About Platonization of Nuclear String Model and of Model of Atoms

December 10, 2023

Matti Pitkänen

orcid:0000-0002-8051-4364.
email: matpitka6@gmail.com,
url: http://tgdtheory.com/public_html/,
address: Rinnekatu 2-4 A 8, 03620, Karkkila, Finland.

Abstract

Platonization of atomic and nuclear physics could be said to be the theme of this work. The construction of electron configurations of atoms and proton and neutron configurations of atomic physics have considerable analogies and the spectra have essentially the same structure in the standard model.

In the TGD framework, the nuclear string model proposes that nuclei form nuclear strings. By quantum classical correspondence energy shells decomposing to subshells containing states with a given angular momentum should have geometric correlates and should correspond to discretization of 2-D surfaces, sphere is a good guess. The first guess is that a nuclear string connecting nucleons is associated with a Platonic solid having nucleons at its vertices.

Therefore Platonic solids as analogs of solid states lattices could provide the discretized space-time and momentum space correlates for energy shells. The nuclear string connecting nucleons would correspond to a Hamilton cycle, which connects the V vertices of the Platonic tessellation and decomposes the edges of the Platonic tessellation to V edges of the cycle and $F-2$ free edges in its complement. The ends of the edges of the Hamilton cycle could contain neutrons and the middle points of the free edges could contain protons (or vice versa in the case of icosahedron). This assignment is suggested by the repulsive Coulomb interaction and explains neutron surplus as well as neutron halos. An alternative, fully symmetric proposal is that Platonic solid and its dual are present. Dual has F vertices at the centers of faces the Platonic solid and V edges connecting them.

Starting from the angular momentum structure of the Periodic Table, one ends up with a detailed model. For low enough angular momentum $l \leq 5$ the states of $j = l \pm 1/2$ multiplets would correspond to different Platonic tessellations for the $F-2$ option. For the F option one would have $l \leq 3$. The increase of h_{eff} increasing the unit of angular momentum would make possible Platonization of higher angular momenta. The twistor lift of TGD explains why fermions with opposite spins correspond to different points of the Platonic solid identified as discretizations of the twistor sphere of discretized momentum twistor space of M^4. Could this construction generalize to arbitrary Lie groups using the analogs of Platonic solids defined by the discrete subspaces of the coset spaces of the group and its Cartan algebra?

The application to nuclear physics, motivated by the tritium beta decay anomaly, leads to a detailed formulation of a model of nucleus in terms of monopole flux tubes. Flux tubes are identified as electropions with mass 1 MeV and the model predicts new excitations in the 10 keV range defined by the mass difference between charged and neutral electropion. The model provides a quantitative explanation for the tritium anomaly. Various objections against Platonization lead to a rather radical prediction. There is a holography mapping the nuclear states to generalized atomic states such that protons correspond to electron shells and neutrons to neutrino shells having size scale of 10^{-8} meters, which is fundamental in biology. This model generalizes to a model of atoms and makes several killer predictions distinguishing

the model from the standard view. The lego model for the construction of nuclear states works also for hadrons.

Contents

1 Introduction

The motivation for this article came from the discovery of a doubly magic form of oxygen (see this) with 8 protons and 28 neutrons combined with the fact that it is difficult to understand the presence of so large number of neutrons in the existing view about strong interactions. A more general challenge of understanding atomic nuclei and atoms in the TGD framework. General coordinate invariance implies that particles as 3-surfaces obey almost deterministic holography so that they give rise to analogs of Bohr orbits. This gives a rather precise content for the idea of what atoms and nuclei are classically but is not enough: one must also understand their shell structure geometrically.

The outcome was a significant advance in the understanding of the nuclear string model [K9]. Nuclear string, consisting of flux tubes connecting the neighboring nucleons of the tessellation, provides a new element to the description of bound states. This element is not present in the standard nuclear physics where only short range strong interactions, assumed to be described in terms of meson exchanges at the low energy limit, are assumed to be present at the fundamental level. The flux tube has a string tension and it literally binds together the objects that it connects.

The earlier proposal that string(s) form a kind of 3-D flux tube spaghetti was replaced with the proposal that energy shells have as space-time counterpart a hierarchy of 2-D spaghettis analogous to magnetic bubbles proposed in the model for the evolution of astrophysical objects [L22]. These 2-D surfaces can be identified as lattices/tessellations of 2-D surfaces having nucleons at their nodes. For the sphere as the geometric counterpart of the energy shell, energy minimization suggests Platonic solids for which the maximal nucleon numbers are fixed. Platonic solid as an analog of solid state lattice naturally represents the energy minimum and their construction is analogous to crystal growth.

Nuclear string is assumed to define a Hamilton cycle connecting the V vertices of the Platonic solid and therefore having V edges. Energy minimization and the phenomena of magic proton and neutron numbers and neutron surplus suggest that neutrons are associated with the V edges of the Hamilton cycle and protons with the $F-2$ (F is the number faces of the Platonic solid and number of vertices for its dual) free edges of the tessellation as a graph as analogs to lattice impurities. One could speak of the dual tessellation. The roles of protons could be changed in the case of icosahedron.

These considerations inspire several questions.

1. Is there a holographic relationship between nuclear quantum states with say $Z = N$ and electronic states of atoms as the identical structures of the states spaces for atomic electrons and nucleons moving in harmonic oscillator potential suggests?

2. The description of electron states in atomic physics and proton and neutron states in spherically symmetric harmonic oscillator model nuclei predict essentially the same spectroscopies. Does the structure of the Periodic Table reflect directly the proposed geometric shell structure of nuclei modelled as unions of 2-D spherical shells as lattices (tessellations) consisting of either neutrons and protons? Genuinely 2-D tessellations correspond to Platonic solids and behave like dynamical units. Could one say that Platonia is realized both at the level of nuclear and atomic physics?

1.1 Platonization of atomic and nuclear physics

1. By starting from atomic physics, one ends up with a number theoretic decomposition of the angular momentum states of protons, neutrons and electrons to subsets using finite field

mathematics. The highly non-trivial fact is that for orbital angular momenta not larger than 5, the states with $j = l + 1/2$ and $j = l - 1/2$ correspond to the points of Platonic solids providing classical space-time correlates of j-blocks as geometric energy shells. $j = 1/2$ and $j = -1/2$ correspond to dual Platonic solids for $l \geq 1$. The edges of Hamilton cycle and its dual consisting of the free edges not belonging to the cycle define what I call dual representations. This applies also in nuclear physics.

2. The first objection is that the geometric representation of half-odd integer angular momentum states looks strange. This natural in the twistor representation of angular momentum states as partial waves in the twistor sphere. The dual representation in momentum twistor space corresponds to discrete subsets of points of the twistor sphere identifiable as Platonic solids. These representations have counterparts at the level of M^4 since the twistor sphere corresponds to the space of light-like rays emanating from a given point.

3. The Platonization of the quantum numbers might generalize. The twistor space would be replaced with the space for the selections of quantization axes identified as a coset space of Lie group with its Cartan group and the connection with McKay correspondence [A4, A3, A2, A1] discussed from the TGD point of view in [K11] [L4, L7, L17] is highly suggestive. The reason is that McKay graphs emerge from the reduction of the representations of the rotation group by restricting them to finite discrete subgroups and Platonic solids indeed define this kind of reduction. Finite measurement resolution can be described in terms both number theoretically and in terms of inclusions of hyperfinite factors and also these involve McKay graphs.

1.2 A new view of nuclear physics

The Platonization of nuclear physics suggests that nuclei are engineered from nucleons by connecting them by monopole flux tubes.

1. The notion of tensegrity applies and predicts that in ground states the distances between neutrons (protons) at the energy shell are constant so that one has a Platonic solid as an analog of ordinary 3-D lattice. Neutron surplus for the nuclei suggests that neutrons are associated with the free edges of Platonic cycles outside the Hamilton cycle. For all Hamiltonian cycles except tetrahedron and icosahedron,the number of dual edges is larger than the number edges of the cycle.

2. Nuclear strings as monopole flux tubes connecting the 3-surfaces representing nucleons have a string tension. This gives rise to an attractive interaction between the nucleons and stabilizes the nucleon configurations to Platonic tessellations or their duals involving nucleons at the free edges of the tessellation as a graph. The longitudinal and transversal vibrational degrees of freedom for individual nuclei are the TGD counterparts of vibrational degrees of the harmonic oscillator model of nuclei.

3. Also the dark classical Z^0 and W force could be involved. Weak isospin would replace strong isospin and the counterparts of pions would be scaled down pions corresponding to electron length scale and having mass scale of 1 MeV. This would require a large value of h_{eff} so that intermediate boson Compton length would correspond to nuclear length scale.

   With an inspiration coming from PCAC and CVC and the fact that CP_2 geometry implies a very intimate relationship between color and electroweak interactions, I have already earlier considered the possibility that strong interactions involving strong isospin could have a dual description as dark electroweak interactions.

   I have also asked whether dark weak bosons are possible even in longer scales and whether this could explain chirality selection in living matter. The neutron halo and maybe even other nucleon shells could be dark. A picture of the nucleus emerges in which the flux tube bonds correspond to pions but with a p-adic length scale of the electron. This explains the MeV scale of nuclear excitations and predicts a new scale of order 10 keV assignable to the mass differences of these pions. This leads to a detailed understanding of the tritium anomaly and the correlation of the nuclear decay with the solar X ray flux.

4. Energy minimization implies that in the ground state the neutrons would be pseudo-neutrons, that is protons connected by negatively charged meson-like flux tubes to each other and also to the protons of the inner shells. Pseudo-neutrons would have a dipole moment and the Coulomb potential of the protons of the nucleus would orient them radially and stabilize the position of the halo. Dipole dipole interactions would bind the pseudo-neutrons to the lower energy shells.

   This mechanism might also work for halos as neutron shells. It turns out that the dipole moments must be of order nuclear size, which requires $h_{eff}/h \sim 100$ so that weak Compton length is of order nucleon Compton length so that weak interactions are as strong as em interactions in nucleon scale.

5. One should also understand the dynamics behind neutron halos (see this) located to the periphery of the nucleus. The first guess is that it corresponds to possibly partially filled Platonic tessellations or their duals but having no protons. The reason would be that the proton charge implies instability. Note however that there also protonic halos have been found. In the case of the oxygen isotope that motivated these considerations neutron halo would correspond to two shells 8 and 20 (cube and icosahedron). Nuclear strings would provide the needed stabilizing force.

6. The notion of electroweak confinement is a very attractive generalization of color confinement and leads to the notion of dark weak atoms in which the surplus Z^0 charge of the nucleus associated with protons is neutralized by opposite charges of electrons. The size of these atom cannot be smaller than the Compton length of dark weak bosons, and should be at most 10 nm, which corresponds to p-adic length scale $L(151)$ defining a fundamental length scale of biology (cell membrane thickness, the thickness of DNA coil, the size of nucleosomes of DNA,...). This would conform with the chiral selection of living matter.

1.3 Nucleus-atom holography

Platonization and the similarity of the spectra of atoms and nucleons inspires a radical but testable strengthening of nucleus-atom holography allowed by the hierarchy of Planck constants, predicting that atomic electron shells are accompanied by neutrino shells and that there is weak confinement in long scales such that electrons and protons and atomic neutrinos and neutrons are paired to weak singlets. The tritium beta decay anomaly [C2], having no generally accepted explanation in the standard model, finds a possible quantitative explanation and thus gives a direct support for the proposal. This would mean that the myth of elusive neutrinos must be given up. The technological implications are obvious.

The obvious objection is that the existence of Hamiltonian tessellations of atomic electrons looks implausible in the standard physics framework. In particular, the existence of monopole flux tubes connecting electrons does not lok plausible. Their string tension would however allow quantum classical correspondence for many electrons not possible in standard view due to the fact the repulsive interaction energy between electrons with the same orbital radius behaves like Z^4 for large Z and is much larger than the attractive binding energy with nucleus.

1.4 Recipe for hadrons

The successful construction of nuclei encourages the question whether hadrons could be obtained by a similar lego construction. Ordinary mesons would correspond to monopole flux tubes. Assume that

1. quark masses are given by by p-adic mass calculations [K2],

2. the p-adic length scale hypothesis is satisfied but quarks and mesons can correspond several p-adic length scales,

3. family replication phenomenon corresponds to the topological mixing of 2-D partonic 2-surfaces for quarks [K1],

4. only baryonic (but not mesonic) quarks suffer topological mixing.

With these assumptions one ends up to a model predicting with a few percent accuracy the masses of light mesons and hadrons. The constituent quarks would correspond to nucleon + flux tube composites and quark sea could be identified as light quarks with p-adic length scale possibly given by electronic p-adic length scale.

2 Platonic solids and nuclear string model

In the sequel the nuclear string model in its original form is discussed first. After that its modification leading to a proposal for the "Platonization" of nuclear and also atomic physics is discussed. The model provides an explanation of neutron surplus and a model for neutron halos. Also a generalization of the notion of atoms suggested by tritium beta decay anomaly and originally motivated by the idea that nuclear holography should be very precise and map proton and neutron states to electron and neutrino states of atoms is proposed. It however turned out that this notion of the atom differs from this naive expectation. These atoms are predicted to have a size scale about 10^{-8} meters, which is a fundamental biological length scale.

2.1 Nuclear string model

The nuclear string model [K10, K9] describes nuclei as string like structures with nucleons connected by color magnetic flux tubes whose length is of order electron Compton length about 10^{-12} meters and even longer and thus much longer than the size scale of nuclei themselves which is below 10^{-14} meters. Color magnetic monopole flux tubes define the color magnetic body of the nucleus.

Each flux tube portion of the nuclear string has a colored quark and antiquark at its ends and is a two-sheeted structure with a monopole flux running along the the Minkowskian sheets of flux tubes as thickened cosmic strings and flowing from Minkowskian sheet to another along Euclidian wormhole contact. Also elementary particles have this kind of flux tubes connecting two wormhole contacts as building bricks and the flux tubes connecting nucleons to connected structures provides a model for nucleus.

If the net color of the quark pair is non-vanishing and also nucleons have color, color confinement binds the nucleons to the nuclear string. The flux tubes bonds could be also color neutral and represent entangled quark and antiquark as a meson-like state. Quark and antiquark would bind to the nucleonic quark with color force.

The flux tube has string tension and energy minimization fixes the ground state configuration of nucleons to be a lattice like structure with constant lengths between the flux tubes. This feature has no counterpart in standard nuclear models. The notion of tensegrity (see this) to which the name of Buckminster Fuller (see this) is associated, is suggestive. Nuclei can be visualized as structures analogous to plants with nuclei taking the role of seed and color magnetic body of much larger size taking the role of plant with color flux tubes however returning back to another nucleon inside the nucleus. This picture also applies to condensed matter lattices.

One can criticize this model of flux tube for the lack of details. What does the monopole flux tube mean: does it correspond to a monopole flux inside an underlying larger space-time sheet with a Minkowskian signature of the induced metric? The problem with this definition is due to the fact that the cross section of the flux tube is a closed surface (which makes possible magnetic fields in absence of currents solving one of the basic problems of cosmology) and does not have a boundary as flux tube with a disk-like cross section has. The only reasonable identification of the flux tube seems to be as a perturbation of an infinitely thin cosmic string of form $X^2 \times Y^2 \subset M^4 \times CP_2$ having 2-D M^4 projection to give a flux tube with 4-D M^4 projection. The wormhole contacts connect two monopole flux tubes of this kind having a distance measured using CP_2 radius as a unit. The space-time would consist of these kinds of flux tubes at the fundamental level. Concerning the local physics, the approximation of a finite flux tube region as M^4 is excellent. The same approximation is made in general relativity and interpreted in terms of Equivalence Principle.

The new physics assumed by the model predicts several new effects, which can be used to explain various anomalies related to nuclear physics. The theoretical view of dark matter as phases of ordinary particles with non-standard value of Planck constant predicts the existence of dark protons and nucleons. This leads to a model of "cold fusion" [K6] [L3, L9] in terms of dark fusion producing dark nuclei with much smaller binding energy per nucleon than for ordinary nuclei, which can spontaneously decay to ordinary nuclei liberating almost all nuclear binding energy in the process. Dark fusion could be also involved with the formation of protostars and elements heavier than Fe could be created outside stellar cores by dark fusion [L6]. The dark nuclei formed as sequences of dark protons play a central role in TGD inspired quantum biology and provide a model for genetic code [L18, L1, L10, L12, L19].

2.2 Platonic tessellations with Hamilton cycles as a geometrization of nuclear shell model

The starting point idea is that Platonic tessellations could provide a geometric correlate for energy shells as discretized 2-sphere 2-D surface of hyperbolic 3-space defining mass shell in $H^3 \subset M^4 \subset M^8$ mapped by $M^8 - H$ duality to light-cone proper time constant surface $H^{\subset} M^4 \subset M^4 \subset H = M^4 \times CP_2$. Nucleus as a union of 3-D lattice-like structures with nuclei connected by nuclear string would be replaced with a union of 2-D energy shells represented by Platonic solids.

2.2.1 Nuclear strings as Hamilton sub-cycles

Nuclear strings would be realized in terms of Hamilton sub-cycles.

1. Nuclear string as Hamilton sub-cycle selects a subset of V (number of vertices) vertices of the Platonic tessellation. The number of vertices of sub-tesselation is the same as its edges. If the numbers of neutrons and protons are the same, and the Hamilton sub-tessellations are if the interaction energy is minimized assuming that protons and neutrons on top of each other have large attractive interaction energy. For full shells this is always the case.

2. The Hamilton sub-cycles for protons and neutrons can be different. The isometries of the Platonic solid in general give rise to different nuclear strings but do not affect the tesselations so that Hamilton cycles represent an additional degrees of freedom. A given Hamilton cycle or sub-cycle has an isotropy group leaving it invariant.

3. A natural requirement is that the Hamilton subcycle bounds a connected subset of the entire tessellation. For Platonic tessellation with Hamilton cycle, the number of free edges is $Efree = F - 2$. This follows from Euler's formula $V - E + F = 2$. A connected sub-tessellation with V vertices and F faces has $E_{free} = F - 1$ free edges. The latter formula follows from the identity $V - E + F = -1$ for polygons and one can easily check it easily for polygons which have no free edges.

   It turns out to be natural to assign to the middle points of the free edges particles as analogs of impurity atoms. The $F - 2$ edges define vertices for an analog of dual Platonic solid with 2 vertices removed.

What can one say about the Hamilton cycles for various Platonic solids?

1. Tetrahedron has only a single Hamilton cycle apart from isometries. For a dodecahedron the full non-oriented Hamilton cycle is unique as already Hamilton found. For an icosahedron the number of Hamilton cycles is rather large.

2. What about cube and octahedron? One can easily convince that both have several non-isometric Hamilton cycles. In the case of a cube, one can consider what the part of the full Hamilton cycle at a selected quadrilateral face can be. It can consist of opposite edges or include 3 edges. This is true also for the opposite face so that these two basic forms not related by rotation are possible. The same conclusion holds true for an octahedron. Also now can consider the square at the middle of the octahedron and make the same observation.

3. Icosahedron allows a large number of cycles to have Z_6, Z_4, $Z_{2,rot}$, $Z_{2,refl}$ or trivial group as an invariance group of the cycle. These cycles play a key role in the model of bioharmony leading also to a model of genetic code [L1, L10, L12, L19]. The presence of several Hamilton cycles implies several nuclear strings characterized by the isotropy group. Note that the Mg nucleus with $Z = N = 12$ corresponds to an icosahedron.

For the Hamilton cycles the number of cycle edges is equal to the number V of vertices. For the Platonic solid the total number E of edges is $E = V + F - 2$, where F is the number of face. For a given tessellation with a Hamilton cycle the number of free edges is $E - V = F - 2$. The free edges could be used to attach surplus neutrons as analogs of lattice impurities.

1. For a tetrahedron one has $E = 6 > V = 4$ so that 2 edges do not belong to the full cycle. The intersection of two cycles has at least 2 common edges. For the octahedron one has $V = 6$ and $F = 8$ giving $E = 12$ so that at 6 edges are not used. The intersection of two cycles could have no common edges. For a cube, the dual of an octahedron, one has $V = 8$ and $F = 6$ so that the intersection of two cycles has at least 4 edges.

2. For an icosahedron one has $V = 12, F = 20$ and $E = 30$, so that 18 edges remain unused. This must relate to the large number of non-equivalent Hamilton cycles. Two cycles leave at least 6 unused edges. For a dodecahedron with $V = 20$ and $F = 12$, 10 edges remain unused and the intersection of two cycles has at least 10 edges.

2.2.2 Tessellation with Hamilton cycle and its dual

The simplest form of the idea is that fermions of a given angular momentum multiplet or its sub-multiplet are added at the vertices of the Platonic tessellation. The presence of the Hamilton cycle must have some function. Its presence allows it to divide the edges of the tessellation to V edges belonging to the cycle and $F - 2$ free edges in the complement of the cycle.

This suggests that the notion of impurity atoms generalizes. One can and to the centers of free edges of say neutron tessellation protons. This tessellation is analogous to a tessellation for the dual of the Platonic solid but missing 2 vertices and will be called dual tessellation in the sequel. Similar lack of 2 nodes was found in the model of DNA based on icosa-tetrahedral tessellation of hyperbolic 3-space [L19].

For the dual of Platonic solid, the roles of vertices and faces are changed. This suggests that dual tessellation might have an interpretation as a sequence of faces of the tessellation defined by the Hamilton cycle. In the model of bioharmony [L18, L1, L10, L12, L19] bioharmony have as building bricks the 20 3-chords associated with the 20 faces of the icosahedron and the 4-chords assignable tetrahedron. The icosahedral harmony is determined by a quint cycle assigned with the icosahedral Hamilton cycle.

2.2.3 Platonic solids as geometric counterparts for energy shells

$M^8 - H$ duality and quantum classical correspondence motivate the question whether the energy shells could have as classical correlate 2-D surfaces with which the circular Bohr orbits for the ground states could be associated. The Platonic counterparts for energy shells should make sense also in atomic physics. Flux tubes defining the closed nuclear strings as Hamilton sub-cycles would provide a geometric description of interactions having no counterpart in standard physics. Without them the classical description would be based on single-particle Bohr orbits.

At quantum level, one would have wave functions both in the space of the energy shells and in the space of Hamilton cycles. The wave function in the space of energy shells depends on parameters like radius of the sphere carrying the tessellation.

Also the shapes of the geometric energy shells and even their topology could vary and these 2-surfaces could be also seen as spatial analogs of 2-D Fermi surfaces of condensed matter physics. $M^8 - H$ duality implies their existence also at the level of condensed matter. In the following only the metric spheres will be considered.

2.2.4 Alternative identifications for the role of Platonic tesselations

One can consider several options for the identification of the role of Platonic tessellations. Also 1-D tessellations such as 1, and 2 could be considered. They could correspond to sub-tessellations of Platonic tessellations if the idea about crystal growth is accepted.

1. The magic numbers for protons and neutrons are independent and have a stabilizing effect. "Independent" could mean that neutrons and protons form their own Hamilton cycles for a

given tessellation and perhaps even separate tessellations. Protons and neutrons could be on top of each other to maximize the attractive strong interaction energy between protons and neutrons.

In the case of ^{4}He this cannot be true unless one allows edges of tessellation as degenerate sub-tessellations or even as a full tessellation or assumes that the tetrahedral Hamilton cycle contains both protons and neutrons. Second problem is that there are excess neutrons.

2. Proton and neutron tessellations are identical but not on top of each other and therefore not ideal concerning the minimization of the repulsive interaction energy. The proton and neutron tessellations would tend to stabilize each other if the nucleon distances are not too large. This could be true for excited states. The tessellations could be even different.

3. Protons and neutrons are associated with the same tessellation and Hamilton cycle but for instance neutrons are associated with the vertices and protons are associated with the centers of $F-2$ free edges not belonging to the Hamilton cycle. One could regard these edges as analog for the realization of dual Platonic solid but lacking 2 vertices.

   For instance, in the case of tetrahedral tessellations there would be at most 2 protons and 4 neutrons and He^4 could correspond to this kind of tessellation. Additional neutrons would correspond to neutron halo. The prediction would be that the number of neutrons is in general larger than protons except in the case of icosahedron: one has $(V, F-2) \in \{(4,2),(6,6),(8,4),(12,18),(20,10)\}$ for $\{T, O, C, I, D\}$.

   The existence of both neutron and proton halos, such as oxygen halos having number of nucleons corresponding to the number of vertices of Hamilton solid, strongly suggests that tessellations which have only neutrons or protons are possible. Proton halos are rare and this could be due to the lack of stability.

4. For the heavier nuclei some neutron and proton type shells, which are not necessarily paired, would appear several times. Neutron and proton halos could correspond to this kind of shells. One must understand how they can be stable.

The proposed picture could be criticized for a breaking of symmetry between the Platonic solid and its dual. It would be nice if the dual of the dual vertices defined by the centers of the free edges would have F rather than $F-2$ vertices. One can indeed consider also the possibility that Platonic solids and their duals are in the same role and can appear simultaneously as tessellations of the same sphere. The dual tessellation would permute the numbers of vertices and faces and the numbers of edges would be the same. Each tessellation would have its own Hamiltonian cycle. The vertices of the dual tessellation would be at the centers of the faces of the tessellation and edges would connect the centers of the faces of tessellation. Also for self dual tessellation one would have separate vertices for the dual. In the case of nuclei protons *resp.* neutrons would be at the vertices and tessellation *resp.* its dual. Whether the intersections of the edges of the cycles have some physical meaning remains an interesting question.

For this option one has the list $(V, F) \in \{(4,4),(6,8),(8,6),(12,20),(20,12)\}$ for $\{T, O, C, I, D\}$. For $1,2,3,4,5$ one has $(2j, 2j+2) \in \{(2,4),(2,6),(6,8),(8,10),(10,12)\}$. For $l=4$ Platonic solid with 10 vertices does not exist. On the other hand, Periodic Table contains no atoms with $l \geq 3$ and one can argue that for atoms having l-blocks with $l \geq 3$ the value of h_{eff} must increase so that the unit of angular momentum increases. For the $(V, F-2)$ option has $(V, F-2) \in \{(4,2),(6,6),(8,4),(12,18),(20,10)\}$ so that in this case one can handle also $l=4$ and $l=5$. For Rydberg atoms large angular momenta are possible and in both cases one can argue that the increase of $\hbar_{eff}$ defining the unit of angular momentum saves the situation for large angular momenta. Same could be true for highly deformed nuclei with large angular momentum.

2.3 The analogy between nuclear shell model and model of atoms as a guideline

The following arguments describe how one ends up with the proposal about Platonization of the physics related to the angular momentum quantization. These arguments generalize to the analog of Platonization of much more general quantum numbers. The analogs of Platonic solids would be discrete spaces defined as orbits of discrete subgroups of G acting in the coset space G/H of a compact Lie-group G with its Cartan subgroup H defining the space of choices for the quantization axes.

2.3.1 Correspondence with the nuclear shell model

What can one say of the connection with the nuclear shell model?

1. At the classical level, the binding energies for the nucleons at a given spherical shell could determine the radius of the shell.

2. At the quantum level, the single-particle wave functions of the shell model are replaced with wave functions for the radius of the shell meaning the presence of a coherent quantum state analogous to Bose-Einstein condensate. For rotation symmetry effective potential, these wave functions would be analogs of harmonic oscillator wave functions. They have similar symmetries: even the counterpart of the Runge-Lenz vector exists.

3. There is also a wave function in the space of Hamilton cycles and corresponds to the discretized rotational degrees of freedom and to the existence of several non-equivalent cycles. The discretization poses a bound on the angular momentum assignable to these wave functions from above just like lattice puts bound on momenta of particles in lattice. One can also consider single-particle wave functions at the discrete Platonic solid. Also for them the counterpart of angular momentum is bounded.

4. Neutron halos could correspond to neutron shells with a large radius expected to have a rather small binding energy per nucleon. In the case of oxygen with 28 neutrons one would have a pair 8p+8n of full proton and neutron shells 8p+8n plus neutron halo consisting of 8+12 neutrons. 12 corresponds to an icosahedron and allows large degeneracy due to the existence of a large number of icosahedral Hamilton cycles. This makes the icosahedron unique but also the octahedron and cube have this degeneracy.

2.3.2 Periodic Table as a carrier of information about nuclear spectrum

The construction of electronic states for atoms is a process very similar to the construction of the proton and neutron states in the shell model.

1. For a spherically symmetric harmonic oscillator potential and Coulomb potential, the spectra are actually essentially the same since there is a principal quantum number with the same set of angular momentum states in both cases. Therefore the periodic system should be realized for nuclear states of both neutrons and protons and atomic electrons. This suggests that the Platonic solids serve as geometric correlates for both the energy shells of atoms, protons and neutrons.

2. In the Periodic Table of atoms (see this), the rows for ground states decompose to s-blocks, p-blocks, d-blocks and f-blocks according to the value $l = 0, 1, 2, 3$ of angular momentum and contain 2,6,10, or 14 elements. Something very similar should occur also for nuclear ground states with varying Z.

   (a) The first row corresponds to nonmetals and is associated with nucleons, D, T, and He isotopes.

   (b) The second *resp.* third 8-row beginning with Li *resp.* Na consists of s-block and p-block. They would correspond naturally to the union of 2- and 6-shells. The proton and neutron shells would be identical with electron shells. 2-blocks correspond to metals.

   (c) For the fourth *resp.* fifth rows beginning with K *resp.* Rb, there is d-block between the ends of these rows having 10 nucleons. It would correspond to d-block and angular momentum $L = 2$. This could correspond to 4+6 that is T and O. It turns out that this interpretation is consistent with the assumption that electronic spin structure is identifical with the nucleonic spin structure at 4+6. First s-block is filled after which 4+6=10 are filled. After this the 6 is filled.

(d) The next two rows beginning with Cs *resp.* Fr have between s-block and p-block d-block with 10 elements an f-block with 14 elements.

(e) The repeated appearance of 2+6 shell would quite precisely correspond to the attribute "Periodic" in the "Periodic Table". Atoms and nuclei would define a local quantum Platonia, one might say!

2.3.3 Could $j = l \pm 1/2$-blocks correspond to Platonic solids?

How could one assign Platonic tessellations or their duals to the l-blocks, or rather their decompositions to $j = l \pm 1/2$ blocks?

1. The total number of states for l-block is $2(2l + 1)$. This number cannot correspond to the number V of vertices of a Platonic solid except in the case of octahedra with $V = 6$. What about the dual tessellations? The value of $F - 2$ equals $2, 6, 4, 18, 10$ for T,O,C,I to be compared with the numbers (2,6,10, 14) of elements in angular momentum blocks. The success is only partial.

2. It is essential that one has many-fermion states. One has a tensor product of irreducible representations l and $s = 1/2$, which decomposes to angular momentum representations $l \pm 1/2$ with $2l$ and $2l + 2$. The crucial observation is that the numbers $6 = 2 + 4$, $10 = 4 + 6$ and $14 = 6 + 8$ for $l = 1, 2, 3$ correspond to the numbers of vertices for HCs or dual HCs for Platonic solid and its dual. $6 = 2 + 4$ corresponds to the HC and its dual for self-dual tetrahedron, $10 = 4 + 6$ corresponds to a dual of HC for cube and HC for octahedron and $14 = 6 + 8$ corresponds to HC for cube and dual HC for octahedron.

3. This sequence could be continued. For $l = 4$ corresponds (8,10) which corresponds to the Hamilton cycle for cube and dual dodecahedron. Cube and dodecahedron do not however correspond to duals as Platonic solids. $l = 5$ corresponds to (10,12) that is dual of the Hamilton cycle for dodecahedron and cycle for icosahedron. $l = 9$ corresponds to $(18, 20)$ that is dual of the Hamilton cycle for icosahedron and cycle for dodecahedron. $l = 9$ is clearly exceptional and this might relate to the decomposition of $3 \times 3 = 5 + 3 + 1$ implying that only the orders of cyclic groups of Platonic solids appear in the sum. This is not the case for $l = 6, 7, 8$.

4. The group theoretic interpretation would be that Platonic solids provide a representation for the half-odd integer angular momentum states resulting in the decomposition of the tensor products of angular momentum multiplets and with spin half representation. One can think that one has angular momentum representation of the tensor product $(2l+1) \otimes 2$ with spinors at the points of a sphere S^2 defining Platonic solid. The rotations of a fixed spinor by the elements of the discrete subgroup of the Platonic solid give rise to the states for $l - 1/2$ or $l + 1/2$ multiplet. There is an upper bound for the values of l since for large values of l the representations are not irreducible with respect to the symmetry group of the Platonic solid.

5. What about the s-block having only two states? They could correspond to fermions at the opposite ends of a degenerate Platonic solid consisting of two points and have Z_2 as its isometries. The Hamilton cycle is not closed since both fermions are at the same point.

There is an objection against this view. For $l \geq 5$ it does not work. Non-Archimedean solids might be a partial cure (fullerene with 60 vertices is the basic example) but does not look attractive. An alternative cure would be that a phase transition increasing the value of h_{eff} occurs for large enough values of j. This would scale the unit of angular momentum $\hbar_{eff}$ and higher values of l would become possible. Highly deformed high mass nuclei with large angular momentum are indeed possible: nuclear spins up to $j = 64\hbar$ are known to exist. High spin nuclei could necessarily involve Platonic solids with a large value of h_{eff} and the value of l would give a criterion for the h_{eff} increasing phase transition. A given nucleus could also involve Platonic solids with several values of h_{eff}.

2.3.4 Does nuclear-atomic holography make sense?

The above observation raises the question whether the electronic and nuclear j-blocks have similar geometric counterparts? Could one speak of holography mapping nuclear states to the electronic states? However, there are also differences.

1. In the atomic case one has only electrons whereas in the case of nuclei one has protons and neutrons.
2. One cannot avoid considerations related to electric and magnetic fluxes, which would be naturally color magnetic in the case of the nuclei. Leptons have no color but could color magnetic flux tubs make sense also for atoms or should one just speak of monopole flux tubes assignable to electromagnetic flux.

   The electronic shells are analogous to conducting surfaces so that radial radial electric flux tubes should receive the net electric flux from lower shells along electric flux tubes and mediate the reduced flux to the larger shell. Magnetic fluxes parallel to the shell surface are expected in the Maxwellian picture but can one assume that monopole flux tubes connect atomic electrons? The TGD view of space-time could allow this. Flux tubes would be thickened closed cosmic strings connected by wormhole contacts at their ends.

If this picture generalizes to nuclear physics, the phenomenological basic properties of atoms would have counterparts at the level of nuclei and one would have a geometric description of both atomic and nuclear reactions in terms of the geometric shells.

Even more, Platonization could makes sense quite generally. The analogues of Platonic solids would be discrete coset spaces of a compact Lie-group with its Cartan subgroup defining the space of quantization axes. A good guess is that Platonization could relate closely to the mysterious McKay correspondence [A4, A3, A2, A1] discussed from the TGD point of view in [K11] [L4, L7, L17]. McKay correspondence is also associated with the inclusion of the hyperfinite factors [K14, K8] providing a realization of the finite measurement resolution as also the number theory based discretization does.

3 Comparing the standard picture of nuclei to TGD picture

It is instructive to compare the standard picture of nuclei to the TGD picture. In the following I will describe various ideas in the order in which they emerged but dropping away some of the worst sidetracks.

3.1 Some questions

It is good to start with some questions.

1. The problem with the Platonic hypothesis in its simplest form is that the interpretation for 1 or 2 protons or neutrons as a Platonic sub-tessellation is not possible since there exists no closed 2-D Hamilton subcycle of Platonic tessellation with 1 or 2 neutrons. Should one accept the sub-tessellations of a circle with 1 and 2 neutrons or protons and regular polygons? The symmetry groups of regular polygons in plane appear in the hierarchy of inclusions of hyper-finite factors (HFFs). For them the triangle group Z_3 is the smallest group. 1 or 2 surplus neutrons is the problem.

   What binds the surplus neutrons of the circular tessellation together? Is the TGD analogue of strong interaction based of strong isospin and meson exchanges enough? The same problem is encountered also for the 2-D tessellations of protons and neutrons in general. What would binds the circular tessellation to the rest of the nucleus?
2. The TGD based model for the planetary system [L22, L23] provides some support for the idea of having tessellations of various dimensions. The model predicts both 2-D spherical mass distributions assignable to 2-D magnetic bubbles, 1-D objects such as Saturnus rings associated with circular magnetic flux tubes and planets as point-like objects. Single surplus neutron might allow interpretation as the analog of a planet and the neutron halo with more than 2 neutrons could be analogous to the ring of Saturn defining a plane.

3. Platonic solids are compact analogies of condensed matter lattices. This suggests that the surplus neutrons could be analogous to impurity atoms in an atomic lattice. Platonic tessellations and their sub-tessellations contain free edges and the neutron could attach in the middle of the free edge of a protonic tessellation by two color flux tubes. This would give an attractive contribution to the binding energy. In the same way, proton could attach in the middle of free edge of neutron tessellation. In this case, the repulsive Coulomb force would favor the neutron surplus.

3.2 Trying to realize the standard nuclear physics view in TGD framework

Consider first a TGD view based on standard low energy nuclear physics involving only strong interactions coupling to string isospin and meson exchanges. Note that the harmonic oscillator model for nuclei cannot be satisfactorily deduced from standard view of strong interactions nor from QCD which fails to be perturbative in nuclear scales.

1. The strong force would be mediated along color flux tube portions connecting neighboring nucleons, which could be meson-like states of quantum entangling quark and antiquark at their ends. Color magnetic monopole flux tubes assumed to have a pair of quark and antiquark at its ends would define a partial Hamilton cycle going through all nuclei as a connected subset of the vertices of the Platonic solid. Note that in the case of cube the sub-Hamilton cycles have an even number of nucleons whereas for other Platonic solids the number can be odd.

2. This picture would conforms with the standard low energy description of the strong interactions based on the notion of strong isospin and mesons as mediators of strong interaction. The standard strong interaction between neutrons and protons in short scales is attractive whereas identical nucleons have a repulsive interaction. The standard strong interaction weakens exponentially with distance. In this picture maximization of the distances for identical nucleons would take place when proton and neutron tessellations could have the same nodes but possibly different nuclear strings.

   However, the neutron surplus suggests that in Platonian picture proton and neutron tessellations are effectively duals of each other. The vertices of the tesselation connected by sub-Hamilton cycle would carry protons and the middle points of the free edges of subtesselation would carry neutrons except in the case of icosahedron. This picture would indeed explain neutron surplus.

3. In the standard picture, the total repulsive interaction energy between nucleons must be minimized. This is true if the radius of the shell is fixed as a constraint and if the distances are the same for nearby nucleons and conforms with the breaking of rotational symmetry to a discrete subgroup analogous to the breaking of translational symmetry in condensed matter. One can however ask why the repulsie interaction does not lead to the explosion of the shell. For full geometric shells the tessellation would correspond to Platonic solid and this assumption could make sense also for the partially filled shells. This requires that the tessellation exists in some sense even without nucleons. One option is that the induced geometry has the symmetries of the tessellation.

   The nucleons of a given type, neutron or proton, at the corresponding shell would form a dynamical unit as a tessellation involving the nuclear string as a possibly partial Hamilton cycle. Nucleon, deuteron and Helium nuclei both protons and neutrons form an exception and would have the same tessellation with 4 points (tetrahedron).

4. One could construct nucleons by using pairs of partially filled shells as building bricks. This process would be analogous to crystal growth. If the shells correspond to the same Platonic solid they minimize energy. If they are different this is not the case.

   For an even number of protons/neutrons one can have a union of shells/Platonic solids. To obtain nucleons with an odd number of nucleons of p and/or n one should build holes to the completely filled tessellations or add nucleons to partially filled tessellations. Let us see whether this makes sense.

3.3 An improved TGD view

One can criticize the standard picture.

1. It is difficult to understand how neutron halos and their proton counterparts are possible in this framework. There is a difficulty to understand why the shells do not explode by the repulsive interaction between identical nucleons.

2. 3-D harmonic oscillator model treating neutrons and protons as independent particles works quite satisfactorily but it is not understood how the effective harmonic oscillator potential could follow from strong interactions.

Consider now the improved TGD based view.

1. In the TGD framework, the monopole flux tubes have a string tension determined by volume action and Kähler action to which the 6-D Kähler action for the twistorial reduces in dimensional reduction necessary to have a representation of 6-D twistor space as 6-surface in the product of twistor spaces of M^4 and CP_2. The string tensions of the flux tubes are identical. There are 1-D harmonic oscillator states associated with their length variations just as in the case of ordinary spring. The elastic constant is determined by string tension.

   The transversal oscillations give 2-D stringy spectrum so that one has a 3-D harmonic oscillator but with possibly different elastic constants in longitudinal and transversal degrees of freedom. In an equilibrium situation the sum of the energies of strings connected to a given nucleon can be approximated with harmonic oscillator potentials at the equilibrium points.

2. Instead of residing in a central harmonic oscillator potential, each nucleon would reside in a local harmonic oscillator potential with minimum at its position. There are harmonic oscillator wave functions for the flux tube lengths labelled by integers concentrated around classical flux tube lengths proportional to $\sqrt{n}$. If the integers n_i for nucleons are different the system deviates from Platonic solid since the distance between nucleons are not the same. This situation is not stable so that collective radial excitations in which the radius of the sphere associated with Bohr orbits is scaled up as the harmonic oscillator quantum number increases, are favored. The size of these energy shells depends on n in the same way as in the harmonic oscillator model.

3. What could be the relationship to the TGD counterpart of non-perturbative QCD. Color magnetic flux tubes could define a possible counterpart for the monopole flux tubes. Their ends containing quark and antiquarks would attach to the colored quarks of nucleons by color interaction. In absence of dual edges, two edges are associated with a nucleon so that by a proper ordering of the quark antiquark pairs along the cycle so that one has has a sequence of identically oriented $q\overline{q}$ pairs , the antiquark and quark associated with each nucleon can form a color single binding in this way the flux tube ends together.

   What about the dual tessellation with F-2 vertices? Can a particle at the centers of free edges bind by color flux tube to two vertices of the Hamilton cycle such that both the vertices remain color singlets? If the direction of the $q-\overline{q}$:s are different at these vertices one can have qqq singlet and $\overline{qqq}$ singlets analogous to baryons at the ends of the dual bond. Their color confinement would bind the end of the flux tube to the nucleons involved. This would conform with the interpretation as an analogy of lattice defect. These nucleon-like states should be light and the darkness of the quarks and p-adic length scale hypothesis suggest that could be possible. It is also absolutely essential that the color group is $SU(3)$ since for $SU(n)$, n larger than 3 one could not obtain color singlets as quark triplets.

4. There is a general objection against this model. Consider deuteron pn as the simplest example. The binding of p and n to an end of a flux tube in the proposed way is not possible since one cannot have a Hamiltonian sub-cycle in this case so that one can have only a single quark and antiquark at the ends. Is this enough and if so, could this relate to the small binding energy of deuteron? More generally, the nuclei containing 1 or 2 protons are problematic since for them one cannot have a Hamiltonian sub-cycle.

5. The notion of tensegrity (see this) to which the name of Buckminster Fuller (see this) is associated, would be appropriate in nuclear physics. Buckminster fullerene is a truncated icosahedron (see this), which is a soccer ball-like structure consisting of 20 pentagons and 12 hexagons with each pentagon having 5 hexagons as nearest neighbors and vice versa so that it is a more general than Platonic solid. One cannot of course exclude this kind of structures and it would be interesting to no how many Hamiltonian cycles they allow.

The geometric shell structure and energy minimization should provide some understanding about magic nuclear numbers 2, 8, 20, 28, 50, 82, 126 appearing in the nuclear shell model.

1. In the model based on paired identical Platonic solids, the magic number 2 could correspond to a tetrahedron with the pairing ^{4}He= 2p+2n, that is alpha particle. Cubic Hamilton cycle C would correspond to magic number 8 and dodecahedral Hamilton cycle D could correspond to magic number 20. These numbers could also correspond to unions of several tessellations or of their duals. The higher magic numbers in the list could correspond to the unions of paired shells with holes.

2. If the Periodic Table is taken as a starting point, neutrons and protons should correspond to tessellations and their duals. In this case the magic numbers should be expressible as sums of vertex numbers $V = \in \{4, 6, 8, ..\}$ and numbers $F - 2 \in \{2, 6, 4, ..\}$.

   In the presence of missing nucleons (analogs of electron holes in atomic physics), one could assume a closed cycle which is not maximal. There are however very many decompositions of the magic numbers as sums of vertex numbers of Platonic solids so that more information is needed. Periodic Table could provide this kind of information.

3.4 Why nuclei have a neutron surplus and neutron halos?

One can distinguish between neutron surplus which all nuclides tend to have and neutron halo in the nuclear periphery consisting of only neutrons. In the following teh proposed Platonic model is assumed.

3.4.1 Why light enough nuclei with $N = Z$ tend to have largest abundance and why there is neutron surplus

1. Already for light nuclei there are lonely neutrons, whose strong isospin is not neutralized by protons in the simplest model. As already explained, the attractive force due to the monopole flux tubes could stabilize the Hamiltonian cycle and its dual formed by protons and neutrons.

   The difference $\Delta = N - Z$ between neutron number and proton number, whose maximal value increases gradually. Basically this might reflect the effect of the repulsive Coulomb force between protons. There are also neutron halos at the nuclear periphery. Also neutron halos could correspond to Platonic solids.

2. For light nuclei the most abundant stable nuclei have $N = Z$. In the picture based on strong interactions mediated by strong isospin, this could be understood. It is not clear whether the TGD based explanation for binding based on color flux tubes is consistent with this finding. Platonic solid has V vertices and $F - 2$ free edges. The numbers of free edges are given by $N_{free} = F - 2 \in \{2, 6, 4, 18, 10\}$ for $\{T, O, C, I, D\}$, where the shorthand notations should be obvious. The numbers of Hamiltonian edges are $N_H) = V \in \{4, 6, 8, 12, 20\}$ for $\{T, O, C, I, D\}$. $V < F - 2$ for all Platonic solids except T and I. The neutron surplus suggests that protons and neutrons are assigned in such a way that neutron surplus is possible for the full shells.

3. Depending on Platonic solid, the maximum number of surplus neutrons would be either $V - F + 2$ or $F - 2 - V$, where F is the number of faces. The proton impurities at free edges are possible for T and I. T would have 4 vertice for neutrons and 2 free edges for protons. There would be at most two surplus neutrons for ^{4}He and according to my very old source (Nuclear physics by Robert Howard) this prediction is correct. The modern data show that there can be neutron halos of 4 and 6 neutrons (T and O). This would require neutron halo-

For the octahedron one has $V = F - 2 = 6$ and in this case one can have $N = Z$ for the completely filled shell. This would correspond to $l = 1$ states assignable to O appearing for all energy shells (Periodic Table for nuclei). For $l = 2$ appearing at the fourth row there are 10 states decomposing to 4 and 6 states with different planar parities are assignable to Platonic solids T and O. In this case maximally filled T would have 2 protons at free edges and 4 neutrons at vertices. Neutron surplus would be 2.

For $l = 3$ there are 14 states, which can be assigned O and C. For completely filled O $N = Z$ can be understood. Maximally filled C one would have 8 neutrons and 4 protons for the maximally filled state. The neutron surplus would be 4. It seems that this picture is consistent with the facts that there are stable states with neutron surplus and that the neutron surplus increases with the mass number.

3.4.2 An attempt to understand neutron and proton halos

The tables of nuclides (see this) demonstrate that already already hydrogen isotope with 6 neutrons exist: octahedral tessellation with neutrons is the simplest interpretation. Also 4-neutrons and 6-neutrons do exist and could correspond to T- and O halos. He with mass number 10 could be also regarded as having 6-neutron O halo. Li with 9 neutrons exists. Also now 6-neutron halo would be the explanation. It is also known that 2-neutron halos exist.

What is encouraging is that the halos for lightest nuclides correspond to full Platonic shells quite generally. If this is true, it would be easier to test whether the nuclear states are consistent with the predictions of TGD. There are also proton halos. Both neutron and proton halos are difficult to understand in the standard nuclear physics framework where meson exchanges give repulsive identical nucleons.

Neutron halo is located to the periphery of the nucleus. The first guess is that it corresponds to neutron shells unpaired with proton shells. In the case of the oxygen isotope that motivated these considerations neutron halo would correspond to two shells 8 and 20 (cube and icosahedron). The tension of the nuclear string tension would bind the neutrons to form a halo having a rather large value of principal quantum number n. Neutron halo would be analogous to Rydberg states.

The problem is that in the standard model there is no obvious strong or electromagnetic force that would bind the neutron halo to the inner shells. One should understand what binds the neutron halo to the rest of the nucleus. Strong interactions between protons and neutrons based on strong isospin seem to be excluded as also electromagnetic interactions if halo neutrons are identical with free neutrons.

TGD suggests several new physics mechanisms explaining the stability of the halo and its binding to the main nucleus.

1. The stability of the neutron and proton halos could be due to string tension of the monopole flux tubes binding the neutrons and also neutrons to a mechanically stable system. The flux tubes would have colored quark and antiquark at their ends. The mechanism binding neutrons and protons to shells by color flux tubes has been already discussed. Electromagnetic interaction between quark and quark or antiquark with opposite sign of electromagnetic charge could also help to fix the flux tube to the nucleons at its end. Color and electromagnetic interactions would only take care of attaching the ends of the flux tube to its end nucleons.

   If monopole flux tubes are in question they would be actually pairs of meson-like flux tubes connecting the two nucleons created in the reconnection U-shaped flux tube tentacles of the nucleons. The flux tube pairs could be gauge boson like entities having fluxes at two space-time sheets flowing between Minkowskian space-time sheets through Euclidian wormhole contacts. This mechanism is the basic mechanism of catalysis according to the TGD based view of biocatalysis. These flux tubes pairs could be also interpreted as virtual gauge bosons.

   The condition that the dark gluon Compton length is not smaller than the size scale for color bound objects given by the analog of Bohr radius for an atom-like entity consisting of two shells with color charges Q_c and $-Q_c$. Both dark gluon Compton length and the dark Bohr radius are proportional to $\hbar_{eff}$ which disappears from the formula. This gives formula

$L_g = L_p/2Q_c^2\alpha_s$ which for $Q_g = 1$ and $\alpha_s = .1$ would given $L_g = 5L_p$ which conforms with QCD Λ about .2 GeV. At the confinement limit the value of α_s increases and L_g decreases.

2. Could dark classical Z^0 force be involved? For this option weak isospin would have the same role as strong isospin. This would require a large value of h_{eff}. Could weak boson Compton length be scaled to a nuclear Compton length scale? I have actually proposed that something this could take place even in longer scales and explain chirality selection in living matter. The neutron halo would be dark and at the magnetic body of the nucleus. The condition that Z_0 boson Compton length is not smaller than the Bohr radius of the weak atom formed by the halo and inner shells would give $L_p/2N^2\alpha_{ew} = L_W$. The value of N would be unrealistically large.

3. An option inspired by the model for dark proton flux tubes providing a mechanism behind "cold" fusion is that halo neutrons are pseudo-neutrons, which are are actually protons to which flux negatively charged meson-like flux tube bonds connecting them to the protons of inner shells or to the neutrons of the halo are associated.

   (a) These pseudo-neutrons would be electric dipoles and the electromagnetic attraction between these dipoles could bind the pseudo neutron halo to the inner shells. Note that the value of the electric dipole moment of a free neutron is consistent with zero: according to Wikipedia, one has $d = (0.0 \pm 1.1) \times 10^{-28} e$ m. This mechanism would be a variant of gluon binding mechanism.

   (b) I have asked whether actually all nuclear neutrons could be this kind of effective neutrons. Could neutron shells consist of possibly dark protons connected by negatively charged protons, which would stabilize them. Or do the strong interactions between proton and neutron shells stabilize the shells by the mechanism already considered?

   (c) The partially conserved axial current hypothesis (PCAC) and the conserved vector current hypothesis (CVC) hypothesis involve a poorly understood connection between the parameters of weak and strong interactions. I have asked whether the low energy strong interactions could be also understood in terms of weak interactions but with weak interaction Compton length taken to be of the order of nucleon Compton length. This would conform with the fact that in TGD all gauge fields at the fundamental level are expressibles in terms of CP_2 spinor connection and there are only 4 fundamental field-like variables. This would be the case if the nuclear nucleons are actually dark in the TGD sense.

   (d) The attractive interaction between the nuclear Coulomb field and the dipoles forming the shell of halo neutrons would orient the dipoles radially. A simple Bohr model based estimate for the interaction energy and quantization of angular momentum demonstrates that the value of effective Planck constant must be roughly 100 times the ordinary value in the case of oxygen halos so that weak boson Compton length would be of the order of nucleon Compton length and halo neutrons would be dark. Note that the unit of angular momentum is $\hbar_{eff}$ so that neutron halo could possess rather large angular momentum.

   (e) A related proposal is that weak interactions become as strong as em interactions below nuclear scale and bind halo neutrons to the protons of the inner shells. Weak interaction at the nucleon scale could provide a dual description for the binding of the neutrons and protons of the proton and neutron shells. Weak interactions cannot however bind protons and neutrons to a halo.

3.4.3 A more detailed model for halo binding in terms of pseudo-neutrons

To get a quantitative grasp on the situation , one can model the halo of pseudo-neutrons with pseudo-neutron identified as a dark proton + negatively charged mesonlike particle represented as a monopole flux tube with quark and antiquark at its ends. It must be emphasized that this option as a model of neutron halo is only one possible option. It will be however found that the assumption of pseudo-neutrons as analogs of p-π^- pairs could provide insights about the values of nuclear binding energy and the 10 keV energy scale that would be associated with low energy excitations of nuclei assumed in the proposal for the explanation of the tritium beta decay anomaly.

1. The negatively flux tubes connect the proton with an inner Platonic shell and also with other protons of the tessellation in which case it is part of Hamilton cycle. The dipole moment of "neutron" is $p = ea$, where are is the length of the pion-like flux tube. The mass of the flux tube should be rather small and of the order of nuclear binding energy.

2. Suppose that the Platonic shell behaves like an almost rigid body able rotate and contract or expand in the radial direction. This suggests that the motion in which only rotation takes place can be described by modelling the shell as a particle with mass equal to $M = N_n m_n$, where N_n is the number of neutrons. The dipole potential is given by $\Phi_c = -p \cdot E_c$, where $E_c = Ze/R^2$ is the radial electric field created by protons. Assuming that dipoles are radial one obtains for the Coulomb force $F_c = -2Ze^2 a/R^3$. Mechanical equilibrium condition reads as $F_c = Mv^2/R = L^2/MR^3$, where $L = MvR$ is the quantized angular momentum component. Assume that L is quantized as $L^2 = l(l+1)\hbar_{eff}^2$.

   The condition gives

   $$L^2 = l(l+1)\hbar_{eff}^2 = 2Ze^2 aM = 8\pi\alpha Z N_n \hbar^2 \frac{a}{L_c(n)} .$$

   This condition says nothing about the value of R. This scaling invariance is due to the neglect of the dynamics related to the distance between the nucleons. One expects that there is some equilibrium distance determined by the elastic constant assignable to the flux tube bonds of the Hamilton cycle. In the simplest situation R would be determined by this dynamics alone. This dynamics is also sensitive to the shape of the Hamilton cycle.

   This gives an estimate for $r = a/L_c(n)$ as

   $$r \equiv \frac{a}{L_c(n)} = \frac{\hbar_{eff}}{\hbar}^2 \times \frac{l(l+1)}{8\pi\alpha Z N_n} .$$

   For $l = 1$ and $\hbar_{eff} = \hbar$ this gives $r \simeq .17$. One can solve $\hbar_{eff}/\hbar = 7.7$ if r is given

   $$\frac{\hbar_{eff}}{\hbar} = \sqrt{\frac{4\pi\alpha Z N_n \times r}{l(l+1)}} .$$

   For $Z = 8$ and $N_n = 8$ and $r = 1$ this would give $\frac{\hbar_{eff}}{\hbar} \simeq 2.423$. For $r = 10$ one would have $\hbar_{eff}/\hbar = 7.7$. These estimates do not support the idea that weak bosons are dark and have Compton length near nucleon Compton length. This estimate could however make sense for the inner shells.

3. The halo radius is a model dependent and rather formal parameter and estimated from the mass formula for the nuclei assuming that the densities of the nuclei are constant. This gives $R = R_0 A^{1/3}$, $R_0 = 1.25$ fm giving $R = R_0 A^{1/3}$, $R_0 = 1.25$ fm and is is larger than for the nuclei without the halo. One can argue that if the halo is dark, the dark protons reside at or are associated with the magnetic body of the system, which is expected to have a large size just as in the case of biological dark matter. In the case of the gravitational magnetic body "association" is the correct word and means that an ordinary particle touches the gravitational magnetic body.

   The TGD based model for "cold fusion" [K6] [L3, L9] leads to the estimate for the Compton length of dark proton. The estimate is rather near to electron Compton length so that the value of $\hbar_{eff}(n)$ would be about 2^{11}. The idea that the Compton size of dark halo protons is of order electron size looks weird. I have proposed that only the color magnetic body of the dark proton mediating color interaction by gluons is large whereas the proton itself has the normal size and assignable to the nuclear periphery but that the flux tubes would be long loops. One can compare the nucleus to a seed of a plant and the magnetic body to the plant itself. This assumption is similar to that made of the dark atomic valence electrons which would be actually analogous to dark halo neutrons. If neutron halos or rather, their magnetic bodies, are large, then nuclei with neutron halo could be optimal concerning "cold fusion".

If one assumes that the Compton radius of dark weak boson is about $L_c = 2.4 \times 10^{-15}$ meters, one must have $\hbar_{eff}(W)/\hbar \sim 10^2 \sim 2^{13}$. This would give $L_c(n) = 1.7 \times 10^{-12}$ m, which is roughly one half of the electron Compton length $L_c(e) = 2.43 \times 10^{-12}$ m. These observations inspire the question whether the condition $\hbar_{eff}(n)/\hbar \sim (\hbar_{eff}(W)/\hbar)^2/2$ could be satisfied.

3.4.4 Doubly magic mystery

Recently I learned of a nuclear physics finding, which is interesting from the point of view of the TGD based model for nuclei in terms nuclear strings that I managed to develop to a rather detailed form quite [L20]. This model extends to a model of both atoms and hadrons based on the same general basic ideas and makes rather non-trivial and testable predictions.

1. The first basic notion is nuclear string identifiable as a monopole flux tube. The nucleus would consist of one or more nuclear strings and they would define Hamiltonian cycle going through all vertices of the Platonic solid assignable to j-shell with angular momentum $j = l \pm 1/2$ and number of states $N_- = 2l$ or $N_+ = 2l + 2$.

2. The fact that flux tubes involve also Coulomb flux does not allow closed Hamiltonian cycles: these are possible only for electromagnetically neutral systems.

   One can however eliminate one edge from the cycle. This kind of quasicycle for which the flux tube arrives point A from another Platonic solid and flows through all the remaining points of the cycle and returns to a neighboring point B and continues to a neighboring Platonic solid. This kind of quasicycles can be generated when the flux tube portions defining edges of closed cycles form reconnections: in this case however both the incoming and outgoing flux tube would correspond to missing edges. This would also allow degenerate quasi cycles with 2 vertices required by the model.

   One can ask whether one should one allow Hamiltonian paths in which incoming and outgoing fluxes are not associated with neighboring vertices. Their number is obviously larger than the number of cycles.

3. Both the harmonic oscillator potential used in the simplest nuclear model and Coulombic potential used in the model of atoms are characterized by a principal quantum number n such that l=1,2,..,n orbital angular momenta are realized for it.

   This motivates the idea of nucleus-atom holography meaning that the protonic states of the nucleus correspond to the states of electrons of the atom. This also leads to a speculative question whether neutrinos could play the role of neutrons in atoms.

One can imagine two options for how the particles are assigned to Hamiltonian cycles.

1. For the first option the particles could be assigned to V vertices of the platonic solid or to the V edges of the Hamilton cycle: if the quasi-Hamilton cycles are allowed, then only the vertices are allowed. The numbers of vertices are given by $V = 4, 6, 8, 12, 20$ for T,O,C,I,D.

   It is interesting to look in detail at the assignments of states at different n-shells.

   (a) $n = 0$: l=0. There are 2 states. Degenerate Platonic solid as diametrically opposite points of the sphere.

   (b) $n = 1$: l=0,1. There are 2+2+4 states. 2 degenerate Platonic solids + T

   (c) $n = 2$: l=0,1,2: 4+6 additional states to n=1-shell. Additional T and O.

   (d) $n = 3$: 6+8 additional states to n=2-shell. Additional O and C.

   (e) $n = 6$: 8+10 additional states to n=3 shell. 6 corresponds to C. 10 has no counterpart as a single platonic solid. 10 → T +O.

(f) $n = 6$: 10 +12 states: 12 corresponds to I. 10→ T+O.

2. For the second option one assigns particles at the centers of complementary edges which by definition do not belong to the Hamiltonian cycle. There are $F - 2$ complementary edges.

   One has $F - 2 \in \{2, 6, 4, 18, 10\}$ for T,O,C,D,O.

   (a) $n = 0$: The 2 states correspond to T.
   (b) $n = 1$: The 2+4=6 additional states correspond to T+C:
   (c) $n = 2$: l=0,1,2: The 4+6=10 additional states correspond to C+O.
   (d) $n = 3$: There are 6+8 additional states. 6 corresponds to O but 8 corresponding to j=3+1/2 is missing. T+O would give 2+6=8 and C+C would give 4+4=8. j=3+1/2 cannot therefore correspond to a single platonic solid.

   One could worry about the fact that the magic number N=20 does not find an explanation in this picture. Rather, $N = 18$ would correspond to 3 full shells. As if $l = 0$ doublet would stabilize $N = 18$ state. Why should this be the case? Interestingly, $N = 18$ is atomic magic number.

   Energy shell can be defined in terms of an energy gap to the next state with a higher energy and this suggests that the discrepancy relates to the fact that $l = 0$ state of $n = 3$ shell are near to the energy of the highest state of the $n = 2$.

   Spin-orbit interaction comes first into mind since it distinguishes between the energies for a given value of n and comes first to mind. $L \cdot S$ term is vanishing and its spin-orbit interaction is therefore expected to be smallest for $l = 0$ state. In the case of atom, the interaction energy is nonvanishing since it involves expectation value of $1/2dV/dr$, where V is in the atomic case Coulomb potential, in $l = 0$ state and gives a term proportional to 1/l which at the limit $l \rightarrow 0$ gives a non-vanishing net result. In the case of a nucleus, the harmonic oscillator potential would give vanishing interaction energy.

   The $F - 2$ option does not require the somewhat questionable degenerate Platonic solid but the V option works also for $n = 3$.

3. One can ask whether the notion of n-shell could allow a description in terms of Platonic solids? In atoms l=0 and l=1 shells for n=1 shells give 2+2+4 =8 states, which could be assigned to the 8 vertices of the cube. $F - 2 = 8$ is not satisfied by any Platonic solid.

   $l = 0, 1, 2$ shells for the $n = 2$-shell correspond to 2+ 2+4+4+6= 8+10=18 states assignable to the n=2 shell. These 18 states cannot be assigned to the vertices of a single Platonic solid. These states can be however assigned with the complementary edges of the icosahedron with $F - 2 = 18$. It would look however strange to assign the $n = 2$ shell to complementary edges of the icosahedron and $n = 1$ shell to the vertices of the cube.

With this background one can try to answer the question whether the recent findings, suggested to involve new nuclear physics, could help to test and even fix the details of the TGD based model.

1. The nucleus ^{28}O has 8 protons and 20 neutrons and is doubly magic and should be therefore stable. It has 12 surplus halo neutrons and decays to a state with 8 surplus neutrons plus 4 neutrons with a life-time about 10^{-21} seconds. The 12 surplus neutrons in the halo cannot correspond to a full shell. This could explain the short life-time.

2. ^{28}O decays by emitting 4 neurons to ^{24}O with 8 surplus neutrons. This state should be rather stable.

What could TGD say about this?

1. The reason why one cannot apply the magic nucleus rule could be that halo neutrons are different from the core neurons and must be treated separately. A possible reason is that the halo neutrons correspond to a non-standard value of $h_{eff} = nh_0 > h$. This can occur also for the valence electrons of rare earth metals.

2. The 12 surplus neutrons in the halo do not correspond to a full n-shell. Both V and $F-2$ options are doomed to fail if the stability corresponds to a full n-shell.

   The ordinary 8 neutrons of ^{16}O could correspond to a full $n=1$ shell. 8+4=12 halo neutrons would naturally correspond to a partially filled $n=2$ shell having 8+4+6=18 neutrons. This does not depend on whether one has V or $F-2$ option.

3. The 8 halo neutrons have the same quantum numbers as the full $n=1$ shell, which suggests stability. This conforms with the experimental findings. 4 neutrons, which correspond to $j=3/2$-plet could be assigned with the complementary edges of the cube but cannot form a full shell since the 6 $j=5/2$-plet is missing.

3.5 Three objections against Platonization

There are several objections against the proposed Platonization deduced on basis of the period table, which lead to a deeper understanding of what is involved.

3.5.1 The first objection

The first objection against Platonization is the geometric representation of spin 1/2 states. States with 1/2 and -1/2 states correspond to different points of the sphere S^2. In standard picture fermions with an opposite spin and same angular momentum state correspond to the same point of 3-space. Twistor lift of TGD allows to circumvent this objection.

1. The twistor lift in $M^4 \times CP_2$ [K12] [L2, L8, L5, L15, L16] represents the states with various total angular momentum, even those with half-odd integer spin, as partial waves at the twistor sphere. By M^8-H duality, these angular momentum eigenstates can be identified as points of twistor space of the twistor sphere S^2 associated with M^4 light-cone and parametrizing light-like rays emanating from a vertex of light-cone of M^4. Light-like ray would characterize the direction of momentum in the massless case. Its length could characterize the angular momentum projection in the direction of the quantization axis. For a massive case, the notion of the rest system is well-defined and the quantization axis of angular momentum need not be parallel to the momentum and angular momentum also has an orbital part.

   The rather obvious idea is that the twistor sphere corresponds to the sphere with which Platonic solids are associated and that the discretization of angular momentum and spin gives rise to a Platonic tessellation or its dual for the twistor sphere as already explained.

   Geometrically the points of twistor space of M^4 correspond to an S^2 bundle of M^4 such that the point of S^2 fiber corresponds to a light-like ray emanating from the point of M^4. The bundle is non-trivial since the points of M^4 with a light-like distance have the connecting light-ray as a common light-like ray so that the twistor spheres intersect at this point. Platonic solids would be discretizations of the twistor sphere providing a geometric classical representation for the partial waves in the twistor sphere.

2. The twistor representation of the angular momentum eigenstates of spin 1/2 fundamental particles as partial waves should therefore have a discrete dual representation analogous to momentum space representation of ordinary partial waves in M^4. M^8-H duality, which generalizes the momentum position duality, generalizes to the duality between corresponding twistor spaces and the representation of angular momenta at twistor sphere at the level of H should be obtained by M^8-H duality from its counterpart at the level of M^8.

   M^8 twistor space is the generalization of the M^8_c as an analog of momentum space as dual of the position space $M^4 \times CP_2$. It is discretized since in number theoretic physics 4-momenta are at discretized mass shells such that the momentum components of physical states are integers when the momentum unit defined by the causal diamond (CD) contains the space-time surfaces in zero energy ontology (ZEO). The angular momentum values associated with $j=l\pm 1/2$ multiplets are discrete and should correspond to the points of the twistor sphere for the twistor space of $M^4 \subset M^8$.

When the particle is massive one can indeed select quantization axes freely and the angular momentum vectors would correspond to points of the Platonic solid connected by the Hamilton cycle or be associated with the middle-points of the $F-2$ free edges defining analog of dual Platonic solid with two vertices removed.

3. Physical intuition suggests that the points twistor sphere can be also mapped to points of a sphere of the mass shell $H^3 \subset M^4 \subset M_c^8$. Twistor sphere would have the Platonic tessellation at each point of M^4 but only at certain points of H^3 some point of the tesselation would be "active", that is,would contain a fermion.

   In TGD all states are massless in the 8-D sense so that the existence of this representation is suggestive. The contribution of the point of the twistor of $E^4 \subset M^8$ to the momentum could make the ray of $H^3 \subset M^4 \subset M^8$ a light-like ray of M^8. Indeed, by a suitable choice of $M^4 \subset M^8$, any massless state of M^8 massive state has a light-like projection to M^4.

3.5.2 The second objection

The second objection is based on the existence of tessellation. Does tessellation exist even in absence of neutrons and protons as a kind of a plan or a template for the formation of the Platonic crystal? How could it be realized physically? One location for the template would be at the level of the magnetic body of the system having a larger value of h_{eff}.

The idea about magnetic body as controller of ordinary matter is is actually central in TGD inspired quantum biology which leads to the model of genetic code and DNA in terms of icosa-tetrahedral tessellation of hyperbolic 3-space H^3 [L12, L19]. The coding would mean that the symmetries of the fundamental region of H^3 tessellation would be the same as those of the Platonic solid.

3.5.3 The third objection

There is also a strong objection against the atomic realization of Platonization. In the standard view about atoms, one would have only the electrons assigned with $F-2$ free edges of the tessellation. Can one really say that the tessellation and the Hamiltonian cycle is present if there are no counterparts of the neutrons located at the nodes of the Platonic solid?

1. The easy but unsatisfactory option is to forget the idea about the existence of the geometric realization of the energy shells and the notion of Platonization in the case of atoms.

2. The mad scientist option is to ask what if the counterparts of neutrons do actually exist in atoms. The only possibility is that they are neutrinos, which bind to neutrons of the nucleus in the same way as electrons bind to the protons. This would realize nucleus-atom holography in a very strong sense. Atomic states would be holographic images of nuclear states: I have discussed this information-theoretically attractive idea for hadrons and also its generalization in [L21].

3. The Standard model does not allow this since the weak length scale is quite too short. In the TGD framework weak interactions with large enough value of h_{eff} could have weak length scale, which is of order atomic length scale or even longer could become in rescue. Below the weak scale electroweak gauge bosons would be massless and this would make it possible to construct electroweak singlets by binding neutrons and neutrinos with opposite weak isospin together by using monopole flux tubes connecting neutrons and neutrons and neutrinos to "weak mesons". Electrons and protons with opposite weak isospin would form electroweak singlets in the same way and the holography between nuclei and atoms would be very precise.

4. In fact, TGD proposes electroweak confinement as a possible interpretation of electroweak massivation and the model for elementary particles in terms of screening neutrinos involving also right-handed neutrino [K3, K4] to take care that fermion number comes out correctly. In biology the hypothesis that the electroweak scale can be of order of the scale of size scale the basic information molecules (DNA, proteins), cell membrane scale, or even cell size scale, would explain large parity breaking effects such as chiral selection and of course

predict the long length scale quantum coherence explaining the coherence of living matter impossible to understand in the biology as chemistry only paradigm.

5. There is also a second argument in favor of the proposal. TGD allows to consider also the proposal that leptons are bound states of 3 antiquarks in very small scale, say that of single wormhole contact and of order CP_2 size scale [L11]. This would trivialize the puzzle of matter-antimatter asymmetry and is favored as the simplest possible reduction of elementary particles to the bound states of fermions and antifermions. Leptons would represent antimatter and the twistor lift of TGD indeed predicts a small CP violation, which could favor the condensation of quarks to baryons and antiquarks to leptons [L11]. The atoms could be seen as consisting of equal amounts of matter and antimatter.

Besides its apparent craziness, a reasonable looking justification for rejecting this hypothesis is that it looks completely untestable, at least in the framework of the standard model. But is the situation so gloomy in the TGD Universe?

1. Weak confinement in its strongest form means that the electron-proton pairs and neutron-neutrino pairs form electroweak singlets. The experimental situation is opposite to that in the case of hadron physics where one wants to see the quarks. The quark structure becomes visible only by using high enough collision energies. In atomic physics we see without any difficulty the electrons and neutrons and protons and electrons when the energies involved with the interactions of atoms are above say few keV, which corresponds to weak scale of order atomic length scale.

   The challenge is to detect the weak confinement. This is possible by using weak interactions at low energies. One should observe the analogs of hadronic reactions. A basic example would be the transformation of the atomic electroweak singlet n+$\nu \rightarrow$ p+e. The needed energy scale would be of the order of keV if the weak Compton length is of order atomic scale.

4 Possible generalization of the view of nuclear strong interactions

In the sequel a view of nuclear interactions based on the notion of monopole flux tubes is developed. Also the generalization of this view to atomic atomic physics is considered.

4.1 Are color interactions needed to describe nuclear strong interactions?

The assumption of pseudo-neutrons as analogs of p-π^- pairs could provide insights about the values of nuclear binding energy and also the 10 keV energy scale associated with low energy excitations of nuclei required by the proposal for the explanation of the tritium beta decay anomaly.

It is good to start from the problems of existing models.

(a) The old-fashioned model of nuclei assumes pion exchanges as the origin of nuclear force. The basic problem of this view is that since gluons have vanishing electroweak quantum numbers and it is difficult to understand why charged pions as mediators of strong interactions could appear.

(b) Second problem is that the mass of the pion is about 140 MeV and much larger than the 1 MeV scale of the nuclear excitations. In the harmonic oscillator model this scale is just assumed. Intriguingly, the neutron proton mass difference is also near 1 MeV. Also the electropions, explaining in the TGD framework [K13] the anomalous production of electron-positron pairs in heavy ion collisions discovered already at seventies, have mass very nearly equal to $2m_e = 1$ MeV. There is also evidence for muopions and taupions but all this is forgotten since there is no room for these particles in the standard model since their existence would have been observed in weak boson decays. This problem is circumvented if electropions are dark in the TGD sense.

In TGD color is realized as color partial waves in CP_2 degrees of freedom rather than as spin-like quantum numbers and both quarks electrons also allow colored excitations. In particular, leptons allow octet excitations. The anomalous electron-positron pairs would result in the decays of electropions identified as the bound states of colored electron and positron. Also the triplets of color octet leptons allow color singlets analogous to baryons and in the model of nucleus these triplets would occur at vertices from which free edge begins. If this were the case, then colored lepton states could be an essential part of nuclear physics. Note however that one can also consider a model of electropions based on scaled variants of quarks with mass scale defined by the p-adic length scale $L(127)$ of electron.

An attractive proposal is that the pion-like color bonds in nuclei correspond to electropions or their quark-antiquark counterparts and that the mass of the neutral electropion is 1 MeV. In the following I will talk about electropion also when it consists of quark and antiquark.

(a) One can estimate the mass of the charged electropion from the mass of neutral electropion by scaling the mass difference of pion which is $\Delta m(\pi) = m(\pi^0) - m(\pi^{\pm}) = (139.6 - 135.0) = 4.6$ MeV. By scaling this with $m(\pi_L)/m(\pi) \simeq 1/140$ one obtains $\Delta m(\pi_L) \simeq 33$ keV, which could correspond to the 10 keV energy scale.

(b) Nucleon- flux tube pairings of type $n - \pi_L^0$ and $p - \pi_L^-$ (pseudo-neutron) and $p - \pi_L^0$ and $n - \pi_L^+$ (pseudo-proton) pairings are possible. The energy differences between these nucleon-flux tube pairs would naturally correspond to the n-p mass difference of about 1 MeV.

The first guess is that in nuclear ground states with a minimum energy pseudo neutron $p - \pi_L^-$ and genuine proton $p - \pi_L^0$ are favored. The excited states with excitation energies in 10 keV scale contain genuine protons $p - \pi_L^0$ and pseudo protons $n - \pi_L^-$. Also the excited states $n - \pi_L^+$ can be excited to states $n - \pi_L^0$ with excitation energy in the 10 keV range.

4.2 Tritium beta decay anomaly

There exists is a well-established anomaly, so called tritium beta decay anomaly [C2] (see also this) supporting the proposal that Z^0 force and neutrinos could be important even in nuclear scales.

The tritium anomaly appears in the beta decay electron spectrum of tritium at the lower end of the electron energy spectrum for decays T=$H^3 \rightarrow He^3$+ e+ν. In the standard model, this beta decay should correspond to the nuclear decay n$\rightarrow$ p+W^- with W^- decaying to e+$\overline{\nu}$ pair so that the tritium anomaly remains a mystery. The Kurie plot is linear near the endpoint where neutrino energy goes to zero and overshoots at the endpoint (by conservation laws the overshoot should not occur). This leads to a parametrization in which neutrino mass squared is negative. There is also a narrow bump in Kurie plot starting 5-10 eV below the endpoint.

4.2.1 An explanation of tritium anomaly in terms of neutron-neutrino pairing

The first explanation of the tritium beta decay anomaly would be in terms of neutron-neutrino pairing by classical Z^0 force to form what might be called neutrino atoms.

(a) In TGD, this anomaly could correspond in a reasonable approximation to an atomic rather than nuclear process proceeding as n+$\nu \rightarrow$ p+e instead of rather than n $\rightarrow$ p+e +$\overline{\nu}$. Here p has nuclear binding energy and ν has the analog of atomic binding energy. For n+ν bound state neutrino energy is small and if neutrinos have a small mass, the neutrino bound state energy is much smaller than neutrino mass. The contribution of neutrinos to the total energy of the initial and final state nuclei can be neglected. The rest masses of the initial and final states are $m(H^3) = 2809.257$ MeV and $m(H^3) = 2809.239$ MeV. The difference of these energies is $\Delta m = m(He^3) - M(H^3) = -.018$ MeV and very small and the liberated energy does not go to electron and make it relativistic but reduces the strong binding energy by $-\Delta m$. Therefore the transition n+$\nu \rightarrow$ p+e might proceed as a transition between atomic states, at least if H^3 is an excited state.

As a consequence, the kinematics of the decay is effectively the same as that for the beta decay if the final state antineutrino has a negative energy $-m_\nu$ so that these transitions look like an anomaly at the end of beta decay spectrum.

If the decay is identified as an ordinary beta decay, the energy of the neutrino is given by $E = \sqrt{p^2 + m_\nu^2}$ in terms of its momentum p. This cannot give a negative energy for neutrino and the best one can achieve is $E = 0$. This would require tachyonic neutrinos.

(b) One can estimate the change of the neutrino bound state energy in the transition by using energy conservation: $m(H^3) + E(\nu, H^3) = m(He^3) + E(\nu, He^3)$. This gives $\Delta E(\nu) \equiv E(\nu, H^3) - E(\nu, He^3) = m(He^3) - m(H^3) = .018$ MeV. This suggests rather a large scale for the neutrino binding energies, which is in conflict with the intuition that neutrinos have a small mass and have a Z^0 Coulomb energy much smaller than its mass.

A possible solution of the problem is that H^3 is in an excited state with excitation energy in the range 1-10 keV. TGD leads to the proposal that these kinds of states exist and could relate to the existence of dark pseudo neutrinos which can be regarded as composites of dark proton and monopole flux tube carrying electron charge. This would explain why the solar X ray anomaly meaning the variation of nuclear decay rates correlating with the solar X ray flux suggests that X rays excite these states of atoms. Also the tritium beta decay anomaly shows annual variation (see this) suggesting that solar X rays generate excited states of H^3 for which the reaction is possible leading to unbound state of electron and He^3.

(c) That one sees a bump of width about 1-10 eV instead of a sharp peak in the spectrum could reflect the presence of different bound states of final state electrons. The scale of the electronic binding energies is indeed consistent with this (hydrogen ground state binding energy is $E_H = 13.6$ eV giving for He^3 energy of $4E_H = 54.2$ eV).

It will be however found that this model is not the only model that one can imagine.

2. There exists also a second tritium related anomaly: anomalously low levels of thermonuclear tritium have been observed in the study of underground water movement in a chalk aquifier [C3] (see this). The anomalously low levels of tritium could be due to the transformations n+$\nu \rightarrow$ p+e if the inverse process has a lower rate. This could be due to the kinematics of the process: there would be less phase space volume for the reversal of the process p+e $\rightarrow$ n+ν.

This mechanism is quite general and predicts the occurrence of nuclear transmutations based on the transformations of n+ ν pairs to p+e pairs or vice versa increasing the nuclear charge as a low energy. In biology these processes could be of special importance where there exists evidence that heavier elements appear as a kind of biofusion [C1, C4].

This kind of transmutations might be involved also with "cold fusion" discussed from the TGD point of view in [K6] [L3, L9]. If it is possible to assign to a nucleus neutron halo, this process might allow it to generate nuclei with a higher value of nuclear charge and a new form of low energy alchemy would become possible. There is no need to emphasize its potential technological significance.

This picture would suggests that the myth of elusive neutrinos could be wrong at low energies. The existence of long ranged weak interactions could also allow to improve the understanding of the neutrino-matter interactions.

4.2.2 Alternative view of the tritium beta decay anomaly

Does the proposed mechanism possibly explaining the tritium anomaly have alternatives? What really happens in the tritium beta decay? Can one understand the 10 keV scale in the anomalous tritium beta decay? How can the X-ray flux from the Sun amplify the beta decay anomaly? One can also ask whether the proposed idea about duality between dark weak interactions and strong interactions could allow a concrete quantitative formulation.

1. One should transform $\pi_L^{\pm}$ bond to π_L^0 bond or vice versa by emission of W boson but this changes the charge of H^3 unless the W decays to e-ν pair. The exchange of W boson between nucleon and leptopion that is quark/lepton of the corresponding bond involves energy change of about 1 MeV in the process and is considerably larger than 10 keV scale for X rays. A possible mechanism inducing transition between these states would be a variant of beta decay involving a spontaneous beta decay of pseudo-neutron decaying as $n - \pi_L^0 \rightarrow p - \pi_L^0 + +W^-$ followed by $W^- \rightarrow e^- + \overline{\nu}$.

   In the spontaneous decays of $p - \pi_L^0 \rightarrow p - \pi_L^- + +W^+$ and $n - \pi_L^+ \rightarrow n - -\pi_L^0 + W^+$ genuine and pseudo proton the scale of energy change is is 10 keV and these transitions could be involved with the tritium beta anomaly.

2. I have already considered a possible mechanism for tritium beta decay involving neutrino atoms and transition $n + \nu \rightarrow p + e$. Could one consider alternative mechanisms or at least analogous transitions.

   The spontaneous decay $n - \pi_L^0 \rightarrow n - \pi_L^+ + W^-$ is kinematically possible whereas $p - \pi_L^- \rightarrow p - \pi_L^+ + W^-$ is not allowed by energy conservation.

   One can imagine also a variant of beta decay involving a spontaneous beta decay of pseudo-neutron decaying as $p - \pi_L^0 \rightarrow p - \pi_L^- + W^+$ and $n - \pi_L^+ \rightarrow n - \pi_L^0 + W^+$ followed by $W^+ \rightarrow e^+ + \nu$. Now one would however have W^+ rather than W^- in the final state.

   The energy of W^- and of $e^- + \overline{\nu}$ is constrained by the mass difference $\Delta m(\pi_L) \simeq 33$ keV and by energy conservation. The mass difference $m(H^3) - m(He^3) = 18$ keV is but pseudo neutrons . This beta decay could explain the tritium anomaly instead of $n + \nu \rightarrow p + e^-$ for the generalized atom.

4.2.3 Why does tritium beta anomaly correlate with the X-ray flux from the Sun?

The proposed model of the beta anomaly in terms of a decay $p - \pi_L^0 \rightarrow p - \pi_L^- + W^+$ does not yet explain the correlation of the beta decay anomaly with X ray emission from the Sun. Could X ray absorption with X ray energy equal to the excitation energy induce reverse dark weak transitions $p - \pi_L^- \rightarrow p - \pi_L^0$? A possible mechanism would be following:

1. The absorption of X-ray by π_L^- (π_L^+) occurs first and increases the energy of $p - \pi_L^-$ but does not induce its decay if the energy of X-ray is not much larger than $\Delta m(\pi_L)$. X ray can be also absorbed by $n - \pi_L^+$.

2. After this the exchange of dark W$^-$ between π_L^- and induces the transition $p - \pi_L^- \rightarrow n - \pi_L^0$. In the same way, $n - \pi_L^+$ can be transformed to $p - \pi_L^0$.

4.3 A model for generalized atoms involving both electrons and neutrinos

The explanation of the tritium anomaly in terms of a transition $p + \nu \rightarrow p + e^-$ gave a good motivation for developing a model of dark weak atoms in which the screening of nuclear charges of both proton and neutron takes place. This idea would be very natural for a mad holographic mathematician loving symmetries. The best way to proceed is by making questions.

4.3.1 What does one mean with weak charges?

One must distinguish between two weak charges: namely the weak charges associated with W boson exchanges and Z^0 exchanges.

1. Protons and neutrons have opposite W charges identifiable as weak isospin. They also have non-vanishing Z^0 charges due to the mixing of the neutral $SU(2)$ boson with U(1) boson caused by the electroweak symmetry breaking. Z^0 weak charge is given for protons as $Q_W(p) = 1 - 4sin^2(\theta_W)Q_{em} \simeq .041$ and for electrons as $Q_W(e) = -Q_W(p) \simeq -0.041$. For neutrons *resp.* neutrinos having no em charge, the weak charge is -1 *resp.* +1. Note that about 25 nuclear protons generate weak Z^0 charge near a unity.

   ^{25}Mn and ^{26}Fe are good candidates for nuclei for which the proton weak charge can be shielded almost completely by a single dark neutron. For a larger number of neutrons about $N = 4 \times 10^6$ in the case of a neutrino atom the protonic Z^0 charge which cannot be screened by neutrinos is smaller than the unit charge. Therefore no separate screening of proton charge by electrons is necessary.

2. If the weak charges of the nucleons couple independently to the classical Z^0 gauge field, also nuclei have a weak Z^0 charge to which the contribution of protons is rather small. Nuclear screening of the weak Z^0 charge is not possible whereas the isospins can sum up to zero so that the W boson weak charge of a $Z = N$ nucleus vanishes. For the Z^0 screening to occur, neutrino atoms are necessary. In this case, electrons automatically screen the protonic nuclear charge and neutrinos the neutron charge so that the e-ν atoms have the same structure as nuclei.

Consider the W charges first.

1. The nucleus has non-vanishing W weak charge (weak isospin) for $N \neq Z$ so that screening of weak isospin inside the nucleus is not possible. Most nuclei have several isotopes so that this condition is satisfied for most isotopes. The most interesting nuclei are long-lived and stable nuclei.

2. The study of the table of nuclides appearing in a very old text book published 1963 gives some idea about the situation. Typically the most abundant stable nuclei have $N = Z$. Only He^3 is stable and has a neutron deficit. There can be stable isotopes with neutron surplus of 1 unit for nuclei lighter than O and heavier than H^3 (, which has two surplus neutrons and is stable). Stable nuclei heavier than N can have a surplus varying from 1 to 2 and O is the first nucleus having 3 stable isotopes. For Ca the number of stable isotopes is 4.

   There are also stable nuclei with a maximal abundance with $N > Z$. ^{3}Li^7, ^{11}Na^{23}, ^{15}P^{31}, ^{17}Cl^{36} ,^{19}K^{39}, ^{26}Fe^{55} are biologically interesting.

Consider next the Z^0 charges.

1. Nuclei themselves cannot carry out Z^0 screening but for neutral atoms the protons and electrons have opposite Z^0 charges so that the first guess is that screening of protonic Z^0 charge could occur automatically for ordinary neutral atoms. This would require dark protons and electrons. This is not possible since also now the condition that weak bosons are dark and have Compton length longer than the Bohr radius of atom-like structure.

2. If the size scale is longer than the size of the atom, atom + nucleus can be approximated as a point-like weak Z^0 charge equal to $-N$. Nonrelativistic model for neutrino atom assuming massive neutrino would predict that its Bohr radius is given by $a_W = (\hbar_{eff}(\nu)/\hbar)(1/2N^2\alpha_W)L_\nu$, where $L_\nu = \hbar/m(\nu)$ is neutrino Compton length for $\hbar$ and $\alpha_W = \alpha/sin^2(\theta_W)$.

3. Dark weak bosons are involved and one can argue that the dark weak scale $L_W(dark)(\hbar_{eff}(W)/\hbar)\times L_W$ should not be smaller than the size of neutrino atom characterized by Bohr radius a_W. But what does one mean with L_W? L_W is the scale below which the Z^0 boson is effectively massless and one can argue that inside this scale is infinite with the flux tubes, just as for

photons. Does this mean that the condition is not needed. In longer scales dark weak bosons have a scaled up weak scale. This argument sounds admittedly tricky.

What if one requires $L_W > a_W$? For $\hbar_{eff}(\nu) = \hbar_{eff}(W)$ this would give $a_W = (1/22N^2 g_W^2 \alpha_W) L_\nu \leq L_W$ giving $m(\nu) = m_W = 2N^2 g_W(e)\alpha_W) m_W$. For $N = 1$ this would give $m_\nu = 2\alpha_W m_W$, which is larger than nucleon mass for $\alpha_W \sim \alpha/sin^2(\theta_W)$. This does not conform with the fact that neutrino mass is very small.

The solution of the problem is simple: give up the idea that the neutrino atom corresponds to an ordinary atom! Assume instead that it corresponds to a block of condensed matter analogous to an atom and require that weak Z^0 charge of neutrons of the effective nucleus to be so large that the condition holds true! This is in accordance with the theoretician friendly nature stating that the phase transition occurs when the perturbation series does not converge anymore. Below this condition will be considered explicitly.

4. Neutrino mass of .1 eV would correspond to the ordinary Compton length $L(\nu) \sim 10\ \mu$m which is a typical cell length scale. $L_W \sim a_W$ requires $\hbar_{eff}(W)/\hbar \sim a_W/L_W \sim 10^7$. The neutrino Bohr radius is about $a_W = (sin^2(\theta_W)/2N^2\alpha)L(\nu) \sim .32/N^2$ mm. For $N = 1$ corresponding to D and He^3. It should be noted that the particle independent gravitational Compton length of Earth is about $GM_E/2 \simeq 5$ mm if the gravitational Planck constant satisfies the formula first proposed by Nottale. For larger values of Z, one obtains shorter size scales. For $Z = 20$ (Ca) one has $a_W = 1\mu$m, the scale of the cell nucleus.

5. A simultaneous W screening and Z^0 screening are not possible in the general case. Z^0 screening requires in general more neutrinos than electrons and the surplus neutrinos give a non-vanishing weak isospin coupling to classical W boson fields. This does not occur for hydrogen.

6. Ions would also be ions with respect to the Z^0 charge. This might have some relevance in biology.

4.3.2 The screening of Z^0 charges of neutrons by neutrinos

I have not yet specified precisely what neutrino atoms could be.

1. Since the Bohr radius of neutrino atom, proportional to $1/N^2$, would be larger than the neutrino Compton length of order 10^{-5} m for nuclear Z^0 charges, it its difficult to believe that the neutrinos could screen single nuclear weak charge. Rather, they should screen nuclear weak charges for a large enough number of nuclei so that the large neutron number decreases the Bohr radius.

2. There is also a second condition essential for the general holography like relationship proposed in [L21]. The idea is that the phase transition increasing the value of $\hbar$ to $\hbar_{eff}$ occurs only when the perturbation series using coupling constant strength $Q_1Q_2g^2/4\pi\hbar$ does not converge. The increase of $\hbar_{eff}$ makes the series converging. One might say that the TGD Universe is theoretician friendly.

3. There is also a consistency condition. The dark weak Compton length for the flux tubes connecting neutrons of nuclei to neutrinos must not be shorter than the Bohr radius of the neutrino atom. This gives a condition which fixes the size of the neutrino atom if one assumes that the counterpart of the nucleus corresponds to a block of condensed matter. Both the dark compton length L_W and the Bohr radius are proportional $\hbar_{eff}/\hbar$ to the condition so that it disappears from the condition and one has

$$L_W \geq a_\nu(N) = \frac{L_\nu}{2\alpha_W N^2} \quad .$$

This fixes the value of neutron number N:

$$N \geq \sqrt{\frac{1}{2\alpha_W}} \times \sqrt{\frac{L_\nu}{L_W}} \quad .$$

By using $\alpha_{ew} = \alpha_{em}/sin^2(theta_W)$, where one has $\alpha_{em} \simeq 1/137$ and $sin^2(\theta_W) \simeq 0.231$, and one obtains

$$N \geq \sqrt{\frac{137 \times sin^2(\theta_W)}{2}} \times \sqrt{\frac{L_\nu}{L_W}} \ .$$

For $m_\nu \simeq .1$ eV one would have $L_\nu = 1.24 \times 10^{-5}$ m. L_W is for Z^0 with weak boson mass ~ 100 GeV equal to $10^{-12} L_\nu$ so that one has $N \simeq \sqrt{137/8} \times 10^6 \simeq 4 \times 10^6$.

It must be emphasized that this condition applies only in the case of weak atoms. It is quite possible to have a shorter weak scale L_W but in this case there would be no weak atoms.

4. Assuming that the neutrons of the neutrino atom correspond to dark neutrons of a spherical mass blob with radius R consisting of atoms with radius $r = 10^{-10}$ obtains having average number of N_{nucl} neutrons $N = (R/r)^3 N_{nucl}$. This gives

$$\frac{R}{r} = (\frac{N}{N_{nucl}})^{1/3} \simeq 10^2 \times (\frac{4}{N_{nucl}})^{1/3}$$

The dependence on the value of N_{nucl} is rather weak so that the radius is near 10^{-8} m, which is p-adic length scale $L(151)$ and fundamental scale in biology. For instance, cell membrane thickness and thickness of multiply coiled DNA double strand and the nucleosomes along DNA have this length scale. This suggests that neutrino atoms might play a key role in making living matter a macroscopic quantum system. It is rather remarkable that the fundamental biological size scale is expressible in terms of fundamental constants and the density of neutrons in condensed matter.

The result does not formally say anything about the value of $\hbar_{eff}$ except the dark weak boson scale $\hbar_{eff}/\hbar L_W$ equal to neutrino Bohr radius, must be longer than R. This gives the condition $\hbar_{eff}/\hbar \geq R/L_W \geq 10^9$.

4.3.3 The screening of protonic Z^0 charges by dark electrons in weak atoms

One can develop a similar argument for the screening of the protonic Z^0 charge by atomic electrons.

1. The argument is in this case the same as above. Only the weak charge is $\pm.041$ units and the Bohr radius of the neutrino is replaced with the Bohr radius for electrons. The nucleus is replaced with a large enough number of nuclei. The condition that the Compton length of dark weak bosons is not smaller than the dark Bohr radius of the atom. Now this gives the condition $L_W \geq L_e/Z^2 Q_Z^2 \alpha_{ew}$, where $Q_Z \simeq \pm.041$ is the weak Z^0 charge of electron/proton and $L_0 \simeq 2.2 \times 10^{-12}$ m is Compton length of electron. This gives $Z \geq (1/2\alpha_{ew} Q_Z^2)^{1/2} \times (L_e/L_W)^{1/2}$ would give $Z \sim 2.5 \times 10^6$ for the total em charge of the block.

2. Z is roughly half of the number $N \sim 4 \times 10^6$ of the previous estimate for neutrino screening. For higher dark electronic orbitals this would require the absence of the screening electrons in the interior. Except for the s-wave states, the Z^0 screening would be analogous to the usual atomic screening of the electromagnetic charge. It however seems that this phase represents a new phase of matter. The value of Weinberg angle fixing the value of weak Z^0 charges of e and ν must be responsible for this and biology suggests anthropic explanation. In the TGD framework number evolution could explain this.

3. The matter could be partially ionized ordinary condensed matter carrying net em charge and there also weak charge, at least if nuclear protons are dark with respect to weak interactions. From this one can estimate the parameter R/R_0 and, somewhat surprisingly, it corresponds to the size of 10^{-8} meters if one assumes total ionization. One can also assume a partial ionization as in bio-matter (in Pollack effect every fourth proton goes dark magnetic flux tube). The radius R scales as $R \propto \epsilon^{1/3}$ as a function of ionization ϵ. Therefore dark atoms can contain both electrons and neutrons could screen both the weak charges of neutrons and protons if the ionization degree is about $\epsilon \simeq 1/8$. An interestinn question is whether this

true for exclusion zones in Pollack effect. They could have a structure similar to that for the nuclei involving both edges of the Hamilton path and its edges carrying neutrinos *resp.* electrons.

4. The proposed picture about neutrino atoms and their electronic counterparts does not affect the argument explaining the tritium anomaly. In fact the essentially same estimate for the radii of neutrino atoms and their electronic counterparts would allow both p-n and e-p pairs be dark. The final state e-p pairs could also have ordinary value of Planck constant. In this case the decay would occur to an atomic orbital with radius about 10^{-8} meters, which correspond roughly to the principal quantum number $n = 10$.

Neutrino atoms would be rather large and this means that condensed matter blocks of size 10^{-8} could be regarded as Z^0 plasma but with nuclei having so large Compton length that they form a macroscopically quantum coherent phase analogous to liquid Helium. Neutrino superconductivity and superfluidity are highly suggestive. This would give support for the TGD proposal and classical Z^0 force could play a central role in condensed matter and in hydrodynamics [L13, L14]. For instance, hydrodynamical vortices could be interpreted in terms of Z^0 superfluidity.

4.4 What Platonic vision allows to say about nuclear dynamics?

Only the static aspects of Platonic vision are discussed hitherto. Platonicity allows the orbital angular momenta $l \leq 5$ and as a special case also $l = 9$ (dodecahedron and the dual edges in icosahedron) but one cannot completely exclude also some higher values of l. The interesting question is that Platonicity could predict selection rules for nuclear reactions.

How to describe the interacting many-nucleon and many-atom states encountered in scattering? It is rather easy to guess the basic principle from the vision of the interaction of biomolecules in biocatalysis and from the interaction of closed strings in string models.

1. Biomolecules must first find each other and after this they must become close to each other to react and the energy needed to overcome the potential energy wall must come from some source. U-shaped flux tube tentacles with a large value of h_{eff} would reconnect and form a pair of flux tubes connecting the molecules after which the value of h_{eff} would be reduced and force the molecules near each other. In this process energy would be liberated and kick the molecules over the potential energy wall and the reaction could proceed.

2. The extended flux tube edges connecting two subsequent nucleons of the Hamilton cycle assigned with a Platonic shell with a given value of n and j would define the tentacle-like entities. A similar extension is possible also for the dual edges. The lengthening of a tentacle preserving its magnetic energy would involve a temporary increase of h_{eff} and a reduction of string tension. The reconnection of the edge tentacles would allow for the nuclei to find each other. This would induce a fusion of the Hamiltonian cycles to a Hamiltonian cycle of the composite graph. After that h_{eff} would be reduced and the liberated energy would allow the systems to overcome the potential wall and the nuclear reaction would proceed.

3. Can one assume that the entire Platonic solid and the Hamiltonian cycle are present for partially filled j-blocks? This would require that the free vacancies are realized as pre-existing geometric entities. The information j fixes the j-block as Platonic solid and fermion statistics and energy minimization forces the nuclei to fill a fixed j-block. Fermion statistics could thus force the existence of Platonic solid as its geometric counterpart. Hamilton sub-cycles, possibly even several, for partially filled shells must be assumed. Subcycles must define connected regions, which poses strong constraints on the order in which the free vacancies.

What can one conclude from these assumptions?

1. The initial states of the nuclear reaction can be regarded as tensor products $j_{i,1} \otimes j_{i,2}$ of j-blocks. Final states are tensor products $j_{f,1} \otimes j_{f,2} ... \otimes j_{f,n}$, of $n \geq 2$ j-blocks. These tensor products must contain common states for the reaction to proceed and the assumption that the values of $j_{f,i}$ are consistent with the Platonic solids, poses conditions on the values of $j_{f,i}$.

2. $l = 5$ and $l = 9$ and possibly some higher values of l, define elementary shells and at the fundamental level the reactions would occur between pairs of elementary shells and proceed by the proposed re-connection mechanism. The composite Hamiltonian cycle formed in the reaction is not in general elementary but should transform to a union of elementary cycles belonging to outgoing nuclei. In "topolocally elastic scattering", the shells would fuse temporarily and emerge as unchanged.

3. The conservation of the sum $V_{a,1} + V_{a,2}$ for the numbers $V_{a,i}$ of "active" vertices containing a nucleon and the number $E_{a,1} + E_{a,2}$ of "active" free edges containing a nucleon corresponds to the conservation of nucleon numbers. Active vertices and free edges would be shared by different final state Platonic solids. The total numbers of active vertices and activ free edges is conserved in the reconnection but after that topological reactions modifying the face types of tessellations could occur as analogs of phase transitions changing the face type of solid lattice. These conditions pose constraints on the Platonic solids possible in the final states.

   For the most general option, the graph formed by the edges and vertices of the intermediate graph as a fusion of Hamiltonian sub-cycles would rearrange to form a Hamiltonian sub-cycle of some Platonic solid. The process would be analogous to melting of a crystal followed by a condensation to a new crystal. A stronger assumption is that the total numbers of vertices and free edges for Platonic solids are conserved.

4. Can one assume something about the dynamics of faces? The assumption that the total number of faces is conserved implies that the number of free edges equal to $F-2$ is conserved. The face type (triangle, pentagon, or square) is the same for a given Platonic solid. An even stronger assumption would be that the total number of faces of a given type is conserved and looks unrealistic.

   The reactions in which a single Platonic solid appears in the final state would be strongly restricted by the conservation of the vertex number. Tetrahedron, octahedron, and icosahedron have triangular faces. If $V_1 + V_2$ is conserved, the reactions icosahedron (12 vertices, 18 free edges) $\leftrightarrow$ 2 octahedrons (6 vertices, 6 free edges), icosahedron $\leftrightarrow$ 3 tetrahedrons (4 vertices, 2 free edges). The number of free edges is not conserved. It seems that the most general option is the most realistic one.

   If the numbers of different face types are conserved, in reactions involving incoming Platonic solids with different faces the outcome should consist of similar solids or contain Archimedean solids, which can have several face types. There is however no deep reason for the conservation of numbers of different face types.

4.5 Could the notions of Platonization and tensegrity make sense in atomic physics?

It is easy to invent an objection against the idea of holographic correspondence between nuclear and atomic physics, which requires that the notions of Platonization and tensegrity make sense also in atomic physics. Tensegrity requires that electrons in atoms are connected by monopole flux tubes. Can one invent any justification for this kind of crazy idea?

1. Hydrogen atom was the brilliant success of atomic physics. It was generalized by treating in the lowest order approximation electrons as independent entities experiencing only the Coulomb force of the nucleus. However, at the classical level this does not seem to make sense since the mutual Coulomb interaction at the shell with the same value of the principal quantum number n and same value of l should have roughly the same orbital radius $r_n = n^2/Z^2a_0$. The order of magnitude for their total repulsive Coulomb energy has a lower bound about $Z(Z-1) \times /r_n = Z(Z-1)Z^2/(2a_0n^2)$. Here only the nearest neighbor interactions are counted.

   The classical interaction energy behaves like Z^4 for large values of Z whereas the attractive interaction with the nucleus behaves classically like Z^2! Does it really make sense to assume that the interactions of electrons can be treated as a small perturbation?

2. If one takes quantum classical correspondence seriously, one must ask whether there exists an interaction, which would work against the repulsive Coulomb interaction and prevent the explosion of the energy shell (or the angular momentum shell $j = l \pm 1/2$). Here the string tension of the monopole flux tubes could come in rescue and provide the force preventing the explosion. It would contribute to the total energy a constant amount and the effect would be visible for atoms with $Z > 1$. In a good approximation the system would behave like a rigid body. String tension would also give rise to vibrational modes whose existence would serve as a killer test for the proposal. One has a good reason to expect that these energies are rather small as compared to atomic energies.

In the Platonic model it is possible to calculate the repulsive Coulomb interaction energy also exactly since the tesselation contains the points of the Hamiltonian cycle with V vertices and its dual with $F - 2$ vertices at free edges connecting neighboring vertices of the Hamiltonian cycle which are not nearest neighbors along the cycle.

1. For a full electron electron shell as a Platonic solid defining a tessellation of the sphere, the distances of electrons at full shell would be constant, which would make the estimation of the contribution of electron interaction energy very simple. The dominating contribution to the Coulomb interaction would come from nearest neighbor interactions between electrons of the Hamiltonian cycle and between Hamiltonan electrons and electrons of dual edges. The sum of the repulsive interaction of the shell containing only Hamiltonian edges would be constant.

2. A lower bound for this contribution to the repulsive interaction energy would come from nearest neighbor interactions and would be of order $E_{Coul} = k_1 Z^2/2a_0n^2 + k_2 Z^2/2a_0n^2$, where one has $(k_1, k_2) = (2(l + 1)x_1, 2lx_2)$. Where $x_i = 1$ for the Hamiltonian cycle and $x_i = 1/2$ for its dual if the distance between electron of cycle and electron of its dual is half of the distance between Hamiltonian electrons. For $l = 1$ one has $(k_1, k_2) = (4, 2)$ that is the Hamilton cycle and its dual for the tetrahedron. For $l = 2$ one has $(6, 4)$ that is Hamiltonian cycle for octahedron and its dual for cube. For $l = 3$ ones $(8, 6)$ that is the Hamiltonian cycle for the cube and its dual for octahedron.

   There is also the interaction energy between different shells and if the wave functions for the orientation of the shells are allowed, the calculations are more complex. The selection of a common quantization axis of angular momentum eliminates this degree of freedom.

In the lowest order approximation, the dynamics would effectively reduce to single-particle level since the Platonic tessellation would be a rigid body-like system having only the radial degree of freedom plus degrees of freedom related to orientation. The wave functions for electrons at the vertices of the Platonic solid would be obtained by the operations of the symmetric group of the Platonic solid, which is a discrete subgroup of the rotation group and the rotation would give a superpositions of the harmonics belonging to the multiplet $j = l \pm 1/2$. Antisymmetrization would leave only the products of wave functions with different values of j_z.

The monopole flux tubes could serve as correlates for the pairing of valence electrons. p-Adically scaled down electropions [K13] could also appear as molecular bonds. Note that electropion mass is rather precisely 2 electron masses. Evidence exists for muo-pions and tau-pions and also their p-adically scaled down variants could appear as bonds. Chemical bonds could correspond to these scaled down pions. Tensegrity is indeed a very natural concept in molecular physics.

4.6 Could the predicted new atomic physics kill the proposal?

New atomic physics is predicted and it can turn out to be fatal. I hasten to confess that the following speculations reflect my rudimentary knowledge of details of atomic physics. The new conceptual element are flux tubes, which can be regarded as springs with mass and elastic constant (string tension).

The first question concerns electric fields in the flux tube picture.

1. If there are only flux tubes present, the electric fluxes must run along them (a more conservative option is that fluxes flow to a large space-time sheet). Perhaps the most natural interpretation is that the localization of electric fluxes to flux tubes induces a constraint force

due to the space-time geometry, something completely new. If so, one can argue that the dynamics for the flux tubes carrying also electric flux automatically describes the repulsive Coulomb force subject to geometrodynamic constraints.

An important implication is that the Hamiltonian cycles of j-blocks must reconnect to the Hamiltonian cycles of other j-blocks and to the nucleus. The Hamiltonian cycles of the entire atom must fuse to a single large cycle, which can be closed for a neutral atom, and would correspond to closed monopole flux tube starting from the atomic nucleus. Each charge along the cycle contributes to the electric flux flowing in the monopole flux tube.

It has been proposed [?] (`https://cutt.ly/eneociZ`) that molecular bonds could be interpreted as electric flux tubes. This proposal is discussed from TGD point of view in [K7]. If the atoms of the molecule are ionized the Hamiltonian cycles of atoms must be reconnect by U-shaped tentacles and ionic bonds would correspond to flux tubes and presumably all chemical bonds.

Consider next the mass of the flux tube.

1. Flux tubes connecting neighboring charges could be p-adically scaled electropions with mass smaller than the mass 1 MeV of electropions and would contribute to the mass of the atom. In the case of nuclei scaled hadronic pions between nucleons having mass of order MeV are replaced by p-adically scaled elctropions. Note that electropions have mass of 1 MeV. In the case of atoms, their scaled variants should have a considerably smaller mass, which would naively correspond to the atomic p-adic length scale and mass scale of 1-10 keV. Note that 10 keV would be the scale of proposed nuclear excitation energies supported by nuclear physics X-ray anomalies. One can argue that the mass corresponds to the atomic p-adic length scale $L(137)$ as a natural length scale for the flux tube gives and would be of order $m \sim keV$.

2. One the other hand, one could argue that the mass should be very small because, to my best knowledge, standard atomic physics works very well. However, the additive contribution of these masses does not affect the electronic bound state energies but only the total mass of the system. I do not know whether anyone has studied the possible dependence of the total mass of atom on the number of electrons? Does it contain an additive contribution increasing by one unit at each step along the row of the periodic table as an additional flux tube appears to the Hamilton cycle. These contributions could be also interpreted as contributions of the repulsive interactions of electrons to the energy.

As in the case of nuclei, the atomic flux tubes would act as springs, i.e. harmonic oscillators. This predicts a spectrum of excited states with scale determined by the elastic constant k or equivalent ground state oscillation frequency ω_0.

1. If ω_0 is large enough, the excitation energies would be greater than the ionization energy and there would be no detectable effects. The naive argument that ω_0 corresponds to the atomic length scale $L(137)$ as a natural length scale for the flux tube gives $\omega_0 \sim 1keV$. This energy scale would be for light atoms with $Z \leq 9$ (Oxygen) larger than the ionization energy $E = Z^2 \times 13.7$ eV so that photons causing excitation would cause ionization.

2. An equally naive scaling from nuclear scale to atomic scale would suggest that the value of ω_0 is scaled from $\hbar\omega_0 = 1$ MeV by the ratio $L(113/L(137) = 2^{-12}$ of nuclear and atomic length scales to about $\omega_0 = .25$ keV. This is not far from the above estimate.

3. How to deal with atoms with a small number of electrons, in particular Helium with 2 electrons? $j = 2$ j-blocks are special in the sense that they do not allow sub-Hamiltonian cycle. Could the flux tube connecting the electrons be absent in this case so that only the repulsive electronic contribution would be present? Note also that the repulsive interaction energy between electrons would be smaller than the attractive interaction energy of electrons for atoms with $Z = 2$. If this picture is correct, new atomic physics would emerge when j-block contains more than 2 electrons.

   One can also consider the possibility that the coupling to photons is weak enough, perhaps by the condition that the photon must transform first to dark photon. The behavior of multi-electron atoms in a radiation field whose photons have a low energy must have been studied.

One could also imagine that the flux tubes form a $h_{eff} \geq h$ quantum coherent state, in which there are $n = h_{eff}/h$ flux tubes forming the sub-tessellation of Platonic tessellation for a given j-block with vertices connected by flux tubs. Here n would be the number electrons in the j-block. The excitation energy $E = \hbar_{eff}\omega_0$ is scaled by $\hbar_{eff}/\hbar = n$.

1. If all flux tubes associated with atom were excited at once as a phase transition, the required excitation energy would be rather large for large enough n and the excitations by photons might be possible without ionizing the atom.
2. The atoms at the left end of the row are the problem for this option and more generally, the atoms at the left end of each j-blocks. One expects that the flux tube length depends on the value of the principal quantum number N labelling the rows since the size of Platonic solid must increase with n like n^2. Can one assume that the mass of the spring does not depend on the row? If the elastic constant k does not depend on the row, one could consider a simultaneous collective excitation of all flux tubes so that the binding energy could increase enough.

5 A new view of hadrons inspired by the nuclear physics model

The proposed revision of the view of the construction of nuclei makes it possible to get rid of the non-perturbative nuclear nightmare and is extremely simple. The recent QCD based view of hadron physics leads to a similar non-perturbatove nightmare. Could the proposed lego brick picture of nuclear physics extend to the level of hadrons?

1. Could the original Gell-Mann model baryons consisting of heavy constituent quarks find a justification. Could the difference between constituent quarks and current quarks be due to different p-adic mass scales?
2. Could the masses of hadrons be just sums of quark masses and masses of bonds which are not far from pion masses. p-Adic mass calculations [K2] lead to a formula for the mass squared of leptons and quarks. For a given p-adic mass scale mass squared is in the lowest order approximation integer: $m^2 = Am_p^2$, where m_p is the p-adic mass scale. Apart from the effects due to CKM mixing, the values of these integers for leptons (e, ν_e) and the quarks are

   $$(A(e), A(\nu), A(u), A(d), A(c), A(s), A(t), A(b)) = (5, 4, 5, 8, 14, 17, 65, 68) \ .$$

   These values represent lower bounds and perturbative p-adic second order contribution to A can be at most one unit. What is remarkable is that the masses of electron and u quark for the same p-adic length scale are identical in the lowest order and electron and neutrino masses are nearly identical. The large electron-neutrino mass difference would be due to different p-adic mass scales.

   If the p-adic length scales of u,d, and s quarks are same, their mass ratios in the lowest order are $m(u)/m(d) = \sqrt{5}8$ and $m(s)/m(d) = \sqrt{17/8}$.
3. Can one predict the masses of pions and lightest baryons p, n, Λ from this input by assuming that the masses of quarks and pion-like bonds are additive and selecting the p-adic mass scale to be that associated with Gaussian Mersenne prime corresponding to $k = 113$, to $k = 109$ or to $k = 107$ assigned with light baryons?

5.1 Masses of mesons

One can start by estimating the masses of mesons

5.1.1 Pion masses

Consider first the masses of pions.

1. If quark masses are additive, obtains identical masses $m(\pi^{\pm})/m_p = m(\pi^0) = (\sqrt{8}+\sqrt{5})\times m(k) = (\sqrt{8/5}+1)m_e$, $m_e = .5$ MeV in lowest for neutral and charge pions. $m(k$ is the p-adic mass scale $p \simeq 2^k$, most naturally $k = 113$. Note that in the case of neutral pions averaging over the pairs $u\overline{u}$ and $d\overline{d}$ is involved.

2. Since one has $A(u) = A(e) = 5$, for $k = 113$, which has been identified a the nuclear p-adic length scales, the mass of u quark is obtained by scaling the mass of electron by the factor $2^{(127-113)/2} = 2^7$. This gives $m(u(113)) \simeq 64$ MeV. The mass of the d quark would be $m(d) = \sqrt{8}5m(u) \simeq 80.9$ MeV. For $k = 113$, pion mass would be $m(\pi) = m(u)+m(d) =$ 144.9 MeV, which is quite near to 140 MeV for neutral pions. Note that there would be no Cabibbo mixing for pions.

5.1.2 Masses of light mesons containing strange and possibly also other quarks

In the TGD framework the heavy quarks can appear as light quarks, at least when bound to mesons. This raises the question whether c, t, and b quarks could also appear in light neutral mesons.

Neutral and charged kaon masses are predicted to be the same. Kaon masses are $m(K^{\pm}) = 494$ MeV and $m(K^0) = 498$ MeV. For $K^+ = u\overline{s}$ mass one obtains $m(K) = (\sqrt{17/5}+\sqrt{8/5})\times 2^{\Delta k+1/2)}m_e$, $\Delta k = 127-k$. For $k = 110$, one obtains $m(K) = 514$ MeV which is by 4 per cent larger than K^- mass 494 MeV. One might argue that $\Delta k = 17$, which is somewhat alarming since this brings in half-octave $\sqrt{2}$ or tritonus. Interestingly, tritonus was once cursed by the church as the devil's interval.

What about masses of $m(\eta) = 548$ MeV and $m(\eta') = 957.8$ MeV? The Gell-Mann model with flavor SU(3) symmetry predicts that one has two η type mesons corresponding to quark decompositions

$$\begin{aligned} \eta_1 &= \frac{1}{\sqrt{3}}(u\overline{u}+d\overline{d}s\overline{s}) \ , \\ \eta_2 &= \frac{1}{\sqrt{6}}(u\overline{u}+d\overline{d}-2s\overline{s}) \ . \end{aligned}$$

η_1 is clearly SU(3) singlet so that η_2 should correspond to η in absence of SU(3) symmetry breaking, which mixes the mesons. This mixing is necessary also in the Gell-Mann model and is rather small. The masses predicted in absence of mixing are

$$\begin{aligned} m_1 &= \frac{2}{3}(m(u)+m(d)+m(s)) = \frac{2}{3}(1+\sqrt{8/5}+\sqrt{17/5})m(u) = \frac{2}{3}(m(u)+m(d)+m(s)) = \frac{2}{3}(1+\sqrt{8/5}+\sqrt{17/5} \\ m_2 &= \frac{1}{3}(1+\sqrt{8/5}+4\sqrt{17/5})2^{\Delta k}m(e) \\ per. \end{aligned}$$

For $k = k(\eta) = 110$, the predicted masses are $m_1 = 496$ MeV and $m_2 = 581$ MeV. m_2 is 6 per cent higher than $m(\eta) = 548$ MeV. A mixing by angle η is required. Using the notions $c^2 = cos^2(\eta)$ and $s^2 = 1-c^2$, the condition $c^2m_2+s^2m_1 = m(eta)$ gives $s^2 = (m(eta)-m_2)/m_2$. The rather large value of $s \simeq .6231$ reflects the fact that m_1 and m_2 are rather near to each other.

This allows us to predict the mass of η' assuming that η and η' are orthogonal irrespective of whether their p-adic length scales (kind of scaling invariance). A good guess for the p-adic scaling is by factor 2 meaning $k(\eta') = 108$. The predicted mass $m(\eta') = c^2m_1+s^2m_2 = 1058$ MeV and 10 per cent larger than the real mass $m(\eta') = 949$ MeV.

A possible reason for the discrepancy is that the mixing with $c\overline{c}$ pair is not taken into account. Since the ratio of c and s masses is $\sqrt{14/17}$, this would reduce the prediction for the masses of η and η'. If η is $SU(4)$ singlet, the mass m_1 is given as

$$m_1 = (1\sqrt{8/5}+\sqrt{17/5}+\sqrt{147/5})\times 2^{\Delta k/2}m(e) \quad , \quad \Delta k = 127-k = \quad .$$

This would give $m_1 = 464$ MeV, which is 10 per cent lower than for SU(3) singlet: the situation would get worse since c mass is smaller than s mass for a given p-adic length scale.

What if one also includes the third (t,b) quark generation? The coefficients (A(t),A(b)) are predicted to be $(A(t), A(b)) = (65, 68)$. For a given p-adic prime, the presence of $t\bar{t}$ and $b\bar{b}$ pairs in η and η' would increase their masses since the ratio of $m(b)/m(d)$ is equal to 2. It turns out that the mass of η_1 increases by a factor $1.4 \simeq \sqrt{2}$ so that the p-adic scale must be reduced from $k = 110$ to $k = 111$ so that the mass of η_1 would not be changed and one avoids the tritonus in the case of η although it remains in K.

If η' corresponds to η_1 with 3 all 3 generations and to p-adic length scale $k = 109$, one obtains $m(\eta') = m(\eta_1) = 938$ MeV. The error would be 29 MeV, that is 3 percent. To estimate the mass of η_2 an explicit expression for it would be required. Orthogonality with η_1 does not fix η_2 completely. In any case η_2 would correspond to η. One possibility is η_2 does not include heavy quarks: recall that in this case the prediction would be too small by 6 per cent. A small mixing also with $t\bar{t}$ and $b\bar{b}$ would improve the situation.

5.2 Masses of proton, neutron and Λ

Consider next the masses of the proton, neutron and Λ.

1. The flux tube contribution to the mass would be the sum of the masses identifiable as pions and would be given by $m(tubes) = 3 \times 140 = 420$ MeV, that is $m_p/2$, exactly one half of proton mass. The simplex model assumes that the quark contribution is the same as the meson contribution. This suggests mass equipartion or a kind of dynamical supersymmetry relating pion and quark contributions to the mass of the nucleon. The masses of proton and neutron $m(p) = 2m(u)+m(d) = (2+\sqrt{17/5})m_e$ and $m(n) = m(u)+2m(d) = (1+2\sqrt{5})m(u)$ so that neutron proton mass splitting would be quite too large.

2. The first candidate for the solution of the problem is provided by the same mechanism as used to minimized energy in the construction of nuclei: $n - \pi_L^0$ with a larger mass were replaced by $p-\pi_L^-$ pairs, where π_L^- has the mass of electropion (quarks correspond to $k = 127$ characterizing also electron). One can replace d quarks with $u - \pi^-$ pairs so that the masses uud and udd are identical. The contribution of quarks to the total mass of the nucleon would be $3m(u)/2 = 193$ MeV for $k = 113$. For $k = 111$ the contribution is 384 MeV and by $\Delta m = 36$ MeV smaller than the nucleon mass $\simeq 940$ MeV.

   Intriguingly, if the mass equals to the average mass $m(u, k = 111) + m(d, k = 111))/2 = m(\pi(k = 113))$ of u and d quarks, $k = 111$ gives the same contribution as pions and one obtains proton mass correctly. The masses of nucleons would come out correctly apart from differences relating to pion charge which is 4 MeV. The masses of n *resp.* p is 939.57 MeV *resp.* 938.27 and the mass difference is 1.3 MeV.

3. Could this be achieved by the TGD counterpart of CKM mixing [K1, K2, K5, K3, K4], which is certainly present. In TGD, CKM mixing is caused by the different topological mixings of the partonic 2-surfaces at which quarks reside. In a good approximation, the mixing is present only for the lowest quark genera ($g = 0$ (sphere), which corresponds to u (d) and $g = 1$ (torus), which corresponds to c (s). CKM mixing would be essentially the difference of the topological mixings. In the case of Cabibbo mixing, the mixing angle θ_c would be different $\theta_c = \theta_u - \theta_d$ of the topological mixing angles θ_u and θ_d.

   The condition is that the mixing of u quark and scaled down c quarks is such that the light mixed state has mass $m(\pi(k = 111)) = 2m(\pi)$. One would have

   $$c_u^2 + s_c^2\sqrt{14/5}m(u(k = 111)) = m(\pi(k = 113)) \equiv m(\pi) \ .$$

   Here one has $(c_u, s_u) = (cos(\theta_u), sin(\theta_u)$ and $m(u(k = 111)) = 128$ MeV and $m(\pi) = 140$ MeV. This gives $s_u^2 = m(\pi)-m(u(111))/m(u(111)/(\sqrt{14/5}-1)$ giving $s_u = \pm.1392$. For the Cabibbo angle $s_c = .2250$ this gives $s_d = s_u \mp s_c$. For positive s_u this gives $s_d = -0.0858$. In [K5] I have discussed a model for the topological mixing of quarks assuming that mass squared values are averages of different mass squared values of the topologically mixed particles with a given p-adic length scale. In the recent case, the mixing cannot occur for the mass squared values: this would lead to a negative value for s_c^2.

4. This proposal resembles the Gell-Mann model in which constituent quarks would give the entire mass of the nucleon. The situation is the same now if the constituent quarks are identified as quark-flux tube pairs. The QCD inspired view replaces constituent quarks with current quarks and divides them to valence quarks and sea quarks. Due to the technical problems of the non-perturbative QCD one cannot build a concrete model. Current quark masses would be in the range 5-10 MeV.

   In the TGD framework, valence quarks could correspond to the quarks with mass scale $k = 111$ and sea quarks would have small p-adic mass scale. Nuclear physics suggests electron mass scale as a mass scale of sea quarks: in this case the current quark masses would be $m(u) = m_e$ and $m(d) = \sqrt{8/5}m_e$. The total sea quark mass would be measured in few MeVs: of order .1 per cent.

5. In case that the topological mixing does not completely take care of the equipartition of the pion and quark contributions to the mass, the missing $\Delta m \leq 36$ MeV could be assigned to the light sea quarks and corresponds to 3.8 per cent of the total mass of the nucleon. The estimates for this contribution vary but are few percent of nucleon mass. It is also known that sea quarks carry only a very small longitudinal momentum fraction and valence quarks carry 1/3 of longitudinal momentum. This would conform with the interpretation of the valence quarks as $q - \pi$-structures and sea quarks as light quarks of mass of order electron mass appearing as bonds in nuclei. They could correspond to flux loops with length of order electron's p-adic length scale $L(127)$, which is of the order of electron Compton length.

6. Can one understand the mass $m(\Lambda) = 1116$ MeV of Λ baryon containing also strange quark s? The mass difference $m(\Lambda) - m(n) \simeq 178$ MeV cannot correspond to the mass difference $m(s) - m(d)$, which in absence of topological mixing would be maximal and in this case given by $(\sqrt{17/5} - \sqrt{8}5)m(u, 111) \simeq 81$ MeV. This is too small to explain the $\Lambda - n$ mass difference.

   Could energy minimization be achieved by replacing the $s-\pi^0$ pair with $p-K^-$ pair solve the problem? Kaon mass is 493 MeV so that $m(\Lambda)-m(p)$ would be equal to $m(K)-m(\pi) \simeq 353$ MeV for $k = 113$. This is too large by a factor 2. For $k = 115$ one would obtain mass difference 176.1 MeV to be compared with real mass difference $m(\Lambda) - m(n) \simeq 178$ MeV!

If this picture is correct, the p-adic length scale hypothesis would make it possible to build mesons from quarks with masses predicted by and various mesons whose masses would be sums of quark masses in the case of charged mesons and averages of sums in the case of neutral mesons. It is essential that quarks and mesons can exist in several p-adic mass scales related by a power of 2. Different topological mixings for U and D type quarks would explain CKM mixing. In mesons topological mixing would not occur.

6 Summary and outlook

In the 3-D standard harmonic oscillator model, the states of protons and neutrons treated as independent particles correspond to orbitals: they are characterized by the principal quantum number n and the angular momentum l in the range $[0, n]$ and spin as single-particle quantum numbers. The oscillations in the length of the flux tube and the transversal stringy excitations of the flux tube would correspond to 3-D oscillations in the oscillator model. The difference is that the nucleons are bound by pion-like color flux tubes with string tension to form lattice-like structures on the energy shells which correspond in ground state to Platonic solids.

In the TGD based model also other quantum numbers are obtained, and they could relate to, for example, the energy excitations of the 10 keV scale which is suggested by the effect of X rays on nuclear decay rates and are needed to explain the tritium anomaly.

The energy shells are filled with neutrons and protons, and a structure similar to that of the periodic table is obtained, but now for protons and neutrons separately. This lattice-like structure is realized with the help of Platonic solids for which the distance between neighboring lattice points are the same. The states corresponding to the given l correspond to j-blocks $j = l \pm 1/2$, which can be assigned with Platonic solids: $j = l + 1/2$ *resp.* $j = l - 1/2$ corresponds to Hamiltonian cycle *resp.* its dual, and the states of these multiplets.

There are two options. For the $F-2$ option, nucleons are assigned either to the vertices or to the center points of the edges of the edge complement of the Hamiltonian cycle. For the F option they are assigned either to the vertices of the Platonic solid or its dual assignable to the same sphere. Both Hamiltonian cycles are involved. The growth of the principal quantum $n = 0, 1, 2, ...$ corresponds to the growth of the energy shell like $\sqrt{n}$.

In the standard model, the harmonic oscillator potential is introduced as an "effective" potential that would sum up all interactions. In TGD, the harmonic oscillator potential would correspond to the oscillator potential associated with the monopole flux tubes acting as springs, i.e. the spring force. The concept of tensegrity is natural. Energy minimization determines the configuration at the energy shell as lattice-like structure leads to Platonic solids and possibly also more general structures such as buckminsterfullerene having also constant distances between the vertices.

Also a detailed quantitative realization of the idea about the duality of the descriptions of strong interactions in terms of color interactions and weak interactions, very natural in the TGD framework, emerges. u quark and electron have same mass for a given p-adic prime so that the quark analog of electropion would be scaled down pion. The p-adic length scale of quark would depend on hadron and mass predictions are correct with few percent.

The monopole flux tube bonds are identified as pions and nucleons can correspond to pairs of a nucleon and neutral pion or pair of a nucleon and charged pion with the same total charge. Scaled down pions correspond to the MeV scale of nuclear strong interactions . The mass differences of scaled down pions are of order 10 keV, which would define a new energy scale explaining the correlation of nuclear decay rates with the X-ray flux from the Sun.

This picture generalizes in two directions. The monopole flux tubes identifiable as electropions would be present in atomic physics and define pairings of atomic electrons making it possible to understand the stability of many-electron atoms classically. Valence bonds and other chemical bonds could relate to these flux tube bonds. The model of hadrons in turn brings in besides pions also other mesons as flux tube bonds in hadrons.

p-Adic fractality inspires several questions. Could the scaled down variants of kaon and other mesons, even strange and charmed, appear in nuclear physics and define low energy excitations of nuclei? Could even scaled down leptopions with long enough p-adic length scale appear as bonds between atomic electrons in atomic and molecular physics? Could valence bond and pairing of valence electrons reduce to monopole flux tube identifiable as leptopion? Could even electrons appear with several p-adic length scales and could the heavy electrons of condensed matter physics actually be p-adically scaled up variants of ordinary electrons?

One ends up with a proposal for a unification of strong and weak interactions in terms of topological geometrodynamics. In hadronic phase the color gauge interactions are absent by color confinement. In hadronic collisions the collision energy allows to increase the value of h to h_{eff} for quarks, which become dark. This reduces the value of color coupling strength and guarantees the convergence of the perturbation theory.

The mathematically oriented reader has certainly noticed that I have talked only about Platonic solids. They have only one kind of face and their vertices are related by a discrete subgroup of the rotation group. The vertices of 13 Archimedean solids (see this) are related in the same way but they have 2 or 3 kinds of faces. Symmetries are broken in the sense that the symmetry group is transitive only in subsets of similar faces.

As already "Archimedean" suggests, Archimedean solids are obtained from Platonic solids in some way. 5 Archimedean solids are obtained from Platonic solids by truncation. If k faces meet at the vertex, the cutting of k-polygon creates a k-face. By appropriately choosing the size of the k-polygon, the edge lengths are constant so that one has an Archimedean solid. For instance, Buckminsterfullerene is a truncated icosahedron having 12 pentagonal faces and 20 hexagonal faces: similar faces have no common edges. $(V, E) = (60, 30)$ implies that the number of free edges for the Hamiltonian cycle is 30.

Could also Archimedean solids have a role, at least in the nuclear physics context where highly deformed high spin nuclei appear, in the description of large l states which in general do not allow Platonic solids as a description? Indeed, the edges of Archimedean solids have the same length and therefore they could correspond to dynamical equilibria if the tension for the edges determines the equilibrium. One can define Hamiltonian cycles for Archimedean solids and this gives V Hamiltonian edges and $E - V$ free edges and the proposed Platonization could have an Archimedean variant. The question whether Archimedean solids might have application to nuclear physics must be left open. The second, to me more attractive, option is that the value of h_{eff} increases so that also the unit of angular momentum increases for some Platonic solid appearing in the nucleus.

There are several challenges to be met.

1. The first challenge is to test whether the structure of the periodic table can be identified for protons *resp.* neutrons with a period of 8 protons *resp.* neutrons. Is it possible to identify the equivalents of l-blocks s, p-,d-,f-,..? Do full shells correspond to stable nuclides (analogous to noble gases). In the periodic table, the chemical properties of an element are determined by its valence electrons. Now, the valence protons would be in the place of the valence electrons. When the value of Z is increased, l-blocks should be observed, i.e. a periodic 2+6 structure, repeating 10, 14,... This should be the case based on the standard model.

2. The periodicity for the periodic table was derived chemically from the chemical behavior of elements. This should be true for nuclei as well.

   For instance, the scattering cross sections of nucleons from the full Platonic solids should be small in analogy to chemical inertness with noble gases. Non-full shells could have a nucleus that is reactive just like metals in the periodic table (the first 2 elements of the row) and at the other end of the row have a nuclei with a few missing protons or neutrons as analogues of elements such as chlorine, F, phosphorus, oxygen. A large cross section, for example, a proton-nucleus scattering would be a signature, and it might be that it could be found in the tables.

3. A further challenge is the construction of a concrete quantitative model for the proposed view of nuclear strong interactions as dark weak interactions. This includes also the testing of the proposed model for mesons and baryons based on the generalization of the model of nuclei.

REFERENCES

Mathematics

[A1] Butin F. Branching Law for the Finite Subgroups of $SL(4, C)$. *Ukrainian Mathematical Journal*, 67:1484–1497, 2016. Available at: `https://link.springer.com/article/10.1007/s11253-016-1167-8`.

[A2] Butin F and Perets GS. Branching law for finite subgroups of $SL(3, C)$ and McKay correspondence. *Journal of Group Theory, De Gruyter*, 17(2):191–251, 2013. Available at: `https://hal.archives-ouvertes.fr/hal-00412643/document`.

[A3] McKay J. Cartan matrices, finite groups of quaternions, and kleinian singularities. *Proc AMS*, 1981. Available at: `https://tinyurl.com/ydygjgge`.

[A4] Reid M. McKay correspondence. Available at: `https://arxiv.org/abs/alg-geom/9702016`.

Particle and Nuclear Physics

[C1] Kervran CL. *Biological transmutations, and their applications in chemistry, physics, biology, ecology, medicine, nutrition, agriculture, geology.* Swan House Publishing Co., 1972.

[C2] Lobashev VM et al. In Maalampi J Enqvist K, Huitu K, editor, *Neutrino 96*, Singapore, 1996. Word Scientific.

[C3] Foster SSD. The Chalk groundwater tritium anomaly — A possible explanation. *Journal of Hydrology*, 25(1–2):159–165, 1975. Available at: `https://www.sciencedirect.com/science/article/abs/pii/0022169475900451`.

[C4] Bird C Tompkins P. *The secret life of plants.* Harper & Row, New York, 1973.

Books related to TGD

[K1] Pitkänen M. Construction of elementary particle vacuum functionals. In *p-Adic Physics*. Available at: https://tgdtheory.fi/pdfpool/elvafu.pdf, 2006.

[K2] Pitkänen M. Massless states and particle massivation. In *p-Adic Physics*. Available at: https://tgdtheory.fi/pdfpool/mless.pdf, 2006.

[K3] Pitkänen M. New Particle Physics Predicted by TGD: Part I. In *p-Adic Physics*. Available at: https://tgdtheory.fi/pdfpool/mass4.pdf, 2006.

[K4] Pitkänen M. New Particle Physics Predicted by TGD: Part II. In *p-Adic Physics*. Available at: https://tgdtheory.fi/pdfpool/mass5.pdf, 2006.

[K5] Pitkänen M. p-Adic Particle Massivation: Hadron Masses. In *p-Adic Length Scale Hypothesis and Dark Matter Hierarchy*. Available at: https://tgdtheory.fi/pdfpool/mass3.pdf, 2006.

[K6] Pitkänen M. Cold Fusion Again. In *Hyper-finite Factors and Dark Matter Hierarchy: Part II*. Available at: https://tgdtheory.fi/pdfpool/coldfusionagain.pdf, 2019.

[K7] Pitkänen M. Criticality and dark matter: part III. In *Hyper-finite Factors and Dark Matter Hierarchy: Part I*. Available at: https://tgdtheory.fi/pdfpool/qcritdark3.pdf, 2019.

[K8] Pitkänen M. Evolution of Ideas about Hyper-finite Factors in TGD. In *Hyper-finite Factors and Dark Matter Hierarchy: Part II*. Available at: https://tgdtheory.fi/pdfpool/vNeumannnew.pdf, 2019.

[K9] Pitkänen M. Nuclear String Hypothesis. In *Hyper-finite Factors and Dark Matter Hierarchy: Part II*. Available at: https://tgdtheory.fi/pdfpool/nuclstring.pdf, 2019.

[K10] Pitkänen M. TGD and Nuclear Physics. In *Hyper-finite Factors and Dark Matter Hierarchy: Part II*. Available at: https://tgdtheory.fi/pdfpool/padnucl.pdf, 2019.

[K11] Pitkänen M. TGD view about McKay Correspondence, ADE Hierarchy, Inclusions of Hyperfinite Factors, $M^8 - H$ Duality, SUSY, and Twistors. In *Towards M-Matrix: Part II*. Available at: https://tgdtheory.fi/pdfpool/MCKaygeneral.pdf, 2019.

[K12] Pitkänen M. The classical part of the twistor story. In *Towards M-Matrix: Part II*. Available at: https://tgdtheory.fi/pdfpool/twistorstory.pdf, 2019.

[K13] Pitkänen M. The Recent Status of Lepto-hadron Hypothesis. In *Hyper-finite Factors and Dark Matter Hierarchy: Part II*. Available at: https://tgdtheory.fi/pdfpool/leptc.pdf, 2019.

[K14] Pitkänen M. Was von Neumann Right After All? In *Hyper-finite Factors and Dark Matter Hierarchy: Part I*. Available at: https://tgdtheory.fi/pdfpool/vNeumann.pdf, 2019.

Articles about TGD

[L1] Pitkänen M. Geometric theory of harmony. Available at: https://tgdtheory.fi/public_html/articles/harmonytheory.pdf., 2014.

[L2] Pitkänen M. How the hierarchy of Planck constants might relate to the almost vacuum degeneracy for twistor lift of TGD? Available at: https://tgdtheory.fi/public_html/articles/hgrtwistor.pdf., 2016.

[L3] Pitkänen M. Cold fusion, low energy nuclear reactions, or dark nuclear synthesis? Available at: https://tgdtheory.fi/public_html/articles/krivit.pdf., 2017.

[L4] Pitkänen M. Could McKay correspondence generalize in TGD framework? Available at: https://tgdtheory.fi/public_html/articles/McKay.pdf., 2017.

[L5] Pitkänen M. Questions about twistor lift of TGD. Available at: https://tgdtheory.fi/public_html/articles/twistquestions.pdf., 2017.

[L6] Pitkänen M. Solar Metallicity Problem from TGD Perspective. Available at: https://tgdtheory.fi/public_html/articles/darkcore.pdf., 2019.

[L7] Pitkänen M. TGD view about McKay Correspondence, ADE Hierarchy, and Inclusions of Hyperfinite Factors. Available at: https://tgdtheory.fi/public_html/articles/McKay.pdf., 2019.

[L8] Pitkänen M. Twistors in TGD. Available at: https://tgdtheory.fi/public_html/articles/twistorTGD.pdf., 2019.

[L9] Pitkänen M. Could TGD provide new solutions to the energy problem? Available at: https://tgdtheory.fi/public_html/articles/proposal.pdf., 2020.

[L10] Pitkänen M. How to compose beautiful music of light in bio-harmony? https://tgdtheory.fi/public_html/articles/bioharmony2020.pdf., 2020.

[L11] Pitkänen M. Can one regard leptons as effectively local 3-quark composites? https://tgdtheory.fi/public_html/articles/leptoDelta.pdf., 2021.

[L12] Pitkänen M. Is genetic code part of fundamental physics in TGD framework? Available at: https://tgdtheory.fi/public_html/articles/TIH.pdf., 2021.

[L13] Pitkänen M. TGD and Condensed Matter. https://tgdtheory.fi/public_html/articles/TGDcondmatshort.pdf., 2021.

[L14] Pitkänen M. TGD and Quantum Hydrodynamics. https://tgdtheory.fi/public_html/articles/TGDhydro.pdf., 2021.

[L15] Pitkänen M. About TGD counterparts of twistor amplitudes: part I. https://tgdtheory.fi/public_html/articles/twisttgd1.pdf., 2022.

[L16] Pitkänen M. About TGD counterparts of twistor amplitudes: part II. https://tgdtheory.fi/public_html/articles/twisttgd2.pdf., 2022.

[L17] Pitkänen M. McKay Correspondence from Quantum Arithmetics Replacing Sum and Product with Direct Sum and Tensor Product? . https://tgdtheory.fi/public_html/articles/McKayGal.pdf., 2022.

[L18] Pitkänen M. TGD view about water memory and the notion of morphogenetic field . https://tgdtheory.fi/public_html/articles/watermorpho.pdf., 2022.

[L19] Pitkänen M. About tessellations in hyperbolic 3-space and their relation to the genetic code . https://tgdtheory.fi/public_html/articles/tessellationH3.pdf., 2023.

[L20] Pitkänen M. About Platonization of Nuclear String Model and of Model of Atoms. https://tgdtheory.fi/public_html/articles/nuclatomplato.pdf., 2023.

[L21] Pitkänen M. About the TGD based views of family replication phenomenon and color confinement. https://tgdtheory.fi/public_html/articles/emuanomaly.pdf., 2023.

[L22] Pitkänen M. Magnetic Bubbles in TGD Universe: Part I. https://tgdtheory.fi/public_html/articles/magnbubble1.pdf., 2023.

[L23] Pitkänen M. Magnetic Bubbles in TGD Universe: Part II. https://tgdtheory.fi/public_html/articles/magnbubble2.pdf., 2023.