ different particle types with e_r particles having d_r internal degrees of freedom. The factor replacing $1/(1 - p^{-s})$ for the general prime p is given by $\prod_{r=1}^{g} 1/(1 - p^{-e_r d_r s})$.

3. There are only finite number of ramified primes p having $e_r > 1$ for some r and they correspond to primes dividing the so called discriminant D of the irreducible polynomial P defining the extension. $D \ mod \ p$ obviously vanishes if D is divisible by p. For second order polynomials $P = x^2 + bx + c$ equals to the familiar $D = b^2 - 4c$ and in this case the two roots indeed co-incide. For quadratic extensions with $D = b^2 - 4c > 0$ the ramified primes divide D.

   Remark: Resultant $R(P, Q)$ and discriminant $D(P) = R(P, dP/dx)$ are elegant tools used by number theorists to study extensions of rationals defined by irreducible polynomials (see `http://tinyurl.com/oyumsnk` and `http://tinyurl.com/p67rdgb`). From Wikipedia articles one finds an elegant proof for the facts that $R(P, Q)$ is proportional to the product of differences of the roots of P and Q, and D to the product of squares for the differences of distinct roots. $R(P, Q) = 0$ tells that two polynomials have a common root. $D \ mod \ p = 0$ tells that polynomial and its derivative have a common root so that there is a degenerate root modulo p and the prime is indeed ramified. For modulo p reduction of P the vanishing of $D(P) \ mod \ p$ follows if D is divisible by p. There exists clearly only a finite number of primes of this kind.

   Most primes are unramified and one has maximum number n of factors in the decomposition and $e_r = 1$: maximum splitting of p occurs. The factor $1/(1 - p^{-s})$ is replaced with its n:th power $1/(1 - p^{-s})^n$. The geometric interpretation is that space-time sheet is replaced with n-fold covering and each sheet gives one factor in the power. It is also possible to have a situation in which no splitting occurs and one as $e_r = 1$ for one prime $P_r = p$. The factor is in this case equal to $1/(1 - p^{-ns})$.

From Wikipedia (see `http://tinyurl.com/yckfjgpk`) one learns that for Galois extensions L/K the ratio ζ_L/ζ_K is so called Artin L-function of the regular representation (group algebra) of Galois group factorizing in terms of irreps of $Gal(L/K)$ is *holomorphic* (no poles!) so that ζ_L must have also the zeros of ζ_K. This holds in the special case $K = Q$. Therefore extension of rationals can only bring new zeros but no new poles!

1. This result is quite far reaching if one accepts the hypothesis about super-symplectic conformal weights as zeros of ζ_K and the conjecture about coupling constant evolution. In the case of $\zeta_{F,K}$ this means new poles meaning new conformal weights due to increased complexity and a modification of the conjecture for the coupling constant evolution due to new primes in extension. The outcome looks physically sensible.

2. Quadratic field $Q(\sqrt{m})$ serves as example. Quite generally, the factorization of rational primes to the primes of extension corresponds to the factorization of the minimal polynomial for the generating element θ for the integers of extension and one has $p = P_i^{e_i}$, where e_i is ramification index. The norm of p factorizes to the produce of norms of $P_i^{e_i}$.

   Rational prime can either remain prime in which case $x^2 - m$ does not factorize mod p, split when $x^2 - m$ factorizes mod P, or ramify when it divides the discriminant of $x^2 - m = 4m$. From this it is clear that for unramfied primes the factors in ζ are replaced by either $1/(1-p^{-s})^2$ or $1/(1-p^{-2s}) = 1/(1-p^{-s})(1+p^{-s})$. For a finite number of unramified primes one can have something different.

For Gaussian primes with $m = -1$ one has $e_r = 1$ for $p\ mod\ 4 = 3$ and $e_r = 2$ for $p =\ mod\ 4 = 1$. z_K therefore decomposes into two factors corresponding to primes $p\ mod\ 4 = 3$ and $p\ mod\ 4 = 3$. One can extract out Riemann zeta and the remaining factor

$$\prod_{p\ mod\ 4=3} \frac{1}{(1-p^{-s})} \times \prod_{p\ mod\ 4=1} \frac{1}{(1+p^{-s})}$$

should be holomorphic and without poles but having possibly additional zeros at critical line. That ζ_K should possess also the poles of ζ as poles looks therefore highly non-trivial.

10.7.9 Other applications of NTU

NTU in the strongest form says that all numbers involved at "basic level" (whatever this means!) of adelic TGD are products of roots of unity and of power of a root of e. This is extremely powerful physics inspired conjecture with a wide range of possible mathematical applications.

1. For instance, vacuum functional defined as an exponent of action for preferred externals would be number of this kind. One could define functional integral as adelic operation in all number fields: essentially as sum of exponents of action for stationary preferred extremals since Gaussian and metric determinants potentially spoiling NTU would cancel each other leaving only the exponent.
2. The implications of NTU for the zeros of Riemann zeta [L13] will be discussed in more detail in the Appendix. Suffice it to say that the observations about Fourier transform for the distribution of loci of non-trivial zeros of zeta together with NTU leads to explicit proposal for the algebraic for of zeros of zeta. The testable proposal is that zeros decompose to disjoint classes $C(p)$ labelled by primes p and the condition that p^{iy} is root of unity in given class $C(p)$.
3. NTU generalises to all Lie groups. Exponents $exp(in_iJ_i/n)$ of lie-algebra generators define generalisations of number theoretically universal group elements and generate a discrete subgroup of compact Lie group. Also hyperbolic "phases" based on the roots $e^{m/n}$ are possible and make possible discretized NTU versions of all Lie-groups expected to play a key role in adelization of TGD.

   NTU generalises also to quaternions and octonions and allows to define them as number theoretically universal entities. Note that ordinary p-adic variants of quaternions and octonions do not give rise to a number field: inverse of quaternion can have vanishing p-adic variant of norm squared satisfying $\sum_n x_n^2 = 0$.

   NTU allows to define also the notion of Hilbert space as an adelic notion. The exponents of angles characterising unit vector of Hilbert space would correspond to roots of unity.

10.7.10 Going to the roots of p-adicity

The basic questions raised by the p-adic mass calculations concern the origin of preferred p-adic primes and of p-adic length scale hypothesis. One can also ask whether there might be a natural origin for p-adicity at the level of WCW.

Preferred primes as ramified primes for extensions of rationals?

Preferred primes as ramified primes for extensions of rationals?

The intuitive feeling is that the notion of preferred prime is something extremely deep and to me the deepest thing I know is number theory. Does one end up with preferred primes in number theory? This question brought to my mind the notion of *ramification of primes* (`http://tinyurl.com/hddljlf`) (more precisely, of prime ideals of number field in its extension), which happens only for special primes in a given extension of number field, say rationals. Ramification is completely analogous to the degeneracy of some roots of polynomial and corresponds to criticality if the polynomial corresponds to criticality (catastrophe theory of Thom is one application). Could this be the mechanism assigning preferred prime(s) to a given elementary system, such as elementary particle? I have not considered their role earlier also their hierarchy is highly relevant in the number theoretical vision about TGD.

1. Stating it very roughly (I hope that mathematicians tolerate this sloppy language of physicist): as one goes from number field K, say rationals Q, to its algebraic extension L, the original prime ideals in the so called *integral closure* (http://tinyurl.com/js6fpvr) over integers of K decompose to products of prime ideals of L (prime ideal is a more rigorous manner to express primeness). Note that the general ideal is analog of integer.

   Integral closure for integers of number field K is defined as the set of elements of K, which are roots of some monic polynomial with coefficients, which are integers of K having the form $x^n + a_{n-1}x^{n-1} + ... + a_0$. The integral closures of both K and L are considered. For instance, integral closure of algebraic extension of K over K is the extension itself. The integral closure of complex numbers over ordinary integers is the set of algebraic numbers.

   Prime ideals of K can be decomposed to products of prime ideals of L: $P = \prod P_i^{e_i}$, where e_i is the ramification index. If $e_i > 1$ is true for some i, *ramification* occurs. P_i:s in question are like co-inciding roots of polynomial, which for in thermodynamics and Thom's catastrophe theory corresponds to criticality. Ramification could therefore be a natural aspect of quantum criticality and ramified primes P are good candidates for preferred primes for a given extension of rationals. Note that the ramification make sense also for extensions of given extension of rationals.

2. A physical analogy for the decomposition of ideals to ideals of extension is provided by decomposition of hadrons to valence quarks. Elementary particles becomes composite of more elementary particles in the extension. The decomposition to these more elementary primes is of form $P = \prod P_i^{e(i)}$, the physical analog would be the number of elementary particles of type i in the state (http://tinyurl.com/h9528pl). Unramified prime P would be analogous a state with e fermions. Maximally ramified prime would be analogous to Bose-Einstein condensate of e bosons. General ramified prime would be analogous to an e-particle state containing both fermions and condensed bosons. This is of course just a formal analogy.

3. There are two further basic notions related to ramification and characterizing it. *Relative discriminant* is the ideal divided by all ramified ideals in K (integer of K having no ramified prime factors) and relative different for P is the ideal of L divided by all ramified P_i:s (product of prime factors of P in L). These ideals represent the analogs of product of preferred primes P of K and primes P_i of L dividing them. These two integers ideals would characterize the ramification.

In TGD framework the extensions of rationals (http://tinyurl.com/h9528pl) and p-adic number fields (http://tinyurl.com/zq22tvb) are unavoidable and interpreted as an evolutionary hierarchy physically and cosmological evolution would gradually proceed to more and more complex extensions. One can say that string world sheets and partonic 2-surfaces with parameters of defining functions in increasingly complex extensions of prime emerge during evolution. Therefore ramifications and the preferred primes defined by them are unavoidable. For p-adic number fields the number of extensions is much smaller for instance for $p > 2$ there are only 3 quadratic extensions.

How could ramification relate to p-adic and adelic physics and could it explain preferred primes?

1. Ramified p-adic prime $P = P_i^e$ would be replaced with its e:th root P_i in p-adicization. Same would apply to general ramified primes. Each un-ramified prime of K is replaced with $e = K : L$ primes of L and ramified primes P with $\#\{P_i\} < e$ primes of L: the increase of algebraic dimension is smaller. An interesting question relates to p-adic length scale. What happens to p-adic length scales. Is p-adic prime effectively replaced with e:th root of p-adic prime: $L_p \propto p^{1/2}L_1 \to p^{1/2e}L_1$? The only physical option is that the p-adic temperature for P would be scaled down $T_p = 1/n \to 1/ne$ for its e:th root (for fermions serving as fundamental particles in TGD one actually has $T_p = 1$). Could the lower temperature state be more stable and select the preferred primes as maximimally ramified ones? What about general ramified primes?

2. This need not be the whole story. Some algebraic extensions would be more favored than others and p-adic view about realizable imaginations could be involved. p-Adic pseudo constants are expected to allow p-adic continuations of string world sheets and partonic 2-surfaces to 4-D preferred extremals with number theoretic discretization. For real continuations the situation is more difficult. For preferred extensions - and therefore for corresponding ramified primes - the number of real continuations - realizable imaginations - would be especially large.

   The challenge would be to understand why primes near powers of 2 and possibly also of other small primes would be favored. Why for them the number of realizable imaginations would be especially large so that they would be winners in number theoretical fight for survival?

Can one make this picture more concrete? What kind of algebraic extensions could be considered?

1. In p-adic context a proper definition of counterparts of angle variables as phases allowing definition of the analogs of trigonometric functions requires the introduction of algebraic extension giving rise to some roots of unity. Their number depends on the angular resolution. These roots allow to define the counterparts of ordinary trigonometric functions - the naïve generalization based on Taylors series is not periodic - and also allows to defined the counterpart of definite integral in these degrees of freedom as discrete Fourier analysis. For the simplest algebraic extensions defined by x^n-1 for which Galois group is abelian are are unramified so that something else is needed. One has decomposition $P=\prod P_i^{e(i)}$, $e(i)=1$, analogous to n-fermion state so that simplest cyclic extension does not give rise to a ramification and there are no preferred primes.

2. What kind of polynomials could define preferred algebraic extensions of rationals? Irreducible polynomials are certainly an attractive candidate since any polynomial reduces to a product of them. One can say that they define the elementary particles of number theory. Irreducible polynomials have integer coefficients having the property that they do not decompose to products of polynomials with rational coefficients. It would be wrong to say that only these algebraic extensions can appear but there is a temptation to say that one can reduce the study of extensions to their study. One can even consider the possibility that string world sheets associated with products of irreducible polynomials are unstable against decay to those characterize irreducible polynomials.

3. What can one say about irreducible polynomials? Eisenstein criterion (`http://tinyurl.com/47kxjz` states following. If $Q(x)=\sum_{k=0,..,n} a_k x^k$ is n:th order polynomial with integer coefficients and with the property that there exists at least one prime dividing all coefficients a_i except a_n and that p^2 does not divide a_0, then Q is irreducible. Thus one can assign one or more preferred primes to the algebraic extension defined by an irreducible polynomial Q of this kind - in fact any polynomial allowing ramification. There are also other kinds of irreducible polynomials since Eisenstein's condition is only sufficient but not necessary.

   Furthermore, in the algebraic extension defined by Q, the prime ideals P having the above mentioned characteristic property decompose to an n:th power of single prime ideal P_i: $P=P_i^n$. The primes are maximally/completely ramified.

   A good illustration is provided by equations $x^2+1=0$ allowing roots $x_\pm=\pm i$ and equation $x^2+2px+p=0$ allowing roots $x_\pm=-p\pm\sqrt{p}p-1$. In the first case the ideals associated with $\pm i$ are different. In the second case these ideals are one and the same since $x_+ == -x_- + p$: hence one indeed has ramification. Note that the first example represents also an example of irreducible polynomial, which does not satisfy Eisenstein criterion. In more general case the n conditions on defined by symmetric functions of roots imply that the ideals are one and same when Eisenstein conditions are satisfied.

4. What is so nice that one could readily construct polynomials giving rise to given preferred primes. The complex roots of these polymials could correspond to the points of partonic 2-surfaces carrying fermions and defining the ends of boundaries of string world sheet. It must be however emphasized that the form of the polynomial depends on the choices of the complex coordinate. For instance, the shift $x\rightarrow x+1$ transforms $(x^n-1)/(x-1)$ to a polynomial satisfying the Eisenstein criterion. One should be able to fix allowed coordinate changes in such a way that the extension remains irreducible for all allowed coordinate changes.

Already the integral shift of the complex coordinate affects the situation. It would seem that only the action of the allowed coordinate changes must reduce to the action of Galois group permuting the roots of polynomials. A natural assumption is that the complex coordinate corresponds to a complex coordinate transforming linearly under subgroup of isometries of the embedding space.

In the general situation one has $P = \prod P_i^{e(i)}$, $e(i) \geq 1$ so that aso now there are prefered primes so that the appearance of preferred primes is completely general phenomenon.

The origin of p-adic length scale hypothesis?

p-Adic length scale hypothesis emerged from p-adic length scale hypothesis. A possible generalization of this hypothesis is that p-adic primes near powers of prime are physically favored. There indeed exists evidence for the realization of 3-adic time scale hierarchies in living matter [I8] (`http://tinyurl.com/jbh9m27`) and in music both 2-adicity and 3-adicity could be present: this is discussed in TGD inspired theory of music harmony and genetic code [K87]. See also [L27, L20].

One explanation would be that for preferred primes the number of p-adic space-time sheets representable also as real space-time sheets is maximal. Imagined worlds would be maximally realizable. Preferred p-adic primes would correspond to ramified primes for extensions with the property that the number of realizable imaginations is especially large for them. Why primes satisfying p-adic length scale hypothesis or its generalization would appear as ramified primes for extensions, which are winners in number theoretical evolution?

Also the weak form of NMP (WNMP) applying also to the purely number theoretic form of NMP [K65] might come in rescue here.

1. Entanglement negentropy for a NE [K65] characterized by n-dimensional projection operator is the $log(N_p(n)$ for some p whose power divides n. The maximum negentropy is obtained if the power of p is the largest power of prime divisor of p, and this can be taken as definition of number theoretical entanglement negentropy (NEN). If the largest divisor is p^k, one has $N = k \times log(p)$. The entanglement negentropy per entangled state is $N/n = klog(p)/n$ and is maximal for $n = p^k$. Hence powers of prime are favoured, which means that p-adic length scale hierarchies with scales coming as powers of p are negentropically favored and should be generated by NMP. Note that $n = p^k$ would define a hierarchy of $h_{eff}/h = p^k$. During the first years of h_{eff} hypothesis I believe that the preferred values obey $h_{eff} = r^k$, r integer not far from $r = 2^{11}$. It seems that this belief was not totally wrong.

2. If one accepts this argument, the remaining challenge is to explain why primes near powers of two (or more generally p) are favoured. $n = 2^k$ gives large entanglement negentropy for the final state. Why primes $p = n_2 = 2^k - r$ would be favored? The reason could be following. $n = 2^k$ corresponds to $p = 2$, which corresponds to the lowest level in p-adic evolution since it is the simplest p-adic topology and farthest from the real topology and therefore gives the poorest cognitive representation of real PE as p-adic PE (Note that $p = 1$ makes formally sense but for it the topology is discrete).

3. WNMP [K65, K116] suggests a more feasible explanation. The density matrix of the state to be reduced is a direct sum over contributions proportional to projection operators. Suppose that the projection operator with largest dimension has dimension n. Strong form of NMP would say that final state is characterized by n-dimensional projection operator. WNMP allows "free will" so that all dimensions $n - k$, $k = 0, 1, ...n - 1$ for final state projection operator are possible. 1-dimensional case corresponds to vanishing entanglement negentropy and ordinary state function reduction isolating the measured system from external world.

4. The negentropy of the final state per state depends on the value of k. It is maximal if $n-k$ is power of prime. For $n = 2^k = M_k + 1$, where M_k is Mersenne prime $n-1$ gives the maximum negentropy and also maximal p-adic prime available so that this reduction is favoured by NMP. Mersenne primes would be indeed special. Also the primes $n = 2^k - r$ near 2^k produce large entanglement negentropy and would be favored by NMP.

5. This argument suggests a generalization of p-adic length scale hypothesis so that $p = 2$ can be replaced by any prime.

10.8 What could be the role of complexity theory in TGD?

I have many times wondered what could be the role of chaos theory or better in TGD. In fact, I would prefer to talk about complexity theory since the chaos in the sense as it is used is only apparent and very different from thermodynamical chaos.

Wikipedia article (see `http://tinyurl.com/qexmowa`) gives a nice summary about the history of chaos theory and I repeat only some main points here. Chaos theory has roots already at the end of 18the century by the works of Poincare (non-periodic orbits in 3-body system) and Hadamard (free particle gliding frictionlessly on surface of constant negative curvature, "Hadamard billiard". In this case all trajectories are unstable diverging exponentially from each other: this is characterized by positive Lyapunov exponent.

Chaos theory got is start from ergodic theory (see `http://tinyurl.com/pfcrz4c`) studying dynamical systems with the original motivation coming from statistical physics. For instance, spin glasses are a representative example of non-ergodic system in which the trajectory of point does not go arbitrary near to every point. The study of non-linear differential equations George David Birkhoff, Andrey Nikolaevich Kolmogorov, Mary Lucy Cartwright and John Edensor Littlewood, and Stephen Smale provides was purely mathematical study of chaotic systems. Smale discovered strange attractor at which periodic orbits form a dense set. Chaos theory was formalized around 1950. At this time it was also discovered that finite-D linear systems do not allow chaos.

The emergence of computers meant breakthrough. Much of chaos theory involves repeated iteration of simple mathematical formulas. Edward Lorentz was a pioneer of chaos theory working with weather prediction and accidentally discovered initial value sensitivity. Benard Mandelbrot discovered fractality and Mitchell Feigenbaum the universality of chaos for iteration of functions of real variable.

Chaotic systems are as far from integrable systems as one could imagine: all orbits are cycles in integrable Hamiltonian dynamics. There are good reasons to suspect that TGD Universe is completely integrable classically. Chaos theory however describes also the emergence of complexity through phase transition like steps - period n-tupling and most importantly by period doubling for iteration of maps.

Chaotic (or actually extremely complex and only apparently chaotic) systems seem to be the diametrical opposite of completely integrable systems about which TGD is a possible example. There is however also something common: in completely integrable classical systems all orbits are cyclic and in chaotic systems they form a dense set in the space of orbits. Furthermore, in chaotic systems the approach to chaos occurs via steps as a control parameter is changed. Same would take place in adelic TGD fusing the descriptions of matter and cognition.

In TGD Universe the hierarchy of extensions of rationals inducing finite-dimensional extension of p-adic number fields defines a hierarchy of adelic physics and provides a natural correlate for evolution. Galois groups and ramified primes appear as characterizers of the extensions. The sequences of Galois groups could characterize an evolution by phase transitions increasing the dimension of the extension associated with the coordinates of "world of classical worlds" (WCW) in turn inducing the extension used at space-time and Hilbert space level. WCW decomposes to sectors characterized by Galois groups G_3 of extensions associated with the 3-surfaces at the ends of space-time surface at boundaries of causal diamond (CD) and G_4 characterizing the space-time surface itself. G_3 (G_4) acts on the discretization and induces a covering structure of the 3-surface (space-time surface). If the state function reduction to the opposite boundary of CD involves localization into a sector with fixed G_3, evolution is indeed mapped to a sequence of G_3s.

Also the cognitive representation defined by the intersection of real and p-adic surfaces with coordinates of points in an extension of rationals evolve. The number of points in this representation becomes increasingly complex during evolution. Fermions at partonic 2-surfaces connected by fermionic strings define a tensor network, which also evolves since the number of fermions can change.

The points of space-time surface invariant under non-trivial subgroup of Galois group define singularities of the covering, and the positions of fermions at partonic surfaces could correspond to these singularities - maybe even the maximal ones, in which case the singular points would be rational. There is a temptation to interpret the p-adic prime characterizing elementary particle as a ramified prime of extension having a decomposition similar to that of singularity so that category theoretic view suggests itself.

One also ends up to ask how the number theoretic evolution could select preferred p-adic primes satisfying the p-adic length scale hypothesis as a survivors in number theoretic evolution, and ends up to a vision bringing strongly in mind the notion of conserved genes as analogy for conservation of ramified primes in extensions of extension. $h_{eff}/h = n$ has natural interpretation as divisor of the order of Galois group of extension. The generalization of $\hbar_{gr} = GMm/v_0 = \hbar_{eff}$ hypothesis to other interactions is discussed in terms of number theoretic evolution as increase of G_3, and one ends up to surprisingly concrete vision for what might happen in the transition from prokaryotes to eukaryotes.

10.8.1 Basic notions of chaos theory

It is good to begin by summarizing the basic concepts of chaos theory. Again Wikipedia article (see `http://tinyurl.com/qexmowa`) gives a more detailed representation and references. Citing Wikipedia freely: Within the apparent randomness of chaotic complex systems there are patterns, constant feedback loops, repetition, self-similarity, fractals, self-organization and there is sensitivity to initial conditions (butterfly effect) implying the loss of predictability although chaotic systems as such are deterministic.

Basic prerequisites for chaotic dynamics

Wikipedia article lists three basic conditions for chaotic dynamics. Dynamics must **a)** be sensitive to initial conditions, **b)** allow topological mixing, **c)** have dense set of periodic orbits.

1. Sensitivity to initial conditions.

   Mathematical formulation for the sensitivity to initial conditions can be formulated by perturbation theory for differential equations. The rate of separation of images of points initially near to each other increases exponentially as $exp(\lambda t)$ in initial value sensitive situation and the approximation fails soon. Lyapunov exponent λ characterizes the time evolution of the difference. In multi-dimensional case there are several Lyapunov exponents but the largest one is often enough to characterize the situation.

2. Topological mixing (transitivity).

   This notion corresponds to everyday intuition about mixing. For instance, the flow defined by a vector field mixes the marker completely with the fluid. Iteration of simple scaling operation is initial value sensitive but does not cause topological mixing. In 1-D case all points larger than one approach to infinity and smaller than 1 to zero so that the behavior is extremely simple.

3. Dense set of periodic orbits.

   Periodic orbits should form a dense set in the space of orbits: every point of space is approached arbitrarily closely by a periodic orbit. In completely integrable system all orbits would be periodic orbits so that the difference of these systems is very delicate and one can wonder whether the conditions a) and b) follow from this delicate difference. One can also ask whether there might be a deep connection between completely integrable and chaotic systems.

   Sharkovkii's theorem states that any 1-D system with dynamics determined by iteration of a continuous function of real argument exhibits a regular cycle of period 3 exhibits all other cycles. This theorem can be generalized further (see `http://tinyurl.com/l7q3rah`). Introduce Sharkovskii ordering of integers as union of sets consisting of odd integers multiplied by powers of 2. The generalization of the theorem states that if n is a peri89od and precedes k in Sharkovskii ordering then k is prime period (it is not a multiple of smaller period).

   The theorem holds true for reals but not for periodic functions at circle which are encountered for iterations defined by powers of cyclic group elements. The discrete subgroup of hyperbolic subgroups of Lie groups do not have not cycles at all.

Strange attractors and Julia sets

Logistic map $x \rightarrow kx(1-x)$ is chaotic everywhere but many systems are chaotic only in a subset of phase space. An interesting situation arises when the chaotic behavior takes place at attractor, since all initial positions in the basic of the attractor lead to the attractor and to a chaotic behavior. Lorentz attractor is a well-known example of strange attractor (see Wikipedia article for illustration). It contains dense sets of both periodic and aperiodic orbits.

Julia set (see `http://tinyurl.com/l8jl5ne`) is the boundary of the basin of attraction in chaotic systems defined by iteration of a rational function of complex argument mapping complex plane to itself. Both Julia sets and strange attractors have a fractal structure.

Strange attractors can appear only in spaces with dimension $D \geq 3$. Poincare-Bendixon theorem states that 2-D differential equations on Euclidian plane have very regular behavior. In non-Euclidian geometry situation changes and the hyperbolic character of the geometry implying initial value sensitivity of geodesic motion is the reason for this. Also infinite-D linear systems can exhibit chaotic behavior.

10.8.2 How to assign chaos/complexity theory with TGD?

Completely integrable systems can be seen as a diametric opposite of chaotic systems. If classical TGD indeed represents a completely integrable system meaning that space-time surfaces as preferred extremals can be constructed explicitly, one might think that chaos theory need not have much to do with classical TGD. Chaos is however the end product of transitions making the system more complex, and it might well be that the understanding about the emergence of complexity in chaotic systems could help to develop the vision about emergence of complexity in TGD. Note also that periodic orbit are dense in chaotic systems so that diametrical opposites might actually meet.

The most relevant TGD based ingredients used in the sequel are following: WCW [K93]; strong form of holography (SH) [K121], quantum classical correspondence (QCC), zero energy ontology (ZEO) [K69], dark matter as hierarchy of phases with effective Planck constant $h_{eff}/h = n$ [K38, **?**, K79], p-Adic physics as physics of cognition [K73, K65, K83] [L34], adelic physics [L34] fusing the physics of matter and cognition by integrating reals and extensions of various p-adic number fields induced by an extension of rationals to a larger structure, and the notions of adelic manifold and associated cognitive representation [L25] , Negentropy Maximization Principle (NMP) [K65] satisfied automatically in statistical sense in adelic physics [L34].

Complexity in TGD

Complexity is often taken to mean computational complexity for classical computations. Complexity as it is understood in the sequel relates very closely cognition. Too complex looks chaotic since our cognitive abilities do not allow to discern too complex patterns. Hence complexity theory should characterize cognitive representations whatever they are.

Number theoretic vision about TGD serves as the guideline here.

1. In adelic TGD [K54] cognitive representations correspond to the intersections of real space-time surfaces and their p-adic variants obeying same field equations and representing correlates for cognition. In these intersections the coordinates of points belong to an extension of rationals defining adele [L25].

   One ends up with a generalization of the notion of manifold to adelic manifold. Intersection defines a common discrete spine consisting of points with coordinates in the extension of rationals defining the adele. These poins are shared by the real and p-adic variants of the adelic manifold. I have called this manifold also monadic manifold since there is strong resemblance with the ideas of Leibniz. In real sector this manifold differs from ordinary manifold in that the open sets are labelled by a discrete set of points in the intersection.

   In TGD framework it is essential that the spine of the space-time surface consists of points of embedding space for which it is convenient to use preferred coordinates.

2. Complexity corresponds roughly to the dimension of extension of rationals defining the adeles. p-Adic differential equations are non-deterministic due to the existence of p-adic pseudo constants depending on finite number of p-adic digits of the p-adic number. This non-determinism is identified as a correlate for imagination. p-Adic variants of space-time surfaces are not uniquely determined this means finite cognitive resolution.

By SH [K118] the data associated with string world sheets, partonic 2-surfaces, and discretization allow to construct space-time surfaces as preferred extremals of the action principle defining classical TGD and to find the Kähler function for WCW geometry. It is quite well possible that the data allowing to construct p-adic space-time surfaces does not allow continuation to a preferred extremal: all imaginations are not realizable!

The algebraic dimension of the extension could be relevant for the ability of mathematical cognition to imagine spaces with dimension higher than that for the real 3-space. Besides the extensions of p-adics induced by algebraic extensions of rationals also those induced by some root of e are algebraically finite-dimensional. One can imagine also other extensions involving transcendentals in real sense but it is not clear whether there are finite dimensional extensions among them. The finiteness of cognition suggests that only these extensions can be allowed. All imaginations are not realizable!

3. Extension is characterized partially by Galois group (see `http://tinyurl.com/mrvqhz2`) acting as automorphisms meaning that Galois group permutes the roots of the n:th order polynomials defining extensions of rationals via their non-rational roots. So called ramified primes (see `http://tinyurl.com/m32nvcz` and `http://tinyurl.com/oh7tgsw`) provide additional characteristics.

   Iteration cycles appearing in complexity theory for iteration of functions and repeated action of an element Galois group defining a finite Abelian group are mathematically similar notions. Now only cycles are present whereas chaotic systems have aperiodic orbits. The cyclic subgroups of Galois group do not seem to have an natural realization as iterative dynamics except in quantum sense meaning that cyclic orbits are replaced with wave functions labelled by number theoretic integer valued "momenta" for the action of the analog of Cartan subgroup as maximal commutative subgroup for the Galois group. The maximal Abelian Galois group is analog of Cartan subgroup for Galois group of algebraic numbers and states are in its irreducible representations.

   Remark: What is interesting that for polynomials with order larger than 4, one cannot write closed analytic expressions for the roots of the polynomials. This obviously means a fundamental limitation on symbolic cognitive representations provided by explicit formulas. The realization of was a huge step in the evolution of mathematics. Could also the emergence of Galois groups with order larger at space-time level than 5 have meant cognitive revolution - probably at much lower level in the hierarchy? Could this relate also to the fact that space-time dimension is $D = 4$ and thus imaginable using 4-D algebraic extension of rationals?

   A possible measure for the cognitive complexity is the dimension of the Galois group of the extension. One can speak also about the complexity of the Galois group itself - the non-Abelianity of Galois group brings in additional complexity. The number of generators and number of relations between them serve as a measure for complexity of Galois group.

   Extension of rationals is also characterized by so called ramified primes and should have a profound physical meaning. p-Adic length scale hypothesis states that physically preferred primes are near powers of 2 and perhaps also other small primes. Could they correspond to ramified primes. Why just these ramified primes would be survivors in the number theoretic evolution, is the fascinating question to be addressed later.

4. The increase of the dimension of extension or complexity of its Galois group corresponds naturally to evolution interpreted as emergence of algebraic complexity and evolutionary paths could be seen as sequences of inclusions for Galois groups. Chaos would correspond to the limit when the extension of rationals approaches to infinite sub-field of algebraic numbers - say maximal Abelian extension of rationals - so that the number of points in the cognitive representation becomes infinite.

   The Galois group of algebraic numbers - the magic Absolute Group - would characterize this limit as a kind of never achievable mathematical enlightenment. A more practical definition would be that external system is experienced as complex, when its number theoretical complexity exceeds that of the conscious observer so that it is impossible to form a faithful cognitive representation about the system. Note that these cognitive representations could be formulated as homomorphisms between Galois groups. This would suggest a rather nice category theoretical picture about cognitive representations in the self hierarchy.

5. Galois group acts on the cognitive representation associated with the space-time sheet and in general gives n-fold covering of the space-time sheet: n is naturally the dimension of the extension and thus a divisor of the order of Galois group since Galois group acts on the discretization and implies n-sheeted structure for it and therefore also for the space-time surface.

   The value of the effective Planck constant assigned with dark matter as phases of ordinary matter $h_{eff}/h = n$ was identified from very beginning as number of sheets for some kind of covering space of embedding space. n would correspond to a divisor for the order of Galois group for discretized embedding space consisting of points with coordinates in extension of rational. The increase of h_{eff} corresponds to the emergence of also cognitive complexity. Physically it is accompanied by the emergence of quantum coherence and non-locality in increasingly long scales.

General vision about evolution as emergence of complexity

Evolution would mean emergence of number theoretical complexity. Evolutionary paths would naturally correspond to sequences of inclusions (note that recent view allows also temporary "de-evolutions" but in statistical sense evolution occurs). There are infinitely many evolutionary pathways of this kind.

There is a strong resemblance with the inclusion sequences of hyper-finite factors of type II_1 (HHFs) for von Neumann algebras [K120] also playing a central role in TGD and assignable to a fractal hierarchy of isomorphic sub-algebras of super-symplectic algebra associated with the isometries of WCW and related Kac-Moody algebras. It is difficult to believe that this could be an accident.

Evolution must mean a discrete time evolution of some kind - most naturally by non-deterministic quantum version of discrete dynamics, which can be deterministic only in statistical sense. By QCC this evolution should have classical correlates at space-time level. ZEO and TGD inspired theory of consciousness, which can be regarded as a generalization of quantum measurement theory in ZEO, is essential in attempts to concretize this intuition.

1. Galois group codes for the complexity and evolution means the emergence of increasingly complex Galois groups assignable to spacetime surface in a sector of WCW for which WCW coordinates are in corresponding extension of rationals. One can say that evolution defines a path in the space of sectors of WCW characterized by Galois groups. Although the space-time dynamics is expected to be integrable, the notion of complexity still has meaning, and ultimate chaos would emerge at the limit of algebraic numbers as extension of rationals.

2. One can assign Galois group G_5 to space-time surface. Suppose that one an assign Galois groups $G_3 \subset G_4$ with the 3-surfaces at the ends of space-time surfaces at boundaries of CD. This point will be discused below in more detail.

3. At quantum level conscious entities - selves - correspond to sequences off steps consisting of unitary evolution followed by a localization in the moduli space of CD. State function reduction to the opposite boundary of CD means death of self and re-incarnation of self with opposite arrow of time: also this means localization to a definite sector of WCW [L34, L32]. The sequence of pairs of selves and their time reversals associated with the opposite boundaries of CD (, which itself increases in size) defines a candidate for the non-deterministic quantum analog of iteration in complexity theory.

4. There is a temptation to assume that for the passive boundary of CD all 3-surfaces in quantum superposition have same G_3 - the G_3 that emerged in the first state function reduction to the passive boundary when this self was born. G_3 so would be automatically measured observable and sequence of reductions would define a sequence of G_3s analogous to iteration sequence and also to evolution.

   But can one assume that G_3 is measured automatically in the re-incarnation of self as its time-reversal [K7, K54]? Could only some charateristics of G_3 - say order $n = h_{eff}/h$ - be measured? Also ramified primes characteristize extensions and their measurement is also possible and proposed to characterize elementary particles: they do not fix G_3. These uncertainties are not relevant for the general vision.

5. For the active boundary one would have a superposition of 3-surfaces with different Galois groups and the sequence of the steps consisting of unitary evolution followed by a localization in the moduli space of CDs including also a localization in clock time determined by distance between the tips of CD. Also this would give to quantal discrete dynamics. Also now one can wonder whether Galois group is measured or not. If not, one would have a dispersion like process in the space of Galois groups labelling sectors of WCW.

6. Also the evolution of the tensor net defined by fermionic strings connecting the positions of fermions at partonic 2-surfaces would define a discrete dynamics in the space of these networks both at classical and quantum level [L21]. The dynamics of many-fermion states would determine this evolution.

In the sequel this picture is discussed in more detail.

How can one assign an extension of rationals to WCW, embedding space, and a region of space-time surface?

What fixes the extension used at both WCW level, embedding space level, and space-time level? The natural assumption is that the extension used for WCW coordinates induces the extension used at embedding space level and space-time level. At the level of space-time surfaces WCW coordinates appear as moduli (parameters) characterizing preferred extremals and would have values in an extension of rationals characterizing the adele by inducing the extensions of p-adic sectors.

1. The simplest option is that the extension is dictated by WCW. Preferred WCW coordinates - made possible by maximal isometries and fixed apart from the isometries of WCW - are in the extension: this makes the space of allowed 3-surfaces discrete. This in turn induces a constraint on space-time surfaces: WCW coordinates define parameters characterizing the space-time surface as a preferred extremal. One could use also other coordinates of WCW but these would not be optimal as cognitive representations.

   This applies also at the level of embedding space. Contrary to what I first thought, it is not actually absolutely necessary to use preferred space-time coordinates (subset of embedding space coordinates) since cognitive representation depends on coordinates in finite measurement resolution: consider only spherical and Cartesian coordinates with given resolution defining different discretizations. The preferred coordinates would be preferred because they are cognitively optimal.

2. Real embedding space is replaced with a discrete set of points of H with preferred coordinates in an extension of rationals. The direct identification of the points of extension as real numbers with p-adic numbers is extremely discontinuous although it would respect algebraic symmetries. The situation is saved by the lower dimensionality of space-time surfaces for which the set of points with coordinates in extension is discrete and even finite in the generic case. The surface $x^n + y^n = z^n$ has only one rational point for $n > 2$! $D = 4 < 8$ for space-time surfaces automatically brings in finite measurement resolution and cognitive resolution induced directly from the restriction on WCW parameters.

   SH has as data the intersection plus string world sheets (SH). String world sheets are in the intersection of reality and p-adicities defined by rational functions with coefficients of polynomials in extension, and makes sense both in real and p-adic sense. To these initial data one can assign as a preferred extremal of Kähler action a smooth p-adic space-time surface such that each point is contained in an open set consisting of points with p-adic coordinates having norm smaller than some power of p. This extremal is not unique in the p-adic sectors. In real sector it might not exist at all as already discussed.

3. 3-surface is seen as pair of 3-surfaces assigned to the ends of the space-time surface at boundaries of CD. WCW coordinates parameterize this pair and correspond to extension in 4-D sense. These parameters are expected to decompose to sets of parameters characterizing the 3-D members of pair and parameters characterizing the connecting space-time surface unless it is unique. If so, one can assign to the initial and final 3-surfaces subsets of WCW coordinates.

The extensions associated with the ends of CD would be extensions in 3-D sense and sub-extensions of the extension in 4-D sense. Hence one can say that classical space-time evolution connecting initial and final 3-surfaces can modify the extension, its Galois group, and therefore also $h_{eff}/h = n$. This would be the classical view a about number theoretic evolution and also about quantum critical fluctuation changing the value of $h_{eff}/h = n$.

4. The extension of rationals for WCW coordinates induces the cognitive representation posing constraints of p-adic space-time surfaces. Adelic sub-WCW consisting of preferred extremals inside given CD decomposes to sectors characterized by an extension of rationals and evolution should correspond number theoretically to a path in the space of WCW sectors.

   This is a restriction on p-adic space-time sheets and thus cognition: the larger the number of points in the intersection, the more precise the cognitive representation is. The increase of the dimension of extension implies that the number of points of cognitive representation increases and it becomes more precise. The cognitive abilities of the system evolve. p-Adic pseudo constants allow imagination but also make the representation imprecise in scales below that defined by the cognitive representation. The continuation to smooth p-adic surface would however explain the highly non-trivial fact that we automatically tend to associate continuous structures with discrete data.

5. The fermions at partonic 2-surfaces are at positions for which preferred space-time coordinates are in extension and can be said to actualize the cognitive representation. It turns out that these positions could naturally correspond to the singularities of the space-time surfaces as n-fold covering in the sense that the dimension of the orbit of Galois group would be reduced at these points.

Can one assign the analog of discrete dynamics to TGD at fundamental level?

Could one assign a discrete symbolic dynamics to classical and quantum TGD?

At classical level the dynamics would correspond to space-time surface connecting the boundaries of CD and 3-surfaces at them. As already explained, the WCW coordinates characterizing space-time surface as a preferred extremal correspond to what might be called Galois group in 4-D sense. These coordinates decompose to coordinates characterizing the coordinates at the 3-surfaces at the ends of of space-time at boundaries of CD in extensions characterized by Galois groups in 3-D sense - the initial and final Galois group. The classical evolutionary step would be a step leading from the initial fo final Galois group serving as classical correlate for quantum evolution.

What about quantum level?

1. One expects that zero energy state in general is a superposition of space-time surfaces with different Galois groups in 4-D sense, G_4. The Galois groups in 3-D sense - G_3 - assignable to the ends of space-time surface would be sub-groups of G_4. If the first state function reduction to the opposite boundary of CD involves a localization to a sector of WCW having same G_3 at passive boundary for all 3-surfaces in the superposition.

   Subsequent reductions at opposite boundaries would define evolutionary pathway in the space of Galois groups G_3 leading in statistical sense to the increase of complexity.

2. The original vision was that Negentropy Maximization Principle (NMP) [K65] is needed as a separate principle to guarantee evolution but adelic physics implies it in statistical sense automatically [L34]. There is infinite number of extensions more complex than given one and only finite number of them less complex.

3. At quantum level the basic notion is self. It corresponds to a discrete sequence steps consisting of unitary evolution followed by a localization in the moduli space of CDs. This would correspond to a dispersion in WCW to sectors characterized by different Galois groups G_4 and G_3 associated with the 3-surface at active boundary. As explained, the state function reduction to the opposite boundary of CD analogous to a halting of quantum computation would correspond to a localization to a sector with definite Galois group G_3.

4. These time discrete time evolutions are non-deterministic unlike the dynamical evolutions studied in chaos theory defined by differential equations or iteration of function. The sequence of unitary time evolutions involving localization in the moduli of CD would however give rise to a quantum analog of iteration and one can ask whether the quantum counterparts for the

notions of cycle, super-stable cycle etc... could make sense for the quantum superpositions of 4-surfaces involved. One expects dispersion in the space of Galois groups so that this idea does not look promising. One can also wonder if the sequence of unitary transformations could lead to some kind of asymptotic self-organization pattern before the first state function reduction to the opposite boundary of CD.

It is natural to consider also the evolution of the cognitive representation itself both at the space-time level and forced by the change of the many-fermion state and at quantum level.

1. For a given preferred extremal cognitive representation defines a discrete set of points in an extension of rationals and the number of points in the extension increases as it grows. The positions of fermions at partonic 2-surfaces define the nodes of a graph with strings connecting fermions at different partonic 2-surfaces serving as edges. Evolution of fermionic state changes the topology of this network by adding vertices and changing the connection.

   One can assign a complexity theory to these graphs. A connection with tensor nets [L21] emerging in the description of quantum complexity is highly suggestive. The nodes of the tensor net would correspond to fermions at partonic 2-surfaces. As the number of fermions increases, the complexity of this network increases and also the space-time surface itself becomes more complex. The maximum number of fermions increases with the dimension of extension.

   An interesting proposal is that fermion lines are accompanied by magnetic flux tubes taking the role of wormholes in ER-EPR correspondence (see `http://tinyurl.com/hzqlo6r`), which emerged more than half decade after its TGD analog. The discrete evolution of many-fermion state in state function reductions in the fermionic sector induces the evolution of this network.

2. In the case of graphs one can speak about various kinds of cycles, in particular Hamiltonian cycles going through all points of graph and having no self-intersections. Interestingly, Hamiltonian cycles for icosahedron (here the isometry group of icosahedron is involved as an additional structure) lead to a vision about genetic code and music harmonies [L11].

3. An interesting question concerns the extensions of rationals having as Galois group the isometry groups of Platonic solids: they probably exist. One can also consider the counterparts of Galois groups as discrete subgroups of the Galois group $SO(3)$ of quaternions. They emerge naturally for algebraic discretizations of M^4 regarded as a subspace of complexified quaternions with time axis identified as the real axis for quaternions (for $M^8 - H$ correspondence [K105, K119] see `http://tinyurl.com/mdvazmr`). Platonic solids correspond to *finite* discretizations with finite isometry groups belonging to a hierarchy of finite discrete subgroups of $SO(3)$ labelling the hierarchy of inclusions of HFFs: a connection between HFFs and quaternions is suggestive. For HFFs Platonic solids are in unique role in the sense that only for them the action of $SO(3)$ is genuinely 3-D. In Mac Kay correspondence they correspond to exceptional groups.

   For this generalization evolution would correspond to evolution in the space of Galois groups for finite-dimensional extensions of rational valued quaternions. p-Adic quaternions do not however form a field since p-adic quaternion can have vanishing norm squared.

4. The wave functions in the Galois group G reduce to wave functions in its coset space G/H if they are invariant under subgroup H. One can also perform the analog of second quantization for fermions in Galois group labelling the space-time sheets (or those of 3-space). In the model of harmony based on Hamilton's cycles the notes of 12-note scale would correspond to vertices of icosahedron obtained as coset space of I/Z_5, where I is icosahedral group with 60 elements. 3-chords of the harmony for a given Hamiltonian cycle would correspond to faces, which are triangles. Single particle fermion states localized at vertices (points of coset space) would correspond to notes of the scale and 3-fermion states localized at vertices of triangle to allowed 3-chords. The observation that one can understand the degeneracies of vertebrate genetic code by introducing besides icosahedron also tetrahedron suggests that both music and genetic code could relate directly to cognition described number theoretically.

5. It is also known that graphs can be identified as representations for Boolean statements (see `http://tinyurl.com/myrkhny`). Many-fermion states represent in TGD framework quantum Boolean statements with fermion number taking the role of bit. Could it be that this graphs

indeed represent entanglement many-fermion states having interpretation as quantum Boolean statements?

Can one imagine a quantum counterpart of iteration cycle? The space-time sheets can be seen as covering spaces with the number of sheets equal to the order $n = h_{eff}/h$ of Galois group. This gives additional discrete degrees of freedom and one could have wave functions in Galois group and also in its cyclic subgroup. These might serve as quantum counterparts for iteration cycles. An open question is whether n is always accompanied by $1/n$ fractionization of quantum numbers so that dark elementary particles would have same quantum numbers as ordinary ones but could be said to decompose to n pieces corresponding to sheets of covering.

One can also imagine that the cycles appear in the statistical description. At this limit one obtains deterministic kinetic equations and by their non-linearity one expects that they exhibit chaotic behavior in the usual sense.

Why would primes near powers of two (or small primes) be important?

p-Adic length scale hypothesis states that physically preferred p-adic primes come as primes near prime powers of two and possibly also other small primes. Does this have some analog to complexity theory, period doubling, and with the super-stability associated with period doublings?

Also ramified primes characterize the extension of rationals and would define naturally preferred primes for a given extension.

1. Any rational prime p can be decomposes to a product of powers P_i^k of primes P_i of extension given by $p = \prod_i P_i^{k_i}$, $\sum k_i = n$. If one has $k_i \neq 1$ for some i, one has ramified prime. Prime p is Galois invariant but ramified prime decomposes to lower-dimensional orbits of Galois group formed by a subset of $P_i^{k_i}$ with the same index k_i . One might say that ramified primes are more structured and informative than un-ramified ones. This could mean also representative capacity.

2. Ramification has as its analog criticality leading to the degenerate roots of a polynomial or the lowering of the rank of the matrix defined by the second derivatives of potential function depending on parameters. The graph of potential function in the space defined by its arguments and parameters if n-sheeted singular covering of this space since the potential has several extrema for given parameters. At boundaries of the n-sheeted structure some sheets degenerate and the dimension is reduced locally . Cusp catastrophe with 3-sheets in catastrophe region is standard example about this.

   Ramification also brings in mind super-stability of n-cycle for the iteration of functions meaning that the derivative of n:th iterate $f(f(...)(x) \equiv f^{n)}(x)$ vanishes. Superstability occurs for the iteration of function $f = ax + bx^2$ for $a = 0$.

3. I have considered the possibility that the n-sheeted coverings of the space-time surface are singular in that the sheet co-incide at the ends of space-time surface or at some partonic 2-surfaces. One can also consider the possibility that only some sheets or partonic 2-surfaces co-incide.

   The extreme option is that the singularities occur only at the points representing fermions at partonic 2-surfaces. Fermions could in this case correspond to different ramified primes. The graph of $w = z^{1/2}$ defining 2-fold covering of complex plane with singularity at origin gives an idea about what would be involved. This option looks the most attractive one and conforms with the idea that singularities of the coverings in general correspond to isolated points. It also conforms with the hypothesis that fermions are labelled by p-adic primes and the connection between ramifications and Galois singularities could justify this hypothesis.

4. Category theorists love structural similarities and might ask whether there might be a morphism mapping these singularities of the space-time surfaces as Galois coverings to the ramified primes so that sheets would correspond to primes of extension appearing in the decomposition of prime to primes of extension.

   Could the singularities of the covering correspond to the ramification of primes of extension? Could this degeneracy for given extension be coded by a ramified prime? Could quantum criticality of TGD favour ramified primes and singularities at the locations of fermions at partonic 2-surfaces?

Could the fundamental fermions at the partonic surfaces be quite generally localize at the singularities of the covering space serving as markings for them? This also conforms with the assumption that fermions with standard value of Planck constants corresponds to 2-sheeted coverings.

5. What could the ramification for a point of cognitive representation mean algebraically? The covering orbit of point is obtained as orbit of Galois group. For maximal singularity the Galois orbit reduces to single point so that the point is rational. Maximally ramified fermions would be located at rational points of extension. For non-maximal ramifications the number of sheets would be reduced but there would be several of them and one can ask whether only maximally ramified primes are realized. Could this relate at the deeper level to the fact that only rational numbers can be represented in computers exactly.
6. Can one imagine a physical correlate for the singular points of the space-time sheets at the ends of the space-time surface? Quantum criticality as analogy of criticality associated with super-stable cycles in chaos theory could be in question. Could the fusion of the space-time sheets correspond to a phenomenon analogous to Bose-Einstein condensation? Most naturally the condensate would correspond to a fractionization of fermion number allowing to put n fermions to point with same M^4 projection? The largest condensate would correspond to a maximal ramification $p = P_i^n$.

Why ramified primes would tend to be primes near powers of two or of small prime? The attempt to answer this question forces to ask what it means to be a survivor in number theoretical evolution. One can imagine two kinds of explanations.

1. Some extensions are winners in the number theoretic evolution, and also the ramified primes assignable to them. These extensions would be especially stable against further evolution representing analogs of evolutionary fossils. As proposed earlier, they could also allow exceptionally large cognitive representations that is large number of points of real space-time surface in extension.
2. Certain primes as ramified primes are winners in the sense the further extensions conserve the property of being ramified.
   (a) The first possibility is that further evolution could preserve these ramified primes and only add new ramified primes. The preferred primes would be like genes, which are conserved during biological evolution. What kind of extensions of existing extension preserve the already existing ramified primes. One could naïvely think that extension of an extension replaces P_i in the extension for $P_i = Q_{ik}^{k_i}$ so that the ramified primes would remain ramified primes.
   (b) Surviving ramified primes could be associated with a exceptionally large number of extensions and thus with their Galois groups. In other words, some primes would have strong tendency to ramify. They would be at criticality with respect to ramification. They would be critical in the sense that multiple roots appear.
   Can one find any support for this purely TGD inspired conjecture from literature? I am not a number theorist so that I can only go to web and search and try to understand what I found. Web search led to a thesis (see `http://tinyurl.com/mkhrssy`) studying Galois group with prescribed ramified primes.
   The thesis contained the statement that not every finite group can appear as Galois group with prescribed ramification. The second statement was that as the number and size of ramified primes increases more Galois groups are possible for given pre-determined ramified primes. This would conform with the conjecture. The number and size of ramified primes would be a measure for complexity of the system, and both would increase with the size of the system.
   (c) Of course, both mechanisms could be involved.

Why ramified primes near powers of 2 would be winners? Do they correspond to ramified primes associated with especially many extension and are they conserved in evolution by subsequent extensions of Galois group. But why? This brings in mind the fact that $n = 2^k$-cycles becomes super-stable and thus critical at certain critical value of the control parameter. Note also that ramified primes are analogous to prime cycles in iteration. Analogy with the evolution of genome is also strongly suggestive.

$h_{eff}/h = n$ hypothesis and Galois groups

The natural hypothesis is that $h_{eff}/h = n$ equals to dimension of the extension of rationals in the case that it gives the number of sheets of the covering assignable to the space-time surfaces. The stronger hypothesis is that $h_{eff}/h = n$ is associated with flux tubes and is proportional to the quantum numbers associated with the ends.

1. The basic idea is that Mother Nature is theoretician friendly. As perturbation theory breaks down, the interaction strength expressible as a product of appropriate charges divided by Planck constant, is reduced in the phase transition $\hbar \to \hbar_{eff}$.
2. In the case of gravitation $GMm \to= GMm(h/h_{eff})$. Equivalence Principle is satisfied if one has $\hbar_{eff} = \hbar_{gr} = GMm/v_0$, where v_0 is parameter with dimensions of velocity and of the order of some rotation velocity associated with the system. If the masses move with relativistic velocities the interaction strength is proportional to the inner product of four-momenta and therefore to Lorentz boost factors for energies in the rest system of the entire system. In this case one must assume quantization of energies to satisfy the constraint or a compensating reduction of v_0. Interactions strength becomes equal to $\beta_0 = v_0/c$ having no dependence on the masses: this brings in mind the universality associated with quantum criticality.
3. The hypothesis applies to all interactions. For electromagnetism one would have the replacements $Z_1Z_2\alpha \to Z_1Z_2\alpha(h/h_{em})$ and $\hbar_{em} = Z_1Z_2\alpha/\beta_0$ giving Universal interaction strength. In the case of color interactions the phase transition would lead to the emergence of hadron and it could be that inside hadrons the valence quark have $h_{eff}/h = n > 1$. In this case one could consider a generalization in which the product of masses is replaced with the inner product of four-momenta. In this case quantization of energy at either or both ends is required. For astrophysical energies one would have effective energy continuum.

This hypothesis suggests the interpretation of $h_{eff}/h = n$ as either the dimension of the extension or the order of its Galois group. If the extensions have dimensions n_1 and n_2, then the composite system would be n_2-dimensional extension of n_1-dimensional extension and have dimension $n_1 \times n_2$. This could be also true for the orders of Galois groups. This would be the case if Galois group of the entire system is free group generated by the G_1 and G_2. One just takes all products of elements of G_1 and G_2 and assumes that they commute to get $G_1 \times G_2$.

Consider gravitation as example.

1. The dimension of the extension should coincide with $\hbar_{eff}/\hbar = n = \hbar_{gr}/\hbar = GMm/v_0\hbar$. The transition occurs only if the value of $\hbar_{gr}/\hbar$ is larger than one. One can say that the dimension of the extension is proportional the product of masses using as unit Planck mass. Rather large extensions are involved and the number of sheets in the Galois covering is huge.

   Note that it is difficult to say how larger Planck constants are actually involved since by gravitational binding the classical gravitational forces are additive and by Equivalence principle same potential is obtained as sum of potentials for splitting of masses into pieces. Also the gravitational Compton length $\lambda_{gr} = GM/v_0$ for m does not depend on m at all so that all particles have same $\lambda_{gr} = GM/v_0$ irrespective of mass (note that v_0 is expressed using units with $c = 1$).

   The maximally incoherent situation would correspond to ordinary Planck constant and the usual view about gravitational interaction between particles. The extreme quantum coherence would mean that both M and m behave as single quantum unit. In many-sheeted space-time this could be understood in terms of a picture based on flux tubes. The interpretation for the degree of coherence is discussed in terms of flux tube connections mediating gravitational flux is discussed in [**?**].
2. h_{gr}/h would be the dimension of the extension, and there is a temptation to associate with the product of masses the product $n = n_1n_2$ of dimensions n_i associated masses M and m at least in some situations.

   The problem is that the dimension of the extension associated with m would be smaller than 1 for masses $m < m_P/\sqrt{\beta_0}$. Planck mass is about 1.3×10^{19} proton masses and corresponds to a blob of water with size scale 10^{-4} meters - size scale of a large neuron so that only above these scale gravitational quantum coherence would be possible. For $v_0 < 1$ it would seem that even in the case of large neurons one must have more than one neurons. Maybe pyramidal neurons could satisfy the mass constraint and would represent higher level of conscious as compared to other neurons and cells. The giant neurons discovered by the group led by Christof Koch in the brain of of mouse having axonal connections distributed over the entire brain might fulfil the constraint (see `http://tinyurl.com/gvwggsc`).

3. It is difficult to avoid the idea that macroscopic quantum gravitational coherence for multicellular objects with mass at least that for the largest neurons could be involved with biology. Multicellular systems can have mass above this threshold for some critical cell number. This might explain the dramatic evolutionary step distinguishing between prokaryotes (mono-cellulars consisting of Archaea and bacteria including also cellular organelles and cells with sub-critical size) and eukaryotes (multi-cellulars).

4. I have proposed an explanation of the fountain effect appearing in super-fluidity and apparently defying the law of gravity. In this case m was assumed to be the mass of ^4He atom in case of super-fluidity to explain fountain effect [**?**]. The above arguments however allow to ask whether anything changes if one allows the blobs of superfluid to have masses coming as a multiple of $m_P/\sqrt{\beta_0}$. One could check whether fountain effect is possible for super-fluid volumes with mass below $m_P/\sqrt{\beta_0}$.

What about h_{em}? In the case of super-conductivity the interpretation of h_{em}/h as product of orders of Galois groups would allow to estimate the number $N = Q/2e$ of Cooper pairs of a minimal blob of super-conducting matter from the condition that the order of its Galois group is larger than integer. The number $N = Q/2e$ is such that one has $2N\sqrt{\alpha/\beta_0} = n$. The condition is satisfied if one has $\alpha/\beta_0 = q^2$, with $q = k/2l$ such that N is divisible by l. The number of Cooper pairs would be quantized as multiples of l. What is clear that em interaction would correspond to a lower level of cognitive consciousness and that the step to gravitation dominated cognition would be huge if the dark gravitational interaction with size of astrophysical systems is involved [K79]. Many-sheeted space-time allows this in principle.

These arguments support the view that quantum information theory indeed closely relates not only to gravitation but also other interactions. Speculations revolving around blackhole, entropy, and holography, and emergence of space would be replaced with the number theoretic vision about cognition providing information theoretic interpretation of basic interactions in terms of entangled tensor networks [L21]. Negentropic entanglement would have magnetic flux tubes (and fermionic strings at them) as topological correlates. The increase of the complexity of quantum states could occur by the "fusion" of Galois groups associated with various nodes of this network as macroscopic quantum states are formed. Galois groups and their representations would define the basic information theoretic concepts. The emergence of gravitational quantum coherence identified as the emergence of multi-cellulars would mean a major step in biological evolution.

10.9 Why The Non-trivial Zeros Of Riemann Zeta Should Reside At Critical Line?

The following argument shows that the troublesome looking "1/2" in the non-trivial zeros of Riemann zeta can be understood as being necessary to allow a hermitian realization of the radial scaling generator rd/dr at light-cone boundary, which in the radial light-like radial direction corresponds to half-line R^+. Its presence allows unitary inner product and reduces the situation to that for ordinary plane waves on real axis. For preferred extremals strong form of holography poses extremely strong conditions expected to reduce the scaling momenta $s = 1/2 + iy$ to the zeros of zeta at critical line. RH could be also seen as a necessary condition for the existence of super-symplectic representations and thus for the existence of the "World of Classical Worlds" as a mathematically well-defined object. We can thank the correctness of Riemann's hypothesis for our existence!

10.9.1 What Is The Origin Of The Troublesome 1/2 In Non-trivial Zeros Of Zeta?

Riemann Hypothess (RH) states that the non-trivial (critical) zeros of zeta lie at critical line $s = 1/2$. It would be interesting to know how many physical justifications for why this should be the case has been proposed during years. Probably this number is finite, but very large it certainly is. In Zero Energy Ontology (ZEO) forming one of the cornerstones of the ontology of quantum TGD, the following justification emerges naturally.

1. The "World of Classical Worlds" (WCW) consisting of space-time surfaces having ends at the boundaries of causal diamond (CD), the intersection of future and past directed light-cones times CP_2 (recall that CDs form a fractal hierarchy). WCW thus decomposes to sub-WCWs and conscious experience for the self associated with CD is only about space-time surfaces in the interior of CD: this is a trong restriction to epistemology, would philosopher say.

   Also the light-like orbits of the partonic 2-surfaces define boundary like entities but as surfaces at which the signature of the induced metric changes from Euclidian to Minkowskian. By holography either kinds of 3-surfaces can be taken as basic objects, and if one accepts strong form of holography, partonic 2-surfaces defined by their intersections plus string world sheets become the basic entities.

2. One must construct tangent space basis for WCW if one wants to define WCW Kähler metric and gamma matrices. Tangent space consists of allowed deformations of 3-surfaces at the ends of space-time surface at boundaries of CD, and also at light-like parton orbits extended by field equations to deformations of the entire space-time surface. By strong form of holography only very few deformations are allowed since they must respect the vanishing of the elements of a sub-algebra of the classical symplectic charges isomorphic with the entire algebra. One has almost 2-dimensionality: most deformations lead outside WCW and have zero norm in WCW metric.

3. One can express the deformations of the space-like 3-surface at the ends of space-time using a suitable function basis. For CP_2 degrees of freedom color partial waves with well defined color quantum numbers are natural. For light-cone boundary $S^2 \times R^+$, where R^+ corresponds to the light-like radial coordinate, spherical harmonics with well defined spin are natural choice for S^2 and for R^+ analogs of plane waves are natural. By scaling invariance in the light-like radial direction they look like plane waves $\psi_s(r) = r^s = exp(us)$, $u = log(r/r_0)$, $s = x + iy$. Clearly, u is the natural coordinate since it replaces R^+ with R natural for ordinary plane waves.

4. One can understand why $Re[s] = 1/2$ is the only possible option by using a simple argument. One has super-symplectic symmetry and conformal invariance extended from 2-D Riemann surface to metrically 2-dimensional light-cone boundary. The natural scaling invariant integration measure defining inner product for plane waves in R^+ is $du = dr/r = dlog(r/r_0)$ with u varying from $-\infty$ to $+\infty$ so that R^+ is effectively replaced with R. The inner product must be same as for the ordinary plane waves and indeed is for $\psi_s(r)$ with $s = 1/2 + iy$ since the inner product reads as

   $$\langle s_1, s_2\rangle \equiv \int_0^\infty \overline{\psi_{s_1}}\psi_{s_2} dr = \int_0^\infty exp(i(y_1 - y_2) r^{-x_1-x_2} dr \quad .$$

   For $x_1 + x_2 = 1$ one obtains standard delta function normalization for ordinary plane waves:

   $$\langle s_1, s_2\rangle \int \int_{-\infty}^\infty exp[i(y_1 - y_2)u] du \propto \delta(y_1 - y_2) \quad .$$

   If one requires that this holds true for all pairs (s_1, s_2), one obtains $x_i = 1/2$ for all s_i. Preferred extremal condition gives extremely powerful additional constraints and leads to a quantisation of $s = -x - iy$: the first guess is that non-trivial zeros of zeta are obtained: $s = 1/2 + iy$. This identification would be natural by generalised conformal invariance. Thus RH is physically extremely well motivated but this of course does not prove it.

5. The presence of the real part $Re[s] = 1/2$ in the eigenvalues of scaling operator apparently breaks hermiticity of the scaling operator. There is however a compensating breaking of hermiticity coming from the fact that real axis is replaced with half-line and origin is pathological. What happens that real part 1/2 effectively replaces half-line with real axis and obtains standard plane wave basis. Note also that the integration measure becomes scaling invariant - something very essential for the representations of super-symplectic algebra. For $Re[s] = 1/2$ the hermicity conditions for the scaling generator rd/dr in R^+ reduce to those for the translation generator d/du in R.

10.9.2 Relation To Number Theoretical Universality And Existence Of WCW

This relates also to the number theoretical universality and mathematical existence of WCW in an interesting manner.

1. If one assumes that p-adic primes p correspond to zeros $s = 1/2 + y$ of zeta in 1-1 manner in the sense that $p^{iy(p)}$ is root of unity existing in all number fields (algebraic extension of p-adics) one obtains that the plane wave exists for p at points $r = p^n$. p-Adically wave function is discretized to a delta function distribution concentrated at $(r/r_0) = p^n$- a logarithmic lattice. This can be seen as space-time correlate for p-adicity for light-like momenta to be distinguished from that for massive states where length scales come as powers of $p^{1/2}$. Something very similar is obtained from the Fourier transform of the distribution of zeros at critical line (Dyson's argument), which led to a the TGD inspired vision about number theoretical universality [L13] (see `http://tinyurl.com/y7gl4huo`).
2. My article "Strategy for Proving Riemann Hypothesis" (`http://tinyurl.com/yd7k46ar`) [L2] written for 12 years ago ((for a slightly improved version see `http://tinyurl.com/ydcfkxwr`) relies on coherent states instead of eigenstates of Hamiltonian. The above approach in turn absorbs the problematic 1/2 to the integration measure at light cone boundary and conformal invariance is also now central.
3. Quite generally, I believe that conformal invariance in the extended form applying at metrically 2-D light-cone boundary (and at light-like orbits of partonic 2-surfaces) could be central for understanding why physics requires RH and maybe even for proving RH assuming it is provable at all in existing standard axiomatic system. For instance, the number of generating elements of the extended supersymplectic algebra is infinite (rather than finite as for ordinary conformal algebras) and generators are labelled by conformal weights defined by zeros of zeta (perhaps also the trivial conformal weights). $s = 1/2 + iy$ guarantees that the real parts of conformal weights for all states are integers. By conformal confinement the sum of ys vanishes for physical states. If some weight is not at critical line the situation changes. One obtains as net conformal weights all multiples of x shifted by all half odd integer values. And of course, the realisation as plane waves at boundary of light-cone fails and the resulting loss of unitary makes things too pathological and the mathematical existence of WCW is threatened.
4. The existence of non-trivial zeros outside the critical line could thus spoil the representations of super-symplectic algebra and destroy WCW geometry. RH would be crucial for the mathematical existence of the physical world! And the physical worlds exist only as mathematical objects in TGD based ontology: there are no physical realities behind the mathematical objects (WCW spinor fields) representing the quantum states. TGD inspired theory of consciousness tells that quantum jumps between the zero energy states give rise to conscious experience, and this is in principle all that is needed to understand what we experience.

10.10 Why Mersenne primes are so special?

Mersenne primes are central in TGD based world view. p-Adic thermodynamics combined with p-adic length scale hypothesis stating that primes near powers of two are physically preferred provides a nice understanding of elementary particle mass spectrum. Mersenne primes $M_k = 2^k - 1$, where also k must be prime, seem to be preferred. Mersenne prime labels hadronic mass scale (there is now evidence from LHC for two new hadronic physics labelled by Mersenne and Gaussian Mersenne), and weak mass scale. Also electron and tau lepton are labelled by Mersenne prime. Also Gaussian Mersennes $M_{G,k} = (1+i)^k - 1$ seem to be important. Muon is labelled by Gaussian Mersenne and the range of length scales between cell membrane thickness and size of cell nucleus contains 4 Gaussian Mersennes!

What gives Mersenne primes so special physical status? I have considered this problem many times during years. The key idea is that natural selection is realized in much more general sense than usually thought, and has chosen them and corresponding p-adic length scales. Particles characterized by p-adic length scales should be stable in some well-defined sense.

Since evolution in TGD corresponds to generation of information, the obvious guess is that Mersenne primes are information theoretically special. Could the fact that $2^k - 1$ represents almost k bits be of significance? Or could Mersenne primes characterize systems, which are information theoretically especially stable? In the following a more refined TGD inspired quantum information theoretic argument based on stability of entanglement against state function reduction, which would be fundamental process governed by Negentropy Maximization Principle (NMP) and requiring no human observer, will be discussed.

10.10.1 How to achieve stability against state function reductions?

TGD provides actually several ideas about how to achieve stability against state function reductions. This stability would be of course marvellous fact from the point of view of quantum computation since it would make possible stable quantum information storage. Also living systems could apply this kind of storage mechanism.

1. p-Adic physics leads to the notion of negentropic entanglement (NE) for which number theoretic entanglement entropy is negative and thus measures genuine, possibly conscious information assignable to entanglement (ordinary entanglement entropy measures the lack of information about the state of either entangled system). NMP favors the generation of NE. NE can be however transferred from system to another (stolen using less diplomatically correct expression!), and this kind of transfer is associated with metabolism. This kind of transfer would be the most fundamental crime: biology would be basically criminal activity! Religious thinker might talk about original sin.

   In living matter NE would make possible information storage. In fact, TGD inspired theory of consciousness constructed as a generalization of quantum measurement theory in Zero Energy Ontology (ZEO) identifies the permanent self of living system (replaced with a more negentropic one in biological death, which is also a reincarnation) as the boundary of CD, which is not affected in subsequent state function reductions and carries NE. The changing part of self - sensory input and cognition - can be assigned with opposite changing boundary of CD.

2. Also number theoretic stability can be considered. Suppose that one can assign to the system some extension of algebraic numbers characterizing the WCW coordinates ("world of classical worlds") parametrizing the space-time surface (by strong form of holography (SH) the string world sheets and partonic 2-surfaces continuable to 4-D preferred extremal) associated with it.

   This extension of rationals and corresponding algebraic extensions of p-adic numbers would define the number fields defining the coefficient fields of Hilbert spaces. Assume that you have an entangled system with entanglement coefficients in this number field. Suppose you want to diagonalize the corresponding density matrix. The eigenvalues belong in general case to a larger algebraic extension since they correspond to roots of a characteristic polynomials assignable to the density matrix. Could one say, that this kind of entanglement is stable (at least to some degree) against state function reduction since it means going to an eigenstate which does not belong to the extension used? Reader can decide!

3. Hilbert spaces are like natural numbers with respect to direct sum and tensor product. The dimension of the tensor product is product mn of the dimensions of the tensor factors. Hilbert space with dimension n can be decomposed to a tensor product of prime Hilbert spaces with dimensions which are prime factors of n. In TGD Universe state function reduction is a dynamical process, which implies that the states in state spaces with prime valued dimension are stable against state function reduction since one cannot even speak about tensor product

decomposition, entanglement, or reduction of entanglement. These state spaces are quantum indecomposable and would be thus ideal for the storage of quantum information!

Interestingly, the system consisting of k qubits have Hilbert space dimension $D = 2^k$ and is thus maximally unstable against decomposition to $D = 2$-dimensional tensor factors! In TGD Universe NE might save the situation. Could one imagine a situation in which Hilbert space with dimension $M_k = 2^k - 1$ stores the information stably? When information is processed this state space would be mapped isometrically to 2^k-dimensional state space making possible quantum computations using qubits. The outcome of state function reduction halting the computation would be mapped isometrically back to M_k-D space. Note that isometric maps generalizing unitary transformations are an essential element in the proposal for the tensor net realization of holography and error correcting codes [L21]. Can one imagine any concrete realization for this idea? This question be considered in the sequel.

10.10.2 How to realize $M_k = 2^k - 1$-dimensional Hilbert space physically?

One can imagine at least three physical realizations of $M_k = 2^k - 1$-dimensional Hilbert space.

1. The set with k elements has 2^k subsets. One of them is empty set and cannot be physically realized. Here the reader might of course argue that if they are realized as empty boxes, one can realize them. If empty set has no physical realization, the wave functions in the set of non-empty subsets with $2^k - 1$ elements define $2^k - 1$-dimensional Hilbert space. If $2^k - 1$ is Mersenne prime, this state state space is stable against state function reductions since one cannot even speak about entanglement!

   To make quantum computation possible one must map this state space to 2^k dimensional state space by isometric embedding. This is possible by just adding a new element to the set and considering only wave functions in the set of subsets containing this new element. Now also the empty set is mapped to a set containing only this new element and thus belongs to the state space. One has 2^k dimensions and quantum computations are possible. When the computation halts, one just removes this new element from the system, and the data are stored stably!

2. Second realization relies on k bits represented as spins such that $2^k - 1$ is Mersenne prime. Suppose that the ground state is spontaneously magnetized state with $k+l$ parallel spins, with the l spins in the direction of spontaneous magnetization and stabilizing it. $l > 1$ is probably needed to stabilize the direction of magnetization: $l \leq k$ suggests itself as the first guess. Here thermodynamics and a model for spin-spin interaction would give a better estimate.

   The state with the k spins in direction opposite to that for l spins would be analogous to empty set. Spontaneous magnetization disappears, when a sufficient number of spins is in direction opposite to that of magnetization. Suppose that k corresponds to a critical number of spins in the sense that spontaneous magnetization occurs for this number of parallel spins. Quantum superpositions of $2^k - 1$ states for k spins would be stable against state function reduction also now.

   The transformation of the data to a processable form would require an addition of $m \geq 1$ spins in the direction of the magnetization to guarantee that the state with all k spins in direction opposite to the spontaneous magnetization does not induce spontaneous magnetization in opposite direction. Note that these additional stabilizing spins are classical and their direction could be kept fixed by a repeated state function reduction (Zeno effect). One would clearly have a critical system.

3. Third realization is suggested by TGD inspired view about Boolean consciousness. Boolean logic is represented by the Fock state basis of many-fermion states. Each fermion mode defines one bit: fermion in given mode is present or not. One obtains 2^k states. These states have different fermion numbers and in ordinary positive energy ontology their realization is not possible.

   In ZEO situation changes. Fermionic zero energy states are superpositions of pairs of states at opposite boundaries of CD such that the total quantum numbers are opposite. This applies to fermion number too. This allows to have time-like entanglement in which one has superposition of states for which fermion numbers at given boundary are different. This kind of states might be realized for super-conductors to which one at least formally assigns coherent state of Cooper pairs having ill-defined fermion number.

Now the non-realizable state would correspond to fermion vacuum analogous to empty set. Reader can of course argue that the bosonic degrees of freedom assignable to the space-time surface are still present. I defend this idea by saying that the purely bosonic state might be unstable or maybe even non-realizable as vacuum state and remind that also bosons in TGD framework consists of pairs of fundamental fermions.

If this state is effectively decoupled from the rest of the Universe, one has 2^k-1-dimensional state space and states are stable against state function reduction. Information processing becomes possible by adding some positive energy fermions and corresponding negative energy fermions at the opposite boundaries of CD. Note that the added fermions do not have time-like quantum entanglement and do not change spin direction during time evolution.

The proposal is that Boolean consciousness is realized in this manner and zero energy states represents quantum Boolean thoughts as superposition of pairs $(b_1 \otimes b_2)$ of positive and negative energy states and having identification as Boolean statements $b_1 \to b_2$. The mechanism would allow both storage of thoughts as memories and their processing by introducing the additional fermion.

10.10.3 Why Mersenne primes would be so special?

Returning to the original question "Why Mersenne primes are so special?". A possible explanation is that elementary particle or hadron characterized by a p-adic length scale $p = M_k = 2^k - 1$ both stores and processes information with maximal effectiveness. This would not be surprising if p-adic physics defines the physical correlates of cognition assumed to be universal rather than being restricted to human brain.

In adelic physics p-dimensional Hilbert space could be naturally associated with the p-adic adelic sector of the system. Information storage could take place in $p = M_k = 2^k - 1$ phase and information processing (cognition) would take place in 2^k-dimensional state space. This state space would be reached in a phase transition $p = 2^k - 1 \to 2$ changing effective p-adic topology in real sector and genuine p-adic topology in p-adic sector and replacing padic length scale $\propto \sqrt{p} \simeq 2^{k/2}$ with k-nary 2-adic length scale $\propto 2^{k/2}$.

Electron is characterized by the largest not completely super-astrophysical Mersenne prime M_{127} and corresponds to $k = 127$ bits. Intriguingly, the secondary p-adic time scale of electron corresponds to .1 seconds defining the fundamental biorhythm of 10 Hz.

This proposal suffers from deficiencies. It does not explain why Gaussian Mersennes are also special. Gaussian Mersennes correspond ordinary primes near power of 2 but not so near as Mersenne primes do. Neither does it explain why also more general primes $p \simeq 2^k$ seem to be preferred. Furthermore, p-adic length scale hypothesis generalizes and states that primes near powers of at least small primes q: $p \simeq q^k$ are special at least number theoretically. For instance, $q = 3$ seems to be important for music experience and also $q = 5$ might be important (Golden Mean)

Could it be that the proposed model relying on criticality generalizes. There would be $p < 2^k$-dimensional state space allowing isometric embedding to 2^k-dimensional space such that the bit configurations orthogonal to the image would be unstable in some sense. Say against a phase transition changing the direction of magnetization. One can imagine the variants of above described mechanism also now. For $q > 2$ one should consider pinary digits instead of bits but the same arguments would apply (except in the case of Boolean logic).

10.10.4 Brain and Mersenne integers

I received a link to an interesting the article "Brain Computation Is Organized via Power-of-Two-Based Permutation Logic" by Kun Xie, Grace E. Fox, Jun Liu, Cheng Lyu, Jason C. Lee, Hui Kuang, Stephanie Jacobs, Meng Li, Tianming Liu, Sen Song and Joe Z. Tsien in Frontiers in Systems Neuroscience [?]see `http://tinyurl.com/zfymqrq`).

The proposed model is about how brain classifies neuronal inputs. The following represents my attempt to understand the model of the article.

1. One can consider a situation in which one has n inputs identifiable as bits: bit could correspond to neuron firing or not. The question is however to classify various input combinations. The obvious criterion is how many bits are equal to 1 (corresponding neuron fires). The input combinations in the same class have same number of firing neurons and the number of subsets with k elements is given by the binomial coefficient $B(n,k) = n!/k!(n-k)!$. There are clearly $n-1$ different classes in the classification since no neurons firing is not a possible observation. The conceptualization would tell how many neurons fire but would not specify which of them.
2. To represent these bit combinations one needs $2^n - 1$ neuron groups acting as unit representing one particular firing combination. These subsets with k elements would be mapped to neuron cliques with k firing neutrons. For given input individual firing neurons ($k = 1$) would represent features, lowest level information. The n cliques with $k = 2$ neurons would represent a more general classification of input. One obtains $M_n = 2^n - 1$ combinations of firing neurons since the situations in which no neurons are firing is not counted as an input.
3. If all neurons are firing then all the however level cliques are also activated. Set theoretically the subsets of set partially ordered by the number of elements form an inclusion hierarchy, which in Boolean algebra corresponds to the hierarchy of implications in opposite direction. The clique with all neurons firing correspond to the most general statement implying all the lower level statements. At k:th level of hierarchy the statements are inconsistent so that one has $B(n,k)$ disjoint classes.

The $M_n = 2^n - 1$ (Mersenne number) labelling the algorithm is more than familiar to me.

1. For instance, electron's p-adic prime corresponds to Mersenne prime $M_{127} = 2^{127} - 1$, the largest not completely super-astrophysical Mersenne prime for which the mass of particle would be extremely small. Hadron physics corresponds to M_{107} and M_{89} to weak bosons and possible scaled up variant of hadron physics with mass scale scaled up by a factor 512 ($= 2^{(107-89)/2}$). Also Gaussian Mersennes seem to be physically important: for instance, muon and also nuclear physics corresponds to $M_{G,n} = (1+i)^n - 1$, $n = 113$.
2. In biology the Mersenne prime $M_7 = 2^7 - 1$ is especially interesting. The number of statements in Boolean algebra of 7 bits is 128 and the number of statements that are consistent with given atomic statement (one bit fixed) is $2^6 = 64$. This is the number of genetic codons which suggests that the letters of code represent 2 bits. As a matter of fact, the so called Combinatorial Hierarchy $M(n) = M_{M(n-1)}$ consists of Mersenne primes $n = 3, 7, 127, 2^{127} - 1$ and would have an interpretation as a hierarchy of statements about statements about ... It is now known whether the hierarchy continues beyond M_{127} and what it means if it does not continue. One can ask whether M_{127} defines a higher level code - memetic code as I have called it - and realizable in terms of DNA codon sequences of 21 codons [L20] (see `http://tinyurl.com/jukyq6y`).
3. The Gaussian Mersennes $M_{G,n}$ $n = 151, 157, 163, 167$, can be regarded as a number theoretical miracles since the these primes are so near to each other. They correspond to p-adic length scales varying between cell membrane thickness 10 nm and cell nucleus size 2.5 μm and should be of fundamental importance in biology. I have proposed that p-adically scaled down variants of hadron physics and perhaps also weak interaction physics are associated with them.

I have made attempts to understand why Mersenne primes M_n and more generally primes near powers of 2 seem to be so important physically in TGD Universe.

1. The states formed from n fermions form a Boolean algebra with 2^n elements, but one of the elements is vacuum state and could be argued to be non-realizable. Hence Mersenne number $M_n = 2^n - 1$. The realization as algebra of subsets contains empty set, which is also physically non-realizable. Mersenne primes are especially interesting as sine the reduction of statements to prime nearest to M_n corresponds to the number $M_n - 1$ of physically representable Boolean statements.
2. Quantum information theory suggests itself as explanation for the importance of Mersenne primes since M_n would correspond the number of physically representable Boolean statements of a Boolean algebra with n-elements. The prime $p \leq M_n$ could represent the number of elements of Boolean algebra representable p-adically [L27] (see `http://tinyurl.com/gp9mspa`).

3. In TGD Fermion Fock states basis has interpretation as elements of quantum Boolean algebra and fermionic zero energy states in ZEO expressible as superpositions of pairs of states with same net fermion numbers can be interpreted as logical implications. WCW spinor structure would define quantum Boolean logic as “square root of Kähler geometry”. This Boolean algebra would be infinite-dimensional and the above classification for the abstractness of concept by the number of elements in subset would correspond to similar classification by fermion number. One could say that bosonic degrees of freedom (the geometry of 3-surfaces) represent sensory world and spinor structure (many-fermion states) represent that logical thought in quantum sense.
4. Fermion number conservation would seem to represent an obstacle but in ZEO it can circumvented since zero energy states can be superpositions of pair of states with opposite fermion number F at opposite boundaries of causal diamond (CD) in such a way that F varies. In state function reduction however localization to single value of F is expected to happen usually. If superconductors carry coherent states of Cooper pairs, fermion number for them is ill defined and this makes sense in ZEO but not in standard ontology unless one gives up the super-selection rule that fermion number of quantum states is well-defined.

One can of course ask whether primes n defining Mersenne primes (see `http://tinyurl.com/l3lxe2n`) could define preferred numbers of inputs for subsystems of neurons. This would predict $n = 2, 3, 5, 7, 13, 17, 19, 31, 67, 127, 257, ..$ define favoured numbers of inputs. $n = 127$ would correspond to memetic code.

10.11 Number Theoretical Feats and TGD Inspired Theory of Consciousness

Number theoretical feats of some mathematicians like Ramanujan remain a mystery for those believing that brain is a classical computer. Also the ability of idiot savants - lacking even the idea about what prime is - to factorize integers to primes challenges the idea that an algorithm is involved. In this article I discuss ideas about how various arithmetical feats such as partitioning integer to a sum of integers and to a product of prime factors might take place. The ideas are inspired by the number theoretic vision about TGD suggesting that basic arithmetics might be realized as naturally occurring processes at quantum level and the outcomes might be “sensorily perceived”. One can also ask whether zero energy ontology (ZEO) could allow to perform quantum computations in polynomial instead of exponential time.

The indian mathematician Srinivasa Ramanujan is perhaps the most well-known example about a mathematician with miraculous gifts. He told immediately answers to difficult mathematical questions - ordinary mortals had to hard computational work to check that the answer was right. Many of the extremely intricate mathematical formulas of Ramanujan have been proved much later by using advanced number theory. Ramanujan told that he got the answers from his personal Goddess. A possible TGD based explanation of this feat relies on the idea that in zero energy ontology (ZEO) quantum computation like activity could consist of steps consisting quantum computation and its time reversal with long-lasting part of each step performed in reverse time direction at opposite boundary of causal diamond so that the net time used would be short at second boundary.

The adelic picture about state function reduction in ZEO suggests that it might be possible to have direct sensory experience about prime factorization of integers [L26]. What about partitions of integers to sums of primes? For years ago I proposed that symplectic QFT is an essential part of TGD. The basic observation was that one can assign to polygons of partonic 2-surface - say geodesic triangles - Kähler magnetic fluxes defining symplectic invariance identifiable as zero modes. This assignment makes sense also for string world sheets and gives rise to what is usually called Abelian Wilson line. I could not specify at that time how to select these polygons. A very natural manner to fix the vertices of polygon (or polygons) is to assume that they correspond ends of fermion lines which appear as boundaries of string world sheets. The polygons would be fixed rather uniquely by requiring that fermions reside at their vertices.

The number 1 is the only prime for addition so that the analog of prime factorization for sum is not of much use. Polygons with $n = 3, 4, 5$ vertices are special in that one cannot decompose them to non-degenerate polygons. Non-degenerate polygons also represent integers $n > 2$. This inspires the idea about numbers $\{3, 4, 5\}$ as "additive primes" for integers $n > 2$ representable as non-degenerate polygons. These polygons could be associated many-fermion states with negentropic entanglement (NE) - this notion relate to cognition and conscious information and is something totally new from standard physics point of view. This inspires also a conjecture about a deep connection with arithmetic consciousness: polygons would define conscious representations for integers $n > 2$. The splicings of polygons to smaller ones could be dynamical quantum processes behind arithmetic conscious processes involving addition.

10.11.1 How Ramanujan did it?

Lubos Motl wrote recently a blog posting (`http://tinyurl.com/zduu72p`) about $P \neq NP$ computer in the theory of computation based on Turing's work. This unproven conjecture relies on a classical model of computation developed by formulating mathematically what the women doing the hard computational work in offices at the time of Turing did. Turing's model is extremely beautiful mathematical abstraction of something very every-daily but does not involve fundamental physics in any manner so that it must be taken with caution. The basic notions include those of algorithm and recursive function, and the mathematics used in the model is mathematics of integers. Nothing is assumed about what conscious computation is and it is somewhat ironic that this model has been taken by strong AI people as a model of consciousness!

1. A canonical model for classical computation is in terms of Turing machine, which has bit sequence as inputs and transforms them to outputs and each step changes its internal state. A more concrete model is in terms of a network of gates representing basic operations for the incoming bits: from this basic functions one constructs all recursive functions. The computer and program actualize the algorithm represented as a computer program and eventually halts - at least one can hope that it does so. Assuming that the elementary operations require some minimum time, one can estimate the number of steps required and get an estimate for the dependence of the computation time as function of the size of computation.

2. If the time required by a computation, whose size is characterized by the number N of relevant bits, can be carried in time proportional to some power of N and is thus polynomial, one says that computation is in class P. Non-polynomial computation in class NP would correspond to a computation time increasing with N faster than any power of N, say exponentially. Donald Knuth, whose name is familiar for everyone using Latex to produce mathematical text, believes on $P = NP$ in the framework of classical computation. Lubos in turn thinks that the Turing model is probably too primitive and that quantum physics based model is needed and this might allow $P = NP$.

What about quantum computation as we understand it in the recent quantum physics: can it achieve $P = NP$?

1. Quantum computation is often compared to a superposition of classical computations and this might encourage to think that this could make it much more effective but this does not seem to be the case. Note however that the amount of information represents by N qubits is however exponentially larger than that represented by N classical bits since entanglement is possible. The prevailing wisdom seems to be that in some situations quantum computation can be faster than the classical one but that if $P = NP$ holds true for classical computation, it holds true also for quantum computations. Presumably because the model of quantum computation begins from the classical model and only (quantum computer scientists must experience this statement as an insult - apologies!) replaces bits with qubits.

2. In quantum computer one replaces bits with entangled qubits and gates with quantum gates and computation corresponds to a unitary time evolution with respect to a discretized time parameter constructed in terms of fundamental simple building bricks. So called tensor networks realize the idea of local unitary in a nice manner and has been proposed to defined error correcting quantum codes. State function reduction halts the computation. The outcome is non-deterministic but one can perform large number of computations and deduce from the distribution of outcomes the results of computation.

What about conscious computations? Or more generally, conscious information processing. Could it proceed faster than computation in these sense of Turing? To answer this question one must first try to understand what conscious information processing might be. TGD inspired theory of consciousnesss provides one a possible answer to the question involving not only quantum physics but also new quantum physics.

1. In TGD framework Zero energy ontology (ZEO) replaces ordinary positive energy ontology and forces to generalize the theory of quantum measurement. This brings in several new elements. In particular, state function reductions can occur at both boundaries of causal diamond (CD), which is intersection of future and past direct light-cones and defines a geometric correlate for self. Selves for a fractal hierarchy - CDs within CDs and maybe also overlapping. Negentropy Maximization Principle (NMP) is the basic variational principle of consciousness and tells that the state function reductions generate maximum amount of conscious information. The notion of negentropic entanglement (NE) involving p-adic physics as physics of cognition and hierarchy of Planck constants assigned with dark matter are also central elements.
2. NMP allows a sequence of state function reductions to occur at given boundary of diamond-like CD - call it passive boundary. The state function reduction sequence leaving everything unchanged at the passive boundary of CD defines self as a generalized Zeno effect. Each step shifts the opposite - active - boundary of CD "upwards" and increases its distance from the passive boundary. Also the states at it change and one has the counterpart of unitary time evolution. The shifting of the active boundary gives rise to the experienced time flow and sensory input generating cognitive mental images - the "Maya" aspect of conscious experienced. Passive boundary corresponds to permanent unchanging "Self".
3. Eventually NMP forces the first reduction to the opposite boundary to occur. Self dies and reincarnates as a time reversed self. The opposite boundary of CD would be now shifting "downwards" and increasing CD size further. At the next reduction to opposite boundary re-incarnation of self in the geometric future of the original self would occur. This would be re-incarnation in the sense of Eastern philosophies. It would make sense to wonder whose incarnation in geometric past I might represent!

Could this allow to perform fast quantal computations by decomposing the computation to a sequence in which one proceeds in both directions of time? Could the incredible feats of some "human computers" rely on this quantum mechanism (see `http://tinyurl.com/hk5baty`). The indian mathematician Srinivasa Ramanujan (see `http://tinyurl.com/l42q7a2`) is the most well-known example of a mathematician with miraculous gifts. He told immediately answers to difficult mathematical questions - ordinary mortals had to hard computational work to check that the answer was right. Many of the extremely intricate mathematical formulas of Ramanujan have been proved much later by using advanced number theory. Ramanujan told that he got the answers from his personal Goddess.

Might it be possible in ZEO to perform quantally computations requiring classically non-polynomial time much faster - even in polynomial time? If this were the case, one might at least try to understand how Ramanujan did it although higher levels selves might be involved also (did his Goddess do the job?).

1. Quantal computation would correspond to a state function reduction sequence at fixed boundary of CD defining a mathematical mental image as sub-self. In the first reduction to the opposite boundary of CD sub-self representing mathematical mental image would die and quantum computation would halt. A new computation at opposite boundary proceeding to opposite direction of geometric time would begin and define a time-reversed mathematical mental image. This sequence of reincarnations of sub-self as its time reversal could give rise to a sequence of quantum computation like processes taking less time than usually since one half of computations would take place at the opposite boundary to opposite time direction (the size of CD increases as the boundary shifts).
2. If the average computation time is same at both boundaries, the computation time would be only halved. Not very impressive. However, if the mental images at second boundary - call it A - are short-lived and the selves at opposite boundary B are very long-lived and represent

very long computations, the process could be very fast from the point of view of A! Could one overcome the $P \neq NP$ constraint by performing computations during time-reversed re-incarnations?! Short living mental images at this boundary and very long-lived mental images at the opposite boundary - could this be the secret of Ramanujan?

3. Was the Goddess of Ramanujan - self at higher level of self-hierarchy - nothing but a time reversal for some mathematical mental image of Ramanujan (Brahman=Atman!), representing very long quantal computations! We have night-day cycle of personal consciousness and it could correspond to a sequence of re-incarnations at some level of our personal self-hierarchy. Ramanujan tells that he met his Goddess in dreams. Was his Goddess the time reversal of that part of Ramanujan, which was unconscious when Ramanujan slept? Intriguingly, Ramanujan was rather short-lived himself - he died at the age of 32! In fact, many geniuses have been rather short-lived.

4. Why the alter ego of Ramanujan was Goddess? Jung intuited that our psyche has two aspects: anima and animus. Do they quite universally correspond to self and its time reversal? Do our mental images have gender?! Could our self-hierarchy be a hierarchical collection of anima and animi so that gender would be something much deeper than biological sex! And what about Yin-Yang duality of Chinese philosophy and the ka as the shadow of persona in the mythology of ancient Egypt?

10.11.2 Symplectic QFT, $\{3, 4, 5\}$ as Additive Primes, and Arithmetic Consciousness

For years ago I proposed that symplectic QFT is an essential part of TGD [K20, K105]. The basic observation was that one can assign to polygons of partonic 2-surface - say geodesic triangles - Kähler magnetic fluxes defining symplectic invariance identifiable as zero modes. This assignment makes sense also for string world sheets and gives rise to what is usually called Abelian Wilson line. I could not specify at that time how to select these polygons in the case of partonic 2-surfaces.

The recent proposal of Maldacena and Arkani-Hamed [B47] (see http://tinyurl.com/ych26gcm) that CMB might contain signature of inflationary cosmology as triangles and polygons for which the magnitude of n-point correlation function is enhanced led to a progress in this respect. In the proposal of Maldacena and Arkani-Hamed the polygons are defined by momentum conservation. Now the polygons would be fixed rather uniquely by requiring that fermions reside at their vertices and momentum conservation is not involved.

This inspires the idea about numbers $\{3, 4, 5\}$ as "additive primes" for integers $n > 2$ representable as non-degenerate polygons. Geometrically one could speak of prime polygons not decomposable to lower non-degenerate polygons. These polygons are different from those of Maldacena and Arkani-Hamed and would be associated many-fermion states with negentropic entanglement (NE) - this notion relates to cognition and conscious information and is something totally new from standard physics point of view. This inspires also a conjecture about a deep connection with arithmetic consciousness: polygons would define representations for integers $n > 2$. The splicings of polygons to smaller ones could be dynamical quantum processes behind arithmetic conscious processes involving addition. I have already earlier considered a possible counterpart for conscious prime factorization in the adelic framework [L26].

Basic ideas of TGD inspired theory of conscious very briefly

Negentropy Maximization Principle (NMP) is the variational principle of consciousness in TGD framework. It says that negentropy gain in state function reduction (quantum jump re-creating Universe) is maximal. State function reduction is basically quantum measurement in standard QM and sensory qualia (for instance) could be perhaps understood as quantum numbers of state resulting in state function reduction. NMP poses conditions on whether this reduction can occur. In standard ontology it would occur always when the state is entangled: reduction would destroy the entanglement and minimize entanglement entropy. When cognition is brought in, the situation changes.

The first challenge is to define what negentropic entanglement (NE) and negentropy could mean.

1. In real physics without cognition one does not have any definition of negentropy: on must define negentropy as reduction of entropy resulting as conscious entity gains information. This kind of definition is circular in consciousness theory.
2. In p-adic physics one can define number theoretic entanglement entropy with same basic properties as ordinary Shannon entropy. For some p-adic number fields this entropy can be negative and this motivates an interpretation as conscious information related to entanglement - rather to the ignorance of external observer about entangled state. The prerequisite is that the entanglement probabilities belong to an an extension of rationals inducing a finite-dimensional extension of rationals. Algebraic extensions are such extensions as also those generate by a root of e (e^p is p-adic number in Q_p).

A crucial step is to fuse together sensory and cognitive worlds as different aspects of existence.

1. One must replace real universe with adelic one so that one has real space-time surfaces and their p-adic variants for various primes p satisfying identical field equations. These are related by strong form of holography (SH) in which 2-D surfaces (string world sheets and partonic 2-surfaces) serve as "space-time genes" and obey equations which make sense both p-adically in real sense so that one can identify them as points of "world of classical worlds" (WCW).
2. One can say that these 2-surfaces belong to intersection of realities and p-adicities - intersection of sensory and cognitive. This demands that the parameters appearing in the equations for 2-surface belong algebraic extension of rational numbers: the interpretation is that this hierarchy of extensions corresponds to evolutionary hierarchy. This also explains imagination in terms of the p-adic space-time surfaces which are not so unique as the real one because of inherent non-determinism of p-adic differental equations. What can be imagined cannot be necessarily realized. You can continued p-adic 2-surface to 4-D surface but not to real one.

There is also second key assumption involved.

1. Hilbert space of quantum states is *same* for real and p-adic sectors of adelic world: for instance, tensor product would lead to total nonsense since there would be both real and p-adic fermions. This means same quantum state and same entanglement but seen from sensory and various cognitive perspectives. This is the basic idea of adelicity: the p-adic norms of rational number characterize the norm of rational number. Now various p-adic conscious experiences characterize the quantum state.
2. Real perspective sees entanglement always as entropic. For some finite number number of primes p p-adic entanglement is however negentropic. For instance, for entanglement probabilities $p_i = 1/N$, the primes appearing as factors of N are such information carrying primes. The presence of these primes can make the entanglement stable. The total entropy equal to the sum of negative real negentropy + various p-adic negentropies can be positive and cannot be reduced in the reduction so that reduction does not occur at all! Entanglement is stabilized by cognition and the randomness of state function reduction tamed: matter has power over matter!
3. There is analogy with the reductionism-holism dichotomy. Real number based view is reductionistic: information is obtained when the entangled state is split into un-entangled part. p-Adic number based view is holistic: information is inthe negentropic entanglement and can be seen as abstraction or rule. The superposition of state pairs represents a rule with state pairs (a_i, b_i) representing the instance of the rule A↔B. Maximal entanglement defined by entanglement probabilities $p_i = 1/N$ makes clear the profound distinction between these views. In real sector the negentropy is negative and smallest possible. In p-adic sector the negentropy is maximum for p-adic primes appearing as factors of N and total negentropy as their sum is large. NE allows to select unique state basis if the probabilities p_i are different.

   For $p_i = 1/N$ one can choose any unitary related state basis since unit matrix is invariant under unitary transformations. From the real point of view the ignorance is maximal and entanglement entropy is indeed maximal. For instance, in case of Schrödinger cat one could choose the cat's state basis to be any superposition of dead and alive cat and a state orthogonal to it. From p-adic view information is maximal. The reports of meditators, in particular Zen buddhists, support this interpretation. In "enlightened state" all discriminations disappear:

it does not make sense to speak about dead or alive cat or anything between these two options. The state contains information about entire state - not about its parts. It is not information expressible using language relying on making of distinctions but silent wisdom.

How do polygons emerge in TGD framework?

The duality defined by strong form of holography (SH) has 2 sides. Space-time side (bulk) and boundary side (string world sheets and partonic 2-surfaces). 2-D half of SH would suggest a description based on string world sheets and partonic 2-surfaces. This description should be especially simple for the quantum states realized as spinor fields in WCW ("world of classical worlds"). The spinors (as opposed to spinor fields) are now fermionic Fock states assignable to space-time surface defining a point of WCW. TGD extends ordinary 2-D conformal invariance to super-symplectic symmetry applying at the boundary of light-cone: note that given boundary of causal diamond (CD) is contained by light-cone boundary.

1. The correlation functions at embedding space level for fundamental objects, which are fermions at partonic 2-surfaces could be calculated by applying super-symplectic invariance having conformal structure. I have made rather concrete proposals in this respect. For instance, I have suggested that the conformal weights for the generators of supersymplectic algebra are given by poles of fermionic zeta $\zeta_F(s) = \zeta(s)/\zeta(2s)$ and thus include zeros of zeta scaled down by factor 1/2 [L14]. A related proposal is conformal confinement guaranteeing the reality of net conformal weights.
2. The conformally invariant correlation functions are those of super-symplectic CFT at light-cone boundary or its extension to CD. There would be the analog of conformal invariance associated with the light-like radial coordinate r_M and symplectic invariance associated with CP_2 and sphere S^2 localized with respect to r_M analogous to the complex coordinate in ordinary conformal invariance and naturally continued to hypercomplex coordinate at string world sheets carrying the fermionic modes and together with partonic 2-surfaces defining the boundary part of SH.

Symplectic invariants emerge in the following manner. Positive and negative energy parts of zero energy states would also depend on zero modes defined by super-symplectic invariants and this brings in polygons. Polygons emerge also from four-momentum conservation. These of course are also now present and involve the product of Lorentz group and color group assignable to CD near its either boundary. It seems that the extension of Poincare translations to Kac-Moody type symmetry allows to have full Poincare invariance (in its interior CD looks locally like $M^4 \times CP_2$).

1. One can define the symplectic invariants as magnetic fluxes associated with S^2 and CP_2 Kähler forms. For string world sheets one would obtain non-integrable phase factors. The vertices of polygons defined by string world sheets would correspond to the intersections of the string world sheets with partonic 2-surfaces at the boundaries of CD and at partonic 2-surfaces defining generalized vertices at which 3 light-like 3-surfaces meet along their ends.
2. Any polygon at partonic 2-surface would also allow to define such invariants. A physically natural assumption is that the vertices of these polygons are realized physically by adding fermions or antifermions at them. Kähler fluxes can be expressed in terms of non-integrable phase factors associated with the edges. This assumption would give the desired connection with quantum physics and fix highly uniquely but not completely the invariants appearing in physical states.

The correlated polygons would be thus naturally associated with fundamental fermions and a better analogy would be negentropically entangled n-fermion state rather than corresponding to maximum of the modulus of n-point correlation function. Hierarchy of Planck constants makes these states possible even in cosmological scales. The point would be that negentropic entanglement assignable to the p-adic sectors of WCW would be in key role.

Symplectic invariants and Abelian non-integrable phase factors

Consider now the polygons assignable to many-fermion states at partonic 2-surfaces.

1. The polygon associated with a given set of vertices defined by the position of fermions is far from unique and different polygons correspond to different physical situations. Certainly one must require that the geodesic polygon is not self-intersecting and defines a polygon or set of polygons.
2. Geometrically the polygon is not unique unless it is convex. For instance, one can take regular n-gon and add one vertex to its interior. The polygon can be also constructed in several ways. From this one obtains a non-convex $n+1$-gon in $n+1$ ways.
3. Given polygon is analogous with Hamiltonian cycle connecting all points of given graph. Now one does not have graph structure with edges and vertices unless one defines it by nearest neighbor property. Platonic solids provide an example of this kind of situation. Hamiltonian cycles [A10, A26] are key element in the TGD inspired model for music harmony leading also to a model of genetic code [K87] [L11].
4. One should somehow fix the edges of the polygon. For string world sheets the edges would be boundaries of string world sheet. For partonic 2-surfaces the simplest option is that the edges are geodesic lines and thus have shortest possible length. This would bring in metric so that the idea about TGD as almost topological QFT would be realized.

One can distinguish between two cases: single polygon or several polygons.

1. One has maximal entanglement between fundamental fermions, when the vertices define single polygon. One can however have several polygons for a given set of vertices and in this case the coherence is reduced. Minimal correlations correspond to maximal number of 3-gons and minimal number of 4-gons and 5-gons.
2. For large $h_{eff} = n \times h$ the partonic 2-surfaces can have macroscopic and even astrophysical size and one can consider assigning many-fermion states with them. For instance, anyonic states could be interpreted in this manner. In this case it would be natural to consider various decompositions of the state to polygons representing entangled fermions.

The definition of symplectic invariant depends on whether one has single polygon or several polygons.

1. In the case that there are several polygons not containing polygons inside them (if this the case,then the complement of polygon must satisfy the condition) one can uniquely identify the interior of each polygon and assign a flux with it. Non-integrable phase factor is well-defined now. If there is only single polygon then also the complement of polygon could define the flux. Polygon and its complement define fluxes Φ and $\Phi_{tot} - \Phi$.
2. If partonic 2-surface carries monopole Kähler charge Φ_{tot} is essentially $n\pi$, where n is magnetic monopole flux through the partonic 2-surface. This is half integer - not integer: this is key feature of TGD and forces the coupling of Kähler gauge potential to the spinors leading to the quantum number spectrum of standard model. The exponent can be equal to -1 for half-odd integer.

   This problem disappears if both throats of the wormhole contact connecting the space-time sheets with Minkowski signature give their contribution so that two minus-signs give one plus sign. Elementary particles necessarily consist of wormhole contacts through which monopole flux flows and runs along second space-time sheet to another contact and returns along second space-time sheet so that closed monopole flux tube is obtained. The function of the flux must be single valued. This demands that it must reduce to the cosine of the integer multiple of the flux and identifiable as as the real part of the integer power of magnetic flux through the polygon.

The number theoretically deepest point is geometrically completely trivial.

1. Only $n > 2$-gons are non-degenerate and 3-, 4- and 5-gons are prime polygons in the sense that they cannot be sliced to lower polygons. Already 6-gon decomposes to 2 triangles.
2. One can wonder whether the appearance of 3 prime polygons might relate to family replication phenomenon for which TGD suggests an explanation in terms of genus of the partonic 2-surface [K25]. This does not seem to be the case. There is however other three special integers: namely 0,1, and 2.

The connection with family replication phenomenon could be following. When the number of handles at the parton surface exceeds 2, the system forms entangled/bound states describable in terms of polygons with handles at vertices. This would be kind of phase transition. Fundamental fermion families with handle number 0,1,2 would be analogous to integers 0,1,2 and the anyonic many-handle states with NE would be analogous to partitions of integers $n > 2$ represented by the prime polygons. They would correspond to the emergence of p-adic cognition. One could not assign NE and cognition with elementary particles but only to more complex objects such as anyonic states associated with large partonic 2-surfaces (perhaps large because they have large Planck constant $h_{eff} = n \times h$) [K80].

Integers $(3, 4, 5)$ as "additive primes" for integers $n \geq 3$: a connection with arithmetic consciousness

The above observations encourage a more detailed study of the decomposition of polygons to smaller polygons as a geometric representation for the partition of integers to a sum of smaller integers. The idea about integers $(3, 4, 5)$ as "additive primes" represented by prime polygons is especially attractive. This leads to a conjecture about NE associated with polygons as quantum correlates of arithmetic consciousness.

1. Motivations

The key idea is to look whether the notion of divisibility and primeness could have practical value in additive arithmetics. 1 is the only prime for addition in general case. $n = 1 + 1 + ...$ is analogous to p^n and all integers are "additive powers" of 1.

What happens if one considers integers $n \geq 3$? The basic motivation is that $n \geq 3$ is represented as a non-degenerate n-gon for $n \geq 3$. Therefore geometric representation of these primes is used in the following. One cannot split triangles from 4-gon and 5-gon. But already for 6-gon one can and obtains 2 triangles. Thus $\{3, 4, 5\}$ would be the additive primes for $n \geq 3$ represented as prime polygons.

The n-gons with $n \in \{3, 4, 5\}$ appear as faces of the Platonic solids! The inclusions of von Neumann algebras known as hyperfinite factors of type II_1 central in TGDs correspond to quantum phases $exp(\pi/n)$ $n = 3, 4, 5....$ Platonic solids correspond to particular finite subgroups of 3-D rotation group, which are in one-one correspondence with simply laced Lie-groups (ADE). There is also a direct connection with the classification of $\mathcal{N} = 2$ super-conformal theories, which seem to be relevant for TGD.

I cannot resist the temptation to mention also a personal reminiscence about a long lasting altered state of consciousness about 3 decades ago. I called it Great Experience and it boosted among other things serious work in order to understand consciousness in terms of quantum physics. One of the mathematical visions was that number 3 is in some sense fundamental for physics and mathematics. I also precognized infinite primes and much later indeed discovered them. I have repeatedly returned to the precognition about number 3 but found no really convincing reason for its unique role although it pops up again and again in physics and mathematics: 3 particle families, 3 colors for quarks, 3 spatial dimensions, 3 quaternionic imaginaryunits, triality for octonions, to say nothing about the role of trinity in mystics and religions. The following provides the first argument for the special role of number 3 that I can take seriously.

2. Partition of integer to additive primes

The problem is to find a partition of an integer to additive primes $3, 4, 5$. The problem can be solved using a representation in terms of $n > 2$-gons as a geometrical visualization. Some general aspects of the representation.

1. The detailed shape of n-gons in the geometric representation of partitions does not matter: they just represent geometrically a partition of integer to a sum. The partition can be regarded as a dynamical process. n-gons splits to smaller n-gons producing a representation for a partition $n = \sum_i n_i$. What this means is easiest to grasp by imagining how polygon can be decomposed to smaller ones. Interestingly, the decompositions of polytopes to smaller ones - triangulations - appear also in Grassmannian twistor approach to $\mathcal{N} = 4$ super Yang Mills theory.

2. For a given partition the decomposition to n-gons is not unique. For instance, integer 12 can be represented by 3 4-gons or 4 3-gons. Integers $n \in \{3,4,5\}$ are special and partitions to these n-gons are in some sense maximal leading to a maximal decoherence as quantum physicist might say.

   The partitions are not unique and there is large number of partitions involving 3-gons,4-gons, 5-gons. The reason is that one can split from n-gons any n_1-gon with $n_1 < n$ except for $n = 3,4,5$.

3. The daydream of non-mathematician not knowing that everything has been very probably done for aeons ago is that one could chose n_1 to be indivisible by 4 and 5, n_2 indivisible by 3 and 5 and n_3 indivisible by 3 and 4 so that one might even hope for having a unique partition. For instance, double modding by 4 and 5 would reduce to double modding of $n_1 \times 3$ giving a non-vanishing result, and one might hope that n_1, n_2 and n_3 could be determined from the double modded values of n_i uniquely. Note that for $n_i \in \{1,2\}$ the number $n = 24 = 2\times3+2\times4+2\times5$ playing key role in string model related mathematics is the largest integer having this kind of representation. One should numerically check whether any general orbit characterized by the above formulas contains a point satisfying the additional number theoretic conditions.

   Therefore the task is to find partitions satisfying these indivisibility conditions. It is however reasonable to consider first general partitions.

4. By linearity the task of finding general partitions (forgetting divisibility conditions) is analogous to that of finding of solutions of non-homogenous linear equations. Suppose that one has found a partition

$$n = n_1 \times 3 + n_2 \times 4 + n_3 \times 5 \leftrightarrow (n_1, n_2, n_3) \ . \tag{10.11.1}$$

   This serves as the analog for the special solution of non-homogenous equation. One obtains a general solutions of equation as the sum $(n_1 + k_1, n_2 + k_1, n_3 + k_3)$ of the special solution and general solution of homogenous equation

$$k_1 \times 3 + k_2 \times 4 + k_3 \times 5 = 0 \ . \tag{10.11.2}$$

   This is equation of plane in N^3 - 3-D integer lattice.
   Using $4 = 3 + 1$ and $5 = 3 + 2$ this gives equations

$$k_2 + 2 \times k_3 = 3 \times m \ , \quad k_1 - k_3 + 4 \times m = 0 \ , \quad m = 0, 1, 2, ... \tag{10.11.3}$$

5. There is periodicity of $3 \times 4 \times 5 = 60$. If (k_1, k_2, k_3, m) is allowed deformation, one obtains a new one with same divisibility properties as the original one as $(k_1 + 60, k_2 - 120, k_3 + 60, m)$. If one does not require divisibility properties for all solutions, one obtains much larger set of solutions. For instance $(k_1, k_2, k_3) = m \times (1, -2, 1)$ defines a line in the plane containing the solutions. Also other elementary moves than (1,-2,1) are possible.

One can identify very simple partitions deserving to be called standard partitions and involve mostly triangles and minimal number of 4- and 5-gons. The physical interpretation is that the coherence is minimal for them since mostly the quantum coherent negentropically entangled units are minimal triangles.

1. One starts from n vertices and constructs n-gon. For number theoretic purposes the shape does not matter and the polygon can be chosen to be convex. One slices from it 3-gons one by one so that eventually one is left with $k \equiv n \ mod \ 3 == 0, 1$ or 2 vertices. For $k = 0$ no further operations are needed. For $k = 1$ *resp.* $k = 2$ one combines one of the triangles and edge associated with 1 *resp.* 2 vertices to 4-gon *resp.* 5-gon and is done. The outcome is one of the partitions

$$n = n_1 \times 3 \ , \quad n = n_1 \times 3 + 4, n = n_1 \times 3 + 5 \tag{10.11.4}$$

These partitions are very simple, and one can easily calculate similar partitions for products and powers. It is easy to write a computer program for the products and powers of integers in terms of these partitions.

2. There is however a uniqueness problem. If n_1 is divisible by 4 or 5 - $n_1 = 4 \times m_1$ or $n_1 = 5 \times m_1$ - one can interpret $n_1 \times 3$ as a collection of m_1 4-gons or 5-gons. Thus the geometric representation of the partition is not unique. Similar uniqueness condition must apply to n_2 and n_3 and is trivially true in above partitions.

   To overcome this problem one can pose a further requirement. If one wants n_1 to be indivisible by 4 and 5 one can transform 2 or or 4 triangles and existing 4-gon or 5-gon or 3 or 6 triangles to 4-gons and 5-gons.

   (a) Suppose $n = n_1 \times 3 + 4$. If n_1 divisible by 4 *resp.* 5 or both, $n_1 - 2$ is not and 4-gon and 2 3-gons can be transformed to 2 5-gons: $(n_1, 1, 0) \rightarrow (n_1 - 2, 0, 2)$. If $n_1 - 2$ is divisible by 5, $n_1 - 3$ is not divisible by either 4 or 5 and 3 triangles can be transformed to 4-gon and 5-gon: $(n_1, 1, 0) \rightarrow (n_1 - 3, 2, 1)$.

   (b) Suppose $n = n_1 \times 3 + 5$. If n_1 divisible by 4 *resp.* 5 or both, $n_1 - 1$ is not and triangle and 5-gon can be transformed to 2 4-gons: $(n_1, 0, 1) \rightarrow (n_1 - 1, 2, 0)$. If $n_1 - 1$ is divisible by 4 or 5, $n_1 - 3$ is not and 3 triangles and 5-gon can be transformed to 2 5-gons and 4-gon: $(n_1, 0, 1) \rightarrow (n_1 - 3, 1, 2)$.

   (c) For $n = n_1 \times 3$ divisible by 4 or 5 or both one can remove only $m \times 3$ triangles, $m \in \{1, 2\}$ since only in these case the resulting $m \times 3$ (9 or 18) vertices can partitioned to a union of 4-gon and 5-gon or of 2 4-gons and 2 5-gons: $(n_1, 0, 0) \rightarrow (n_1 - 3, 1, 1)$ or $(n_1, 0, 0) \rightarrow (n_1 - 6, 2, 2)$.

   These transformations seem to be the minimal transformations allowing to achieve indivisibility by starting from the partition with maximum number of triangles and minimal coherence.

Some further remarks about the partitions satisfying the divisibility conditions are in order.

1. The multiplication of n with partition (n_1, n_2, n_3) satisfying indivisibility conditions by an integer m not divisible by $k \in \{3, 4, 5\}$ gives integer with partition $m \times (n_1, n_2, n_3)$. Note also that if n is not divisible by $k \in \{3, 4, 5\}$ the powers of n, n^k has partition $n^{k-1} \times (n_1, n_2, n_3)$ and this could help to solve Diophantine equations.

2. Concerning the uniqueness of the partition satisfying the indivisibility conditions, the answer is negative. $8 = 3 + 5 = 4 + 4$ is the simplest counter example. Also the m-multiples of 8 such that m is indivisible by 2,3,4,5 serve as counter examples. 60-periodicity implies that for sufficiently large values of n the indivisibility conditions do not fix the partition uniquely. (n_1, n_2, n_3) can be replaced with $(n_1 + 60 + n_2 - 120, n_3 + 60)$ without affecting divisibility properties.

3. Intriguing observations related to 60-periodicity

60-periodicity seems to have deep connections with both music consciousness and genetic code if the TGD inspired model of genetic code is taken seriously code [K87] [L11].

1. The TGD inspired model for musical harmony and genetic involves icosahedron with 20 triangular faces and tetrahedron with 4 triangular faces. The 12 vertices of icosahedron correspond to the 12 notes. The model leads to the number 60. One can say that there are 60 +4 DNA codons and each 20 codon group is 60=20+20+20 corresponds to a subset of aminoacids and 20 DNAs assignable to the triangles of icosahedron and representing also 3-chords of the associated harmony. The remaining 4 DNAs are associated with tetrahedron.

   Geometrically the identification of harmonies is reduced to the construction of Hamiltonian cycles - closed isometrically non-equivalent non-self-intersecting paths at icosahedron going through all 12 vertices. The symmetries of the Hamiltonian cycles defined by subgroups of the icosahedral isometry group provide a classification of harmonies and suggest that also genetic code carries additional information assignable to what I call bio-harmony perhaps related to the expression of emotions - even at the level of biomolecules - in terms of "music" defined as sequences 3-chords realized in terms of triplets of dark photons (or notes) in 1-1 correspondence with DNA codons in given harmony.

2. Also the structure of time units and angle units involves number 60. Hour consists of 60 minutes, which consists of 60 seconds. Could this accident somehow reflect fundamental aspects of cognition? Could we be performing sub-conscious additive arithmetics using partitions of n-gons? Could it be possible to "see" the partitions if they correspond to NE?

4. Could additive primes be useful in Diophantine mathematics?

The natural question is whether it could be number theoretically practical to use "additive primes" $\{3, 4, 5\}$ in the construction of natural numbers $n \geq 3$ rather than number 1 and successor axiom. This might even provide a practical tool for solving Diophantine equations (it might well be that mathematicians have long ago discovered the additive primes).

The most famous Diophantine equation is $x^n + y^n = z^n$ and Fermat's theorem - proved by Wiles - states that for $n > 2$ it has no solutions. Non-mathematician can naïvely ask whether the proposed partition to additive primes could provide an elementary proof for Fermat's theorem and continue to test the patience of a real mathematician by wondering whether the parition for a sum of powers $n > 2$ could be always different from that for single power $n > 2$ perhaps because of some other constraints on the integers involved?

5. Could one identify quantum physical correlates for arithmetic consciousness?

Even animals and idiot savants can do arithmetics. How this is possible? Could one imagine physical correlates for arithmetic consciousness for which product and addition are the fundamental aspects? Is elementary arithmetic cognition universal and analogous to direct sensory experience. Could it reduce at quantum level to a kind of quantum measurement process quite generally giving rise to mental images as outcomes of quantum measurement by repeated state function reduction lasting as long as the corresponding sub-self (mental image) lives?

Consider a partition of integer to a product of primes first. I have proposed a general model for how partition of integer to primes could be experienced directly [L26]. For negentropically entangled state with maximal possible negentropy having entanglement probabilities $p_i = 1/N$, the negentropic primes are factors of N and they could be directly "seen" as negentropic p-adic factors in the adelic decomposition (reals and extensions of various p-adic number fields defined by extension of rationals defined the factors of adele and space-time surfaces as preferred extremals of Kähler action decompose to real and p-adic sectors).

What about additive arithmetics?

1. The physical motivation for n-gons is provided symplectic QFT [K20, K105], which is one aspect of TGD forced by super symplectic conformal invariance having structure of conformal symmetry. Symplectic QFT would be analogous to conformal QFT. The key challenge is to identify symplectic invariants on which the positive and negative energy parts of zero energy states can depend. The magnetic flux through a given area of 2-surface is key invariant of this kind. String world sheet and partonic 2-surfaces are possible identifications for the surface containing the polygon.

   Both the Kähler form associated with the light-cone boundary, which is metrically sphere with constant radius r_M (defining light-like radial coordinate) and the induced Kähler form of CP_2 define these kind of fluxes.

2. One can assign fluxes with string world sheets. In this case one has analog of magnetic flux but over a surface with metric signature (1,-1). Fluxes can be also assigned as magnetic fluxes with partonic 2-surfaces at which fundamental fermions can be said to reside. n fermions defining the vertices at partonic 2-surface define naturally an n-gon or several of them. The interpretation would be as Abelian Wilson loop or equvalently non-integrable phase factor.

3. The polygons are not completely unique but this reflect the possibility of several physical states. n-gon could correspond to NE. The imaginary exponent of Kähler magnetic flux Φ through n-gon is symplectic invariant defining a non-integrable phase factor and defines a multiplicative factor of wave function. When the state decomposes to several polygons, one can uniquely identify the interior of the polygon and thus also the non-integrable phase factor.

   There is however non-uniqueness, when one has only single n-gon since also the complement of n-gon at partonic 2-surface containing now now polygons defines n-gon and the corresponding flux is $\Phi_{tot} - \Phi$. The flux Φ_{tot} is quantized and equal to the integer valued magnetic charge times 2π. The total flux disappears in the imaginary exponent and the non-integrable phase factor for the complementary polygon reduces to complex conjugate of that for polygon. Uniqueness allows only the cosine for an integer multiple of the flux.

The non-integrable phase factor assignable to fermionic polygon would give rise to a correlation between fermions in zero modes invariant under symplectic group. The correlations defined by the n-gons at partonic 2-surfaces would be analogous to that in momentum space implied by the momentum conservation forcing the momenta to form a closed polygon but having totally different origin.

Could it be that the wave functions representing collections of n-gons representing partition of integer to a sum could be experienced directly by people capable of perplexing mathematical feats. The partition to a sum would correspond to a geometric partition of polygon representing partition of positive integer $n \geq 3$ to a sum of integers. Quantum physically it would correspond to NE as a representation of integer.

This might explain number theoretic miracles related to addition of integers in terms of direct "seeing". The arithmetic feats could be dynamical quantum processes in which polygons would decompose to smaller polygons, which would be directly "seen". This would require at least two representations: the original polygon and the decomposed polygon resulting in the state function reduction to the opposite boundary of CD. An ensemble of arithmetic sub-selves would seem to be needed. NMP does not seem to favour this kind of partition since negentropy is reduced but if its time reversal occurs in geometric time direction opposite to that of self it might look like partition for the self having sub-self as mental image.

10.12 p-Adicizable discrete variants of classical Lie groups and coset spaces in TGD framework

In TGD framework p-adicization and adelization are carried out at all levels of geometry: embedding space, space-time and WCW. Adelization at the level of state spaces requires that it is common from all sectors of the adele and has as coefficient field an extension of rationals allowing both real and p-adic interpretations: the sectors of adele give only different views about the same quantum state.

In the sequel the recent view about the p-adic variants of embedding space, space-time and WCW is discussed. The notion of finite measurement resolution reducing to number theoretic existence in p-adic sense is the fundamental notion. p-Adic geometries replace discrete points of discretization with p-adic analogs of monads of Leibniz making possible to construct differential calculus and formulate p-adic variants of field equations allowing to construct p-adic cognitive representations for real space-time surfaces.

This leads to a beautiful construction for the hierarchy of p-adic variants of embedding space inducing in turn the construction of p-adic variants of space-time surfaces. Number theoretical existence reduces to conditions demanding that all ordinary (hyperbolic) phases assignable to (hyperbolic) angles are expressible in terms of roots of unity (roots of e).

For $SU(2)$ one obtains as a special case Platonic solids and regular polygons as preferred p-adic geometries assignable also to the inclusions of hyperfinite factors [K120, K39]. Platonic solids represent idealized geometric objects ofthe p-adic world serving as a correlate for cognition as contrast to the geometric objects of the sensory world relying on real continuum.

In the case of causal diamonds (CDs) - the construction leads to the discrete variants of Lorentz group $SO(1,3)$ and hyperbolic spaces $SO(1,3)/SO(3)$. The construction gives not only the p-adicizable discrete subgroups of $SU(2)$ and $SU(3)$ but applies iteratively for all classical Lie groups meaning that the counterparts of Platonic solids are countered also for their p-adic coset spaces. Even the p-adic variants of WCW might be constructed if the general recipe for the construction of finite-dimensional symplectic groups applies also to the symplectic group assignable to $\Delta CD \times CP_2$.

The emergence of Platonic solids is very remarkable also from the point of view of TGD inspired theory of consciousness and quantum biology. For a couple of years ago I developed a model of music harmony [K87] [L11] relying on the geometries of icosahedron and tetrahedron. The basic observation is that 12-note scale can be represented as a closed curve connecting nearest number points (Hamiltonian cycle) at icosahedron going through all 12 vertices without self intersections. Icosahedron has also 20 triangles as faces. The idea is that the faces represent 3-chords for a given harmony characterized by Hamiltonian cycle. Also the interpretation terms of 20 amino-acids identifiable and genetic code with 3-chords identifiable as DNA codons consisting of three letters is highly suggestive.

One ends up with a model of music harmony predicting correctly the numbers of DNA codons coding for a given amino-acid. This however requires the inclusion of also tetrahedron. Why icosahedron should relate to music experience and genetic code? Icosahedral geometry and its dodecahedral dual as well as tetrahedral geometry appear frequently in molecular biology but its appearance as a preferred p-adic geometry is what provides an intuitive justification for the model of genetic code. Music experience involves both emotion and cognition. Musical notes could code for the points of p-adic geometries of the cognitive world. The model of harmony in fact generalizes. One can assign Hamiltonian cycles to any graph in any dimension and assign chords and harmonies with them. Hence one can ask whether music experience could be a form of p-adic geometric cognition in much more general sense.

The geometries of biomolecules brings strongly in mind the geometry p-adic space-time sheets. p-Adic space-time sheets can be regarded as collections of p-adic monad like objects at algebraic space-time points common to real and p-adic space-time sheets. Monad corresponds to p-adic units with norm smaller than unit. The collections of algebraic points defining the positions of monads and also intersections with real space-time sheets are highly symmetric and determined by the discrete p-adicizable subgroups of Lorentz group and color group. When the subgroup of the rotation group is finite one obtains polygons and Platonic solids. Bio-molecules typically consists of this kind of structures - such as regular hexagons and pentagons - and could be seen as cognitive representations of these geometries often called sacred! I have proposed this idea long time ago and the discovery of the recipe for the construction of p-adic geometries gave a justification for this idea.

10.12.1 p-Adic variants of causal diamonds

To construct p-adic variants of space-time surfaces one must construct p-adic variants of the embedding space. The assumption that the p-adic geometry for the embedding space induces p-adic geometry for sub-manifolds implies a huge simplification in the definition of p-adic variants of preferred extremals. The natural guess is that real and p-adic space-time surfaces gave algebraic points as common: so that the first challenge is to pick the algebraic points of the real space-time surface. To define p-adic space-time surface one needs field equations and the notion of p-adic continuum and by assigning to each algebraic point a p-adic continuum to make it monad, one can solve p-adic field equations inside these monads.

The idea of finite measurement resolution suggests that the solutions of p-adic field equations inside monads are arbitrary. Whether this is consistent with the idea that same solutions of field equations can be interpreted either p-adically or in real sense is not quite clear. This would be guaranteed if the p-adic solution has same formal representation as the real solution in the vicinity of given discrete point - say in terms of polynomials with rational coefficients and coordinate variables which vanish for the algebraic point.

Real and p-adic space-time surfaces would intersect at points common to all number fields for given adele: cognition and sensory worlds intersect not only at the level of WCW but also at the level of space-time. I had already considered giving up the latter assumption but it seems to be necessary at least for string world sheets and partonic 2-surfaces if not for entire space-time surfaces.

General recipe

The recipe would be following.

1. One starts from a discrete variant of $CD \times CP_2$ defined by an appropriate discrete symmetry groups and their subgroups using coset space construction. This discretization consists of points in finite-dimensional extension of p-adics induced by an extension of rationals. These points are assumed to be in the intersection of reality and p-adicities at space-time level - that is common for real and p-adic space-time surfaces. Cognitive representations in the real

world are thus discrete and induced by the intersection. This is the original idea which I was ready to give up as the vision about discretization at WCW level allowing to solve all problems related to symmetries emerged. At space-time level the p-adic discretization reduces symmetry groups to their discrete subgroups: cognitive representations unavoidably break the symmetries. What is important the distance between discrete p-adic points labelling monads is naturally their real distance. This fixes metrically real-p-adic/sensory-cognitive correspondence.

2. One replaces each point of this discrete variant $CD \times CP_2$ with p-adic continuum defined by an algebraic extension of p-adics for the adele considered so that differentiation and therefore also p-adic field equations make sense. The continuum for given discrete point of $CD_d \times CP_{2,d}$ defines kind of Leibnizian monad representing field equations p-adically. The solution decomposes to p-adically differentiable pieces and the global solution of field equations makes sense since it can be interpreted in terms of pseudo-constants. p-Adicization means discretization but with discrete points replaced with p-adic monads preserving also the information about local behavior. The loss of well-ordering inside p-adic monad reflects its loss due to the finiteness of measurement resolution.

3. The distances between monads correspond to their distances for real variant of $CD \times CP_2$. Are there natural restrictions on the p-adic sizes of monads? Since p-adic units are in question that size in suitable units is $p^{-N} < 1$. It would look natural that the p-adic size of the is smaller than the distance to the nearest monad. The denser the discretization is, the larger the value of N would be. The size of the monad decreases at least like $1/p$ and for large primes assignable to elementary particles ($M_{127} = 2^{127} - 1$) is rather small. The discretizations of the subgroups share the properties of the group invariant geometry of groups so that they are to form a regular lattice like structure with constant distance to nearest neighbors. At the embedding level therefore p-adic geometries are extremely symmetric. At the level of space-time geometries only a subset of algebraic points is picked and the symmetry tends to be lost.

CD degrees of freedom

Consider first CD degrees of freedom.

1. For M^4 one has 4 linear coordinates. Should one p-adicize these or should one discretize CDs defined as intersections of future and past directed light-cones and strongly suggested by ZEO. CD seems to represent the more natural option. The construction of a given CD suggests that one should replace the usual representation of manifold as a union of overlapping regions with intersection of two light-cones with coordinates related in the intersection as in the case of ordinary manifold: $\cup \rightarrow \cap$.

2. For a given light-cone one must introduce light-cone proper time a, hyperbolic angle η and two angle coordinates (θ, ϕ). Light-cone proper time a is Lorentz invariant and corresponds naturally to an ordinary p-adic number of more generally to a p-adic number in algebraic extension which does not involve phases.

   The two angle coordinates (θ, ϕ) parameterizing S^2 can be represented in terms of phases and discretized. The hyperbolic coordinate can be also discretized since e^p exists p-adically, and one obtains a finite-dimensional extension of p-adic numbers by adding roots of e and its powers. e is completely exceptional in that it is p-adically an algebraic number.

3. This procedure gives a discretization in angle coordinates. By replacing each discrete value of angle by p-adic continuum one obtains also now the monad structure. The replacement with continuum means the replacement

$$U_{m,n} \equiv exp(i2\pi m/n) \rightarrow U_{m,n} \times exp(i\phi) \ , \tag{10.12.1}$$

   where ϕ is p-adic number with norm $p^{-N} < 1$ It can also belong to an algebraic extension of p-adic numbers. Building the monad is like replacing in finite measurement the representative point of measurement resolution interval with the entire interval. By finite measurement resolution one cannot fix the order inside the interval. Note that one obtains a hierarchy of subgroups depending on the upper bound p^{-n} for the modulus. For $p \ mod \ 4 = 1$ imaginary unit exist as ordinary p-adic number and for $p \ mod \ 4 = 3$ in an extension including $\sqrt{-1}$.

4. For the hyperbolic angle one has

$$E_{m,n} \equiv exp(m/n) \rightarrow E_{m,n} \times exp(\eta) \tag{10.12.2}$$

with the ordinary p-adic number η having norm $p^{-N} < 1$. Lorentz symmetry is broken to a discrete subgroup: this could be interpreted in terms of finite cognitive resolution. Since e^p is p-adic number also hyperbolic angle has finite number of values and one has compactness in well-defined sense although in real context one has non-compactness.

In cosmology this discretization means quantization of redshift and thus recession velocities. A concise manner to express the discretization to say that the cosmic time constant hyperboloids are discrete variants of Lobatchevski spaces $SO(3,1)/SO(3)$. The spaces appear naturally in TGD inspired cosmology.

5. The coordinate transformation relating the coordinates in the two intersecting coordinate patches maps hyperbolic and ordinary phases to each other as such. Light-cone proper time coordinates are related in more complex manner. $a_+^2 = t^2 - r^2$ and $a_-^2 = (t-T)^2 - r^2$ are related by $a_+^2 - a_-^2 = 2tT - T^2 = 2a_+cosh(\eta)T - T^2$.

This leads to a problem unless one allows a_+ and a_- to belong to an algebraic extension containing the roots of e making possible to define hyperbolic angle. The coordinates $a_\pm$ can also belong to a larger extension of p-adic numbers. The expectation is that one obtains an infinite hierarchy of algebraic extensions of rationals involving besides the phases also other non-Abelian extension parameters. It would seem that the Abelian extension for phases and the extension for a must factorize somehow. Note also that the expression of a_+ in terms of a_- given by

$$a_+ = -cosh(\eta)T \pm \sqrt{sinh^2(\eta)T^2 + a_-^2} \ . \tag{10.12.3}$$

This expression makes sense p-adically for all values of a_- if one can expand the square root as a converning power series with respect to a_-. This is true if $a_-/sinh(\eta)T$ has p-adic norm smaller than 1.

6. What about the boundary of CD which corresponds to a coordinate singularity? It seems that this must be treated separately. The boundary has topology $S^2 \times R_+$ and S^2 can be p-adicized as already explained. The light-like radial coordinate $r = asinh(\eta)$ vanishes identically for finite values of $sinh(\eta)$. Should one regard r as ordinary p-adic number? Or should one think that entire light-one boundary corresponds to single point $r = 0$? The discretization of r is in powers of a roots of e is very natural so that each power $E_{m,n}$ corresponds to a p-adic monad. If now powers $E_{m,n}$ are involved, one obtains just the monad at $r = 0$.

The construction of quantum TGD leads to the introduction of powers $exp(log(r/r_0)s)$, where s is zero of Riemann Zeta [L14]. These make sense p-adically if $u = log(r/r_0)$ has p-adic norm smaller than unity and s makes sense p-adically. The latter condition demanding that the zeros are algebraic numbers is quite strong.

10.12.2 Construction for $SU(2)$, $SU(3)$, and classical Lie groups

In the following the detailed construction for $SU(2)$, $SU(3)$, and classical Lie groups will be sketched.

Subgroups of $SU(2)$ having p-adic counterparts

In the case $U(1)$ the subgroups defined by roots of unity reduce to a finite group Z_n. What can one say about p-adicizable discrete subgroups of $SU(2)$?

1. To see what happens in the case of $SU(2)$ one can write $SU(2)$ element explicitly in quaternionic matrix representation

$$(\theta, n) \equiv cos(\theta)Id + sin(\theta)\sum_i n_i I_i \ . \tag{10.12.4}$$

   Here Id is quaternionic real unit and I_i are quaternionic imaginary units. $n = (n_1, n_2, n_2)$ is a unit vector representable as $(cos(\phi), sin(\phi)cos(\psi), sin(\phi)sin(\psi))$. This representation exists p-adically if the phases $exp(i\theta)$, $exp(i(\phi)$ and $exp(i\psi)$ exist p-adically so that they must be roots of unity.

   The geometric interpretation is that n defines the direction of rotation axis and θ defines the rotation angle.

2. This representation is not the most general one in p-adic context. Suppose that one has two elements of this kind characterized by (θ_i, n_i)such that the rotation axes are different. From the multiplication table of quaternions one has for the product (θ_{12}, n_{12}) of these

$$cos(\theta_{12}) \ = \ cos(\theta_1)cos(\theta_2) - sin(\theta_1)sin(\theta_2)n_1 \cdot n_2 \ . \tag{10.12.5}$$

   This makes sense p-adically if the inner product $cos(\chi) \equiv n_1 \cdot n_2$ corresponds to root of unity in the extension of rationals used. Therefore the angle between the rotation axes is number theoretically quantized in order that p-adicization works.

   One can solve θ_{12} from the above equation in real context but in the general case it does not correspond to $U_{m,n}$. This is not however a problem from p-adic point of view. The reduction to a root of unity is true only in some special cases. For $n_1 = n_2$ the group generated by the products reduces a discrete $Z_n \subset U(1)$ generated by a root of unity. If n_1 and n_2 are orthogonal the angle between rotation axes corresponds trivially to a root of unity. In this case one has the isometries of cube. For other Platonic solids the angles between rotation axes associated with various $U(1)$ subgroups generating the entire sub-group are fixed by their geometries. The rotation angles correspond to $n = 3$ for tetrahedron and icosahedron and $n = 5$ dodecahedron and for $n = 3$. There is also duality between cube and octahedron and icosahedron and dodecahedron.

3. Platonic solids can be geometrically seen as discretized variants of $SU(2)$ and it seems that they correspond to finite discrete subgroups of $SU(2)$ defining $SU(2)_d$. Platonic sub-groups appear in the hierarchy of Jones inclusions. The other finite subgroups of $SU(2)$ appearing in this hierarchy act on polygons of plane and being generated by Z_n and rotations around the axes of plane and would naturally correspond to discrete U(1) sub-groups of $SU(2)$ and in a well-defined sense to a degenerate situation. By Mc-Kay correspondence all these groups correspond to ADE type Lie groups. These subgroups define finite discretizations of $SU(2)$ and S^2. p-Adicization would lead directly to the hierarchy of inclusions assigned also with the hierarchy of sub-algebras of super-symplectic algebra characterized by the hierarchy of Planck constants.

4. There are also p-adicizable discrete subgroups, which are infinite. By taking two rotations with angles which correspond to root of unity with rotation axes, whose mutual angle corresponds to root of unity one can generate an infinite discrete subgroup of $SU(2)$ existing in p-adic sense. More general discrete U(1) subgroups are obtained by taking n rotation axes with mutual angles corresponding to roots of unity and generating the subgroup from these. In case of Platonic solids this gives a finite subgroup.

Construction of p-adicizable discrete subgroups of CP_2

The construction of p-adic CP_2 proceeds along similar lines.

1. In the original ultra-naïve approach the local p-adic metric of CP_2 is obtained by a purely formal replacement of the ordinary metric of CP_2 with its p-adic counterpart and it defines the CP_2 contribution to induced metric. This makes sense since Kähler function is rational

function and components of CP_2 metric and spinor connection are rational functions. This allows to formulate p-adic variants of field equations. This description is however only local. It says nothing about global aspects of CP_2 related to the introduction of algebraic extension of p-adic numbers.

One should be able to realize the angle coordinates of CP_2 in a physically acceptable manner. The coordinates of CP_2 can be expressed by compactness in terms of trigonometric functions, which suggests a realization of them as phases for the roots of unity. The number of points depends on the Abelian extension of rationals inducing that of p-adics which is chosen. This gives however only discrete version of p-adic CP_2 serving as a kind of spine. Also the flesh replacing points with monads is needed.

2. A more profound approach constructs the algebraic variants of CP_2 as discrete versions of the coset space $CP_2 = SU(3)/U(2)$. One restricts the consideration to an algebraic subgroup of $SU(3)_d$ with elements, which are 3×3 matrices with components, which are algebraic numbers in the extension of rationals. Since they are expressible in terms of phases one can express them in terms of roots of unity. In the same manner one identifies $U(2)_d \subset SU(3)_d$. $CP_{2,d}$ is the coset space $SU(3)_D/U(2)_d$ of these. The representative of a given coset is a point in the coset and expressible in terms of roots of unity.

3. The construction of the p-adicizable subgroups of $SU(3)$ suggests a generalization. Since $SU(3)$ is 8-D and Cartan algebra is 2-D the coset space is 6-dimensional flag-manifold $F = SU(3)/U(1) \times U(1)$ with coset consisting of elements related by automorphism $g \equiv hgh^{-1}$. F defines the twistor space of CP_2 characerizing the choices for the quantization axes of color quantum numbers. The points of F should be expressible in terms of phase angles analogous to the angle defining rotation axis in the case of SU(2).

   In the case of $SU(2)$ n U(1) subgroups with specified rotation axes with p-adically existing mutual angles are considered. The construction as such generates only $SU(2)_d$ subgroup which can be trivially extended to $U(2)_d$. The challenge is to proceed further.

   Cartan decomposition of the Lie algebra (see `http://tinyurl.com/y7cjbm4c`) seems to provide a solution to the problem. In the case of $SU(3)$ it corresponds to the decomposition to U(2) sub-algebra and its complement. One could use the decomposition $G = KAK$ where K is maximal compact subgroup. A is exponentiation of the maximal Abelian subalgebra, which is 3-dimensional for CP_2. By Abelianity the p-adicization of A in terms of roots of unity simple. The image of A in G/K is totally geodesic sub-manifold. In the recent case one has $G/Ki = CP_2$ so that the image of A is geodesic sphere S^2. This decomposition implies the representation using roots of unity. The construction of discrete p-adicizable subgroups of $SU(n)$ for $n > 3$ would continue iteratively.

4. Since the construction starts from $SU(2)$, $U(1)$, and Abelian groups, and proceeds iteratively it seems that Platonic solids have counterparts for all classical Lie groups containing $SU(2)$. Also level p-adicizable discrete coset spaces have analogous of Platonic solids.

The results imply that $CD \times CP_2$ is replaced by a discrete set of p-adic monads at a given level of hierarchy corresponding to the finite cognitive resolution.

Generalization to other groups

The above argument demonstrates that p-adicization works iteratively for $SU(n)$ and thus for $U(n)$. For finite-dimensional symplectic group $Sp(n, R)$ the maximal compact sub-group is $U(n)$ so that KAK construction should work also now. $SO(n)$ can be regarded as subgroup of $SU(n)$ so that the p-adiced discretrized variants of maximal compact subgroups should be constructible and KAK give the groups. The inspection of the table of the Wikipedia article (see `http://tinyurl.com/j44639q`) encourages the conjecture that the construction of $SU(n)$ and $U(n)$ generalizes to all classical Lie groups.

This construction could simplify enormously also the p-adicization of WCW and the theory would discretize even in non-compact degrees of freedom. The non-zero modes of WCW correspond to the symplectic group for $\delta M^4 \times CP_2$, and one might hope that the p-adicization works also at the limit of infinite-dimensional symplectic group with $U(\infty)$ taking the role of K.

10.13 Some layman considerations related to the fundamentals of mathematics

I am not a mathematician and therefore should refrain from consideration of anything related to fundamentals of mathematics. In the discussions with Santeri Satama I could not avoid the temptation to break this rule. I however feel that I must confess my sins and in the following I will do this.

1. Gödel's problematics is shown to have a topological analog in real topology, which however disappears in p-adic topology which raises the question whether the replacement of the arithmetics of natural numbers with that of p-adic integers could allow to avoid Gödel's problematics.
2. Number theory looks from the point of view of TGD more fundamental than set theory and inspires the question whether the notion of algebraic number could emerge naturally from TGD. There are two ways to understand the emergence of algebraic numbers: the hierarchy of infinite primes in which ordinary primes are starting point and the arithmetics of Hilbert spaces with tensor product and direct sum replacing the usual arithmetic operations. Extensions of rationals give also rise to cognitive variants of n-D spaces.
3. The notion of empty set looks artificial from the point of view of physicist and a possible cure is to take arithmetics as a model. Natural numbers would be analogous to nonempty sets and integers would correspond to pairs of sets (A, B), $A \subset B$ or $B \subset A$ with equivalence $A, B) \equiv (A \cup C, B \cup C)$. Empty set would correspond to pairs (A, A). In quantum context the generalization of the notion of being member of set $a \in A$ suggests a generalization: being an element in set would generalize to being single particle state which in general is de-localized to the set. Subsets would correspond to many-particle states. The basic operation would be addition or removal of element represented in terms of oscillator operator. The order of elements of set does not matter: this would generalize to bosonic and fermionic many particle states and even braid statistics can be considered. In bosonic case one can have multiple points - kind of Bose-Einstein condensate.
4. One can also start from finite-D Hilbert space and identify set as the collection of labels for the states. In infinite-D case there are two cases corresponding to separable and non-separable Hilbert spaces. The condition that the norm of the state is finite without infinite normalization constants forces selection of de-localized discrete basis in the case of a continuous set like reals. This inspires the question whether the axiom of choice should be given up. One possibility is that one can have only states localized to finite or at least discrete set of points which correspond points with coordinates in an extension of rationals.

10.13.1 Geometric analog for Gödel's problematics

Godel's problematics involves statements which cannot be proved to be true or false or are simultaneously true and false. This problematics has also a purely geometric analog in terms of set theoretic representation of Boolean algebras when real topology is used but not when p-adic topology is used.

The natural idea is that Boolean algebra is realized in terms of open sets such that the negation of statement corresponds to the complement of the set. In p-adic topologies open sets are simultaneously also closed and there are no boundaries: this makes them and - more generally Stone spaces - ideal for realizing Boolean algebra set theoretically. In real topology the complement of open set is closed and therefore not open and one has a problem.

Could one circumvent the problem somehow?

1. If one replaces open sets with their closures (the closure of open set includes also its boundary, which does not belong to the open set) and closed complements of open sets, the analog of Boolean algebra would consist of closed sets. Closure of an open set and the closure of its open complement - stament and its negation - share the common boundary. Statement and its negation would be simultaneously true at the boundary. This strange situation reminds of Russell's paradox but in geometric form.

2. If one replaces the closed complements of open sets with their open interiors, one has only open sets. Now the sphere would represent statement about which one cannot say whether it is true or false. This would look like Gödelian sentence but represented geometrically.

   This leads to an already familiar conclusion: p-adic topology is natural for the geometric correlates of cognition, in particular Boolean cognition. Real topology is natural for the geometric correlates of sensory experience.

3. Gödelian problematics is encountered already for arithmetics of natural numbers although naturals have no boundary in the discrete topology. Discrete topology does not however allow well-ordering of natural numbers crucial for the definition of natural number. In the induced real topology one can order them and can speak of boundaries of subsets of naturals. The ordering of natural numbers by size reflects the ordering of reals: it is very difficult to think about discrete without implicitly bringing in the continuum.

   For p-adic integers the induced topology is p-adic. Is Gödelian problematics is absent in p-adic Boolean logic in which set and its complement are both open and closed. If this view is correct, p-adic integers might replace naturals in the axiomatics of arithmetics. The new element would be that most p-adic integers are of infinite size in real sense. One has a natural division of them to cognitively representable ones finite also in real sense and non-representable ones infinite in real sense. Note however that rationals have periodic pinary expansion and can be represented as pairs of finite natural numbers.

In algebraic geometry Zariski topology in which closed sets correspond to algebraic surfaces of various dimensions, is natural. Open sets correspond to their complements and are of same dimension as the embedding space. Also now one encounters asymmetry. Could one say that algebraic surfaces characterize "representable" (="geometrically provable"?) statements as elements of Boolean algebra and their complements the non-representable ones? 4-D space-time (as possibly associative/co-associative) algebraic variety in 8-D octonionic space would be example of representable statement. Finite unions and intersections of algebraic surfaces would form the set of representable statements. This new-to-me notion of representability is somehow analogous to provability or demonstrability.

10.13.2 Number theory from quantum theory

Could one define or at least represent the notion of number using the notions of quantum physics? A natural starting point is hierarchy of extensions of rationals defining hierarchy of adeles. Could one obtain rationals and their extensions from simplest possible quantum theory in which one just constructs many particle states by adding or removing particles using creation and annihilation operators?

How to obtain rationals and their extensions?

Rationals and their extensions are fundamental in TGD. Can one have quantal construction for them?

1. One should construct rationals first. Suppose one starts from the notion of finite prime as something God-given. At the first step one constructs infinite primes as analogs for many-particle states in super-symmetric arithmetic quantum field theory [K103]. Ordinary primes label states of fermions and bosons. Infinite primes as the analogs of free many-particle states correspond to rationals in a natural manner.

2. One obtains also analogs of bound states which are mappable to irreducible polynomials, whose roots define algebraic numbers. This would give hierarchy of algebraic extensions of rationals. At higher levels of the hierarchy one obtains also analogs of prime polynomials with number of variables larger than 1. One might say that algebraic geometry has quantal representation. This might be very relevant for the physical representability of basic mathematical structures.

Arithmetics of Hilbert spaces

The notions of prime and divisibility and even basic arithmetics emerge also from the tensor product and direct sum for Hilbert spaces. Hilbert spaces with prime dimension do not decompose to tensor products of lower-dimensional Hilbert spaces. One can even perform a formal generalization of the dimension of Hilbert space so that it becomes rational and even algebraic number.

For some years ago I indeed played with this thought but at that time I did not have in mind reduction of number theory to the arithemetics of Hilbert spaces. If this really makes sense, numbers could be replaced by Hilbert spaces with product and sum identified as tensor product and direct sum!

Finite-dimensional Hilbert space represent the analogs of natural numbers. The analogs of integers could be defined as pairs (m,n) of Hilbert spaces with spaces (m,n) and $(m+r,n+r)$ identified (this space would have dimension $m-n$. This identification would hold true also at the level of states. Hilbert spaces with negative dimension would correspond to pairs with $(m-n)<0$: the canonical representives for m and $-m$ would be $(m,0)$ and $(0,m)$. Rationals can be defined as pairs (m,n) of Hilbert spaces with pairs (m,n) and (km,kn) identified. These identifications would give rise to kind of gauge conditions and canonical representatives for m and $1/m$ are $(m,1)$ and $(1,m)$.

What about Hilbert spaces for which the dimension is algebraic number? Algebraic numbers allow a description in terms of partial fractions and Stern-Brocot (S-B) tree (see `http://tinyurl.com/yb6ldekq` and `http://tinyurl.com/yc6hhboo`) containing given rational number once. S-B tree allows to see information about algebraic numbers as constructible by using an algorithm with finite number of steps, which is allowed if one accepts abstraction as basic aspect of cognition. Algebraic number could be seen as a periodic partial fraction defining an infinite path in S-B tree. Each node along this path would correspond to a rational having Hilbert space analog. Hilbert space with algebraic dimension would correspond to this kind of path in the space of Hilbert spaces with rational dimension. Transcendentals allow identification as non-pediodic partial fraction and could correspond to non-periodic paths so that also they could have Hilbert spaces counterparts.

How to obtain the analogs higher-D spaces?

Algebraic extensions of rationals allow cognitive realization of spaces with arbitrary dimension identified as algebraic dimension of extension of rationals.

1. One can obtain n-dimensional spaces (in algebraic sense) with integer valued coordinates from n-D extensions of rationals. Now the n-tuples defining numbers of extension and differing by permutations are not equivalent so that one obtains n-D space rather than n-D space divided by permutation group S_n. This is enough at the level of cognitive representations and could explain why we are able to imagine spaces of arbitrary dimension although we cannot represent them cognitively.
2. One obtains also Galois group and orbits of set A of points of extension under Galois group as $G(A)$. One obtains also discrete coset spaces G/H and alike. These do not have any direct analog in the set theory. The hierarchy of Galois groups would bring in discrete group theory automatically. The basic machinery of quantum theory emerges elegantly from number theoretic vision.
3. In octonionic approach to quantum TGD one obtains also hierarchy of extensions of rationals since space-time surface correspond zero loci for RE or IM for octonionic polynomials obtained by algebraic continuation from real polynomials with coeffficients in extension of rationals [**?**].

10.13.3 Could quantum set theory make sense?

In the following my view point is that of quantum physicist fascinated by number theory and willing to reduce set theory to what could be called called quantum set theory. It would follow from physics as generalised number theory (adelic physics) and have ordinary set theory as classical correlate.

1. From the point of quantum physics set theory and the notion of number based on set theory look somewhat artificial constructs. Nonempty set is a natural concept but empty set and

set having empty set as element used as basic building brick in the construction of natural numbers looks weird to me.

2. From TGD point of view it would seem that number theory plus some basic pieces of quantum theory might be more fundamental than set theory. Could set theory emerge as a classical correlate for quantum number theory already considered and could quantal set theory make sense?

Quantum set theory

What quantum set theory could mean? Suppose that number theory-quantum theory connection really works. What about set theory? Or perhaps its quantum counterpart having ordinary set theory as a classical correlate?

1. A purely quantal input to the notion of set would be replacement of points delocalized states in the set. A generic single particle quantum state as analog of element of set would not be localized to a single element of set. The condition that the state has finite norm implies in the case of continuous set like reals that one cannot have completely localized states. This would give quantal limitation to the axiom of choice. One can have any discrete basis of state functions in the set but one cannot pick up just one point since this state would have infinite norm.

   The idea about allowing only say rationals is not needed since there is infinite number of different choices of basis. Finite measurement resolution is however unvoidable. An alternative option is restriction of the domains of wave functions to a discrete set of points. This set can be chosen in very many ways and points with coordinates in extension of rationals are very natural and would define cognitive representation.

2. One can construct also the analogs of subsets as many-particle states. The basic operation would be addition/removal of a particle from quantum state represented by the action of creation/annihilation operator.

   Bosonic states would be invariant under permutations of single particle states just like set is the equivalence class for a collection of elements $(a_1, ..., a_n)$ such that any two permutations are equivalent. Quantum set theory would however bring in something new: the possibility of both bosonic and fermionic statistics. Permutation would change the state by phase factor -1. One would have fermionic and bosonic sets. For bosonic sets one could have multiplet elements ("Bose-Einstein condensation"): in the theory of surfaces this could allow multiple copies of the same surface. Even braid statistics is possible. The phase factor in permutation could be complex. Even non-commutative statistics can be considered.

   Many particle states formed from particles, which are not identical are also possible and now the different particle types can be ordered. On obtains n-ples decomposing to ordered K-ple of n_i-ples, which are consist of identical particles and are quantum sets. One could talk about K-sets as a generalization of set as analogs of classical sets with K-colored elements. Group theory would enter into the picture via permutation groups and braid groups would bring in braid statistics. Braids strands would have K colors.

How to obtain classical set theory?

How could one obtain classical set theory?

1. Many-particle states represented algebraically are detected in lab as sets: this is quantum classical correspondence. This remains to me one of the really mysterious looking aspects in the interpretation of quantum field theory. For some reason it is usually not mentioned at all in popularizations. The reason is probably that popularization deals typically with wave mechanics but not quantum field theory unless it is about Higgs mechanism, which is the weakest part of quantum field theory!

2. From the point of quantum theory empty set would correspond to vacuum. It is not observable as such. Could the situation change in the presence of second state representing the environment? Could the fundamental sets be always *non-empty* and correspond to states with non-vanishing particle number. Natural numbers would correspond to eigenvalues of an observable telling the cardinality of set. Could representable sets be like natural numbers?

3. Usually integers are identified as pairs of natural numbers (m, n) such that integer corresponds to $m - n$. Could the set theoretic analog of integer be a pair (A, B) of sets such that A is subset of B or vice versa? Note that this does not allow pairs with disjoint members. (A, A) would correspond to empty set. This would give rise to sets (A, B) and their "antisets" (B, A) as analogs of positive and negative integers.

   One can argue that antisets are not physically realizable. Sets and antisets would have as analogs two quantizations in which the roles of oscillator operators and their hermitian conjugates are changed. The operators annihilating the ground state are called annilation operators. Only either of these realization is possible but not both simultaneously.

   In ZEO one can ask whether these two options correspond to positive and negative energy parts of zero energy states or to the states with state function reduction at either boundary of CD identified as correlates for conscious entities with opposite arrows of geometric time (generalized Zeno effect).

4. The cardinality of set, the number of elements in the set, could correspond to eigenvalue of observable measuring particle number. Many-particle states consisting of bosons or fermions would be analogs for sets since the ordering does not matter. Also braid statistics would be possible.

   What about cardinality as a p-adic integer? In p-adic context one can assign to integer m, integer $-m$ as $m \times (p-1) \times (1 + p + p^2 + ...)$. This is infinite as real integer but finite as p-adic integer. Could one say that the antiset of m-element as analog of negative integer has cardinality $-m = m(p-1)(1+p+p^2+..)$. This number does not have cognitive representation since it is not finite as real number but is cognizable.

   One could argue that negative numbers are cognizable but not cognitively representable as cardinality of set? This representation must be distinguished from cognitive representations as a point of embedding space with coordinates in extension of rationals. Could one say that antisets and empty set as its own antiset can be cognized but cannot be cognitively represented?

Nasty mathematician would ask whether I can really start from Hilbert space of state functions and deduce from this the underlying set. The elements of set itself should emerge from this as analogs of completely localized single particle states labelled by points of set. In the case of finite-dimensional Hilbert space this is trivial. The number of points in the set would be equal to the dimension of Hilbert space. In the case of infinite-D Hilbert space the set would have infinite number of points.

Here one has two views about infinite set. One has both separable (infinite-D in discrete sense: particle in box with discrete momentum spectrum) and non-separable (infinite-D in real sense: free particle with continuous momentum spectrum) Hilbert spaces. In the latter case the completely localized single particle states would be represented by delta functions divided by infinite normalization factors. They are routinely used in Dirac's bra-ket formalism but problems emerge in quantum field theory.

A possible solution is that one weakens the axiom of choice and accepts that only discrete points set (possibly finite) are cognitively representable and one has wave functions localized to discrete set of points. A stronger assumption is that these points have coordinates in extension of rationals so that one obtains number theoretical universality and adeles. This is TGD view and conforms also with the identification of hyper-finite factors of type II_1 as basic algebraic objects in TGD based quantum theory as opposed to wave mechanics (type I) and quantum field theory (type III). They are infinite-D but allow excellent approximation as finite-D objects.

This picture could relate to the notion of non-commutative geometry, where set emerges as spectrum of algebra: the points of spectrum label the ideals of the integer elements of algebra.

10.14 Abelian Class Field Theory And TGD

The context leading to the discovery of adeles (`http://tinyurl.com/64pgerm`) was so called Abelian class field theory. Typically the extension of rationals means that the ordinary primes decompose to the primes of the extension just like ordinary integers decompose to ordinary primes. Some primes can appear several times in the decomposition of ordinary non-square-free integers and similar phenomenon takes place for the integers of extension. If this takes place one says that the original prime is ramified. The simplest example is provided Gaussian integers $Q(i)$. All odd primes are unramified and primes $p\ mod\ 4 = 1$ they decompose as $p = (a + ib)(a - ib)$ whereas primes $p\ mos\ 4 = 3$ do not decompose at all. For $p = 2$ the decomposition is $2 = (1 + i)(1 - i) = -i(1 + i)^2 = i(1 - i)^2$ and is not unique $\{\pm 1, \pm i\}$ are the units of the extension. Hence $p = 2$ is ramified.

There goal of Abelian class field theory (see http://tinyurl.com/y8aefmg2) is to understand the complexities related to the factorization of primes of the original field. The existence of the isomorphism between ideles modulo rationals - briefly ideles - and maximal Abelian Galois Group of rationals (MAGG) is one of the great discoveries of Abelian class field theory. Also the maximal - necessarily Abelian - extension of finite field G_p has Galois group isomorphic to the ideles. The Galois group of $G_p(n)$ with p^n elements is actually the cyclic group Z_n. The isomorphism opens up the way to study the representations of Abelian Galois group and also those of the AGG. One can indeed see these representations as special kind of representations for which the commutator group of AGG is represented trivially playing a role analogous to that of gauge group.

This framework is extremely general. One can replace rationals with any algebraic extension of rationals and study the maximal Abelian extension or algebraic numbers as its extension. One can consider the maximal algebraic extension of finite fields consisting of union of all all finite fields associated with given prime and corresponding adele. One can study function fields defined by the rational functions on algebraic curve defined in finite field and its maximal extension to include Taylor series. The isomorphisms applies in al these cases. One ends up with the idea that one can represent maximal Abelian Galois group in function space of complex valued functions in $GL_e(A)$ right invariant under the action of $GL_e(Q)$. A denotes here adeles.

In the following I will introduce basic facts about adeles and ideles and then consider a possible realization of the number theoretical vision about quantum TGD as a Galois theory for the algebraic extensions of classical number fields with associativity defining the dynamics. This picture leads automatically to the adele defined by p-adic variants of quaternions and octonions, which can be defined by posing a suitable restriction consistent with the basic physical picture provide by TGD.

10.14.1 Adeles And Ideles

Adeles and ideles are structures obtained as products of real and p-adic number fields. The formula expressing the real norm of rational numbers as the product of inverses of its p-adic norms inspires the idea about a structure defined as produc of reals and various p-adic number fields.

Class field theory (http://tinyurl.com/y8aefmg2) studies Abelian extensions of global fields (classical number fields or functions on curves over finite fields), which by definition have Abelian Galois group acting as automorphisms. The basic result of class field theory is one-one correspondence between Abelian extensions and appropriate classes of ideals of the global field or open subgroups of the ideal class group of the field. For instance, Hilbert class field, which is maximal unramied extension of global field corresponds to a unique class of ideals of the number field. More precisely, reciprocity homomorphism generalizes the quadratic resiprocity for quadratic extensions of rationals. It maps the idele class group of the global field defined as the quotient of the ideles by the multiplicative group of the field - to the Galois group of the maximal Abelian extension of the global field. Each open subgroup of the idele class group of a global field is the image with respect to the norm map from the corresponding class field extension down to the global field.

The idea of number theoretic Langlands correspondence, [K52, A46, A45]. is that n-dimensional representations of Absolute Galois group correspond to infinite-D unitary representations of group $Gl_n(A)$. Obviously this correspondence is extremely general but might be highly relevant for TGD, where embedding space is replaced with Cartesian product of real embedding space and its p-adic variants - something which might be related to octonionic and quaternionic variants of adeles. It seems however that the TGD analogs for finite-D matrix groups are analogs of local gauge groups or Kac-Moody groups (in particular symplectic group of $\delta M^4_+ \times CP_2$) so that quite heavy generalization of already extremely abstract formalism is expected.

The following gives some more precise definitions for the basic notions.

1. Prime ideals of global field, say that of rationals, are defined as ideals which do not decompose to a product of ideals: this notion generalizes the notion of prime. For instance, for p-adic numbers integers vanishing mod p^n define an ideal and ideals can be multiplied. For Abelian extensions of a global field the prime ideals in general decompose to prime ideals of the extension, and the decompostion need not be unique: one speaks of ramification. One of the challenges of tjhe class field theory is to provide information about the ramification. Hilbert class field is define as the maximal unramified extension of global field.
2. The ring of integral adeles (see `http://tinyurl.com/64pgerm`) is defined as $A_Z = R \times \hat{Z}$, where $\hat{Z} = \prod_p Z_p$ is Cartesian product of rings of p-adic integers for all primes (prime ideals) p of assignable to the global field. Multiplication of element of A_Z by integer means multiplication in all factors so that the structure is like direct sum from the point of view of physicist.
3. The ring of rational adeles can be defined as the tensor product $A_Q = Q \otimes_Z A_Z$. $_Z$ means that in the multiplication by element of Z the factors of the integer can be distributed freely among the factors $\hat{Z}$. Using quantum physics language, the tensor product makes possible entanglement between Q and A_Z.
4. Another definition for rational adeles is as $R \times \prod_p' Q_p$: the rationals in tensor factor Q have been absorbed to p-adic number fields: given prime power in Q has been absorbed to corresponding Q_p. Here all but finite number of Q_p elements ar p-adic integers. Note that one can take out negative powers of p_i and if their number is not finite the resulting number vanishes.The multiplication by integer makes sense but the multiplication by a rational does not smake sense since all factors Q_p would be multiplied.
5. Ideles are defined as invertible adeles (`http://tinyurl.com/yc3yrcxx`Idele class group). The basic result of the class field theory is that the quotient of the multiplicative group of ideles by number field is homomorphic to the maximal Abelian Galois group!

10.14.2 Questions About Adeles, Ideles And Quantum TGD

The intriguing general result of class field theory (`http://tinyurl.com/y8aefmg2`) is that the the maximal Abelian extension for rationals is homomorphic with the multiplicative group of ideles. This correspondence plays a key role in Langlands correspondence.

Does this mean that it is not absolutely necessary to introduce p-adic numbers? This is actually not so. The Galois group of the maximal abelian extension is rather complex objects (absolute Galois group, AGG, defines as the Galois group of algebraic numbers is even more complex!). The ring $\hat{Z}$ of adeles defining the group of ideles as its invertible elements homeomorphic to the Galois group of maximal Abelian extension is profinite group (`http://tinyurl.com/y9d8vro7`). This means that it is totally disconnected space as also p-adic integers and numbers are. What is intriguing that p-dic integers are however a continuous structure in the sense that differential calculus is possible. A concrete example is provided by 2-adic units consisting of bit sequences which can have literally infinite non-vanishing bits. This space is formally discrete but one can construct differential calculus since the situation is not democratic. The higher the pinary digit in the expansion is, the less significant it is, and p-adic norm approaching to zero expresses the reduction of the insignificance.

1. Could TGD based physics reduce to a representation theory for the Galois groups of quaternions and octonions?

Number theoretical vision about TGD raises questions about whether adeles and ideles could be helpful in the formulation of TGD. I have already earlier considered the idea that quantum TGD could reduce to a representation theory of appropriate Galois groups. I proceed to make questions.

1. Could real physics and various p-adic physics on one hand, and number theoretic physics based on maximal Abelian extension of rational octonions and quaternions on one hand, define equivalent formulations of physics?
2. Besides various p-adic physics all classical number fields (reals, complex numbers, quaternions, and octonions) are central in the number theoretical vision about TGD. The technical problem is that p-adic quaternions and octonions exist only as a ring unless one poses some additional

conditions. Is it possible to pose such conditions so that one could define what might be called quaternionic and octonionic adeles and ideles?

It will be found that this is the case: p-adic quaternions/octonions would be products of rational quaternions/octonions with a p-adic unit. This definition applies also to algebraic extensions of rationals and makes it possible to define the notion of derivative for corresponding adeles. Furthermore, the rational quaternions define non-commutative automorphisms of quaternions and rational octonions at least formally define a non-associative analog of group of octonionic automorphisms [K105, K119].

3. I have already earlier considered the idea about Galois group as the ultimate symmetry group of physics. The representations of Galois group of maximal Abelian extension (or even that for algebraic numbers) would define the quantum states. The representation space could be group algebra of the Galois group and in Abelian case equivalently the group algebra of ideles or adeles. One would have wave functions in the space of ideles.

   The Galois group of maximal Abelian extension would be the Cartan subgroup of the absolute Galois group of algebraic numbers associated with given extension of rationals and it would be natural to classify the quantum states by the corresponding quantum numbers (number theoretic observables).

   If octonionic and quaternionic (associative) adeles make sense, the associativity condition would reduce the analogs of wave functions to those at 4-dimensional associative sub-manifolds of octonionic adeles identifiable as space-time surfaces so that also space-time physics in various number fields would result as representations of Galois group in the maximal Abelian Galois group of rational octonions/quaternions. TGD would reduce to classical number theory! One can hope that WCW spinor fields assignable to the associative and co-associative space-time surfaces provide the adelic representations for super-conformal algebras replacing symmetries for point like objects.

   This of course involves huge challenges: one should find an adelic formulation for WCW in terms octonionic and quaternionic adeles, similar formulation for WCW spinor fields in terms of adelic induced spinor fields or their octonionic variants is needed. Also zero energy ontology, causal diamonds, light-like 3-surfaces at which the signature of the induced metric changes, space-like 3-surfaces and partonic 2-surfaces at the boundaries of CDs, $M^8 - H$ duality, possible representation of space-time surfaces in terms of of O_c-real analytic functions (O_c denotes for complexified octonions), etc. should be generalized to adelic framework.

4. Absolute Galois group is the Galois group of the maximal algebraic extension and as such a poorly defined concept. One can however consider the hierarchy of all finite-dimensional algebraic extensions (including non-Abelian ones) and maximal Abelian extensions associated with these and obtain in this manner a hierarchy of physics defined as representations of these Galois groups homomorphic with the corresponding idele groups.

5. In this approach the symmetries of the theory would have automatically adelic representations and one might hope about connection with Langlands program [K52], [K52, A46, A45].

2. Adelic variant of space-time dynamics and spinorial dynamics?

As an innocent novice I can continue to pose stupid questions. Now about adelic variant of the space-time dynamics based on the generalization of Kähler action discussed already earlier but without mentioning adeles ([K122]).

1. Could one think that adeles or ideles could extend reals in the formulation of the theory: note that reals are included as Cartesian factor to adeles. Could one speak about adelic space-time surfaces endowed with adelic coordinates? Could one formulate variational principle in terms of adeles so that exponent of action would be product of actions exponents associated with various factors with Neper number replaced by p for Z_p. The minimal interpretation would be that in adelic picture one collects under the same umbrella real physics and various p-adic physics.

2. Number theoretic vision suggests that 4: th/8: th Cartesian powers of adeles have interpretation as adelic variants of quaternions/ octonions. If so, one can ask whether adelic quaternions

and octonions could have some number theoretical meaning. Adelic quaternions and octonions are not number fields without additional assumptions since the moduli squared for a p-adic analog of quaternion and octonion can vanish so that the inverse fails to exist at the light-cone boundary which is 17-dimensional for complexified octonions and 7-dimensional for complexified quaternions. The reason is that norm squared is difference $N(o_1) - N(o_2)$ for $o_1 \oplus io_2$. This allows to define differential calculus for Taylor series and one can consider even rational functions. Hence the restriction is not fatal.

If one can pose a condition guaranteeing the existence of inverse for octonionic adel, one could define the multiplicative group of ideles for quaternions. For octonions one would obtain non-associative analog of the multiplicative group. If this kind of structures exist then four-dimensional associative/co-associative sub-manifolds in the space of non-associative ideles define associative/co-associative adeles in which ideles act. It is easy to find that octonionic ideles form 1-dimensional objects so that one must accept octonions with arbitrary real or p-adic components.

3. What about equations for space-time surfaces. Do field equations reduce to separate field equations for each factor? Can one pose as an additional condition the constraint that p-adic surfaces provide in some sense cognitive representations of real space-time surfaces: this idea is formulated more precisely in terms of p-adic manifold concept [K122] (see the appendix of the book). Or is this correspondence an outcome of evolution?

   Physical intuition would suggest that in most p-adic factors space-time surface corresponds to a point, or at least to a vacuum extremal. One can consider also the possibility that same algebraic equation describes the surface in various factors of the adele. Could this hold true in the intersection of real and p-adic worlds for which rationals appear in the polynomials defining the preferred extremals.

4. To define field equations one must have the notion of derivative. Derivative is an operation involving division and can be tricky since adeles are not number field. The above argument suggests this is not actually a problem. Of course, if one can guarantee that the p-adic variants of octonions and quaternions are number fields, there are good hopes about well-defined derivative. Derivative as limiting value $df/dx = \lim(f(x+dx) - f(x))/dx$ for a function decomposing to Cartesian product of real function $f(x)$ and p-adic valued functions $f_p(x_p)$ would require that $f_p(x)$ is non-constant only for a finite number of primes: this is in accordance with the physical picture that only finite number of p-adic primes are active and define "cognitive representations" of real space-time surface. The second condition is that dx is proportional to product $dx \times \prod dx_p$ of differentials dx and dx_p, which are rational numbers. dx goes to xero as a real number but not p-adically for any of the primes involved. dx_p in turn goes to zero p-adically only for Q_p.

5. The idea about rationals as points common to all number fields is central in number theoretical vision. This vision is realized for adeles in the minimal sense that the action of rationals is well-defined in all Cartesian factors of the adeles. Number theoretical vision allows also to talk about common rational points of real and various p-adic space-time surfaces in preferred coordinate choices made possible by symmetries of the embedding space, and one ends up to the vision about life as something residing in the intersection of real and p-adic number fields. It is not clear whether and how adeles could allow to formulate this idea.

6. For adelic variants of embedding space spinors Cartesian product of real and p-adc variants of embedding spaces is mapped to their tensor product. This gives justification for the physical vision that various p-adic physics appear as tensor factors. Does this mean that the generalized induced spinors are infinite tensor products of real and various p-adic spinors and Clifford algebra generated by induced gamma matrices is obtained by tensor product construction? Does the generalization of massless Dirac equation reduce to a sum of d'Alembertians for the factors? Does each of them annihilate the appropriate spinor? If only finite number of Cartesian factors corresponds to a space-time surface which is not vacuum extremal vanishing induced Kähler form, Kähler Dirac equation is non-trivial only in finite number of adelic factors.

3. Objections leading to the identification of octonionic adeles and ideles

The basic idea is that appropriately defined invertible quaternionic/octonionic adeles can be regarded as elements of Galois group assignable to quaternions/octonions. The best manner to proceed is to invent objections against this idea.

1. The first objection is that p-adic quaternions and octonions do not make sense since p-adic variants of quaternions and octonions do not exist in general. The reason is that the p-adic norm squared $\sum x_i^2$ for p-adic variant of quaternion, octonion, or even complex number can vanish so that its inverse does not exist.
2. Second objection is that automorphisms of the ring of quaternions (octonions) in the maximal Abelian extension are products of transformations of the subgroup of $SO(3)$ (G_2) represented by matrices with elements in the extension and in the Galois group of the extension itself. Ideles separate out as 1-dimensional Cartesian factor from this group so that one does not obtain 4-field (8-fold) Cartesian power of this Galois group.

One can define quaternionic/octonionic ideles in terms of rational quaternions/octonions multiplied by p-adic number. For adeles this condition produces non-sensical results.

1. This condition indeed allows to construct the inverse of p-adic quaternion/octonion as a product of inverses for rational quaternion/octonion and p-adic number. The reason is that the solutions to $\sum x_i^2 = 0$ involve always p-adic numbers with an infinite number of pinary digits - at least one and the identification excludes this possibility. The ideles form also a group as required.
2. One can interpret also the quaternionicity/octonionicity in terms of Galois group. The 7-dimensional non-associative counterparts for octonionic automorphisms act as transformations $x \to gxg^{-1}$. Therefore octonions represent this group like structure and the p-adic octonions would have interpretation as combination of octonionic automorphisms with those of rationals.
3. One cannot assign to ideles 4-D idelic surfaces. The reason is that the non-constant part of all 8-coordinates is proportional to the same p-adic valued function of space-time point so that space-time surface would be a disjoint union of effectively 1-dimensional structures labelled by a subset of rational points of M^8. Induced metric would be 1-dimensional and induced Kähler and spinor curvature would vanish identically.
4. One must allow p-adic octonions to have arbitrary p-adic components. The action of ideles representing Galois group on these surfaces is well-defined. Number field property is lost but this feature comes in play as poles only when one considers rational functions. Already the Minkowskian signature forces to consider complexified octonions and quaternions leading to the loss of field property. It would not be surprising if p-adic poles would be associated with the light-like orbits of partonic 2-surfaces. Both p-adic and Minkowskian poles might therefore be highly relevant physically and analogous to the poles of ordinary analytic functions. For instance, n-point functions could have poles at the light-like boundaries of causal diamonds and at light-like partonic orbits and explain their special physical role.

   The action of ideles in the quaternionic tangent space of space-time surface would be analogous to the action of of adelic linear group $Gl_n(A)$ in n-dimensional space.
5. Adelic variants of octonions would be Cartesian products of ordinary and various p-adic octonions and would define a ring. Quaternionic 4-surfaces would define associative local sub-rings of octonion-adelic ring.

Chapter 11

Philosophy of Adelic Physics

11.1 Introduction

I have developed during last 39 years a proposal for unifying fundamental interactions which I call "Topological Geometrodynamics" (TGD). During last twenty years TGD has expanded to a theory of consciousness and quantum biology and also p-adic and adelic physics have emerged as one thread in the number theoretical vision about TGD.

Since Quantum TGD and physical arguments have served as basic guidelines in the development of p-adic ideas, the best way to introduce the subject of p-adic physics, is by describing first TGD briefly.

In this article I will consider the p-adic aspects of TGD - the first thread of the number theoretic vision - as I see them at this moment.

1. I will describe p-adic mass calculations based on p-adic generalization of thermodynamics and super-conformal invariance [K59, K25] with number theoretical existence constrains leading to highly non-trivial and successful physical predictions. Here the notion of canonical identification mapping p-adic mass squared to real mass squared emerges and is expected to be key player of adelic physics and allow to map various invariants from p-adics to reals and vice versa.

2. I will propose the formulation of p-adicization of real physics and adelization meaning the fusion of real physics and various p-adic physics to single coherent whole by a generalization of number concept fusing reals and p-adics to larger structure having algebraic extension of rationals as a kind of intersection.

   The existence of p-adic variants of definite integral, Fourier analysis, Hilbert space, and Riemann geometry is far form obvious, and various constraints lead to the idea of NTU and finite measurement resolution realized in terms of number theory. Maybe the only way to overcome the problems relies on the idea that various angles and their hyperbolic analogs are replaced with their exponentials and identified as roots of unity and roots of e existing in finite-dimensional algebraic extension of p-adic numbers. Only group invariants - typically squares of distances and norms - are mapped by canonical identification from p-adic to real realm and various phases are mapped to themselves as number theoretically universal entities.

   Another challenge is the correspondence between real and p-adic physics at various levels: space-time level, embedding space level, and WCW level. Here the enormous symmetries of WCW and those of embedding space are in crucial role. Strong form of holography (SH) allows a correspondence between real and p-adic space-time surfaces induced by algebraic continuation from string world sheets and partonic 2-surface, which can be said to be common to real and p-adic space-time surfaces.

3. In the last section I will describe the role of p-adic physics in TGD inspired theory of consciousness. The key notion is Negentropic entanglement (NE) characterized in terms of number theoretic entanglement negentropy (NEN). Negentropy Maximization Principle (NMP) would force the growth of NE. The interpretation would be in terms of evolution as increase of negentropy resources - Akashic records as one might poetically say. The newest finding is

that NMP in statistical sense follows from the mere fact that the dimension of extension of rationals defining adeles increases unavoidably in statistical sense - separate NMP would not be necessary.

In the sequel I will use some shorthand notations for key principles and key notions. Quantum Field Theory (QFT); Relativity Principle (RP); Equivalence Principle (EP); General Coordinate Invariance (GCI); World of Classical Worlds (WCW); Strong Form of GCI (SGCI); Strong Form of Holography (SH); Preferred Extremal (PE); Zero Energy Ontology (ZEO); Quantum Criticality (QC); Hyper-finite Factor of Type II_1 (HFF); Number Theoretical Universality (NTU); Canonical Identification (CI); Negentropy Maximization Principle (NMP); Negentropic entanglement (NE); Number Theoretical Entanglement Negentropy (NEN); are the most often occurring acronyms.

11.2 TGD briefly

This section gives a brief summary of classical and quantum TGD, which to my opinion is necessary for understanding the number theoretic vision.

11.2.1 Space-time as 4-surface

TGD forces a new view about space-time as 4-surface of 8-D imbedding space. This view is extremely simple locally but by its many-sheetedness topologically much more complex than GRT space-time.

Energy problem of GRT as starting point

The physical motivation for TGD was what I have christened the energy problem of General Relativity [K123, L19].

1. The notion of energy is ill-defined because the basic symmetries of empty space-time are lost in the presence of gravity. The presence of matter curves empty Minkowski space M^4 so that its isometries realized as transformations leaving the distances between points and thus shapes of 4-D objects invariant are lost. Noether's theorem states that symmetries and conservation laws correspond to each other. Hence conservation laws are lost and conserved quantities are ill-defined. Usually this is not seen a practical problem since gravitation is so weak interaction.
2. The proposed way out of the problem is based on the assumption that space-times are imbeddable as 4-surfaces to some 8-dimensional space $H = M^4 \times S$ by replacing the points of 4-D empty Minkowski space with 4-D very small internal space S. The space S is unique from the requirement that the theory has the symmetries of standard model: $S = CP_2$, where CP_2 is complex projective space with 4 real dimensions [K123]. Isometries of space-time are replaced with those of imbedding space. Noether's theorem predicts the classical conserved charges for given general coordinate invariant (GCI) action principle.

Also now the curvature of space-time codes for gravitation. Equivalence Principle (EP) and General Coordinate Invariance (GCI) of GRT augmented with Relativity Principle (RT) of SRT remain the basic principles. Now however the number of solutions to field equations - preferred extremals (PEs) - is dramatically smaller than in Einstein's theory [K8, K14].

1. An unexpected bonus was geometrization classical fields of standard model for $S = CP_2$. Also the space-time counterparts for field quanta emerge naturally but this requires a profound generalization of the notion of space-time: the topological inhomogenities of space-time surface are identified as particles. This means a further huge reduction for dynamical field like variables at the level of single space-time sheet. By general coordinate invariance (GCI) only four imbedding space coordinates appear as variables analogous to classical fields: in a typical GUT their number is hundreds.

2. CP_2 also codes for the standard model quantum numbers in its geometry in the sense that electromagnetic charge and weak isospin emerge from CP_2 geometry: the corresponding symmetries are not isometries so that electroweak symmetry breaking is coded already at this level. Color quantum numbers correspond to the isometries of CP_2 defining an unbroken symmetry: this also conforms with empirical facts. The color of TGD however differs from that in standard model in several aspects and LHC has began to exhibit these differences via the unexpected behavior of what was believed to be quark gluon plasma [K67]. The conservation of baryon and lepton numbers follows as a prediction. Leptons and quarks correspond to opposite chiralities for imbedding space spinors.

3. What remains to be explained in standard model is family replication phenomenon for leptons and quarks. Both quarks and leptons appear as three families identical apart from having different masses. The conjecture was is that fermion families correspond to different topologies for 2-D surfaces characterized by genus telling the number g (genus) of handles attached to sphere to obtain the surface: sphere, torus, ... The 2-surfaces are identified as "partonic 2-surfaces" whose orbits are light-like 3-surface at which the signature of the induced metric of space-time surface transforms from Minkowskian to Euclidian. The partonic orbits replace the lines of Feynman diagrams in TGD Universe in accordance with the replacement of point-like particle with 2-surface.

   Only the three lowest genera are observed experimentally. A possible explanation is in terms of conformal symmetries: the genera $g \leq 2$ allow always Z_2 as a subgroup of conformal symmetries (hyper-ellipticity) whereas higher genera in general do not. The handles of partonic 2-surfaces could form analogs of unbound many-particle states for $g > 2$ with a continuous spectrum of mass squared but for $g = 2$ form a bound state by hyper-ellipticity [K25].

4. Later further arguments in favor of$H = M^4 \times CP_2$ have emerged. One of them relates to twistorialization and twistor lift of TGD [L8, K41, K10]. 4-D Minkowski space is unique space-time with Minkowskian signature of metric in the sense that it allows twistor structure. This is a problem in attempts to introduce twistors to General Relativity Theory (GRT) and a serious obstacle in the quantization based on twistor Grassmann approach, which has demonstrate its enormous power in the quantization of gauge theories. In TGD framework one can ask whether one could lift also the twistor structure to the level of H. M^4 has twistor structure and so does also CP_2: which is the only Euclidian 4-manifold allowing twistor space, which is also a Kähler manifold! This led to the notion of twistor lift of TGD inducing rather recent breakthrough in the understanding of TGD.

TGD can be also seen as a generalization of hadronic string model - not yet superstring model since this model became fashionable two years after the thesis about TGD [K2]. Later it has become clear that string like objects, which look like strings but are actually 3-D are basic stuff of TGD Universe and appear in all scales [K29, K8]. Also strictly 2-D string world sheets popped up in the formulation of quantum TGD (analogy with branes) [**?**]o that one can say that string model in 4-D space-time is part of TGD.

Concluding, TGD generalizes standard model symmetries and provides an incredibly simple proposal for a dynamics: only 4 classical field variables and in fermionic sector only quark and lepton like spinor fields. The basic objection against TGD looks rather obvious in the light of after-wisdom. One loses linear superposition of fields, which holds in good approximation in ordinary field theories, which are almost linear. The solution of the problem to be discussed later relies on the notion many-sheeted space-time [L19].

Many-sheeted space-time

The replacement of the abstract manifold geometry of general relativity with the geometry of 4-surfaces brings in the shape of surface as seen from the perspective of 8-D space-time as additional degrees of freedom giving excellent hopes of realizing the dream of Einstein about geometrization of fundamental interactions.

The work with the generic solutions of the field equations assignable to almost any variational principle satisfying GCI led soon to the realization that the topological structure of space-time in this framework is much more richer than in GRT.

1. Space-time decomposes into space-time sheets of finite size. This led to the identification of physical objects that we perceive around us as space-time sheets. The original identification of space-time sheet was as a surface of in H with outer boundary. For instance, the outer boundary of the table would be where that particular space-time sheet ends (what "ends" means is not however quite obvious!). We would directly see the complex topology of many-sheeted space-time! Besides sheets also string like objects and elementary particle like objects appear so that TGD can be regarded also as a generalization of string models obtained by replacing strings with 3-D surfaces.

   It turned that boundaries are probably excluded by boundary conditions. Rather, two sheets with boundaries must be glued along their boundaries together to get double covering. Sphere can be seen as simplest example of this kind of covering: northern and southern hemispheres are glued along equator together.

2. The original vision was that elementary particles are topological inhomogenities glued to these space-time sheets using topological sum contacts. This means drilling a hole to both sheets and connecting with a very short cylinder. 2-dimensional illustration should give the idea. In this conceptual framework material structures and shapes would not be due to some mysterious substance in slightly curved space-time but reduce to space-time topology just as energy- momentum currents reduce to space-time curvature in GRT.

   This view has gradually evolved to much more detailed picture. Elementary particles have wormhole contacts as basic building bricks. Wormhole contact is very small region with *Euclidian (!)* signature of the induced metric connecting two Minkowskian space-time sheets with light-like boundaries carrying spinor fields and there particle quantum numbers. Wormhole contact carries magnetic monopole flux through it and there must be second wormhole contact in order to have closed lines of magnetic flux. Particle world lines are replaced with 3-D light-like surfaces - orbits of partonic 2-surfaces - at which the signature of the induced metric changes.

   One might describe particle as a pair of magnetic monopoles with opposite charges. With some natural assumptions the explanation for the family replication phenomenon in terms of the genus g of the partonic 2-surface is not affected. Bosons emerge as fermion anti-fermion pairs with fermion and anti-fermion at the opposite throats of the wormhole contact. In principle family replication phenomenon should have bosonic analog. This picture assigns to particles strings connecting the two wormhole throats at each space-time sheet so that string model mathematics becomes part of TGD.

The notion of classical field differs in TGD framework in many respects from that in Maxwellian theory.

1. In TGD framework fields do not obey linear superposition and all classical fields are expressible in terms of four imbedding space coordinates in given region of space-time surface. Superposition for classical fields is replaced with *superposition of their effects* [K106, K123] - in full accordance with operationalism. Particle can topologically condense simultaneously to several space-time sheets by generating topological sum contacts (not stable like the wormhole contacts carrying magnetic monopole flux and defining building bricks of particles). Particle "experiences" the superposition of the effects of the classical fields at various space-time sheets rather than the superposition of the fields.

   It is also natural to expect that at macroscopic length scales the physics of classical fields (to be distinguished from that for field quanta) can be explained using only four primary field like variables. Electromagnetic gauge potential has only four components and classical electromagnetc fields give and excellent description of physics. This relates directly to electroweak symmetry breaking in color confinement which in standard model imply the effective absence of weak and color gauge fields in macroscopic scales. TGD however predicts that copies of hadronic physics and electroweak physics could exist in arbitrary long scales [K66] and there are indications that just this makes living matter so different as compared to inanimate matter.

2. The notion of induced gauge field means that one induces electroweak gauge potentials defining so called spinor connection at space-time surface (induction of bundle structure). Induction boils down locally to a projection of the imbedding space vectors representing the spinor

connection. The classical fields at the imbedding space level are non-dynamical and fixed and extremely simple: one can say that one has generalization of constant electric field and magnetic fields in CP_2. The dynamics of the 3-surface however implies that induced fields can form arbitrarily complex field patterns. This is essentially dynamics of shadows.

Induced gauge fields are not equivalent with ordinary free gauge fields. For instance, the attempt to represent constant magnetic or electric field as a space-time time surface has a limited success. Only a finite portion of space-time carrying this field allows realization as 4-surface. I call this topological field quantization. The magnetization of electric and magnetic fluxes is the outcome. Also gravitational field patterns allowing imbedding are very restricted: one implication is that topological with over-critical mass density are not globally imbeddable. This would explain why the mass density in cosmology can be at most critical. This solves one of the mysteries of GRT based cosmology [K98].

Quite generally, the field patterns are extremely restricted: not only due to imbeddability constraint but also due to the fact that by SH only very restricted set of space-time surfaces can appear solutions of field equations: I speak of preferred extremals (PEs) [K8, K14, L19]. One might speak about archetypes at the level of physics: they are in quite strict sense analogies of Bohr orbits in atomic physics: this is implies by the realization of GCI. This kind of simplicity does not conform with what we observed. The way out is many-sheeted space-time. Although fields do not superpose, particles experience the superposition of effects from the archetypal field configurations (superposition is replaced with set theoretic union).

3. The important implication is that one can assign to each material system a field identity since electromagnetic and other fields decompose to topological field quanta. Examples are magnetic and electric flux tubes and flux sheets and topological light rays representing light propagating along tube like structure without dispersion and dissipation making em ideal tool for communications [K77]. One can speak about field body or magnetic body of the system.

   Field body indeed becomes the key notion distinguishing TGD inspired model of quantum biology from competitors but having applications also in particle physics since also leptons and quarks possess field bodies. The is evidence for the Lamb shift anomaly of muonic hydrogen [C2] and the color magnetic body of u quark whose size is somewhat larger than the Bohr radius could explain the anomaly [K67]. The magnetic flux tubes of magnetic body carry monopole fluxes existing without generating currents. In cosmology the flux tubes assignable to the remnants of cosmic strings make possible long range magnetic fields in all scales impossible in standard cosmology. Also super-conductivity is proposed to rely on dark $h_{eff} = n \times h$ Cooper pairs at pairs of flux tubes carrying monopole flux.

GRT and gauge theory limit of TGD is obtained as an approximation.

1. GRT/gauge theory type description is an approximation obtained by lumping together the space-time sheets to single region of M^4, with gravitational fields and gauge potentials as sums of corresponding induced field quantities at space-time surface geometrized in terms of geometry of H. Gravitational field corresponds to the deviation of the induced metric from Minkowski metric using M^4 coordinates for space-time surface so that the description applies only in long length scale limit.

   Space-time surface has both Minkowskian and Euclidian regions. Euclidian regions are identified in terms of what I call generalized scattering/twistor diagrams. The 3-D boundaries between Euclidian and Minkowskian regions have degenerate induced 4-metric and I call them light-like orbits of partonic 2-surfaces or light-like wormhole throats analogous to blackhole horizons. The interiors of blackholes are replaced with the Euclidian regions and every physical system is characterized by this kind of region.

   Lumping of sheets together implies that global conservation laws cannot hold exactly true for the resulting GRT type space-time. Equivalence Principle (EP) as Einstein's equations stating conservation laws locally follows as a local remnant of Poincare invariance.

2. Euclidian regions are identified as slightly deformed pieces of CP_2 connecting two Minkowskian space-time regions. Partonic 2-surfaces defining their boundaries are connected to each other by magnetic flux tubes carrying monopole flux.

   Wormhole contacts connect two Minkowskian space-time sheets already at elementary particle level, and appear in pairs by the conservation of the monopole flux. Flux tube can be visualized as a highly flattened square traversing along and between the space-time sheets involved. Flux tubes are accompanied by fermionic strings carrying fermion number. Fermionic strings give rise to string world sheets carrying vanishing induced em charged weak fields (otherwise em charge would not be well-defined for spinor modes). String theory in space-time surface becomes part of TGD. Fermions at the ends of strings can get entangled and entanglement can carry information.

3. Strong form of GCI (SGCI) states that light-like orbits of partonic 2-surfaces on one hand and space-like 3-surfaces at the ends of causal diamonds on the other hand provide equivalent descriptions of physics. The outcome is that partonic 2-surfaces and string world sheets at the ends of CD can be regarded as basic dynamical objects.

   Strong form of holography (SH) states the correspondence between quantum description based on these 2-surfaces and 4-D classical space-time description, and generalizes AdS/CFT correspondence. One has huge super-symplectic symmetry algebra acting as isometries of WCW with conformal structure [K28, K93, K121], conformal algebra of light-cone boundary extending the ordinary conformal algebra, and ordinary Kac-Moody and conformal symmetries of string world sheets. This explains why 10-D space-time can be replaced with ordinary space-time and 4-D Minkowski space can be replaced with partonic 2-surfaces and string world sheets. This holography looks very much like the one we are accustomed with!

11.2.2 Zero energy ontology (ZEO)

In standard ontology of quantum physics physical states are assumed to have positive energy. In zero energy ontology (ZEO) [K69] physical states decompose to pairs of positive and negative energy states such that the net values of the conserved quantum numbers vanish. The interpretation of these states in ordinary ontology would be as transitions between initial and final states, physical events.

ZEO and positive energy ontology

ZEO is consistent with the crossing symmetry of QFTs meaning that the final states of the quantum scattering event can be described formally as negative energy states. As long as one can restrict the consideration to either positive or negative energy part of the state ZEO is consistent with positive energy ontology. This is the case when the observer characterized by a particular CD studies the physics in the time scale of much larger CD containing observer's CD as a sub-CD. When the time scale sub-CD of the studied system is much shorter that the time scale of sub-CD characterizing the observer, the interpretation of states associated with sub-CD is in terms of quantum fluctuations.

ZEO solves the problem, which emerges in any theory assuming symmetries giving rise to conservation laws. The problem is that the theory itself is not able to characterize the values of conserved quantum numbers of the initial state of say cosmology. In ZEO this problem disappears since in principle any zero energy state is obtained from any other state by a sequence of quantum jumps without breaking of conservation laws. The fact that energy is not conserved in GRT based cosmologies can be also understood since each CD is characterized by its own conserved quantities. As a matter fact, one must speak about average values of conserved quantities since one can have a quantum superposition of zero energy states with the quantum numbers of the positive energy part varying over some range.

At the level of principle the implications are quite dramatic. In quantum jump as recreation replacing the quantum Universe with a new one it is possible to create entire sub-universes from vacuum without breaking the fundamental conservation laws. From the point of view of consciousness theory the important implication is that "free will" is consistent with the laws of physics. This makes obsolete the basic arguments in favor of materialistic and deterministic world view.

Zero energy states are located inside causal diamond (CD)

By quantum classical correspondence zero energy states must have space-time and imbedding space correlates.

1. Positive and negative energy parts of zero energy state reside at future and past light-like boundaries of causal diamond (CD) identified as intersection of future and past directed light-cones and visualizable as double cone. The analog of CD in cosmology is big bang followed by big crunch. Penrose diagrams provide an excellent 2-D visualization of the notion. CDs form a fractal hierarchy containing CDs within CDs. Disjoint CDs are possible and CDs could also intersect.

   The interpretation of CD in TGD inspired theory of consciousness is as an imbedding space correlate for perceptive field of conscious entity: the contents of conscious experience is about the region defined by CD. At the level of space-time sheets the experience come from space-time sheets in the interior of CD. Whether the sheets can be assumed to continue outside CD is still unclear.

   Quantum measurement theory must be modified in ZEO since state function reduction can happen at both boundaries of CD and the reduced states at opposite boundaries are related by time reversal. One can also have quantum superposition of CDs changing between reductions at active boundary followed by localization in the moduli space of CDs with the tip of passive boundary fixed. Quantum measurement theory generalizes to a theory of consciousness with continuous entity identified as a sequence of state function reductions at active (changing) boundary of CD [K7].

2. By number theoretical universality (NTU) the temporal distances between the tips of the intersecting light-cones are assumed to come as integer multiples $T = m \times T_0$ of a fundamental time scale T_0 defined by CP_2 size R as $T_0 = R/c$. p-Adic length scale hypothesis [K71, K119] motivates the stonger hypothesis that the distances tend to come as as octaves of T_0: $T = 2^n T_0$. One prediction is that in the case of electron this time scale is .1 seconds defining the fundamental biorhythm. Also in the case u and d quarks the time scales correspond to biologically important time scales given by 10 ms for u quark and by and 2.5 ms for d quark [K9] . This means a direct coupling between microscopic and macroscopic scales.

11.2.3 Quantum physics as physics of classical spinor fields in WCW

The notions of Kähler geometry of "World of Classical Worlds" (WCW) and WCW spinor structure are inspired by the vision about the geometrization of the entire quantum theory.

Motivations for WCW

The notion of "World of Classical Worlds" (WCW) [K50, K28, K93] was forced by the failure of both path integral approach and canonical quantization in TGD framework. The idea is that the Kähler function defining WCW Kähler geometry is determined by the real part of an action S determining space-time dynamics and receiving contributions from both Minkowskian and Euclidian regions of space-time surface X^4 (note that $\sqrt{g_4}$ is proportional to imaginary unit in Minkoskian regions).

1. If S is space-time volume both canonical quantization and path integral would make sense at least formally since in principle one could solve the time derivatives of four imbedding space coordinates as functions of canonical momentum densities (general coordinate invariance allows to eliminate four coordinates). The calculation of path integral is however more or less hopeless challenge in practice.

2. A mere space-time volume as action is however not physically attractive. This was thought to leave under consideration only Kähler action S_K - Maxwell action for the induced Kähler form expressible in terms of gauge potential defined by the induced Kähler gauge potential of CP_2. This action has however a huge vacuum degeneracy. Any space-time surface with at most 2-D CP_2 projection, which is Lagrangian sub-manifold of CP_2, is vacuum extremal. Symplectic transformations acting like U(1) gauge transformations generate new vacuum extremals. They however fail to act as symmetries of non-vacuum extremals so that gauge invariance is not in question: the deviation of the induced metric from flat metric is the reason for the failure. This degeneracy is assumed to give rise to what might be called 4-D spin glass degeneracy meaning that the landscape for the maxima of Kähler function is fractal.

3. Canonical quantization fails because by the extreme non-linearity of the action principle making it is impossible to solve time derivatives explicitly in terms of canonical momentum

densities. The problem is especially acute for the canonical imbedding of empty Minkowski space to $M^4 \times CP_2$. The action is vanishing up to fourth order in imbedding space coordinates so that canonical momentum densities vanish identically and there is no hope of defining propagator in path integral approach. The mechanical analog would be criticality around which the potential reduces to $V \propto x^4$. Quantum criticality is indeed a basic aspect of TGD Universe.

The hope held for a long time was that WCW geometry allowing to get rid of path integral would solve the problems. One could however worry about vacuum degeneracy implying that WCW metric becomes extremely degenerate for vacuum extremals and also holography becomes extremely non-unique for them. Also the expected feailure of perturbative approach around M^4 is troublesome.

WCW and twistor lift of TGD

During last year this picture has indeed changed thanks to what might be called twistor lift of TGD [L8, K41, K10] inspired by twistor Grassmann approach to supersymmetric gauge theories [B22]. Remarkably, twistor lift would provide automatically the fundamental couplings of standard model and GRT and also the scale assigned to GUTs as CP_2 radius. PEs would be both extremals of Kähler action and minimal surfaces.

1. The basic observation is E^4, and its Euclidian compactification S^4 and CP_2 are completely unique in that they allow twistor space with Kähler structure [A61]. This was discovered by Hitchin at roughly the same time as I discovered TGD! This generalizes to M^4 having a generalization of ordinary Kähler structure to what I have called Hamilton-Jacobi structure by decomposition $M^4 = M^2 \times E^2$, where M^2 allows hypercomplex structure [L8, K41]. One can consider also integral distributions of tangent decompositions $M^4 = M^2(x) \times E^2(x)$, depending on position. The twistor space has a double fibration by S^2 with base spaces identifiable as M^4 and conformal compactification of M^4 for which metric is defined only up to conformal scaling. The first fibration $M^4 \times S^2$ with a well-defined metric would correspond to the classical TGD.

2. Both Newton's constant G and cosmological constant Λ emerge from twistor lift in M^4 factor. The radius of S^2 is identified in terms of Planck length $l_P = \sqrt{G}$. For CP_2 factor, the radius corresponds to the radius of CP_2 geodesic sphere. 4-D Kähler action can be lifted to 6-D Kähler action only for $M^4 \times CP_2$ so that TGD would be completely unique both mathematically and physically. The twistor space of CP_2 is flag-manifold $SU(3)/U(1) \times U(1)$ having interpretation as the space for the choices of quantization axis of color isospin and hypercharge. This choice could correspond to a selection of Eguchi-Hanson complex coordinates for CP_2 by fixing their phase angles in which isospin and hypercharge rotations induce shifts.

3. The physically motivated conjecture is that the PEs can be lifted to their 6-D twistor bundles with S^2 serving as a fiber, that one induce the twistor structure and the outcome is equal to the twistor structure of space-time surface, and that this condition is at least part of the PE property. This would correspond to the solution of massless wave equations in terms of twistors in the original twistor approach of Penrose [B53]. The analog of spontaneous compactification would lead to 4-D action equal to Kähler action plus volume term. One could of course postulate this action directly without mentioning twistors at all.

   The coefficient of the volume term would correspond to dark energy density characterized by cosmological constant Λ being extremely small in cosmological scales. It removes vacuum degeneracy although the situation remains highly non-perturbative. This can be combined with the earlier conjecture that cosmological constant Λ behaves as $\Lambda \propto 1/p$ under p-adic coupling constant evolution so that Λ would be large in primordial cosmology.

4. The generic extremals of space-time action would depend on coupling parameters, which does not fit with the number theoretic vision inspiring speculations that space-time surface can be seen as quaternionic sub-manifolds of 8-D octonionic space-time [K105], satisfying quaternion analyticity [K41], or a 4-D generalization of holomorphy. By SH the extremals are however "preferred". What could this imply?

Intriguingly, all known non-vacuum extremals and also CP_2 type vacuum extremals having null-geodesic as M^4 projection are extremals of both Kähler action and volume term separately! The dynamics for volume term and Kähler action effectively decouple and coupling constants do not appear at all in field equations. The twistor lift would only select minimal surface amongst vacuum extremals, modify the Kähler function of WCW identifiable as exponent for the real part of action, and provide a profound mathematical and physical motivation for cosmological constant Λ remaining mysterious GRT framework. One could even hope that preferred extremals are nothing but minimal surface extremals of Kähler action with the vanishing conditions for some sub-algebra of super-symplectic algebra satisfied automatically!

The analog of decoupling of Kähler action and volume term should take place also for induced spinors. This is expected if mere analyticity properties make spinor modes solutions of modified Dirac equations. This is true in 2-D case Hamilton-Jacobi structure should guarantee this in 4-D case [K121, K41].

PEs depend on coupling parameters only via boundary conditions stating the vanishing of Noether charges for a sub-algebra of super-symplectic algebra and its commutator with entire algebra. Also the conservation conditions at 3-D light-like surfaces at which the signature of metric changes imply dependence on coupling parameters. These conditions allow the transfer of classical charges between Minkowskian and Euclidian regions necessary to understand momentum exchange between particles and environment classically only if Kähler couplings strength is complex - otherwise there is no exchange of conserved quantities since their real *resp.* imaginary at the two sides [L14]. Interestingly, also in twistor Grassmann approach the massless poles in propagators are complex.

This picture conforms with the conjecture that discrete p-adic evolution of the Kähler coupling strength in subset of primes near prime powers of two corresponds to complex zeros of zeta [L14]. This conforms also with the conjectured discreteness of p-adic coupling constant evolution by phase transitions changing the values of coupling parameters. One implication is that all loop corrections in functional integral vanish.

5. In path integral approach quantum TGD would be extremely non-perturbative around extremals for which Kähler action vanishes. Same is true also in WCW approach. The cure would be provided by the hierarchy of Planck constants $h_{eff}/h = n$, which effectively scales Λ down to Λ/n. n would be the number sheets of the M^4 covering defined by the space-time surface: the action of Galois group for the number theoretic discretization of space-time surface could give rise to this covering. The finiteness of the volume term in turn forces ZEO: the volume of space-time surface is indeed finite due to the finite size of CD.

Consider now the delicacies of this picture.

1. Should assign also to M^4 the analog of symplectic structure giving an additional contribution to the induced Kähler form? The symmetry between M^4 and CP_2 suggests this, and this term could be highly relevant for the understanding of the observed CP breaking and matter antimatter asymmetry [L35]. Poincare invariance is not lost since the needed moduli space for M^4 Kähler forms would be the moduli space of CDs forced by ZEO in any case, and M^4 Kähler form would serve as the correlate for fixing rest system and spin quantization axis in quantum measurement.

2. Also induced spinor fields are present. The well-definedness of electro-magnetic charge for the spinor modes forces in the generic case the localization of the modes of induced spinor fields at string world sheets (and possibly to partonic 2-surfaces) at which the induced charged weak gauge fields and possibly also neutral Z^0 gauge field vanish. The analogy with branes and super-symmetry force to consider two options.

   Option I: The *fundamental* action principle for space-time surfaces contains besides 4-D action also 2-D action assignable to string world sheets, whose topological part (magnetic flux) gives rise to a coupling term to Kähler gauge potentials assignable to the 1-D boundaries of string world sheets containing also geodesic length part. Super-symplectic symmetry demands that modified Dirac action has 1-, 2-, and 4-D parts: spinor modes would exist at both string boundaries, string world sheets, and space-time interior. A possible interpretation for the interior modes would be as generators of space-time super-symmetries [K96].

This option is not quite in the spirit of SH and string tension appears as an additional parameter. Also the conservation of em charge forces 2-D string world sheets carrying vanishing induced W fields and this is in conflict with the existence of 4-D spinor modes unless they satisfy the same condition. This looks strange.

Option II: Stringy action and its fermionic counterpart are effective actions only and justified by SH. In this case there are no problems of interpretation. SH requires only that the induced spinor fields at string world sheets determine them in the interior much like the values of analytic function at curve determine it in an open set of complex plane. At the level of quantum theory the scattering amplitudes should be determined by the data at string world sheets. If induced W fields at string world sheets are vanishing, the mixing of different charge states in the interior of X^4 would not make itself visible at the level of scattering amplitudes! In this case 4-D spinor modes do not define space-time super-symmetries.

3. Why the string world sheets coding for effective action should carry vanishing weak gauge fields? If M^4 has the analog of Kähler structure [L35], one can speak about Lagrangian sub-manifolds in the sense that the sum of the symplectic forms of M^4 and CP_2 projected to Lagrangian sub-manifold vanishes. Could the induced spinor fields for effective action be localized to generalized Lagrangian sub-manifolds? This would allow both string world sheets and 4-D space-time surfaces but SH would select 2-D Lagrangian manifolds. At the level of effective action the theory would be incredibly simple.

   Induced spinor fields at string world sheets could obey the "dynamics of avoidance" in the sense that *both* the induced weak gauge fields W, Z^0 and induced Kähler form (to achieve this U(1) gauge potential must be sum of M^4 and CP_2 parts) would vanish for the regions carrying induced spinor fields. They would coupleonly to the *induced em field (!)* given by the R_{12} part of CP_2 spinor curvature [K15] for $D = 2, 4$. For $D = 1$ at boundaries of string world sheets the coupling to gauge potentials would be non-trivial since gauge potentials need *not* vanish there. Spinorial dynamics would be extremely simple and would conform with the vision about symmetry breaking of weak group to electromagnetic gauge group.

   The projections of canonical currents of Kähler action to string world sheets would vanish, and the projections of the 4-D modified gamma matrices would define just the induced 2-D metric. If the induced metric of space-time surface reduces to an orthogonal direct sum of string world sheet metric and metric acting in normal space, the flow defined by 4-D canonical momentum currents is parallel to string world sheet. These conditions could define the "boundary" conditions at string world sheets for SH.

This admittedly speculative picture has revolutionized the understanding of both classical and quantum TGD during last year. [K41, K10, L19]. In particular, the construction of single-sheeted PEs as minimal surfaces allows a kind of lego like engineering of more complex PEs [L18]. The minimal surface equations generalize Laplace equation of Newton's gravitational theory to non-linear massless d'Alembert equation with gravitational self-coupling. One obtains the analog of Schwartschild solution and radiative solutions describing also gravitational radiation [L19]. An open question is whether classical theory makes sense if also the analog of Kähler form in M^4 is allowed.

Identification of WCW

The notion of WCW [K50, K28, K93] was inspired by the super-space approach of Wheeler in which 3-geometries are the basic geometric entities.

1. In TGD framework 3-surfaces take this role. Einstein's program for geometrizing classical physics is generalized to a geometrization of entire quantum physics. Hermitian conjugation corresponds to complex conjugation in infinite-dimensional context so that WCW must have Kähler geometry. The geometrization of fermionic statistics/oscillator operators is in terms of gamma matrices of WCW expressible as linear combinations of oscillator operators for second quantized induced spinor field. Formally purely classical spinor modes of WCW represent many fermion states as functionals of 3-surface. One can also interpret gamma matrices as generators of super-conformal symmetries in accordance with the fact that also SUSY involves Clifford algebra.

In ZEO the entanglement coefficients between positive and negative energy parts of zero energy states determine the S-matrix so that S-matrix would be coded by the modes of WCW spinor fields. Twistor approach to TGD [K41] suggests that the S-matrix reduces completely to the symmetries defined by the multi-local (locus corresponds to partonic 2-surface) generators of the Yangian associated with the super-symplectic algebra.

2. ZEO forces to identify 3-surfaces as pairs of 3-surfaces with members at the opposite boundaries of CD. SH reduces them to a collection of partonic 2-surfaces at boundaries of CD plus number theoretic discretization in space-time interior. Basic geometric objects are pairs of initial and final states (coordinates for both in mechanical analogy) rather than initial states with initial value conditions (coordinates and velocities in mechanical analogy) and initial value problem transforms to boundary value problem. Processes rather than states become the basic elements of ontology: this has far reaching consequences in biology and neuroscience.

3. The realization of GCI requires that the definition of WCW Kähler function assigns to a "physically" 3-surface a unique 4-surface for 4-D general coordinate transformations to act: "physically" could mean "apart from transformations acting as gauge transformations" not affecting the action and conserved classical charges. The outcome is holography.

4. Strong form of holography (SH) would emerge as follows. The condition that light-like 3-surfaces defining boundaries between Euclidian and Minkowskian regions are basic geometric entities equivalent with pairs of space-like 3-surfaces at the ends of given causal diamond CD implies SH: partonic 2-surfaces and their 4-D tangent space data should code the physics. One could also speak about almost/effective 2-dimensionality. Tangent space data could in turn be coded by string world sheets. Number theoretical discretitization of space-time interior with preferred coordinates in the extension of rationals could give meaning for "almost".

5. Kähler metric is expressible both in terms of second derivatives of Kähler function K [K50] and as anticommutators of WCW gamma matrices expressible as linear combinations of fermionic oscillator operators. This suggests a close relationship between space-time dynamics and spinor dynamics.

   Super-symplectic symmetry between the action defining space-time surfaces (Kähler action plus volume term) and modified Dirac action would realize this relationship. This is achieved if the modified gamma matrices are defined by the canonical momentum currents of 2-D action associated with string world sheets. These currents are parallel to the string world sheets. This implies the analog of AdS/CFT correspondence requiring only that induced spinor modes at string world sheets determine them in space-time interior (this is like analytic continuation). The localization of spinor modes at string world sheets is *not* required as I believed first.

The geometry of loop spaces developed by Freed [A43] serves as a model in the construction of WCW Kähler geometry [K93].

1. The existence of loop space Riemann connection requires maximal isometry group identifiable as Kac-Moody group so that Killing vector fields span the entire tangent space of the loop space.

2. In TGD framework the properties of Kähler action lead to the idea that WCW is union of homogenous or even symmetric spaces of symplectic algebra acting at the boundary of $\delta CD \subset \delta CD_+ \cup \delta CD_-$, $\delta CD_\pm \subset \delta M^4_\pm \times CP_2$. ZEO requires that the conserved quantum numbers for physical states are opposite for the positive and negative energy parts of the states at the two opposite boundary parts of CD. The symmetric spaces G/H in the union are labelled by zero modes, which do not appear in the line element as differentials but only as parameters of the metric. Conserved Noether charges of isometries and symplectic invariants of examples of zero modes as also the super-symplectic Noether charges invariant under complex conjugation of WCW coordinates.

3. Homogenous spaces of the symplectic group G are obtained by dividing by a subgroup H. An especially attractive option is suggested by the fractal structure of the symplectic algebra containing an infinite hierarchy of sub-algebras G_n for which conformal weights are $n > 0$-multiples of those for G. For this option $H = G_n$ is isomorphic to G and one could have infinite hierarchies of inclusions analogous to the hierarchy of inclusions of hyperfinite factors of type II_1 (HFFs). PE property requires almost 2-dimensionality and elimination of huge

number of degrees of freedom. The natural condition is that the Noether charges of G_n vanish at the ends of CD. A stronger condition is that also the Noether charges for $[G, G_n]$ vanish. This implies effective normal algebra property and G/G_n acts effectively like group.

The inclusion of HFFs would define measurement resolution with included factor acting like gauge algebra. Measurement resolution would be naturally determined by the number theoretic discretization of the space-time surface so that physics as geometry and number theory visions would meet each other.

4. This inclusion hierarchy can be identified in terms of quantum criticality (QC). The transitions $n \to kn$ increasing the value of $n > 0$ reduce QC since pure gauge symmetries are reduced, and new physical super-symplectic degrees of freedom emerge. QC also requires that Kähler couplings strength analogous to temperature is analogous to critical temperature so that the quantum theory is uniquely defined if their is only one critical temperature. Spectrum for α_K seems more plausible and the possibility that Kähler coupling strength depends on the level of the number theoretical hierarchy defined by the allowed extensions of rationals can be considered [L14].

WCW spinor structure

The basic idea is geometrization of quantum states by identifying them as modes of WCW spinor fields [K121, K93]. This requires definition of WCW spinors and WCW spinor structure, WCW gamma matrices and Dirac operator, etc..

The starting point is the definition of WCW gamma matrices using a representation analogous to the usual vielbein representation as linear combinations of flat space gamma matrices. The conceptual leap is the observation that there is no need to assume that the counterparts of flat space gamma matrices have vectorial quantum numbers. Instead, they are identified as fermionic oscillator operators for second quantized free induced spinor fields at space-time surface.

This allows geometrization of the fermionic statistics since WVW spinors for a given 3-surface are analogous to fermionic Fock states. One can also say that spinor structure follows as a square root of metric and also that the spinor basis defines a geometric correlate of Boolean mind [K24]. The dependence of WCW spinor field on 3-surface represents the bosonic degrees of freedom not reducible to many-fermion states. For instance, most of hadron mass would be associated with these degrees of freedom.

Quantum TGD involves Dirac equations at space-time level, imbedding space level, and level of WCW. The dynamics of the induced spinor fields is related by super-symmetry to the action defining space-time surfaces as preferred extremals. [K121, K93].

1. The gamma matrices in the equation - modified gamma matrices - are determined by contractions of the canonical momentum currents of Kähler action with the imbedding space gamma matrices. The localization at string world sheets for which only induced neutral weak fields or only em field are non-vanishing is accompanied by the integrability condition that various conserved currents run along string world sheets: one can speak of sub-flow. I
2. Modified Dirac equation can be solved exactly just like in the case of string models using holomorphy and the properties of complexified modified gamma matrices. This is expected to be true also in 4-D case by Hamilton-Jacobi structure. If the dynamics of avoidance is realized the modified Dirac equation would be essentially free Dirac equation and holomorphy would allow to solve it.

At the level of WCW one obtains also the analog of massless Dirac equation as the analog of super Virasoro conditions of Super Virasoro algebra.

1. The fermionic counterparts of super-conformal gauge conditions assignable with sub-algebra G_n of supersymplectic conformal symmetry associated with the both light-cone boundary (light-like radial coordinate), with conformal symmetries of light-cone boundary, and with string world sheets.
2. The ground states of supersymplectic representations satisfy massless imbedding space Dirac equation in imbedding space so that Dirac equations in WCW, in imbedding space, and at string world sheets are involved. In twistorialization also massless M^8 Dirac equation emerges in the tangent space M^8 of imbedding space assignable to the partonic 2-surfaces

and generalizes the 4-D light-likeness with its 8-D counterpart applying to states with M^4 mass. Here octonionic representation of imbedding space gamma matrices emerges naturally and allows to speak about 8-D analogs of Pauli's sigma matrices [L8].

11.2.4 Quantum criticality, measurement resolution, and hierarchy of Planck constants

The notions of quantum criticality (QC), finite measurement resolution, and hierarchy of Planck constants proposed to give rise to dark matter as phases of ordinary matter are central for TGD [?, K120, K38].

These notions relate closely to the strong form of holography (SH) implied by strong form of general coordinate invariance (SGCI). In adelic physics all this would relate closely to the hierarchy of extensions of rationals serving as a correlate for number theoretical evolution.

Finite measurement resolution and fractal inclusion hierarchy of super-symplectic algebras

The fractal hierarchy of isomorphic sub-algebras of supersymplectic algebra - call it g - defines an excellent candidate for the realization of finite the measurement resolution. Similar hierarchies can be assigned also for the extended super-conformal algebra assignable with light-like boundaries of CD and with Kac-Moody and conformal algebras assignable to string world sheets.

An interesting possibility is that the the conformal weights assignable to infinitesimal scaling operator of the light-like radial coordinate of light-cone boundary correspond to zeros of Riemann zeta [K119] [L13]. A kind of dual spectrum would correspond to conformal weights that correspond to logarithms for powers of primes. One can identify the conformal weight as negative of the pole of fermionic zeta $z_F = \zeta(s)/\zeta(2s)$ natural in TGD framework. The real part of conformal weight for the generators is $h_R = -1/4$ for "non-trivial" poles and positive integer $h = n > 0$ for "trivial" poles. There is also a pole for $h = -1$. Hence one obtains tachyonic ground states, which must be assumed also in p-adic mass calculations [K59].

Also the generators of Yangian algebra [L8] integrating the algebras assignable to various partonic 2-surfaces to a multi-local algebra are labelled by a non-negative integer n analogous to conformal weight and telling the number of partonic 2-surfaces involved with the action of the generator. Also this algebra has similar fractal hierarchy of sub-algebras so that the considerations that follow might apply also to it. Now that number of partonic 2-surface would play the role of measurement resolution.

As noticed, there are also other algebras, which allow conformal hierarchy if one can restrict the conformal weights to be non-negative. The first of them generates generalized conformal transformations of light-cone boundary depending on light-like radial coordinate as parameter: also now radial conformal weights for generators can have zeros of zeta as spectrum. As a special case one obtains infinite-dimensional group of isometries of light-cone boundary. Second one corresponds to ordinary conformal and Kac-Moody symmetries for induced spinor fields acting on string world sheets. Also here similar hierarchy of sub-algebras can be considered. In the following argument one restricts to super-symplectic algebra assumed to act as isometries of WCW.

Consider now how the finite measurement resolution could be realized as an infinite hierarchy of super-symplectic gauge symmetry breakings. The physical picture relies on quantum criticality of TGD Universe. The levels of the hierarchy labelled by positive integer n and a ball at the top of ball at... serves as a convenient metaphor.

1. The sub-algebra g_n for which conformal weights of generators (whose commutators give the sub-algebra) are positive integer multiples for those of the entire algebra g defines the algebra acting as pure gauge algebra defining a sub-group of symplectic group. The action of g_n as gauge algebra would mean that it affects on degrees of freedom below the measurement resolution. One can assign to this algebra a coset space G/G_n of the entire symplectic group G and of subgroup G_n. This coset space would describe the dynamical degrees of freedom. If the subgroup were a normal subgroup, the coset space would be a group. This is not the case now since the commutator $[g, g_n]$ of the entire algebra with the sub-algebra does not belong to g_n.

However, if one poses stronger - physically very attractive - gauge conditions stating that not only g_n but also the commutator algebra $[g, g_n]$ annihilates the physical states and that corresponding classical Noether charges vanish, one obtains effectively a normal subgroup and one has good hopes that coset space acts effectively as group, which is finite-dimensional as far as conformal weights are considered.

2. $n > 0$ is essential for obtaining effective normal algebra property. Without this assumption the commutator $[g, g_n]$ would be entire g. If the spectrum of supersymplectic conformal weights is integer valued it is not obvious why one should pose the restriction $n \geq 1$.

3. In this framework pure conformal invariance could reduce to a finite-dimensional gauge symmetry. A possible interpretation would be in terms of Mc-Kay correspondence [A70] assigning to the inclusions of HFFs labelled by integer $n \geq 3$ a hierarchy of simply laced Lie-groups. Since the included algebra would naturally correspond to degrees of freedom not visible in the resolution used, the interpretation as a dynamical gauge group is suggestive. The dynamical gauge group could correspond to n-dimensional Cartan algebra acting in conformal degrees of freedom identifiable as a simply laced Lie group. This would assign a infinite hierarchy of dynamical gauge symmetries to the broken conformal gauge invariance acting as symmetries of dark matter. This still leaves infinite number of degrees of freedom assignable to the imbedding space Hamiltonians and spectrum generated by zeros of zeta but this might have interpretation in terms of gauging so that additional vanishing conditions for Noether charges are suggestive.

Dark matter as large phases with large gravitational Planck constant $h_{eff} = h_{gr}$

D. Da Rocha and Laurent Nottale [E1] have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive [K97, K78].

1. The proposal is that a Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests that astrophysical systems are at some levels of the hierarchy of space-time sheets macroscopic quantum systems and that only the generalizations of Bohr orbits are involved. The space-time sheets in question would carry dark matter.

2. Nottale's hypothesis would predict a gigantic value of $\hbar_{gr}$. Equivalence Principle and the independence of gravitational Compton length $\Lambda_{gr} = \hbar_{gr}/m = GM/v_0 = 2r_S/v_0$ (typically astrophysical scale) on mass m implies however that one can restrict the values of mass m to masses of microscopic objects so that $\hbar_{gr}$ would be much smaller. Large $\hbar_{gr}$ could provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets, which is quantum coherent in the required time scale [K97].

   One could criticize the hypothesis since it treats the masses M and m asymmetrically: this is only apparently true [**?**].

3. It is natural to assign the values of Planck constants postulated by Nottale to the space-time sheets mediating gravitational interaction and identifiable as magnetic flux tubes (quanta). The cross section of the flux tube corresponds to a sphere $S^2_i \subset CP_2$, $i = I, II$ [K10]. S^2_I is homologically non-trivial carrying Kähler magnetic monopole flux. S^2_{II} is homologically trivial carrying vanishing Kähler magnetic flux but non-vanishing electro-weak flux [K10].

   The flux tubes of type I have both Kähler magnetic energy and dark energy due to the volume action. Flux tubes of type II would have only the volume energy. Both flux tubes could be remnants of cosmic string phase of primordial cosmology. The energy of these flux quanta would be correlated for galactic dark matter and volume action and also magnetic tension would give rise to negative "pressure" forcing accelerated cosmological expansion. This leads to a rather detailed vision about the evolution of stars and galaxies identified as bubbles of ordinary and dark matter inside flux tubes identifiable also as dark energy.

4. Both theoretical consistency and certain experimental findings from astrophysics [E3, E4] and biology [K23, K12] suggest the identification $h_{eff} = n \times h = h_{gr}$. The large value of h_{gr} can be seen as a manner to reduce the string tension of fermionic strings so that gravitational (in fact all!) bound states can be described in terms of strings connecting the partonic 2-surfaces defining particles (analogous to AdS/CFT description) [K93]. The values $h_{eff}/h = n$ can be interpreted in terms of a hierarchy of breakings of super-conformal symmetry in which the super-conformal generators act as gauge symmetries only for a sub-algebras with conformal weights coming as multiples of n. Macroscopic quantum coherence in astrophysical scales is implied. If also modified Dirac action is present, part of the interior degrees of freedom associated with the fermionic part of conformal algebra become physical.

   Fermionic oscillator operators could generate super-symmetries and sparticles could correspond to dark matter with $h_{eff}/h = n > 1$. One implication would be that at least part if not all gravitons would be dark and be observed only through their decays to an ordinary high frequency graviton ($E = hf_{high} = h_{eff}f_{low}$) or to a bunch of n low energy gravitons.

Hierarchies of quantum criticalities, Planck constants, and dark matters

Quantum criticality is one of the corner stone assumptions of TGD. In the original approach the value of Kähler coupling strength α_K together with CP_2 radius R fixed quantum TGD and is analogous to critical temperature. Twistor lift [K10] brings in additional coupling constant Λ obeying p-adic coupling constant evolution and Planck length l_G, which like CP_2 radius would not obey coupling constant evolution (as also G). The values of these parameters should be fixed by quantum criticality. What else does quantum criticality mean is however far from obvious, and I have pondered the notion repeatedly both from the point of view of mathematical description and phenomenology [K50, K121, K93].

1. Criticality is characterized by long range correlations and sensitivity to external perturbations and living systems define an excellent example of critical systems - even in the scale of populations since without sensitivity and long range correlations cultural evolution and society would not be possible. For a physicist with the conceptual tools of existing theoretical physics the recent information society in which the actions of people at different side of globe are highly correlated, should look like a miracle.

2. The hierarchy of Planck constants with dark matter identified as phases of ordinary matter with non-standard value $h_{eff} = n \times h$ of Planck constant is one of the "almost-predictions" of TGD is definitely something essentially new physics. The phase transition transforming ordinary matter to dark matter in this sense generates long range quantal correlations and even macroscopic quantum coherence.

   Finding of a universal mechanism generating dark matter have been a key challenge during last ten years. Could quantum criticality having classical or perhaps even thermodynamical criticality as its correlate be always accompanied by the generation of dark matter? If this were the case, the recipe would be stupifyingly simple: create a critical system! Dark matter would be everywhere and we would have observed its effects for centuries! Magnetic flux tubes (possibly carrying monopole flux) define the space-time correlates for long range correlations at criticality and would carry the dark matter. They are indeed key players in TGD inspired quantum biology.

3. Change of symmetry is assigned with criticality as also conformal symmetry (in 2-D case). In TGD framework conformal symmetry is extended and infinite hierarchy of breakings of conformal symmetry so that a sub-algebras of various conformal algebras with conformal weights coming as integer multiples of integer n defining h_{eff} would occur.

4. Phase separation is what typically occurs at criticality and one should understand also this. The strengthening of this hypothesis with the assumption $h_{eff} = h_{gr}$, where $h_{gr} = GMm/v_0$ is is the gravitational Planck constant originally introduced by Nottale [K79, ?]. In the formula v_0 has dimensions of velocity, and will be proposed to be determined by a condition relating the size of the system with mass M to the radius within which the wave function of particle m with $h_{eff} = h_{gr}$ is localized in the gravitational field of M.

The condition $h_{eff} = h_{gr}$ implies that the integer n in h_{eff} is proportional to the mass of the particle. The implication is that particles with different masses reside at flux tubes with different Planck constant and separation of phases indeed occurs.

5. What is remarkable is that neither gravitational Compton length nor cyclotron energy spectrum depends on the mass of the particle. This universality could play key role in living matter. One can assign Planck constant also to other interactions such as electromagnetic interaction so that one would have $h_{em} = Z_1 Z_2 e^2/v_0$. The phase transition could take place when the perturbation series based on the coupling strength $\alpha = Z_1 Z_2 e^2/\hbar$ ceases to converge. In the new phase perturbation series would converge since the coupling strength is proportional to $1/h_{eff}$. Hence criticality and separation into phases serve as criteria as one tries to see whether the earlier proposals for the mechanisms giving rise to large h_{eff} phases make sense. One can also check whether the systems to which large h_{eff} has been assigned are indeed critical.

One example of criticality is super-fluidity. Superfluids exhibit rather mysterious looking effects such as fountain effect [D8] and what looks like quantum coherence of superfluid containers, which should be classically isolated. These findings serve as a motivation for the proposal that genuine superfluid portion of superfluid corresponds to a large h_{eff} phase near criticality at least and that also in other phase transition like phenomena a phase transition to dark phase occurs near the vicinity [?].

But how does quantum criticality relate to number theory and adelic physics? $h_{eff}/h = n$ has been identified as the number of sheets of space-time surface identified as a covering space of some kind. Number theoretic discretization defining the "spine" for a monadic space-time surface [L25] defines also a covering space with Galois group for an extension of rationals acting as covering group. Could n be identifiable as the order for a sub-group of Galois group? If this is the case, the proposed rule for h_{eff} changing phase transitions stating that the reduction of n occurs to its factor would translate to spontaneous symmetry breaking for Galois group and spontaneous - symmetry breakings indeed accompany phase transitions.

TGD variant of AdS/CFT duality

AdS/CFT duality [B37] has provided a powerful approach in the attempts to understand the non-perturbative aspects of super-string theories. The duality states that conformal field theory in n-dimensional Minkowski space M^n identifiable as a boundary of $n+1$-dimensional space AdS_{n+1} is dual to a string theory in $AdS_{n+1} \times S^{9-n}$.

As a mathematical discovery AdS/CFT duality is extremely interesting but it seems that it need not have much to do with physics as such. From TGD point of view the reason is obvious: the notion of conformal invariance is quite too limited. In TGD framework conformal invariance is extended to a super-symplectic symmetry in $\delta M^4_\pm \times CP_2$, whose Lie-algebra has the structure of conformal algebra. Also ordinary super-conformal symmetries associated with string world sheets are present as well as generalization of 2-D conformal symmetries to their analogs at light-cone boundary and light-like orbits of partonic 2-surfaces. In this framework AdS/CFT duality is expected to be modified.

The matrix elements $G_{K\overline{L}}$ of Kähler metric of WCW can be expressed in two manners. As contractions of the derivatives $\partial_K \partial_{\overline{L}} K$ of the Kähler function of WCW with isometry generators or as anticommutators $\{\Gamma_K, \Gamma_{\overline{L}}\}$ of WCW gamma matrices identified as supersymplectic Noether super charges assignable to fermionic strings connecting partonic 2-surfaces. Kähler function is identified as real part of the action: if coupling parameters are real it reduces to the action for the Euclidian space-time regions with 4-D CP_2 projection and otherwise contains contributions from both Minkowskian and Euclidian regions. The action defines the modified gamma matrices appearing in modified Dirac action as contractions of canonical momentum currents with imbedding space gamma matrices.

This observation suggests that there is a super-symmetry between action and modified Dirac action. The problem is that induced spinor fields naive of SH and also well-definedness of em charge demand the localization of induced spinor modes at 2-D string world sheets. This simply cannot be true. On the other hand, SH only requires that the data about induced spinor fields and space-time surface at the string world sheets is enough to construct the modes in space-time interior.

This leaves two options if one assumes that SH is exact (recall however that the number theoretic interpretation for the hierarchy of Planck constants suggests that the number-theoretic spin of monadic space-time surface represents additional discrete data needed besides that assignable to string world sheets to describe dark matter). As found in the section 11.2.3, there are two options.

Option I: The analog of brane hierarchy is realized at the level of fundamental action. There is a separate fundamental 2-D action assignable with string world sheets - area and topological magnetic flux term - as also world line action assignable to the boundaries of string world sheets. By previous argument string tension should be determined by the value of the cosmological constant Λ obeying -adic coupling constant evolution rather than by G: otherwise there is no hope about gravitationally bound states above Planck scale. String tension would appear as an additional fundamental coupling parameter (perhaps fixed by quantum criticality). This option does not quite conform with the spirit of SH.

Option II: 4-D space-time action and corresponding modified Dirac action defining fundamental actions are expressible as effective actions assignable to string world sheets and their boundaries. String world sheet effective action action could be expressible as string area for the effective metric defined by the anti-commutators of modified gamma matrices at string world sheet. If the sum of the induced Kähler forms of M^4 and CP_2 vanishes at string world sheets the effective metric would be the induced 2-D metric: this together with the observed CP breaking could provide a justification for the introduction of the analog of Kähler form in M^4. String tension would be dynamical rather than determined by l_P and depend on Λ, l_P, R and α_K. This representation of Kähler action would be one aspect of the analog of AdS/CFT duality in TGD framework.

Both options would allow to understand how strings connecting partonic 2-surfaces give rise to the formation of gravitationally bound states. Bound states of macroscopic size are possible only if one allows hierarchy of Planck constants and this is required also by the (extremely) small value of Λ (in cosmic scales).

Consider the concerete realizations for this vision.

1. SGCI requires effective 2-dimensionality. In given UV and IR resolutions partonic 2-surfaces and string world sheets are assignable to a finite hierarchy of CDs inside CDs with given CD characterized by a discrete scale coming as an integer multiple of a fundamental scale (essentially CP_2 size). Λ would closely relate to the size scale of CD. String world sheets have boundaries consisting of either light-like curves in induced metric at light-like wormhole throats and space-like curves at the ends of CD whose M^4 projections are light-like. These braids carrying fermionic quantum numbers intersect partonic 2-surfaces at discrete points.
2. This implies a rather concrete analogy with $AdS_5 \times S_5$ duality, which describes gluons as open strings. In zero energy ontology (ZEO) string world sheets are indeed a fundamental notion and the natural conjecture is that these surfaces are minimal surfaces, whose area by quantum classical correspondence depends on the quantum numbers of the external particles.

String tension of gravitational flux tubes

For Planckian cosmic strings only quantum gravitational bound states of length of order Planck length are possible. There must be a mechanism reducing the string tension. The *effective* string tension assignable to magnetic flux tubes must be inversely proportional to $1/h_{eff}^2$, $h_{eff} = n \times h = h_{gr} = 2\pi GMm/v_0$ in order to obtain gravitationally bound states in macroscopic length scales identified as structures for which partonic 2-surfaces are connected by flux tubes accompanied by fermionic strings.

The reason is that the size scale of (quantum) gravitationally bound states of masses M and m is given by grvitational Compton length $\Lambda_{gr} = GM/v_0$ [K97, K79, **?**] assignable to the gravitational flux tubes connecting the masses M and m. If the string tension is of order Λ_{gr}^2 this is achieved since the typical length of string would be Λ_{gr}. Gravitational string tension must be therefore of order $T_{gr} \sim 1/\Lambda_{gr}^2$. How could this be achieved? One can imagine several options and here only the option based on the assumptions

1. Twistor lift makes sense.
2. Fundamental action is 4-D for both space-time and fermionic degrees of freedom and 2-D string world sheet action is an effective action realizing SH. Note effective action makes also possible braid statistics, which does not make sense at fundamental level.

3. Also M^4 carries the analog of Kähler form and the sum of induced Kähler forms from M^4 and CP_2 vanishes at string world sheets and also weak gauge fields vanishes at string world sheets leaving only em field.

is considered since it avoids all the objections that I have been able to invent.

For the twistor lift of TGD [K10] predicting cosmological constant Λ depending on p-adic length scale $\Lambda \propto 1/p$ the gravitational strings would be naturally homologically trivial cosmic strings. These vacuum extremals of Kähler action transform to minimal surface extremals with string tension given by $\rho_{vac}S$, where ρ_{vac} the density of dark energy assignable to the volume term of the action and S the transverse area of the flux tube. One should have $\rho_{vac}S = 8\pi\Lambda S/G = 1/\Lambda_{gr}^2$ so that one would have

$$8\pi\Lambda S = \frac{G}{\Lambda_{gr}^2} \quad .$$

Λ for flux tubes (characterizing the size of CDs containing them) would depend on the gravitational coupling Mm.

11.2.5 Number theoretical vision

Physics as infinite-D spinor geometry of WCW and physics as generalized number theory are the two basic vision about TGD. The number theoretical vision involves three threads [K104, K105, K103].

1. The first thread [K104] involves the notion of number theoretical universality NTU: quantum TGD should make sense in both real and p-adic number fields (and their algebraic extensions induced by extensions of rationals). p-Adic number fields are needed to understand the space-time correlates of cognition and intentionality [K71, K42, K73].

   p-Adic mass calcuations lead to the notion of a p-adic length scale hierarchy quantifying the notion of the many-sheeted space-time [K71, K42]. One of the first applications was the calculation of elementary particle masses [K59]. The basic predictions are only weakly model independent since only p-adic thermodynamics for Super Virasoro algebra are involved. Not only the fundamental mass scales would reduce to number theory but also particle masses are predicted correctly under rather mild assumptions and are exponentially sensitive to the p-adic length scale predicted by p-adic length scale hypothesis. Also predictions such as the possibility of neutrinos to have several mass scales were made on the basis of number theoretical arguments and have found experimental support [K59, K25].

2. Second thread [K105] is inspired by the dimensions $D = 1, 2, 4, 8$ of the basic objects of TGD and assumes that classical number fields are in a crucial role in TGD. 8-D imbedding space would have octonionic structure and space-time surfaces would have associative (quaternionic) tangent space or normal space. String world sheets could correspond to commutative surfaces. Also the notion of M^8-H-duality is part of this thread and states that quaternionic 4-surfaces of M^8 containing preferred M^2 in its tangent space can be mapped to PEs in H by assigning to the tangent space CP_2 point parametrizing it. M^2 could be replaced by integrable distribution of $M^2(x)$. If PEs are also quaternionic one has also $H-H$ duality allowing to iterate the map so that PEs form a category. Also quaternion analyticity of PEs is a highly attractive hypothesis [L8]. For instance, it might be possible to interpret string world sheets and partonic 2-surfaces appearing in strong form of holography (SH) as co-dimension 2 surfaces analogous to poles of analytic function in complex plane. Light-like 3-surfaces might be seen as analogs of cuts. The coding of analytic function by its singularities could be seen as analog of SH.

3. The third thread [K103] corresponds to infinite primes and leads to several speculations. The construction of infinite primes is structurally analogous to a repeated second quantization of a supersymmetric arithmetic quantum field theory with free particle states characterized by primes. The many-sheeted structure of TGD space-time could reflect directly the structure of infinite prime coding it. Space-time point would become infinitely structured in various p-adic senses but not in real sense (that is cognitively) so that the vision of Leibniz about monads reflecting the external world in their structure is realized in terms of algebraic holography. Space-time becomes algebraic hologram and realizes also Brahman=Atman idea of Eastern philosophies.

11.3 p-Adic mass calculations and p-adic thermodynamics

p-Adic mass calculations carried for the first time around 1995 were the stimulus eventually leading to the number theoretical vision as a kind dual for the geometric vision about TGD. In this secton I will roughly describe the calculations [K25, K59] and the questions and challenges raised by them.

11.3.1 p-Adic numbers

Like real numbers, p-adic numbers (`http://tinyurl.com/hmgqtoh`) can be regarded as completions of the rational numbers to a larger number field [K42]. Each prime p defines a p-adic number field allowing the counterparts of the usual arithmetic operations.

1. The basic difference between real and p-adic numbers is that p-adic topology is ultra-metric. Ultrametricity means that the distance function $d(x,y)$ (the counterpart of $|x-y|$ in the real context) satisfies the inequality

$$d(x,z) \leq Max\{d(x,y), d(y,z)\} \ ,$$

(Max(a, b) denotes maximum of a and b) rather than the usual triangle inequality

$$d(x,z) \leq d(x,y) + d(y,z) \ .$$

2. The topology defined by p-adic numbers is compact-open. Hence the generalization of manifold obtained by gluing together n-balls fails because smallest open n-balls are just points and one has totally disconnected topology.

3. p-Adic numbers are not well-ordered like real numbers. Therefore one cannot assign orientation to the p-adic number line. This in turn leads to difficulties with attempts to define definite integrals and the notion of differential form although indefinite integral is well-defined. These difficulties serve as important guidelines in the attempts to understand what p-adic physics is and also how to fuse real and various p-adic physics to a larger structure.

4. p-Adic numbers allow an expansion in powers of p analogous to the decimal expansion

$$x = \sum_{n\geq 0} x_n p^n \ ,$$

and the number of terms in the expansion can be infinite so that p-adic number need not be finite as a real number. The norm of the p-adic number (counterpart of $|x|$ for real numbers) is defined as

$$N_p(x) = \sum_{n\geq 0} x_n p^n = p^{-n_0} \ ,$$

and depends only very weakly on p-adic number. The ultra-metric distance function can be defined as $d_p(x,y) = N_p(x-y)$.

5. p-Adic numbers allow a generalization of the differential calculus. The basic rules of the p-adic differential calculus are the same as those of the ordinary differential calculus. There is however one important new element: the set of the functions having vanishing p-adic derivative consists of so called pseudo constants, which are analogs of real valued piecewise constant functions. In the real case only constant functions have vanishing derivative. This implies that p-adic differential equations are non-deterministic. This non-determinism is identified as a counterpart of the non-determinism of cognition and imagination [K73].

11.3.2 Model of elementary particle

p-Adic mass calculations [K25, K59] rely heavily on a topological model for elementary particle and it is appropriate to describe it before going to the summary of calculations.

Family replication phenomenon topologically

One of the basic ideas of TGD approach to particle physics has been genus-generation correspondence: boundary components of the 3-surface should be carriers of elementary particle numbers and the observed particle families should correspond to various boundary topologies.

With the advent of zero energy ontology (ZEO) this picture has changed somewhat.

1. The wormhole throats identified as light-like 3-surfaces at with the induced metric of the space-time surface changes its signature from Minkowskian to Euclidian correspond to the light-like orbits of partonic 2-surfaces. One cannot of course exclude the possibility that also boundary components allow to satisfy boundary conditions without assuming vacuum extremal property of nearby space-time surface.

   The intersections of the wormhole throats with the light-like boundaries of causal diamonds (CDs) identified as intersections of future and past directed light cones ($CD \times CP_2$ is actually in question but I will speak about CDs) define special partonic 2-surfaces and the conformal moduli of these partonic 2-surfaces appear in the elementary particle vacuum functionals [K25] naturally. A modification of the original simple picture came from the proposed identification of physical particles as bound states of two wormhole contacts connected by tubes carrying monopole fluxes.

2. For generalized scattering diagrams stringy trouser vertices are replaced with vertices at which the ends of light-like wormhole throats meet. This vertex is the analog of 3-vertex for Feynman diagrams in particle physics lengths scales and for the biological replication (DNA and even cell) in macroscopic length scales.

   In this picture the interpretation of the analog of trouser vertex is in terms of propagation of same particle along two different paths. This interpretation is mathematically natural since vertices correspond to 2-manifolds rather than singular 2-manifolds, which are just splitting to two disjoint components. Second complication comes from the weak form of electric-magnetic duality forcing to identify physical particles as weak strings with magnetic monopoles at their ends and one should understand also the possible complications caused by this generalization.

These modifications force to consider several options concerning the identification of light fermions and bosons and one can end up with a unique identification only by making some assumptions. Masslessness of all wormhole throats - also those appearing in internal lines - and dynamical $SU(3)$ symmetry for particle generations are attractive general enough assumptions of this kind. Bosons and their possible spartners would correspond to wormhole contacts with fermion and anti-fermion at the throats of the contact. The expectation was tht free fermions and their possible spartners correspond to CP_2 type vacuum extremals with single wormhole throat. It however turned however that dynamical $SU(3)$ symmetry forces to identify massive (and possibly topologically condensed) fermions as pairs of (g, g) type wormhole contacts. The existence of higher boson families would mean breaking of quark and lepton universality and there are indications for this kind of anomaly [K66] .

The notion of elementary particle vacuum functional

Obviously one must know something about the dependence of the elementary particle state functionals on the geometric properties of the boundary component and in the sequel an attempt to construct what might be called elementary particle vacuum functionals (EPVFs), is made. The basic assumptions underlying the construction are the following ones [K25].

1. EPVFs depend on the geometric properties of the two-surface X^2 representing elementary particle.
2. EPVFs possess extended Diff invariance: all 2-surfaces on the orbit of the 2-surface X^2 correspond to the same value of the vacuum functional. This condition is satisfied if vacuum functionals have as their argument, not X^2 as such, but some 2- surface Y^2 belonging to the unique orbit of X^2 (determined by the principle selecting PE as a generalized Bohr orbit [K50, K8, K14]) and determined in general coordinate invariant manner.
3. ZEO allows to select uniquely the partonic 2-surface as the intersection of the wormhole throat at which the signature of the induced 4-metric changes with either the upper or lower

boundary of $CD\times CP_2$. This is essential since otherwise one one could not specify the vacuum functional uniquely.

4. Vacuum functionals possess conformal invariance and therefore for a given genus depend on a finite number of variables specifying the conformal equivalence class of Y^2.
5. Vacuum functionals satisfy the cluster decomposition property: when the surface Y^2 degenerates to a union of two disjoint surfaces (particle decay in string model inspired picture), vacuum functional decomposes into a product of the vacuum functionals associated with disjoint surfaces.
6. EPVFs are stable against the decay $g\rightarrow g_1+g_2$ and one particle decay $g\rightarrow g-1$. This process corresponds to genuine particle decay only for stringy diagrams. For generalized scattering diagrams the interpretation is in terms of propagation along two different paths simultaneously.

In [K25] the construction of EPVFs is described in detail. This requires some basic concepts related to the description of the space of the conformal equivalence classes of Riemann surfaces and the concept of hyper-ellipticity. Since theta functions will play a central role in the construction of the vacuum functionals, also their basic properties are needed. Also possible explanations for the experimental absence of the higher fermion families are considered. Concerning p-adic mass calculations, the key question is how to construct p-adic variants of EPVFs.

11.3.3 p-Adic mass calculations

p-Adic thermodynamics

Consider first the basic ideas of p-adic thermodynamics.

1. p-Adic valued mass squared is identified as as thermal mass in p-adic thermodynamics. Boltzmann weights $exp(-E/T)$ do not make sense if one just replaces exponent function with the p-adic variant of its Taylor series. The reason is that $exp(x)$ has p-adic norm equal to 1 for all acceptable values of the argument x (having p-adic norm smaller than one) so that partition function does not have the usual exponential convergence property. Nothing however prevents from consider Boltzmann weights as powers p^n making sense for integer values of n. Here the p-adic norm approaches zero for $n\rightarrow+\infty$: thus the correspondences $e^{-E/T}\leftrightarrow p^{E/T_p}$.

   The values of E/T_p must be quantized to integers. This is guaranteed if E is integer valued in suitable unit of energy and $1/T_p$ has integer valued spectrum using same unit for T_p. Super-conformal invariance guarantees integer valued spectrum of E, which in the recent case corresponds to mass squared. These number theoretical conditions are very powerful and lead to the quantization of also thermal mass squared for given p-adic prime p.
2. The p-adic mass squared is mapped to real number by canonical identification $I:\sum x_np^n\rightarrow\sum x_np^{-n}$ or its variant for rationals. Canonical identification is continuous and maps powers of p^n to their inverses. One modification of canonical identification maps rationals m/n in their representation in which m and n have no common divisors to $I(m)/I(n)$. The predictions of calculations depend in some cases on which variant one uses but rational option looks the most reasonable choice.
3. p-Adic length scale hypothesis states that preferred p-adic primes correspond to powers of 2: $p\simeq 2^k$, but smaller than 2^k. The values of k form with $p=2^k-1$ is prime - Mersenne prime - are especially favored. The nearer the prime p to 2^k, the more favored p is physically. One justification for the hypothesis is that preferred primes have been selected by an evolutionary process.
4. It turns out that p-adic temperature is $T_p=1$ for fermions. For gauge bosons $T_p\leq 1/2$ seems to be necessary assumption for gauge bosons implying that the contribution to mass squared is very small so that super-symplectic contribution assignable to the wormhole magnetic flux tube dominates for weak bosons. For canonical identification $m/n\rightarrow I(m)/I(n)$ second order contribution to fermionic mass squared is very small.
5. The large values of p-adic prime p guarantee that the p-adic thermodynamics convervges extremely rapidly. For $m/n\rightarrow I(m)/I(n)$ already the second order contribution is extremely

small since the expansion for the real mass squared is in terms of $1/p$ and for electron with $p = M_{127}$ one has $p \sim 10^{38}$. Hence the calculations are essentially exact and errors are those of the model. It is quite possible that calculations could be done exactly using exact expressions for the super-symplectic partition functions generalized to p-adic context. The success of the p-adic mass calculations is especially remarkable because p-adic length scale hypothesis $p \simeq 2^k$ predicts exponential sensitivity of the particle mass scale on k.

Symmetries

The number theoretical existence of p-adic thermodynamics requires powerful symmetries to guarantee integer valued spectrum for the thermalized contribution to the mass squared.

1. Super-conformal symmetry with integer valued conformal weights for Virasoro scaling generator L_0 is essential because it predicts in string models that mass squared is apart from ground state contribution integer valued in suitable units. In TGD framework fermionic string world sheets are characterized by super-conformal symmetry. This gives the p-adic thermodynamics assumed in the calculations. One could however assign Super Virasoro algebra also to super-symplectic algebra having its analog as sub-algebra with positive integer conformal weights. Same applies to the extended conformal algebra of light-cone boundary.

2. TGD however predicts also generalization of conformal symmetry associated with light-cone boundary involving ordinary complex conformal weights and the conformal weight associated with the light-like radial coordinate. For the latter conformal weights for the generators of supersymmetry might be given by $h = -s_n/2$. s_n zero of zeta or pole $h = -s = -1$ of zeta.

   Also super-symplectic symmetries would have similar radial spectrum of conformal weights. Conformal confinement requiring that the conformal weights of states are real implies that the spectrum of conformal weights for physical states consists of non-negative integers as for ordinary superconformal invariance.

   It is not clear whether thermalization occurs in these degrees of freedom except perhaps for trivial conformal weights. These degrees of freedom need not therefore contribute to thermal masses of leptons and quarks but would give dominating contribution to hadron masses and weak boson masses. The negative conformal weights predicted by $h = -s/2$ hypothesis predicts that ground state weight is negative for super-symplectic representations and must be compensated for massless states.

   The assumption that ground state conformal weight is negative and thus tachyonic is essential in case of p-adic mass calculations [K59], and only for massless particles (graviton, photon, gluons) it vanishes or is of order $O(1/p)$. This could be achieved if the ground state of super-symplectic representation has $h = 0$.

3. Modular invariance [K25] assignable to partonic 2-surfaces is a further assumption similar to that made made in string models. This invariance means that for a given genus the dynamical degrees of freedom of the partonic 2-surface correspond to finite-dimensional space of Teichmueller parameters. For genus $g = 0$ this space is trivial.

   Also modular invariance for string world sheets can be considered. By SH the information needed in mass calculations should be assignable to partonic 2-surfaces: the assumption is that one can assign this information to single partonic 2-surface. Stringy contribution would be seen only in scattering amplitudes.

   This might be true only effectively: the recent view about elementary particles is that they are pairs of wormhole contacts connected by flux tubes defining a closed monopole flux and wormhole throats of contact have same genus for light states. Furthermore the quantum numbers of particle are associated with single throat for fermions and with opposite throats of single contact for bosons. The second wormhole contact would carry neutralizing weak charges to realize the finite range of weak interactions as "weak confinement".

   The number of genera is infinite and one must understand why only three quark and lepton generations are observed. An attractive explanation is in terms of symmetry. For the three lowest genera the partonic 2-surfaces are always hyper-elliptic and have thus global conformal Z_2 symmetry. For higher genera this is not true always and EPVFs constructed from the assumption of modular invariance vanish for the hyper-elliptic surfaces. This suggests that the higher genera are very massive or can be interpreted as many-particle states of handles, which are not bound states but have continuous mass squared.

Contributions to mass squared

There are several contributions to the p-adic thermal mass squared come from the degrees of freedom, which are thermalized.

Super-conformal degrees of freedom associated with string world sheets are certainly thermalized. p-Adic mass calculations strongly suggest that the number of super-conformal tensor factors is $N = 5$ but also $N = 4$ and $N = 6$ can be considered marginally.

I have considered several identifications of tensor factors and not found a compelling alternative. If one assumes that super-symplectic degrees of freedom do not contribute to the thermal mass, string world sheets should explain masses of elementary fermions. Here charged lepton masses are the test bench. One other hand, if super-symplectic degrees of freedom contribute one obtains additional tensor factor assignable to $h = -s/2$, s trivial zero of zeta). Only one tensor factor emerges since Hamiltonians correspond to the products of functions of the coordinates of light-cone boundary and CP_2).

1. $SU(2)_L \times U(1)$ gives 2 tensor factors. $SU(3)$ gives 1 tensor factor. The two transversal degrees of freedom for string world sheet suggest 2 degrees of freedom corresponding to Abelian group E^2. Rotations however transforms these degrees to each other so that 1 tensor factor should emerge. This gives 4 tensor factors. Could it correspond to the degrees of freedom parallel to string at its end assignable to wormhole throat? Could normal vibrations of partonic 2-surface? This would $N = 5$ tensor factors. Another possibility is that the fifth tensor factor comes from super-symplectic Super-Virasoro algebra defined by trivial conformal weights.

2. Super-symplectic contributions need not be present for ordinary elementary fermions. For weak bosons they could give string tension assignable to the magnetic flux tube connecting the wormhole contacts. It is not clear whether this contribution is thermalized. This contribution might be present only for the phases with $h_{eff} = n \times h$. This contribution would dominate in hadron masses.

3. Color degrees of freedom contribute to the ground state mass squared since ground state corresponds to an imbedding space spinor mode massless in 8-D sense. The mass squared contribution corresponds to an eigenvalue of CP_2 spinor d'Alembertian. Its eigenvalues correspond to color multiplets and only the covariantly constant right handed neutrino is color singlet. For the other modes the color representation is non-trivial and depends on weak quantum numbers of the fermion. The construction of the massless state from a tachyonic ground state with conformal weight $h_{vac} = -3$ must involve colored super-Kac Moody generators compensating for the anomalous color charge so that one obtains color single for leptons and color triplet for quarks as massless state.

4. Modular degrees of freedom give a contribution depending on the genus g of the partonic 2-surface. This contribution is estimated by considering p-adic variants of elementary particle vacuum functionals Ω_{vac} [K59] expressible as products of theta functions with the structure of partition function. Theta functions are expressible as sums of exponent functions $exp(X)$ with X defined as a contraction of the matrix Ω_{ij} defined by Teichmueller parameters between integer valued vectors.

   In ZEO the interpretation of Ω_{vac} is as a complex square root of partition functional (quantum theory as complex square root of thermodynamics in ZEO). The integral of $|\Omega|^2$ over allowed moduli has interpretation as partition function. The exponential $exp(Re(X)) = p^{Re(X)/log(p)}$ has interpretation as an exponential of "Hamiltonian" defined by the vacuum conformal weight defined by moduli. $T = log(p)$ is identified as p-adic temperature as in ordinary p-adic thermodynamics.

   NTU requires that the integration over the moduli parameters reduces to a sum over number theoretically universal moduli parameters. The exponents $exp(X)$ must exist p-adically. PE property alone could guarantee this. The exponentials appearing in theta functions should reduce to products $p^k p^{iy} = exp(k/log(p))p^{iy}$ with k is integer and p^{iy} a root of unity. The vacuum expectation value of $Re(X)$ contributing to the mass squared is obtained from the standard formula as logarithmic temperature derivative of the "integral" $\int |\Omega_{vac}|^2$. The formula is same as for the Super-Virasoro contributions apart from the integration reducing to a sum.

The considerations of the section 11.4.2 [L13] suggest that for given p-adic prime p the exponent $k+iy$ corresponds to a linear combinations of poles of fermionic zeta $z_F(s) = \zeta(s)/\zeta(2s)$ in the class $C(p)$ with non-negative integer coefficients. This class corresponds essentially to the conformal weights of a fractal sub-algebra of super-symplectic algebra. It could give rise also to the complex values of action so that Riemann zeta would define the core of TGD.

The general dependence of the contribution of genus g to mass squared on g follows from the functional form of EPVF as a product theta functions serving as building brick partition functions apart from overall multiplicative constant and gives a nice agreement with the observed charged lepton mass ratios. The basic feature of the formula is exponential dependence on g.

5. The super-symplectic stringy contribution assignable to the magnetic flux tube dominates for weak bosons and is analogous to the stringy contribution to the hadron masses.

p-Adic mass calculations leave open several questions. What is the precise origin of preferred p-adic primes and of p-adic length scale hypothesis? How to understand the preferred number $N = 5$ of Super-Kac-Moody tensor factors? How to calculate the contribution of super-symplectic degrees of freedom - are they thermalized? Why only 3 lowest genera are light and what are the masses of the predicted bosonic higher genera implying breaking of fermion universality.

11.3.4 p-Adic length scale hypothesis

p-Adic length scale hypothesis [K1, K71] has served as a basic hypothesis of p-adic TGD for several years. This hypothesis states that the scales $L_p = \sqrt{p}l$, $l = 1.376 \cdot 10^4\sqrt{G}$ are fundamental length scale at p-adic condensate level p. The original interpretation of the hypothesis was following:

1. Above the length scale L_p p-adicity sets on and effective course grained space-time or imbedding space topology is p-adic rather than ordinary real topology. Imbedding space topology seems to be more appropriate identification.
2. The length scale L_p serves as a p-adic length scale cutoff for the quantum field theory description of particles. This means that space-time begins to look like Minkowski space so that the QFT $M^4 \to CP_2$ becomes a realistic approximation. Below this length scale string like objects and other particle like 3-surfaces are important.
3. It is un-natural to assume that just single p-adic field would be chosen from the infinite number of possibilities. Rather, there is an infinite number of cutoff length scales. To each prime p there corresponds a cutoff length scale L_p above which p-adic quantum field theory $M^4 \to CP_2$ makes sense and one has a hierarchy of p-adic QFTs. These different p-adic field theories correspond to different hierarchically levels possibly present in the topological condensate. Hierarchical ordering $< p_1 < p_2 < ...$ means that only the surface $p_1 < p_2$ can condense on the surface p_2. The condensed surface can in practice be regarded as a point like particle at level p_2 described by the p-adic conformal field theory below length scale L_{p_2}.

The recent view inspired by adelic physics is that preferred p-adic primes correspond to so called ramified primes for the algebraic extension of rationals defining the adele [K119]. Weak form of Negentropy Maximization Principle (WNMP) [K65] in turn allows to conclude that the length scales corresponding to powers of primes are preferred. Therefore p-adic length scale hypothesis generalizes. There is evidence for 3-adic time scales in biology [I7, I8] and 3-adic time scales can be also assigned with Pythagorean scale in geometric theory of harmony [K87] [L11].

11.3.5 Mersenne primes and Gaussian Mersennes are special

Mersenne primes and their complex counterparts Gaussian Mersennes pop up in p-adic mass calculations and both elementary particle physics, biology [K84], and astrophysics and cosmology [K63] provide support for them.

Mersenne primes

One can also consider the milder requirement that the exponent $\lambda = 2^{\epsilon L_0}$ represents trivial scaling represented by unit in good approximation for some p-adic topology. Not surprisingly, this is the case for $L_0 = mp^k$ since by Fermat's theorem $a^p \ mod \ p = 1$ for any integer a, in particular $a = 2$. This is also the case for $L_0 = mk$ such that $2^k \ mod \ p = 1$ for p prime. This occurs if $2^k - 1$ is Mersenne prime: in this case one has $2^{L_0} = 1$ modulo p so that the sizes of the fractal sub-algebras are exponentially larger than the sizes of $L_0 \propto p^n$ algebras. Note that all scalings a^{L_0} are near to unity for $L_0 = p^n$ whereas now only $a = 2$ gives scalings near unity for Mersenne primes. Perhaps this extended fractality provides the fundamental explanation for the special importance of Mersenne primes.

In this case integrated scalings 2^{L_0} leave the states almost invariant so that even a stronger form of the breaking of the exact conformal invariance would be in question in the super-symplectic case. The representation would be defined by the generators for which conformal weights are odd multiples of n ($M_n = 2^n - 1$) and L_{-kn}, $k > 0$ would generate zero norm states only in order $O(1/M_n)$.

Especially interesting is the hierarchy of primes defined by the so called Combinatorial Hierarchy resulting from TGD based model for abstraction process. The primes are given by $2, 3, 7 = 2^3 - 1, 127 = 2^7 - 1, 2^{127} - 1, ..$: $L_0 = n \times 127$ would correspond to M_{127}-adicity crucial for the memetic code.

Gaussian Mersennes are also special

If one allows also Gaussian primes then the notion of Mersenne prime generalizes: Gaussian Mersennes are of form $(1 \pm i)^n - 1$. In this case one could replace the scaling operations by scaling combined with a twist of $\pi/4$ around some symmetry axis: $1 + i = \sqrt{2}exp(i\pi/4)$ and generalized p-adic fractality would mean that for certain values of n the exponentiated operation consisting of n basic operations would be very near to unity.

1. The integers k associated with the lowest Gaussian Mersennes are following: $2, 3, 5, 7, 11, 19, 29, 47, 73, 79, 113$. $k = 113$ corresponds to the p-adic length scale associated with the atomic nucleus and muon. Thus all known charged leptons, rather than only e and τ, as well as nuclear physics length scale, correspond to Mersenne primes in the generalized sense.

2. The primes $k = 151, 157, 163, 167$ define perhaps the most fundamental biological length scales: $k = 151$ corresponds to the thickness of the cell membrane of about ten nanometers and $k = 167$ to cell size about 2.56 μm. This observation also suggests that cellular organisms have evolved to their present form through four basic evolutionary stages. This also encourages to think that $\sqrt{2}exp(i\pi/4)$ operation giving rise to logarithmic spirals abundant in living matter is fundamental dynamical symmetry in bio-matter.

   Logarithmic spiral provides the simplest model for biological growth as a repetition of the basic operation $\sqrt{2}exp(i\pi/4)$. The naive interpretation would be that growth processes consist of $k = 151, 157, 163, 167$ steps involving scaling by $\sqrt{2}$. This however requires the strange looking assumption that growth starts from a structure of size of order CP_2 length. Perhaps this exotic growth process is associated with pair of MEs or magnetic flux tubes of opposite time orientation and energy emergenging CP_2 sized region in a mini big bang type process and that the resulting structure serves as a template for the biological growth.

3. $k = 239, 241, 283, 353, 367, 379, 457$ associated with the next Gaussian Mersennes define astronomical length scales. $k = 239$ and $k = 241$ correspond to the p-adic time scales .55 ms and 1.1 ms: basic time scales associated with nerve pulse transmission are in question. $k = 283$ corresponds to the time scale of 38.6 min. An interesting question is whether this period could define a fundamental biological rhythm. The length scale $L(353)$ corresponds to about 2.6×10^6 light years, roughly the size scale of galaxies. The length scale $L(367) \simeq \times 3.3 \times 10^8$ light years is of same order of magnitude as the size scale of the large voids containing galaxies on their boundaries (note the analogy with cells). $T(379) \simeq 2.1 \times 10^{10}$ years corresponds to the lower bound for the order of the age of the Universe. $T(457) \sim 10^{22}$ years defines a completely superastronomical time and length scale.

11.3.6 Questions

The proposed picture leaves open several questions.

1. Could the descriptions by both real and p-adic thermodynamics be possible? Could they be equivalent (possibly in finite measurement resolution) as is suggested by NTU? The consistency of these descriptions would imply temperature quantization and p-adic length scale hypothesis not possible in purely real context.
2. What could the extension of conformal symmetry to supersymplectic symmetry mean? One possible view is that super-symplectic symmetries correspond to dark degrees of freedom and that only the super-symplectic ground states with negative conformal weights affect the p-adic thermodynamics, which applies only to fermionic degrees of freedom at string world sheets. Super-symplectic degrees of freedom would give the dominant contribution to hadron masses and could contribute also to weak gauge boson masses. $N = 5$ for the needed number of tensor factors is however a strong constraint and perhaps most naturally obtained when also the super-symplectic Virasoro associated with the trivial zeros of zeta is thermalized.
3. What happens in dark sectors. Preferred extremal property is proposed to mean that the states are annihilated by super-symplectic sub-algebra isomorphic to the original algebra and its commutator with the entire algebra. The conjecture is that this gives rise to Kac-Moody algebras as dynamical symmetries - maybe ADE type algebras, whose Dynkin diagrams characterize the inclusion of HFFs. Does this give an additional tensor factor to super-Virasoro algebra?
4. Superconformal symmetry true in the sense that Super Virasoro conditions hold true. Partition function however depends on mass squared only rather than the entire scaling generator L_0 as thought erratically in the first formulation of p-adic calculation. This does not mean breaking of conformal invariance. Super Virasoro conditions hold true although partition function is for the vibrational part of L_0 determining the mass squared spectrum.

11.4 p-Adicization and adelic physics

This section is devoted to the challenges related to p-adicization and adelization of physics in which the correspondence between real and p-adic numbers via canonical identification serves as the basic building brick. Also the problems associated with p-adic variants of integral, Fourier analysis, Hilbert space, and Riemann geometry should be solved in a way respecting fundamental symmetries and their p-adic variants must be met. The notion of number theoretical universality (NTU) plays a key role here. One should also answer to questions about the origin of preferred primes and p-adic length scale hypothesis.

11.4.1 Challenges

The basic challenges encountered are construction of the p-adic variants of real number based physics, understanding their relationship to real physics, and the fusion of various physics to single coherent whole.

The p-adicization of real physics is not just a straightforward formal generalization of scattering amplitudes of existing theories but requires a deeper understanding of the physics involved. The interpretation of p-adic physics as correlate for cognition and imagination is an important guideline and will be discussed l in more detail in separate section.

Definite integral and Fourier analysis are basic elements of standard physics and their generalization to the p-adic context defines a highly non-trivial challenge. Also the p-adic variants of Riemann geometry and Hilbert space are suggestive. There are however problems.

1. There are problems associated with p-adic definite integral. Riemann sum does not make sense since it approaches zero if the p-adic norm of discretization unit approaches zero. The problems are basically due to the absence of well-orderedness essential for the definition of definite integral and differential forms and their integrals.

   Residue integration might make sense in finite angle resolution. For algebraic extension containing $e^{i\pi/n}$ the number theoretically universal approximation $i\pi = n(e^{i\pi/n} - 1)$ could be used. In twistor approach integrations reduce to multiple residue integrations and since twistor approach generalizes in TGD framework, this approach to integration is very attractive.

Positivity is a central notion in twistor Grassmannian approach [B24]. Since canonical identification maps p-adic numbers to non-negative real numbers, there is a strong temptation to think that positivity relates to NTU [L15].

2. There are problems with Fourier analysis. The naive generalization of trigonometric functions by replacing e^{ix} with its p-adic counterpart is not physical. Same applies to e^x. Algebraic extensions are needed to get roots of unity ad e as counterparts of the phases and discretization is necessary and has interpretation in terms of finite resolution for angle/phase and its hyperbolic counterpart.

3. The notion of Hilbert space is problematic. The naive generalization of Hilbert space norm square $|x|^2 = \sum x_n \overline{x}_n$ for state $(x_1, x_2, ...)$ can vanish p-adically. Also here NTU could help. State would contain as coefficients only roots of e and unity and only the overall factor could be p-adic number. Coefficients could be restricted to the algebraic numbers generating the algebraic extension of rational numbers and would not contain powers of p or even ordinary p-adic numbers expect in the overall normalization factor.

Second challenge relates to the relationship between real and p-adic physics. Canonical identification (CI) $\sum x_n p^n \rightarrow \sum x_n p^{-n}$ or some of its variants should play an important role. CI is expected to map the invariants appearing in scattering amplitudes to their real counterparts.

1. Real and p-adic variants of space-time surfaces should exist and relate to each other somehow. Is this relationship local and involve CI at space-time level or imbedding space level? Or is it only a global and non-local assignment of preferred real extremals to their p-adic counterparts? Or is between these extreme options and involves algebraic discretization of the space-time surface weakening the strong form of SH as already proposed? How do real and p-adic imbedding spaces relate to each other and can this relationship induce local correspondence between preferred extremals (PEs) [K8, K14, K10]?

2. NTU in some sense is a highly suggestive approach to these questions and would suggest that canonical identification applies to isometry invariants whereas angles and hyperbolic angles, or rather the corresponding "phases" belonging to an extension of p-adics containing roots of e and roots of unity are mapped to themselves. Note that the roots of e define extensions of rationals, which induce finite dimensional algebraic extensions of p-adic numbers. This would make possible to define imbedding space in accordance with NTU. Also the Hilbert space could be defined by requiring that its points correspond to number theoretically universal angles expressible in terms of roots of unity.

3. What about real and p-adic variants of WCW? Are they needed at all? Or could their existence be used as a powerful constraint on real physics? The representability of WCW as a union of infinite-dimensional symmetric spaces labelled by zero modes suggests that the same description applies at the level of WCW and imbedding space.

One cannot circumvent the question about how to generalize functional integral from real WCW to p-adic WCWs. In particular, what is the p-adic variant of the action defining the dynamics of space-time surfaces. In the case of exponent of action the p-adic variant could be defined by assuming algebraic universality: again the roots of e and of unity would be in central role. Also the Kähler structure of WCW implying that Gaussian and metric determinants cancel each other in functional integral, would be absolutely crucial.

One must remember that the exponents of action for scattering amplitudes for the stationary phase extremal cancel from the path integral representation of scattering amplitudes. Also now this mechanism would allow to get rid of the poorly defined exponent for single minimum. If there is sum over scattering amplitudes assignable to different maxima, normalization sould give ratios of these exponents for different extrema/maxima and only these ratios should belong to the extension of rationals.

The zero modes of WCW metric are invariants of supersymplectic group so that canonical identification could relate their real and p-adic variants. Zero modes could break NTU and would be behind p-adic thermodynamics and dependence of mass scale on p-adic prime.

The third challenge relates to the fusion of p-adic physics and real physics to a larger structure. Here a generalization of number concept obtained by glueing reals and various p-adics together along an extension of rational numbers inducing the extensions of p-adic numbers is highly suggestive. Adeles associated with the extension of rationals are highly attractive and closely related notion. Real and various p-adic physics would be correlates for sensory and cognitive aspects of the same universal physics rather than separate physics in this framework. One important implication of this view is that real entropy and p-adic negentropies characterize the same entanglement with coefficients in an extension of rationals.

NTU for hyperbolic and ordinary phases is definetely the central idea. How the invariance of angles under conformal transformations does relate to this? Could one perhaps define a discretized version of conformal symmetry preserving the phases defined by the angles between vectors assignable with the tangent spaces of discretized geometric structures and thus respecting NTU? Of should one apply conformal symmetry at Lie algebra level only?

11.4.2 NTU and the correspondence between real and p-adic physics

p-Adic real correspondence is certainly the basic problem of p-adicization and adelization. One can make several general questions about p-adic real correspondence and canonical identification inspired by p-adic mass calculations.

How generally p-adic real correspondence does apply? Could canonical identification for group invariants combined with direct identification of ordinary and hyperbolic phases identified as roots of unity and e apply at WCW and imbedding space level having maximally symmetric geometries? Could this make sense even at space-time level as a correspondence induced from imbedding space level [L25]? Does canonical identification apply locally for the discretizations of space-time surface or only globally for the parameters characterizing PEs (string world sheets and partonic 2-surfaces by SH), which are general coordinate invariant and Poincare invariant quantities?

The following vision seems to be the most feasible one found hitherto.

1. Preservation of symmetries and continuity compete. Lorenz transformations do not commute with canonical identification. This suggests that canonical identification applies only to Lorentz invariants formed from quantum numbers. This is enough in the case of scattering amplitudes. Canonical identification applies only to isometry invariants at the level of WCW and the phases/exponents of ordinary/hyperbolic angles correspond to numbers in the algebraic extension common to extensions of rationals and various p-adics.

2. Canonical identification applies at the level of momentum space and maps p-adic Lorentz invariants of scattering amplitudes to their real counterparts. Phases of angles and their hyperbolic counterparts should correspond to parameters defining extension and should be mapped as such to their p-adic counterparts.

3. The constraints coming from GCI and symmetries do not allow local correspondence but allow to consider its discretized version at space-time leve induced by the correspondence at the level of imbedding space.

   This requires the restriction of isometries and other symmetries to algebraic subgroups defined by the extension of rationals. This would imply reduction of symmetry due to finite cognitive/measurement resolution and should be acceptable. If one wants to realize the ideas about imagination, discretization must be applied also for the space-time interior meaning partial breaking of SH and giving rise to dark matter degrees freedom in TGD sense. SH could apply in real sector for realizable imaginations only. Note that the number of algebraic points of space-time surface is expected to be relatively small.

The correspondence must be considered at the level of imbedding space, space-time, and WCW.

1. At the level of imbedding space p-adic–real correspondence is induced by points in extension of rationals and is totally discontinuous. This requires that space-time dimension is smaller than imbedding space dimension.

2. At space-time level the correspondence involves field equations derivable from a local variational principle make sense also p-adically although the action itself is ill-defined as 4-D integral. The notion of p-adic PE makes sense by strong form of holography applied to 2-surfaces in the intersection. p-Adically however only the vanishing of Noether currents for a sub-algebra of the super-symplectic algebra might make sense. This condition is stronger than the vanishing of Noether charges defined by 3-D integrals.

3. Correspondence at the level of WCW can make sense and reduces to that for string world sheets and partonic 2-surfaces by SH. Real and p-adic 4-surfaces would be obtained by algebraic continuation as PEs from 2-surfaces by assuming that the space-time surface contains subset of points of imbedding space belonging to the extension of rationals [L25]. p-Adic pseudo constants make p-adic continuation easy. Real continuation need not exist always. p-Adic WCW would be considerably larger than real WCW and make possible a predictive quantum theory of imagination and cognition.

   What I have called intersection of realities and p-adicities can be identified as the set of 2-surfaces plus algebraic discretization of space-time interior. Also the values of induced spinor fields at the points of discretization must be given. The parameters characterizing the extremals (say coefficients of polynomials) - WCW coordinates - would be in extension of rationals inducing a finite-D extension of p-adic number fields.

   The hierarchy of algebraic extensions induces an evolutionary hierarchy of adeles. The interpretation could be as a mathematical correlate for cosmic evolution realized at the level of the core of WCW defined by the intersection? 2-surfaces could be called space-time genes.

4. Also the p-adic variant Kähler action or at least the exponent of Kähler action defining vacuum functional should be obtainable by algebraic continuation. The weakest condition states that the ratios of action exponents for the maxima of Kähler function to the sum of action exponents for maxima belong to the extension. Without this condition the hopes of satisfying NTU seem rather meager.

11.4.3 NTU at space-time level

What about NTU at space-time level? NTU requires a correspondence between real and p-adic numbers and the details of this corresponds have been a long standing problem.

1. The recent view about the correspondence between real PEs to their p-adic counterparts does not demand discrete local correspondence assumed in the earlier proposal [K122]. The most abstract approach would give up the local correspondence at space-time level altogether, and restrict the preferred coordinates of WCW (having maximal group of isometries) to numbers in the extension of rationals considered. WCW would be discretized.

   Intuitively a more realistic view is a correspondence at space-time level in the sense that real and p-adic space-time sheets intersect at points belonging to the extension of rationals and defining "cognitive representations". Only some p-adic space-time surfaces would have real counterpart.

2. The strongest form of NTU would require that the allowed points of imbedding space belonging an extension of rationals are mapped as such to corresponding extensions of p-adic number fields (no canonical identification). At imbedding space level this correspondence would be extremely discontinuous. The "spines" of space-time surfaces would however contain only a subset of points of extension, and a natural resolution length scale could emerge and prevent the fluctuation. This could be also seen as a reason for why space-times surfaces must be 4-D. The fact that the curve $x^n + y^n = z^n$ has no rational points for $n > 2$, raises the hope that the resolution scale could emerge spontaneously.

3. The notion of monadic geometry discussed in detail in [L25] would realize this idea. Define first a number theoretic discretization of imbedding space in terms of points, whose coordinates in group theoretically preferred coordinate system belong to the extension of rationals considered. One can say that these algebraic points are in the intersection of reality and various p-adicities. Overlapping open sets assigned with this discretization define in the real sector a covering by open sets. In p-adic sector compact-open-topology allows to assign with each point 8^{th} Cartesian power of algebraic extension of p-adic numbers. These compact open

sets define analogs for the monads of Leibniz and p-adic variants of field equations make sense inside them.

The monadic manifold structure of H is induced to space-time surfaces containing discrete subset of points in the algebraic discretization with field equations defining a continuation to space-time surface in given number field, and unique only in finite measurement resolution. This approach would resolve the tension between continuity and symmetries in p-adic–real correspondence: isometry groups would be replaced by their sub-groups with parameters in extension of rationals considered and acting in the intersection of reality and p-adicities.

The Galois group of extension acts non-trivially on the "spines" of space-time surfaces. Hence the number theoretical symmetries act as physical symmetries and define the orbit of given space-time surface as a kind of covering space. The coverings assigned to the hierarchy of Planck constants would naturally correspond to Galois coverings and dark matter would represent number theoretical physics.

This would give rise to a kind of algebraic hierarchy of adelic 4-surfaces identifiable as evolutionary hierarchy: the higher the dimension of the extension, the higher the evolutionary level.

11.4.4 NTU and WCW

p-Adic–real correspondence at the level of WCW

It has not been obvious whether one should perform p-adicization and adelization at the level of WCW. Minimalist could argue that scattering amplitudes are all we want and that their p-adicization and adelization by algebraic continuation can be tolerated only if it can give powerful enough constraints on the amplitudes.

1. The anti-commutations for fermionic oscillator operators are number theoretically universal. Supersymmetry suggests that also WCW bosonic degrees of freedom satisfy NTU. This could mean that the coordinates of p-adic WCW consist of super-symplectic invariants mappable by canonical identification to their real counterparts plus phases and their hyperbolic counterparts expressible as genuinely algebraic numbers common to all number fields. This kind of coordinates are naturally assignable to symmetric spaces [L25].

2. Kähler structure should be mapped from p-adic to real sector and vice versa. Vacuum functional identified as exponent of action should be NTU. Algebraic continuation defined by SH involves p-adic pseudo constants. All p-adic continuations by SH should correspond to the same value of exponent of action obtained by algebraic continuation from its real value. The degeneracy associated with p-adic pseudo-constants would be analogous to gauge invariance - imagination in TGD inspired theory of consciousness.

3. Ist it possible have NTU for WCW functional integration? Or is it enough to realize NTU for scattering amplitudes only. What seems clear that functional integral must reduce to a discrete sum. Physical intuition suggests a sum over maxima of Kähler function forming a subset of PEs representing stationary points. One cannot even exclude the possibility that the set of PEs is discrete and that one can sum over all of them.

   Restriction to maximum/stationary phase approximation gives rise to sum over exponents multiplied with Gaussian determinants. The determinant of Kähler metric however cancels the Gaussian determinants, and one obtains only a sum over the exponents of action.

   The breaking of strong NTU could happen: consider only p-adic mass calculations. This breaking is however associated with the parts of quantum states assignable to the boundaries of CD, not with the vacuum functional.

NTU for functional integral

Number theoretical vision relies on NTU. In fermionic sector NTU is necessary: one cannot speak about real and p-adic fermions as separate entities and fermionic anti-commutation relations are indeed number theoretically universal.

What about NTU in case of functional integral? There are two opposite views.

1. One can define p-adic variants of field equations without difficulties if preferred extremals are minimal surface extremals of Kähler action so that coupling constants do not appear in the solutions. If the extremal property is determined solely by the analyticity properties as it is for various conjectures, it makes sense independent of number field. Therefore there would be no need to continue the functional integral to p-adic sectors. This in accordance with the philosophy that thought cannot be put in scale. This would be also the option favored by pragmatist.
2. Consciousness theorist might argue that also cognition and imagination allow quantum description. The supersymmetry NTU should apply also to functional integral over WCW (more precisely, its sector defined by CD) involved with the definition of scattering amplitudes.

1. Key observations

The general vision involves some crucial observations.

1. Only the expressions for the scatterings amplitudes should should satisfy NTU. This does not require that the functional integral satisfies NTU.
2. Since the Gaussian and metric determinants cancel in WCW Kähler metric the contributions form maxima are proportional to action exponentials $exp(S_k)$ divided by the $\sum_k exp(S_k)$. Loops vanish by quantum criticality.
3. Scattering amplitudes can be defined as sums over the contributions from the maxima, which would have also stationary phase by the double extremal property made possible by the complex value of α_K. These contributions are normalized by the vacuum amplitude.

   It is enough to require NTU for $X_i = exp(S_i)/\sum_k exp(S_k)$. This requires that $S_k - S_l$ has form $q_1 + q_2 i\pi + q_3 log(n)$. The condition brings in mind homology theory without boundary operation defined by the difference $S_k - S_l$. NTU for both S_k and $exp(S_k)$ would only values of general form $S_k = q_1 + q_2 i\pi + q_3 log(n)$ for S_k and this looks quite too strong a condition.
4. If it is possible to express the 4-D exponentials as single 2-D exponential associated with union of string world sheets, vacuum functional disappears completely from consideration! There is only a sum over discretization with the same effective action and one obtains purely combinatorial expression.

2. What does one mean with functional integral?

The definition of functional integral in WCW is one of the key technical problems of quantum TGD [K119]. NTU states that the integral should be defined simultaneously in all number fields in the intersection of real and p-adic worlds defined by string world sheets and partonic 2-surfaces with WCW coordinates in algebraic extension of rationals and allowing by strong holography continuation to 4-D space-time surface. NTU is powerful constraint and could help in this respect.

1. Path integral is not in question. Rather, the functional integral is analogous to Wiener integral and perhaps allows identification as a genuine integral in the real sector. In p-adic sectors algebraic continuation should give the integral and here number theoretical universality gives excellent hopes. The integral would have exactly the same form in real and p-adic sector and expressible solely in terms of algebraic numbers characterizing algebraic extension and finite roots of e and roots of unity $U_n = exp(i2\pi/n)$ in algebraic extension of p-adic numbers.

   Since vacuum functional $exp(S)$ is exponential of complex action S, the natural idea is that only rational powers e^q and roots of unity and phases $exp(i2\pi q)$ are involved and there is no dependence on p-adic prime p! This is true in the integer part of q is smaller than p so that one does not obtain e^{kp}, which is ordinary p-adic number and would spoil the number theoretic universality. This condition is not possible to satisfy for all values of p unless the value of Kähler function is smaller than 2. One might consider the possibility that the allow primes are above some minimum value.

   The minimal solution to NTU conditions is that the ratios of action exponentials for maxima of Kähler function to the sum of these exponentials belong to the extension of rationals considered.

2. What does one mean with functional integral? TGD is expected to be an integrable in some sense. In integrable QFTs functional integral reduces to a sum over stationary points of the action: typically only single point contributes - at least in good approximation.

   For real α_K and Λ vacuum functional decomposes to a product of exponents of real contribution from Euclidian regions ($\sqrt{g_4}$ real) and imaginary contribution Minkowskian regions ($\sqrt{g_4}$ imaginary). There would be no exchange of momentum between Minkowskian and Euclidian regions. For complex values of α_K [L14] situation changes and Kähler function as real part of action receives contributions from both Euclidian and Minkowskian regions. The imaginary part of action has interpretation as analog of Morse function and action as it appears in QFTs. Now saddle points must be considered.

   PEs satisfy extremely strong conditions [K8, K10]. All classical Noether charges for a sub-algebra associated with super-symplectic algebra and isomorphic to the algebra itself vanish at both ends of CD. The conformal weights of this algebra are $n > 0$-ples of those for the entire algebra. What is fascinating that the condition that the preferred extremals are minimal surface extremals of Kähler action could solve these conditions and guarantee also NTU at the level of space-time surfaces. Supersymplectic boundary conditions at the ends of CD would however pose number theoretic conditions on the coupling parameters. In p-adic case the conditions should reduce to purely local conditions since p-adic charges are not well-defined as integrals.

3. In TGD framework one is constructing zero energy states rather calculating the matrix elements of S-matrix in terms of path integral. This gives certain liberties but a natural expectation is that functional integral as a formal tool at least is involved.

   Could one *define* the functional integral as a discrete sum of contributions of standard form for the preferred extremals, which correspond to maxima in Euclidian regions and associated stationary phase points in Minkowskian regions? Could one assume that WCW spinor field is concentrated along single maximum/stationary point.

   Quantum classical correspondence suggests that in Cartan algebra isometry charges are equal to the quantal charges for quantum states expressible in number theoretically universal manner in terms of fermionic oscillator operators or WCW gamma matrices? Even stronger condition would be that classical correlation functions are identical with quantal ones for allowed space-time surfaces in the quantum superposition. Could the reduction to a discrete sum be interpreted in terms of a finite measurement resolution?

4. In QFT Gaussian determinants produce problems because they are often poorly defined. In the recent case they could also spoil the NTU based on the exceptional properties of e. In the recent case however Gaussian determinant and metric determinant for Kähler metric cancel each other and this problem disappears. One could obtain just a sum over products of roots of e and roots of unity. Thus also Kähler structure seems to be crucial for the dream about NTU.

11.4.5 Breaking of NTU at the level of scattering amplitudes

NTU in strong sense could be broken at the level of scattering amplitudes. At space-time level the breaking does not look natural in the recent framework. Consider only p-adic mass calculations predicting that mass scale depends on p-adic prime. Also for the action strong form of NTU might fail for small p-adic primes since the value of the real part of action would be larger than than p. Should one allow this? What does one actually mean with NTU in the case of action?

Canonical identification is an important element of p-adic mass calculations and might also be needed to map p-adic variants of scattering amplitudes to their real counterparts. The breaking of NTU would take place, when the canonical real valued image of the p-adic scattering amplitude differs from the real scattering amplitude. The interpretation would be in terms of finite measurement resolution. By the finite measurement/cognitive resolution characterized by p one cannot detect the difference.

The simplest form of the canonical identification is $x = \sum_n x_n p^n \to \sum x_n p^{-n}$. Product xy and sum $x + y$ do not in general map to product and sum in canonical identification. The interpretation would be in terms of a finite measurement resolution: $(xy)_R = x_R y_R$ and $(x+y)_R = x_R + y_R$ only modulo finite measurement resolution. p-Adic scattering amplitudes are obtained by algebraic continuation from the intersection by replacing algebraic number valued parameters (such as momenta) by general p-adic numbers. The real images of these amplitudes under canonical identification are in general not identical with real scattering amplitudes the interpretation being in terms of a finite measurement resolution.

In p-adic thermodynamics NTU in the strong sense fails since thermal masses depend on p-adic mass scale. NTU can be broken by the fermionic matrix elements in the functional integral so that the real scattering amplitudes differ from the canonical images of the p-adic scattering amplitudes. For instance, the elementary particle vacuum functionals in the space of Teichmueller parameters for the partonic 2-surfaces and string world sheets should break NTU [K25].

11.4.6 NTU and the spectrum of Kähler coupling strength

During years I have made several attempts to understand coupling evolution in TGD framework. The most convincing proposal has emerged rather recently and relates the spectrum of $1/\alpha_K$ to that for the zeros of Riemann zeta [L14] and to the evolution of the electroweak U(1) couplings strength.

1. The first idea dates back to the discovery of WCW Kähler geometry defined by Kähler function defined by Kähler action (this happened around 1990) [K50]. The only free parameter of the theory is Kähler coupling strength α_K analogous to temperature parameter α_K postulated to be is analogous to critical temperature. Whether only single value or entire spectrum of of values α_K is possible, remained an open question.

   About decade ago I realized that Kähler action is *complex* receiving a real contribution from space-time regions of Euclidian signature of metric and imaginary contribution from the Minkoswkian regions. Euclidian region would give Kähler function and Minkowskian regions analog of QFT action of path integral approach defining also Morse function. Zero energy ontology (ZEO) [K123] led to the interpretation of quantum TGD as complex square root of thermodynamics so that the vacuum functional as exponent of Kähler action could be identified as a complex square root of the ordinary partition function. Kähler function would correspond to the real contribution Kähler action from Euclidian space-time regions. This led to ask whether also Kähler coupling strength might be complex: in analogy with the complexification of gauge coupling strength in theories allowing magnetic monopoles. Complex α_K could allow to explain CP breaking. I proposed that instanton term also reducing to Chern-Simons term could be behind CP breaking.

   The problem is that the dynamics in Minkowskian and Euclidian regions decouple completely and if Euclidian regions serve as space-time correlates for physical objects, there would be no exchanges of classical charges between physical objects. Should one conclude that α_K must be complex?

2. p-Adic mass calculations for 2 decades ago [K59] inspired the idea that length scale evolution is discretized so that the real version of p-adic coupling constant would have discrete set of values labelled by p-adic primes. The simple working hypothesis was that Kähler coupling strength is renormalization group (RG) invariant and only the weak and color coupling strengths depend on the p-adic length scale. The alternative ad hoc hypothesis considered was that gravitational constant is RG invariant. I made several number theoretically motivated ad hoc guesses about coupling constant evolution, in particular a guess for the formula for gravitational coupling in terms of Kähler coupling strength, action for CP_2 type vacuum extremal, p-adic length scale as dimensional quantity. Needless to say these attempts were premature and a hoc.

3. The vision about hierarchy of Planck constants $h_{eff} = n \times h$ and the connection $h_{eff} = h_{gr} = GMm/v_0$, where $v_0 < c = 1$ has dimensions of velocity [?] forced to consider very seriously the hypothesis that Kähler coupling strength has a spectrum of values in one-one correspondence with p-adic length scales. A separate coupling constant evolution associated with h_{eff} induced by $\alpha_K \propto 1/\hbar_{eff} \propto 1/n$ looks natural and was motivated by the idea that Nature is theoretician friendly: when the situation becomes non-perturbative, Mother Nature comes in rescue and an h_{eff} increasing phase transition makes the situation perturbative again.

   Quite recently the number theoretic interpretation of coupling constant evolution [K119] [L13] in terms of a hierarchy of algebraic extensions of rational numbers inducing those of p-adic number fields encouraged to think that $1/\alpha_K$ has spectrum labelled by primes and values of h_{eff}. Two coupling constant evolutions suggest themselves: they could be assigned to length scales and angles which are in p-adic sectors necessarily discretized and describable using only algebraic extensions involve roots of unity replacing angles with discrete phases.

4. Few years ago the relationship of TGD and GRT was finally understood [K114]. GRT space-time is obtained as an approximation as the sheets of the many-sheeted space-time of TGD are replaced with single region of space-time. The gravitational and gauge potential of sheets add together so that linear superposition corresponds to set theoretic union geometrically. This forced to consider the possibility that gauge coupling evolution takes place only at the level of the QFT approximation and α_K has only single value. This is nice but if true, one does not have much to say about the evolution of gauge coupling strengths.

5. The analogy of Riemann zeta function with the partition function of complex square root of thermodynamics suggests that the zeros of zeta have interpretation as inverses of complex temperatures $s = 1/\beta$. Also $1/\alpha_K$ is analogous to temperature. This led to a radical idea to be discussed in detail in the sequel.

   Could the spectrum of $1/\alpha_K$ reduce to that for the zeros of Riemann zeta or - more plausibly - to the spectrum of poles of fermionic zeta $\zeta_F(ks) = \zeta(ks)/\zeta(2ks)$ giving for $k = 1/2$ poles as zeros of zeta and as point $s = 2$? ζ_F is motivated by the fact that fermions are the only fundamental particles in TGD and by the fact that poles of the partition function are naturally associated with quantum criticality whereas the vanishing of ζ and varying sign allow no natural physical interpretation.

   The poles of $\zeta_F(s/2)$ define the spectrum of $1/\alpha_K$ and correspond to zeros of $\zeta(s)$ and to the pole of $\zeta(s/2)$ at $s = 2$. The trivial poles for $s = 2n$, $n = 1, 2, ..$ correspond naturally to the values of $1/\alpha_K$ for different values of $h_{eff} = n \times h$ with n even integer. Complex poles would correspond to ordinary QFT coupling constant evolution. The zeros of zeta in increasing order would correspond to p-adic primes in increasing order and UV limit to smallest value of poles at critical line. One can distinguish the pole $s = 2$ as extreme UV limit at which QFT approximation fails totally. CP_2 length scale indeed corresponds to GUT scale.

6. One can test this hypothesis. $1/\alpha_K$ corresponds to the electroweak U(1) coupling strength so that the identification $1/\alpha_K = 1/\alpha_{U(1)}$ makes sense. One also knows a lot about the evolutions of $1/\alpha_{U(1)}$ and of electromagnetic coupling strength $1/\alpha_{em} = 1/[cos^2(\theta_W)\alpha_{U(1)}$. What does this predict?

   It turns out that at p-adic length scale $k = 131$ ($p \simeq 2^k$ by p-adic length scale hypothesis, which now can be understood number theoretically [K119]) fine structure constant is predicted with .7 per cent accuracy if Weinberg angle is assumed to have its value at atomic scale! It is difficult to believe that this could be a mere accident because also the prediction evolution of $\alpha_{U(1)}$ is correct qualitatively. Note however that for $k = 127$ labelling electron one can reproduce fine structure constant with Weinberg angle deviating about 10 per cent from the measured value of Weinberg angle. Both models will be considered.

7. What about the evolution of weak, color and gravitational coupling strengths? Quantum criticality suggests that the evolution of these couplings strengths is universal and independent of the details of the dynamics. Since one must be able to compare various evolutions and combine them together, the only possibility seems to be that the spectra of gauge coupling strengths are given by the poles of $\zeta_F(w)$ but with argument $w = w(s)$ obtained by a global conformal transformation of upper half plane - that is Möbius transformation (see `http://tinyurl.com/gwjs85b`) with real coefficients (element of $GL(2, R)$) so that one as $\zeta_F((as + b)/(cs + d))$. Rather general arguments force it to be and element of $GL(2, Q)$, $GL(2, Z)$ or maybe even $SL(2, Z)$ ($ad - bc = 1$) satisfying additional constraints. Since TGD predicts several scaled variants of weak and color interactions, these copies could be perhaps parameterized by some elements of $SL(2, Z)$ and by a scaling factor K.

   Could one understand the general qualitative features of color and weak coupling contant evolutions from the properties of corresponding Möbius transformation? At the critical line there can be no poles or zeros but could asymptotic freedom be assigned with a pole of $cs + d$ and color confinement with the zero of $as + b$ at real axes? Pole makes sense only if Kähler action for the preferred extremal vanishes. Vanishing can occur and does so for massless extremals characterizing conformally invariant phase. For zero of $as + b$ vacuum function would be equal to one unless Kähler action is allowed to be infinite: does this make sense?. One can however hope that the values of parameters allow to distinguish between weak and color interactions. It is certainly possible to get an idea about the values of the parameters of the transformation and one ends up with a general model predicting the entire electroweak coupling constant evolution successfully.

To sum up, the big idea is the identification of the spectra of coupling constant strengths as poles of $\zeta_F((as + b/)(cs + d))$ identified as a complex square root of partition function with motivation coming from ZEO, quantum criticality, and super-conformal symmetry; the discretization of the RG flow made possible by the p-adic length scale hypothesis $p \simeq k^k$, k prime; and the assignment of complex zeros of ζ with p-adic primes in increasing order. These assumptions reduce the coupling constant evolution to four real rational or integer valued parameters (a, b, c, d). In the sequel this vision is discussed in more detail.

11.4.7 Other applications of NTU

NTU in the strongest form says that all numbers involved at "basic level" (whatever this means!) of adelic TGD are products of roots of unity and of power of a root of e. This is extremely powerful physics inspired conjecture with a wide range of possible mathematical applications.

1. For instance, vacuum functional defined as an exponent of action for preferred externals would be number of this kind. One could define functional integral as adelic operation in all number fields: essentially as sum of exponents of action for stationary preferred extremals since Gaussian and metric determinants potentially spoiling NTU would cancel each other leaving only the exponent.
2. The implications of NTU for the zeros of Riemann zeta [L13] will be discussed in more detail in the Appendix. Suffice it to say that the observations about Fourier transform for the distribution of loci of non-trivial zeros of zeta together with NTU leads to explicit proposal for the algebraic for of zeros of zeta. The testable proposal is that zeros decompose to disjoint classes $C(p)$ labelled by primes p and the condition that p^{iy} is root of unity in given class $C(p)$.
3. NTU generalises to all Lie groups. Exponents $exp(in_iJ_i/n)$ of lie-algebra generators define generalisations of number theoretically universal group elements and generate a discrete subgroup of compact Lie group. Also hyperbolic "phases" based on the roots $e^{m/n}$ are possible and make possible discretized NTU versions of all Lie-groups expected to play a key role in adelization of TGD.

   NTU generalises also to quaternions and octonions and allows to define them as number theoretically universal entities. Note that ordinary p-adic variants of quaternions and octonions do not give rise to a number field: inverse of quaternion can have vanishing p-adic variant of norm squared satisfying $\sum_n x_n^2 = 0$.

   NTU allows to define also the notion of Hilbert space as an adelic notion. The exponents of angles characterising unit vector of Hilbert space would correspond to roots of unity.

11.4.8 Going to the roots of p-adicity

The basic questions raised by the p-adic mass calculations concern the origin of preferred p-adic primes and of p-adic length scale hypothesis. One can also ask whether there might be a natural origin for p-adicity at the level of WCW.

Preferred primes as ramified primes for extensions of rationals?

Preferred primes as ramified primes for extensions of rationals?

The intuitive feeling is that the notion of preferred prime is something extremely deep and to me the deepest thing I know is number theory. Does one end up with preferred primes in number theory? This question brought to my mind the notion of *ramification of primes* (`http://tinyurl.com/hddljlf`) (more precisely, of prime ideals of number field in its extension), which happens only for special primes in a given extension of number field, say rationals. Ramification is completely analogous to the degeneracy of some roots of polynomial and corresponds to criticality if the polynomial corresponds to criticality (catastrophe theory of Thom is one application). Could this be the mechanism assigning preferred prime(s) to a given elementary system, such as elementary particle? I have not considered their role earlier also their hierarchy is highly relevant in the number theoretical vision about TGD.

1. Stating it very roughly (I hope that mathematicians tolerate this sloppy language of physicist): as one goes from number field K, say rationals Q, to its algebraic extension L, the original prime ideals in the so called *integral closure* (`http://tinyurl.com/js6fpvr`) over integers of K decompose to products of prime ideals of L (prime ideal is a more rigorous manner to express primeness). Note that the general ideal is analog of integer.

   Integral closure for integers of number field K is defined as the set of elements of K, which are roots of some monic polynomial with coefficients, which are integers of K having the form $x^n + a_{n-1}x^{n-1} + ... + a_0$. The integral closures of both K and L are considered. For instance, integral closure of algebraic extension of K over K is the extension itself. The integral closure of complex numbers over ordinary integers is the set of algebraic numbers.

   Prime ideals of K can be decomposed to products of prime ideals of L: $P = \prod P_i^{e_i}$, where e_i is the ramification index. If $e_i > 1$ is true for some i, *ramification* occurs. P_i:s in question are like co-inciding roots of polynomial, which for in thermodynamics and Thom's catastrophe theory corresponds to criticality. Ramification could therefore be a natural aspect of quantum criticality and ramified primes P are good candidates for preferred primes for a given extension of rationals. Note that the ramification make sense also for extensions of given extension of rationals.

2. A physical analogy for the decomposition of ideals to ideals of extension is provided by decomposition of hadrons to valence quarks. Elementary particles becomes composite of more elementary particles in the extension. The decomposition to these more elementary primes is of form $P = \prod P_i^{e(i)}$, the physical analog would be the number of elementary particles of type i in the state (`http://tinyurl.com/h9528pl`). Unramified prime P would be analogous a state with e fermions. Maximally ramified prime would be analogous to Bose-Einstein condensate of e bosons. General ramified prime would be analogous to an e-particle state containing both fermions and condensed bosons. This is of course just a formal analogy.

3. There are two further basic notions related to ramification and characterizing it. *Relative discriminant* is the ideal divided by all ramified ideals in K (integer of K having no ramified prime factors) and relative different for P is the ideal of L divided by all ramified P_i:s (product of prime factors of P in L). These ideals represent the analogs of product of preferred primes P of K and primes P_i of L dividing them. These two integers ideals would characterize the ramification.

In TGD framework the extensions of rationals (`http://tinyurl.com/h9528pl`) and p-adic number fields (`http://tinyurl.com/zq22tvb`) are unavoidable and interpreted as an evolutionary hierarchy physically and cosmological evolution would gradually proceed to more and more complex extensions. One can say that string world sheets and partonic 2-surfaces with parameters of defining functions in increasingly complex extensions of prime emerge during evolution. Therefore ramifications and the preferred primes defined by them are unavoidable. For p-adic number fields the number of extensions is much smaller for instance for $p > 2$ there are only 3 quadratic extensions.

How could ramification relate to p-adic and adelic physics and could it explain preferred primes?

1. Ramified p-adic prime $P = P_i^e$ would be replaced with its e:th root P_i in p-adicization. Same would apply to general ramified primes. Each un-ramified prime of K is replaced with $e = K : L$ primes of L and ramified primes P with $\#\{P_i\} < e$ primes of L: the increase of algebraic dimension is smaller. An interesting question relates to p-adic length scale. What happens to p-adic length scales. Is p-adic prime effectively replaced with e:th root of p-adic prime: $L_p \propto p^{1/2}L_1 \rightarrow p^{1/2e}L_1$? The only physical option is that the p-adic temperature for P would be scaled down $T_p = 1/n \rightarrow 1/ne$ for its e:th root (for fermions serving as

fundamental particles in TGD one actually has $T_p = 1$). Could the lower temperature state be more stable and select the preferred primes as maximimally ramified ones? What about general ramified primes?

2. This need not be the whole story. Some algebraic extensions would be more favored than others and p-adic view about realizable imaginations could be involved. p-Adic pseudo constants are expected to allow p-adic continuations of string world sheets and partonic 2-surfaces to 4-D preferred extremals with number theoretic discretization. For real continuations the situation is more difficult. For preferred extensions - and therefore for corresponding ramified primes - the number of real continuations - realizable imaginations - would be especially large.

   The challenge would be to understand why primes near powers of 2 and possibly also of other small primes would be favored. Why for them the number of realizable imaginations would be especially large so that they would be winners in number theoretical fight for survival?

Can one make this picture more concrete? What kind of algebraic extensions could be considered?

1. In p-adic context a proper definition of counterparts of angle variables as phases allowing definition of the analogs of trigonometric functions requires the introduction of algebraic extension giving rise to some roots of unity. Their number depends on the angular resolution. These roots allow to define the counterparts of ordinary trigonometric functions - the naive generalization based on Taylors series is not periodic - and also allows to defined the counterpart of definite integral in these degrees of freedom as discrete Fourier analysis. For the simplest algebraic extensions defined by $x^n - 1$ for which Galois group is abelian are are unramified so that something else is needed. One has decomposition $P = \prod P_i^{e(i)}$, $e(i) = 1$, analogous to n-fermion state so that simplest cyclic extension does not give rise to a ramification and there are no preferred primes.

2. What kind of polynomials could define preferred algebraic extensions of rationals? Irreducible polynomials are certainly an attractive candidate since any polynomial reduces to a product of them. One can say that they define the elementary particles of number theory. Irreducible polynomials have integer coefficients having the property that they do not decompose to products of polynomials with rational coefficients. It would be wrong to say that only these algebraic extensions can appear but there is a temptation to say that one can reduce the study of extensions to their study. One can even consider the possibility that string world sheets associated with products of irreducible polynomials are unstable against decay to those characterize irreducible polynomials.

3. What can one say about irreducible polynomials? Eisenstein criterion (`http://tinyurl.com/47kxjz` states following. If $Q(x) = \sum_{k=0,..,n} a_k x^k$ is n:th order polynomial with integer coefficients and with the property that there exists at least one prime dividing all coefficients a_i except a_n and that p^2 does not divide a_0, then Q is irreducible. Thus one can assign one or more preferred primes to the algebraic extension defined by an irreducible polynomial Q of this kind - in fact any polynomial allowing ramification. There are also other kinds of irreducible polynomials since Eisenstein's condition is only sufficient but not necessary.

   Furthermore, in the algebraic extension defined by Q, the prime ideals P having the above mentioned characteristic property decompose to an n:th power of single prime ideal P_i: $P = P_i^n$. The primes are maximally/completely ramified.

   A good illustration is provided by equations $x^2 + 1 = 0$ allowing roots $x_\pm = \pm i$ and equation $x^2 + 2px + p = 0$ allowing roots $x_\pm = -p \pm \sqrt{p}p - 1$. In the first case the ideals associated with $\pm i$ are different. In the second case these ideals are one and the same since $x_+ == -x_- + p$: hence one indeed has ramification. Note that the first example represents also an example of irreducible polynomial, which does not satisfy Eisenstein criterion. In more general case the n conditions on defined by symmetric functions of roots imply that the ideals are one and same when Eisenstein conditions are satisfied.

4. What is so nice that one could readily construct polynomials giving rise to given preferred primes. The complex roots of these polymials could correspond to the points of partonic 2-surfaces carrying fermions and defining the ends of boundaries of string world sheet. It must be however emphasized that the form of the polynomial depends on the choices of the complex

coordinate. For instance, the shift $x \rightarrow x+1$ transforms $(x^n-1)/(x-1)$ to a polynomial satisfying the Eisenstein criterion. One should be able to fix allowed coordinate changes in such a manner that the extension remains irreducible for all allowed coordinate changes.

Already the integral shift of the complex coordinate affects the situation. It would seem that only the action of the allowed coordinate changes must reduce to the action of Galois group permuting the roots of polynomials. A natural assumption is that the complex coordinate corresponds to a complex coordinate transforming linearly under subgroup of isometries of the imbedding space.

In the general situation one has $P = \prod P_i^{e(i)}$, $e(i) \geq 1$ so that aso now there are prefered primes so that the appearance of preferred primes is completely general phenomenon.

The origin of p-adic length scale hypothesis?

p-Adic length scale hypothesis emerged from p-adic length scale hypothesis. A possible generalization of this hypothesis is that p-adic primes near powers of prime are physically favored. There indeed exists evidence for the realization of 3-adic time scale hierarchies in living matter [I8] (`http://tinyurl.com/jbh9m27`) and in music both 2-adicity and 3-adicity could be present: this is discussed in TGD inspired theory of music harmony and genetic code [K87]. See also [L27, L20].

One explanation would be that for preferred primes the number of p-adic space-time sheets representable also as real space-time sheets is maximal. Imagined worlds would be maximally realizable. Preferred p-adic primes would correspond to ramified primes for extensions with the property that the number of realizable imaginations is especially large for them. Why primes satisfying p-adic length scale hypothesis or its generalization would appear as ramified primes for extensions, which are winners in number theoretical evolution?

Also the weak form of NMP (WNMP) applying also to the purely number theoretic form of NMP [K65] might come in rescue here.

1. Entanglement negentropy for a NE [K65] characterized by n-dimensional projection operator is the $log(N_p(n)$ for some p whose power divides n. The maximum negentropy is obtained if the power of p is the largest power of prime divisor of p, and this can be taken as definition of number theoretical entanglement negentropy (NEN). If the largest divisor is p^k, one has $N = k \times log(p)$. The entanglement negentropy per entangled state is $N/n = klog(p)/n$ and is maximal for $n = p^k$. Hence powers of prime are favoured, which means that p-adic length scale hierarchies with scales coming as powers of p are negentropically favored and should be generated by NMP. Note that $n = p^k$ would define a hierarchy of $h_{eff}/h = p^k$. During the first years of h_{eff} hypothesis I believe that the preferred values obey $h_{eff} = r^k$, r integer not far from $r = 2^{11}$. It seems that this belief was not totally wrong.
2. If one accepts this argument, the remaining challenge is to explain why primes near powers of two (or more generally p) are favoured. $n = 2^k$ gives large entanglement negentropy for the final state. Why primes $p = n_2 = 2^k - r$ would be favored? The reason could be following. $n = 2^k$ corresponds to $p = 2$, which corresponds to the lowest level in p-adic evolution since it is the simplest p-adic topology and farthest from the real topology and therefore gives the poorest cognitive representation of real PE as p-adic PE (Note that $p = 1$ makes formally sense but for it the topology is discrete).
3. WNMP [K65, K116] suggests a more feasible explanation. The density matrix of the state to be reduced is a direct sum over contributions proportional to projection operators. Suppose that the projection operator with largest dimension has dimension n. Strong form of NMP would say that final state is characterized by n-dimensional projection operator. WNMP allows "free will" so that all dimensions $n-k$, $k = 0, 1, ...n-1$ for final state projection operator are possible. 1-dimensional case corresponds to vanishing entanglement negentropy and ordinary state function reduction isolating the measured system from external world.
4. The negentropy of the final state per state depends on the value of k. It is maximal if $n-k$ is power of prime. For $n = 2^k = M_k + 1$, where M_k is Mersenne prime $n-1$ gives the maximum negentropy and also maximal p-adic prime available so that this reduction is favoured by NMP. Mersenne primes would be indeed special. Also the primes $n = 2^k - r$ near 2^k produce large entanglement negentropy and would be favored by NMP.

5. This argument suggests a generalization of p-adic length scale hypothesis so that $p = 2$ can be replaced by any prime.

11.5 p-Adic physics and consciousness

p-Adic physics as physics of cognition and imagination is an important thread in TGD inspired theory of consciousness. In the sequel I describe briefly the basic of TGD inspired theory of consciousness as generalization of quantum measurement theory to ZEO (ZEO), describe the definition of self, consider the question whether NMP is needed as a separate principle or whether it is implied is in statistical sense by the unavoidable statistical increase of $n = h_{eff}/h$ if identified as a factor of the dimension of Galois group extension of rationals defining the adeles, and finally summarize the vision about how p-adic physics serves as a correlate of cognition and imagination.

11.5.1 From quantum measurement theory to a theory of consciousness

The notion of self can be seen as a generalization of the poorly defined definition of the notion of observer in quantum physics. In the following I take the role of skeptic trying to be as critical as possible.

The original definition of self was as a subsystem able to remain unentangled under state function reductions associated with subsequent quantum jumps. The density matrix was assumed to define the universal observable. Note that a density matrix, which is power series of a product of matrices representing commuting observables has in the generic case eigenstates, which are simultaneous eigenstates of all observables. Second aspect of self was assumed to be the integration of subsequent quantum jumps to coherent whole giving rise to the experienced flow of time.

The precise identification of self allowing to understand both of these aspects turned out to be difficult problem. I became aware the solution of the problem in terms of ZEO (ZEO) only rather recently (2014).

1. Self corresponds to a sequence of quantum jumps integrating to single unit as in the original proposal, but these quantum jumps correspond to state function reductions to a fixed boundary of causal diamond CD leaving the corresponding parts of zero energy states invariant - "small" state function reductions. The parts of zero energy states at second boundary of CD change and even the position of the tip of the opposite boundary changes: one actually has wave function over positions of second boundary (CD sizes roughly) and this wave function changes. In positive energy ontology these repeated state function reductions would have no effect on the state (Zeno effect) but in TGD framework there occurs a change for the second boundary and gives rise to the experienced flow of time and its arrow and self: self is generalized Zeno effect.
2. The first quantum jump to the opposite boundary corresponds to the act of "free will" or birth of re-incarnated self. Hence the act of "free will" changes the arrow of psychological time at some level of hierarchy of CDs. The first reduction to the opposite boundary of CD means "death" of self and "re-incarnation" of time-reversed self at opposite boundary at which the the temporal distance between the tips of CD increases in opposite direction. The sequence of selves and time reversed selves is analogous to a cosmic expansion for CD. The repeated birth and death of mental images could correspond to this sequence at the level of sub-selves.
3. This allows to understand the relationship between subjective and geometric time and how the arrow of and flow of clock time (psychological time) emerge. The average distance between the tips of CD increases on the average as along as state function functions occur repeatedly at the fixed boundary: situation is analogous to that in diffusion. The localization of contents of conscious experience to boundary of CD gives rise to the illusion that universe is 3-dimensional. The possibility of memories made possibly by hierarchy of CDs demonstrates that this is not the case. Self is simply the sequence of state function reductions at same boundary of CD remaining fixed and the lifetime of self is the total growth of the average temporal distance between the tips of CD.

One can identify several rather abstract state function reductions selecting a sector of WCW.

1. There are quantum measurements inducing localization in the moduli space of CDs with passive boundary and states at it fixed. In particular, a localization in the moduli characterizing the Lorentz transform of the upper tip of CD would be measured. The measured moduli characterize also the analog of symplectic form in M^4 strongly suggested by twistor lift of TGD - that is the rest system (time axis) and spin quantization axes. Of course, also other kinds of reductions are possible.
2. Also a localization to an extension of rationals defining the adeles should occur. Could the value of $n = h_{eff}/h$ be observable? The value of n for given space-time surface at the active boundary of CD could be identified as the order of the smallest Galois group containing all Galois groups assignable to 3-surfaces at the boundary. The superposition of space-time surface would not be eigenstate of n at active boundary unless localization occurs. It is not obvious whether this is consistent with a fixe value of n at passive boundary.

   The measured value of n could be larger or smaller than the value of n at the passive boundary of CD but in statistical sense n would increase by the analogy with diffusion on half line defined by non-negative integers. The distance from the origin unavoidably increases in statistical sense. This would imply evolution as increase of maximal value of negentropy and generation of quantum coherence in increasingly longer scales.
3. A further abstract choice corresponds to the the replacement of the roles of active and passive boundary of CD changing the arrow of clock time and correspond to a death of self and re-incarnation as time-reversed self.

Can one assume that these measurements reduce to measurements of a density matrix of either entangled system as assumed in the earlier formulation of NMP, or should one allow both options. This question actually applies to all quantum measurements and leads to a fundamental philosophical questions unavoidable in all consciousness theories.

1. Do all measurements involve entanglement between the moduli or extensions of two CDs reduced in the measurement of the density matrix? Non-diagonal entanglement would allow final states states, which are not eigenstates of moduli or of n: this looks strange. This could also lead to an infinite regress since it seems that one must assume endless hierarchy of entangled CDs so that the reduction sequence would proceed from top to bottom. It looks natural to regard single CD as a sub-Universe.

   For instance, if a selection of quantization axis of color hypercharge and isospin (localization in the twistor space of CP_2) is involved, one would have an outcome corresponding to a quantum superposition of measurements with different color quantization axis!

   Going philosophical, one can also argue, that the measurement of density matrix is only a reaction to environment and does not allow intentional free will.
2. Can one assume that a mere localization in the moduli space or for the extension of rationals (producing an eigenstate of n) takes place for a fixed CD - a kind of self measurement possible for even unentangled system? If there is entanglement in these degrees of freedom between two systems (say CDs), it would be reduced in these self measurements but the outcome would not be an eigenstate of density matrix. An interpretation as a realization of intention would be approriate.
3. If one allows both options, the interpretation would be that state function reduction as a measurement of density matrix is only a reaction to environment and self-measurement represents a realization of intention.
4. Self measurements would occur at higher level say as a selection of quantization axis, localization in the moduli space of CD, or selection of extension of rationals. A possible general rule is that measurements at space-time level are reactions as measurements of density matrix whereas a selection of a sector of WCW would be an intentional action. This because formally the quantum states at the level of WCW are as modes of classical WCW spinor field single particle states.
5. If the selections of sectors of WCW at active boundary of CD commute with observables, whose eigenstates appear at passive boundary (briefly *passive observables*) meaning that time reversal commutes with them - they can occur repeatedly during the reduction sequence and self as a generalized Zeno effect makes sense.

If the selections of WCW sectors at active boundary do not commute with passive observables then volition as a choice of sector of WCW must change the arrow of time. Libet's findings show that conscious choice induces neural activity for a fraction of second before the conscious choice. This would imply the correspondences *"big" measurement changing the arrow of time - self-measurement at the level of WCW - intentional action* and *"small" measurement - measurement at space-time level - reaction.*

Self as a generalized Zeno effect makes sense only if there are active commuting with passive observables. If the passive observables form a maximal set, the new active observables commuting with them must emerge. The increase of the size of extension of rationals might generate them by expanding the state space so that self would survive only as long at it evolves.

Otherwise there would be only single unitary time evolution followed by a reduction to opposite boundary. This makes sense only if the sequence of "big" reductions for sub-selves can give rise to the time flow experienced by self: the birth and death of mental images would give rise to flow of time of self.

A hierarchical process starting from given CD and proceeding downwards to shorter scales and stopping when the entanglement is stable is highly suggestive and favors self measurements. What stability could mean will be discussed in the next section. CDs would be a correlate for self hierarchy. One can say also something about the anatomy and correlates of self hierarchy.

1. Self experiences its sub-selves as mental images and even we would represent mental images of some higher level collective self. Everything is conscious but consciousness can be lost or at least it is not possible to have memory about it. The flow of consciousness for a given self could be due to the quantum jump sequences performed by its sub-selves giving rise to mental images.
2. By quantum classical correspondence self has also space-time correlates. One can visualize sub-self as a space-time sheet "glued" by topological sum to the space-time sheet of self. Subsystem is not described as a tensor factor as in the standard description of subsystems. Also sub-selves of selves can entangle negentropically and this gives rise to a sharing of mental images about which stereo vision would be basic example. Quite generally, one could speak of stereo consciousness. Also the experiences of sensed presence [J8] could be understood as a sharing of mental images between brain hemispheres, which are not themselves entangled. This is possible also between different brains. In the normal situation brain hemispheres are entangled.
3. At the level of 8-dimensional imbedding space the natural correlate of self would be CD (causal diamond). At the level of space-time the correlate would be space-time sheet or light-like 3-surface. The contents of consciousness of self would be determined by the space-time sheets in the interior of CD. Without further restrictions the experience of self would be essentially four-dimensional. Memories would be like sensory experiences except that they would be about the geometric past and for some reason are not usually colored by sensory qualia. For instance .1 second time scale defining sensory chronon corresponds to the secondary p-adic time scale characterizing the size of electron's CD (Mersenne prime M_{127}), which suggests that Cooper pairs of electrons are essential for the sensory qualia.

11.5.2 NMP and self

The view about Negentropy Maximization Principle (NMP) [K65] has co-evolved with the notion of self and I have considered many variants of NMP.

1. The original formulation of NMP was in positive energy ontology and made same predictions as standard quantum measurement theory. The new element was that the density matrix of sub-system defines the fundamental observable and the system goes to its eigenstate in state function reduction. As found, the localizations at to WCW sectors define what might be called self-measurements and identifiable as active volitions rather than reactions.
2. In p-adic physics one can assign with rational and even algebraic entanglement probabilities number theoretical entanglement negentropy (NEN) satisfying the same basic axioms

as the ordinary Shannon entropy but having negative values and therefore having interpretation as information. The definition of p-adic negentropy (real valued) reads as $S_p = -\sum P_k log(|P_k|_p)$, where $|.|_p$ denotes p-adic norm. The news is that $N_p = -S_p$ can be positive and is positive for rational entanglement probabilities. Real entanglement entropy S is always non-negative.

NMP would force the generation of negentropic entanglement (NE) and stabilize it. NNE resources of the Universe - one might call them Akashic records- would steadily increase.

3. A decisive step of progress was the realization is that NTU forces all states in adelic physics to have entanglement coefficients in some extension of rationals inducing finite-D extension of p-adic numbers. The same entanglement can be characterized by real entropy S and p-adic negentropies N_p, which can be positive. One can define also total p-adic negentropy: $N = \sum_p N_p$ for all p and total negentropy $N_{tot} = N - S$.

   For rational entanglement probabilities it is easy to demonstrate that the generalization of adelic theorem holds true: $N_{tot} = N - S = 0$. NMP based on N_{tot} rather than N would not say anything about rational entanglement. For extensions of rationals it is easy to find that $N - S > 0$ is possible if entanglement probabilities are of form X_i/n with $|X_i|_p = 1$ and n integer [L23]. Should one identify the total negentropy as difference $N_{tot} = N - S$ or as $N_{tot} = N$?

   Irrespective of answer, large p-adic negentropy seems to force large real entropy: this nicely correlates with the paradoxical finding that living systems tend to be entropic although one would expect just the oppositecite [L23]: this relates in very interesting manner to the work of biologists Jeremy England [I10]. The negentropy would be cognitive negentropy and not visible for ordinary physics.

4. The latest step in the evolution of ideas NMP was the question whether NMP follows from number theory alone just as second law follows form probability theory! This irritates theoretician's ego but is victory for theory. The dimension n of extension is positive integer and cannot but grow in statistical sense in evolution! Since one expects that the maximal value of negentropy (define as $N - S$) must increase with n. Negentropy must increase in long run.

Number theoretic entanglement can be stable

Number theoretical Shannon entropy can serve as a measure for genuine information assignable to a pair of entanglement systems [K65]. Entanglement with coefficients in the extension is always negentropic if entanglement negentropy comes from p-adic sectors only. It can be negentropic if negentropy is defined as the difference of p-adic negentropy and real entropy.

The diagonalized density matrix need not belong to the algebraic extension since the probabilities defining its diagonal elements are eigenvalues of the density matrix as roots of N:th order polynomial, which in the generic case requires n-dimensional algebraic extension of rationals. One can argue that since diagonalization is not possible, also state function reduction selecting one of the eigenstates is impossible unless a phase transition increasing the dimension of algebraic extension used occurs simultaneously. This kind of NE could give rise to cognitive entanglement.

There is also a special kind of NE, which can result if one requires that density matrix serves a universal observable in state function reduction. The outcome of reduction must be an eigen space of density matrix, which is projector to this subspace acting as identity matrix inside it. This kind NE allows all unitarily related basis as eigenstate basis (unitary transformations must belong to the algebraic extension). This kind of NE could serve as a correlate for "enlightened" states of consciousness. Schrödingers cat is in this kind of state stably in superposition of dead and alive and state basis obtained by unitary rotation from this basis is equally good. One can say that there are no discriminations in this state, and this is what is claimed about "enlightened" states too.

The vision about number theoretical evolution suggests that NMP forces the generation of NE resources as NE assignable to the "passive" boundary of CD for which no changes occur during sequence of state function reductions defining self. It would define the unchanging self as negentropy resources, which could be regarded as kind of Akashic records. During the next "re-incarnation" after the first reduction to opposite boundary of CD the NE associated with the reduced state would serve as new Akashic records for the time reversed self. If NMP reduces to the statistical increase of $h_{eff}/h = n$ the consciousness information contents of the Universe increases in statistical sense. In the best possible world of SNMP it would increase steadily.

Does NMP reduce to number theory?

The heretic question that emerged quite recently is whether NMP is actually needed at all! Is NMP a separate principle or could NMP reduced to mere number theory [K65]? Consider first the possibility that NMP is not needed at all as a separate principle.

1. The value of $h_{eff}/h = n$ should increase in the evolution by the phase transitions increasing the dimension of the extension of rationals. $h_{eff}/h = n$ has been identified as the number of sheets of some kind of covering space. The Galois group of extension acts on number theoretic discretizations of the monadic surface and the orbit defines a covering space. Suppose n is the number of sheets of this covering and thus the dimension of the Galois group for the extension of rationals or factor of it.

2. It has been already noticed that the "big" state function reductions giving rise to death and reincarnation of self could correspond to a measurement of $n = h_{eff}$ implied by the measurement of the extension of the rationals defining the adeles. The statistical increase of n follows automatically and implies statistical increase of maximal entanglement negentropy. Entanglement negentropy increases in statistical sense.

   The resulting world would not be the best possible one unlike for a strong form of NMP demanding that negentropy does increaes in "big" state function reductions. n also decrease temporarily and they seem to be needed. In TGD inspired model of bio-catalysis the phase transition reducing the value of n for the magnetic flux tubes connecting reacting bio-molecules allows them to find each other in the molecular soup. This would be crucial for understanding processes like DNA replication and transcription.

3. State function reduction corresponding to the measurement of density matrix could occur to an eigenstate/eigenspace of density matrix only if the corresponding eigenvalue and eigenstate/eigenspace is expressible using numbers in the extension of rationals defining the adele considered. In the generic case these numbers belong to N-dimensional extension of the original extension. This can make the entanglement stable with respect to state the measurements of density matrix.

   A phase transition to an extension of an extension containing these coefficients would be required to make possible reduction. A step in number theoretic evolution would occur. Also an entanglement of measured state pairs with those of measuring system in containing the extension of extension would make possible the reduction. Negentropy could be reduced but higher-D extension would provide potential for more negentropic entanglement and NMP would hold true in the statistical sense.

4. If one has higher-D eigen space of density matrix, p-adic negentropy is largest for the entire subspace and the sum of real and p-adic negentropies vanishes for all of them. For negentropy identified as total p-adic negentropy SNMP would select the entire sub-space and NMP would indeed say something explicit about negentropy.

Or is NMP needed as a separate principle?

Hitherto I have postulated NMP as a separate principle [K65]. Strong form of NMP (SNMP) states that Negentropy does not decrease in "big" state function reductions corresponding to death and re-incarnations of self.

One can however argue that SNMP is not realistic. SNMP would force the Universe to be the best possible one, and this does not seem to be the case. Also ethically responsible free will would be very restricted since self would be forced always to do the best deed that is increase maximally the negentropy serving as information resources of the Universe. Giving up separate NMP altogether would allow to have also "Good" and "Evil".

This forces to consider what I christened weak form of NMP (WNMP). Instead of maximal dimension corresponding to N-dimensional projector self can choose also lower-dimensional subspaces and 1-D sub-space corresponds to the vanishing entanglement and negentropy assumed in standard quantum measurement theory. As a matter fact, this can also lead to larger negentropy gain since negentropy depends strongly on what is the large power of p in the dimension of the resulting eigen sub-space of density matrix. This could apply also to the purely number theoretical reduction of NMP.

WNMP suggests how to understand the notions of Good and Evil. Various choices in the state function reduction would correspond to Boolean algebra, which suggests an interpretation in terms of what might be called emotional intelligence [K116]. Also it turns out that one can understand how p-adic length scale hypothesis - actually its generalization - emerges from WNMP [K119].

1. One can start from ordinary quantum entanglement. It corresponds to a superposition of pairs of states. Second state corresponds to the internal state of the self and second state to a state of external world or biological body of self. In negentropic quantum entanglement each is replaced with a pair of sub-spaces of state spaces of self and external world. The dimension of the sub-space depends on which pair is in question. In state function reduction one of these pairs is selected and deed is done. How to make some of these deeds good and some bad? Recall that WNMP allows only the possibility to generate NNE but does not force it. WNMP would be like God allowing the possibility to do good but not forcing good deeds.

   Self can choose any sub-space of the subspace defined by $k \leq N$-dimensional projector and 1-D subspace corresponds to the standard quantum measurement. For $k = 1$ the state function reduction leads to vanishing negentropy, and separation of self and the target of the action. Negentropy does not increase in this action and self is isolated from the target: kind of price for sin.

   For the maximal dimension of this sub-space the negentropy gain is maximal. This deed would be good and by the proposed criterion NE corresponds to conscious experience with positive emotional coloring. Interestingly, there are $2^k - 1$ possible choices, which is almost the dimension of Boolean algebra consisting of k independent bits. The excluded option corresponds to 0-dimensional sub-space - empty set in set theoretic realization of Boolean algebra. This could relate directly to fermionic oscillator operators defining basis of Boolean algebra - here Fock vacuum would be the excluded state. The deed in this sense would be a choice of how loving the attention towards system of external world is.

2. A map of different choices of k-dimensional sub-spaces to k-fermion states is suggestive. The realization of logic in terms of emotions of different degrees of positivity would be mapped to many-fermion states - perhaps zero energy states with vanishing total fermion number. State function reductions to k-dimensional spaces would be mapped to k-fermion states: quantum jumps to quantum states!

   The problem brings in mind quantum classical correspondence in quantum measurement theory. The direction of the pointer of the measurement apparatus (in very metaphorical sense) corresponds to the outcome of state function reduction, which is now 1-D subspace. For ordinary measurement the pointer has k positions. Now it must have $2^k - 1$ positions. To the discrete space of k pointer positions one must assign fermionic Clifford algebra of second quantized fermionic oscillator operators. The hierarchy of Planck constants and dark matter suggests the realization. Replace the pointer with its space-time k-sheeted covering and consider zero energy energy states made of pairs of k-fermion states at the sheets of the n-sheeted covering? Dark matter would be therefore necessary for cognition. The role of fermions would be to "mark" the k space-time sheets in the covering.

The cautious conclusion is that NMP as a separate principle is not necessary and follows in statistical sense from the unavoidable increase of $n = h_{eff}/h$ identified as dimension of extension of rationals define the adeles if this extension or at least the dimension of its Galois group is observable.

11.5.3 p-Adic physics as correlate of cognition and imagination

The items in the following list give motivations for the proposal that p-adic physics could serve as a correlate for cognition and imagination.

1. By the total disconnectedness of the p-adic topology, p-adic world decomposes naturally into blobs, objects. This happens also in sensory perception. The pinary digits of p-adic number

can be assigned to a p-tree. Parisi proposed in the model of spin glass [B31] that p-adic numbers could relate to the mathematical description of cognition and also Khrennikov [J1] has developed this idea. In TGD framework that idea is taken to space-time level: p-adic space-time sheets represent thought bubbles and they correlate with the real ones since they form cognitive reprentations of the real world. SH allows a concrete realization of this.

2. p-Adic non-determinism due to p-adic pseudo constants suggests interpretation in terms of imagination. Given 2-surfaces could allow completion to p-adic preferred extremal but not to a real one so that pure "non-realizable" imagination is in question.

3. Number theoretic negentropy has interpretation as negentropy characterizing information content of entanglement. The superposition of state pairs could be interpreted as a quantum representation for a rule or abstracted association containing its instances as state pairs. Number theoretical negentropy characterizes the relationship of two systems and should not be confused with thermodynamical entropy, which characterizes the uncertainty about the state of single system.

The original vision was that p-adic non-determinism could serve as a correlate for cognition, imagination, and intention. The recent view is much more cautious. Imagination need not completely reduce to p-adic non-determinism since it has also real physics correlates - maybe as partial realizations of SH as in nerve pulse pattern, which does not propagate down to muscles.

A possible interpretation for the solutions of the p-adic field equations would be as geometric correlates of cognition, imagination, and perhaps even intentionality. Plans, intentions, expectations, dreams, and possibly also cognition as imagination in general could have p-adic cognitive space-time sheets as their geometric correlates. A deep principle seems to be involved: incompleteness is the characteristic feature of p-adic physics but the flexibility made possible by this incompleteness is absolutely essential for imagination and cognitive consciousness in general.

The most feasible view is that the intersections of p-adic and real space-time surfaces define cognitive representations of real space-time surfaces (PEs, [K14, K8, K10]). One could also say that real space-time surface represents sensory aspects of conscious experience and p-adic space-time surfaces its cognitive aspects. Both real and p-adics rather than real or p-adics.

The identification of p-adic pseudo constants as correlates of imagination at space-time level is indeed a further natural idea.

1. The construction of PEs by SH from the data at 2-surfaces is like boundary value problem with number theoretic discretization of space-time surface as additional data. PE property in real context implies strong correlations between string world sheets and partonic 2-surfaces by boundary conditions a them. One cannot choose these 2-surfaces completely independently in real context.

2. In p-adic sectors the integration constants are replaced with pseudo-constants depending on finite number of pinary digits of variables depending on coordinates normal to string world sheets and partonic 2-surfaces. The fixing of the discretization of space-time surface would allow to fix the p-adic pseudo-constants. Once the number theoretic discretization of space-time surface is fixed, the p-adic pseudo-constants can be fixed. Pseudo-constant could allow a large number of p-adic configurations involving string world sheets, partonic 2-surfaces, and number theoretic discretization but not allowed in real context.

   Could these p-adic PEs correspond to imaginations, which in general are not realizable? Could the realizable intentional actions belong to the intersection of real and p-adic WCWs? Could one identify non-realistic imaginations as the modes of WCW spinor fields for which 2-surfaces are not extendable to real space-time surfaces and are localized to 2-surfaces? Could they allow only a partial continuation to real space-time surface. Could nerve pulse pattern representing imagined motor action and not proceeding to the level of muscles correspond to a partially real PE?

   Could imagination and problem solving be search for those collections of string world sheets and partonic 2-surfaces, which allow extension to (realization as) real PEs? If so, p-adic physics would be there as an independent aspect of existence and this is just the original idea. Imagination could be realized in state function reduction, which always selects only those 2-surfaces, which allow continuation to real space-time surfaces. The distinction between only imaginable and also realizable would be the extendability by using strong form of holography.

3. An interesting question is why elementary particles are characterized by preferred p-adic primes (primes near powers of 2, in particular Mersenne primes). Could the number of realizable imaginations for these primes be especially large?

I have the feeling that this view allows respectable mathematical realization of imagination in terms of adelic quantum physics. It is remarkable that SH derivable from - you can guess, SGCI (the Big E again!), plays an absolutely central role in it.

11.6 Appendix: Super-symplectic conformal weights and zeros of Riemann zeta

Since fermions are the only fundamental particles in TGD one could argue that the conformal weight of for the generating elements of supersympelectic algebra could be negatives for the poles of fermionic zeta ζ_F. This demands $n > 0$ as does also the fractal hierarchy of supersymplectic symmetry breakings. NTU of Riemann zeta in some sense is strongly suggested if adelic physics is to make sense.

For ordinary conformal algebras there are only finite number of generating elements ($-2 \leq n \leq 2$). If the radial conformal weights for the generators of g consist of poles of ζ_F, the situation changes. ζ_F is suggested by the observation that fermions are the only fundamental particles in TGD.

1. Riemann Zeta $\zeta(s) = \prod_p (1/(1-p^{-s})$ identifiable formally as a partition function $\zeta_B(s)$ of arithmetic boson gas with bosons with energy $log(p)$ and temperature $1/s = 1/(1/2 + iy)$ should be replaced with that of arithmetic fermionic gas given in the product representation by $\zeta_F(s) = \prod_p (1 + p^{-s})$ so that the identity $\zeta_B(s))/\zeta_F(s) = \zeta_B(2s)$ follows. This gives

$$\frac{\zeta_B(s)}{\zeta_B(2s)} \ .$$

$\zeta_F(s)$ has zeros at zeros s_n of $\zeta(s)$ and at the pole $s = 1/2$ of $zeta(2s)$. $\zeta_F(s)$ has poles at zeros $s_n/2$ of $\zeta(2s)$ and at pole $s = 1$ of $\zeta(s)$.

The spectrum of $1/T$ would be for the generators of algebra $\{(-1/2 + iy)/2, n > 0, -1\}$. In p-adic thermodynamics the p-adic temperature is $1/T = 1/n$ and corresponds to "trivial" poles of ζ_F. Complex values of temperature does not make sense in ordinary thermodynamics. In ZEO quantum theory can be regarded as a square root of thermodynamics and complex temperature parameter makes sense.

2. If the spectrum of conformal weights of the generating elements of the algebra corresponds to poles serving as analogs of propagator poles, it consists of the "trivial" conformal $h = n > 0$- the standard spectrum with $h = 0$ assignable to massless particles excluded - and "non-trivial" $h = -1/4 + iy/2$. There is also a pole at $h = -1$.

Both the non-trivial pole with real part $h_R = -1/4$ and the pole $h = -1$ correspond to tachyons. I have earlier proposed conformal confinement meaning that the total conformal weight for the state is real. If so, one obtains for a conformally confined two-particle states corresponding to conjugate non-trivial zeros in minimal situation $h_R = -1/2$ assignable to N-S representation.

In p-adic mass calculations ground state conformal weight must be $-5/2$ [K59]. The negative fermion ground state weight could explain why the ground state conformal weight must be tachyonic $-5/2$. With the required 5 tensor factors one would indeed obtain this with minimal conformal confinement. In fact, arbitrarily large tachyonic conformal weight is possible but physical state should always have conformal weights $h > 0$.

3. $h = 0$ is not possible for generators, which reminds of Higgs mechanism for which the naïve ground states corresponds to tachyonic Higgs. $h = 0$ conformally confined massless states are necessarily composites obtained by applying the generators of Kac-Moody algebra or super-symplectic algebra to the ground state. This is the case according to p-adic mass calculations [K59], and would suggest that the negative ground state conformal weight can be associated with super-symplectic algebra and the remaining contribution comes from ordinary

super-conformal generators. Hadronic masses, whose origin is poorly understood, could come from super-symplectic degrees of freedom. There is no need for p-adic thermodynamics in super-symplectic degrees of freedom.

11.6.1 A general formula for the zeros of zeta from NTU

Dyson's comment about Fourier transform of Riemann Zeta [A64] (`http://tinyurl.com/hjbfsuv`) is interesting from the point of NTU for Riemann zeta.

1. The numerical calculation of Fourier transform for the imaginary parts iy of zeros $s = 1/2+iy$ of zeta shows that it is concentrated at discrete set of frequencies coming as $log(p^n)$, p prime. This translates to the statement that the zeros of zeta form a 1-dimensional quasicrystal, a discrete structure Fourier spectrum by definition is also discrete (this of course holds for ordinary crystals as a special case). Also the logarithms of powers of primes would form a quasicrystal, which is very interesting from the point of view of p-adic length scale hypothesis. Primes label the "energies" of elementary fermions and bosons in arithmetic number theory, whose repeated second quantization gives rise to the hierarchy of infinite primes [K103]. The energies for general states are logarithms of integers.
2. Powers p^n label the points of quasicrystal defined by points $log(p^n)$ and Riemann zeta has interpretation as partition function for boson case with this spectrum. Could p^n label also the points of the dual lattice defined by iy.
3. The existence of Fourier transform for points $log(p_i^n)$ for any vector y_a in class $C(p)$ of zeros labelled by p requires $p_i^{iy_a}$ to be a root of unity inside $C(p)$. This could define the sense in which zeros of zeta are universal. This condition also guarantees that the factor $n^{-1/2-iy}$ appearing in zeta at critical line are number theoretically universal ($p^{1/2}$ is problematic for Q_p: the problem might be solved by eliminating from p-adic analog of zeta the factor $1-p^{-s}$.
   (a) One obtains for the pair (p_i, s_a) the condition $log(p_i)y_a = q_{ia}2\pi$, where q_{ia} is a rational number. Dividing the conditions for (i,a) and (j,a) gives
   $$p_i = p_j^{q_{ia}/q_{ja}}$$
   for every zero s_a so that the ratios q_{ia}/q_{ja} do not depend on s_a. From this one easily deduce $p_i^M = p_j^N$, where M and N are integers so that one ends up with a contradiction.
   (b) Dividing the conditions for (i,a) and (i,b) one obtains
   $$\frac{y_a}{y_b} = \frac{q_{ia}}{q_{ib}}$$
   so that the ratios q_{ia}/q_{ib} do not depend on p_i. The ratios of the imaginary parts of zeta would be therefore rational number which is very strong prediction and zeros could be mapped by scaling y_a/y_1 where y_1 is the zero which smallest imaginary part to rationals.
   (c) The impossible consistency conditions for (i,a) and (j,a) can be avoided if each prime and its powers correspond to its own subset of zeros and these subsets of zeros are disjoint: one would have infinite union of sub-quasicrystals labelled by primes and each p-adic number field would correspond to its own subset of zeros: this might be seen as an abstract analog for the decomposition of rational to powers of primes. This decomposition would be natural if for ordinary complex numbers the contribution in the complement of this set to the Fourier trasform vanishes. The conditions $(i,a)$and (i,b) require now that the ratios of zeros are rationals only in the subset associated with p_i.

For the general option the Fourier transform can be delta function for $x = log(p^k)$ and the set $\{y_a(p)\}$ contains N_p zeros. The following argument inspires the conjecture that for each p there is an infinite number N_p of zeros $y_a(p)$ in class $C(p)$ satisfying

$$p^{iy_a(p)} = u(p) = e^{\frac{r(p)}{m(p)}i2\pi} \quad ,$$

where $u(p)$ is a root of unity that is $y_a(p) = 2\pi(m(a) + r(p))/log(p)$ and forming a subset of a lattice with a lattice constant $y_0 = 2\pi/log(p)$, which itself need not be a zero.

In terms of stationary phase approximation the zeros $y_a(p)$ associated with p would have constant stationary phase whereas for $y_a(p_i \neq p))$ the phase $p^{iy_a(p_i)}$ would fail to be stationary. The phase e^{ixy} would be non-stationary also for $x \neq log(p^k)$ as function of y.

1. Assume that for $x = qlog(p)$, where q not a rational, the phases e^{ixy} fail to be roots of unity and are random implying the vanishing/smallness of $F(x)$.
2. Assume that for a given p all powers p^{iy} for $y \notin \{y_a(p)\}$ fail to be roots of unity and are also random so that the contribution of the set $y \notin \{y_a(p)\}$ to $F(p)$ vanishes/is small.
3. For $x = log(p^{k/m})$ the Fourier transform should vanish or be small for $m \neq 1$ (rational roots of primes) and give a non-vanishing contribution for $m = 1$. One has

$$F(x = log(p^{k/m}) = \sum_{1\leq a\leq N(p)} e^{k\frac{M(a,p)}{mN(p)}i2\pi} u(p) \ ,$$
$$u(p) = e^{\frac{r(p)}{m(p)}i2\pi} \ .$$

Obviously one can always choose $N(a, p) = N(p)$.

4. For the simplest option $N(p) = 1$ one would obtain delta function distribution for $x = log(p^k)$. The sum of the phases associated with $y_a(p)$ and $-y_a(p)$ from the half axes of the critical line would give

$$F(x = log(p^n)) \propto X(p^n) \equiv 2cos(n\frac{r(p)}{m(p)}2\pi) \ .$$

The sign of F would vary.

5. For $x = log(p^{k/m})$ the value of Fourier transform is expected to be small by interference effects if $M(a, p)$ is random integer, and negligible as compared with the value at $x = log(p^k)$. This option is highly attractive. For $N(p) > 1$ and $M(a, p)$ a random integer also $F(x = log(p^k)$ is small by interference effects. Hence it seems that this option is the most natural one.
6. The rational $r(p)/m(p)$ would characterize given prime (one can require that $r(p)$ and $m(p)$ have no common divisors). $F(x)$ is non-vanishing for all powers $x = log(p^n)$ for $m(p)$ odd. For $p = 2$, also $m(2) = 2$ allows to have $|X(2^n)| = 2$. An interesting ad hoc ansatz is $m(p) = p$ or $p^{s(p)}$. One has periodicity in n with period $m(p)$ that is logarithmic wave. This periodicity serves as a test and in principle allows to deduce the value of $r(p)/m(p)$ from the Fourier transform.

What could one conclude from the data (`http://tinyurl.com/hjbfsuv`)?

1. The first graph gives $|F(x = log(p^k)|$ and second graph displays a zoomed up part of $|F(x = log(p^k)|$ for small powers of primes in the range $[2, 19]$. For the first graph the eighth peak ($p = 11$) is the largest one but in the zoomed graphs this is not the case. Hence something is wrong or the graphs correspond to different approximations suggesting that one should not take them too seriously.

   In any case, the modulus is not constant as function of p^k. For small values of p^k the envelope of the curve decreases and seems to approach constant for large values of p^k (one has $x < 15$ ($e^{15} \simeq 3.3 \times 10^6$).
2. According to the first graph $|F(x)|$ decreases for $x = klog(p) < 8$, is largest for small primes, and remains below a fixed maximum for $8 < x < 15$. According to the second graph the amplitude decreases for powers of a given prime (say $p = 2$). Clearly, the small primes and their powers have much larger $|F(x)|$ than large primes.

There are many possible reasons for this behavior. Most plausible reason is that the sums involved converge slowly and the approximation used is not good. The inclusion of only 10^4 zeros would show the positions of peaks but would not allow reliable estimate for their intensities.

1. The distribution of zeros could be such that for small primes and their powers the number of zeros is large in the set of 10^4 zeros considered. This would be the case if the distribution of zeros $y_a(p)$ is fractal and gets "thinner" with p so that the number of contributing zeros scales down with p as a power of p, say $1/p$, as suggested by the envelope in the first figure.

2. The infinite sum, which should vanish, converges only very slowly to zero. Consider the contribution $\Delta F(p^k, p_1)$ of zeros not belonging to the class $p_1 \neq p$ to $F(x = log(p^k)) = \sum_{p_i} \Delta F(p^k, p_i)$, which includes also $p_i = p$. $\Delta F(p^k, p_i)$, $p \neq p_1$ should vanish in exact calculation.

   (a) By the proposed hypothesis this contribution reads as

   $$\begin{aligned} \Delta F(p, p_1) &= \sum_a cos\left[X(p^k, p_1)(M(a, p_1) + \frac{r(p_1)}{m(p_1)})2\pi)\right] \quad . \\ X(p^k, p_1) &= \frac{log(p^k)}{log(p_1)} \quad . \end{aligned}$$

   Here a labels the zeros associated with p_1. If p^k is "approximately divisible" by p^1 in other words, $p^k \simeq np_1$, the sum over finite number of terms gives a large contribution since interference effects are small, and a large number of terms are needed to give a nearly vanishing contribution suggested by the non-stationarity of the phase. This happens in several situations.

   (b) The number $\pi(x)$ of primes smaller than x goes asymptotically like $\pi(x) \simeq x/log(x)$ and prime density approximately like $1/log(x) - 1/log(x)^2$ so that the problem is worst for the small primes. The problematic situation is encountered most often for powers p^k of small primes p near larger prime and primes p (also large) near a power of small prime (the envelope of $|F(x)|$ seems to become constant above $x \sim 10^3$).

   (c) The worst situation is encountered for $p = 2$ and $p_1 = 2^k - 1$ - a Mersenne prime and $p_1 = 2^{2^k} + 1$, $k \leq 4$ - Fermat prime. For $(p, p_1) = (2^k, M_k)$ one encounters $X(2^k, M_k) = (log(2^k)/log(2^k - 1)$ factor very near to unity for large Mersennes primes. For $(p, p_1) = (M_k, 2)$ one encounters $X(M_k, 2) = (log(2^k - 1)/log(2) \simeq k$. Examples of Mersennes and Fermats are $(3, 2), (5, 2), (7, 2), (17, 2), (31, 2), (127, 2), (257, 2), ...$ Powers 2^k, $k = 2, 3, 4, 5, 7, 8, ..$ are also problematic.

   (d) Also twin primes are problematic since in this case one has factor $X(p = p_1 + 2, p_1) = \frac{log(p_1+2)}{log(p_1)}$. The region of small primes contains many twin prime pairs: (3,5), (5,7), (11,13), (17,19), (29,31),....

   These observations suggest that the problems might be understood as resulting from including too small number of zeros.

3. The predicted periodicity of the distribution with respect to the exponent k of p^k is not consistent with the graph for small values of prime unless the periodic $m(p)$ for small primes is large enough. The above mentioned effects can quite well mask the periodicity. If the first graph is taken at face value for small primes, $r(p)/m(p)$ is near zero, and $m(p)$ is so large that the periodicity does not become manifest for small primes. For $p = 2$ this would require $m(2) > 21$ since the largest power $2^n \simeq e^{15}$ corresponds to $n \sim 21$.

To summarize, the prediction is that for zeros of zeta should divide into disjoint classes $\{y_a(p)\}$ labelled by primes such that within the class labelled by p one has $p^{iy_a(p)} = e^{(r(p)/m(p))i2\pi}$ so that has $y_a(p) = [M(a, p) + r(p)/m(p))]2\pi/log(p)$.

11.6.2 More precise view about zeros of Zeta

There is a very interesting blog post by Mumford (`http://tinyurl.com/zemw27o`), which leads to much more precise formulation of the idea and improved view about the Fourier transform hypothesis: the Fourier transform or its generalization must be defined for all zeros, not only the non-trivial ones and trivial zeros give a background term allowing to understand better the properties of the Fourier transform.

Mumford essentially begins from Riemann's "explicit formula" in von Mangoldt's form.

$$\sum_p \sum_{n \geq 1} log(p)\delta_{p^n}(x) = 1 - \sum_k x^{s_k - 1} - \frac{1}{x(x^2 - 1)} \quad ,$$

where p denotes prime and s_k a non-trivial zero of zeta. The left hand side represents the distribution associated with powers of primes. The right hand side contains sum over cosines

$$\sum_k x^{s_k-1} = 2\frac{\sum_k cos(log(x)y_k)}{x^{1/2}} \quad ,$$

where y_k ithe imaginary part of non-trivial zero. Apart from the factor $x^{-1/2}$ this is just the Fourier transform over the distribution of zeros.

There is also a slowly varying term $1-\frac{1}{x(x^2-1)}$, which has interpretation as the analog of the Fourier transform term but sum over trivial zeros of zeta at $s=-2n$, $n>0$. The entire expression is analogous to a "Fourier transform" over the distribution of all zeros. Quasicrystal is replaced with union on 1-D quasicrystals.

Therefore the distribution for powers of primes is expressible as "Fourier transform" over the distribution of both trivial and non-trivial zeros rather than only non-trivial zeros as suggested by numerical data to which Dyson [A64] referred to (`http://tinyurl.com/hjbfsuv`). Trivial zeros give a slowly varying background term large for small values of argument x (poles at $x=0$ and $x=1$ - note that also $p=0$ and $p=1$ appear effectively as primes) so that the peaks of the distribution are higher for small primes.

The question was how can one obtain this kind of delta function distribution concentrated on powers of primes from a sum over terms $cos(log(x)y_k)$ appearing in the Fourier transform of the distribution of zeros.

Consider $x=p^n$. One must get a constructive interference. Stationary phase approximation is in terms of which physicist thinks. The argument was that a destructive interference occurs for given $x=p^n$ for those zeros for which the cosine does not correspond to a real part of root of unity as one sums over such y_k: random phase approximation gives more or less zero. To get something nontrivial y_k must be proportional to $2\pi \times n(y_k)/log(p)$ in class $C(p)$ to which y_k belongs. If the number of these y_k:s in $C(p)$ is infinite, one obtains delta function in good approximation by destructive interference for other values of argument x.

The guess that the number of zeros in $C(p)$ is infinite is encouraged by the behaviors of the densities of primes one hand and zeros of zeta on the other hand. The number of primes smaller than real number x goes like

$$\pi(x) = N(primes < x) \sim \frac{x}{log(x)}$$

in the sense of distribution. The number of zeros along critical line goes like

$$N(zeros < t) = (t/2\pi) \times log(\frac{t}{2\pi})$$

in the same sense. If the real axis and critical line have same metric measure then one can say that the number of zeros in interval T per number of primes in interval T behaves roughly like

$$\frac{N(zeros < T)}{N(primes < T)} = log(\frac{T}{2\pi}) \times \frac{log(T)}{2\pi}$$

so that at the limit of $T\rightarrow\infty$ the number of zeros associated with given prime is infinite. This asumption of course makes the argument a poor man's argument only.

11.6.3 Possible relevance for TGD

What this speculative picture from the point of view of TGD?

1. A possible formulation for NTU for the poles of fermionic Riemann zeta $\zeta_F = \zeta(s)/\zeta(2s)$ could be as a condition that is that the exponents $p^{ks_a(p)/2} = p^{k/4}p^{iky_a(p)/2}$ exist in a number theoretically universal manner for the zeros $s_a(p)$ for given p-adic prime p and for some subset of integers k. If the proposed conditions hold true, exponent reduces $p^{k/4}e^{k(r(p/m(p)i2\pi}$ requiring that k is a multiple of 4. The number of the non-trivial generating elements of super-symplectic algebra in the monomial creating physical state would be a multiple of 4. These monomials would have real part of conformal weight -1. Conformal confinement suggests that these monomials are products of pairs of generators for which imaginary parts cancel.

2. Quasi-crystal property might have an application to TGD. The functions of light-like radial coordinate appearing in the generators of supersymplectic algebra could be of form r^s, s zero of zeta or rather, its imaginary part. The eigenstate property with respect to the radial scaling rd/dr is natural by radial conformal invariance.

   The idea that arithmetic QFT assignable to infinite primes is behind the scenes in turn suggests light-like momenta assignable to the radial coordinate have energies with the dual spectrum $log(p^n)$. This is also suggested by the interpretation of ζ as square root of thermodynamical partition function for boson gas with momentum $log(p)$ and analogous interpretation of ζ_F.

   The two spectra would be associated with radial scalings and with light-like translations of light-cone boundary respecting the direction and light-likeness of the light-like radial vector. $log(p^n)$ spectrum would be associated with light-like momenta whereas p-adic mass scales would characterize states with thermal mass. Note that generalization of p-adic length scale hypothesis raises the scales defined by p^n to a special physical position: this might relate to ideal structure of adeles.

3. Finite measurement resolution suggests that the approximations of Fourier transforms over the distribution of zeros taking into account only a finite number of zeros might have a physical meaning. This might provide additional understand about the origins of generalized p-adic length scale hypothesis stating that primes $p \simeq p_1^k$, p_1 small prime - say Mersenne primes - have a special physical role.

Chapter i

Appendix

A-1 Introduction

Originally this appendix was meant to be a purely technical summary of basic facts but in its recent form it tries to briefly summarize those basic visions about TGD which I dare to regarded stabilized. I have added illustrations making it easier to build mental images about what is involved and represented briefly the key arguments. This chapter is hoped to help the reader to get fast grasp about the concepts of TGD.

The basic properties of embedding space and related spaces are discussed and the relationship of CP_2 to the standard model is summarized. The basic vision is simple: the geometry of the embedding space $H = M^4 \times CP_2$ geometrizes standard model symmetries and quantum numbers. The assumption that space-time surfaces are basic objects, brings in dynamics as dynamics of 3-D surfaces based on the induced geometry. Second quantization of free spinor fields of H induces quantization at the level of H, which means a dramatic simplification.

The notions of induction of metric and spinor connection, and of spinor structure are discussed. Many-sheeted space-time and related notions such as topological field quantization and the relationship many-sheeted space-time to that of GRT space-time are discussed as well as the recent view about induced spinor fields and the emergence of fermionic strings. Also the relationship to string models is discussed briefly.

Various topics related to p-adic numbers are summarized with a brief definition of p-adic manifold and the idea about generalization of the number concept by gluing real and p-adic number fields to a larger book like structure analogous to adele [L31, L30]. In the recent view of quantum TGD [L52], both notions reduce to physics as number theory vision, which relies on $M^8 - H$ duality [L42, L43] and is complementary to the physics as geometry vision.

Zero energy ontology (ZEO) [L41] [K124] has become a central part of quantum TGD and leads to a TGD inspired theory of consciousness as a generalization of quantum measurement theory having quantum biology as an application. Also these aspects of TGD are briefly discussed.

A-2 Embedding space $M^4 \times CP_2$

Space-times are regarded as 4-surfaces in $H = M^4 \times CP_2$ the Cartesian product of empty Minkowski space - the space-time of special relativity - and compact 4-D space CP_2 with size scale of order 10^4 Planck lengths. One can say that embedding space is obtained by replacing each point m of empty Minkowski space with 4-D tiny CP_2. The space-time of general relativity is replaced by a 4-D surface in H which has very complex topology. The notion of many-sheeted space-time gives an idea about what is involved.

Fig. 1. Embedding space $H = M^4 \times CP_2$ as Cartesian product of Minkowski space M^4 and complex projective space CP_2. `http://tgdtheory.fi/appfigures/Hoo.jpg`

Denote by M^4_+ and M^4_- the future and past directed lightcones of M^4. Denote their intersection, which is not unique, by CD. In zero energy ontology (ZEO) [L41, L45] [K124] causal diamond

(CD) is defined as cartesian product $CD \times CP_2$. Often I use CD to refer just to $CD \times CP_2$ since CP_2 factor is relevant from the point of view of ZEO.

Fig. 2. Future and past light-cones M^4_+ and M^4_-. Causal diamonds (CD) are defined as their intersections. `http://tgdtheory.fi/appfigures/futurepast.jpg`

Fig. 3. Causal diamond (CD) is highly analogous to Penrose diagram but simpler. `http://tgdtheory.fi/appfigures/penrose.jpg`

A rather recent discovery was that CP_2 is the only compact 4-manifold with Euclidian signature of metric allowing twistor space with Kähler structure. M^4 is in turn is the only 4-D space with Minkowskian signature of metric allowing twistor space with Kähler structure [A61] so that $H = M^4 \times CP_2$ is twistorially unique.

One can loosely say that quantum states in a given sector of "world of classical worlds" (WCW) are superpositions of space-time surfaces inside CDs and that positive and negative energy parts of zero energy states are localized and past and future boundaries of CDs. CDs form a hierarchy. One can have CDs within CDs and CDs can also overlap. The size of CD is characterized by the proper time distance between its two tips. One can perform both translations and also Lorentz boosts of CD leaving either boundary invariant. Therefore one can assign to CDs a moduli space and speak about wave function in this moduli space.

In number theoretic approach it is natural to restrict the allowed Lorentz boosts to some discrete subgroup of Lorentz group and also the distances between the tips of CDs to multiples of CP_2 radius defined by the length of its geodesic. Therefore the moduli space of CDs discretizes. The quantization of cosmic recession velocities for which there are indications, could relate to this quantization.

A-2.1 Basic facts about CP_2

CP_2 as a four-manifold is very special. The following arguments demonstrate that it codes for the symmetries of standard models via its isometries and holonomies.

CP_2 as a manifold

CP_2, the complex projective space of two complex dimensions, is obtained by identifying the points of complex 3-space C^3 under the projective equivalence

$$(z^1, z^2, z^3) \equiv \lambda(z^1, z^2, z^3) \ . \tag{A-2.1}$$

Here λ is any non-zero complex number. Note that CP_2 can be also regarded as the coset space $SU(3)/U(2)$. The pair z^i/z^j for fixed j and $z^i \neq 0$ defines a complex coordinate chart for CP_2. As j runs from 1 to 3 one obtains an atlas of three coordinate charts covering CP_2, the charts being holomorphically related to each other (e.g. CP_2 is a complex manifold). The points $z^3 \neq 0$ form a subset of CP_2 homoeomorphic to R^4 and the points with $z^3 = 0$ a set homeomorphic to S^2. Therefore CP_2 is obtained by "adding the 2-sphere at infinity to R^4".

Besides the standard complex coordinates $\xi^i = z^i/z^3$, $i = 1, 2$ the coordinates of Eguchi and Freund [A49] will be used and their relation to the complex coordinates is given by

$$\begin{aligned} \xi^1 &= z + it \ , \\ \xi^2 &= x + iy \ . \end{aligned} \tag{A-2.2}$$

These are related to the "spherical coordinates" via the equations

$$\begin{aligned} \xi^1 &= rexp(i\frac{(\Psi + \Phi)}{2})cos(\frac{\Theta}{2}) \ , \\ \xi^2 &= rexp(i\frac{(\Psi - \Phi)}{2})sin(\frac{\Theta}{2}) \ . \end{aligned} \tag{A-2.3}$$

The ranges of the variables r, Θ, Φ, Ψ are $[0, \infty], [0, \pi], [0, 4\pi], [0, 2\pi]$ respectively.

Considered as a real four-manifold CP_2 is compact and simply connected, with Euler number Euler number 3, Pontryagin number 3 and second $b = 1$.

Fig. 4. CP_2 as manifold. `http://tgdtheory.fi/appfigures/cp2.jpg`

Metric and Kähler structure of CP_2

In order to obtain a natural metric for CP_2, observe that CP_2 can be thought of as a set of the orbits of the isometries $z^i \rightarrow exp(i\alpha)z^i$ on the sphere S^5: $\sum z^i\bar{z}^i = R^2$. The metric of CP_2 is obtained by projecting the metric of S^5 orthogonally to the orbits of the isometries. Therefore the distance between the points of CP_2 is that between the representative orbits on S^5.

The line element has the following form in the complex coordinates

$$ds^2 = g_{a\bar{b}} d\xi^a d\bar{\xi}^b \ , \tag{A-2.4}$$

where the Hermitian, in fact Kähler metric $g_{a\bar{b}}$ is defined by

$$g_{a\bar{b}} = R^2 \partial_a \partial_{\bar{b}} K \ , \tag{A-2.5}$$

where the function K, Kähler function, is defined as

$$\begin{aligned} K &= log(F) \ , \\ F &= 1 + r^2 \ . \end{aligned} \tag{A-2.6}$$

The Kähler function for S^2 has the same form. It gives the S^2 metric $dzd\bar{z}/(1+r^2)^2$ related to its standard form in spherical coordinates by the coordinate transformation $(r, \phi) = (tan(\theta/2), \phi)$.

The representation of the CP_2 metric is deducible from S^5 metric is obtained by putting the angle coordinate of a geodesic sphere constant in it and is given

$$\frac{ds^2}{R^2} = \frac{(dr^2 + r^2\sigma_3^2)}{F^2} + \frac{r^2(\sigma_1^2 + \sigma_2^2)}{F} \ , \tag{A-2.7}$$

where the quantities σ_i are defined as

$$\begin{aligned} r^2\sigma_1 &= Im(\xi^1 d\xi^2 - \xi^2 d\xi^1) \ , \\ r^2\sigma_2 &= -Re(\xi^1 d\xi^2 - \xi^2 d\xi^1) \ , \\ r^2\sigma_3 &= -Im(\xi^1 d\bar{\xi}^1 + \xi^2 d\bar{\xi}^2) \ . \end{aligned} \tag{A-2.8}$$

R denotes the radius of the geodesic circle of CP_2. The vierbein forms, which satisfy the defining relation

$$s_{kl} = R^2 \sum_A e_k^A e_l^A \ , \tag{A-2.9}$$

are given by

$$\begin{aligned} e^0 &= \tfrac{dr}{F} \ , \quad & e^1 &= \tfrac{r\sigma_1}{\sqrt{F}} \ , \\ e^2 &= \tfrac{r\sigma_2}{\sqrt{F}} \ , \quad & e^3 &= \tfrac{r\sigma_3}{F} \ . \end{aligned} \tag{A-2.10}$$

The explicit representations of vierbein vectors are given by

$$\begin{array}{lllllll} e^0 & = & \frac{dr}{F} \ , & e^1 & = & \frac{r(sin\Theta cos\Psi d\Phi+sin\Psi d\Theta)}{2\sqrt{F}} & , \\ e^2 & = & \frac{r(sin\Theta sin\Psi d\Phi-cos\Psi d\Theta)}{2\sqrt{F}} \ , & e^3 & = & \frac{r(d\Psi+cos\Theta d\Phi)}{2F} & . \end{array} \quad \text{(A-2.11)}$$

The explicit representation of the line element is given by the expression

$$ds^2/R^2 = \frac{dr^2}{F^2} + \frac{r^2}{4F^2}(d\Psi + cos\Theta d\Phi)^2 + \frac{r^2}{4F}(d\Theta^2 + sin^2\Theta d\Phi^2) \ . \quad \text{(A-2.12)}$$

From this expression one finds that at coordinate infinity $r = \infty$ line element reduces to $\frac{r^2}{4F}(d\Theta^2 + sin^2\Theta d\Phi^2)$ of S^2 meaning that 3-sphere degenerates metrically to 2-sphere and one can say that CP_2 is obtained by adding to R^4 a 2-sphere at infinity.

The vierbein connection satisfying the defining relation

$$de^A = -V^A_B \wedge e^B \ , \quad \text{(A-2.13)}$$

is given by

$$\begin{array}{llllll} V_{01} & = & -\frac{e^1}{r} \ , & V_{23} & = & \frac{e^1}{r} \ , \\ V_{02} & = & -\frac{e^2}{r} \ , & V_{31} & = & \frac{e^2}{r} \ , \\ V_{03} & = & (r - \frac{1}{r})e^3 \ , & V_{12} & = & (2r + \frac{1}{r})e^3 \ . \end{array} \quad \text{(A-2.14)}$$

The representation of the covariantly constant curvature tensor is given by

$$\begin{array}{llllll} R_{01} & = & e^0 \wedge e^1 - e^2 \wedge e^3 \ , & R_{23} & = & e^0 \wedge e^1 - e^2 \wedge e^3 \ , \\ R_{02} & = & e^0 \wedge e^2 - e^3 \wedge e^1 \ , & R_{31} & = & -e^0 \wedge e^2 + e^3 \wedge e^1 \ , \\ R_{03} & = & 4e^0 \wedge e^3 + 2e^1 \wedge e^2 \ , & R_{12} & = & 2e^0 \wedge e^3 + 4e^1 \wedge e^2 \ . \end{array} \quad \text{(A-2.15)}$$

Metric defines a real, covariantly constant, and therefore closed 2-form J

$$J = -is_{a\bar{b}}d\xi^a d\bar{\xi}^b \ , \quad \text{(A-2.16)}$$

the so called Kähler form. Kähler form J defines in CP_2 a symplectic structure because it satisfies the condition

$$J^k_{\ r} J^{rl} = -s^{kl} \ . \quad \text{(A-2.17)}$$

The condition states that J and g give representations of real unit and imaginary units related by the formula $i^2 = -1$.

Kähler form is expressible locally in terms of Kähler gauge potential

$$J = dB \ , \quad \text{(A-2.18)}$$

where B is the so called Kähler potential, which is not defined globally since J describes homological magnetic monopole.

$dJ = ddB = 0$ gives the topological half of Maxwell equations (vanishing of magnetic charges and Faraday's induction law) and self-duality $*J = J$ reduces the remaining equations to $dJ = 0$. Hence the Kähler form can be regarded as a curvature form of a $U(1)$ gauge potential B carrying a magnetic charge of unit $1/2g$ (g denotes the gauge coupling).

The magnetic flux of J through a 2-surface in CP_2 is proportional to its homology equivalence class, which is integer valued. The explicit representations of J and B are given by

$$\begin{aligned} B &= 2re^3 \ , \\ J &= 2(e^0 \wedge e^3 + e^1 \wedge e^2) = \frac{r}{F^2}dr \wedge (d\Psi + cos\Theta d\Phi) + \frac{r^2}{2F}sin\Theta d\Theta \wedge d\Phi \ . \end{aligned} \tag{A-2.19}$$

The vierbein curvature form and Kähler form are covariantly constant and have in the complex coordinates only components of type (1, 1).

Useful coordinates for CP_2 are the so called canonical (or symplectic or Darboux) coordinates in which the Kähler potential and Kähler form have very simple expressions

$$\begin{aligned} B &= \sum_{k=1,2} P_k dQ_k \ , \\ J &= \sum_{k=1,2} dP_k \wedge dQ_k \ . \end{aligned} \tag{A-2.20}$$

The relationship of the canonical coordinates to the "spherical" coordinates is given by the equations

$$\begin{aligned} P_1 &= -\frac{1}{1+r^2} \ , \\ P_2 &= -\frac{r^2 cos\Theta}{2(1+r^2)} \ , \\ Q_1 &= \Psi \ , \\ Q_2 &= \Phi \ . \end{aligned} \tag{A-2.21}$$

Spinors In CP_2

CP_2 doesn't allow spinor structure in the conventional sense [A40]. However, the coupling of the spinors to a half odd multiple of the Kähler potential leads to a respectable spinor structure. Because the delicacies associated with the spinor structure of CP_2 play a fundamental role in TGD, the arguments of Hawking are repeated here.

To see how the space can fail to have an ordinary spinor structure consider the parallel transport of the vierbein in a simply connected space M. The parallel propagation around a closed curve with a base point x leads to a rotated vierbein at x: $e^A = R^A_B e^B$ and one can associate to each closed path an element of $SO(4)$.

Consider now a one-parameter family of closed curves $\gamma(v) : v \in (0,1)$ with the same base point x and $\gamma(0)$ and $\gamma(1)$ trivial paths. Clearly these paths define a sphere S^2 in M and the element $R^A_B(v)$ defines a closed path in $SO(4)$. When the sphere S^2 is contractible to a point e.g., homologically trivial, the path in $SO(4)$ is also contractible to a point and therefore represents a trivial element of the homotopy group $\Pi_1(SO(4)) = Z_2$.

For a homologically nontrivial 2-surface S^2 the associated path in $SO(4)$ can be homotopically nontrivial and therefore corresponds to a nonclosed path in the covering group Spin(4) (leading from the matrix 1 to -1 in the matrix representation). Assume this is the case.

Assume now that the space allows spinor structure. Then one can parallel propagate also spinors and by the above construction associate a closed path of Spin(4) to the surface S^2. Now, however this path corresponds to a lift of the corresponding $SO(4)$ path and cannot be closed. Thus one ends up with a contradiction.

From the preceding argument it is clear that one could compensate the non-allowed -1-factor associated with the parallel transport of the spinor around the sphere S^2 by coupling it to a gauge potential in such a way that in the parallel transport the gauge potential introduces a compensating -1-factor. For a $U(1)$ gauge potential this factor is given by the exponential

$exp(i2\Phi)$, where Φ is the magnetic flux through the surface. This factor has the value -1 provided the $U(1)$ potential carries half odd multiple of Dirac charge $1/2g$. In case of CP_2 the required gauge potential is half odd multiple of the Kähler potential B defined previously. In the case of $M^4 \times CP_2$ one can in addition couple the spinor components with different chiralities independently to an odd multiple of $B/2$.

Geodesic sub-manifolds of CP_2

Geodesic sub-manifolds are defined as sub-manifolds having common geodesic lines with the embedding space. As a consequence the second fundamental form of the geodesic manifold vanishes, which means that the tangent vectors h^k_α (understood as vectors of H) are covariantly constant quantities with respect to the covariant derivative taking into account that the tangent vectors are vectors both with respect to H and X^4.

In [A75] a general characterization of the geodesic sub-manifolds for an arbitrary symmetric space G/H is given. Geodesic sub-manifolds are in 1-1-correspondence with the so called Lie triple systems of the Lie-algebra g of the group G. The Lie triple system t is defined as a subspace of g characterized by the closedness property with respect to double commutation

$$[X,[Y,Z]] \in t \ \text{ for } \ X,Y,Z \in t \ . \qquad \text{(A-2.22)}$$

$SU(3)$ allows, besides geodesic lines, two nonequivalent (not isometry related) geodesic spheres. This is understood by observing that $SU(3)$ allows two nonequivalent $SU(2)$ algebras corresponding to subgroups $SO(3)$ (orthogonal 3×3 matrices) and the usual isospin group $SU(2)$. By taking any subset of two generators from these algebras, one obtains a Lie triple system and by exponentiating this system, one obtains a 2-dimensional geodesic sub-manifold of CP_2.

Standard representatives for the geodesic spheres of CP_2 are given by the equations

$$S^2_I \ : \ \xi^1 = \bar{\xi}^2 \ \text{ or equivalently } \ (\Theta = \pi/2, \Psi = 0) \ ,$$

$$S^2_{II} \ : \ \xi^1 = \xi^2 \ \text{ or equivalently } \ (\Theta = \pi/2, \Phi = 0) \ .$$

The non-equivalence of these sub-manifolds is clear from the fact that isometries act as holomorphic transformations in CP_2. The vanishing of the second fundamental form is also easy to verify. The first geodesic manifold is homologically trivial: in fact, the induced Kähler form vanishes identically for S^2_I. S^2_{II} is homologically nontrivial and the flux of the Kähler form gives its homology equivalence class.

A-2.2 CP_2 geometry and Standard Model symmetries

Identification of the electro-weak couplings

The delicacies of the spinor structure of CP_2 make it a unique candidate for space S. First, the coupling of the spinors to the $U(1)$ gauge potential defined by the Kähler structure provides the missing $U(1)$ factor in the gauge group. Secondly, it is possible to couple different H-chiralities independently to a half odd multiple of the Kähler potential. Thus the hopes of obtaining a correct spectrum for the electromagnetic charge are considerable. In the following it will be demonstrated that the couplings of the induced spinor connection are indeed those of the GWS model [B41] and in particular that the right handed neutrinos decouple completely from the electro-weak interactions.

To begin with, recall that the space H allows to define three different chiralities for spinors. Spinors with fixed H-chirality $e = \pm 1$, CP_2-chirality l, r and M^4-chirality L, R are defined by the condition

$$\begin{aligned} \Gamma\Psi &= e\Psi \ , \\ e &= \pm 1 \ , \end{aligned} \qquad \text{(A-2.23)}$$

where Γ denotes the matrix $\Gamma_9 = \gamma_5 \otimes \gamma_5$, $1 \otimes \gamma_5$ and $\gamma_5 \otimes 1$ respectively. Clearly, for a fixed H-chirality CP_2- and M^4-chiralities are correlated.

The spinors with H-chirality $e = \pm 1$ can be identified as quark and lepton like spinors respectively. The separate conservation of baryon and lepton numbers can be understood as a consequence of generalized chiral invariance if this identification is accepted. For the spinors with a definite H-chirality one can identify the vielbein group of CP_2 as the electro-weak group: $SO(4)$ having as its covering group $SU(2)_L \times SU(2)_R$.

The covariant derivatives are defined by the spinorial connection

$$A = V + \frac{B}{2}(n_+ 1_+ + n_- 1_-) \ . \tag{A-2.24}$$

Here V and B denote the projections of the vielbein and Kähler gauge potentials respectively and $1_{+(-)}$ projects to the spinor H-chirality $+(-)$. The integers $n_\pm$ are odd from the requirement of a respectable spinor structure.

The explicit representation of the vielbein connection V and of B are given by the equations

$$\begin{array}{lllllll} V_{01} & = & -\frac{e^1}{r} \ , & V_{23} & = & \frac{e^1}{r} \ , \\ V_{02} & = & -\frac{e^2}{r} \ , & V_{31} & = & \frac{e^2}{r} \ , \\ V_{03} & = & (r - \frac{1}{r})e^3 \ , & V_{12} & = & (2r + \frac{1}{r})e^3 \ , \end{array} \tag{A-2.25}$$

and

$$B = 2re^3 \ , \tag{A-2.26}$$

respectively. The explicit representation of the vielbein is not needed here.

Let us first show that the charged part of the spinor connection couples purely left handedly. Identifying Σ_3^0 and Σ_2^1 as the diagonal (neutral) Lie-algebra generators of $SO(4)$, one finds that the charged part of the spinor connection is given by

$$A_{ch} = 2V_{23}I_L^1 + 2V_{13}I_L^2 \ , \tag{A-2.27}$$

where one have defined

$$\begin{aligned} I_L^1 &= \frac{(\Sigma_{01} - \Sigma_{23})}{2} \ , \\ I_L^2 &= \frac{(\Sigma_{02} - \Sigma_{13})}{2} \ . \end{aligned} \tag{A-2.28}$$

A_{ch} is clearly left handed so that one can perform the identification of the gauge potential as

$$W^\pm = \frac{2(e^1 \pm ie^2)}{r} \ , \tag{A-2.29}$$

where $W^\pm$ denotes the charged intermediate vector boson.

The covariantly constant curvature tensor is given by

$$\begin{array}{l} R_{01} = -R_{23} = e^0 \wedge e^1 - e^2 \wedge e^3 \ , \\ R_{02} = -R_{31} = e^0 \wedge e^2 - e^3 \wedge e^1 \ , \\ R_{03} = 4e^0 \wedge e^3 + 2e^1 \wedge e^2 \ , \\ R_{12} = 2e^0 \wedge e^3 + 4e^1 \wedge e^2 \ . \end{array} \tag{A-2.30}$$

The charged part of the curvature tensor is left handed.

This is to be compared with the Weyl tensor, which defines a representation of quaternionic imaginary units.

$$
\begin{array}{l}
W_{03} = W_{12} \equiv 2I_3 = 2(e^0 \wedge e^3 + e^1 \wedge e^2) \ , \\
W_{01} = W_{23} \equiv I_1 = -e^0 \wedge e^1 - e^2 \wedge e^3 \ , \\
W_{02} = W_{31} \equiv I_2 = -e^0 \wedge e^2 - e^3 \wedge e^1 \ .
\end{array} \quad \text{(A-2.31)}
$$

The charged part of the Weyl tensor is right-handed and that the relative sign of the two terms in the curvature tensor and Weyl tensor are opposite.

Consider next the identification of the neutral gauge bosons γ and Z^0 as appropriate linear combinations of the two functionally independent quantities

$$
\begin{array}{rcl}
X & = & re^3 \ , \\
Y & = & \frac{e^3}{r} \ ,
\end{array} \quad \text{(A-2.32)}
$$

appearing in the neutral part of the spinor connection. We show first that the mere requirement that photon couples vectorially implies the basic coupling structure of the GWS model leaving only the value of Weinberg angle undetermined.

To begin with let us define

$$
\begin{array}{rcl}
\bar{\gamma} & = & aX + bY \ , \\
\bar{Z}^0 & = & cX + dY \ ,
\end{array} \quad \text{(A-2.33)}
$$

where the normalization condition

$$ad - bc = 1 \ ,$$

is satisfied. The physical fields γ and Z^0 are related to $\bar{\gamma}$ and $\bar{Z}^0$ by simple normalization factors.

Expressing the neutral part of the spinor connection in term of these fields one obtains

$$
\begin{array}{rcl}
A_{nc} & = & [(c+d)2\Sigma_{03} + (2d-c)2\Sigma_{12} + d(n_+1_+ + n_-1_-)]\bar{\gamma} \\
 & + & [(a-b)2\Sigma_{03} + (a-2b)2\Sigma_{12} - b(n_+1_+ + n_-1_-)]\bar{Z}^0 \ .
\end{array} \quad \text{(A-2.34)}
$$

Identifying Σ_{12} and $\Sigma_{03} = 1 \times \gamma_5 \Sigma_{12}$ as vectorial and axial Lie-algebra generators, respectively, the requirement that γ couples vectorially leads to the condition

$$c = -d \ . \quad \text{(A-2.35)}$$

Using this result plus previous equations, one obtains for the neutral part of the connection the expression

$$A_{nc} = \gamma Q_{em} + Z^0(I_L^3 - sin^2\theta_W Q_{em}) \ . \quad \text{(A-2.36)}$$

Here the electromagnetic charge Q_{em} and the weak isospin are defined by

$$
\begin{array}{rcl}
Q_{em} & = & \Sigma^{12} + \frac{(n_+1_+ + n_-1_-)}{6} \ , \\
I_L^3 & = & \frac{(\Sigma^{12} - \Sigma^{03})}{2} \ .
\end{array} \quad \text{(A-2.37)}
$$

The fields γ and Z^0 are defined via the relations

$$
\begin{array}{rcl}
\gamma & = & 6d\bar{\gamma} = \frac{6}{(a+b)}(aX + bY) \ , \\
Z^0 & = & 4(a+b)\bar{Z}^0 = 4(X - Y) \ .
\end{array} \quad \text{(A-2.38)}
$$

The value of the Weinberg angle is given by

$$sin^2\theta_W = \frac{3b}{2(a+b)} \ , \qquad \text{(A-2.39)}$$

and is not fixed completely. Observe that right handed neutrinos decouple completely from the electro-weak interactions.

The determination of the value of the Weinberg angle is a dynamical problem. The original approach was based on the assumption that it makes sense to talk about electroweak action defined at fundamental level and introduce a symmetry breaking by adding an additional term proportional to Kähler action. The recent view is that Kähler action plus volume term defines the fundamental action.

The Weinberg angle is completely fixed if one requires that the electroweak action contains no cross term of type γZ^0. This leads to a definite value for the Weinberg angle.

One can however add a symmetry breaking term proportional to Kähler action and this changes the value of the Weinberg angle. As a matter fact, color gauge action identifying color gauge field as proportional to $H^A J_{\alpha\beta}$ is proportional to Kähler action. A possible interpretation would be as a sum of electroweak and color gauge interactions.

To evaluate the value of the Weinberg angle one can express the neutral part F_{nc} of the induced gauge field as

$$F_{nc} = 2R_{03}\Sigma^{03} + 2R_{12}\Sigma^{12} + J(n_+1_+ + n_-1_-) \ , \qquad \text{(A-2.40)}$$

where one has

$$\begin{aligned} R_{03} &= 2(2e^0\wedge e^3 + e^1\wedge e^2) \ , \\ R_{12} &= 2(e^0\wedge e^3 + 2e^1\wedge e^2) \ , \\ J &= 2(e^0\wedge e^3 + e^1\wedge e^2) \ , \end{aligned} \qquad \text{(A-2.41)}$$

in terms of the fields γ and Z^0 (photon and Z- boson)

$$F_{nc} = \gamma Q_{em} + Z^0(I_L^3 - sin^2\theta_W Q_{em}) \ . \qquad \text{(A-2.42)}$$

Evaluating the expressions above, one obtains for γ and Z^0 the expressions

$$\begin{aligned} \gamma &= 3J - sin^2\theta_W R_{12} \ , \\ Z^0 &= 2R_{03} \ . \end{aligned} \qquad \text{(A-2.43)}$$

For the Kähler field one obtains

$$J = \frac{1}{3}(\gamma + sin^2\theta_W Z^0) \ . \qquad \text{(A-2.44)}$$

Expressing the neutral part of the symmetry broken YM action

$$\begin{aligned} L_{ew} &= L_{sym} + fJ^{\alpha\beta}J_{\alpha\beta} \ , \\ L_{sym} &= \frac{1}{4g^2}Tr(F^{\alpha\beta}F_{\alpha\beta}) \ , \end{aligned} \qquad \text{(A-2.45)}$$

where the trace is taken in spinor representation, in terms of γ and Z^0 one obtains for the coefficient X of the γZ^0 cross term (this coefficient must vanish) the expression

$$
\begin{aligned}
X &= -\frac{K}{2g^2} + \frac{fp}{18} \ , \\
K &= Tr\left[Q_{em}(I_L^3 - sin^2\theta_W Q_{em})\right] \ , \qquad \text{(A-2.46)}
\end{aligned}
$$

This parameter can be calculated by substituting the values of quark and lepton charges and weak isospins.

In the general case the value of the coefficient K is given by

$$
K = \sum_i \left[-\frac{(18 + 2n_i^2)sin^2\theta_W}{9} \right] \ , \qquad \text{(A-2.47)}
$$

where the sum is over the spinor chiralities, which appear as elementary fermions and n_i is the integer describing the coupling of the spinor field to the Kähler potential. The cross term vanishes provided the value of the Weinberg angle is given by

$$
sin^2\theta_W = \frac{9\sum_i 1}{(fg^2 + 2\sum_i (18 + n_i^2))} \ . \qquad \text{(A-2.48)}
$$

In the scenario where both leptons and quarks are elementary fermions the value of the Weinberg angle is given by

$$
sin^2\theta_W = \frac{9}{(\frac{fg^2}{2} + 28)} \ . \qquad \text{(A-2.49)}
$$

The bare value of the Weinberg angle is 9/28 in this scenario, which is not far from the typical value 9/24 of GUTs at high energies [B7]. The experimental value at the scale length scale of the electron can be deduced from the ratio of W and Z boson masses as $sin^2\theta_W = 1 - (m_W/m_Z)^2 \simeq .22290$. This ratio and also the weak boson masses depend on the length scale.

If one interprets the additional term proportional to J as color action, one could perhaps interpret the value of Weinberg angle as expressing a connection between strong and weak coupling constant evolution. The limit $f \to 0$ should correspond to an infinite value of color coupling strength and at this limit one would have $sin^2\theta_W = \frac{9}{28}$ for $f/g^2 \to 0$. This does not make sense since the Weinberg angle is in the standard model much smaller in QCD scale Λ corresponding roughly to pion mass scale. The Weinberg angle is in principle predicted by the p-adic coupling constant evolution fixed by the number theoretical vision of TGD.

One could however have a sum of electroweak action, correction terms changing the value of Weinberg angle, and color action and coupling constant evolution could be understood in terms of the coupling parameters involved.

Electroweak symmetry breaking

One of the hardest challenges in the development of the TGD based view of weak symmetry breaking was the fact that classical field equations allow space-time surfaces with finite but arbitrarily large size. For a fixed space-time surface, the induced gauge fields, including classical weak fields, are long ranged. On the other hand, the large mass for weak bosons would require a short correlation length. How can one understand this together with the fact that a photon has a long correlation length?

In zero energy ontology quantum states are superpositions of space-time surfaces as analogs of almost unique Bohr orbits of particles identified as 3-D surfaces. For some reason the superposition should be such that the quantum averages of weak gauge boson fields vanish below the weak scale whereas the quantum average of electromagnetic fields is non-vanishing.

This is indeed the case.

1. The supersymplectic symmetries form isometries of the world of classical worlds (WCW) and they act in CP_2 degrees of freedom as symplectic transformations leaving the CP_2 symplectic form J invariant and therefore also its contribution to the electromagnetic field since this part is the same for all space-time surfaces in the superposition of space-time surfaces as a representation of supersymplectic isometry group (as a special case a representation of color group).
2. In TGD, color and electroweak symmetries acting as holonomies are not independent and for the $SU(2)_L$ part of induced spinor connection the symplectic transformations induces $SU(2)_L \times U(1)_R$ gauge transformation. This suggests that the quantum expectations of the induced weak fields over the space-time surfaces vanish above the quantum coherence scale. The averages of W and of the left handed part of Z^0 should therefore vanish.
3. $\langle Z^0 \rangle$ should vanish. For $U(1)_R$ part of Z^0, the action of gauge transformation is trivial in gauge theory. Now however the space-time surface changes under symplectic transformations and this could make the average of the right-handed part of Z^0 vanishing. The vanishing of the average of the axial part of the Z^0 is suggested by the partially conserved axial current hypothesis.

One can formulate this picture quantitatively.

1. The electromagnetic field [L55] contains, besides the induced Kähler form, also the induced curvature form R_{12}, which couples vectorially. Conserved vector current hypothesis suggests that the average of R_{12} is non-vanishing. One can express the neutral part of the induced gauge field in terms of induced spinor curvature and Kähler form J as

$$
\begin{aligned}
R_{03} &= 2(2e^0 \wedge e^3 + e^1 \wedge e^2) = J + 2e^0 \wedge e^3 \ , \\
J &= 2(e^0 \wedge e^3 + e^1 \wedge e^2) \ , \\
R_{12} &= 2(e^0 \wedge e^3 + 2e^1 \wedge e^2) = 3J - 2e^0 \wedge e^3 \quad , \qquad \text{(A-2.50)}
\end{aligned}
$$

2. The induced fields γ and Z^0 (photon and Z- boson) can be expressed as

$$
\begin{aligned}
\gamma &= 3J - sin^2\theta_W R_{12} \ , \\
Z^0 &= 2R_{03} = 2(J + 2e^0 \wedge e^3) \qquad \text{(A-2.51)} \\
per. & \qquad \text{(A-2.52)}
\end{aligned}
$$

The condition $\langle Z^0 \rangle = 0$ gives $2\langle e^0 \wedge e^3 \rangle = -2J$ and this in turn gives $\langle R_{12} \rangle = 4J$. The average over γ would be

$$
\langle \gamma \rangle = (3 - 4sin^2\theta_W)J \ .
$$

For $sin^2\theta_W = 3/4 \ langle \gamma \rangle$ would vanish.

The quantum averages of classical weak fields quite generally vanish. What about correlation functions?

1. One expects that the correlators of classical weak fields as color invariants, and perhaps even symplectic invariants, are non-vanishing below the Compton length since in this kind of situation the points in the correlation function belong to the same 3-surface representing particle, such as hadron.
2. The intuitive picture is that in longer length scales one has disjoint 3-surfaces with a size scale of Compton length. If the states associated with two disjoint 3-surfaces are separately color invariant there are no correlations in color degrees of freedom and correlators reduce to the products of expectations of classical weak fields and vanish. This could also hold when the 3-surfaces are connected by flux tube bonds.

   Below the Compton length weak bosons would thus behave as correlated massless fields. The Compton lengths of weak bosons are proportional to the value of effective Planck constant h_{eff} and in living systems the Compton lengths are proposed to be even of the order of cell size. This would explain the mysterious chiral selection in living systems requiring large parity violation.

3. What about the averages and correlators of color gauge fields? Classical color gauge fields are proportional to the products of Hamiltonians of color isometries induced Kähler form and the expectations of color Hamiltonians give vanishing average above Compton length and therefore vanishing average. Correlators are non-vanishing below the hadron scale. Gluons do not propagate in long scales for the same reason as weak bosons. This is implied by color confinement, which has also classical description in the sense that 3-surfaces have necessarily a finite size.

   A large value of h_{eff} allows colored states even in biological scales below the Compton length since in this kind of situation the points in the correlation function belong to the same 3-surface representing particle, such as dark hadron.

Discrete symmetries

The treatment of discrete symmetries C, P, and T is based on the following requirements:

1. Symmetries must be realized as purely geometric transformations.
2. Transformation properties of the field variables should be essentially the same as in the conventional quantum field theories [B10] .

The action of the reflection P on spinors of is given by

$$\Psi \rightarrow P\Psi = \gamma^0 \otimes \gamma^0 \Psi \ . \tag{A-2.53}$$

in the representation of the gamma matrices for which γ^0 is diagonal. It should be noticed that W and Z^0 bosons break parity symmetry as they should since their charge matrices do not commute with the matrix of P.

The guess that a complex conjugation in CP_2 is associated with T transformation of the physicist turns out to be correct. One can verify by a direct calculation that pure Dirac action is invariant under T realized according to

$$\begin{aligned} m^k &\rightarrow T(M^k) \ , \\ \xi^k &\rightarrow \bar{\xi}^k \ , \\ \Psi &\rightarrow \gamma^1\gamma^3 \otimes 1\Psi \ . \end{aligned} \tag{A-2.54}$$

The operation bearing closest resemblance to the ordinary charge conjugation corresponds geometrically to complex conjugation in CP_2:

$$\begin{aligned} \xi^k &\rightarrow \bar{\xi}^k \ , \\ \Psi &\rightarrow \Psi^\dagger\gamma^2\gamma^0 \otimes 1 \ . \end{aligned} \tag{A-2.55}$$

As one might have expected symmetries CP and T are exact symmetries of the pure Dirac action.

A-3 Induction procedure and many-sheeted space-time

Since the classical gauge fields are closely related in TGD framework, it is not possible to have space-time sheets carrying only single kind of gauge field. For instance, em fields are accompanied by Z^0 fields for extremals of Kähler action.

Classical em fields are always accompanied by Z^0 field and some components of color gauge field. For extremals having homologically non-trivial sphere as a CP_2 projection em and Z^0 fields are the only non-vanishing electroweak gauge fields. For homologically trivial sphere only W fields are non-vanishing. Color rotations does not affect the situation.

For vacuum extremals all electro-weak gauge fields are in general non-vanishing although the net gauge field has U(1) holonomy by 2-dimensionality of the CP_2 projection. Color gauge field has $U(1)$ holonomy for all space-time surfaces and quantum classical correspondence suggest a weak form of color confinement meaning that physical states correspond to color neutral members of color multiplets.

A-3.1 Induction procedure for gauge fields and spinor connection

Induction procedure for gauge potentials and spinor structure is a standard procedure of bundle theory. If one has embedding of some manifold to the base space of a bundle, the bundle structure can be induced so that it has as a base space the imbedded manifold, whose points have as fiber the fiber if embedding space at their image points. In the recent case the embedding of space-time surface to embedding space defines the induction procedure. The induced gauge potentials and gauge fields are projections of the spinor connection of the embedding space to the space-time surface (see `http://tgdtheory.fi/appfigures/induct.jpg`).

Induction procedure makes sense also for the spinor fields of embedding space and one obtains geometrization of both electroweak gauge potentials and of spinors. The new element is induction of gamma matrices which gives their projections at space-time surface.

As a matter fact, the induced gamma matrices cannot appear in the counterpart of massless Dirac equation. To achieve super-symmetry, Dirac action must be replaced with Kähler-Dirac action for which gamma matrices are contractions of the canonical momentum currents of Kähler action with embedding space gamma matrices. Induced gamma matrices in Dirac action would correspond to 4-volume as action.

Fig. 9. Induction of spinor connection and metric as projection to the space-time surface. `http://tgdtheory.fi/appfigures/induct.jpg`.

A-3.2 Induced gauge fields for space-times for which CP_2 projection is a geodesic sphere

If one requires that space-time surface is an extremal of Kähler action and has a 2-dimensional CP_2 projection, only vacuum extremals and space-time surfaces for which CP_2 projection is a geodesic sphere, are allowed. Homologically non-trivial geodesic sphere correspond to vanishing W fields and homologically non-trivial sphere to non-vanishing W fields but vanishing γ and Z^0. This can be verified by explicit examples.

$r = \infty$ surface gives rise to a homologically non-trivial geodesic sphere for which e_0 and e_3 vanish imply the vanishing of W field. For space-time sheets for which CP_2 projection is $r = \infty$ homologically non-trivial geodesic sphere of CP_2 one has

$$\gamma = (\frac{3}{4} - \frac{sin^2(\theta_W)}{2})Z^0 \simeq \frac{5Z^0}{8} \ .$$

The induced W fields vanish in this case and they vanish also for all geodesic sphere obtained by $SU(3)$ rotation.

$Im(\xi^1) = Im(\xi^2) = 0$ corresponds to homologically trivial geodesic sphere. A more general representative is obtained by using for the phase angles of standard complex CP_2 coordinates constant values. In this case e^1 and e^3 vanish so that the induced em, Z^0, and Kähler fields vanish but induced W fields are non-vanishing. This holds also for surfaces obtained by color rotation. Hence one can say that for non-vacuum extremals with 2-D CP_2 projection color rotations and weak symmetries commute.

A-3.3 Many-sheeted space-time

TGD space-time is many-sheeted: in other words, there are in general several space-sheets which have projection to the same M^4 region. Second manner to say this is that CP_2 coordinates are many-valued functions of M^4 coordinates. The original physical interpretation of many-sheeted space-time time was not correct: it was assumed that single sheet corresponds to GRT space-time and this obviously leads to difficulties since the induced gauge fields are expressible in terms of only four embedding space coordinates.

Fig. 10. Illustration of many-sheeted space-time of TGD. `http://tgdtheory.fi/appfigures/manysheeted.jpg`

Superposition of effects instead of superposition of fields

The first objection against TGD is that superposition is not possible for induced gauge fields and induced metric. The resolution of the problem is that it is effects which need to superpose, not the fields.

Test particle topologically condenses simultaneously to all space-time sheets having a projection to same region of M^4 (that is touches them). The superposition of effects of fields at various space-time sheets replaces the superposition of fields.This is crucial for the understanding also how GRT space-time relates to TGD space-time, which is also in the appendix of this book).

Wormhole contacts

Wormhole contacts are key element of many-sheeted space-time. One does not expect them to be stable unless there is non-trivial Kähler magnetic flux flowing through then so that the throats look like Kähler magnetic monopoles.

Fig. 11. Wormhole contact. `http://tgdtheory.fi/appfigures/wormholecontact.jpg`

Since the flow lines of Kähler magnetic field must be closed this requires the presence of another wormhole contact so that one obtains closed monopole flux tube decomposing to two Minkowskian pieces at the two space-time sheets involved and two wormhole contacts with Euclidian signature of the induced metric. These objects are identified as space-time correlates of elementary particles and are clearly analogous to string like objects.

The relationship between the many-sheeted space-time of TGD and of GRT space-time

The space-time of general relativity is single-sheeted and there is no need to regard it as surface in H although the assumption about representability as vacuum extremal gives very powerful constraints in cosmology and astrophysics and might make sense in simple situations.

The space-time of GRT can be regarded as a long length scale approximation obtained by lumping together the sheets of the many-sheeted space-time to a region of M^4 and providing it with an effective metric obtained as sum of M^4 metric and deviations of the induced metrics of various space-time sheets from M^4 metric. Also induced gauge potentials sum up in the similar manner so that also the gauge fields of gauge theories would not be fundamental fields.

Fig. 12. The superposition of fields is replaced with the superposition of their effects in many-sheeted space-time. `http://tgdtheory.fi/appfigures/fieldsuperpose.jpg`

Space-time surfaces of TGD are considerably simpler objects that the space-times of general relativity and relate to GRT space-time like elementary particles to systems of condensed matter physics. Same can be said about fields since all fields are expressible in terms of embedding space coordinates and their gradients, and general coordinate invariance means that the number of bosonic field degrees is reduced locally to 4. TGD space-time can be said to be a microscopic description whereas GRT space-time a macroscopic description. In TGD complexity of space-time topology replaces the complexity due to large number of fields in quantum field theory.

Topological field quantization and the notion of magnetic body

Topological field quantization also TGD from Maxwell's theory. TGD predicts topological light rays ("massless extremals (MEs)") as space-time sheets carrying waves or arbitrary shape propagating with maximal signal velocity in single direction only and analogous to laser beams and carrying light-like gauge currents in the generi case. There are also magnetic flux quanta and electric flux quanta. The deformations of cosmic strings with 2-D string orbit as M^4 projection gives rise to magnetic flux tubes carrying monopole flux made possible by CP_2 topology allowing homological Kähler magnetic monopoles.

Fig. 13. Topological quantization for magnetic fields replaces magnetic fields with bundles of them defining flux tubes as topological field quanta. `http://tgdtheory.fi/appfigures/field.jpg`

The imbeddability condition for say magnetic field means that the region containing constant magnetic field splits into flux quanta, say tubes and sheets carrying constant magnetic field. Unless one assumes a separate boundary term in Kähler action, boundaries in the usual sense are forbidden except as ends of space-time surfaces at the boundaries of causal diamonds. One obtains typically pairs of sheets glued together along their boundaries giving rise to flux tubes with closed cross section possibly carrying monopole flux.

These kind of flux tubes might make possible magnetic fields in cosmic scales already during primordial period of cosmology since no currents are needed to generate these magnetic fields: cosmic string would be indeed this kind of objects and would dominated during the primordial period. Even superconductors and maybe even ferromagnets could involve this kind of monopole flux tubes.

A-3.4 Embedding space spinors and induced spinors

One can geometrize also fermionic degrees of freedom by inducing the spinor structure of $M^4 \times CP_2$.

CP_2 does not allow spinor structure in the ordinary sense but one can couple the opposite H-chiralities of H-spinors to an $n = 1$ ($n = 3$) integer multiple of Kähler gauge potential to obtain a respectable modified spinor structure. The em charges of resulting spinors are fractional (integer valued) and the interpretation as quarks (leptons) makes sense since the couplings to the induced spinor connection having interpretation in terms electro-weak gauge potential are identical to those assumed in standard model.

The notion of quark color differs from that of standard model.

1. Spinors do not couple to color gauge potential although the identification of color gauge potential as projection of $SU(3)$ Killing vector fields is possible. This coupling must emerge only at the effective gauge theory limit of TGD.
2. Spinor harmonics of embedding space correspond to triality $t = 1$ ($t = 0$) partial waves. The detailed correspondence between color and electroweak quantum numbers is however not correct as such and the interpretation of spinor harmonics of embedding space is as representations for ground states of super-conformal representations. The wormhole pairs associated with physical quarks and leptons must carry also neutrino pair to neutralize weak quantum numbers above the length scale of flux tube (weak scale or Compton length). The total color quantum numbers or these states must be those of standard model. For instance, the color quantum numbers of fundamental left-hand neutrino and lepton can compensate each other for the physical lepton. For fundamental quark-lepton pair they could sum up to those of physical quark.

The well-definedness of em charge is crucial condition.

1. Although the embedding space spinor connection carries W gauge potentials one can say that the embedding space spinor modes have well-defined em charge. One expects that this is true for induced spinor fields inside wormhole contacts with 4-D CP_2 projection and Euclidian signature of the induced metric.
2. The situation is not the same for the modes of induced spinor fields inside Minkowskian region and one must require that the CP_2 projection of the regions carrying induced spinor field is such that the induced W fields and above weak scale also the induced Z^0 fields vanish in order to avoid large parity breaking effects. This condition forces the CP_2 projection to be 2-dimensional. For a generic Minkowskian space-time region this is achieved only if the spinor modes are localized at 2-D surfaces of space-time surface - string world sheets and possibly also partonic 2-surfaces.
3. Also the Kähler-Dirac gamma matrices appearing in the modified Dirac equation must vanish in the directions normal to the 2-D surface in order that Kähler-Dirac equation can be satisfied. This does not seem plausible for space-time regions with 4-D CP_2 projection.
4. One can thus say that strings emerge from TGD in Minkowskian space-time regions. In particular, elementary particles are accompanied by a pair of fermionic strings at the opposite space-time sheets and connecting wormhole contacts. Quite generally, fundamental fermions would propagate at the boundaries of string world sheets as massless particles and wormhole contacts would define the stringy vertices of generalized Feynman diagrams. One obtains geometrized diagrammatics, which brings looks like a combination of stringy and Feynman diagrammatics.
5. This is what happens in the the generic situation. Cosmic strings could serve as examples about surfaces with 2-D CP_2 projection and carrying only em fields and allowing delocalization of spinor modes to the entire space-time surfaces.

A-3.5 About induced gauge fields

In the following the induced gauge fields are studied for general space-time surface without assuming the preferred extremal property (Bohr orbit property). Therefore the following arguments are somewhat obsolete in their generality.

Space-times with vanishing em, Z^0, or Kähler fields

The following considerations apply to a more general situation in which the homologically trivial geodesic sphere and extremal property are not assumed. It must be emphasized that this case is possible in TGD framework only for a vanishing Kähler field.

Using spherical coordinates (r, Θ, Ψ, Φ) for CP_2, the expression of Kähler form reads as

$$\begin{aligned} J &= \frac{r}{F^2}dr \wedge (d\Psi + cos(\Theta)d\Phi) + \frac{r^2}{2F}sin(\Theta)d\Theta \wedge d\Phi \ , \\ F &= 1 + r^2 \ . \end{aligned} \quad \text{(A-3.1)}$$

The general expression of electromagnetic field reads as

$$\begin{aligned} F_{em} &= (3+2p)\frac{r}{F^2}dr \wedge (d\Psi + cos(\Theta)d\Phi) + (3+p)\frac{r^2}{2F}sin(\Theta)d\Theta \wedge d\Phi \ , \\ p &= sin^2(\Theta_W) \ , \end{aligned} \quad \text{(A-3.2)}$$

where Θ_W denotes Weinberg angle.

1. The vanishing of the electromagnetic fields is guaranteed, when the conditions

$$\begin{aligned} \Psi &= k\Phi \ , \\ (3+2p)\frac{1}{r^2F}(d(r^2)/d\Theta)(k + cos(\Theta)) &+ (3+p)sin(\Theta) = 0 \ , \end{aligned} \quad \text{(A-3.3)}$$

hold true. The conditions imply that CP_2 projection of the electromagnetically neutral space-time is 2-dimensional. Solving the differential equation one obtains

$$\begin{aligned} r &= \sqrt{\frac{X}{1-X}} \ , \\ X &= D\left[|\frac{k+u}{C}|\right]^{\epsilon} \ , \\ u &\equiv cos(\Theta) \ , \ C = k + cos(\Theta_0) \ , \ D = \frac{r_0^2}{1+r_0^2} \ , \ \epsilon = \frac{3+p}{3+2p} \ , \end{aligned} \quad \text{(A-3.4)}$$

where C and D are integration constants. $0 \leq X \leq 1$ is required by the reality of r. $r = 0$ would correspond to $X = 0$ giving $u = -k$ achieved only for $|k| \leq 1$ and $r = \infty$ to $X = 1$ giving $|u + k| = [(1 + r_0^2)/r_0^2)]^{(3+2p)/(3+p)}$ achieved only for

$$sign(u+k) \times [\frac{1+r_0^2}{r_0^2}]^{\frac{3+2p}{3+p}} \leq k+1 \ ,$$

where $sign(x)$ denotes the sign of x.

The expressions for Kähler form and Z^0 field are given by

$$\begin{aligned} J &= -\frac{p}{3+2p}Xdu \wedge d\Phi \ , \\ Z^0 &= -\frac{6}{p}J \ . \end{aligned} \quad \text{(A-3.5)}$$

The components of the electromagnetic field generated by varying vacuum parameters are proportional to the components of the Kähler field: in particular, the magnetic field is parallel to the Kähler magnetic field. The generation of a long range Z^0 vacuum field is a purely TGD based feature not encountered in the standard gauge theories.

2. The vanishing of Z^0 fields is achieved by the replacement of the parameter ϵ with $\epsilon = 1/2$ as becomes clear by considering the condition stating that Z^0 field vanishes identically. Also the relationship $F_{em} = 3J = -\frac{3}{4}\frac{r^2}{F}du \wedge d\Phi$ is useful.

3. The vanishing Kähler field corresponds to $\epsilon = 1, p = 0$ in the formula for em neutral space-times. In this case classical em and Z^0 fields are proportional to each other:

$$\begin{aligned}
Z^0 &= 2e^0 \wedge e^3 = \frac{r}{F^2}(k+u)\frac{\partial r}{\partial u}du \wedge d\Phi = (k+u)du \wedge d\Phi \ , \\
r &= \sqrt{\frac{X}{1-X}} \ , \ X = D|k+u| \ , \\
\gamma &= -\frac{p}{2}Z^0 \ . \qquad \text{(A-3.6)}
\end{aligned}$$

For a vanishing value of Weinberg angle ($p = 0$) em field vanishes and only Z^0 field remains as a long range gauge field. Vacuum extremals for which long range Z^0 field vanishes but em field is non-vanishing are not possible.

The effective form of CP_2 metric for surfaces with 2-dimensional CP_2 projection

The effective form of the CP_2 metric for a space-time having vanishing em,Z^0, or Kähler field is of practical value in the case of vacuum extremals and is given by

$$\begin{aligned}
ds^2_{eff} &= (s_{rr}(\frac{dr}{d\Theta})^2 + s_{\Theta\Theta})d\Theta^2 + (s_{\Phi\Phi} + 2ks_{\Phi\Psi})d\Phi^2 = \frac{R^2}{4}[s^{eff}_{\Theta\Theta}d\Theta^2 + s^{eff}_{\Phi\Phi}d\Phi^2] \ , \\
s^{eff}_{\Theta\Theta} &= X \times \left[\frac{\epsilon^2(1-u^2)}{(k+u)^2} \times \frac{1}{1-X} + 1 - X\right] \ , \\
s^{eff}_{\Phi\Phi} &= X \times \left[(1-X)(k+u)^2 + 1 - u^2\right] \ , \qquad \text{(A-3.7)}
\end{aligned}$$

and is useful in the construction of vacuum embedding of, say Schwartchild metric.

Topological quantum numbers

Space-times for which either em, Z^0, or Kähler field vanishes decompose into regions characterized by six vacuum parameters: two of these quantum numbers (ω_1 and ω_2) are frequency type parameters, two (k_1 and k_2) are wave vector like quantum numbers, two of the quantum numbers (n_1 and n_2) are integers. The parameters ω_i and n_i will be referred as electric and magnetic quantum numbers. The existence of these quantum numbers is not a feature of these solutions alone but represents a much more general phenomenon differentiating in a clear cut manner between TGD and Maxwell's electrodynamics.

The simplest manner to avoid surface Kähler charges and discontinuities or infinities in the derivatives of CP_2 coordinates on the common boundary of two neighboring regions with different vacuum quantum numbers is topological field quantization, 3-space decomposes into disjoint topological field quanta, 3-surfaces having outer boundaries with possibly macroscopic size.

Under rather general conditions the coordinates Ψ and Φ can be written in the form

$$\begin{aligned}
\Psi &= \omega_2 m^0 + k_2 m^3 + n_2\phi + \text{Fourier expansion} \ , \\
\Phi &= \omega_1 m^0 + k_1 m^3 + n_1\phi + \text{Fourier expansion} \ . \qquad \text{(A-3.8)}
\end{aligned}$$

m^0,m^3 and ϕ denote the coordinate variables of the cylindrical M^4 coordinates) so that one has $k = \omega_2/\omega_1 = n_2/n_1 = k_2/k_1$. The regions of the space-time surface with given values of the vacuum parameters ω_i,k_i and n_i and m and C are bounded by the surfaces at which space-time surface becomes ill-defined, say by $r > 0$ or $r < \infty$ surfaces.

The space-time surface decomposes into regions characterized by different values of the vacuum parameters r_0 and Θ_0. At $r = \infty$ surfaces n_2,ω_2 and m can change since all values of Ψ correspond to the same point of CP_2: at $r = 0$ surfaces also n_1 and ω_1 can change since all values of Φ correspond to same point of CP_2, too. If $r = 0$ or $r = \infty$ is not in the allowed range space-time surface develops a boundary.

This implies what might be called topological quantization since in general it is not possible to find a smooth global embedding for, say a constant magnetic field. Although global embedding exists it decomposes into regions with different values of the vacuum parameters and the coordinate u in general possesses discontinuous derivative at $r = 0$ and $r = \infty$ surfaces. A possible manner to avoid edges of space-time is to allow field quantization so that 3-space (and field) decomposes into disjoint quanta, which can be regarded as structurally stable units a 3-space (and of the gauge field). This doesn't exclude partial join along boundaries for neighboring field quanta provided some additional conditions guaranteeing the absence of edges are satisfied.

For instance, the vanishing of the electromagnetic fields implies that the condition

$$\Omega \equiv \frac{\omega_2}{n_2} - \frac{\omega_1}{n_1} = 0 \ , \qquad \text{(A-3.9)}$$

is satisfied. In particular, the ratio ω_2/ω_1 is rational number for the electromagnetically neutral regions of space-time surface. The change of the parameter n_1 and n_2 (ω_1 and ω_2) in general generates magnetic field and therefore these integers will be referred to as magnetic (electric) quantum numbers.

A-4 The relationship of TGD to QFT and string models

The recent view of the relationship of TGD to QFT and string models has developed slowly during years and it seems that in a certain sense TGD means a return to roots: instead of QFT like description involving path integral one would have wave mechanics for 3-surfaces.

A-4.1 TGD as a generalization of wave mechanism obtained by replacing point-like particles with 3-surfaces

The first vision of TGD was as a generalization of quantum field theory (string models) obtained by replacing pointlike particles (strings) as fundamental objects with 3-surfaces.

The later work has revealed that TGD could be seen as a generalization of the wave mechanism based on the replacement of a point-like particle with 3-D surface. This is due to holography implied by general coordinate invariance. The definition of the metric of the "world of classical worlds" (WCW) must assign a unique or at least almost unique space-time surface to a given 3-surface. This 4-surface is analogous to Bohr orbit so that also Bohr orbitology becomes an exact part of quantum physics. The failure of strict determinism forces to replace 3-surfaces with 4-surfaces and this leads to zero energy ontology (ZEO) in which quantum states are superpositions of space-time surfaces [K50, K28, K93] [L46, L52].

Fig. 5. TGD replaces point-like particles with 3-surfaces. `http://tgdtheory.fi/appfigures/particletgd.jpg`

A-4.2 Extension of superconformal invariance

The fact that light-like 3-surfaces are effectively metrically 2-dimensional and thus possess generalization of 2-dimensional conformal symmetries with light-like radial coordinate defining the analog of second complex coordinate suggests that this generalization could work and extend the super-conformal symmetries to their 4-D analogs.

The boundary $\delta M^4_+ = S^2 \times R_+$- of 4-D light-cone M^4_+ is also metrically 2-dimensional and allows extended conformal invariance. Also the group of isometries of light-cone boundary and of light-like 3-surfaces is infinite-dimensional since the conformal scalings of S^2 can be compensated by S^2-local scaling of the light-like radial coordinate of R_+. These simple facts mean that 4-dimensional Minkowski space and 4-dimensional space-time surfaces are in a completely unique position as far as symmetries are considered.

In fact, this leads to a generalization of the Kac-Moody type symmetries of string models. $\delta M^4_+ \times CP_2$ allows huge supersymplectic symmetries for which the radial light-like coordinate of δM^4_+ plays the role of complex string coordinate in string models. These symmetries are assumed to act as isometries of WCW.

A-4.3 String-like objects and strings

String like objects obtained as deformations of cosmic strings $X^2 \times Y^2$, where X^2 is minimal surface in M^4 and Y^2 a holomorphic surface of CP_2 are fundamental extremals of Kähler action having string world sheet as M^4 projections. Cosmic strings dominate the primordial cosmology of the TGD Universe and the inflationary period corresponds to the transition to radiation dominated cosmology for which space-time sheets with 4-D M^4 projection dominate.

Also genuine string-like objects emerge from TGD. The conditions that the em charge of modes of induces spinor fields is well-defined requires in the generic case the localization of the modes at 2-D surfaces -string world sheets and possibly also partonic 2-surfaces. This in Minkowskian space-time regions.

Fig. 6. Well-definedness of em charge forces the localization of induced spinor modes to 2-D surfaces in generic situations in Minkowskian regions of space-time surface. `http://tgdtheory.fi/appfigures/fermistring.jpg`

A-4.4 TGD view of elementary particles

The TGD based view about elementary particles has two key aspects.

1. The space-time correlates of elementary particles are identified as pairs of wormhole contacts with Euclidean signature of metric and having 4-D CP_2 projection. Their throats behave effectively as Kähler magnetic monopoles so that wormhole throats must be connected by Kähler magnetic flux tubes with monopole flux so that closed flux tubes are obtained.
2. At the level of H Fermion number is carried by the modes of the induced spinor field. In space-time regions with Minkowski signature the modes are localized at string world sheets connecting the wormhole contacts.

Fig. 7. TGD view about elementary particles. a) Particle orbit corresponds to a 4-D generalization of a world line or b) with its light-like 3-D boundary (holography). c) Particle world lines have Euclidean signature of the induced metric. d) They can be identified as wormhole contacts. e) The throats of wormhole contacts carry effective Kähler magnetic charges so that wormhole contacts must appear as pairs in order to obtain closed flux tubes. f) Wormhole contacts are accompanied by fermionic strings connecting the throats at the same sheet: the strings do not extend inside the wormhole contacts. `http://tgdtheory.fi/appfigures/elparticletgd.jpg`

Particle interactions involve both stringy and QFT aspects.

1. The boundaries of string world sheets correspond to fundamental fermions. This gives rise to massless propagator lines in generalized Feynman diagrammatics. One can speak of "long" string connecting wormhole contacts and having a hadronic string as a physical counterpart. Long strings should be distinguished from wormhole contacts which due to their superconformal invariance behave like "short" strings with length scale given by CP_2 size, which is 10^4 times longer than Planck scale characterizing strings in string models.
2. Wormhole contact defines basic stringy interaction vertex for fermion-fermion scattering. The propagator is essentially the inverse of the superconformal scaling generator L_0. Wormhole contacts containing fermion and antifermion at its opposite throats behave like virtual bosons so that one has BFF type vertices typically.
3. In topological sense one has 3-vertices serving as generalizations of 3-vertices of Feynman diagrams. In these vertices 4-D "lines" of generalized Feynman diagrams meet along their 3-D ends. One obtains also the analogs of stringy diagrams but stringy vertices do not have

the usual interpretation in terms of particle decays but in terms of propagation of particles along two different routes.

Fig. 8. a) TGD analogs of Feynman and string diagrammatics at the level of space-time topology. b) The 4-D analogs of both string diagrams and QFT diagrams appear but the interpretation of the analogs stringy diagrams is different. `http://tgdtheory.fi/appfigures/tgdgraphs.jpg`

A-5 About the selection of the action defining the Kähler function of the "world of classical worlds" (WCW)

The proposal is that space-time surfaces correspond to preferred extremals of some action principle, being analogous to Bohr orbits, so that they are almost deterministic. The action for the preferred extremal would define the Kähler function of WCW [K50, K93].

How unique is the choice of the action defining WCW Kähler metric? The problem is that twistor lift strongly suggests the identification of the preferred extremals as 4-D surfaces having 4-D generalization of complex structure and that a large number of general coordinate invariant actions constructible in terms of the induced geometry have the same preferred extremals.

A-5.1 Could twistor lift fix the choice of the action uniquely?

The twistor lift of TGD [L33] [L46, L48, L49] generalizes the notion of induction to the level of twistor fields and leads to a proposal that the action is obtained by dimensional reduction of the action having as its preferred extremals the counterpart of twistor space of the space-time surface identified as 6-D surface in the product $T(M^4) \times T(CP_2)$ twistor spaces of $T(M^4)$ and $T(CP_2)$ of M^4 and CP_2. Only M^4 and CP_2 allow a twistor space with Kähler structure [A61] so that TGD would be unique. Dimensional reduction is forced by the condition that the 6-surface has S^2-bundle structure characterizing twistor spaces and the base space would be the space-time surface.

1. Dimensional reduction of 6-D Kähler action implies that at the space-time level the fundamental action can be identified as the sum of Kähler action and volume term (cosmological constant). Other choices of the action do not look natural in this picture although they would have the same preferred extremals.
2. Preferred extremals are proposed to correspond to minimal surfaces with singularities such that they are also extremals of 4-D Kähler action outside the singularities. The physical analogue are soap films spanned by frames and one can localize the violation of the strict determinism and of strict holography to the frames.
3. The preferred extremal property is realized as the holomorphicity characterizing string world sheets, which generalizes to the 4-D situation. This in turn implies that the preferred extremals are the same for any general coordinate invariant action defined on the induced gauge fields and induced metric apart from possible extremals with vanishing CP_2 Kähler action.

   For instance, 4-D Kähler action and Weyl action as the sum of the tensor squares of the components of the Weyl tensor of CP_2 representing quaternionic imaginary units constructed from the Weyl tensor of CP_2 as an analog of gauge field would have the same preferred extremals and only the definition of Kähler function and therefore Kähler metric of WCW would change. One can even consider the possibility that the volume term in the 4-D action could be assigned to the tensor square of the induced metric representing a quaternionic or octonionic real unit.

Action principle does not seem to be unique. On the other hand, the WCW Kähler form and metric should be unique since its existence requires maximal isometries.

Unique action is not the only way to achieve this. One cannot exclude the possibility that the Kähler gauge potential of WCW in the complex coordinates of WCW differs only by a complex gradient of a holomorphic function for different actions so that they would give the same Kähler form for WCW. This gradient is induced by a symplectic transformation of WCW inducing a

U(1) gauge transformation. The Kähler metric is the same if the symplectic transformation is an isometry.

Symplectic transformations of WCW could give rise to inequivalent representations of the theory in terms of action at space-time level. Maybe the length scale dependent coupling parameters of an effective action could be interpreted in terms of a choice of WCW Kähler function, which maximally simplifies the computations at a given scale.

1. The 6-D analogues of electroweak action and color action reducing to Kähler action in 4-D case exist. The 6-D analog of Weyl action based on the tensor representation of quaternionic imaginary units does not however exist. One could however consider the possibility that only the base space of twistor space $T(M^4)$ and $T(CP_2)$ have quaternionic structure.

2. Kähler action has a huge vacuum degeneracy, which clearly distinguishes it from other actions. The presence of the volume term removes this degeneracy. However, for minimal surfaces having CP_2 projections, which are Lagrangian manifolds and therefore have a vanishing induced Kähler form, would be preferred extremals according to the proposed definition. For these 4-surfaces, the existence of the generalized complex structure is dubious.

   For the electroweak action, the terms corresponding to charged weak bosons eliminate these extremals and one could argue that electroweak action or its sum with the analogue of color action, also proportional Kähler action, defines the more plausible choice. Interestingly, also the neutral part of electroweak action is proportional to Kähler action.

Twistor lift strongly suggests that also M^4 has the analog of Kähler structure. M^8 must be complexified by adding a commuting imaginary unit i. In the E^8 subspace, the Kähler structure of E^4 is defined in the standard sense and it is proposed that this generalizes to M^4 allowing also generalization of the quaternionic structure. M^4 Kähler structure violates Lorentz invariance but could be realized at the level of moduli space of these structures.

The minimal possibility is that the M^4 Kähler form vanishes: one can have a different representation of the Kähler gauge potential for it obtained as generalization of symplectic transformations acting non-trivially in M^4. The recent picture about the second quantization of spinors of $M^4 \times CP_2$ assumes however non-trivial Kähler structure in M^4.

A-5.2 Two paradoxes

TGD view leads to two apparent paradoxes.

1. If the preferred extremals satisfy 4-D generalization of holomorphicity, a very large set of actions gives rise to the same preferred extremals unless there are some additional conditions restricting the number of preferred extremals for a given action.

2. WCW metric has an infinite number of zero modes, which appear as parameters of the metric but do not contribute to the line element. The induced Kähler form depends on these degrees of freedom. The existence of the Kähler metric requires maximal isometries, which suggests that the Kähler metric is uniquely fixed apart from a conformal scaling factor Ω depending on zero modes. This cannot be true: galaxy and elementary particle cannot correspond to the same Kähler metric.

Number theoretical vision and the hierarchy of inclusions of HFFs associated with supersymplectic algebra actings as isometries of WcW provide equivalent realizations of the measurement resolution. This solves these paradoxes and predicts that WCW decomposes into sectors for which Kähler metrics of WCW differ in a natural way.

The hierarchy subalgebras of supersymplectic algebra implies the decomposition of WCW into sectors with different actions

Supersymplectic algebra of $\delta M^4_+ \times CP_2$ is assumed to act as isometries of WCW [L52]. There are also other important algebras but these will not be discussed now.

1. The symplectic algebra A of $\delta M^4_+ \times CP_2$ has the structure of a conformal algebra in the sense that the radial conformal weights with non-negative real part, which is half integer, label the elements of the algebra have an interpretation as conformal weights.

The super symplectic algebra A has an infinite hierarchy of sub-algebras [L52] such that the conformal weights of sub-algebras $A_{n(SS)}$ are integer multiples of the conformal weights of the entire algebra. The superconformal gauge conditions are weakened. Only the subalgebra $A_{n(SS)}$ and the commutator $[A_{n(SS)}, A]$ annihilate the physical states. Also the corresponding classical Noether charges vanish for allowed space-time surfaces.

This weakening makes sense also for ordinary superconformal algebras and associated Kac-Moody algebras. This hierarchy can be interpreted as a hierarchy symmetry breakings, meaning that sub-algebra $A_{n(SS)}$ acts as genuine dynamical symmetries rather than mere gauge symmetries. It is natural to assume that the super-symplectic algebra A does not affect the coupling parameters of the action.

2. The generators of A correspond to the dynamical quantum degrees of freedom and leave the induced Kähler form invariant. They affect the induced space-time metric but this effect is gravitational and very small for Einsteinian space-time surfaces with 4-D M^4 projection.

   The number of dynamical degrees of freedom increases with $n(SS)$. Therefore WCW decomposes into sectors labelled by $n(SS)$ with different numbers of dynamical degrees of freedom so that their Kähler metrics cannot be equivalent and cannot be related by a symplectic isometry. They can correspond to different actions.

Number theoretic vision implies the decomposition of WCW into sectors with different actions

The number theoretical vision leads to the same conclusion as the hierarchy of HFFs. The number theoretic vision of TGD based on M^8-H duality [L52] predicts a hierarchy with levels labelled by the degrees $n(P)$ of rational polynomials P and corresponding extensions of rationals characterized by Galois groups and by ramified primes defining p-adic length scales.

These sequences allow us to imagine several discrete coupling constant evolutions realized at the level H in terms of action whose coupling parameters depend on the number theoretic parameters.

1. Coupling constant evolution with respect to $n(P)$

The first coupling constant evolution would be with respect to $n(P)$.

1. The coupling constants characterizing action could depend on the degree $n(P)$ of the polynomial defining the space-time region by M^8-H duality. The complexity of the space-time surface would increase with $n(P)$ and new degrees of freedom would emerge as the number of the rational coefficients of P.

2. This coupling constant evolution could naturally correspond to that assignable to the inclusion hierarchy of hyperfinite factors of type II_1 (HFFs). I have indeed proposed [L52] that the degree $n(P)$ equals to the number $n(braid)$ of braids assignable to HFF for which super symplectic algebra subalgebra $A_{n(SS)}$ with radial conformal weights coming as $n(SS)$-multiples of those of entire algebra A. One would have $n(P) = n(braid) = n(SS)$. The number of dynamical degrees of freedom increases with n which just as it increases with $n(P)$ and $n(SS)$.

3. The actions related to different values of $n(P) = n(braid) = n(SS)$ cannot define the same Kähler metric since the number of allowed space-time surfaces depends on $n(SS)$.

   WCW could decompose to sub-WCWs corresponding to different actions, a kind of theory space. These theories would not be equivalent. A possible interpretation would be as a hierarchy of effective field theories.

4. Hierarchies of composite polynomials define sequences of polynomials with increasing values of $n(P)$ such that the order of a polynomial at a given level is divided by those at the lower levels. The proposal is that the inclusion sequences of extensions are realized at quantum level as inclusion hierarchies of hyperfinite factors of type II_1.

   A given inclusion hierarchy corresponds to a sequence $n(SS)_i$ such that $n(SS)_i$ divides $n(SS)_{i+1}$. Therefore the degree of the composite polynomials increases very rapidly. The values of $n(SS)_i$ can be chosen to be primes and these primes correspond to the degrees of so called prime polynomials [L50] so that the decompositions correspond to prime factorizations of integers. The "densest" sequence of this kind would come in powers of 2 as $n(SS)_i = 2^i$. The corresponding p-adic length scales (assignable to maximal ramified primes for given $n(SS)_i$) are expected to increase roughly exponentially, say as 2^{r2^i}. $r = 1/2$ would give a subset of scales $2^{r/2}$ allowed by the p-adic length scale hypothesis. These transitions would be very rare.

A theory corresponding to a given composite polynomial would contain as sub-theories the theories corresponding to lower polynomial composites. The evolution with respect to $n(SS)$ would correspond to a sequence of phase transitions in which the action genuinely changes. For instance, color confinement could be seen as an example of this phase transition.

5. A subset of p-adic primes allowed by the p-adic length scale hypothesis $p \simeq 2^k$ defining the proposed p-adic length scale hierarchy could relate to n_S changing phase transition. TGD suggests a hierarchy of hadron physics corresponding to a scale hierarchy defined by Mersenne primes and their Gaussian counterparts [K66, K67]). Each of them would be characterized by a confinement phase transition in which n_S and therefore also the action changes.

2. Coupling constant evolutions with respect to ramified primes for a given value of $n(P)$

For a given value of $n(P)$, one could have coupling constant sub-evolutions with respect to the set of ramified primes of P and dimensions $n = h_{eff}/h_0$ of algebraic extensions. The action would only change by U(1) gauge transformation induced by a symplectic isometry of WCW. Coupling parameters could change but the actions would be equivalent.

The choice of the action in an optimal manner in a given scale could be seen as a choice of the most appropriate effective field theory in which radiative corrections would be taken into account. One can interpret the possibility to use a single choice of coupling parameters in terms of quantum criticality.

The range of the p-adic length scales labelled by ramified primes and effective Planck constants h_{eff}/h_0 is finite for a given value of $n(SS)$.

The first coupling constant evolution of this kind corresponds to ramified primes defining p-adic length scales for given $n(SS)$.

1. Ramified primes are factors of the discriminant $D(P)$ of P, which is expressible as a product of non-vanishing root differents and reduces to a polynomial of the n coefficients of P. Ramified primes define p-adic length scales assignable to the particles in the amplitudes scattering amplitudes defined by zero energy states.

   P would represent the space-time surface defining an interaction region in $N-$-particle scattering. The N ramified primes dividing $D(P)$ would characterize the p-adic length scales assignable to these particles. If $D(P)$ reduces to a single ramified prime, one has elementary particle [L50], and the forward scattering amplitude corresponds to the propagator.

   This would give rise to a multi-scale p-adic length scale evolution of the amplitudes analogous to the ordinary continuous coupling constant evolution of n-point scattering amplitudes with respect to momentum scales of the particles. This kind of evolutions extend also to evolutions with respect to $n(SS)$.

2. According to [L50], physical constraints require that $n(P)$ and the maximum size of the ramified prime of P correlate.

   A given rational polynomial of degree $n(P)$ can be always transformed to a polynomial with integer coefficients. If the integer coefficients are smaller than $n(P)$, there is an upper bound for the ramified primes. This assumption also implies that finite fields become fundamental number fields in number theoretical vision [L50].

3. p-Adic length scale hypothesis [L53] in its basic form states that there exist preferred primes $p \simeq 2^k$ near some powers of 2. A more general hypothesis states that also primes near some powers of 3 possibly also other small primes are preferred physically. The challenge is to understand the origin of these preferred scales.

   For polynomials P with a given degree $n(P)$ for which discriminant $D(P)$ is prime, there exists a maximal ramified prime. Numerical calculations suggest that the upper bound depends exponentially on $n(P)$.

Could these maximal ramified primes satisfy the p-adic length scale hypothesis or its generalization? The maximal prime defines a fixed point of coupling constant evolution in accordance with the earlier proposal. For instance, could one think that one has $p \simeq 2^k$, $k = n(SS)$? Each p-adic prime would correspond to a p-adic coupling constant sub-evolution representable in terms of symplectic isometries.

Also the dimension n of the algebraic extension associated with P, which is identified in terms of effective Planck constant $h_{eff}/h_0 = n$ labelling different phases of the ordinary matter behaving like dark matter, could give rise to coupling constant evolution for given $n(SS)$. The range of allowed values of n is finite. Note however that several polynomials of a given degree can correspond to the same dimension of extension.

Number theoretic discretization of WCW and maxima of WCW Kähler function

Number theoretic approach involves a unique discretization of space-time surface and also of WCW. The question is how the points of the discretized WCW correspond to the preferred extremals.

1. The exponents of Kähler function for the maxima of Kähler function, which correspond to the universal preferred extremals, appear in the scattering amplitudes. The number theoretical approach involves a unique discretization of space-time surfaces defining the WCW coordinates of the space-time surface regarded as a point of WCW.

   In [L52] it is assumed that these WCW points appearing in the number theoretical discretization correspond to the maxima of the Kähler function. The maxima would depend on the action and would differ for ghd maxima associated with different actions unless they are not related by symplectic WCW isometry.

2. The symplectic transformations of WCW acting as isometries are assumed to be induced by the symplectic transformations of $\delta M^4_+ \times CP_2$ [K50, K28]. As isometries they would naturally permute the maxima with each other.

A-6 Number theoretic vision of TGD

Physics as number theory vision is complementary to the physics as geometry vision and has developed gradually since 1993. Langlands program is the counterpart of this vision in mathematics [L51].

The notion of p-adic number fields emerged with the motivation coming from the observation that elementary particle mass scales and mass ratios could be understood in terms of the so-called p-adic length scale hypothesis [K70, K59, K25]. The fusion of the various p-adic physics leads to what I call adelic physics [L31, L30]. Later the hypothesis about hierarchy of Planck constants labelling phases of ordinary matter behaving like dark matter emerged [K31, K32, K33, K33].

Eventually this led to that the values of effective Planck constant could be identified as the dimension of an algebraic extension of rationals assignable to polynomials with rational coefficients. This led to the number theoretic vision in which so-called $M^8 - H$ duality [L42, L43] plays a key role. M^8 (actually a complexification of real M^8) is analogous to momentum space so that the duality generalizes momentum position duality for point-like particles. M^8 has an interpretation as complexified octonions.

The dynamics of 4-surfaces in M^8 is coded by polynomials with rational coefficients, whose roots define mass shells H^3 of $M^4 \subset M^8$. It has turned out that the polynomials satisfy stringent additional conditions and one can speak of number theoretic holography [L50, L51]. Also the ordinary 3→ 4 holography is needed to assign 4-surfaces with these 3-D mass shells. The number theoretic dynamics is based on the condition that the normal space of the 4-surface in M^8 is associative (quaternionic) and contains a commutative complex sub-space. This makes it possible to assign to this surface space-time surface in $H = M^4 \times CP_2$.

At the level of H the space-time surfaces are by holography preferred extremals and are assumed to be determined by the twistor lift of TGD [L33] giving rise to an action which is sum of the Kähler action and volume term. The preferred extremals would be minimal surfaces analogous to soap films spanned by frames. Outside frames they would be simultaneous extremals of the Kähler action, which requires a generalization of the holomorphy characterizing string world sheets.

In the following only p-adic numbers and hierarchy of Planck constants will be discussed.

A-6.1 p-Adic numbers and TGD

p-Adic number fields

p-Adic numbers (p is prime: 2, 3, 5, ...) can be regarded as a completion of the rational numbers using a norm, which is different from the ordinary norm of real numbers [A37]. p-Adic numbers are representable as power expansion of the prime number p of form

$$x = \sum_{k\geq k_0} x(k)p^k,\ x(k) = 0,....,p-1 \ . \quad \text{(A-6.1)}$$

The norm of a p-adic number is given by

$$|x| = p^{-k_0(x)} \ . \quad \text{(A-6.2)}$$

Here $k_0(x)$ is the lowest power in the expansion of the p-adic number. The norm differs drastically from the norm of the ordinary real numbers since it depends on the lowest pinary digit of the p-adic number only. Arbitrarily high powers in the expansion are possible since the norm of the p-adic number is finite also for numbers, which are infinite with respect to the ordinary norm. A convenient representation for p-adic numbers is in the form

$$x = p^{k_0}\varepsilon(x) \ , \quad \text{(A-6.3)}$$

where $\varepsilon(x) = k + ...$ with $0 < k < p$, is p-adic number with unit norm and analogous to the phase factor $exp(i\phi)$ of a complex number.

The distance function $d(x, y) = |x - y|_p$ defined by the p-adic norm possesses a very general property called ultra-metricity:

$$d(x, z) \leq max\{d(x, y), d(y, z)\} \ . \quad \text{(A-6.4)}$$

The properties of the distance function make it possible to decompose R_p into a union of disjoint sets using the criterion that x and y belong to same class if the distance between x and y satisfies the condition

$$d(x, y) \leq D \ . \quad \text{(A-6.5)}$$

This division of the metric space into classes has following properties:

1. Distances between the members of two different classes X and Y do not depend on the choice of points x and y inside classes. One can therefore speak about distance function between classes.
2. Distances of points x and y inside single class are smaller than distances between different classes.
3. Classes form a hierarchical tree.

Notice that the concept of the ultra-metricity emerged in physics from the models for spin glasses and is believed to have also applications in biology [B31]. The emergence of p-adic topology as the topology of the effective space-time would make ultra-metricity property basic feature of physics.

Canonical correspondence between p-adic and real numbers

The basic challenge encountered by p-adic physicist is how to map the predictions of the p-adic physics to real numbers. p-Adic probabilities provide a basic example in this respect. Identification via common rationals and canonical identification and its variants have turned out to play a key role in this respect.

1. Basic form of the canonical identification

There exists a natural continuous map $I : R_p \rightarrow R_+$ from p-adic numbers to non-negative real numbers given by the "pinary" expansion of the real number for $x \in R$ and $y \in R_p$ this correspondence reads

$$\begin{aligned} y &= \sum_{k>N} y_k p^k \rightarrow x = \sum_{k<N} y_k p^{-k} \quad , \\ y_k &\in \{0, 1, .., p-1\} \quad . \end{aligned} \tag{A-6.6}$$

This map is continuous as one easily finds out. There is however a little difficulty associated with the definition of the inverse map since the pinary expansion like also decimal expansion is not unique ($1 = 0.999...$) for the real numbers x, which allow pinary expansion with finite number of pinary digits

$$\begin{aligned} x &= \sum_{k=N_0}^{N} x_k p^{-k} \quad , \\ x &= \sum_{k=N_0}^{N-1} x_k p^{-k} + (x_N - 1)p^{-N} + (p-1)p^{-N-1} \sum_{k=0,..} p^{-k} \quad . \end{aligned} \tag{A-6.7}$$

The p-adic images associated with these expansions are different

$$\begin{aligned} y_1 &= \sum_{k=N_0}^{N} x_k p^k \quad , \\ y_2 &= \sum_{k=N_0}^{N-1} x_k p^k + (x_N - 1)p^N + (p-1)p^{N+1} \sum_{k=0,..} p^k \\ &= y_1 + (x_N - 1)p^N - p^{N+1} \quad , \end{aligned} \tag{A-6.8}$$

so that the inverse map is either two-valued for p-adic numbers having expansion with finite pinary digits or single valued and discontinuous and non-surjective if one makes pinary expansion unique by choosing the one with finite pinary digits. The finite pinary digit expansion is a natural choice since in the numerical work one always must use a pinary cutoff on the real axis.

2. The topology induced by canonical identification

The topology induced by the canonical identification in the set of positive real numbers differs from the ordinary topology. The difference is easily understood by interpreting the p-adic norm as a norm in the set of the real numbers. The norm is constant in each interval $[p^k, p^{k+1})$ (see **Fig. A-6.1**) and is equal to the usual real norm at the points $x = p^k$: the usual linear norm is replaced with a piecewise constant norm. This means that p-adic topology is coarser than the usual real topology and the higher the value of p is, the coarser the resulting topology is above a given length scale. This hierarchical ordering of the p-adic topologies will be a central feature as far as the proposed applications of the p-adic numbers are considered.

Ordinary continuity implies p-adic continuity since the norm induced from the p-adic topology is rougher than the ordinary norm. p-Adic continuity implies ordinary continuity from right as is clear already from the properties of the p-adic norm (the graph of the norm is indeed continuous from right). This feature is one clear signature of the p-adic topology.

Fig. 14. The real norm induced by canonical identification from 2-adic norm. `http://tgdtheory.fi/appfigures/norm.png`

The linear structure of the p-adic numbers induces a corresponding structure in the set of the non-negative real numbers and p-adic linearity in general differs from the ordinary concept of linearity. For example, p-adic sum is equal to real sum only provided the summands have no common pinary digits. Furthermore, the condition $x +_p y < max\{x, y\}$ holds in general for the p-adic sum of the real numbers. p-Adic multiplication is equivalent with the ordinary multiplication only provided that either of the members of the product is power of p. Moreover one has $x \times_p y < x \times y$ in general. The p-Adic negative -1_p associated with p-adic unit 1 is given by $(-1)_p = \sum_k (p-1)p^k$ and defines p-adic negative for each real number x. An interesting possibility is that p-adic linearity might replace the ordinary linearity in some strongly nonlinear systems so these systems would look simple in the p-adic topology.

These results suggest that canonical identification is involved with some deeper mathematical structure. The following inequalities hold true:

$$\begin{aligned} (x+y)_R &\leq x_R + y_R \ , \\ |x|_p|y|_R \leq (xy)_R &\leq x_R y_R \ , \end{aligned} \tag{A-6.9}$$

where $|x|_p$ denotes p-adic norm. These inequalities can be generalized to the case of $(R_p)^n$ (a linear vector space over the p-adic numbers).

$$\begin{aligned} (x+y)_R &\leq x_R + y_R \ , \\ |\lambda|_p|y|_R \leq (\lambda y)_R &\leq \lambda_R y_R \ , \end{aligned} \tag{A-6.10}$$

where the norm of the vector $x \in T_p^n$ is defined in some manner. The case of Euclidian space suggests the definition

$$(x_R)^2 = (\sum_n x_n^2)_R \ . \tag{A-6.11}$$

These inequalities resemble those satisfied by the vector norm. The only difference is the failure of linearity in the sense that the norm of a scaled vector is not obtained by scaling the norm of the original vector. Ordinary situation prevails only if the scaling corresponds to a power of p.

These observations suggests that the concept of a normed space or Banach space might have a generalization and physically the generalization might apply to the description of some non-linear systems. The nonlinearity would be concentrated in the nonlinear behavior of the norm under scaling.

3. Modified form of the canonical identification

The original form of the canonical identification is continuous but does not respect symmetries even approximately. This led to a search of variants which would do better in this respect. The modification of the canonical identification applying to rationals only and given by

$$I_Q(q = p^k \times \frac{r}{s}) = p^k \times \frac{I(r)}{I(s)} \tag{A-6.12}$$

is uniquely defined for rationals, maps rationals to rationals, has also a symmetry under exchange of target and domain. This map reduces to a direct identification of rationals for $0 \leq r < p$ and $0 \leq s < p$. It has turned out that it is this map which most naturally appears in the applications. The map is obviously continuous locally since p-adically small modifications of r and s mean small modifications of the real counterparts.

Canonical identification is in a key role in the successful predictions of the elementary particle masses. The predictions for the light elementary particle masses are within extreme accuracy same for I and I_Q but I_Q is theoretically preferred since the real probabilities obtained from p-adic ones by I_Q sum up to one in p-adic thermodynamics.

4. Generalization of number concept and notion of embedding space

TGD forces an extension of number concept: roughly a fusion of reals and various p-adic number fields along common rationals is in question. This induces a similar fusion of real and p-adic embedding spaces. Since finite p-adic numbers correspond always to non-negative reals n-dimensional space R^n must be covered by 2^n copies of the p-adic variant R_p^n of R^n each of which projects to a copy of R_+^n (four quadrants in the case of plane). The common points of p-adic and real embedding spaces are rational points and most p-adic points are at real infinity.

Real numbers and various algebraic extensions of p-adic number fields are thus glued together along common rationals and also numbers in algebraic extension of rationals whose number belong to the algebraic extension of p-adic numbers. This gives rise to a book like structure with rationals and various algebraic extensions of rationals taking the role of the back of the book. Note that Neper number is exceptional in the sense that it is algebraic number in p-adic number field Q_p satisfying $e^p\ mod\ p = 1$.

Fig. 15. Various number fields combine to form a book like structure. `http://tgdtheory.fi/appfigures/book.jpg`

For a given p-adic space-time sheet most points are literally infinite as real points and the projection to the real embedding space consists of a discrete set of rational points: the interpretation in terms of the unavoidable discreteness of the physical representations of cognition is natural. Purely local p-adic physics implies real p-adic fractality and thus long range correlations for the real space-time surfaces having enough common points with this projection.

p-Adic fractality means that M^4 projections for the rational points of space-time surface X^4 are related by a direct identification whereas CP_2 coordinates of X^4 at these points are related by I, I_Q or some of its variants implying long range correlates for CP_2 coordinates. Since only a discrete set of points are related in this manner, both real and p-adic field equations can be satisfied and there are no problems with symmetries. p-Adic effective topology is expected to be a good approximation only within some length scale range which means infrared and UV cutoffs. Also multi-p-fractality is possible.

The notion of p-adic manifold

The notion of p-adic manifold is needed in order to fuse real physics and various p-adic physics to a larger structure which suggests that real and p-adic number fields should be glued together along common rationals bringing in mind adeles. The notion is problematic because p-adic topology is totally disconnected implying that p-adic balls are either disjoint or nested so that ordinary definition of manifold using p-adic chart maps fails. A cure is suggested to be based on chart maps from p-adics to reals rather than to p-adics (see the appendix of the book)

The chart maps are interpreted as cognitive maps, "thought bubbles".

Fig. 16. The basic idea between p-adic manifold. `http://tgdtheory.fi/appfigures/padmanifold.jpg`

There are some problems.

1. Canonical identification does not respect symmetries since it does not commute with second pinary cutoff so that only a discrete set of rational points is mapped to their real counterparts by chart map arithmetic operations which requires pinary cutoff below which chart map takes rationals to rationals so that commutativity with arithmetics and symmetries is achieved in finite resolution: above the cutoff canonical identification is used

2. Canonical identification is continuous but does not map smooth p-adic surfaces to smooth real surfaces requiring second pinary cutoff so that only a discrete set of rational points is mapped to their real counterparts by chart map requiring completion of the image to smooth preferred extremal of Kähler action so that chart map is not unique in accordance with finite measurement resolution

3. Canonical identification violates general coordinate invariance of chart map: (cognition-induced symmetry breaking) minimized if p-adic manifold structure is induced from that for p-adic embedding space with chart maps to real embedding space and assuming preferred coordinates made possible by isometries of embedding space: one however obtains several in-equivalent p-adic manifold structures depending on the choice of coordinates: these cognitive representations are not equivalent.

A-6.2 Hierarchy of Planck constants and dark matter hierarchy

Hierarchy of Planck constants was motivated by the "impossible" quantal effects of ELF em fields on vertebrate cyclotron energies $E = hf = \hbar \times eB/m$ are above thermal energy is possible only if $\hbar$ has value much larger than its standard value. Also Nottale's finding that planetary orbits migh be understood as Bohr orbits for a gigantic gravitational Planck constant.

Hierachy of Planck constant would mean that the values of Planck constant come as integer multiples of ordinary Planck constant: $h_{eff} = n \times h$. The particles at magnetic flux tubes characterized by h_{eff} would correspond to dark matter which would be invisible in the sense that only particle with same value of h_{eff} appear in the same vertex of Feynman diagram.

Hierarchy of Planck constants would be due to the non-determism of the Kähler action predicting huge vacuum degeneracy allowing all space-time surfaces which are sub-manfolds of any $M^4 \times Y^2$, where Y^2 is Lagrangian sub-manifold of CP_2. For agiven Y^2 one obtains new manifolds Y^2 by applying symplectic transformations of CP_2.

Non-determinism would mean that the 3-surface at the ends of causal diamond (CD) can be connected by several space-time surfaces carrying same conserved Kähler charges and having same values of Kähler action. Conformal symmetries defined by Kac-Moody algebra associated with the embedding space isometries could act as gauge transformations and respect the light-likeness property of partonic orbits at which the signature of the induced metric changes from Minkowskian to Euclidian (Minkowskianb space-time region transforms to wormhole contact say). The number of conformal equivalence classes of these surfaces could be finite number n and define discrete physical degree of freedom and one would have $h_{eff} = n \times h$. This degeneracy would mean "second quantization" for the sheets of n-furcation: not only one but several sheets can be realized.

This relates also to quantum criticality postulated to be the basic characteristics of the dynamics of quantum TGD. Quantum criticalities would correspond to an infinite fractal hierarchy of broken conformal symmetries defined by sub-algebras of conformal algebra with conformal weights coming as integer multiples of n. This leads also to connections with quantum criticality and hierarchy of broken conformal symmetries, p-adicity, and negentropic entanglement which by consistency with standard quantum measurement theory would be described in terms of density matrix proportional $n \times n$ identity matrix and being due to unitary entanglement coefficients (typical for quantum computing systems).

Formally the situation could be described by regarding space-time surfaces as surfaces in singular n-fold singular coverings of embedding space. A stronger assumption would be that they are expressible as as products of n_1 -fold covering of M^4 and n_2-fold covering of CP_2 meaning analogy with multi-sheeted Riemann surfaces and that M^4 coordinates are n_1-valued functions and CP_2 coordinates n_2 -valued functions of space-time coordinates for $n = n_1 \times n_2$. These singular coverings of embedding space form a book like structure with singularities of the coverings localizable at the boundaries of causal diamonds defining the back of the book like structure.

Fig. 17. Hierarchy of Planck constants. http://tgdtheory.fi/appfigures/planckhierarchy.jpg

A-6.3 $M^8 - H$ duality as it is towards the end of 2021

The view of $M^8 - H$ duality (see Appendix **??**) has changed considerably towards the end 2021 [L46] after the realization that this duality is the TGD counterpart of momentum position duality of wave mechanics, which is lost in QFTs. Therefore M^8 and also space-time surface is analogous to momentum space. This forced us to give up the original simple identification of the points $M^4 \subset M^4 \times E^4 = M^8$ and of $M^4 \times CP_2$ so that it respects Uncertainty Principle (UP).

The first improved guess for the duality map was the replacement with the inversion $p^k \rightarrow m^k = \hbar_{eff} p^k/p^2$ conforming in spirit with UP but turned out to be too naive.

The improved form [L46] of the $M^8 - H$ duality map takes mass shells $p^2 = m^2$ of $M^4 \subset M^8$ to cds with size $L(m) = \hbar_{eff}/m$ with a common center. The slicing by mass shells is mapped to a Russian doll like slicing by cds. Therefore would be no CDs in M^8 contrary to what I believed first.

Quantum classical correspondence (QCC) inspires the proposal that the point $p^k \in M^8$ is mapped to a geodesic line corresponding to momentum p^k starting from the common center of cds. Its intersection with the opposite boundary of cd with size $L(m)$ defines the image point. This is not yet quite enough to satisfy UP but the additional details [L46] are not needed in the sequel.

The 6-D brane-like special solutions in M^8 are of special interest in the TGD inspired theory of consciousness. They have an M^4 projection which is $E = E_n$ 3-ball. Here E_n is a root of the real polynomial P defining $X^4 \subset M^8_c$ (M^8 is complexified to M^8_c) as a "root" of its octonionic continuation [L42, L43]. E_n has an interpretation as energy, which can be complex. The original interpretation was as moment of time. For this interpretation, $M^8 - H$ duality would be a linear identification and these hyper planes would be mapped to hyperplanes in $M^4 \subset H$. This motivated the term "very special moment in the life of self" for the image of the $E = E_n$ section of $X^4 \subset M^8$ [L39]. This notion does not make sense at the level M^8 anymore.

The modified $M^8 - H$ duality forces us to modify the original interpretation [L46]. The point $(E_n, p = 0)$ is mapped $(t_n = \hbar_{eff}/E_n, 0)$. The momenta (E_n, p) in $E = E_n$ plane are mapped to the boundary of cd and correspond to a continuous time interval at the boundary of CD: "very special moment" becomes a "very special time interval".

The quantum state however corresponds to a set of points corresponding to quark momenta, which belong to a cognitive representation and are therefore algebraic integers in the extension determined by the polynomial. These active points in E_n are mapped to a discrete set at the boundary of cd(m). A "very special moment" is replaced with a sequence of "very special moments".

So called Galois confinement [L44] forces the total momenta for bound states of quarks and antiquarks to be rational integers invariant under Galois group of extension of rationals determined by the polynomial P [L46]. These states correspond to states at boundaries of sub-CDs so that one obtains a hierarchy. Galois confinement provides a universal number theoretic mechanism for the formation of bound states.

A-7 Zero energy ontology (ZEO)

ZEO is implied by the holography forced in the TGD framework by general coordinate invariance.

A-7.1 Basic motivations and ideas of ZEO

The following gives a brief summary of ZEO [L41] [K124].

1. In ZEO quantum states are not 3-dimensional but superpositions of 4-dimensional deterministic time evolutions connecting ordinary initial 3-dimensional states. By holography they are equivalent to pairs of ordinary 3-D states identified as initial and final states of time evolution. One can say that in the TGD framework general coordinate invariance implies holography and the slight failure of its determinism in turn forces ZEO.

   Quantum jumps replace this state with a new one: a superposition of deterministic time evolutions is replaced with a new superposition. Classical determinism of individual time evolution is not violated and this solves the basic paradox of quantum measurement theory. There are two kinds of quantum jumps: ordinary ("big") state function reductions (BSFRs) changing the arrow of time and "small" state function reductions (SSFRs) (weak measurements) preserving it and giving rise to the analog of Zeno effect [L41].

2. To avoid getting totally confused it is good to emphasize some aspects of ZEO.

   (a) ZEO does not mean that physical states in the usual 3-D sense as snapshots of time evolution would have zero energy state pairs defining zero energy states as initial and final states have same conserved quantities such as energy. Conservation implies that one can adopt the conventions that the values of conserved quantities are opposite for these states so that their sum vanishes: one can think that incoming and outgoing particles come from geometric past and future is the picture used in quantum field theories.

(b) ZEO means two times: subjective time as sequence of quantum jumps and geometric time as space-time coordinate. These times are identifiable but are strongly correlated.

3. In BSFRs the arrow of time is changed and the time evolution in the final state occurs backwards with respect to the time of the external observer. BSFRs can occur in all scales since TGD predicts a hierarchy of effective Planck constants with arbitrarily large values. There is empirical support for BSFRs.

   (a) The findings of Minev et al [L37] in atomic scale can be explained by the same mechanism [L37]. In BSFR a final zero energy state as a superposition of classical deterministic time evolutions emerges and for an observer with a standard arrow of time looks like a superposition of deterministic smooth time evolutions leading to the final state. Interestingly, once this evolution has started, it cannot be stopped unless one changes the stimulus signal inducing the evolution in which case the process does not lead to anywhere: the interpretation would be that BSFR back to the initial state occurs!

   (b) Libets' experiments about active aspects of consciousness [J2] can be understood. Subject person raises his finger and neural activity starts before the conscious decision to do so. In the physicalistic framework it is thought to lead to raising of the finger. The problem with the explanation is that the activity beginning .5 seconds earlier seems to be dissipation with a reversed arrow of time: from chaotic and disordered to ordered at around .15 seconds. ZEO explanation is that macroscopic quantum jump occurred and generated a signal proceeding backwards in time and generated neural activity and dissipated to randomness.

   (c) Earthquakes involve a strange anomaly: they are preceded by ELF radiation. One would expect that they generate ELF radiation. The identification as BSFR would explain the anomaly [L38]. In biology the reversal of the arrow of time would occur routinely and be a central element of biological self-organization, in particular self-organized quantum criticality (see [L40, L56].

A-7.2 Some implications of ZEO

ZEO has profound implications for understanding self-organization and self-organized quantum criticality in terms of dissipation with non-standard arrow of time looking like generation of structures [L40, L56]. ZEO could also allow understanding of what planned actions - like realizing the experiment under consideration - could be.

1. Second law in the standard sense does not favor - perhaps even not allow - realization of planned actions. ZEO forces a generalization of thermodynamics: dissipation with a non-standard arrow of time for a subsystem would look like self-organization and planned action and its realization.

   Could most if not all planned action be like this - induced by BSFR in the geometric future and only apparently planned? There would be however the experience of planning and realizing induced by the signals from geometric future by a higher level in the hierarchy of conscious entities predicted by TGD! In long time scales we would be realizing our fates or wishes of higher level conscious entities rather than agents with completely free will.

2. The notion of magnetic body (MB) serving as a boss of ordinary matter would be central. MB carries dark matter as $h_{eff} = nh_0$ phases of ordinary matter with n serving as a measure for algebraic complexity of extension of rationals as its dimension and defining a kind of universal IQ. There is a hierarchy of these phases and MBs labelled by extension of rationals and the value of n.

   MBs would form a hierarchy of bosses - a realization for master slave hierarchy. Ordinary matter would be at the bottom and its coherent behavior would be induced from quantum coherence at higher levels. BSFR for higher level MB would give rise to what looks like planned actions and experienced as planned action at the lower levels of hierarchy. One could speak of planned actions inducing a cascade of planned actions in shorter time scales and eventually proceeding to atomic level.

A-8 Some notions relevant to TGD inspired consciousness and quantum biology

Below some notions relevant to TGD inspired theory of consciousness and quantum biology.

A-8.1 The notion of magnetic body

Topological field quantization inspires the notion of field body about which magnetic body is especially important example and plays key role in TGD inspired quantum biology and consciousness theory. This is a crucial departure fromt the Maxwellian view. Magnetic body brings in third level to the description of living system as a system interacting strongly with environment. Magnetic body would serve as an intentional agent using biological body as a motor instrument and sensory receptor. EEG would communicated the information from biological body to magnetic body and Libet's findings from time delays of consciousness support this view.

The following pictures illustrate the notion of magnetic body and its dynamics relevant for quantum biology in TGD Universe.

Fig. 18. Magnetic body associated with dipole field. `http://tgdtheory.fi/appfigures/fluxquant.jpg`

Fig. 19. Illustration of the reconnection by magnetic flux loops. `http://tgdtheory.fi/appfigures/reconnect1.jpg`

Fig. 20. Illustration of the reconnection by flux tubes connecting pairs of molecules. `http://tgdtheory.fi/appfigures/reconect2.jpg`

Fig. 21. Flux tube dynamics. a) Reconnection making possible magnetic body to "recognize" the presence of another magnetic body, b) braiding, knotting and linking of flux tubes making possible topological quantum computation, c) contraction of flux tube in phase transition reducing the value of h_{eff} allowing two molecules to find each other in dense molecular soup. `http://tgdtheory.fi/appfigures/fluxtubedynamics.jpg`

A-8.2 Number theoretic entropy and negentropic entanglement

TGD inspired theory of consciousness relies heavily p-Adic norm allows an to define the notion of Shannon entropy for rational probabilities (and even those in algebraic extension of rationals) by replacing the argument of logarithm of probability with its p-adic norm. The resulting entropy can be negative and the interpretation is that number theoretic entanglement entropy defined by this formula for the p-adic prime minimizing its value serves as a measure for conscious information. This negentropy characterizes two-particle system and has nothing to do with the formal negative negentropy assignable to thermodynamic entropy characterizing single particle. Negentropy Maximization Principle (NMP) implies that number theoretic negentropy increases during evolution by quantum jumps. The condition that NMP is consistent with the standard quantum measurement theory requires that negentropic entanglement has a density matrix proportional to unit matrix so that in 2-particle case the entanglement matrix is unitary.

Fig. 22. Schrödinger cat is neither dead or alive. For negentropic entanglement this state would be stable. `http://tgdtheory.fi/appfigures/cat.jpg`

A-8.3 Life as something residing in the intersection of reality and p-adicities

In TGD inspired theory of consciousness p-adic space-time sheets correspond to space-time correlates for thoughts and intentions. The intersections of real and p-adic preferred extremals consist of points whose coordinates are rational or belong to some extension of rational numbers in preferred embedding space coordinates. They would correspond to the intersection of reality and various p-adicities representing the "mind stuff" of Descartes. There is temptation to assign life to the intersection of realities and p-adicities. The discretization of the chart map assigning to real space-time surface its p-adic counterpart would reflect finite cognitive resolution.

At the level of "world of classical worlds" (WCW) the intersection of reality and various p-adicities would correspond to space-time surfaces (or possibly partonic 2-surfaces) representable in terms of rational functions with polynomial coefficients with are rational or belong to algebraic extension of rationals.

The quantum jump replacing real space-time sheet with p-adic one (vice versa) would correspond to a buildup of cognitive representation (realization of intentional action).

Fig. 23. The quantum jump replacing real space-time surface with corresponding p-adic manifold can be interpreted as formation of though, cognitive representation. Its reversal would correspond to a transformation of intention to action. `http://tgdtheory.fi/appfigures/padictoreal.jpg`

A-8.4 Sharing of mental images

The 3-surfaces serving as correlates for sub-selves can topologically condense to disjoint large space-time sheets representing selves. These 3-surfaces can also have flux tube connections and this makes possible entanglement of sub-selves, which unentangled in the resolution defined by the size of sub-selves. The interpretation for this negentropic entanglement would be in terms of sharing of mental images. This would mean that contents of consciousness are not completely private as assumed in neuroscience.

Fig. 24. Sharing of mental images by entanglement of subselves made possible by flux tube connections between topologically condensed space-time sheets associated with mental images. `http://tgdtheory.fi/appfigures/sharing.jpg`

A-8.5 Time mirror mechanism

Zero energy ontology (ZEO) is crucial part of both TGD and TGD inspired consciousness and leads to the understanding of the relationship between geometric time and experience time and how the arrow of psychological time emerges. One of the basic predictions is the possibiity of negative energy signals propagating backwards in geometric time and having the property that entropy basically associated with subjective time grows in reversed direction of geometric time. Negative energy signals inspire time mirror mechanism (see **Fig.** `http://tgdtheory.fi/appfigures/timemirror.jpg` or **Fig.** 24 in the appendix of this book) providing mechanisms of both memory recall, realization of intentational action initiating action already in geometric past, and remote metabolism. What happens that negative energy signal travels to past and is reflected as positive energy signal and returns to the sender. This process works also in the reverse time direction.

Fig. 25. Zero energy ontology allows time mirror mechanism as a mechanism of memory recall. Essentially "seeing" in time direction is in question. `http://tgdtheory.fi/appfigures/timemirror.jpg`

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