Modified Dirac equation and the holography=holomorphy hypothesis

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Abstract

The understanding of the modified equation as a generalization of the massless Dirac equation for the induced spinors of the space-time surface X^4 is far from complete. It is however clear that the modified Dirac equation is necessary.

Two problems should be solved.

1. It is necessary to find out whether the modified Dirac equation follows from the generalized holomorphy alone. The dynamics of the space-time surface is trivialized into the dynamics of the minimal surface thanks to the generalized holomorphy and is universal in the sense that the details of the action are only visible at singularities which define the topological particle vertices. Could holomorphy solve also the modified Dirac equation? The modified gamma matrices depend on the action: could the modified Dirac equation fix the modified gamma matrices and thus also the action or does not universality hold true also for the modified Dirac action?
2. The induction of the second quantized spinor field of H on the space-time surface means only the restriction of the induced spinor field to X^4. This determines the fermionic propagators as H-propagators restricted to X^4. The induced spinor field can be expressed as a superposition of the modes associated with X^4. The modes should satisfy the modified Dirac equation, which should reduce to purely algebraic conditions as in the 2-D case. Is this possible without additional conditions that might fix the action principle? Or is this possible only at lower-dimensional surfaces such as string world sheets?

In this article a proposal for how to meet these challenges is proposed and a holomorphic solution ansatz for the modified Dirac equation is discussed in detail.

Contents

1 Introduction

The understanding of the modified equation as a generalization of the massless Dirac equation for the induced spinors of the space-time surface X^4 [K1, K2] is far from complete. It is however clear that the modified Dirac equation is necessary [L2] and its failure at singularities, analogous to the failure of minimal surface property at them, leads to an identification of fundamental interaction vertices as 2-vertices for the creation of fermion pair in the induced classical electroweak gauge fields.

These singularities are lower-dimensional surfaces are related to the 4-D exotic diffeomorphic structures [A1, A2] and are discussed from the point of view of TGD in [L1]. They can be interpreted as defects of the standard diffeomorphic structure and mean that in the TGD framework particle creation is possible only in dimension $D = 4$.

A fermion-antifermion pair as a topological objects can be said to be created at these singularities. The creation of particles, in the sense that the fermion and antifermion numbers (boson are identified as fermion-antifermion bound states in TGD) are not preserved separately, is only possible in dimension 4, where exotic differentiable structures are possible.

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1. It is necessary to find out whether the modified Dirac equation follows from the generalized holomorphy alone. The dynamics of the space-time surface is trivialized into the dynamics of the minimal surface thanks to the generalized holomorphy and is universal in the sense that the details of the action are only visible at singularities which define the topological particle vertices. Could holomorphy solve also the modified Dirac equation? The modified gamma matrices depend on the action: could the modified Dirac equation fix the modified gamma matrices and thus also the action or does not universality hold true also for the modified Dirac action?

   (a) Let's consider Dirac's equation in M^2 as a simplified example. Denote the light like coordinates (u, v) by $(z, \overline{z})$. The massless Dirac equation reduces to an algebraic condition if the modes are proportional to z^n or $\overline{z}^n$. $\gamma^z \partial_z$ *resp.* $\gamma^{\overline{z}} \partial_{\overline{z}}$ annihilates such a mode if γ^z *resp.* $\gamma^{\overline{z}}$ annihilates the mode.

   (b) These conditions must be generalized to the case of a 4-D space-time surface X^4. Now the complex and Kähler structure are 4-dimensional and holomorphy generalizes. γ^z is generalized to modified gammas Γ^{z_i}, determined by the action principle, which is general coordinate invariant and constructible in terms of the induced geometry. Modified gamma matrices $\Gamma^\alpha = \gamma^k T^\alpha_k$, $T^\alpha_k = \partial L / \partial(\partial_\alpha h^k)$ are contractions of the gamma matrices of H with the canonical impulse currents T^α_k determined by the action density L.

   Irrespective of action, field equations for the space-time surface reduce to the equations of a minimal surface, and are solved by the generalized holomorphy [L4]. The lower-dimensional singularities, at which the minimal surface equations fail, correspond to defects of the standard diffeomorphic structure and are analogs of poles and cuts to analytic functions [L1].

2. The induction of the second quantized spinor field of H on the space-time surface means only the restriction of the induced spinor field to X^4. This determines the fermionic propagators as H-propagators restricted to X^4. The induced spinor field can be expressed as a superposition of the modes associated with X^4. The modes should satisfy the modified Dirac equation, which should reduce by the generalized holomorphy to purely algebraic conditions as in the 2-D case. Is this possible without additional conditions that might fix the action principle? Or is this possible only at lower-dimensional surfaces such as string world sheets?

2 How to meet the challenges

This section begins with an optimistic view of the solution of the problems followed by a critical discussion and detailed proposal for how the generalized holography would solve the modified Dirac equation.

2.1 Optimistic view of how holomorphy solves the modified Dirac equation

Consider first the notations: the coordinates for the 4-surface X^4 are the light-like coordinate pair (u, v) and the complex coordinate pair $(z, \overline{z})$. To simplify the notation, we take the notation $(u, v) \equiv (z_1, \overline{z}_1)$ for the light-like coordinate pair (u, v), so that the coordinates of the space-time surface can be denoted by (z_1, z_2) and $(\overline{z}_1, \overline{z}_2)$. As far as algebra is considered, one can consider E^4 instead of M^4, from which Minkowski's version is obtained by continuing analytically.

1. Let us optimistically assume that the H spinor modes can be expressed as superpositions of conformal X^4 spinor modes, which in their simplest form are products of powers of two "complex" variables $z_i^{n_i}$ or $\overline{z}_i^{n_i}$. Only four different types of modes: $z_1^{n_1} z_2^{n_2}$, $\overline{z}_1^{n_1} z_2^{n_2}$, $z_1^{n_1} \overline{z}_2^{n_2}$ and $\overline{z}_1^{n_1} \overline{z_2}^{n_2}$ should appear.

   The spinor modes of H are plane waves if M^4 has no Kähler structure. Could this mean that the modes can be expressed as products of exponentials $exp(ik_i z_i), exp(ik_i \overline{z}_i)$, $i = 1, 2$. More general analytical functions and their complex conjugates can also be thought of as building blocks of modes. In some cases, the complex coordinate of CP_2 comes into question as well as the complex coordinate of the homologous geodesic sphere.

2. The fermionic oscillator operators associated with X^4 are linear combinations of contributions from different H modes. They satisfy anticommutation relations. It is not clear whether the creation (annihilation) operators for X^4 spinor modes are sums of only creation (annihilation) operators for H spinor modes or wheter for instance sums of the fermion creation operator and the antifermion annihilation operator apppear.

2.2 Objections

Consider now the objections against the optimistic view.

1. Also non-holomorphic modes involving $z_i^{n_1} \overline{z_i}^{n_2}$ could be present and in this case both Γ^{z_i} and $\Gamma^{\overline{z}_i}$ should annihilate the mode. This is not possible unless the metric is degenerate.

2. The spinor modes of CP_2 could make the 4-D holomorphy impossible in the proposed sense. The spinor modes of CP_2 are not holomorphic with respect to the complex coordinates of CP_2 and only the covariantly constant right-handed neutrino satisfies massless Dirac equation in CP_2. Could this imply the presence of X^4 spinor modes, which are not holomorphic (antiholomorphic) with respect to the given coordinate z_i ($\overline{z}_i$) so that the modes involving $z_i^m \overline{z}_i^n$ are possible?

3. The general plane wave basis for M^4 without Kähler form in the transversal degrees of freedom is not consistent with the conformal invariance. Here the sum over this kind of modes should give vanishing non-holomorphic modes.

   Note that the Kähler structure for M^4 adds to the M^4 Dirac equation of H a coupling to the Kähler gauge potential of M^4 and implies a transversal mass squared so that the transversal basis does not consist of plane waves but is an analog of harmonic oscillator basis. Also now the failure of holomorphy takes place.

4. For the massive modes of CP_2 spinors, massivation takes place in M^4 degrees of freedom. This would suggest that the plane waves in longitudinal M^4 degrees of freedom cannot be massless.

   However, $M^8 - H$ duality implies an important difference between TGD and ordinary field theories. The choice of $M^4 \subset M^8$ is not unique and since particles are massless at the level of H one can always choose $M^4 \supset CD$ in such a way that the momentum has only M^4 component and is massless in M^4 sense. Could the holomorphy at the space-time level be seen as the $M^8 - H$ dual of this at the space-time level?

2.3 How could one overcome the objections?

One can consider two ways to overcome these objections.

1. The sum of the contributions of products of M^4 plane waves and CP_2 spinor harmonics is involved and could simply vanish for the non-holomorphic modes. This would look like a mathematical miracle transforming the symmetry under the isometries of H to a conformal symmetry at the level of X^4. This mechanism would not depend on the choice of action although the modified Dirac equation might hold only for a unique action.

2. The 4-D conformal invariance for fermions could degenerate to its 2-D version so that only the modified Dirac equation at 2-D string world sheets would allow conformal modes. Indeed, a longstanding question has been whether this is the case for physical reasons. The restriction of the induced spinors to 2-D string world sheets is consistent with the recent view of scattering amplitudes in which the boundaries of string world sheets at the light-like orbits of partonic 2-surfaces, which are metrically 2-dimensional, carry point-like fermions. If this is really true, then the 4-D conformal invariance would effectively reduce to ordinary conformal invariance.

2.4 Solution of the modified Dirac equations assuming the generalized holomorphy

Consider now the solution of the modified Dirac equation assuming that only holomorphic modes are present.

1. The modified Dirac equation reads a

$$(\Gamma^{z_i}D_{z_i} + \Gamma^{\overline{z}_i}D_{\overline{z}_i})\Psi = 0 \ .$$

   Γ matrices are modified gamma matrices. D_{z_i} denotes covariant derivative. Generalized conformal invariance produces the equations of the minimal surface almost independently of the action. It is however not clear whether in the modified Dirac equation the modified gammas can be replaced by the induced gamma matrices $\Gamma^\alpha = \gamma_k \partial_\alpha h^k$ (action as 4-volume). At least at the singularities that determine the vertices, this does not apply [L2].

2. The solution of the modified Dirac equation should reduce to the generalized holomorphy. This is achieved if one of the operators $D_{\overline{z}_i}$, D_{z_i}, Γ^{z_i}, $\Gamma^{\overline{z}_i}$ annihilates the given mode on the space-time surface. It follows that $\Gamma^{z_i}D_{z_i}$ and $\Gamma^{\overline{z}_i}D_{\overline{z}_i}$ for each index separately annihilate the spinor modes. Either Γ^{z_i} ($\Gamma^{\overline{z}_i}$) or D_{z_i} ($D_{\overline{z}_i}$) would do this.

   Two gamma matrices in the set $\{\Gamma^{z_i}, \Gamma^{\overline{z}_i} | i = 1, 2\}$ must eliminate a given X^4 spinor mode. Since modified gammas depend on the action, this condition might fix the action.

3. There are two cases to consider. The generalized complex structure of the 4-surface X^4 is induced from that of H [L4] or if the space-time surface is a product of Lagrange manifolds $X^2 \times Y^2 \subset M^4 \times CP_2$, is induced from the the complex structures of the 2-D factors associated with their induced metrics [L5].

4. I have proposed that M^4 allows several generalized Kähler structures, which I have called Hamilton-Jacobi structures [L3]. The 4-surface could fix the Hamilton-Jacobi structure from the condition that the modified Dirac equation is valid. Since the modified gammas depend on the action, the annihilation conditions for the modified gamma matrices might fix the choice of the action, and this choice could correlate with the generalized complex structure of X^4.

To sum up, the above considerations are only an attempt to clarify the situation and it is not at all obvious that the generalized holomorphy trivializes the solution of the modified Dirac action.

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