About TGD counterparts of twistor amplitudes: part II

September 20, 2022

Matti Pitkänen

Email: matpitka6@gmail.com.
http://tgdtheory.com/public_html/.
Recent postal address: Rinnekatu 2-4 A 8, 03620, Karkkila, Finland.

Abstract

This article is the second part of the article devoted to the construction of scattering amplitudes in the TGD framework based on the twistor lift of TGD.

External particles are Galois singlets consisting of off-mass shell quarks with mass squared values coming as roots of the polynomial P characterizing the interaction region. External particles are characterized by polynomials P_i satisfying $P_i(0) = 0$. P is identified as the functional composite of P_i since it inherits the masses of incoming particles as their roots. This allows only cyclic permutations of P_i. The scattering event is essentially a re-combination of incoming Galois singlets to new Galois singlets and quarks propagate freely: hence OZI rule generalizes. Also a connection with the dual resonance models emerges.

Unitary, locality, and the failure to find twistorial counterparts of non-planar Feynman diagrams are the basic problems of the twistor Grassmannian approach. Also the non-existence of twistor spaces for most Riemannian manifolds is a problem in GRT framework but in TGD the existence of twistor spaces for M^4 and CP_2 solves this problem. In the TGD framework, the replacement of point-like particles with 3-surfaces leads to the loss of locality at the fundamental level. The analogs of non-planar diagrams are eliminated since only cyclic permutations of P_i are allowed.

This leaves only the problem with unitarity. The TGD counterpart of unitarity realized in terms of Kähler geometry of fermionic state space is very natural in the geometrization of quantum physics. Scattering probabilities are identified as products of covariant and contravariant matrix elements of the metric, and unitary conditions are replaced by the definition of the contravariant metric. Probabilities are complex but real and imaginary parts are separately conserved. The interpretation in terms of Fisher information is possible. Due to the infinite-D character of the state space, the Kähler geometry exists only if it has a maximal group of isometries and is a unique constant curvature geometry. Also the interpretation of this approach in zero energy ontology is discussed.

Objections and critical questions are the best way to make progress and to see which assumptions might not be final. For instance, twistor holomorphy, M^4 conformal symmetry number theoretically, and many other arguments strongly suggest that free quark spinors do not satisfy $D(H)\Psi = 0$ but $D(M^4)\Psi = 0$ and are therefore massless. The propagation of any massive particle along a light-like geodesic is however effectively massless and CP_2 type extremals have light-like M^4 projection so that one must leave this issue open.

There are physical motivations for considering the number theoretic generalizations of the amplitudes. For an iterate of fixed P (say large number of gravitons), the roots of the iterate of P defined virtual mass squared values, approach to the Julia set of P. The construction of scattering amplitudes thus leads to chaos theory at the limit of an infinite number of identical particles. The construction generalizes also to the surfaces defined by real analytic functions and the fermionic variant of Riemann zeta and elliptic functions are discussed as examples.

Contents

1 Introduction

This article is the second part of an article devoted to the construction of scattering amplitudes in the TGD framework based on the twistor lift of TGD. In the first part of the article, the general proposal for the construction of the scattering amplitudes was discussed.

1.1 Scattering as recombination of quarks to Galois singlets

The view about scattering event is as follows.

1. External particles are Galois singlets consisting of off-mass shell massless quarks with mass squared values coming as roots of the polynomial P characterizing the interaction region. External particles are characterized by polynomials P_i satisfying $P_i(0) = 0$. P is identified as the functional composite of P_i since it inherits the roots (mass squared values) of the incoming particles. The TGD view about cognitive state function reduction [L13] allows only cyclic permutations of P_i in the superposition.

2. The scattering event is essentially a re-combination of incoming Galois singlets to new Galois singlets and quarks propagate freely: hence OZI rule generalizes. Also a connection with the dual resonance models emerges. Finiteness is manifest since the integration of virtual moments is restricted to a summation over a finite number of mass shells.

1.2 What about unitarity?

Unitary, locality, and the failure to find twistorial counterparts of non-planar Feynman diagrams are the basic problems of the twistor Grassmannian approach. Also the non-existence of twistor spaces for most Riemannian manifolds is a problem in the GRT framework but disappears in the TGD framework. Even more, the Kähler property of twistor spaces for M^4 and CP_2 makes TGD unique. In the TGD framework, the replacement of point-like particles with 3-surfaces leads to the loss of locality at the fundamental level. The analogs of non-planar diagrams are eliminated since only cyclic permutations of P_i are allowed.

This leaves only the problem with unitarity discussed in the first section of the article. The TGD counterpart of unitarity realized in terms of Kähler geometry of fermionic state space is very natural in the geometrization of quantum physics [L14]. Scattering probabilities are identified as products of covariant and contravariant matrix elements of the metric, and unitary conditions are replaced by the definition of the contravariant metric. Probabilities are complex but real and imaginary parts are separately conserved. The interpretation in terms of Fisher information is possible. Due to the infinite-D character of the state space, the Kähler geometry exists only if it has a maximal group of isometries and is a unique constant curvature geometry. This approach is very natural in zero energy ontology [K14].

1.3 Objections and critical questions

Objections and critical questions are the best way to make progress by making the picture more precise, and allowing us to see which assumptions might not be final. For instance, twistor holomorphy, M^4 conformal symmetry number theoretically, and many other arguments strongly suggest that free quark spinors do not satisfy $D(H)\Psi = 0$ but $D(M^4)\Psi = 0$ and are therefore massless. The propagation of any massive particle along a light-like geodesic is however effectively massless and CP_2 type extremals have light-like M^4 projection so that one must leave this issue open.

1.4 Number theoretical generalizations of scattering amplitudes

Number theoretical generalizations of the scattering amplitudes are also physically highly interesting. For an iterate of a fixed P satisfying $P(0) = 0$ (scattering of a large number of gravitons, say), the roots of the iterate of P defining the virtual mass squared values, approach the Julia set of P. The construction of scattering amplitudes thus leads to chaos theory at the limit of an infinite number of identical particles.

The construction generalizes also to the surfaces defined by real analytic functions and the fermionic variant of Riemann zeta and elliptic functions are discussed as examples. The interpretation of Galois confinement as conformal confinement [K6] is natural in these situations.

2 What about unitarity?

Unitarity is a poorly understood problem of the twistor approach and also of TGD.

2.1 What do we mean with time evolution?

The first questions relate to the identification of the TGD counterpart of S-matrix.

1. Zero energy states correspond to superpositions of pairs of ordinary 3-D states assignable to the opposite boundaries of CD. The simplest assumption corresponds to the idea about state preparation is that the states are unentangled. Unitarity would mean that the 3-D zero energy states at the active boundary of CD are orthogonal if the 3-D states at the passive boundary of CD are orthogonal. The scattering amplitudes considered in this article would naturally correspond to zero energy states. Is there any reason for zero energy states to satisfy this kind of orthogonality?

2. The time evolutions between "small" state function reductions (SSFRs) are assumed to increase the size of CD in a statistical sense at least and affect the states at the active boundary of CD but leave the "visible" part of the state at the passive boundary unaffected. These time evolutions are proposed to correspond to the scalings of CD rather than time translations. In this case unitarity would look a reasonable property.

   The sequence of (ordinary) "big" SFRs (BSFRs) could allow approximate description as being associated with unitary time evolutions with time translations rather than scalings and followed by BSFR changing the arrow of time. The characteristic features of these time evolutions would be polynomial and exponential decay and the relaxation of spin glass would be a key example about time evolution by SSFRs [L20].

2.2 What really occurs in BSFR?

It has been assumed hitherto that a time reversal occurs in BSFR. The assumption that SSFRs correspond to a sequence of time evolutions identified as scalings, forces to challenge this assumption. Could BSFR involve a time reflection T natural for time translations or inversion $I : T \rightarrow 1/T$ natural for the scalings or their combination TI?

I would change the scalings increasing the size of CD to scalings reducing it. Could any of these options: time reversal T, inversion I, or their combination TI take place in BSFRs whereas arrow would remain as such in SSFRs? T (TI) would mean that the active boundary of CD is frozen and CD starts to increase/decrease in size.

There is considerable evidence for T in BSFRs identified as counterparts of ordinary SFRs but could it be accompanied by I?

1. Mere I in BSFR would mean that CD starts to decrease but the arrow of time is not changed and passive boundary remains passive boundary. What comes to mind is blackhole collapse.

   I have asked whether the decrease in size could take place in BSFR and make it possible for the self to get rid of negative subjective memories from the last moments of life, start from scratch and live a "childhood". Could this somewhat ad hoc looking reduction of size actually take place by a sequence of SSFRs? This brings into mind the big bang and big crunch. Could this period be followed by a BSFR involving inversion giving rise to increase of the size of CD as in the picture considered hitherto?

2. If BSFR involves TI, the CD would shift towards a fixed time direction like a worm, and one would have a fixed arrow of time from the point of view of the outsider although the arrow of time would change for sub-CD. This modified option might be consistent with the recent picture, in particular with the findings made in the experiments of Minev *et al* [L7] [L7].

   This kind of shifting must be assumed in the TGD inspired theory of consciousness. For instance, after images as a sequence of time reversed lives of sub-self, do not remain in the geometric past but follow the self in travel through time and appear periodically (when their arrow of time is the same as of self). The same applies to sleep: it could be a period with a reversed arrow of time but the self would shift towards the geometric future during this period: this could be interpreted as a shift of attention towards the geometric future. Also this option makes it possible for the self to have a "childhood"".

3. However, the idea about a single arrow of time does not look attractive. Perhaps the following observation is of relevance. If the arrow of time for sub-CD correlates with that of sub-CD, the change of the arrow of time for CD, would induce its change for sub-CDs and now the sub-CDs would increase in the opposite direction of time rather than decrease.

2.3 Should unitarity be replaced with the Kähler-like geometry of the fermionic state space?

After these preliminaries we can state the key question. Is unitarity possible at all and should it be replaced with some deeper principle? I have considered these questions several times and in [L14] a rather radical solution was proposed.

Assigning an S-matrix to a unitary time evolution works in non-relativistic theory but fails already in the generic QFT and correlation functions replace S-matrix.

1. Einstein's great vision was to geometrize gravitation by reducing it to the curvature of space-time. Could the same recipe work for quantum theory? Could the replacement of the flat Kähler metric of Hilbert space with a non-flat one allow the identification of the analog of unitary S-matrix as a geometric property of Hilbert space? Kähler metric is required to geometrize hermitian conjugation. It turns out that the Kähler metric of a Hilbert bundle determined by the Kähler metric of its base space could replace the unitary S-matrix.

2. An amazingly simple argument demonstrates that one can construct scattering probabilities from the matrix elements of Kähler metric and assign to the Kähler metric a unitary S-matrix assuming that some additional conditions guaranteeing that the probabilities are real and non-negative are satisfied. If the probabilities correspond to the real part of the complex analogs of probabilities, it is enough to require that they are non-negative: complex analogs of probabilities would define the analog of the Teichmüller matrix.

   Teichmüller space parameterizes the complex structures of Riemann surface: could the allowed WCW Kähler metrics - or rather the associated complex probability matrices - correspond to complex structures for some space? By the strong form of holography (SH), the most natural candidate would be Cartesian product of Teichmüller spaces of partonic 2 surfaces with punctures and string world sheets.

3. Under some additional conditions one can assign to Kähler metric a unitary S-matrix but this does not seem necessary. The experience with loop spaces suggests that for infinite-D Hilbert spaces the existence of non-flat Kähler metric requires a maximal group of isometries. Hence one expects that the counterpart of S-matrix is highly unique.

4. In the TGD framework the "world of classical worlds" (WCW) has Kähler geometry allowing spinor structure. WCW spinors correspond to Fock states for second quantized spinors at space-time surface and induced from second quantized spinors of the embedding space. Scattering amplitudes would correspond to the Kähler metric for the Hilbert space bundle of WCW spinor fields realized in zero energy ontology and satisfying Teichmüller condition guaranteeing non-negative probabilities.

5. Equivalence Principle generalizes to the level of WCW and its spinor bundle. In ZEO one can assign also to the Kähler space of zero energy states spinor structure and this strongly suggests an infinite hierarchy of second quantizations starting from space-time level, continuing at the level of WCW, and continuing further at the level of the space of zero energy states. This would give an interpretation for an old idea about infinite primes as an infinite hierarchy of second quantizations of an arithmetic quantum field theory.

6. There is also an objection. The transition probabilities would be given by $P(A,B) = g^{A,\overline{B}}g_{\overline{B},A}$ and the analogs for unitarity conditions would be satisfied by $g^{A,\overline{B}}g_{\overline{B},C} = \delta^A_C$. The problem is that $P(A,B)$ is not real without further conditions. Can one imagine any physical interpretation for the imaginary part of $Im(P(A,B))$?

In this framework, the twistorial scattering amplitudes as zero energy states define the covariant Kähler metric $g_{A\overline{B}}$, which is non-vanishing between the 3-D state spaces associated with the opposite boundaries of CD. $g^{A\overline{B}}$ could be constructed as the inverse of this metric. The problem with the unitarity would disappear.

2.3.1 Explicit expressions for scattering probabilities

The proposed identification of scattering probabilities as $P(A \to B) = g^{A\overline{B}}g_{\overline{B}A}$ in terms of components of the Kähler metric of the fermionic state space.

Contravariant component $g^{A\overline{B}}$ of the metric is obtained as a geometric series $\sum_{n\geq 0} T^n$ from from the deviation $T_{A\overline{B}} = g_{A\overline{B}} - \delta_{A\overline{B}}$ of the covariant metric $g_{A\overline{B}}$ from $\delta A\overline{B}$.

g this is not a diagonal matrix. It is convenient to introduce the notation Z^A, $A \in \{1,...,n\}$ $Z^{\overline{A}} = Z^{n+k}$, $k = n+1,...,2n$. So that the $g_{\overline{B}C}$ corresponds to $g_{k+n,l} = \delta_{k,l} + T_{k,l}$. and one has $g^{A\overline{B}}$ to $g^{k,l+n} = \delta_{k,l} + T^1_{k,l}$.

The condition $g^{A\overline{B}}g_{\overline{B}C} = \delta^A_C$ gives

$$g^{k,l+n}g_{l+n,m} = \delta^k_m \quad . \tag{2.1}$$

giving

$$\sum_l (\delta_{k,l} + T^1_{k,l})(\delta_{l,m} + T_{l,m}) = \delta_{k,m} + (T^1 + T + T^1T)_{km} = \delta_{k,m} \quad . \quad . \tag{2.2}$$

which resembles the corresponding condition guaranteeing unitarity. The condition gives

$$T_1 = -\frac{T}{1+T} = -\sum_{n>1}((-1)^n T^n . \quad . \tag{2.3}$$

The expression for $P(A \to B)$ reads as

$$\begin{array}{l} P(A \to B) = g^{A\overline{B}}g_{\overline{B}A} \\ = [1 - \frac{T}{1+T} + T^\dagger - (\frac{T}{1+T})_{AB}T^\dagger]_{AB} \quad . \end{array} \tag{2.4}$$

It is instructive to compare the situation with unitary S-matrix $S = 1 + T$. Unitarity condition $SS^\dagger = 1$ gives

$$T^\dagger = -\frac{T}{1+T} \quad ,$$

and

$$P(A \to B) = \delta_{AB} + T_{AB} + T^\dagger_{AB} + T^\dagger_{AB}T_{AB} = [\delta_{AB} - (\frac{T}{1+T})_{AB} + T_{AB} - (\frac{T}{1+T})_{AB}T_{AB} \quad .$$

The formula is the same as in the case of Kähler metric.

2.4 Critical questions

One can pose several critical questions helping to further develop the proposed number theoretic picture.

2.4.1 Is mere recombinatorics enough as fundamental dynamics?

Fundamental dynamics as mere re-combination of free quarks to Galois singlets is attractive in its simplicity but might be an over-simplification. Can quarks really continue with the same momenta in each SSFR and even BSFR?

1. For a given polynomial P, there are several Galois singlets with the same incoming integer-valued total momentum p_i. Also quantum superpositions of different Galois singlets with the same incoming momenta p_i but fixed quark and antiquark numbers are in principle possible. One must also remember Galois singlet property in spin degrees of freedom.

2. WCW integration corresponds to a summation over polynomials P with a common ramified prime (RP) defining the p-adic prime. For each P of the Galois singlets have different decomposition to quark momenta.

   One can even consider the possibility that only the total quark number as the difference of quark and antiquark numbers is fixed so that polynomials P in the superposition could correspond to different numbers of quark-antiquark pairs.

3. One can also consider a generalization of Galois confinement by replacing classical Galois singlet property with a Galois-singlet wave function in the product of quark momentum spaces allowing classical Galois non-singlets in the superposition.

   Hydrogen atom serves as an illustration: electron at origin would correspond to classical ground state and s-wave correspond to a state invariant under rotations such that the position of electron is not anymore invariant under rotations. The proposal for transition amplitudes remains as such otherwise.

Note however that the basic dynamics at the level of a single polynomial would be recombinatorics for all these options.

2.4.2 General number theoretic picture of scattering

Only the interaction region has been considered hitherto. One must however understand how the interaction region is determined by the 4-surfaces and polynomials associated with incoming Galois singlets. Also the details of the map of p-adic scatting amplitude to a real one must be understood.

The intuitive picture about scattering is as follows.

1. The incoming particle "i" is characterized by p-adic prime p_i, which is RP for the corresponding 4-surface in M^8. Also the "adelic" option that all RPs characterize the particle, is considered below.

2. The interaction region corresponds to a polynomial P. The integration over WCW corresponds to a sum over several P:s with at least one common RP allowing to map the superposition of amplitudes to real amplitude by canonical identification I: $\sum x_n p^n \to \sum x_n p^{-n}$.

   If one gives up the assumption about a shared RP, the real amplitude is obtained by applying I to the amplitudes in the superposition such that RP varies. Mathematically, this is an ugly option.

3. If there are several shared RPs, in the superposition over polynomials P, one can consider an adelic picture. The amplitude would be mapped by I to a product of the real amplitudes associated with the shared RP:s. This brings in mind the adelic theorem stating that rational number is expressible as a product of the inverses of its p-adic norms. The map I indeed generalizes the p-adic norm as a map of p-adics to reals. Could one say that the real scattering amplitude is a product of canonical images of the p-adic amplitudes for the shared RP:s? Witten has proposed this kind of adelic representation of real string vacuum amplitude.

Whether the adelization of the scattering amplitudes in this manner makes sense physically is far from clear. In p-adic thermodynamics one must choose a single p-adic prime p as RP. Sum over ramified primes for mass squared values would give CP_2 mass scale if there are small p-adic primes present.

The incoming polynomials P_i should determine a unique polynomial P assignable to the interaction regions as CD to which particles arrive. How?

1. The natural requirement would be that P possess the RPs associated with P_i:s. This can be realized if the condition $P_i = 0$ is satisfied and P is a functional composite of polynomials P_i. All permutations π of $1, ..., n$ are allowed: $P = P_{i_1} \circ P_{i_2} \circ ....P_{i_n}$ with $(i_1, ...i_n) = (\pi(1), ..., \pi(n))$. P possesses the roots of P_i.

   Different permutations π could correspond to different permutations of the incoming particles in the proposal for scattering amplitudes so that the formation of area momenta $x_{i+1} = \sum_{k=1}^{i} p_k$ in various orders would corresponds to different orders of functional compositions.

2. Number theoretically, interaction would mean composition of polynomials. I have proposed that so-called cognitive measurements as a model for analysis could be assigned with this kind of interaction [L13, L15]. The preferred extremal property realized as a simultaneous extremal property for both Kähler action and volume action suggests that the classical non-determinism due to singularities as analogs of frames for soap films serves as a classical correlate for quantum non-determinism [L23].

3. If each incoming state "i" corresponds to a superposition of P_i:s with some common RPs, only the RP:s shared by all compositions P from these would appear in the adelic image. If all polynomials P_i are unique (no integration over WCW for incoming particles), the canonical image of the amplitude could be the product over images associated with common RPs.

   The simplest option is that a complete localization in WCW occurs for each external state, perhaps as a result of cognitive state preparation and reduction, so that P has the RP:s of P_is as RP:s and adelization could be maximal.

2.4.3 Do the notions of virtual state, singularity and resonance have counterparts?

Is the proposal physically acceptable? Does the approach allow to formulate the notions of virtual state, singularity and resonance, which are central for the standard approach?

1. The notion of virtual state plays a key role in the standard approach. On-mass-shell internal lines correspond to singularities of S-matrix and in a twistor approach for $\mathcal{N} = 4$ SUSY, they seem to be enough to generate the full scattering amplitudes.

   If only off-mass-shell scattering amplitudes between on- mass-shell states are allowed, one can argue that only the singularities are allowed, which is not enough.

2. Resonance should correspond to the factorization of S-matrix at resonance, when the intermediate virtual state reduces to an on-mass-shell state. Can the approach based on Kähler metric allow this kind of factorization if the building brick of the scattering amplitudes as the deviation of the covariant Kähler metric from the unit matrix $\delta_{A\overline{B}}$ is the basic building bricks and defined between on mass shell states?

   Note that in the dual resonance model, the scattering amplitude is some over contribution of resonances and I have proposed that a proper generalization of this picture could make sense in the TGD framework.

The basic question concerns the number theoretical identification of on-mass-shell and off-mass-shell states.

1. Galois singlets with integer valued momentum components are the natural identification for on-mass-shell states. Galois non-singlet would be off-mass-shell state naturally having complex quark masses and momentum components as algebraic integers.

Virtual states could be arbitrary states without any restriction to the components of quark momentum except that they are in the extension of rationals and the condition coming from momentum conservation, which forces intermediate states to be Galois singlets or products of them.

Therefore momentum conservation allows virtual states as on mass shell states, that is intermediate states, which are Galois singlets but consist of Galois non-singlets identified as off-mass-shell lines. The construction of bound states formed from Galois non-singlets would indeed take place in this way.

2. The expansion of the contravariant part of the scattering matrix $T_1 = T/(1+T)$ appearing in the probability

$$P(A \to B) = g^{A\overline{B}} g_{\overline{B}A} \\ = [1 - \frac{T}{1+T} + T^{\dagger} - (\frac{T}{1+T})_{AB} T^{\dagger}]_{AB} \ .$$

would give a series of analogs of diagrams in which Galois singlets of intermediate states are deformed to non-singlets states.

3. Singularities and resonances would correspond to the reduction of an intermediate state to a product of Galois singlets.

2.4.4 What about the planarity condition in TGD?

The simplest proposal inspired by the experience with the twistor amplitudes is that only planar polygon diagrams are possible since otherwise the area momenta are not well-defined. In the TGD framework, there is no obvious reason for not allowing diagrams involving permutations of external momenta with positive energies *resp.* negative energies since the area momenta $x_{i+1} = \sum_{k=1}^{i} p_k$ are well-defined irrespective of the order. The only manner to uniquely order the Galois singlets as incoming states is with respect to their mass squared values given by integers.

2.4.5 Generalized OZI rule

In TGD, only quarks are fundamental particles and all elementary particles and actually all physical states in the fermionic sector are composites of them. This implies that quark and antiquark numbers are separately conserved in the scattering diagrams and the particle reaction only means the-arrangement of the quarks to a new set of Galois singlets.

At the level of quarks, the scattering would be completely trivial, which looks strange. One would obtain a product of quark propagators connecting the points at mass shells with opposite energies plus entanglement coefficients arranging them at positive and negative energy light-cones to groups which are Galois singlets.

This is completely analogous to the OZI role. In QCD it is of course violated by generation of gluons decaying to quark pairs. In TGD, gauge bosons are also quark pairs so that there is no problem of principle.

There is an objection against this picture.

1. If particle reactions are mere recombinations of Galois singlets with Galois singlets, the quark and antiquark numbers N_q and $N_{\overline{q}}$ of quark and antiquark numbers are separately conserved (as also their difference $N_q - N_{\overline{q}}$). This forbids many reactions, for instance those in which a gauge boson is emitted unless one assumes that many quark states are superpositions of states with a varying total quark number N. This would mean that the extremely simple re-combinatorics picture is lost.

2. Crossing symmetry, which is a symmetry of standard QFTs, suggests a solution to the problem. Crossing symmetry would mean that one can transfer quarks between initial and final states by changing the sign of the quark four-momentum so that momentum conservation is not violated. Crossing means analytic continuation of the scattering amplitude by replacing incoming (outgoing) momentum p with outgoing (incoming) momentum $-p$. The scattering amplitudes for reactions for which the quark number is conserved can be constructed using mere recombinatorics, and the remaining amplitudes would be obtained by crossing.

3. Crossing must respect the Galois singlet property. For instance, the crossing of a single quark destroys Galois singlet. Unless one allows destruction and recombination of Galois singlets, the crossing can apply only to Galois singlets. These rules bring to mind the vanishing of twistor amplitudes when one gluon has negative helicity and the remaining gluons have positive helicity.

2.5 Western and Eastern ontologies of physics

This picture forces us to ask whether something deeper might lurk behind the usual ideas about particle physics in which scattering rates encode the information. Could the imaginary part of $P(A,B)$ have a well-defined physical meaning in some more general framework?

1. In ZEO, single classical time evolution and zero energy state as a pair of initial and final states becomes the basic entity. One can even ask whether it might make sense to speak about probability density for different zero energy states as time evolutions, events.

   Could the "western" view about existing reality evolving in time be replaced with an ontology in which events in both classical sense (zero energy states) and quantum transitions would be what really exists.

   In the "eastern" view, the relevant probabilities would not be for transitions $A \to B$ for a given state A but for the occurrence of these transitions $A \to B$ in given state, whatever its definition might be, and one would measure the relative rates for occurrence for the various transitions $A \to B$.

   The ensemble would not consist of entities A but transitions $A \to B$. In biology and neuroscience, the states are indeed replaced with behaviors. IKn computer science the program, rather than the state of the computer, is the basic notion.

2. In order to develop this picture at the level of scattering amplitudes, one could start from the QFT description for the n-point correlation functions used to construct S-matrix. One adds to the exponent of action a term, which is a combination of small current terms assignable to external particles and calculates functional Taylor series with respect to the small parameters. The Taylor coefficients are identified as n-point functions.

   In QFTs this is regarded as a mere calculational trick and the "state" defined by the exponential as an analog of that in statistical physics is defined by the exponential of action when the values of the parameters vanish.

   One can of course ask what it would mean if these parameters do not vanish. In perturbation theory one actually has this situation. These deformed states look formally like coherent states. Could the physical states at a deeper level correspond to these analogs of coherent states as analogs of thermo-dynamical states?

3. TGD can be formally regarded as a complex square root of thermodynamics, which suggests a generalization of the formulation of quantum theory as algebraic QFT promoted for instance by Connes [A1], and this is what this new interpretation would mean also physically.

4. In the TGD framework, one would add to the exponent of $exp(-K)$ a superposition of oscillator operator monomials of quark oscillator operators creating positive and negative energy parts of the zero energy states with complex coefficients Z_i as parameters and essentially defining coordinates for the Hilbert space. Z_i would be analogous to the complex numbers defining coherent states.

   The exponential can be expanded and fermionic vacuum expectation forces conservation of quark number and the combination of the positive and negative energy parts to give a non-vanishing result. At the limit of infinitely large CD conservation of 4-momentum is obtained.

5. The ordinary transition amplitudes are obtained by performing the limit $Z_i \to 0$, and calculating Taylor coefficients as transition amplitudes. The analog of $G_{A,\overline{B}}$ would be obtained for the analogs 2-point functions having as arguments the parts of zero energy states and $P(A,B) = Re(G_{A,\overline{B}}G_{\overline{B},A})$ would give transition probabilities. For Kähler geometry the analog of probability conservation and unitarity would hold true.

6. That these amplitudes are obtained as second derivatives with respect to the fermionic Hilbert space complex coordinates Z_i and $\overline{Z}_j$ conforms with the interpretation of the exponential containing the additional terms as a generalization of an exponential of Kähler function associated with the fermionic degrees of freedom. Kähler metric indeed corresponds to $\partial_{Z_I}\partial_{\overline{Z}_J}K$, where K is the Kähler function.

7. Could the expressions of higher n-point functions in fermionic degrees of freedom boil down to the curvature tensor and its covariant derivatives so that quantum theory would be geometrized? If one has a constant curvature space, as strongly suggested by the mere existence of infinite-D Kähler metric, then only $G_{A,\overline{B}}$ would be needed so that it is enough to measure only the scattering probabilities (rates at infinite-volume limit for CD).

   Could the parameters Z_i be non-vanishing and define a square root of a thermodynamic state as an analog of a coherent state? If a constant curvature metric is in question, the scattering rates for non-vanishing Z_i could be expressed in terms of those for $Z_i = 0$. Could different phases of quantum theory correlate with the value ranges of the parameters Z_i?

2.6 Connection with the notion of Fisher information

The notion of Fisher information (`https://cutt.ly/GUPvF37`) relates in an interesting manner to the proposed Kähler geometrization of quantum theory.

1. Fisher information matrix F is associated with a probability density function $f(X,Z)$ for random variables X_i depending on the parameters Z_i (Z_i are denoted by θ_i in the Wikipedia article at `https://cutt.ly/GUPvF37`). Matrix F gives information about the $f(X,\theta)$, which must be deduced from the measurements of X. The matrix element F_{ij} is essentially integral over X for the the quantity $\langle \partial_{\theta_i}\partial_{\theta_j} log(f)\rangle$, where $\langle ..\rangle$ denotes the expectation obtained by integrating over X. F_{ij} determines a statistical metric and for complex parameters Z_i one obtains a Kähler metric.

2. In TGD, X would correspond to WCW coordinates and f would be analogous to the vacuum functional $exp(-K)$ but containing also a parameter dependent part defined by the combination of positive and negative energy parts of the fermionic zero energy states. The complex coefficients Z_i *resp.* $\overline{Z}_i$ of monomials of creation *resp.* annihilation operators would define the parameters. Fermionic Kähler metric would have an interpretation as Fisher information, which can be also complex valued.

3. Also the higher derivatives with respect to coefficients of zero energy states would provide information about the vacuum functional. One would have n-point functions for zero energy states possibly reducing to covariant derivatives of the analog curvature tensor. If the space of fermionic zero energy states is analog of a constant curvature space, the scattering amplitudes at the limit $Z_i = 0$ would give all the needed information needed to calculate the scattering amplitudes for $Z_i \neq 0$. $P(A,B)$ would be complex as components of the Fisher information matrix.

4. Basically, the information provided by the scattering amplitudes would be about the generalization of the vacuum functional of WCW including also the fermionic part. Scattering amplitudes would give information Kähler function of the WCW metric and about parameters Z_i.

   The scattering amplitudes indeed correlate strongly with the properties of space-time surfaces determined by polynomials. The p-adic prime p, crucial for the real scattering amplitudes as canonical images of p-adic amplitudes, corresponds to a ramified prime for P and this means localization of the vacuum functional to polynomials having a ramified prime equal to p. The number of Galois singlets in the scattering amplitude means lower bound for the degree of P.

2.7 About the relationship of Kähler approach to the standard picture

The replacement of the notion of unitary S-matrix with Kähler metric of fermionic state space generalizes the notion of unitarity. The rows of the matrix defined by the contravariant metric are orthogonal to the columns of the covariant metric in the inner product $(T \circ U)_{A\overline{B}} = T_{A\overline{C}}\eta^{\overline{C}D}U_{D\overline{B}}$, where $\eta^{\overline{C}D}$ is flat contravariant Kähler metric of state space. Although the probabilities are complex, their real parts sum up to 1 and the sum of the imaginary parts vanishes.

2.7.1 The counterpart of the optical theorem in TGD framework

The Optical Theorem generalizes. In the standard form of the optical theorem $i(T - T^\dagger)_{mm} = 2Im(T) = TT^\dagger_{m,m}$ states that the imaginary part of the forward scattering amplitude is proportional to the total scattering rate. Both quantities are physical observables.

In the TGD framework the corresponding statement

$$T^{A\overline{B}}\eta_{\overline{B}C} + \eta^{A\overline{B}}T_{\overline{B}C} + T^{A\overline{B}}T_{\overline{B}C} = 0 \ . \tag{2.5}$$

Note that here one has $G = \eta + T$, where G and T are hermitian matrices. The correspondence with the standard situation would require the definition $G = \eta + iU$. The replacement $T \to T = iU$, where U is antihermitian matrix, gives

One has

$$i[U^{A\overline{B}}\eta_{\overline{B}C} + \eta^{A\overline{B}}I_{\overline{B}C}] = U^{A\overline{B}}U_{\overline{B}C} \ . \tag{2.6}$$

This statement does not reduce to single condition but gives two separate conditions.

1. The first condition is analogous to Optical Theorem:

$$Im[\eta^{A\overline{B}}U_{C\overline{B}} + U^{A\overline{B}}\eta_{\overline{B}C}] = -Re[U^{A\overline{B}}U_{\overline{B}C}] = Re[U^{A\overline{B}}U_{C\overline{B}}] \ . \tag{2.7}$$

2. Second condition is new and reflects the fact that the probabilities are complex. It is necessary to guarantee that the sum of the probabilities reduces to the sum of their real parts.

$$Re[\eta^{A\overline{B}}U_{C\overline{B}} + U^{A\overline{B}}\eta_{\overline{B}C}] = -Im[U^{A\overline{B}}U_{C\overline{B}}] \ . \tag{2.8}$$

   The challenge would be to find a physical meaning for the imaginary parts of scattering probabilities. This is discussed in [L14]. The idea is that the imaginary parts could make themselves visible in a Markov process involving a power of the complex probability matrix.

In the applications of the optical theorem, the analytic properties of the scattering matrix T make it possible to construct the amplitude as a function of mass shell momenta using its discontinuity at the real axis. Indeed, $2Im(T)$ for the forward scattering amplitude can be identified as the discontinuity $Disc(T)$. In the recent case, this identification would suggests the generalization

$$Disc[T^{A\overline{B}}\eta_{\overline{B}C}] = T^{A\overline{B}}\eta_{\overline{B}C} + \eta^{A\overline{B}}T_{C\overline{B}} \ . \tag{2.9}$$

Therefore covariant and contravariant Kähler metric could be limits of the same analytic function from different sides of the real axis. One assigns the hermitian conjugate of S-matrix to the time reflection. Are covariant and contravariant forms of Kähler metric related by time reversal? Does this mean that T symmetry is violated for a non-flat Kähler metric.

2.7.2 The emergence of QFT type scattering amplitudes at long length scale limit

The basic objection against the proposal for the scattering amplitudes is that they are non-vanishing only at mass shells with $m^2 = n$. A detailed analysis of this objection improves the understanding about how the QFT limit of TGD emerges.

1. The restriction to the mass shells replaces cuts of QFT approach with a discrete set of masses. The TGD counterpart of unitarity and optical theorem holds true at the discrete mass shells.
2. The p-adic mass scale for the reaction region is determined by the largest ramified prime RP for the functional composite of polynomials characterizing the Galois singlets participating in the reaction. For large values of ramified prime RP for the reaction region, the p-adic mass scale increases and therefore the momentum resolution improves.
3. For large enough RP below measurement resolution, one cannot distinguish the discrete sequence of poles from a continuum, and it is a good approximation to replace the discrete set of mass shells with a cut. The physical analogy for the discrete set of masses along the real axis is as a set of discrete charges forming a linear structure. When their density becomes high enough, the description as a line charge is appropriate and in 2-D electrostatistics this replaces the discrete set of poles with a cut.

This picture suggests that the QFT type description emerges at the limit when RP becomes very large. This kind of limit is discussed in the article considering the question whether a notion of a polynomial of infinite degree as an iterate of a polynomial makes sense [L17]. It was found that the number of the roots is expected to become dense in some region of the real line so that effectively the QFT limit is approached.

1. If the polynomial characterizing the scattering region corresponds to a composite of polynomials participating in the reaction, its degree increases with the number of external particles. At the limit of an infinite number of incoming particles, the polynomial approaches a polynomial of infinite degree. This limit also means an approach to a chaos as a functional iteration of the polynomial defining the space-time surface [L11]. In the recent picture, the iteration would correspond to an addition of particles of a given type characterized by a fixed polynomial. Could the characteristic features for the approach of chaos by iteration, say period doubling, be visible in scattering in some situations. Could p-adic length scale hypothesis stating that p-adic primes near powers of two are favored, relate to this.
2. For a large number of identical external particles, the functional composite defining RG involves iteration of polynomials associated with particles of a particular kind, if their number is very large. For instance, the radiation of IR photons and IR gravitons in the reaction increases the degree of RP by adding to P very high iterates of a photonic or gravitonic polynomial.

   Gravitons could have a large value of ramified prime as the approximately infinite range of gravitational interaction and the notion of gravitational Planck constant [L5, L22] originally proposed by Nottale [E1] suggest. If this is the case, graviton corresponds to a polynomial of very high degree, which increases the p-adic length scale of the reaction region and improves the momentum resolution. If the number of gravitons is large, this large RP appears at very many steps of the SFR cascade.

2.7.3 A connection with dual resonance models

There is an intriguing connection with the dual resonances models discussed already in [L6].

1. The basic idea behind the original Veneziano amplitudes (see `http://tinyurl.com/yyhwvbqb`) was Veneziano duality. The 4-particle amplitude of Veneziano was generalized by Yoshiro Nambu, Holber-Beck Nielsen, and Leonard Susskind to N-particle amplitude (see `http://tinyurl.com/yyvkx7as`) based on string picture, and the resulting model was called dual resonance model. The model was forgotten as QCD emerged.
2. Recall that Veneziano duality (1968) was deduced by assuming that scattering amplitude can be described as sum over s-channel resonances or t-channel Regge exchanges and Veneziano duality stated that hadronic scattering amplitudes have a representation as sums over s- or t-channel resonance poles identified as excitations of strings. The sum over exchanges defined by t-channel resonances indeed reduces at larger values of s to Regge form.

3. The resonances have zero width and the imaginary part of the amplitude has a discontinuity only at the resonance poles, which is not consistent with unitarity so that one must force unitarity by hand by an iterative procedure. Further, there were no counterparts for the *sum* of s-, t-, and u-channel diagrams with continuous cuts in the kinematical regions encountered in QFT approach. What puts bells ringing is the u-channel diagrams would be non-planar and non-planarity is the problem of the twistor Grassmann approach.

It is interesting to compare this picture with the twistor Grassman approach and TGD picture.

1. In the TGD framework, one just picks up the residue of what would be analogous to stringy scattering amplitude at mass shells. In the dual resonance models, one keeps the entire amplitude and encounters problems with the unitarity outside the poles. In the twistor Grassmann approach, one assumes that the amplitudes are completely determined by the singularities whereas in TGD they *are* the residues at singularities. At the limit of an infinite-fold iterate the amplitudes approach analogs of QFT amplitudes.
2. In the dual resonance model, the sums over s- and t-channel resonances are the same. This guarantees crossing symmetry. An open question is whether this can be the case also in the TGD framework. If this is the case, the continuum limit of the scattering amplitudes should have a close resemblance with string model scattering amplitudes as the $M^4 \times CP_2$ picture having magnetic flux tubes in a crucial role indeed suggests.
3. In dual resonance models, only the cyclic permutations of the external particles are allowed. As found, the same applies in TGD if the scattering event is a cognitive measurement [L13], only the cyclic permutations of the factors of a fixed functional composite are allowed. Non-cyclic permutations would produce the counterparts of non-planar diagrams and the cascade of cognitive state function reductions could not make sense for all polynomials in the superposition simultaneously. Remarkably, in the twistor Grassmann approach just the non-planar diagrams are the problem.

3 Some useful objections

The details of the proposed construction of the scattering amplitudes starting from twistors are still unclear and the best way to proceed is to invent objections and critical questions.

3.1 How the quark momenta in M^8 and H relate to each other?

The relationship between quark momenta in M^8 and H is not clear. There are four options to consider corresponding to the Dirac propagators in H and M^4 with or without coupling to $A(M^4)$. I assign to these options attributea $D(H,A)$, $D(H)$, $D(M^4,A)$ and $D(M^4)$. For all options something seems to go wrong.

Consider fits the list of criteria that the correct option should satisfy.

1. M^8-H duality suggests the same momentum and mass spectrum for quarks in M^8 and H.
   (a) However, the mass spectrum of color partial waves for quark spinors for $D(H)$ and $D(H,A)$ is very simple and characterized by 2 integers labeling triality $t=1$ representations of $SU(3)$ [L1]. Neither $D(H)$ or $D(H,A)$ allows a mass spectrum as algebraic roots of polynomials and seems to be excluded.
   (b) If M^8-H duality holds true in a strong sense so that these spectra are identical, the only possible conclusion seems to be that the propagator in both M^8 and H is just the M^4 Dirac propagator $D(M^4)$ and that the roots of the polynomial P give the spectrum of off-mass-shell masses. Also tachyonic mass squared values are allowed as roots of P. The real on-shell masses would be associated with Galois singlets.

2. Twistor holomorphy and associativity leave only the $D(M^4)$ option. The couplings to $A(M^4)$ and presence of $D(CP_2)$ spoil these properties. $D(M^4)$ option has very nice features. The integration over the momentum space reduces to a finite summation over virtual mass shells defined by the roots of P and one avoids divergences. This tightens the connection with QFTs. For $D(M^4()$ this nice property is lost. Massless quarks are also consistent with the QCD picture about quarks.

3. The predictions of p-adic mass calculations [K4, K2] were sensitive to the negative ground state conformal weight h_{vac} depending on the electroweak isospin and gave rise to electroweak symmetry breaking. h_{vac} could be generated by conformal generators with weights h coming as algebraic integers determined by P. This favors $D(H)$ and $D(H,A)$. $D(H,A)$ predicts tachyonic ν_R, which was necessary for the calculation. Only $D(H,A)$ survives.

4. For some years ago, I found that the space-time propagators for points of H connected by a light-like geodesic behave like massless propagators irrespective of mass. CP_2 type extermals have a light-like geodesic as an M^4 projection. This would suggest that quarks associated with CP_2 type extremals effectively propagate as massless particles even if one assumes that they correspond to modes of the full H Dirac operator. This allows us to consider $D(H)$ as an alternative. For this option most quarks in the interior of the space-time surface would be extremely massive and practically absent.

5. Suppose that one takes seriously the idea that the situation can be described also by using massless M^8 momenta. This implies that for some choices of $M^4 \subset M^8$ the momentum is parallel to M^4 and therefore massless in 4-D sense. Only the quarks associated with the same M^4 can interact. Hence M^4 can be always chosen so that the on mass-shell 4-momenta are light-like. $D(H,A)$ option could be correct but $D(M^4)$ option would appear as an effective option obtained by a suitable choice of $M^4 \subset M^8$.

6. The consideration of problems related to right-handed neutrino [L19] led to the question whether the quark spinor modes in H are annihilated only by the H d'Alembertian $D^2(H, A(M^4))$ but not by the H Dirac operator [L19]. The assumption that on mass shell H-spinors are annihilated by $D(M^4, A)$ leads to the same outcome.

   D^2 options allow different M^4 chiralities to propagate separately and solves problems related to the notion of right-handed neutrino ν_R (assumed to be 3-antiquark state and modellable using leptonic spinors in H. This also conforms with the right and left-handed character of the standard model couplings. However, the mixing of M^4 chiralities serves as a signature for the massivation and is lost.

   If leptons are allowed as fundamental fermions, $D(H)$ allows ν_R as a spinor mode, which is covariantly constant in CP_2. If leptons are not allowed, one can argue that ν_R as a 3-quark state can be modeled as a mode of H spinor with Kähler coupling yielding correct leptonic charges.

   The M^4 Kähler structure favored by the twistor lift of TGD [L6] implies that ν_R with negative mass squared appears as a mode of $D(H)$. This mode allows the construction of tachyonic ground states. This is lost for $D(M^4)$ with coupling to $A(M^4)$.

   For $D(M^4, A)$, one obtains for all spinor modes states with both positive and negative mass squared from the $J_{kl}\Sigma^{kl}$ term. Physical on-mass- shell states with negative mass squared cannot be allowed. These would however allow to construct tachyonic ground states needed in the p-adic mass calculations. Now the problem is that $D(M^4, A)$ as propagator spoils twistor holomorphy.

7. Since the color group acts as symmetries, one can assume that spinor modes correspond to color partial waves as eigen states of CP_2 spinor d'Alembertian $D^2(CP_2)$. This predicts that different M^4 chiralities propagate independently. $D(M^4)$ and $D(M^4, A)$ options make the same prediction. For the $D(H)$ and $D(H,A)$ option one obtains a mixing of M^4 chiralities having interpretation in terms of massivation.

   For all options the correlation between color and electroweak quantum numbers is "wrong". This is however not a problem for off-mass-shell fundamental quarks since the physical states are obtained as SSA representations.

To sum up, $D(H, A)$ is strongly favored by the p-adic thermodynamics, by the possibility to build the physical quarks using SSA, by the fact that propagators over-light-like distances do not depend on mass, and also by the freedom to choose $M^4 \subset M^8$ in such a manner that on mass shell spinor mode is massless. $D(M^4)$ is strongly favoured by $M^8 - H$ duality (associativity) and by twistor analyticity. Both options seem to be both right and wrong. This suggests that something is wrong with the interpretation of the notion of the Dirac propagator.

1. From the view point of H, M^8 quarks are off-mass-shell whereas from the M^8 point of view they are on-mass-shell. Suppose that off-mass shell quarks in the sense of $D(H, A)$ differ from on-mass-shell quarks only in that they have M^4 momentum $p_{off} = p_{on} + \Delta p$ differing by Δp from the on-mass shell momentum p_{on} with integer components and satisfying mass shell condition for $D(H)$. In CP_2 these states are on-mass-shell. Suppose that p_{off} is on M^8 mass shell determined as a root of P.

   With these assumptions, one can write Dirac operator as $D(H, A, off) = D(H, A, on) + \Delta p^k gamma_k$, whose action to incoming Galois singlets reduces to $D(H, A, off) = \Delta p^k gamma_k = D(M^4)$. This is just the free massless propagator.

2. The propagating entities would be basically solutions of $D(H, A)$ with an off-mass-shell M^4-momentum with Δp having mass. In particular, they are superpositions of components with left- and right-handed M^4 chiralities having opposite CP_2 chiralities and the mixing of M^4 chiralities can be seen as a signature of massivation. On the other hand, $D(M^4)$ does not depend on M^4 chirality. Maybe this option could avoid all objections!

3.2 Can one allow "wrong" correlation between color and electroweak quantum numbers for fundamental quarks?

For CP_2 harmonics, the correlation between color and electroweak quantum numbers is wrong [K4]. Therefore the physical quarks cannot correspond to the solutions of $D^2(H)\Psi = 0$. The same applies also to the solutions of $D(M^4)\Psi = 0$ if one assumes that they belong to irreducible representations of the color group as eigenstates of $D(CP_2)$.

How to construct quark states, which are physical in the sense that they are massless and color-electroweak correlation is correct?

1. The reduction of quark masses to zero requires a tachyonic ground state in p-adic mass calculations [K4]. The assumption that physical states are constructed using quarks, which are on-mass-shell in the M^8 sense but off-mass-shell in the H sense.

   Colored operators with non-vanishing conformal weight are required to make all quark states massless color triplets. This is possible only if the ground state is tachyonic, which gives strong support for M^4 Kähler structure.

2. This is achieved by the identification of physical quarks as states of super-symplectic representations. Also the generalized Kac-Moody algebra assignable to the light-like partonic orbits or both of these representations can be considered. These representations could correspond to inertial and gravitational representations realized at "objective" embedding space level and "subjective" space-time level.

   Supersymplectic generators are characterized by a conformal weight h completely analogous to mass squared. The conformal weights naturally correspond to algebraic integers associated with P. The mass squared values for the Galois singlets are ordinary integers.

3. It is plausible that also massless color triplet states of quarks can be constructed as color singlets. From these one can construct hadrons and leptons as color singlets for a larger extension of rationals. This conforms with the earlier picture about conformal confinement. These physical quarks constructed as states of super-symplectic representation, as opposed to modes of the H spinor field, would correspond to the quarks of QCD.

   One can argue that Galois confinement allows to construct physical quarks as color triplets for some polynomial Q and also color singlets bound states of these with extended Galois group for a higher polynomial $P \circ Q$ and with larger Galois group as representation of group $Gal(P)/Gal(Q)$ allowing representations of a discrete subgroup of color group.

3.3 Can one allow complex quark masses?

One objection relates to unitarity. Complex energies and mass squared values are not allowed in the standard picture based on unitary time evolution.

1. Here several new concepts lend a hand. Galois confinement could solve the problems if one considers only Galois singlets as physical particles. ZEO replaces quantum states with entangled pairs of positive and negative energy states at the boundaries of CD and entanglement coefficients define transition amplitudes.

   The notion of the unitary time evolution is replaced with the Kähler metric in quark degrees of freedom and its components correspond to transition amplitudes. The analog of the time evolution operator assignable to SSFRs corresponds naturally to a scaling rather than time translation and mass squared operator corresponds to an infinitesimal scaling.

2. The complex eigenvalues of mass squared as roots of P be allowed when unitarity at quark level is not required to achieve probability conservation. For complex mass squared values, the entanglement coefficients for quarks would be proportional to mass squared exponents $exp(im^2\lambda)$, λ the scaling parameter analogous to the duration of time evolution. For Galois singlets these exponentials would sum up to imaginary ones so that probability conservation would hold true.

3.4 Are M^8 spinors as octonionic spinors equivalent with H-spinors?

At the level of M^8 octonionic spinors are natural. M^8-H duality requires that they are equivalent with H-spinors. The most natural identification of octionic spinors is as bi-spinors, which have octonionic components. Associativity is satisfied if the components are complexified quaternionic so that they have the same number of components as quark spinors in H. The H spinors can be induced to $X^4 \subset M^8$ by using $M^8 - H$ duality. Therefore the M^8 and H pictures fuse together.

The quaternionicity condition for the octonionic spinors is essential. Octonionic spinor can be expressed as a complexified octonion, which can be identified as momentum p. It is not an on-mass shell spinor. The momenta allowed in scattering amplitudes belong to mass shells defined by the polynomial P. That octonionic spinor has only quaternionic components conforms with the quaternionicity of $X^4 \subset M^8$ eliminating the remaining momentum components and also with the use of $D(M^4)$.

3.5 Two objections against p-adic thermodynamics and their resolution

Unlike the Higgs mechanism, p-adic thermodynamics provides a universal description of massivation involving no other assumptions about dynamics except super-conformal symmetry, which guarantees the existence of p-adic Boltzmann weights.

There are two basic objections against p-adic thermodynamics. The mass calculations require the presence of states with negative conformal weights giving rise to tachyons. Furthermore, by conformal invariance L_0 should annihilate physical states so that all states should have vanishing mass squared! In this article a resolution of these objections, based on the very definition of thermodynamics and on number theoretic vision predicting quark states with discretized tachyonic mass, which are counterparts for virtual states in QFTs, is discussed.

Physical states for the entire Universe would be indeed massless but for subsystems such as elementary particles the thermal expectation of the mass squared is non-vanishing. This conforms with the formula of blackhole entropy stating that it is proportional to the mass square of the blackhole and vanishes for vanishing mass: this would indeed correspond to a pure state.

3.5.1 p-Adic thermodynamics

Number theoretic physics involves the combination of real and various p-adic physics to adelic physics [L2, L3], and classical number fields [K11]. p-Adic mass calculations is a rather successful application of p-adic thermodynamics for the mass squared operator identified as conformal scaling generator L_0. p-Adic thermodynamics can be also understood as a constraint on a real thermodynamics for the mass squared from the condition that it can be also regarded as a p-adic thermodynamics.

The motivation for p-adicization came from p-adic mass calculations [K4, K2].

1. p-Adic thermodynamics for mass squared operator M^2 proportional to scaling generator L_0 of Virasoro algebra. Mass squared thermal mass from the mixing of massless states with states with mass of order CP_2 mass.

2. $exp(-E/T) \rightarrow p^{L_0/T_p}$, $T_p = 1/n$. Partition function p^{L_0/T_p}. p-Adic valued mass squared mapped to a real number by canonical identification $\sum x_n p^n \rightarrow \sum x_n p^{-n}$. Eigenvalues of L_0 must be integers for the Boltzmann weights to exist. Conformal invariance guarantees this.

3. p-adic length scale $L_p \propto \sqrt{p}$ from Uncertainty Principle ($M \propto 1/\sqrt{p}$). p-Adic length scale hypothesis states that p-adic primes characterizing particles are near to a power of 2: $p \simeq 2^k$. For instance, for an electron one has $p = M^{127} - 1$, Mersenne prime. This is the largest not completely super-astrophysical length scale.

   Also Gaussian Mersenne primes $M_{G,n} = (1+i)^n - 1$ seem to be realized (nuclear length scale, and 4 biological length scales in the biologically important range 10 nm,2.5 μm).

4. p-Adic physics [K7] is interpreted as a correlate for cognition. Motivation comes from the observation that piecewise constant functions depending on a finite number of pinary digits have a vanishing derivative. Therefore they appear as integration constants in p-adic differential equations. This could provide a classical correlate for the non-determinism of imagination.

3.5.2 Objections and their resolution

The number theoretic picture leads to a deeper understanding of a long standing objection against p-adic thermodynamics [K4] as a thermodynamics for the scaling generator L_0 of Super Virasoro algebra.

If one requires super-Virasoro symmetry and identifies mass squared with a scaling generator L_0, one can argue that only massless states are possible since L_0 must annihilate these states! All states of the theory would be massless, not only those of fundamental particles as in conformally invariant theories to which twistor approach applies! This looks extremely beautiful mathematically but seems to be in conflict with reality already at single particle level!

The resolution of the objection is that *thermodynamics* is indeed in question.

1. Thermodynamics replaces the state of the entire system with the density matrix for the subsystem and describes approximately the interaction with the environment inducing the entanglement of the particle with it. To be precise, actually a "square root" of p-adic thermodynamics could be in question, with probabilities being replaced with their square roots having also phase factors. The excited states of the entire system indeed are massless [L26].

2. The entangling interaction gives rise to a superposition of products of single particle massive states with the states of environment and the entire mass squared would remain vanishing. The massless ground state configuration dominates and the probabilities of the thermal excitations are of order $O(1/p)$ and extremely small. For instance, for the electron one has $p = M_{127} = 2^{127} - 1 \sim 10^{38}$.

3. In the p-adic mass calculations [K4, K2], the effective environment for quarks and leptons would in a good approximation consist of a wormhole contact (wormhole contacts for gauge bosons and Higgs and hadrons). The many-quark state many-quark state associated with the wormhole throat (single quark state for quarks and 3-quark-state for leptons [L16].

4. In M^8 picture [L9, L10], tachyonicity is unavoidable since the real part of the mass squared as a root of a polynomial P can be negative. Also tachyonic real but algebraic mass squared values are possible. At the H level, tachyonicity corresponds to the Euclidean signature of the induced metric for a wormhole contact.

Tachyonicity is also necessary: otherwise one does not obtain massless states. The super-symplectic states of quarks would entangle with the tachyonic states of the wormhole contacts by Galois confinement.

5. The massless ground state for a particle corresponds to a state constructed from a massive single state of a single particle super-symplectic representation (CP_2 mass characterizes the mass scale) obtained by adding tachyons to guarantee masslessness. Galois confinement is satisfied. The tachyonic mass squared is assigned with wormhole contacts with the Euclidean signature of the induced metric, whose throats in turn carry the fermions so that the wormhole contact would form the nearby environment.

   The entangled state is in a good approximation a superposition of pairs of massive single-particle states with the wormhole contact(s). The lowest state remains massless and massive single particle states receive a compensating negative mass squared from the wormhole contact. Thermal mass squared corresponds to a single particle mass squared and does not take into account the contribution of wormhole contacts except for the ground state.

6. There is a further delicate number theoretic element involved [L19, L23]. The choice of $M^4 \subset M^8$ for the system is not unique. Since M^4 momentum is an M^4 projection of a massless M^8 momentum, it is massless by a suitable choice of $M^4 \subset M^8$. This choice must be made for the environment so that both the state of the environment and the single particle ground state are massless. For the excited states, the choice of M^4 must remain the same, which forces the massivation of the single particle excitations and p-adic massivation.

3.5.3 All physical states are massless!

These arguments strongly suggest that pure states, in particular the state of the entire Universe, are massless. Mass would reflect the statistical description of entanglement using a density matrix. The proportionality between p-adic thermal mass squared (mappable to real mass squared by canonical identification) and the entropy for the entanglement of the subsystem-environment pair is therefore natural.

This proportionality conforms with the formula for the blackhole entropy, which states that the blackhole entropy is proportional to mass squared. Also p-adic mass calculations inspired the notion of blackhole-elementary particle analogy [K10] but without a deeper understanding of its origin.

One implication is that virtual particles are much more real in the TGD framework than in QFTs since they would be building bricks of physical states. A virtual particle with algebraic value of mass squared would have a discrete mass squared spectrum given by the roots of a rational, possibly monic, polynomial and M^8-H duality suggests an association to an Euclidean wormhole contact as the "inner" world of an elementary particle. Galois confinement, universally responsible for the formation of bound states, analogous to color confinement and possibly explaining it, would make these virtual states invisible [L24, L25].

3.5.4 Relationship with Higgs mechanism

Polynomials P have two kinds of solutions depending on whether their roots determine either mass or energy shells. For the energy option a space-time region corresponds by M^8-H duality to a solution spectrum in which the roots correspond to energies rather than mass squared values and light-cone proper time is replaced with linear Minkoski time [L9, L10]. The physical interpretation of the energy shell option has remained unclear.

The energy shell option gives rise to a p-adic variant of the ordinary thermodynamics and requires integer quantization of energy. This option is natural for massless states since scalings leave the mass shell invariant in this case. Scaling invariance and conformal invariance are not violated.

One can wonder what the role of these massless virtual quark states in TQC could be. A good guess is that the two options correspond to phases with broken *resp.* unbroken conformal symmetry. In gauge theories they correspond to phases with broken and unbroken gauge symmetries. The breaking of gauge symmetry indeed induces breaking of conformal symmetry and this breaking is more fundamental.

1. Particle massivation corresponds in gauge theories to symmetry breaking caused by the generation of the Higgs vacuum expectation value. Gauge symmetry breaking induces a breaking of conformal symmetry and particle massivation. In the TGD framework, the generation of entanglement between members of state pairs such that members having opposite values of mass squared determined as roots of polynomial P in the most general case, leads to a breaking of conformal symmetry for each tensor factor and the description in terms of p-adic thermodynamics gives thermal mass squared.

2. What about the situation when energy, instead of mass squared, comes as a root of P. Also now one can construct physical states from massless virtual quarks with energies coming as algebraic integers. Total energies would be ordinary integers. This gives massless entangled states, if the rational integer parts of 4-momenta are parallel. This brings in mind a standard twistor approach with parallel light-like momenta for on-mass shell states. Now however the virtual states can have transversal momentum components which are algebraic numbers (possibly complex) but sum up to zero.

   Quantum entangled states would be superpositions over state pairs with parallel massless momenta. Massless extremals (topological light rays) are natural classical space-time correlates for them. This phase would correspond to the phase with unbroken conformal symmetry.

3. One can also assign a symmetry breaking to the thermodynamic massivation. For the energy option, the entire Galois group appears as symmetry of the mass shell whereas for the mass squared option only the isotropy group does so. Therefore there is a symmetry breaking of the full Galois symmetry to the symmetry defined by the isotropy group. In a loose sense, the real valued argument of P serves as a counterpart of the Higgs field.

   If the symmetry breaking in the model of electroweak interaction corresponds to this kind of symmetry breaking, the isotropy group, which presumably involves also a discrete subgroup of quaternionic automorphisms as an analog of the Galois group. Quaternionic group could act as a discrete subgroup of $SU(2) \subset SU(2)_L \times U(1)$. The hierarchy of discrete subgroups associated with the hierarchy of Jones inclusions assigned with measurement resolution suggests itself. It has the isometry groups of Platonic solids as the groups with genuinely 3-D action. $U(1)$ factor could correspond to Z_n as the isotropy group of the Galois group. In the QCD picture about strong interactions there is no gauge symmetry breaking so that a description based on the energy option is natural. Hadronic picture would correspond to mass squared option and symmetry breaking to the isotropy group of the root.

To sum up, in the maximally symmetric scenario, conformal symmetry breaking would be only apparent, and due to the necessity to restrict to non-tachyonic subsystems using p-adic thermodynamics. Gauge symmetry breaking would be replaced with the replacement of the Galois group with the isotropy group of the root representing mass squared value. The argument of the polynomial defining space-time region would be the analog of the Higgs field.

3.6 Some further comments about the notion of mass

In the sequel some further comments related to the notion of mass are represented.

3.6.1 $M^8 - H$ duality and tachyonic momenta

Tachyonic momenta are mapped to space-like geodesics in H or possibly to the geodesics of X^4 [L9, L10, L21]. This description could allow to describe pair creation as turning of fermion backwards in time [L25]. Tachyonic momenta correspond to points of de Sitter space and are therefore outside CD and would go outside the space-time surface, which is inside CD. Could one avoid this?

1. Since the points of the twistor spaces $T(M^4)$ and $T(CP_2)$ are in 1-1 correspondence, one can use either $T(M^4)$ or $T(CP_2)$ so that the projection to M^4 or CP_2 would serve as the base space of $T(X^4)$. One could use CP_2 coordinates or M^4 coordinates as space-time coordinates if the dimension of the projection is 4 to either of these spaces. In the generic case, both dimensions are 4 but one must be very cautious with genericity arguments which fail at the level of M^8.

2. There are exceptional situations in which genericity fails at the level of H. String-like objects of the form $X^2 \times Y^2 \subset M^4 \subset CP_2$ is one example of this. In this case, X^6 would not define 1-1 correspondence between $T(M^4)$ or $T(CP_2)$.

   Could one use partial projections to M^2 and S^2 in this case? Could $T(X^4)$ be divided locally into a Cartesian product of 3-D M^4 part projecting to $M^2 \subset M^4$ and of 3-D CP_2 part projected to $Y^2 \subset CP_2$.

3. One can also consider the possibility of defining the twistor space $T(M^2 \times S^2)$. Its fiber at a given point would consist of light-like geodesics of $M^2 \times S^2$. The fiber consists of direction vectors of light-like geodesics. S^2 projection would correspond to a geodesic circle $S^1 \subset S^2$ going through a given point of S^2 and its points are parametrized by azimuthal angle Φ. Hyperbolic tangent $tanh(\eta)$ with range $[-1, 1]$ would characterize the direction of a time like geodesic in M^2. At the limit of $\eta \to \pm\infty$ the S^2 contribution to the S^2 tangent vector to length squared of the tangent vector vanishes so that all angles in the range $(0, 2\pi)$ correspond to the same point. Therefore the fiber space has a topology of S^2.

   There are also other special situations such as $M^1 \times S^3$, $M^3 \times S^1$ for which one must introduce specific twistor space and which can be treated in the same way.

During the writing of this article I realized that the twistor space of H defined geometrically as a bundle, which has as H as base space and fiber as the space of light-like geodesic starting from a given point of H need not be equal to $T(M^4) \times T(CP_2)$, where $T(CP_2)$ is identified as $SU(3)/U(1) \times U(1)$ characterizing the choices of color quantization axes.

1. The definition of $T(CP_2)$ as the space of light-like geodesics from a given point of CP_2 is not possible. One could also define the fiber space of $T(CP_2)$ geometrically as the space of geodesics emating from origin at $r = 0$ in the Eguchi-Hanson coordinates [K1] and connecting it to the homologically non-trivial geodesic sphere S^2_G $r = \infty$. This relation is symmetric.

   In fact, all geodesics from $r = 0$ end up to S^2. This is due to the compactness and symmetries of CP_2. In the same way, the geodesics from the North Pole of S^2 end up to the South Pole. If only the endpoint of the geodesic of CP_2 matters, one can always regard it as a point S^2_G.

   The two homologically non-trivial geodesic spheres associated with distinct points of CP_2 always intersect at a single point, which means that their twistor fibers contain a common geodesic line of this kind. Also the twistor spheres of $T(M^4)$ associated with distinct points of M^4 with a light-like distance intersect at a common point identifiable as a light-like geodesic connecting them.

2. Geometrically, a light-like geodesic of H is defined by a 3-D momentum vector in M^4 and 3-D color momentum along CP_2 geodesic. The scale of the 8-D tangent vector does not matter and the 8-D light-likeness condition holds true. This leaves 4 parameters so that $T(H)$ identified in this way is 12-dimensional.

   The M^4 momenta correspond to a mass shell H^3. Only the momentum direction matters so that also in the M^4 sector the fiber reduces to S^2 . If this argument is correct, the space of light-like geodesics at point of H has the topology of $S^2 \times S^2$ and $T(H)$ would reduce to $T(M^4) \times T(CP_2)$ as indeed looks natural.

3.6.2 Conformal confinement at the level of H

The proposal of [L28], inspired by p-adic thermodynamics, is that all states are massless in the sense that the sum of mass squared values vanishes. Conformal weight, as essentially mass squared value, is naturally additive and conformal confinement as a realization of conformal invariance would mean that the sum of mass squared values vanishes. Since complex mass squared values with a negative real part are allowed as roots of polynomials, the condition is highly non-trivial.

$M^8 - H$ duality [L9, L10] would make it natural to assign tachyonic masses with CP_2 type extremals and with the Euclidean regions of the space-time surface. Time-like masses would be assigned with time-like space-time regions. In [?] it was found that, contrary to the beliefs held hitherto, it is possible to satisfy boundary conditions for the action action consisting of the Kähler action, volume term and Chern-Simons term, at boundaries (genuine or between Minkowskian and Euclidean space-time regions) if they are light-like surfaces satisfying also $detg_4 = 0$. Masslessness, at least in the classical sense, would be naturally associated with light-like boundaries (genuine or between Minkowskian and Euclidean regions).

3.6.3 About the analogs of Fermi torus and Fermi surface in H^3

Fermi torus (cube with opposite faces identified) emerges as a coset space of E^3/T^3, which defines a lattice in the group E^3. Here T^3 is a discrete translation group T^3 corresponding to periodic boundary conditions in a lattice.

In a realistic situation, Fermi torus is replaced with a much more complex object having Fermi surface as boundary with non-trivial topology. Could one find an elegant description of the situation?

1. Hyperbolic manifolds as analogies for Fermi torus?

The hyperbolic manifold assignable to a tessellation of H^3 defines a natural relativistic generalization of Fermi torus and Fermi surface as its boundary. To understand why this is the case, consider first the notion of cognitive representation.

1. Momenta for the cognitive representations [L27] define a unique discretization of 4-surface in M^4 and, by M^8-H duality, for the space-time surfaces in H and are realized at mass shells $H^3 \subset M^4 \subset M^8$ defined as roots of polynomials P. Momentum components are assumed to be algebraic integers in the extension of rationals defined by P and are in general complex.

   If the Minkowskian norm instead of its continuation to a Hermitian norm is used, the mass squared is in general complex. One could also use Hermitian inner product but Minkowskian complex bilinear form is the only number-theoretically acceptable possibility. Tachyonicity would mean in this case that the real part of mass squared, invariant under $SO(1,3)$ and even its complexification $SO_c(1,3)$, is negative.

2. The active points of the cognitive representation contain fermion. Complexification of H^3 occurs if one allows algebraic integers. Galois confinement [L27, ?] states that physical states correspond to points of H^3 with integer valued momentum components in the scale defined by CD.

   Cognitive representations are in general finite inside regions of 4-surface of M^8 but at H^3 they explode and involve all algebraic numbers consistent with H^3 and belonging to the extension of rationals defined by P. If the components of momenta are algebraic integers, Galois confinement allows only states with momenta with integer components favored by periodic boundary conditions.

Could hyperbolic manifolds as coset spaces $SO(1,3)/\Gamma$, where Γ is an infinite discrete subgroup $SO(1,3)$, which acts completely discontinuously from left or right, replace the Fermi torus? Discrete translations in E^3 would thus be replaced with an infinite discrete subgroup Γ. For a given P, the matrix coefficients for the elements of the matrix belonging to Γ would belong to an extension of rationals defined by P.

1. The division of $SO(1,3)$ by a discrete subgroup Γ gives rise to a hyperbolic manifold with a finite volume. Hyperbolic space is an infinite covering of the hyperbolic manifold as a fundamental region of tessellation. There is an infinite number of the counterparts of Fermi torus [L18]. The invariance respect to Γ would define the counterpart for the periodic boundary conditions.

   Note that one can start from $SO(1,3)/\Gamma$ and divide by $SO(3)$ since Γ and $SO(3)$ act from right and left and therefore commute so that hyperbolic manifold is $SO(3) \setminus SO(1,3)/\Gamma$.

2. There is a deep connection between the topology and geometry of the Fermi manifold as a hyperbolic manifold. Hyperbolic volume is a topological invariant, which would become a basic concept of relativistic topological physics (`https://cutt.ly/RVsdNl3`).

The hyperbolic volume of the knot complement serves as a knot invariant for knots in S^3. Could this have physical interpretation in the TGD framework, where knots and links, assignable to flux tubes and strings at the level of H, are central. Could one regard the effective hyperbolic manifold in H^3 as a representation of a knot complement in S^3?

Could these fundamental regions be physically preferred 3-surfaces at H^3 determining the holography and M^8-H duality in terms of associativity [L9, L10]. Boundary conditions at the boundary of the unit cell of the tessellation should give rise to effective identifications just as in the case of Fermi torus obtained from the cube in this way.

2 .De Sitter manifolds as tachyonic analogs of Fermi torus do not exist

Can one define the analogy of Fermi torus for the real 4-momenta having negative, tachyonic mass squared? Mass shells with negative mass squared correspond to De-Sitter space $SO(1,3)/SO(1,2)$ having a Minkowskian signature. It does not have analogies of the tessellations of H^3 defined by discrete subgroups of $SO(1,3)$.

The reason is that there are no closed de-Sitter manifolds of finite size since no infinite group of isometries acts discontinuously on de Sitter space: therefore these is no group replacing the Γ in H^3/Γ. (`https://cutt.ly/XVsdLwY`).

3.Do complexified hyperbolic manifolds as analogs of Fermi torus exist?

The momenta for virtual fermions defined by the roots defining mass squared values can also be complex. Tachyon property and complexity of mass squared values are not of course not the same thing.

1. Complexification of H^3 would be involved and it is not clear what this could mean. For instance, does the notion of complexified hyperbolic manifold with complex mass squared make sense.

2. $SO(1,3)$ and its infinite discrete groups Γ act in the complexification. Do they also act discontinuously? p^2 remains invariant if $SO(1,3)$ acts in the same way on the real and imaginary parts of the momentum leaves invariant both imaginary and complex mass squared as well as the inner product between the real and imaginary parts of the momenta. So that the orbit is 5-dimensional. Same is true for the infinite discrete subgroup Γ so that the construction of the coset space could make sense. If Γ remains the same, the additional 2 dimensions can make the volume of the coset space infinite. Indeed, the constancy of $p_1 \cdot p_2$ eliminates one of the two infinitely large dimensions and leaves one.

   Could one allow a complexification of $SO(1,3)$, $SO(3)$ and $SO(1,3)_c/SO(3)_c$? Complexified $SO(1,3)$ and corresponding subgroups Γ satisfy $OO^T = 1$. Γ_c would be much larger and contain the real Γ as a subgroup. Could this give rise to a complexified hyperbolic manifold H^3_c with a finite volume?

3. A good guess is that the real part of the complexified bilinear form $p \cdot p$ determines what tachyonicity means. Since it is given by $Re(p)^2 - Im(p)^2$ and is invariant under $SO_c(1,3)$ as also $Re(p) \cdot Im(p)$, one can define the notions of time-likeness, light-likeness, and space-likeness using the sign of $Re(p)^2 - Im(p^2)$ as a criterion. Note that $Re(p)^2$ and $Im(p)^2$ are separately invariant under $SO(1,3)$.

   The physicist's naive guess is that the complexified analogs of infinite discrete and discontinuous groups and complexified hyperbolic manifolds as analogs of Fermi torus exist for $Re(P^2) - Im(p^2) > 0$ but not for $Re(P^2) - Im(p^2) < 0$ so that complexified dS manifolds do not exist.

4. The bilinear form in H^3_c would be complex valued and would not define a real valued Riemannian metric. As a manifold, complexified hyperbolic manifold is the same as the complex hyperbolic manifold with a hermitian metric (see `https://cutt.ly/qVsdS7Y` and `https://cutt.ly/kVsd3Q2`) but has different symmetries. The symmetry group of the complexified bilinear form of H^3_c is $SO_c(1,3)$ and the symmetry group of the Hermitian metric is $U(1,3)$ containing $SO(1,3)$ as a real subgroup. The infinite discrete subgroups Γ for $U(1,3)$

contain those for $SO(1,3)$. Since one has complex mass squared, one cannot replace the bilinear form with hermitian one. The complex H^3 is not a constant curvature space with curvature -1 whereas H_c^3 could be such in a complexified sense.

3.7 Is pair creation really understood in the twistorial picture?

Twistorialization leads to a beautiful picture about scattering amplitudes at the level of M^8 [L24, L25]. In the simplest picture, scattering would be just a re-organization of Galois singlets to new Galois singlets. Fundamental fermions would move as free particles.

The components of the 4-momentum of virtual fundamental fermion with mass m would be algebraic integers and therefore complex. The real projection of 4-momentum would be mapped by M^8-H duality to a geodesic of M^4 starting from either vertex of the causal diamond (CD) . Uncertainty Principle at classical level requires inversion so that one has $a=\hbar_{eff}/m$, where ab denotes light-cone proper time assignable to either half-cone of CD and m is the mass assignable to the point of the mass shell $H^3 \subset M^4 \subset M^8$.

The geodesic would intersect the $a=\hbar_{eff}/m$ 3-surface and also other mass shells and the opposite light-cone boundaries of CDs involved. The mass shells and CDs containing them would have a common center but Uncertainty Principle at quantum level requires that for each CD and its contents there is an analog of plane wave in CD cm degrees of freedom.

One can however criticize this framework. Does it really allow us to understand pair creation at the level of the space-time surfaces $X^4 \subset H$?

1. All elementary particles consist of fundamental fermions in the proposed picture. Conservation of fermion number allows pair creation occurring for instance in the emission of a boson as fermion-antifermion pair in $f \to f+b$ vertex.
2. The problem is that if only non-space-like geodesics of H are allowed, both fermion and antifermion numbers are conserved separately so that pair creation does not look possible. Pair creation is both the central idea and source of divergence problems in QFTs.
3. One can identify also a second problem: what are the anticommutation relations for the fermionic oscillator operators labelled by tachyonic and complex valued momenta? Is it possible to analytically continue the anticommutators to complexified $M^4 \subset H$ and $M^4 \subset M^8$? Only the first problem will be considered in the following.

Is it possible to understand pair creation in the proposed picture based on twistor scattering amplitudes or should one somehow bring the bff 3-vertex or actually $ff\overline{ff}$ vertex to the theory at the level of quark lines? This vertex leads to a non-renormalizable theory and is out of consideration.

One can first try to identify the key ingredients of the possible solution of the problem.

1. Crossing symmetry is fundamental in QFTs and also in TGD. For non-trivial scattering amplitudes, crossing moves particles between initial and final states. How should one define the crossing at the space-time level in the TGD framework? What could the transfer of the end of a geodesic line at the boundary of CDs to the opposite boundary mean geometrically?
2. At the level of H, particles have CP_2 type extremals -wormhole contacts - as building bricks. They have an Euclidean signature (of the induced metric) and connect two space-time sheets with a Minkowskian signature.

   The opposite throats of the wormhole contacts correspond to the boundaries between Euclidean and Minkowskian regions and their orbits are light-like. Their light-like boundaries, orbits of partonic 2-surfaces, are assumed to carry fundamental fermions. Partonic orbits allow light-like geodesics as possible representation of massless fundamental fermions.

   Elementary particles consist of at least two wormhole contacts. This is necessary because the wormhole contacts behave like Kähler magnetic charges and one must have closed magnetic field lines. At both space-time sheets, the particle could look like a monopole pair.

3. The generalization of the particle concept allows a geometric realization of vertices. At a given space-time sheet a diagram involving a topological 3-vertex would correspond to 3 light-like partonic orbits meeting at the partonic 2-surface located in the interior of X^4. Could the topological 3-vertex be enough to avoid the introduction of the 4-fermion vertex?

Could one modify the definition of the particle line as a geodesic of H to a geodesic of the space-time surface X^4 so that the classical interactions at the space-time surface would make it possible to describe pair creation without introducing a 4-fermion vertex? Could the creation of a fermion pair mean that a virtual fundamental fermion moving along a space-like geodesics of a wormhole throat turns backwards in time at the partonic 3-vertex. If this is the case, it would correspond to a tachyon. Indeed, in M^8 picture tachyons are building bricks of physical particles identified as Galois singlets.

1. If fundamental fermion lines are geodesics at the light-like partonic orbits, they can be light-like but are space-like if there is motion in transversal degrees of freedom.

2. Consider a geodesic carrying a fundamental fermion, starting from a partonic 2-surface at either light-like boundary of CD. As a free fermion, it would propagate to the opposite boundary of CD along the wormhole throat.

   What happens if the fermion goes through a topological 3-vertex? Could it turn backwards in time at the vertex by transforming first to a space-like geodesic inside the wormhole contact leading to the opposite throat and return back to the original boundary of CD? It could return along the opposite throat or the throat of a second wormhole contact emerging from the 3-vertex. Could this kind of process be regarded as a bifurcation so that it would correspond to a classical non-determinism as a correlate of quantum non-determinism?

3. It is not clear whether one can assign a conserved space-like M^4 momentum to the geodesics at the partonic orbits. It is not possible to assign to the partonic 2-orbit a 3-momentum, which would be well-defined in the Noether sense but the component of momentum in the light-like direction would be well-defined and non-vanishing.

   By Lorentz invariance, the definition of conserved mass squared as an eigenvalue of d'Alembertian could be possible. For light-like 3-surfaces the d'Alembertian reduces to the d'Alembertian for the Euclidean partonic 2-surface having only non-positive eigenvalues. If this process is possible and conserves M^4 mass squared, the geodesic must be space-like and therefore tachyonic.

   The non-conservation of M^4 momentum at single particle level (but not classically at n-particle level) would be due to the interaction with the classical fields.

4. In the M^8 picture, tachyons are unavoidable since there is no reason why the roots of the polynomials with integer coefficients could not correspond to negative and even complex mass squared values. Could the tachyonic real parts of mass squared values at M^8 level, correspond to tachyonic geodesics along wormhole throats possibly returning backwards along the another wormhole throat?

How does this picture relate to p-adic thermodynamics [L28] as a description of particle massivations?

1. The description of massivation in terms of p-adic thermodynamics [L28] suggests that at the fundamental level massive particles involve non-observable tachyonic contribution to the mass squared assignable to the wormhole contact, which cancels the non-tachyonic contribution.

   All articles, and for the most general option all quantum states could be massless in this sense, and the massivation would be due the restriction of the consideration to the non-tachyonic part of the mass squared assignable to the Minkowskian regions of X^4.

2. p-Adic thermodynamics would describe the tachyonic part of the state as "environment" in terms of the density matrix dictated to a high degree by conformal invariance, which this description would break. A generalization of the blackhole entropy applying to any system emerges and the interpretation for the fact that blackhole entropy is proportional to

mass squared. Also gauge bosons and Higgs as fermion-antifermion pairs would involve the tachyonic contribution and would be massless in the fundamental description.

3. This could solve a possible and old problem related to massless spin 1 bosons. If they consist of a collinear fermion and antifermion, which are massless, they have a vanishing helicity and would be scalars, because the fermion and antifermion with parallel momenta have opposite helicities. If the fermion and antifermion are antiparallel, the boson has correct helicity but is massive.

   Massivation could solve the problem and p-adic thermodynamics indeed predicts that even photons have a very small thermal mass. Massless gauge bosons (and particles in general) would be possible in the sense that the positive mass squared is compensated by equally small tachyonic contribution.

4. It should be noted however that the roots of the polynomials in M^8 can also correspond to energies of massless states. This phase would be analogous to the Higgs=0 phase. In this phase, Galois symmetries would not be broken: for the massive phase Galois group permutes different mass shells (and different $a = constant$ hyperboloids) and one must restrict Galois symmetries to the isotropy group of a given root. In the massless phase ,Galois symmetries permute different massless momenta and no symmetry breaking takes place.

4 Antipodal duality and TGD

I learned of a new particle physics duality from the popular article "Particle Physicists Puzzle Over a New Duality" published in Quanta Magazine (`https://cutt.ly/jZOaDhd`). The article describes the findings of Dixon et al reported in the article "Folding Amplitudes into Form Factors: An Antipodal Duality" [B1] (`https://cutt.ly/EZOsfGl`) This work relies on the calculations of Goncharov et al published in the article "Classical Polylogarithms for Amplitudes and Wilson Loops" [B4] (`https://cutt.ly/sZOsuu6`).

The calculations of Goncharov et al lead to an explicit formula for the loop contributions to the 6-gluon scattering amplitude in $\mathcal{N} = 4$ SUSY. The new duality is called antipodal duality and relates 6-gluon amplitude for the forward scattering to a 3-gluon form factor of stress tensor analogous to a quantum field describing a scalar particle. This amplitude can be identified as a contribution to the scattering amplitude $h + g \rightarrow g + g$. The result is somewhat mysterious since in the standard model strong and electroweak interactions are completely separate.

4.1 Findings of Dixon et al

Consider first the findings of Dixon et al [B1].

1. One considers [B4] twistor amplitudes in $\mathcal{N} = \triangle$ SUSY. Only the maximally helicity violating amplitudes (MHV) are considered and one restricts the consideration to planar diagrams (to my best understanding, non-planar diagrams are still poorly understood). The contribution of the loop corrections is studied and the number of loops is rather high in the computations checking the claimed result.

   6-gluon forward scattering amplitude and 3-gluon form factor of stress energy tensor regarded as a quantum field are discussed. Conformal invariance fixes the Lorentz invariants appearing in the 6-gluon forward amplitude and in the 3-gluon form factor of stress tensor to be 3 conformally invariant cross ratios formed from the 3 gluon momenta.

   The claimed antipodal duality is found to hold true for each number of loops separately at the limit when one of conformal invariants approaches zero: the interpretation is that momentum exchange between 2 gluons vanishes at this limit. For 6-gluon forward amplitudes, this limit corresponds to in the 3-D space of conformal invariants to the edges of a tetrahedron.

2. $3g \rightarrow 3g$ scattering amplitude is studied at the limit when the scattering is in forward direction. One has effectively 3 gluons but not 3-gluon scattering since there is no momentum conservation constraining the total momentum of 3 gluons except effectively for the forward scattering of the stress tensor.

As far as total quantum numbers are considered, the stress tensor can give rise to a quantum field behaving like Higgs as far as QCD is considered. The surprising finding is that the so-called antipodal duality applied to the 6-gluon amplitude gives a 3-gluon form factor of the stress tensor, which is scalar having no spin and vanishing color quantum numbers.

3. The antipodal transformation is carried for the 6-gluon amplitude in forward direction so that only 3 gluon momenta are involved. One starts from the 6-gluon amplitude constructed using the standard rules, which require that the amplitude involves only cyclic permutations of the gluons (elements of S_6 of the gluons.

   One considers permutation group $S_3 \subset S_6$ acting in the same way on the first 3 first and 3 remaining gluons, and constructs an S_3 singlet as a sum of the amplitudes obtained by applying S_3 transformations. S_3 operations are not allowed in the twistor diagrammatics since only planar amplitudes are considered usually (the construction of twistor counterparts of non-planar amplitudes is not well-understood).

4. One also constructs the 3-gluon form factor of stress energy tensor by using the twistor rules and considers the so-called soft limit at which the sum of the 3 gluon momenta vanishes so that the effectite particle assignable to the stress tensor scatters in the forward direction. It comes as a surprise that this amplitude is related to the amplitude obtained from the forward 6-gluon amplitude by the antipodal transformation.

5. The duality also involves a simple transformation of the 3 conformal invariants formed from the gluon momenta involved to the 3-gluon form factor of the energy momentum tensor. The antipodal duality holds true at the edges of the 2-D tetrahedron surface defined by the image of the 3-gluon form factor in the space of 3 conformal invariants characterizing the 6-gluon forward amplitude.

   The term antipodal derives from the fact that the 6-gluon amplitude can be expressed as a "word" formed from 6 "letters" and the above described transformation reverses the order of the letters.

6. It is conjectured that this result generalizes to large values of n so that antipodal images of $2n$-gluon scattering amplitude in forward direction could correspond to $n-$-gluon form factor for stress tensor energy and this in turn would be associated with scattering of Higgs and n gluons.

4.2 Questions

Since the stress tensor is a scalar, it is not totally surprising that a term proportional to this amplitude contributes to the scattering amplitude $h + g \rightarrow g + g$, where h denotes Higgs particle. What looks somewhat mysterious is that Higgs is an electro-weakly interacting particle and has no direct color interactions. The description of the scattering in the standard model involves electroweak interactions and involves at least one decay of a gluon to a quark pair in turn interacting with the Higgs.

This inspires several questions.

1. Can one consider more general subgroups $S_m \subset S_{2n}$ and by forming S_m singlets construct amplitudes with a physical interpretation?

2. Can one imagine a deep duality between color and electroweak interactions such that $\mathcal{N} = 4$ SUSY would reflect this duality? Could one even think that the strong and electroweak interactions are in some sense dual?

In TGD color interactions and electroweak interactions are related to the isometries and holonomies of $CP2$ and there indeed exists quite a number of pieces of evidence for this kind of duality. However, the possibility that electroweak or color interactions alone could provide a full description of scattering amplitudes looks unrealistic: both electroweak and color quantum numbers are needed. The number-theoretical view of TGD [L21, L3, L24, L25] could however come into rescue.

4.3 In what sense could electroweak and color interactions be dual?

Some kind of duality of electroweak and color interactions is suggested by the antipode duality having an interpretation in terms of Hopf algebras (https://en.wikipedia.org/wiki/Hopf_algebra): antipode generalizes the notion of inverse for an element of algebra.

TGD contains several mysterious looking and not-well understood features suggesting some kind of duality between electroweak and color interactions. What could make this duality possible in the TGD framework, would be the presence of Galois symmetry, which would allow us to describe electroweak or color particle multiplets number-theoretically using representations of the Galois group.

1. The electric-magnetic duality or Montonen-Olive duality (https://en.wikipedia.org/wiki/Montonen\OT1\textendashOlive_duality) is inspired by the homology of CP_2 in TGD [?]. The generalization of this duality in gauge theories relates the perturbative description of gauge interactions for gauge group G to a non-perturbative description in terms of magnetic monopoles associated with the dual gauge group G_L. Langlands duality [?, ?] discussed from the TGD perspective in [?, ?] relates the representations of Galois groups and those of Lie groups, and involves Lie group and its Langlands dual. Therefore gauge groups, magnetic monopoles and the corresponding dual gauge group, and number theory seem to be mathematically related, and TGD suggests a physical realization of this view.

2. The dual groups G and G_L should be very similar but electroweak gauge group $U(2)$ and color group $SU(3)$, albeit naturally related as holonomy and isometry groups of CP_2, do not satisfy this condition. Here the Galois group could come into rescue and provide the missing quantum numbers.

3. Depending on the situation, Galois confinement could relate to color confinement or electroweak confinement. In the context of electric-magnetic duality [K3, K8, K5], I have discussed electroweak confinement and as a possible dual description for the electroweak massivation, involving summation of electroweak $SU(2)$ quantum numbers to zero in the scale of monopole flux tubes assignable to elementary particles. The screening of weak isospin would take place by a pair of neutrino and right-handed neutrino in the Compton scale of weak boson or fermion: $h_{eff} > h$ allows longer scales.

4. Also magnetic charge or flux assignable to the flux tubes could make possible a topological description of color hypercharge topologically whereas color isospin could might have description in terms of weak iosospin. I considered this idea already in my thesis. As a matter of fact, already before the discovery of CP_2 around 1980, I proposed that magnetic (homology-) charges 2,-1,-1 for cP_2 could correspond to em charges 2/3,-1/3,-1/3 of quarks and that quark confinement could be a topological phenomenon. Maybe these almost forgotten ideas might find a place in TGD after all.

Consider now the possible duality between electroweak and color interactions.

4.3.1 H level

At the level of H spinors do not couple classically to gluons and color is not spin-like quantum number.

1. The proposal is that the zero energy states are singlets either with respect to the Galois group or the isotropy group of a given root. Z_3 as a subgroup or possibly normal subgroup of the Galois group would act on the space of fermion momenta for which components are algebraic integers belonging to the extension of rationals defined by P.

2. Color confinement could correspond to Galois confinement. Alternatively, the confinement of color isospin could correspond to Galois confinement whereas the confinement of color hypercharge would have a description in terms of the already mentioned monopole confinement. Both number theoretic and topological color would be invisible.

Could antipodal duality be understood number-theoretically?

1. The antipodal duality produces an S_3 singlet from a twistor amplitude. Could color singlets correspond to Z_3 Galois-singlets and electroweak singlets above Compton scale to Z_2 singlets.

2. Could Z_2 be realized as an exchange of two gluons ordered cyclically in the amplitude? Could one think that S^6 acts as a Galois group or its isotropy group?

   The stress tensor as a Higgs like state is not a doublet. Could one obtain Higgs as a Z_2 doublet by allowing the entire orbit of S_3 but requiring only that Z_3 singlet property holds true?

3. Could all isotropy groups or even all subgroups of S^3 be allowed. Could S_n quite generally have a representation as a Galois group? This picture applies also to $2n$-gluon amplitudes but also more general conditions for Galois singlet property can be imagined.

4.3.2 M^8 level

The roles of color and electroweak quantum numbers are changed in $M^8 - H$ duality [L9, L10].

1. At the level of M^8, complexified octonionic 2-spinors [L8, L9, L10] decompose to the representations of the subgroup $SU(3) \subset G_2$ of octonionic automorphisms as $1 + \overline{1} + 3 + \overline{3}$. One obtains leptons and quarks with spin but electroweak quantum numbers do not appear as spin-like quantum numbers. This would suggests that one should assume both lepton and quark spinors at the level of H although the idea about leptons as 3-quark composites in CP_2 scale is attractive [L16].

   One can however construct octonionic spinor fields $M^4 \times E^4$ with the spinor partial waves belonging to the representations of $SO(4) = SU(2) \times SU(2)$ decomposing to representation of $U(2)$ with quantum numbers having interpretation as orbital angular momentum like electroweak quantum numbers.

2. At the level of 4-surfaces of M^8, weak isospin doublet could correspond to Galois doublet associated with a Z_2 factor of the Galois group.

4.3.3 Twistor space level

Also at the level of twistor spaces, the roles of electroweak and color numbers are changed in $M^8 - H$ duality.

1. At the level of H, $M^4 \times CP_2$ is replaced by the product of the twistor spaces $T(M^4)$ and $T(CP_2) = SU(3)/U(1) \times U(1)$. Since spinors are not involved anymore, electroweak quantum numbers disappear. Number theoretic description should apply. Here Galois subgroup Z_2 could help.

   This suggests that $U(2) \subset SU(3)$ must be interpreted in terms of electroweak quantum numbers. There indeed exists a natural embedding of the holonomy group of CP_2 to its isometry group. At the level of space-time, surface color hyper-charge and isopin could correspond to electroweak hyper-charge and isospin. This works if, for given electroweak quantum numbers, the choice of the quantization axes of color quantum numbers depends on the state so that the regions of space-time surface assignable to a fermion depends on its color quantum numbers in H. This would give a correlation between space-time geometry and quantum numbers.

2. At the level of M^8 the twistor space $T(E^4)$ contains information about weak quantum numbers but no information of color quantum numbers since octonionic spinors are given up. Z_6 as a subgroup of the Galois group could help now.

Also the induced twistor structure at the level of space-time surface in H and at the level of 4-surface in M^8 gives strong consistency conditions.

1. The induced twistor structure for the surface $T(X^4) \subset T(H)$ has S^2 bundle structure characterizing twistor space. This structure is obtained by dimensional reduction to $X^6 = X^4 \times S^2$ locally such that S^2 corresponds to the twistor sphere of both $T(M^4)$ and $T(CP_2)$.

2. For cognitive representations as unique number theoretic discretizations of the space-time surface, the twistor spheres S^2 of $T(M^4)$ *resp.* $T(CP_2)$ must correspond to each other. The point of S^2 represents the direction of the quantization axis and the value $\pm 1/2$ of spin *resp.* color isospin or appropriately normalized color hypercharge respectively.

   For quark triplets this kind of correlation can make sense between spin and color hypercharge only and only at the level of the space-time surface. Since the quantization directions of color isospin are not fixed, only the correlation between representations, rather states, is required and makes sense for quarks. This suggests that color isospin at the space-time level must correspond to Galois quantum number.

3. What about leptons? For leptons color hypercharge vanishes. However, both leptonic and quark-like induced spinors have anomalous hypercharge proportional to electromagnetic charge so that also leptonic spinors would form doublets with respect to anomalous color [K9].

The induced twistor structure for 4-surfaces in M^8 does not correspond to dimensional reduction but one expects an analogous correlation between spin and electroweak quantum numbers induced by the mapping of the twistor spheres S^2 to each other.

1. This correlation spin H-spinors correspond to tensor products of spin and electroweak doublets and all elementary particles are constructed from these.

2. Something seems to be however missing: also M^4 spinors should have a $U(1)$ charge to make the picture completely symmetric. The spinor lift strongly suggests that also M^4 has the analog of Kähler structure [L19] and this would give rise to $U(1)$ charge for M^4 spinors [L4] [K8]. This coupling could give rise to small CP breaking effects at the level of fundamental spinors [L19].

The experimental picture about strong and electroweak interactions suggests that the description of standard model interactions as either color interactions or electroweak interactions combined with a number theoretic/topological description of the missing quantum numbers is enough.

1. In hadron physics, only electroweak quantum numbers are visible. Color could be described using number-theory and topology and also these descriptions might be dual. In the QCD picture at high energies only color quantum numbers are visible and electroweak quantum numbers could be described number-theoretically. For a given particle, electroweak confinement would work above its Compton scale of weak scale.

2. In the old fashioned hadron physics conserved vector current hypothesis (CVC) and partially conserved axial current hypothesis (PCAC) relate hadron physics and electroweak physics in a manner which is not fully understood since also quark confinement is still poorly understood. PCAC reflects the massivation of hadrons and can be also seen as caused by the massivation of quarks and leptons and makes successful predictions. In the TGD framework PCAC is applied to the model of so-called lepto-hadrons [K12].

   One can say that hadronic description uses $SO(4) = SU(2)_L \times U(2)_R$ or rather, $U_{ew}(2)$ as a symmetry group whereas QCD uses $SU(3)$ in accordance with the duality between color and electroweak interactions. This conforms with the $M^8 - H$ duality.

3. What about CP_2 type extremals (wormhole contacts), which have Euclidean metric. Could electroweak spin be described as the spin of an octo-spinor and could M^4 spin be described number-theoretically.

What about leptons? For leptons color hypercharge vanishes. However, both leptonic and quark-like induced spinors have anomalous hypercharge proportional to electromagnetic charge so that also leptonic spinors would form doublets with respect to anomalous color.

5 How could Julia sets and zeta functions relate to Galois confinement?

In this section the limit of large particle number of identical particles for the scattering is considered. It is found that the mass spectrum belongs to the Julia set of an infinitely iterated polynomial defining the many-particle state. Also a generalization replacing polynomials with real analytic functions is discussed and it is found that zeta functions and elliptic functions are especially interesting concerning conformal confinement as analog of Galois confinement.

5.1 The mass spectrum for an iterate of polynomial and chaos theory

Suppose that the number theoretic interaction in the scattering corresponds to a functional composition of the polynomials characterizing the external particles. If the number of the external particles is large, the composite can involve a rather high iterate of a single polynomial. This motivates the study of the scattering of identical particles described by the same polynomial P at the limit of a large particle number. These particles could correspond to elementary particles, in particular IR photons and gravitons. This situation leads to an iteration of a complex polynomial.

If the polynomials satisfy $P(0) = 0$ requiring $P(x) = xP_1(x)$, the roots of P are inherited. In this case fixed points correspond to the points with $P(x) = 1$. Assume also that the coefficients are rational. Monic polynomials are an especially interesting option.

For a k:th iterate of P, the mass squared spectrum is obtained as a union of spectra obtained as images of the spectrum of P under iterates P^{-r}, $r \leq k$, for the inverse of P, which is an n-valued algebraic function if P has degree n. This set is a subset of Fatou set (`https://cutt.ly/hOgq6Yy`)and for polynomials a subset of filled Julia set.

At the limit of large k, the limiting contributions to the spectrum approach a subset of Julia set defined as a P-invariant set which for polynomials is the boundary of the set for which the iteration divergences. The iteration of all roots except $x = 0$ (massless particles) leads to the Julia set asymptotically.

All inverse iterates of the roots of P are algebraic numbers. The Julia set itself is expected to contain transcendental complex numbers. It is not clear whether the inverse iterates at the limit are algebraic numbers or transcendentals. For instance, one can ask whether they could consist of n-cycles for various values of n consisting of algebraic points and forming a dense subset of the Julia set. The fact that the number of roots is infinite at this limit, suggests that a dense subset is in question.

The basic properties of Julia set deserve to be listed.

1. At the real axis , the fixed points satisfying $P(x) = x$ with $|dP/dx| > 1$ are repellers and belong to the Julia set. In the complex plane, the definition of points of the Julia set is $|P(w) - P(z)| \geq |w - z|$ for point w near to z.

2. Julia set is the complement of the Fatou set consisting of domains. Each Fatou domain contains at least one critical point with $dP/dz = 0$. At the real axis, this means that P has maximum or minimum. The iteration of P inside Fatou domain leads to a fixed point inside the Fatou set and inverse iteration to its boundary. The boundaries of Fatou domains combine to form the Julia set. In the case of polynomials, Fatou domains are labeled by the $n - 1$ solutions of $dP/dz = P_1 + zdP_1/dz = 0$.

3. Julia set is a closure of infinitely many periodic repelling orbits. The limit of inverse iteration leads towards these orbits. These points are fixed points for powers P^n of P.

4. For rational functions Julia set is the boundary of a set consisting of points whose iteration diverges to infinity. For polynomials Julia set is the boundary of the so-called filled Julia set consisting of points for which the iterate remains finite.

Chaos theory also studies the dependence of Julia set on the parameters of the polynomials. Mandelbrot fractal is associated to the polynomial $Q(z) = a + z^2$ for which origin is an stable critical point and corresponds to the boundary of the region in a-plane containing origin such that outside the boundary the iteration leads to infinity and in the interior to origin.

The critical points of P with $dP/dz = 0$ for $z = z_{cr}$ located inside Fatou domains are analogous to point $z = 0$ for $Q(z)$ associated with Fatou domains and quadratic polynomial $a + b(z - z_{cr})^2$, $b > 0$, would serve as an approximation. The variation of a is determined by the variation of the coefficients of P required to leave z_{cr} invariant.

Feigenbaum studied iteration of a polynomial $a - x^2$ for which origin is unstable critical point and found that the variation of a leads to a period doubling sequence in which a sequence of 2^n-cycles is generated (`https://cutt.ly/pOgwuqj`). Origin would correspond to an unstable critical point $dP(z)/dz = 0$ belonging to a Julia set.

The physical implications of this picture are highly interesting.

1. For a large number of interacting quarks, the mass squared spectrum of quarks as roots of the iterate of P in the interaction region would approach the Julia set as infinite inverse iterates of the roots of P. This conforms with the idea that the complexity increases with the particle number.

   Galois confinement forces the mass squared spectrum to be integer valued when one uses as a unit the p-adic mass scale defined by the larger ramified prime for the iterate. The complexity manifests itself only as the increase of the microscopic states in interaction regions.

2. Julia set contains a dense set consisting of repulsive n-cycles, which are fixed points of P and the natural expectation is that the mass spectrum decomposes into n-multiplets. Whether all values of n are allowed, is not clear to me. The limit of a large quark number would also mean an approach to (quantum) criticality.

To sum up, it would seem that chaos (or rather complexity-) theory could be an essential part of the fundamental physics of many-quark systems rather than a mere source of pleasures of mathematical aesthetics.

5.2 A possible generalization of number theoretic approach to analytic functions

$M^8 - H$ duality also allows the possibility that space-time surfaces in M^8 are defined as roots of real analytic functions. This option will be considered in this subsection.

5.2.1 Are polynomials 4-surfaces only an approximation

One of the open problems of the number-theoretic vision is whether the space-time surfaces associated with rational or even monic polynomials are an approximation or not.

1. One could argue that the cognitive representations are only a universal discretization obtained by approximating the 4-surface in M^8 by a polynomial. This discretization relies on an extension of rationals and more general than rational discretizations, which however appear via Galois confinement for the momenta of Galois singlets.

   One objection against space-time surfaces as being determined by polynomials in M^8 was that the resulting 4-surfaces in M^8 would bre algebraic surfaces. There seems to be no hope about Fourier analysis. The problem disappeared with the realization that polynomials determine only the 3-surfaces as mass-shells of M^4 and that $M^8 - H$ duality is realized by an explicit formula subject to $I(D) = exp{-K}$ condition.

2. Galois confinement provides a universal mechanism for the formation of bound states. Could evolution be a development of real states for which cognitive representations in terms of quarks become increasingly precise.

   That the quarks defining the active points of the representation are at 3-D mass shells would correspond to holography at the level of M^8. At the level of H they would be at the boundaries of CD. This would explain why we experience the world as 3-dimensional.

Also the 4-surfaces containing quark mass shells defined in terms of roots of arbitrary real analytic functions are possible.

1. Analytic functions could be defined in terms of Taylor or Laurent series. In fact, any representation can be considered. Also now one can consider representation involving only integers, rationals, algebraic numbers, and even reals as parameters playing a role of Taylor coefficients.

2. Does the notion of algebraic integers generalize? The roots of the holomorphic functions defining the meromorphic functions as their ratios define an extension of rationals, which is in the general transcendental. It is plausible that the notion of algebraic integers generalizes and one can assume that quarks have momentum components, which are transcendental integers. One can also define the transcendental analog of Galois confinement.

3. One can form functional composites to construct scattering amplitudes and this would make possible particle reactions between particles characterized by analytic functions. Iteration of analytic functions and approach to chaos would emerge as the functions involved appear very many times as one expects in case of IR photons and gravitons.

What about p-adicization requiring the definition discriminant D and identification of the ramified primes and maximal ramified prime? Under what conditions do these notions generalize?

1. One can start from rational functions. In the case of rational functions R, one can generalize the notion of discriminant and define it as a ratio $D = D_1/D_2$ of discriminants D_1 and D_2 for the polynomials appearing as a numerator and denominator of R. The value of D is finite irrespective of the values of the degrees of polynomials.

2. Analytic functions define function fields. Could a generalization of discriminant exist. If the analytic function is holomorphic, it has no poles so that D could be defined as the product of squares of root differences.

   If the roots appear as complex conjugate pairs, D is real. This is guaranteed if one has $f(\overline{z}) = \overline{f(z)}$. The real analyticity of f guarantees this and is necessary in the case of polynomials. A stronger condition would be that the parameters such as Taylor coefficients are rational.

   If the roots are rationals, the discriminant is a rational number and one can identify ramified primes and p-adic prime if the number of zeros is finite.

3. Meromorphic functions are ratios of two holomorphic functions. If the numbers of zeros are finite, the ratio of the discriminants associated with the numerator and denominator is finite and rational under the same assumptions as for holomorphic functions.

4. $M^8 - H$ duality leads to the proposal that the discriminant interpreted as a p-adic number for p-adic prime defined by the largest ramified prime, is equal to the exponent of $exp(-K)$ of Kähler function for the space-time surface in H.

   If one can assign ramified primes to D, it is possible to interpret D as a p-adic number having a finite real counterpart in canonical identification. For instance, if the roots of zeta are rationals, this could make sense.

5.2.2 Questions related to the emergence of mathematical consciousness

These considerations inspire further questions about the emergence of mathematical consciousness.

1. Could some mathematical entities such as analytic functions have a direct realization in terms of space-time surfaces? Could cognitive processes be identified as a formation of functional composites of analytic functions? They would be analogs of particle reactions in which the incoming particles consist of quarks, which are associated with mass-shells defined by the roots of analytic function.

   These composites would decay to products of polynomials in cognitive measurements involving a cascade of SSFRs reducing the entanglement between a relative Galois group and corresponding normal group acting as Galois group of rationals [L13].

2. Could the basic restriction to cognition come from the Galois confinement: momenta of composite states must be integers using p-adic mass scale as a unit.

   Or could one think that the normal sub-group hierarchies formed by Galois groups actually give rise to hierarchies of states, which are Galois confined for an extension of the Galois group.

   Could these higher levels relate to the emergence of consciousness about algebraic numbers. Could one extend computationalism allow also extensions of rationals and algebraic integers as discussed in [L12].

   Galois confinement for an extension of rationals would be analogous to the replacement of a description in terms of hadrons with that in terms of quarks and mean increase of cognitive resolution. Also Galois confinement could be generalized to its quantum version. One could have many quark states for which wave function in the space of total momenta is Galois singlet whereas total momenta are algebraic integers. S-wave states of a hydrogen atom define an obvious analog.

3. During the last centuries the evolution of mathematical consciousness has made huge steps due to the discovery of various mathematical concepts. Essentially a transformation of rational arithmetics with real analysis and calculus has taken place since the times of Newton. Could these evolutionary explosions correspond to the emergence of space-time surfaces defined by analytic functions or is it that only a conscious awareness about their existence has emerged?

5.2.3 Space-time surfaces defined by zeta functions and elliptic functions

Several physical interpretations of Riemann zeta have been proposed. Zeta has been associated with chaotic systems, and the interpretation of the imaginary parts of the roots of zeta as energies has been considered. Also an interpretation as a formal analog of a partition function has been considered. The interpretation as a scattering amplitude was considered by Grant Remmen [B3] (`https://cutt.ly/TID1kjU`).

1.Conformal confinement as Galois confinement for polynomials?

TGD suggests a totally different kind of approach in the attempts to understand Riemann Zeta. The basic notion is conformal confinement [K6].

1. The proposal is that the zeros of zeta correspond to complex conformal weights $s_n = 1/2+iy_n$. Physical states should be conformally confined meaning that the total conformal weight as the sum of conformal weights for a composite particle is real so that the state would have integer value conformal weight n, which is indeed natural. Also the trivial roots of zeta with $s = -2n$, $n > 0$, could be considered.

2. In $M^8 - H$ duality, the 4-surfaces $X^4 \subset M^8$ correspond to roots of polynomials P. M^8 has an interpretation as an analog of momentum space. The 4-surface involves mass shells $m^2 = r_n$, where r_n is the root of the polynomial P, algebraic complex number in general.

   The 4-surface goes through these 3-D mass-shells having $M^4 \subset M^8$ as a common real projection. The 4-surface is fixed from the condition that it defines $M^8 - H$ duality mapping it to $M^4 \times CP_2$. One can think X^4 as a deformation of M^4 by a local $SU(3)$ element such that the image points are $U(2)$ invariant and therefore define a point of CP_2. $SU(3)$ has an interpretation as octonionic automorphism.

3. Galois confinement states that physical states as many-quark states with quark momenta as algebraic integers in the extension defined by the polynomial have integer valued momentum components in the scale defined by the causal diamond also fixed by the p-adic prime identified as the largest ramified prime associated with the discriminant D of P.

   Mass squared in the stringy picture corresponds to conformal weight so that the mass squared values for quarks are analogous to conformal weights and the total conformal weight is integer by Galois confinement.

2.Conformal confinement for zeta functions

At least formally, TGD also allows a generalization of real polynomials to analytic functions. For a generic analytic function it is not possible to find superpositions of roots that would be integers and this could select Riemann Zeta and possible other analytic functions are those with infinite number of roots since they might allow a large number of bound states and be therefore winners in the number theoretic selection.

Riemann zeta is a highly interesting analytic function in this respect.

1. Actually an infinite hierarchy of zeta functions, one for any extension of rationals and conjectured to have zeros at the critical line, can be considered. Could one regard these zetas as analogous to polynomials with an infinite degree so that the allowed mass squared values for quarks would correspond to the roots of zeta?

2. Conformal confinement [K6] requires integer valued momentum components and total conformal weights as mass squared values. The fact that the roots of zetas appear as complex conjugates allows for a very large number of states with real conformal weights. This is however not enough. The fact that the roots are of the form $z_n = 1/2 + iy_n$ or $z = -2n$ implies that the conformal weights of Galois/conformal singlets are integer-valued and the spectrum is the same as in conformal field theories.

3. Riemann zeta has only a single pole at $s = 1$. Discriminant would be the product $\prod_{m\neq n}(y_m - y_n^2)\prod_{m\neq n} 4(m-n)^2 \prod_{m,n}(4m^2 + y_n^2)$ since the pole gives $D = 1$. D would be infinite.

4. Fermionic zeta $\zeta_F(s) = \zeta(s)/\zeta(2s)$ is analogous to the partition function for fermionic statistics and looks more appropriate in the case of quarks. In this case, the zeros are z_n *resp.* $z_n/2$ and the ratio of determinants would reduce to an infinite power of 2. The ramified prime would be the smallest possible: $p = 2$!

   $D = D_1/D_2$ would be infinite power of 2 and 2-adically zero so that $exp(-K)$ should vanish and Kähler function would diverge. 3-adically it would be infinite power of -1. If one can say that the number of roots is even, one has $D = 1$ 3-adically. Kähler function would be equal to zero, which is in principle possible.

   For Mersenne primes $M_n = 2^n - 1$, 2^n would be equal to $1 + M_n = 1 \ \ modM_n$ and one would obtain an infinite power $1 + M_n$, which is equal to 1 mod M_n. Could this relate to the special role of Mersenne primes?

5. What about Riemann Hypothesis? By $\zeta(\overline{s}) = \overline{zeta}(s)$, the zeros of zeta appear in complex conjugate pairs. By functional equation, also s and $1 - s$ are zeros. Suppose that there is a zero $s_+ = s_0 + iy_n$ with $s_0 \neq 1/2$ in the interval $(0, 1)$. This is accompanied by zeros $\overline{s}_+$, $1 - s_+$, $s_- = 1 - \overline{s}_+$. The sum of these four zeros is equal to $s = 2$. Therefore Galois singlet property does not allow us to say anything about the Riemann hypothesis.

3.Conformal confinement for elliptic functions

Elliptic functions (`https://cutt.ly/dINxAeQ`) provide examples of analytic functions with infinite number of roots forming a doubly periodic lattice and are therefore candidates for analogs of polynomials with infinite degree.

1. Weierstrass $\mathcal{P}(z)$-function $\mathcal{P}(z) = \sum_\lambda 1/(z-\lambda)^2$, where the summation is over the lattice defined by a complex modular parameter τ, is the fundamental elliptic function. The basic objection is that $\mathcal{P}(z)$ is not real analytic. Despite this it is interesting to look at its properties so that conformal weights do not appear in complex conjugate pairs. Therefore it is not clear whether conformal confinement is possible. One can also ask whether the notion of integer could be replaced with that of "modular" integers $m + n\tau$.

2. Elliptic functions are doubly periodic and characterized by the ratio τ of complex periods ω_1 and ω_2. One can assume the convention $\omega_1 = 1$ giving $\omega_2 = \tau$. The roots of the elliptic function for an infinite lattice and complex rational roots are of obvious interest concerning the generalization of Galois/conformal confinement.

3. The fundamental set of zeros is associated with a cell of this lattice. The finite number of zeros (with zero with multiplicity m counted as m zeros) in the cell is the same as the number poles and characterizes partially the elliptic function besides τ.

4. Weierstrass $\mathcal{P}$-function and its derivative $d\mathcal{P}/\lceil\ddagger$ are the building blocks of elliptic functions. A general elliptic function is a rational function of $\mathcal{P}$ and $d\mathcal{P}/\lceil\ddagger$. In even elliptic functions only the even funktion $\mathcal{P}$ appears.

5. The roots of Weierstrass $\mathcal{P}$-function $\mathcal{P}(z) = \sum_\lambda 1/(z-\lambda)^2$ appear in pairs $\pm z$ whereas the double poles at at the points of the modular lattice: see the article "The zeros of the Weierstrass $\mathcal{P}$-function and hypergeometric series" of Duke and Imamoglu [A2] (`https://cutt.ly/uIZSK4T`).

   The roots are given by Eichler-Zagier formula $z_\pm(m,n) = 1/2+m+n\tau\pm z_1$, where z_1 contains an imaginary transcendental part $log(5+2\sqrt{6})/2\pi)$ plus second part, which depends on τ (see formula 6) of `https://cutt.ly/uIZSK4T`).

6. Conformally confined states with conformal weights $h = 1+(m_1+m_2)+(n_1+n_2)\tau$ can be realized as pairs with conformal weights $(z_+(m_1,n_1), z_-(m_2,n_2)$. The condition $n_1 = -n_2$ guarantees integer-valued conformal weights and conformal confinement for a general value of τ.

7. A possible problem is that the total conformal weights can be also negative, which means tachyonicity. This is not a problem also in the case of Riemann zeta if trivial zeros are included.

   As a matter of fact, already at the level of M^8, M^4 Kähler structure implies that right-handed neutrino ν_R is a tachyon [L19]. However, ν_R provides the tachyon needed to construct massless super-symplectic ground states and also allows us to understand why neutrinos can be massive although right-handed neutrinos are not detected. The point is that only the square of Dirac equation in H is satisfied so that different M^4 chiralities can propagate independently.

   In M^8-H duality, non-tachyonicity makes it possible to map the momenta at mass shell to the boundary of CD in H. Hence the natural condition would be that the total conformal weight of a physical state is non-negative.

What about the notion of discriminant and ramified prime? One can assign to the algebraic extensions primes as prime ideals for algebraic integers and this suggests that the generalization of p-adicity and p-adic prime is possible. If this is the case also for transcendental extensions, it would be possible to define transcendental p-adicity.

One can however ask whether the discriminant is rational under some conditions. D could also allow factorization to the primes of the transcendental extension.

1. Elliptic functions are meromorphic and have the same number of poles and zeros in the basic cell so that there are some hopes that the ratio of discriminants is finite and even rational or integer for a suitable choice of the modular parameter τ as the ratio of the periods and the other parameters. Discriminant D as the ratio D_1/D_2 of the discriminants defined by the products of differences of roots and poles could be finite although they diverge separately.

2. For the Weierstrass $\mathcal{P}$-function, the zeros appear as pairs $\pm z_0$ and also as complex conjugate pairs. Complex pairs are required by real analyticity essential for the number theoretical vision. It might be possible to define the notion of ramified prime under some assumptions.

   For $z_+(m,n)$ or $z_-(m,n)$, the defining D_1 in D_1/D_2 would reduce to a product $\prod_{m,n}\Delta_{m,n})^2(\Delta_{m,n}+2z_1)(\Delta_{m,n}-2z_1)$, $\Delta_{m,n} = \Delta m+\Delta n\tau$, which is a complex integer valued if τ has integer components. D_1 would be a product of Gaussian integers.

3. The number of poles and zeros for the basic cell is the same so that D_2 as a product of the pole differences would have an identical general form. For large values of m,n, the factors in the product approach $\Delta_{m,n}$ for both zeros and poles so that the corresponding factors combine to a factor approaching unity.

The double poles of $\mathcal{P}(z) = \sum_\lambda 1/(z-\lambda)^2$ are at points of the lattice. One has $D_2 = \prod_{m,n}\Delta_{m,n})^4$. This gives

$$D = \frac{D_1}{D_2} = \prod_{m,n}(1+\frac{2z_0}{\Delta_{m,n}})(1-\frac{2z_0}{\Delta_{m,n}}) = \prod_{m,n}(1-4(\frac{2z_0}{\Delta_{m,n}})^2) \ .$$

Therefore D is finite and in general complex and transcendental so that the notion of ramified prime does not make sense as an ordinary prime. z_0 contains a transcendental constant term plus a term depending on τ (https://cutt.ly/uIZSK4T). Whether values of τ for which D is rational, might exist, is not clear.

In the number theoretic vision, the construction of many-particle states corresponds to the formation of functional composites of polynomials P. If the condition $P(0) = 0$ is satisfied, the $n - fold$ composite inherits the roots of $n-1$-fold composites and the roots are like conserved genes. If one multiplies zeta functions and elliptic functions by z, one obtains similar families and the formation of composites gives rise to iteration sequences and approach to chaos [L11].

REFERENCES

Mathematics

[A1] Rovelli C Connes A. Von Neumann algebra automorphisms and time-thermodynamics relation in general covariant quantum theories, 1994. Available at: https://arxiv.org/PS_cache/gr-qc/pdf/9406/9406019v1.pdf.

[A2] Duke W and Imamoglu Ö. The zeros of the Weierstrass $\mathcal{P}$-function and hypergeometric series. *Mathematische Annalen*, 340(4):897–905, 2008. Available at: https://cutt.ly/uIZSK4T.

Theoretical Physics

[B1] Dixon LJ et al. Folding Amplitudes into Form Factors: An Antipodal Duality, 2022. Available at: https://arxiv.org/pdf/2112.06243.pdf.

[B2] Minev ZK et al. To catch and reverse a quantum jump mid-flight, 2019. Available at: https://arxiv.org/abs/1803.00545.

[B3] Remmen GN. Amplitudes and the riemann zeta function. *Phys Rev Lett*, 2021. Available at: https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.127.241602.

[B4] Vergu C Goncharov AB, Spradlin M and Volovich A. Classical Polylogarithms for Amplitudes and Wilson Loops. *Phys Rev Lett*, 105(151605), 2010. textnormalAvailable at: https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.105.151605.

Cosmology and Astro-Physics

[E1] Nottale L Da Rocha D. Gravitational Structure Formation in Scale Relativity, 2003. Available at: https://arxiv.org/abs/astro-ph/0310036.

Books related to TGD

[K1] Pitkänen M. Basic Properties of CP_2 and Elementary Facts about p-Adic Numbers. In *Towards M-matrix*. Available at: https://www.tgdtheory.fi/public_html/pdfpool/append.pdf, 2006.

[K2] Pitkänen M. Construction of elementary particle vacuum functionals. In *p-Adic Physics*. Available at: https:/tgdtheory.fi/pdfpool/elvafu.pdf, 2006.

[K3] Pitkänen M. Identification of the WCW Kähler Function. In *Quantum Physics as Infinite-Dimensional Geometry*. Available at: https:/tgdtheory.fi/pdfpool/kahler.pdf., 2006.

[K4] Pitkänen M. Massless states and particle massivation. In *p-Adic Physics*. Available at: https:/tgdtheory.fi/pdfpool/mless.pdf, 2006.

[K5] Pitkänen M. New Particle Physics Predicted by TGD: Part II. In *p-Adic Physics*. Available at: https:/tgdtheory.fi/pdfpool/mass5.pdf, 2006.

[K6] Pitkänen M. Number theoretic vision, Hyper-finite Factors and S-matrix. In *Towards M-Matrix: part I*. Available at: https:/tgdtheory.fi/pdfpool/UandM.pdf, 2012.

[K7] Pitkänen M. *p-Adic length Scale Hypothesis*. Online book. Available at: https://www.tgdtheory.fi/tgdhtml/padphys.html., 2013.

[K8] Pitkänen M. About Preferred Extremals of Kähler Action. In *Physics in Many-Sheeted Space-Time: Part I*. Available at: https:/tgdtheory.fi/pdfpool/prext.pdf, 2019.

[K9] Pitkänen M. Dark valence electrons and color vision. In *TGD based view about living matter and remote mental interactions: Part II*. Available at: https:/tgdtheory.fi/pdfpool/colorvision.pdf, 2019.

[K10] Pitkänen M. p-Adic Physics: Physical Ideas. In *TGD as a Generalized Number Theory: Part I*. Available at: https:/tgdtheory.fi/pdfpool/phblocks.pdf, 2019.

[K11] Pitkänen M. TGD as a Generalized Number Theory: Quaternions, Octonions, and their Hyper Counterparts. In *TGD as a Generalized Number Theory: Part I*. Available at: https:/tgdtheory.fi/pdfpool/visionb.pdf, 2019.

[K12] Pitkänen M. The Recent Status of Lepto-hadron Hypothesis. In *Hyper-finite Factors and Dark Matter Hierarchy: Part II*. Available at: https:/tgdtheory.fi/pdfpool/leptc.pdf, 2019.

[K13] Pitkänen M. The Recent View about Twistorialization in TGD Framework. In *Towards M-Matrix: Part II*. Available at: https:/tgdtheory.fi/pdfpool/smatrix.pdf, 2019.

[K14] Pitkänen M. Zero Energy Ontology. In *Towards M-Matrix: part I*. Available at: https:/tgdtheory.fi/pdfpool/ZEO.pdf, 2021.

Articles about TGD

[L1] Pitkänen M. The Geometry of CP_2 and its Relationship to Standard Model. 2010. Available at: https://tgdtheory.fi/pdfpool/appendb.pdf.

[L2] Pitkänen M. Philosophy of Adelic Physics. In *Trends and Mathematical Methods in Interdisciplinary Mathematical Sciences*, pages 241–319. Springer.Available at: https://link.springer.com/chapter/10.1007/978-3-319-55612-3_11, 2017.

[L3] Pitkänen M. Philosophy of Adelic Physics. Available at: https:/tgdtheory.fi/public_html/articles/adelephysics.pdf., 2017.

[L4] Pitkänen M. Questions about twistor lift of TGD. Available at: https:/tgdtheory.fi/public_html/articles/twistquestions.pdf., 2017.

[L5] Pitkänen M. About the physical interpretation of the velocity parameter in the formula for the gravitational Planck constant . Available at: https:/tgdtheory.fi/public_html/articles/vzero.pdf., 2018.

[L6] Pitkänen M. The Recent View about Twistorialization in TGD Framework. Available at: https:/tgdtheory.fi/public_html/articles/smatrix.pdf., 2018.

[L7] Pitkänen M. Copenhagen interpretation dead: long live ZEO based quantum measurement theory! Available at: https:/tgdtheory.fi/public_html/articles/Bohrdead.pdf., 2019.

[L8] Pitkänen M. TGD view about McKay Correspondence, ADE Hierarchy, Inclusions of Hyper-finite Factors, $M^8 - H$ Duality, SUSY, and Twistors. Available at: https:/tgdtheory.fi/public_html/articles/McKaygeneral.pdf., 2019.

[L9] Pitkänen M. A critical re-examination of $M^8 - H$ duality hypothesis: part I. Available at: https:/tgdtheory.fi/public_html/articles/M8H1.pdf., 2020.

[L10] Pitkänen M. A critical re-examination of $M^8 - H$ duality hypothesis: part II. Available at: https:/tgdtheory.fi/public_html/articles/M8H2.pdf., 2020.

[L11] Pitkänen M. Could quantum randomness have something to do with classical chaos? Available at: https:/tgdtheory.fi/public_html/articles/chaostgd.pdf., 2020.

[L12] Pitkänen M. MIP*= RE: What Could This Mean Physically? Available at: https://tgtheory.fi/public_html/articles/deeptheorem.pdf., 2020.

[L13] Pitkänen M. The dynamics of SSFRs as quantum measurement cascades in the group algebra of Galois group. Available at: https:/tgdtheory.fi/public_html/articles/SSFRGalois.pdf., 2020.

[L14] Pitkänen M. Zero energy ontology, hierarchy of Planck constants, and Kähler metric replacing unitary S-matrix: three pillars of new quantum theory (short version). Available at: https:/tgdtheory.fi/public_html/articles/kahlersm.pdf., 2020.

[L15] Pitkänen M. About the role of Galois groups in TGD framework. https://tgdtheory.fi/public_html/articles/GaloisTGD.pdf., 2021.

[L16] Pitkänen M. Can one regard leptons as effectively local 3-quark composites? https://tgdtheory.fi/public_html/articles/leptoDelta.pdf., 2021.

[L17] Pitkänen M. Does the notion of polynomial of infinite order make sense? https://tgdtheory.fi/public_html/articles/transcendGalois.pdf., 2021.

[L18] Pitkänen M. Is genetic code part of fundamental physics in TGD framework? Available at: https://tgdtheory.fi/public_html/articles/TIH.pdf., 2021.

[L19] Pitkänen M. Neutrinos and TGD. https://tgdtheory.fi/public_html/articles/TGDneutrino.pdf., 2021.

[L20] Pitkänen M. Spin Glasses, Complexity, and TGD. https://tgdtheory.fi/public_html/articles/sg.pdf., 2021.

[L21] Pitkänen M. TGD as it is towards the end of 2021. https://tgdtheory.fi/public_html/articles/TGD2021.pdf., 2021.

[L22] Pitkänen M. Three alternative generalizations of Nottale's hypothesis in TGD framework. https://tgdtheory.fi/public_html/articles/MDMdistance.pdf., 2021.

[L23] Pitkänen M. What could 2-D minimal surfaces teach about TGD? https://tgdtheory.fi/public_html/articles/minimal.pdf., 2021.

[L24] Pitkänen M. About TGD counterparts of twistor amplitudes: part I. https://tgdtheory.fi/public_html/articles/twisttgd1.pdf., 2022.

[L25] Pitkänen M. About TGD counterparts of twistor amplitudes: part II. https://tgdtheory.fi/public_html/articles/twisttgd2.pdf., 2022.

[L26] Pitkänen M. Quantum Gravitation and Topological Quantum Computation. https://tgdtheory.fi/public_html/articles/TQCTGD.pdf., 2022.

[L27] Pitkänen M. Trying to fuse the basic mathematical ideas of quantum TGD to a single coherent whole. https://tgdtheory.fi/public_html/articles/fusionTGD.pdf., 2022.

[L28] Pitkänen M. Two objections against p-adic thermodynamics and their resolution. https://tgdtheory.fi/public_html/articles/padmass2022.pdf., 2022.