What gravitons are and could one detect them in TGD Universe?

April 24, 2024

Matti Pitkänen

orcid:0000-0002-8051-4364.
email: matpitka6@gmail.com,
url: `http://tgdtheory.com/public_html/`,
address: Rinnekatu 2-4 A 8, 03620, Karkkila, Finland.

Abstract

What gravitons are in the TGD framework? This question has teased me for decades. It is easy to understand gravitation at the classical level in the TGD framework but the identification of gravitons has been far from obvious. Second question is whether the new physics provided by TGD could make the detection of gravitons possible?

The stimulus, which led to the ideas related to the TGD based identification of gravitons, to be discussed in the sequel, came from condensed matter physics. There was a highly interesting popular article telling about the work of Liang et al with the title "Evidence for chiral graviton modes in fractional quantum Hall liquids" published in Nature.

The generalized Kähler structure for $M^4 \subset M^4 \times CP_2$ leads to together with holography=generalized holomorphy hypothesis to the question whether the spinor connection of M^4 could have interpretation as gauge potentials with spin taking the role of the gauge charge. The objection is that the induced M^4 spinor connection has a vanishing spinor curvature. If only holomorphies preserving the generalized complex structure are allowed one cannot transform this gauge potential to zero everywhere. This argument can be strengthened by assigning the fundamental vertices with the splitting of closed string-like flux tubes representing elementary particles. The vertices would correspond to the defects of ordinary 4-D smooth structure making possible a theory allowing a creation of fermion pairs. The vielbein part of the induced M^4 spinor connection could not be eliminated by a global general coordinate transformation at the defects.

For the induced vielbein conection of M^4 one would have an analog of topological field theory and the Equivalence Principle at quantum level would state that locally the vielbein part of M^4 spinor connection can be transformed to zero but not globally. The Kähler part of the M^4 spinor connection cannot be transformed away and could give rise to gravitons as monopole flux tubes containing fermion pairs with rotational angular momentum $L = 1$.

This description of gravitons corresponds to gauge-gravitation duality. Gravitons and gauge bosons would be in a completely similar role as far as vertices of generalized Feynman diagrams are considered. The vertex contains besides gauge potential terms also a term proportional to the trace of the second fundamental form at the singularity and vanishing elsewhere. It is identifiable as a generalized acceleration and generalization of the Higgs field. The condition for the vertex generalizes Newton's "F=ma"!

The second question is whether gravitons could be detected in the TGD Universe. It turns out that the FQHE type systems do not allow this but dark protons at the monopole flux tube condensates give rise to a mild optimism.

Contents

1 Introduction

What gravitons are in the TGD framework? This question has teased me for decades. It is easy to understand gravitation at the classical level in the TGD framework but the identification of gravitons has been far from obvious. Second question is whether the new physics provided by TGD could make the detection of gravitons possible?

1.1 Could FQHE make possible detection of gravitons

The initial stimulus, which lead to the ideas related to the TGD based identification of gravitons to be discussed in the sequel came from condensed matter physics. There was a highly interesting popular article (see this) telling about the work of Liang et al with title "Evidence for chiral graviton modes in fractional quantum Hall liquids" published in Nature [D1] (see this).

The abstract of the article helps to gain some idea of what is involved.

> *Exotic physics could emerge from interplay between geometry and correlation. In fractional quantum Hall (FQH) states, novel collective excitations called chiral graviton modes (CGMs) are proposed as quanta of fluctuations of an internal quantum metric under a quantum geometry description.*
>
> *Such modes are condensed-matter analogues of gravitons that are hypothetical spin-2 bosons. They are characterized by polarized states with chirality of +2 or 2, and energy gaps coinciding with the fundamental neutral collective excitations (namely, magnetorotons) in the long-wavelength limit. However, CGMs remain experimentally inaccessible.*
>
> *Here we observe chiral spin-2 long-wavelength magnetorotons using inelastic scattering of circularly polarized lights, providing strong evidence for CGMs in FQH liquids. At*

filling factor v = 1/3, a gapped mode identified as the long-wavelength magnetoroton emerges under a specific polarization scheme corresponding to angular momentum S = 2, which persists at extremely long wavelengths. Remarkably, the mode chirality remains 2 at v = 2/5 but becomes the opposite at v = 2/3 and 3/5. The modes have characteristic energies and sharp peaks with marked temperature and filling-factor dependence, corroborating the assignment of long-wavelength magnetorotons. The observations capture the essentials of CGMs and support the FQH geometrical description, paving the way to unveil rich physics of quantum metric effects in topological correlated systems.

From the abstract of the latter article it is clear that, assuming that quantum gravitation is what the standard thinking would suggest, the work has not produced evidence for genuine gravitons contrary to what the popular article claims. Gravitons in question are "condensed matter gravitons" and would be associated with systems exhibiting fractional quantum Hall effect (FQHE) involving quantum coherent many-electron states.

One can however ask whether the notion of chiral graviton might generalize to the TGD framework and this question turned out to be one of the really good questions which sometimes pop into mind.

1.2 Theoretical models for chiral gravitons

Chern-Simons action is used to model FQHE. This does not however provide a full description of these systems (also viscosity shows quantal features) and it has been proposed that effective 3-dimensional gravitation using effective 3-metric could work. This approach predicts graviton type excitations (chiral gravitons). If the 3-metric has the property that it is degenerate, i.e. has one light-like direction, one has a situation in which Chern-Simons action is extremely natural since it does not involve the metric and all so that one avoids the problems due to the singularity of the contravariant metric.

Haldane has proposed a geometric representation of the fractional quantum Hall effect in the article [B3]. Liou et al have proposed that chiral gravitons are possible in FQH liquids [B2]. Son has proposed what he calls Newton-Cartan Geometry [B1] and Gromow and Son have proposed a bimetric theory of fractional quantum hall states [B4].

Since the bimetric theory is the latest contribution to this kind of models, I include the abstract here to give a more concrete view about what is involved

We present a bimetric low-energy effective theory of fractional quantum Hall (FQH) states that describes the topological properties and a gapped collective excitation, known as the Girvin-Macdonald-Platzman (GMP) mode. The theory consists of a topological Chern-Simons action, coupled to a symmetric rank-2 tensor, and an action a la bimetric gravity, describing the gapped dynamics of a spin-2 mode.

The theory is formulated in curved ambient space and is spatially covariant, which allows us to restrict the form of the effective action and the values of phenomenological coefficients. Using bimetric theory, we calculate the projected static structure factor up to the k 6 order in the momentum expansion.

To provide further support for the theory, we derive the long-wave limit of the GMP algebra, the dispersion relation of the GMP mode, and the Hall viscosity of FQH states. The particle-hole (PH) transformation of the theory takes a very simple form, making the duality between FQH states and their PH conjugates manifest. We also comment on the possible applications to fractional Chern insulators, where closely related structures arise. It is shown that the familiar FQH observables acquire a curious geometric interpretation within the bimetric formalism.

Although chiral gravitons are not genuine gravitons, one can ask whether real gravitons could be observed by using FQH systems.

1. One can ask whether real gravitons could couple to the chiral gravitons in analogy with the coupling of a photon to a vector boson in hadron physics.

2. One can wonder whether real gravitons could be the, not necessarily chiral, TGD analogs of chiral gravitons. As a matter of fact, this led to a consideration of a long standing problem of what gravitons are in the TGD framework. They might indeed ge analogs of chiral gravitons and this view solves several problems of the earlier view.

3. Quantum coherence in a rather long scale is associated FQH system and this might amplify the rate for graviton absorption: proportionality to the number N of electrons would be replaced with a proportionality to N^2, and one can ask wonder whether the extremely small scattering/absorption rate might increase so that it could be detected. Unfortunately, the number of electrons turns out to be too small and also their mass. One can however also consider what I call monopole flux tube condensates with flux tubes carrying dark protons. These play a key role in the TGD inspired quantum biology. Larger mass of proton and the long scale quantum coherence scale lead to a more optimistic view of the detection.

1.3 Could gravitons in TGD be analogs of chiral gravitons and could gravitons be detected in the TGD Universe?

The objection is that the induced M^4 spinor connection has a vanishing spinor curvature. If only holomorphies preserving the generalized complex structure are allowed one cannot transform this gauge potential to zero everywhere. This argument can be strengthened by assigning the fundamental vertices with the splitting of closed string-like flux tubes representing elementary particles. The vertices would correspond to the defects of ordinary 4-D smooth structure making possible a theory allowing a creation of fermion pairs. The vielbein part of the induced M^4 spinor connection could not be eliminated by a global general coordinate transformation at the defects.

For the induced vielbein conection of M^4 one would have an analog of topological field theory and the Equivalence Principle at quantum level would state that locally the vielbein part of M^4 spinor connection can be transformed to zero but not globally. The Kähler part of the M^4 spinor connection cannot be transformed away and could give rise to gravitons as monopole flux tubes containing fermion pairs with rotational angular momentum $L = 1$.

This description of gravitons corresponds to gauge-gravitation duality. Gravitons and gauge bosons would be in a completely similar role as far as vertices of generalized Feynman diagrams are considered. The vertex contains besides gauge potential terms also a term proportional to the trace of the second fundamental form at the singularity and vanishing elsewhere. It is identifiable as a generalized acceleration and generalization of the Higgs field. The condition for the vertex generalizes Newton's "F=ma"!

The second question raised by the article [D1] is whether gravitons could be detected in the TGD Universe. It turns out that the FQHE type systems do not allow this but dark protons at the monopole flux tube condensates give rise to a mild optimism.

2 Brief summary of some basic ideas of TGD

In this section some background of classical and quantum TGD is described and also the question what gravitons are is considered.

2.1 Recent view of classical TGD

Before continuing, it is good to summarize the basic view about classical TGD as it is now.

1. In the TGD framework, one can understand classical gravitation in terms of the induced geometry of the space-time surface $X^4 \subset H = M^4 \times CP_2$. The gravitational constant G should be determined by the square of the CP_2 radius $R \sim 10^4 l_P$, $l_P^2 = G\hbar$. If one accepts the hierarchy of Planck constants $h_{eff} = nh_0$ predicted by the number theoretical vision about TGD [L17], the effective radius of CP_2, which is about 10^4 Planck lengths, would be apart from a numerical scale factor near unity $R_{eff}^2 = (\hbar_{eff}/h_0) l_P^2$.

2. Embeddability to H and the holography forced by the general coordinate invariance, implying that space-time surfaces are analogs of Bohr orbits, poses extremely strong constraints on the space-time surfaces so that they cannot directly correspond to the Einsteinian space-time.

The QFT limit of TGD is obtained by replacing the many-sheeted space-time surface with a single metrically deformed region of M^4 such that gauge potentials are sums of the induced gauge potentials for the space-time sheets. Same applies to the deviations of the induced metric from the M^4 metric. This picture applies in long length scales in which Einsteinian view of space-time works [L11, L12, L15].

3. Holography is realized as a generalized holomorphy [L17, L19]. The twistor lift of TGD [L3, L4, L13] leads to the proposal that M^4 has a generalized Kähler structure, which combines ordinary complex structure and hypercomplex structure to its 4-D generalization so that H also allow generalized complex structure with 1 hypercomplex (light-like) coordinate and 1 complex coordinate for M^4 and two complex coordinates for CP_2. I have christened this generalization of the complex structure as Hamilton-Jacobi structure [L13]. A good guess is that there is a moduli space of Hamilton-Jacobi structures and in the first guess locally equal to a Cartesian product of the moduli space of ordinary complex structures and its hyper-complex analog.

   The generalized complex structure corresponds to the slicings of M^4 and X^4 by complex partonic 2-surfaces and hypercomplex string world sheets which are transversal or possibly even orthogonal locally.

2.2 Chern-Simons-Kähler action

The status of Chern-Simons-Kähler (C-S-K) action is not clear. It if taken to be real as also the remaining part of the action it can contributes to Kähler function and the exponent of vacuum functional. If it is taken imaginary it does not contribute to the Kähler function its exponential and defines a complex phase of the vacuum functional.

1. In the TGD framework Chern-Simons-Kähler action is the only possible action for 3-D light-like surfaces representing light-like orbits of partonic 2-surfaces appearing as interfaces of Euclidean and Minkowskian space-time regions or as boundaries of space-time surfaces. Irrespective of whether it is real or not, one can assume that the field equations expressing conservation of various charges are satisfied. If it is real, the bounary conditions allow the flow of interior Noether charges and complex charges to the boundary or interfaces. Otherwise there is no flow to the boundary.

   At these 3-surfaces 4-D induced metric degenerates to an effectively 3-dimensional metric. The twistor lift of TGD suggests that C-S-K action involves contributions from both CP_2 and M^4 allowing a generalized Kähler structure [L13]. The M^4 contribution allows the assignment of non-vanishing Poincare charges to C-S-K action.

2. By its topological nature, C-S-K action does not involve the induced metric at all. The interior part of action makes itself visible in boundary conditions stating that quantum numbers do not flow out through boundaries and are conserved at light-like interfaces between regions of space-time surface with Euclidean and Minkowski signature [L8].

3. Modified Dirac action is the fermionic counterpart of C-S-K action and is determined uniquely by consistency arguments predicting a far reaching generalization of superconformal symmetry and related Kac-Moody symmetry is used to describe all interactions at elementary particle level [K2] [L19].

   Modified C-S-K Dirac action involves couplings to the induced electroweak gauge potentials. The covariant derivatives contain the CP_2 spinor connection determined by the CP_2 metric. CP_2 scale appears as a counterpart of Planck length and could be equal to Planck length for the minimal value of effective Planck constant $h_{eff} = nh_0$. Also the M^4 part associated with the generalized Kähler structure is present if one accepts a twistor lift of TGD.

4. The light-like surface can also contain many-fermion states and I proposed for a long time ago that at the fundamental level FQHE type systems could correspond to the nanoscopic analogs of partonic 2-surfaces carrying a very large number of electrons [K1]. One possibility is that the partonic surface contains a very large number of handles behaving like particles but this is not the only possibility.

The couplings of this kind of systems to gauge bosons and gravitons would be described as in the case of elementary particles. One would have a sum over scattering amplitudes and quantum coherence would apply. 2-dimensionality would be essential and would raise FQHE type systems in a special role.

2.3 About the QFT limit of TGD

Just for fun, one can also look at the situation from the point of view of Einstein-Yang-Mills type theory, which should emerge as the QFT limit of TGD at which space-time surface can be assumed to have a 4-D M^4 projection so that the modelling of the many-sheeted space-time surfaces as slightly curved M^4 should make sense. The gauge potentials would correspond to the sums of the induced gauge potentials for various space-time sheets. Same would apply to the deviation of the metric from the M^4 metric.

One expects that in the case of effectively 2-D systems, the light-like partonic orbits cannot be completely eliminated even at this limit and FQHE systems could represent systems of this kind. In elementary particle length scales they could be replaced by point-like particles but in the case of multi-electrons states at nanoscopic parton surfaces this does not work.

What happens to the curvature scalar at the limit when the induced 4-metric becomes effectively 3-D?

1. The induced covariant 4-metric becomes degenerate at the partonic orbit and the contravariant metric has some divergent components. $\sqrt{-g_4}$ vanishes at this limit like $1/L$, $L \to 0$.

2. The curvature tensor $R^{\alpha}_{\beta\gamma\delta}$ has dimension zero and could remain finite. Ricci tensor $R^{\alpha\beta}$ and Einstein tensor $G^{\alpha\beta}$ could diverge like $1/L^4$. Curvature scalar could diverge like $1/L^2$. If Einstein's equations hold true, the energy momentum tensor is proportional to the Einstein tensor and could diverge like $1/L^4$. Multiplied with $\sqrt{-g}$ it would diverge like $1/L^3$. This suggests that the limit gives the analog of Chern-Simons-Kähler action or its QFT analog as a delta function like singularity. The modified Dirac action should also have a counterpart, which could be finite since it has vanishing dimension.

3 What gravitons could be in the TGD Universe?

The idea that gravitons could be regarded as $SO(1,3)$ gauge bosons does not look attractive in the standard gauge theory but in TGD the situation is different.

3.1 Could gravitational interaction correspond to the M^4 spinor connection induced to the space-time surface?

In the generalized complex coordinates of H, the spinor connection of M^4 is non-trivial and contributes to the induced spinor connection.

1. The Kähler part corresponds in QFT picture to spin 1 particle coupling to the fermion number. The gauge charges for the vielbein part of the induced M^4 spinor connection are components of spin matrices so that the gauge potentials behave like spin 2 objects. Spin becomes a gauge charge. Could the components of the M^4 spinor connection correspond to gravitons or more general graviton-like states consisting of a single fermion pair as carriers of quantum numbers?

2. The modified gamma matrices $\Gamma^{\alpha} = T^{\alpha}_k \Gamma^k$, where one has $T^{\alpha k} = \partial L/\partial h^k_{\alpha}$ are fixed by the classical action uniquely (by the hermiticity of the modified Dirac operator) and are proportional to what generalizes energy momentum tensor. This implies a generalization of superconformal symmetry and super symplectic and other charges defining the isometry charges of the "world of classical worlds" (WCW) are accompanied by super charges defining the gamma matrices.

What would be especially nice is that the couplings of both gauge bosons and scalar particles and of gravitons would involve the analog of energy momentum tensor! One could see YHF either as a gauge theory or a theory of gravitation so that one would have a different realization of gauge-gravitation duality.

There is however a grave objection. The vielbein part of M^4 spinor connection gives rise to vanishing induced gauge fields since the M^4 metric is trivial. Only the induced Kähler gauge field is non-vanishing. Does this spoil the idea that the induced M^4 spinor connection could describe the coupling of the spinor field to gravitons? One can argue that the describability of gravitation in terms of spin 1 gauge boson assignable to partonic 2-surface is consistent with holography stating that gravitation in the interior of space-time surface is describable in terms of gauge theory at the 3-surfaces where the holographic data are given.

1. One can eliminate the vielbein part of the spinor connecting locally by going to standard generalized complex coordinates corresponding to the $M^2 \times E^2$ slicing for which the spinor connection vanishes. If this coordinate transformation is a generalized holomorphy acting as a generalized symmetry, the transformation cannot be carried out globally and must have singularities analogous to poles and cuts of analytic functions since the topologies of string world sheets and partonic 2-surfaces need not be those associated with pieces M^2 and E^2. Allowing general coordinate transformations, this elimination is always possible locally. The global elimination by a general coordinate transformation is however not possible for string-like objects and deformations of CP_2 type extremals.

   Could have an analog of topological gauge theory for the induced spinor connection so that the M^4 parts of gauge potentials and that they have a physical meaning. Could the possibility to eliminate the spinor connection by a generalized holomorphy locally relate to the quantum counterpart of the Equivalence Principle?

2. Locally only $U(1)$ Kähler gauge potential remains a dynamical degree of freedom. Could it somehow give rise to gravitons? The couplings of the Kähler gauge potential to matter and antimatter are opposite. Does this predict that the graviton associated with it couples with an opposite sign to matter and antimatter and that gravitation is a repulsive force? The spin 2 character of graviton requires that the flux tube assignable to graviton is in $L = 1$ rotational state. Could this make the graviton exchange an attractive interaction and imply that the couplings of gravitons to matter and antimatter are of the same sign?

Could gravitons identified in terms of the M^4 spinor connection serve as the TGD analogs of chiral gravitons. The term chiral graviton suggests that they have definite chirality in the sense that they violate parity symmetry so that their couplings are chiral just like the couplings of weak bosons.

The classical gravitation is not chiral. Chirality might reflect the fact that Chern-Simons action violates chirality as an action for 3-D space. In the TGD, Chern-Simons-Kähler action is defined for a light-like 3-surface whereas reflection acts in M^4. Parity symmetry would require that the holomorphic spinor connection for coordinates associated with Hamilton-Jacobi structure is vectorial.

Could the self-dual Kähler form, having a physical interpretation as a presence of parallel electric and magnetic fields of identical strength, induce a parity violation at the fundamental level? I have proposed that CP could be violated and that this could explain the CP violation in hadron physics and the matter-antimatter asymmetry. Could the TGD counterparts of chiral gravitons couple vectorially but violate the CP symmetry?

3.2 How to identify graviton in the TGD framework?

TGD leads to the identification of all elementary particles in terms of closed monopole flux tubes associated with pairs two parallel space-time sheets. The Euclidean wormhole contacts at the "ends" of the flux tube correspond to light-like orbits of partonic 2-surfaces and would carry point-like fermions serving as building bricks of all elementary particles. In the case of particles with spin smaller than 2, either wormhole contact can carry the spin and electroweak quantum numbers and second wormhole contact possibly carries a neutrino pair neutralizing the weak isospin so that one has a weak analog of confinement. There are also closed half-monopole flux tubes having boundaries [L20] and both these and monopole flux tubes with closed cross section could be important in superconductivity [L2].

The proposal has been that graviton spin reduces to fermionic spin so that both wormhole contacts should carry a fermion pair with spin 1. These kinds of states might well exist but in this picture it is difficult to understand how the expected value of the gravitational constant is coded to the structure of the state formed by the two spin 1 fermion pairs. The second problem is that it is not obvious how the Equivalence Principle could be realized at the level of gravitons in this picture.

The assumption of two fermion pairs is not necessary if the monopole flux tube rotates. One could have fermion and antifermion at the wormhole contacts defining the ends of this string-like object. Angular momentum $L = 0$ would give bosons with spin 0 and 1. $L = 1$ would allow bosons spin 2, 1, and 0 and $L = 2$ would allow bosons with spin $3, 2, 1$.

These observations force us to seriously consider the possibility that the M^4 spinor connection for generalized complex structure defines the gravitational field. This assumption gives additional constraints on possible values of spin.

1. In the case of electroweak couplings the product $\Gamma^\mu A_\mu(CP_2)$ with the spinor field gives a spin 1/2 object. Therefore the fermion and antifermion at the ends of the two string-like objects have spin 1/2 and the fermion can emit only a gauge boson or scalar. One obtains standard model vertices. In [L12] it is shown how gluon vertices emerge from the vertex involving Kähler gauge potential coupling vectorially.

2. The product $\Gamma^\mu A_\mu(M^4)$ involves Kähler and vielbein terms. Kähler term is analogous to ordinary $U(1)$ coupling. In this case $L = 1$ state of the monopole flux tube representing graviton could have spin 2. Also spin 1 and spin 0 states are in principle present but might be elimated by gauge conditions.

   The vielbein term involves a product of M^4 gamma matrices and sigma matrices operating in a spin 1/2 fermion state producing spin 1/2 state. The tensor product gives rise to total spins 3/2 and 1/2. Spin 3/2 configuration could contribute angular momentum $L = 1$ to the rotation of the string-like object with fermion and antifermion at its ends and produce besides $J = 2$ state identifiable as graviton also $J = 1$ state. Whether gauge conditions eliminate this state or whether it represents a new spin 1 boson remains unclear.

3.3 Exotic differential structures in 4 dimensions, particle vertices, and the new view of gravitons

What remains to be understood are the counterparts of the basic vertices of the gauge theory and quantum gravity. One can start from a long standing problem of TGD. Since gauge bosons do not appear as fundamental fields, the prediction seems to be that pair creation is not possible. The net fermion and antifermion numbers would be separately conserved. How to circumvent this problem?

The solution came from the discovery that 4-dimensional space-times are completely unique in the sense that they allow an infinite number of exotic smooth structures [L7]. Apart from a subset of measure zero they are reduced to ordinary differentiable structures. These subsets are physically analogous to defects and the simplest defects are point defects but one can also imagine 1-, 2-, and even 3-D defects. This finding means a serious difficulty for general relativity. Should some kind of cosmic censorship hypothesis deny their existence?

In the TGD framework, an attractive identification of the defects would be as singularities at which the minimal surface property for space-time surfaces as generalized complex surfaces fails. These singularities are analogs of poles and cuts in the complex analysis. In fact, hypercomplex poles are 1-D geodesic lines in M^2 and would correspond to light-like curves in the general case. Therefore 3-D light-like partonic orbits would be analogous to poles. String world sheet could serve as a counterpart for a hypercomplex cut.

The identification

defects of the ordinary smooth structure $\leftrightarrow$ singularities at which the minimal surface property fails $\leftrightarrow$ poles and other singularities where generalized holomorphy fails looks highly attractive. The 3-D light-like orbits of partonic orbits, string world sheets, strings, and points at which light-like orbits of point-like fermions split, could correspond to these singularities identifiable as generalized vertices. It is not clear whether 3-D defects can be space-like 3-surfaces.

These structures could be essential for the definition of creation and annihilation vertices for fermion-antifermion pairs. The intuitive picture is that a fermion turns backwards in time in this kind of vertex.

1. In QFTs a standard approximation is to replace the gauge boson of the vertex with a classical gauge potential. In TGD there are no bosons as fundamental particles and this replacement is necessary. This would correspond to a turning of fermion lines at the orbit of a partonic 2-surface backwards in time which is somehow special. Could this point correspond to a defect of the ordinary differentiable structure which is actually exotic smooth structure?

2. There is also another problem. Modified Dirac action should give rise to all fundamental vertices. At the fermion line the modified Dirac equation is satisfied but it puts modified Dirac action to zero so that the action would be trivial in the gauge theory sense. At the singularity the modified Dirac equation could however fail and one would obtain a delta function like singularity giving the standard classical vertex for the creation of a fermion pair or a particle with spin smaller than 2 as a bound fermion pair. This picture generalizes also to higher-dimensional defects. Interesting quantum physics would be possible only in space-time dimension four!

3. This picture could generalize also to the creation of gravitons if they are analogs of gauge bosons with gauge group $SO(1,3)$ or its compact subgroup as required by unitarity unless one allows infinite-dimensional representations of $SO(1,3)$, which in fact are naturally associated with the causal diamonds (CDs), which are basic objects in zero energy ontology (ZEO) [L14]: the Poincare invariance which is problematic at the level of CD would be realized in the moduli space of CDs.

4. If gravitons are pairs of fermion pairs, the vertex involves two separate vertices in an essential way. This is possible but does not look elegant since two separate gauge boson vertices would be needed. This would conform with the idea that gravitation is in some sense square of a gauge theory but does not look an attractive idea.

In this framework one could understand basic vertices as splitting of two-sheeted closed monopole flux tubes with Euclidean wormhole contacts at ends. The splitting of the flux tube as a generalization of reconnection for closed strings would produce two closed flux tubes. The simplest reconnection would involve creation of a fermion-antifermion pair such that the fermion and antifermion pair go to separate wormhole contacts. The defects of the smooth structure would correspond to situations in which the topology of 3-surface is between two topologies. The pinching of torus to produce two spheres represent the basic example of this.

In the case of a fermion with a neutrino-antineutrino (left- and right-handed neutrinos) pair at the second wormhole contact neutralizing the weak isospin of the fermion as a geometric object, the reconnection would produce a pair of monopole flux tubes. The first one would represent a fermion. The second one would represent a boson with fermion and antifermion at opposite wormhole contacts. If the string does not rotate, the boson has spin 1 or 0 corresponding to a gauge boson and Higgs type scalar or pseudoscalar. If the string rotates one obtains a boson with spin 2 or 1 for the simplest option if the M^4 spinor connection contributes.

One can of course worry about the triviality of the vielbein part of M^4 spinor connection. Maybe it gives only rise to a topological gravitation whereas the Kähler part would give rie to graviton. The failure of the standard smooth structure at the defect could however imply that the elimination of the vielbein spinor connection by a general coordinate transformation fails just at the defect which is 2-D and has complex CP_2 coordinates as more natural coordianates.

3.4 What modified Dirac action is and how it could determine the scattering amplitudes?

Holography=generalized holomorphy property means that minimal surface field equations are true outside singularities for any general coordinate invariant action constructible in terms of the induced geometry. However, the twistor lift of TGD suggests that 6-D Kähler action is the fundamental action. It reduces to 4-D Kähler action plus volume term in the dimensional reduction guaranteeing that the 6-surface can be regarded as a generalization of twistor space having space-time surface as a base-space and 2-sphere.

One can express the induced spinor field obtained as a restriction of the second quantized H spinor field to the space-time surface and it satisfies modified Dirac equation [L19].

Modified Dirac action L_D is defined for the induced spinor fields.

1. It is fixed by the condition of hermiticity stating that the canonical momentum currents appearing in it have a vanishing divergence. If the modified gamma matrices Γ^α are defined by an action S_B defining the space-time surface itself, they are indeed divergenceless by field equations. This implies a generalization of conformal symmetry to the 4-D situation [L13] and the modes of the modified Dirac equation define super-symplectic and generalized conformal charges defining the gamma matrices of WCW [L19].

2. Generalized holomorphy implies that S_B could be chosen to correspond to modified gamma matrices defined by the sum of $L_K + L_V$ or even by L_V defining induced gamma matrices. Which option is more plausible?

3. An attractive guiding physical idea is that the singularities are not actually singularities if exotic smooth structure is introduced. Field equations hold true but with $S_K + S_V$. The singularities would cancel. One would avoid problems with the conservation laws by using exotic smooth structure.

4. At the short distance limit for which α_K is expected to diverge as a U(1) coupling, the action reduces to S_V and the defects would be absent. Only closed cosmic strings and monopole flux tubes would be present but wormhole contacts and string world sheets identifiable as defects are absent: this would be the situation in the primordial cosmology [L18]. Only dark energy as classical energy of the cosmic strings and monopole flux tubes would be present and there would be no elementary particles and elementary particle scattering at this limit.

In [L12] the vertices were discussed and the conclusion was that the vertices are generalizations of ordinary vertices in which induced gauge potentials replaced standard model electroweak gauge potentials. The key idea is that one can overcome the problem due to the fact that gauge fields are not primary fields in TGD is that classical gauge potentials give rise to a pair creation. Strong interaction vertices were also discussed as graviton vertices. The discussion was based on physical intuition and far from rigorous.

While developing this article, I ended up with the idea that the trace of the modified Gamma matrices, which for the volume action is just the trace of the second fundamental form appears as a candidate for a vertex and would be non-vanishing at the singularities of the minimal surfaces so that the identification of the defects of the smooth structure as these singularities and vertices would be natural. What is so remarkable is that the CP_2 part of the second fundamental form has quantum numbers of the Higgs field of the standard model. The M^4 part in turn contains a term proportional to the square of CP_2 radius identifiable as Planck length for the minimal value of $h_{eff} = nh_0$. The effective value CP_2 radius scales $\sqrt{h_{eff}/h_0}$.

The challenge is to construct a mathematical scenario in which these two kinds of couplings emerge naturally. One can from the following tentative physics inspired picture.

1. The vertex which corresponds to a $d < 4$-dimensional surfaces as a defect of the standard smooth structure and as singularity of the space-time as minimal surface.

2. Both gauge potentials and second fundamental form or more generally the covariant divergences for the canonical momentum currents of a more general action, having group theoretical properties of Higgs field in CP_2 degrees of freedom, appear in the vertex and is slashed

between induced spinor and its conjugate. Classical gauge potentials and the analog of the Higgs field make pair creation possible.

3. Fermion and antifermions are not conserved separately at the vertex. Their sum is however conserved. Their separate conservation would be the prediction in absences of the vertices. The conservation law for the total fermion number can be expressed as a vanishing of the divergence of the fermion current $J^\alpha = \overline{\Psi}\Gamma^\alpha\Psi\sqrt{g}$, where Γ^α are modified gamma matrices defined in terms of the action.

4. The vertex serves as a source for the modified Dirac equation since this equation cannot be satisfied at the vertex. This source must be also proportional to a delta function.

5. Intuitively, the pair creation vertex corresponds to a situation in which the fermion line turns back in time directions. This direction could also be some other direction, and in standard perturbation theory it could be even space-like, and should be normal to the singularity as a surface of the space-time surface. This raises the question whether one should identify the spinor fields emerging from the vertex as Ψ and its T- (CP-) conjugate proportional to $\overline{\Psi}$. The ordinary perturbation suggests that nothing special needs to be done.

3.4.1 How to obtain the vertices from the modified Dirac equation

How could one deduce the vertex from the modified Dirac action by using these assumptions.

1. Should one assign a $d < 4$-dimensional fermionic term to the singular surface? Could one add to the Dirac action a total divergence expressing the separate conservation laws for the total fermion number outside the vertices and only the conservation of their sum at the vertex? This divergence can be transformed to an $d < 4$-dimensional integral over the vertex for the flux of the normal component of the non-conserved part of the fermion current through the surface.

2. One can check what comes out from this guess. Let us forget the possible complications due to the reversal of the direction of time at the vertex possibly due to T reflection. The total divergence is given by $\partial_\mu J^\mu$, $J^\mu = \overline{\Psi}\Gamma^\mu\Psi\sqrt{g}$ for the surface representing the particle (it can have dimension smaller than 4 and even correspond to a fermion line).

   One can write the divergence as a covariant divergence and for the contributions involving bilinearas of the creation operators for fermions and antifermions, one obtains a sum of two terms which should not vanish at the singularity whereas the remaining contribution vanishes everywhere. This gives

$$\begin{aligned}\partial_\mu J^\mu &= \textstyle\sum_\pm \overline{\Psi}_\pm[D^\leftarrow + D^\rightarrow + D_\mu\Gamma^\mu]\Psi_\mp]\sqrt{g} \ , \\ D^\leftarrow &= (-i\partial_\mu^\leftarrow + A_\mu)\Gamma^\mu \ , \\ D^\rightarrow &= \Gamma^\mu(i\partial_\mu^\rightarrow + A_\mu) \ ,\end{aligned} \qquad (3.1)$$

   $D_\mu\Gamma^\mu$ corresponds to the generalized Higgs term and is singular at the vertex.

3. What one wants is the following. At the singularity for the $\Psi_\pm\Gamma^\alpha\Psi_\mp$ parts of the current, the contributions of ordinary derivatives, at least of the normal derivatives, cancel each other also at the singularity. The contributions from the gauge potential terms should be nonvanishing at the singularity and should give the gauge couplings so that one would have

$$\partial_\mu J^\mu = \sum_\pm \overline{\Psi}_\pm[A_\mu\Gamma^\mu + \Gamma^\mu A_\mu + D_\mu\Gamma^\mu]\Psi_\mp\sqrt{g} \ . \qquad (3.2)$$

   Kähler gauge potential commutes with the modified gamma matrices so that one obtains what one wants.

4. The Higgs term is singular but the gauge potential term does not seem able to develop a singularity unless the induced gauge potential behaves like a gauge potential of a point charge. Something still goes wrong in the above proposal. The modified Dirac equation is true outside the singularity. Singularity however serves as a fermionic source of the Dirac field. This means that the terms $\overline{\Psi}_\pm D^{\leftarrow}$ and $D^{\rightarrow}\Psi_\mp$ have a delta function like behavior at the singularity just like field equations have a singularity. This of course conforms with the super-symmetry associated with the modified Dirac equation.

   To get correct couplings, the sum of the source terms should give $A_\mu\Gamma^\mu + \Gamma^\mu A_\mu$ times a $d-1$-dimensional delta function associated with the singularity. $\sqrt{g_d}$ must appear in the integration measure to get the dimensions correctly. The delta function would compensate for the reduction of the dimension in the integral appearing in the Dirac action.

   One could worry about the gauge invariance of the scattering amplitudes at the fundamental level. In TGD, the notion of gauge invariance for Kähler gauge potential could be only approximate and correspond to symplectic transformations which are not isometries of the imbedding spaces. Classical gravitation would cause it's breaking as a gauge symmetry.

3.4.2 Critical comments

Can the induced M^4 vielbein connection and/or Kähler gauge potential really produce a realistic quantum theory of gravitation? I have already mentioned the basic objections. The strongest objection is that in the QFT framework spin 1 fields give repulsion/attraction between changes of same/different sign.

1. Induced M^4 spinor connection contains a $U(1)$ (Kähler) part and vielbein part. Kähler part cannot be eliminated by a general coordinate transformation or gauge symmetry so that it defines a candidate for graviton. Note that instead of a genuine gauge symmetry one has an M^4 analog of symplectic symmetry [L13].

   Should the M^4 Kähler gauge potential be counted as a contribution to the $U(1)$ gauge potential of electroweak interactions? Or could it give rise to an independent degree of freedom at the QFT limit and give rise to graviton? This looks like an attractive option.

   In the QFT picture, M^4 Kähler gauge potential would correspond to a spin 1 particle. However, gravitons could correspond to closed monopole flux tubes with $L = 1$ angular momentum associated with the rotation of the flux tube and one would obtain a connection with the string picture. This might relate to the ability to approximate classical gravitation with a Maxwellian gauge theory using Newtonian gravitational potential as a counterpart of electric potential.

2. Could the vielbein part coupling spin give rise to graviton as an analog of spin 1 particle? Also in this case, the simplest option involves $L = 1$ rotational state of the flux tube. For space-time regions representable as graphs $M^4 \rightarrow CP_2$ this contribution can be eliminated by going to linear Minkowski coordinates. Could the mere $J = 2$ property of the graviton guarantee that it is always an attractive interaction and also matter and antimatter attract each other.

   Can one imagine a good reason for restricting the allowed 4-D general coordinate transformation of X^4 to generalized conformal transformations respecting the dynamical Hamilton-Jacobi structure of M^4? Or are global M^4 coordinates impossible in the case of vertices as singularities of minimal surfaces?

   For cosmic strings one must use M^2 light-like coordinate and complex CP_2 coordinate but M^2 light-like coordinate is not possible globally so that the global elimination of the spinor connection by a global general coordinate transformation is not possible. One would have an analog of a nontrivial bundle with a flat connection and therefore an analog of topological QFT.

   At the light-like partonic orbits deformed in M^4 directions the situation is similar. Here the description using QFT from CP_2 to M^4 is the appropriate approach and M^4 deformation gives only a small correction to the CP_2 metric.

3. Can one understand gravitational constant? The vertex term $\Gamma^\mu A_\mu$, where A_μ denotes M^4 spinor connection, contains a contribution from the CP_2 gamma matrices proportional to CP_2 radius squared and would naturally correspond to graviton coupling. Why would the M^4 part of Γ^α proportional to the canonical momentum current $T^{\alpha k}\gamma_k$ be proportional to R^2? This contribution would come from 2-D partonic surfaces as singularities of partonic orbits and if CP_2 coordinates are used, the dominating contribution is proportional to R^2.

To sum up, if $J = 2$ property implies the expected properties of graviton, the Kähler gauge potential of M^4 could give rise to gravitons whereas spin vielbein connection would give rise to a topological gravitation.

3.4.3 More detailed view about the action

The action associated with the singularities involves singular terms coming from the action defining the space-time surfaces as 4-D Bohr orbits. One could also assign a lower-dimensional action with the singularities as independent action. For instance, Chern-Simons-Kähler action can emerge from the Kähler or appear as an independent action. Fermion lines can involve a 1-D Dirac action coupled to induced gauge potentials and string world sheets can involve separate action. It is not clear whether these all actions could emerge from a 4-D action at singularities.

One can consider several options assuming that the exotic smooth structure can be regarded as ordinary smooth structure with defects identifiable as singularities.

Option 1: The first option relies on the assumption that the exponential of the modified Dirac action is imaginary and analogous to the phase defined by the action in QFTs. This is enough in TGD since fermions are the only fundamental particles and bosonic action is a purely classical notion.

1. Volume action is in a very special role in that it represents both the classical dynamics of particles as 3-D surfaces as analogs of geodesic lines, the classical geometrized dynamics of massless fields, and generalizes the Laplace equations of complex analysis.

   This motivates the proposal that only the induced gamma matrices $\Gamma^\alpha g^{\alpha\beta} h^k_\beta \gamma_k$ (no contribution from L_K) corresponding to S_V appear in L_D and the bosonic action $S_B = S_K + S_V + S_I$, where the instant term S_I, reducing to a Chern-Simons-Kähler term, is real, is defined by the twistor lift of TGD.

   If the field equations are satisfied also at the singularities, the contributions from $S_K + S_I$ and S_V cancel each other at the singularity in accordance with the idea that an exotic smooth structure is in question. Both S_K and S_I contributions would have an imaginary phase.

2. Therefore L_V, which involves cosmological constant Λ, would disappear from the scattering amplitudes by the field equations for L_B although it is implicitly present. The number theoretic evolution of the $S_K + S_I$ would make itself visible in the scattering vertices. Outside the singularities both terms vanish separately but at singularities this is not the case. Only lower-D singularities contribute to the scattering amplitudes.

   The number theoretical parameters of the bosonic action determined by the hierarchy of extensions of rationals would parametrize different exotic smooth structures and make themselves visible in the quantum dynamics in this way. S_I would contribute to classical charges and its M^4 part would contribute to the Poincare charges.

3. An objection against this proposal is that the divergence of the modified gamma matrices defined by the $S_K + S_I$ need not be well-defined. It should be proportional to a lower-dimensional delta function located at the singularity.

4. The divergence of $g^{\mu\nu}\partial_\nu$ vertex as the trace of the second fundamental form $D_\alpha h^k \beta$ defined by the covariant derivatives of coordinate gradients, appears in the vertex. The second fundamental form is orthogonal to the space-time surface and can be written as

$$\begin{array}{ll} g^{\mu\nu} D_\nu \partial_\mu h^k = P^k_l H^l \ , & P^k_l = h^k_l - g^{\mu\nu} h^k_\mu h_{lr} h^r_\nu \ , \\ H^k = g^{\alpha\beta}(\partial_\alpha + B^k_\alpha)(g^{\alpha\beta} h^k_\beta \ , & B^k_\alpha = B^k_{lm} h^m_\alpha \ . \end{array} \tag{3.3}$$

P^k_l projects to the normal space of the space-time surface. H^k is covariant derivative of h^k_α and $B^k_\alpha = B^k_{lm} h^m_\alpha$ is the projection of the Riemann connection of H to the space-time surface.

5. This allows a very elegant physical interpretation. In linear Minkowski coordinates for M^4, one has $B^k_\alpha = 0$ but the presence of the CP_2 contribution coming from the orthonormal projection implies that the covariant divergence is non-vanishing and proportional to the radius squared of CP_2. Vertex is proportional to the trace of the second fundamental form, whose CP_2 part is analogous to the Higgs field of the standard model. This field is vanishing in the interior by the minimal surface property in analogy with the generalized Equivalence Principle.

   The trace of the second fundamental form is a generalization of acceleration from 1-D case to 4-D situation so that the interaction vertices are lower-dimensional regions of the space-time surface which experience acceleration. The regions outside the vertices represent massless fields geometrically. At the singularities the Higgs-like field is non-vanishing so that there is mass present. The analog of Higgs vacuum expectation is non-vanishing only at the defects.

   I started more than half a century ago from Newton's "F=ma" and now I discover it as a part of the interaction vertex, which is the core of quantum field theories! As the previous argument demonstrated, besides this Higgs-like contribution to the vertex there is a contribution coming from the gauge potentials. The trace of the second fundamental form would be described in CP_2 degrees of freedom Higgs and in M^4 degrees of freedom gravitation. The generalized acceleration would be due to U(1) force assignable to Kähler action both in M^4 and CP_2 degrees of freedom.

   In some sense, the circle is closing. I started more than half a century ago from Newton's "F=ma" and now I discover it in the interaction vertex, which is the core of quantum field theories! I almost see Newton nodding and smiling and saying "What I said!".

Option 2: Modified gamma matrices are defined by $S_K + S_V + iS_I$ and the real part of the singularity vanishes. The imaginary part cannot vanish simultaneously.

1. The exponent of Kähler function defines a real vacuum functional and K is determined by $S_K + S_V$ whereas the action exponential of QFTs of QFTs defines a phase. In topological QFTs, the contribution of the instanton term $S_{D,I}$ is naturally purely imaginary and could define "imaginary part of the Kähler function K, which does not contribute to the Kähler metric of WCW.

   One can argue that this must be the case also for S_D. Hence the contribution of $S_K + S_V$ to S_D would be real and differ by a multiplication with i from that in QFTs whereas the contribution of iS_I would be imaginary. One must admit that this is not quite logical. Also the contribution to the Noether charges would be imaginary. This does not look physically plausible.

2. One cannot require the vanishing of both the real part and imaginary part of the divergence of the modified gamma matrices at the singularity. The contribution of L_{C-S-K} at the singularity would be non-vanishing and determine scattering amplitudes and imply their universality.

   For the representations of Kac-Moody algebras the coefficient of Chern-Simons action is $k/4\pi$ and allows an interpretation as quantization of α_K as $\alpha_K = 1/k$. Scattering vertices would be universal and determined by an almost topological field theory. Almost comes from the fact that the exponent of S_B defines the vacuum functional.

3. The real exponential $exp(K)$ of the real Kähler function defined by $S_K + S_V$ would be visible in the WCW vacuum functional and bring in an additional dependence on the α_K and cosmological constant Λ, whose number theoretic evolution would fix the evolution of the other coupling strengths. Note that the induced spinor connection corresponds in gauge theories to gauge potentials for which the gauge coupling is absorbed as a multiplicative factor.

There are therefore two options. For both cases $1/\alpha_K = 1/k$ appears in the action.

1. For Option 1 only iS_V appears in S_D and $iS_K + iS_{C-S-K}$ determines the scattering amplitudes for option 2). Exponent of the modified Dirac action defines the analog of the imaginary action exponential of QFTs.

2. For Option 2 for which the entire action defines the modified gamma matrices the iS_{C-S-K} defines the scattering amplitudes and one has an analog of topological QFT. This picture would conform with an old proposal that in some sense TGD is a topological quantum field theory. One can however argue that the treatment of $S_K + S_V$ and S_I in different ways does not conform with QFT treatment and also the Noether charges are a problem.

Some technical remarks are in order.

1. The spinor connection does not disappear from the dynamics at the singularities. It is transformed to components of projected Riemann connection of H appearing in the divergence $D_\alpha T^{\alpha k}_{C-S-K}$.

2. The modified Dirac action must be dimensionless so that the scaling dimension of the induced spinors should be $d = -3/2$ and therefore same as the scaling dimension of M^4 spinors. This looks natural since CP_2 is compact.

   The volume term included in the definition of the induced gamma matrices must be normalized by $1/L_p^4$. L_p is a p-adic length scale and is roughly of order of a biological scale $L_p \sim 10^{-4}$ meters if the scale dependent cosmological constant Λ corresponds to the inverse squared for the horizon radius. One has $1/L_p^4 = 3\Lambda/8\pi G$. This guarantees the expected rather slow coupling constant evolution induced by that of α_K diverging in short scales.

3.5 What singularities can correspond to vertices for fermion pair creation?

It is far from clear whether all singularities have an interpretation in terms of exotic smooth structures. The physical criterion would be that the creation of a fermion pair takes place at the defect and that the minimal surface property fails. Fermions can correspond to induced spinor fields and fermion pairs could be created at surfaces of dimension $d < 4$.

1. For closed two-sheeted cosmic strings and monopole flux tubes, which split by reconnection, the interpretation makes sense and means a generalization of the basic vertex for closed strings. These objects can be 2-sheeted as elementary particles in which case the reconnection would occur in the direction of CP_2. If they are single sheeted, the reconnection would occur in the direction of M^4.

2. 3-D light-like light-partonic orbits appearing as interfaces between Euclidean and Minkowskian space-time regions and as boundaries of space-time surfaces are singularities [L8]. Boundary conditions state that the possible flows of conserved charges from the interior go to the partonic orbit so that the divergence of the Chern-Simons-Kähler canonical momentum current coming from instanton term equals to the sum of the normal components of the canonical currents associated with Kähler action and volume term.

   (a) Chern-Simons action at the light-like partonic orbit coming from the instanton term is well-defined and finite and field equations should not give rise to a singularity except possibly at partonic 2-surfaces, which have been identified as analogs of vertices at which the partonic 2-surface X^2 splits to two.

   (b) At the light-like partonic orbit 4-metric has a vanishing determinant and is therefore effectively 2-D (the light-like components of $g_{uv} = g_{vu}$ of the 4-metric vanish). As a consequence, $\sqrt{g_4}$ vanishes like L^2 at the partonic orbit unless some coordinate gradients diverge.

   The canonical momentum currents for the volume action are proportional to the contravariant induced metric appearing in the trace of the second fundamental form diverging like $1/L^2$ and to $\sqrt{g}$ so that they remain finite.

(c) Kähler action contains the contravariant metric twice and is proportional to $\sqrt{g_4}$. This can give rise to a divergence of type $1/L^2$ unless the boundary conditions make it finite. I have proposed long ago that the electric-magnetic self-duality at the partonic orbit can transform the Kähler action to an instanton term giving Chern-Simons Kähler term. In this case, a separate instanton term would not be needed. In this case everything would be finite at the partonic orbit. Minimal surface property fails in a smooth manner.

The intuitive picture is that the contributions from the normal currents at the partonic orbit and the Chern-Simons term cancel each other and the partonic orbit cannot play a role of a vertex.

(d) The possible presence of $1/L^2$ divergence could however give rise to a 2-D defect and genuine vertex. If it is identified as a creation of a pair of partonic 2-surfaces, the interpretation in terms of a creation of a fermion pair is possible and could be assigned to the splitting of a monopole flux tube.

In accordance with the QFT picture, I have considered the possibility that the 2-D vertex could correspond to a branching of a partonic orbit. In the recent picture it would be accompanied by a creation of a fermion pair. The stringy view however suggests that pair creation occurs in the creation of partonic orbits in the splitting of monopole flux tubes. The stringy view is more attractive.

3. I have also proposed that 1-D singularities identifiable as boundaries of string world sheets and identifiable as fermion lines at the partonic orbits are important. The creation of a pair of fermion lines would give rise to the analogs of gauge theory vertices as 0-D singularities. It is however far from clear whether the stringy singularities are actually present and whether they could correspond to exotic smooth structures. One can imagine two options.

   (a) There are no string world sheets. Monopole flux tubes can be regarded as deformations of cosmic strings. Instead of strings several monopole flux tubes can emerge from a wormhole contact. For the minimal option, monopole flux tubes, CP_2 type extremals, and massless extremals as counterparts of radiation fields are the basic extremals and the splitting of monopole flux tubes gives rise to vertices as defects of the ordinary smooth structure.

   (b) String world sheets appear as singularities of the monopole flux tubes or even more general 4-surfaces and are analogous to wormhole contacts as blow-ups in which a point of X^4 explodes to CP_2 type extremal. I have indeed proposed that a blow-up at which the points of the string world sheet as surface $X^2 \subset X^4$ are replaced with a homologically non-trivial 2-surface $Y^2 \subset CP_2$ takes place. Y^2 could connect two parallel space-time sheets. Could these singularities correspond to defects of exotic smooth structures such that the ends of the string carry fermion number? The vertex for the creation of a pair of fermion and antifermion lines would correspond to a diffeo defect. Note that also these defects could reduce to a splitting of a monopole flux tube so that TGD would generalize the stringy picture.

3.6 The symmetry between gravitational and gauge interactions

The beauty of the proposal is that implies a complete symmetry between gravitational and gauge interactions. Weak interactions and gravitation couple to weak isospin and spin respectively. Color interactions couple to the isometry charges of CP_2 and gravitational interactions coupling to the isometry charges of M^4. The extreme weakness of the gravitation can be understood as the presence of the CP_2 contribution to the induced metric in the gravitational vertices.

Does color confinement have any counterpart at the level of M^4? The idea that physical states have vanishing four-momenta does not look attractive.

1. In ZEO, the finite-D space of causal diamonds (CDs) forms [L14] the backbone of WCW and Poincare invariance and Poincare quantum numbers can be assigned with wave functions in this space. For CD, the infinite-D unitary representations of $SO(1,3)$ satisfying appropriate boundary conditions are a highly attractive identification for the counterparts

of finite-D unitary representations associated with gauge multiplets. The basic objection against gravitation as $SO(1,3)$ gauge theory would fail.

One could replace the spinor fields of H with spinor fields restricted to CD with spinor fields for which M^4 parts sinor nodes as plane waves are replaced with spinor modes in CD labelled by spin and its hyperbolic counterpart assignable to Lorentz boosts with respect to either tip of CD. One could also express these modes as superpositions of the plane wave modes defined in the entire H.

The analog of color confinement would hold true for particles as unitary representations of $SO(1,3)$ in CD. One could say that $SO(1,3)$ appears as an internal isometry group of an observer's perceptive field represented by CD and Poincare group as an external symmetry group treating the observer as a physical object.

2. By separation of variables the spinor harmonics in CD factorize phases depending on the mass of the particle determined by CP_2 and spinor harmonic of hyperbolic 3-space $H^3 = SO(1,3)/SO(3)$. $SO(1,3)$ allows an extremely rich set of representations in the hyperbolic space H^3 analogous to spherical harmonics. A given infinite discrete subgroup $\Gamma \subset SO(1,3)$ defines a fundamental domain of Γ as a double coset space $\Gamma \setminus SO(1,3)/SO(3)$. This fundamental domain is analogous to a lattice cell of condensed matter lattice defined by periodic boundary conditions. The graphics of Escher give an idea about these structures in the case of H^2. The products of wave functions defined in $\Gamma \subset SO(1,3)$ and of wave functions in Γ define a wave function basis analogous to the space states in condensed matter lattice.

3. TGD allows gravitational quantum coherence in arbitrarily long scales and I have proposed that the tessellations of H^3 define the analogs of condensed matter lattices at the level of cosmology and astrophysics [L16]. The unitary representations of $SO(1,3)$ would be central for quantum gravitation at the level of gravitationally dark matter. They would closely relate to the unitary representations of the supersymplectic group of $\delta M^4_+ \times CP_2$ in M^4 degrees of freedom and define their continuations to the entire CD.

4. There exists a completely unique tessellation known as icosa tetrahedral tessellation consisting of icosahedrons, tetrahedrons, and octahedrons glued along boundaries together. I have proposed that it gives rise to a universal realization of the genetic code of which biochemical realizations is only a particular example [L1, L9]. Also this supports a deep connection between biology and quantum gravitation emerging also in classical TGD [L6, L5]. Also electromagnetic long range classical fields are predicted to be involved with long length scale quantum coherence [L10].

The challenge is to understand the implications of this picture for M^8-H duality [L17]. The discretization of M^8 identified as octonions O with the Minkowskian norm defined by $Re(Im(o^2))$ is linear M^8 coordinates natural for octonions. The discretization obtained by the requirement that the coordinates of the points of M^8 (momenta) are algebraic integers in an algebraic extension of rationals would make sense also in p-adic number fields.

In the Robertson-Walker coordinates for the future light-cone M^4_+, sliced by H^3:s. In M^8, the analog of time coordinate is defined by mass and in H by the light-cone proper time. Hyperbolic angle and spherical angles defined the coordinates of H^3. The discretizations defined by the spaces $\Gamma \setminus SO(1,3)/SO(3)$ would define a discretization and one can define an infinite hierarchy of discretizations defined by the discrete subgroups of $SO(1,3)$ with matrix elements belonging to an extension of rationals. This number theoretically universal discretization defines a natural alternative for the linear discretization. Maybe the linear *resp.* non-linear discretization could be assigned to the moduli space of CDs *resp.* CD.

4 Could TGD allow the detection of gravitons?

Could an effectively 2-dimensional system make it possible to observe real gravitons and why should this be the case? What about couplings to quantum coherent many-particle states such as electrons or protons at light-like 3-surfaces assignable to the FQHE in the TGD description?

4.1 Could TGD allow analogs of chiral gravitons?

Could TGD say something interesting about these proposals? Could gravitons identified in terms of the M^4 spinor connection serve as the TGD analogs of chiral gravitons. The term chiral graviton suggests that they have definite chirality in the sense that they violate parity symmetry so that their couplings are chiral just like the couplings of weak bosons.

The classical gravitation is not chiral. Chirality might reflect the fact that Chern-Simons action violates chirality as an action for 3-D space. In the TGD, Chern-Simons-Kähler action is defined for a light-like 3-surface whereas reflection acts in M^4. Parity symmetry would require that the holomorphic spinor connection for coordinates associated with Hamilton-Jacobi structure is vectorial.

Could the self-dual Kähler form, having a physical interpretation as a presence of parallel electric and magnetic fields of identical strength, induce a parity violation at the fundamental level? I have proposed that CP could be violated and that this could explain the CP violation in hadron physics and the matter-antimatter asymmetry. Could the TGD counterparts of chiral gravitons couple vectorially but violate the CP symmetry?

4.2 Is the detection of gravitons possible in FQHE type systems?

Could the interaction vertices of electrons with a real graviton sum up to a very large number of identical amplitudes when the wavelength of graviton is much longer than the size of the partonic 2-surface? One would obtain an analog of diffraction, somewhat like in the TGD based models of the recently discovered gravitational hum identified in the TGD framework as diffraction in astrophysical length scales [L16].

One might hope that the coupling of real gravitons to condensed matter gravitons and/or nanoscale quantum coherence make the graviton absorption amplitude proportional to the square of the total number N of electrons.

1. FQHE occurs in 2-D electron gas and (see this) and the typical densities of electrons are of order $10^{11}/cm^2$. For an area cm^2 one would have $N^2 \sim 10^{22}$.

2. The differential cross section from a particle with mass m [B5] (see this) is

$$\frac{d\sigma}{d\Omega} = \frac{G^2 m^2}{sin^4(\theta)}(cos^8(\theta) + sin^8(\theta)) \ .$$

   For the electron the order of magnitude is $\sigma \sim 10^{-42} l_e^2$, where l_e is electron's Compton length, unless θ is very near to the forward direction. There is no hope of detecting gravitons in this way unless one has analog of forward scattering. Even if quantum coherence occurs, the hopes for detection seem rather meager. If θ is of order 10^{-5}, the order of magnitude for the differential cross section is of the order of l_e^2.

4.3 What about dark protons at the monopole flux tubes and half-monopole flux tubes?

Proton mass is roughly 2000 times larger than electron mass and they are more promising.

1. Could the dark protons at magnetic monopole flux tubes give rise to a similar quantum coherence amplifying the interactions with gravitons. Monopole flux tube condensates involve a very large number of parallel monopole flux tubes, which form a quantum coherent region. Could the quantum coherent scattering of gravitons lead to observable effects via the exchange of momentum with ordinary matter despite the fact that the dark matter is not directly observable using the recent technology.

2. The number of monopole flux tubes corresponds to h_{eff}/h and this can be as large as 10^{14}. This would give a factor of order $N^2 \sim 10^{28}$ to the scattering cross section. In the case of dark protons, one would have a scaling factor $(m_p/m_e)^2 \sim 4 \times 10^6$. This would give a factor of order 10^{34} giving $\sigma \sim 10^{-8} l_e^2 \sim 10^{-2} l_p^2$. Could this make the detection possible?

3. Half-monopole flux tubes appear in the TGD based model for the transition to superconductivity as an intermediate, not yet superconducting, flux tubes carrying dark electrons but not their Cooper pairs [L20]: the pair of dark electron and corresponding hole at the level of ordinary matter replaces the notion of Bogoliubov quasiparticle as a superposition of electron and hole in such a way that the total fermion number is conserved. Half-monopole flux tubes have boundaries, which should be light-like and can be so as static structure in the induced geometry [L8], which could carry dark protons.

   Note that also the light-like surfaces associated with the Quantum Hall systems would be naturally half-monopole flux tubes since electrons in these systems are known to form bound states with magnetic fluxes.

REFERENCES

Theoretical Physics

[B1] Son DT. Newton-Cartan Geometry and the Quantum Hall Effect, 2013. Available at: https://arxiv.org/pdf/1306.0638.pdf.

[B2] Liou et al. Chiral gravitons in fractional quantum hall liquids. *Phys. Rev. Lett.*, 123:146801, 2019. Available at: https://arxiv.org/abs/1106.3375.

[B3] Haldane FDM. Geometrical description of the fractional quantum Hall effect. *Phys. Rev. Lett.*, 107:116801, 2011. Available at: https://arxiv.org/abs/1106.3375.

[B4] Son DT Gromow A. Bimetric theory of fractional Quantum Hall states. *Phys Rev X*, 7(4):041032, 2017. Available at: https://arxiv.org/abs/1705.06739.

[B5] Rothman T and Boughn S. Can Gravitons Be Detected?, 2006. Available at: https://arxiv.org/pdf/gr-qc/0601043.pdf.

Condensed Matter Physics

[D1] Liang J et al. Evidence for chiral graviton modes in fractional quantum Hall liquids. *Nature*, 628:78–83, 2024. Available at: https://www.nature.com/articles/s41586-024-07201-w.

Books related to TGD

[K1] Pitkänen M. Quantum Hall effect and Hierarchy of Planck Constants. In *TGD and Condensed Matter. https://tgdtheory.fi/tgdhtml/BTGDcondmat.html*. Available at: https://tgdtheory.fi/pdfpool/anyontgd.pdf, 2023.

[K2] Pitkänen M. WCW Spinor Structure. In *Quantum Physics as Infinite-Dimensional Geometry. https://tgdtheory.fi/tgdhtml/Btgdgeom.html*. Available at: https://tgdtheory.fi/pdfpool/cspin.pdf, 2023.

Articles about TGD

[L1] Pitkänen M. Is genetic code part of fundamental physics in TGD framework? Available at: https://tgdtheory.fi/public_html/articles/TIH.pdf., 2021.

[L2] Pitkänen M. TGD and Condensed Matter. https://tgdtheory.fi/public_html/articles/TGDcondmatshort.pdf., 2021.

[L3] Pitkänen M. About TGD counterparts of twistor amplitudes: part I. https://tgdtheory.fi/public_html/articles/twisttgd1.pdf., 2022.

[L4] Pitkänen M. About TGD counterparts of twistor amplitudes: part II. https://tgdtheory.fi/public_html/articles/twisttgd2.pdf., 2022.

[L5] Pitkänen M. Comparison of Orch-OR hypothesis with the TGD point of view. https://tgdtheory.fi/public_html/articles/penrose.pdf., 2022.

[L6] Pitkänen M. How animals without brain can behave as if they had brain. https://tgdtheory.fi/public_html/articles/precns.pdf., 2022.

[L7] Pitkänen M. Intersection form for 4-manifolds, knots and 2-knots, smooth exotics, and TGD. https://tgdtheory.fi/public_html/articles/finitefieldsTGD.pdf., 2022.

[L8] Pitkänen M. TGD inspired model for freezing in nano scales. https://tgdtheory.fi/public_html/articles/freezing.pdf., 2022.

[L9] Pitkänen M. About tessellations in hyperbolic 3-space and their relation to the genetic code . https://tgdtheory.fi/public_html/articles/tessellationH3.pdf., 2023.

[L10] Pitkänen M. About long range electromagnetic quantum coherence in TGD Universe. https://tgdtheory.fi/public_html/articles/hem.pdf., 2023.

[L11] Pitkänen M. About Platonization of Nuclear String Model and of Model of Atoms. https://tgdtheory.fi/public_html/articles/nuclatomplato.pdf., 2023.

[L12] Pitkänen M. About the Relationships Between Weak and Strong Interactions and Quantum Gravity in the TGD Universe . https://tgdtheory.fi/public_html/articles/SW.pdf., 2023.

[L13] Pitkänen M. Holography and Hamilton-Jacobi Structure as 4-D generalization of 2-D complex structure. https://tgdtheory.fi/public_html/articles/HJ.pdf., 2023.

[L14] Pitkänen M. New result about causal diamonds from the TGD view point of view. https://tgdtheory.fi/public_html/articles/CDconformal.pdf., 2023.

[L15] Pitkänen M. Symmetries and Geometry of the "World of Classical Worlds" . https://tgdtheory.fi/public_html/articles/wcwsymm.pdf., 2023.

[L16] Pitkänen M. The TGD view of the recently discovered gravitational hum as gravitational diffraction. https://tgdtheory.fi/public_html/articles/gravhum.pdf., 2023.

[L17] Pitkänen M. A fresh look at M^8H duality and Poincare invariance. https://tgdtheory.fi/public_html/articles/TGDcritics.pdf., 2024.

[L18] Pitkänen M. About the Recent TGD Based View Concerning Cosmology and Astrophysics. https://tgdtheory.fi/public_html/articles/3pieces.pdf., 2024.

[L19] Pitkänen M. Modified Dirac equation and the holography=holomorphy hypothesis. https://tgdtheory.fi/public_html/articles/modDir.pdf., 2024.

[L20] Pitkänen M. What happens in the transition to superconductivity? https://tgdtheory.fi/public_html/articles/halfmonopole.pdf., 2024.