Contents

1 How TGD evolved

In an earlier blog discussion Hamed asked about some kind of program for learning TGD in roughly the same manner as I did it myself. I decided to write a brief summary about the basic steps leaving aside the worst side tracks since 35 years means too flat learning curve;-).

I wrote a summary about the very first steps, that is the steps made during the four years before my thesis and related to classical dynamics mostly. I could not avoid mentioning and even briefly explaining notions like the "world of classical worlds" (WCW), quantum TGD, Kähler action, modified Dirac equation, zero energy ontology, etc... since I want to relate the problems that I encountered during the first years of TGD to their solutions which came much later, some of them even during this year. I hope that I find time to write similar summaries about later stages in the evolution of TGD and add them to this text.

This summary does not provide any Golden Road to TGD. It is much more difficult to assimilate ideas of others than to discover and develop ideas by one self. The authority of the original discoverer - such as that of Witten's- helps enormously but I do not possess this kind of authority so that I must trust only on the force of the ideas themselves.

1.1 Basic question and its answer

The basic motivation came from the energy problem of general relativity (GRT). Noether's theorem relates classical conservation laws (conservation of energy, momentum, and angular momentum) to Poincare symmetries of 4-D Minkowski space M^4.

In general relativity (GRT) M^4 is replaced with a curved space time and all these conservation laws as lost in the generic case. One cannot even define the quantities, which would be conserved for M^4. Conservation laws are replaced by a purely local statement saying that energy momentum tensor has a vanishing local divergence: $D_beta T^{\alpha\beta} = 0$. This is true if Einstein's equations $T^{\alpha\beta} = kG^{\alpha\beta}$ are satisfied. This local character is however in conflict with Uncertainty Principle requiring that momenta are non-local quantities defined as integrals over 3-space.

Unless gravitational constant were so small, this would have been regarded as a catastrophe but since perturbation theory around flat M^4 is possible classically (but not quantum mechanically as we know!), the situation has been accepted. In my youthful idealism I could not follow the flock and asked whether it might be possible to have a theory in which gravitation is geometrized as in Einstein's theory but without losing Poincare symmetries?

The answer to the question came at the moment I made it. Assume that space-times are a 4-D surfaces in some higher-dimensional space $M^4 \times S$ and that the action principle giving this surface as a solution of field equations is invariant under Poincare symmetries assignable to M^4 factor. The *absolutely crucial* difference to standard approach is that the symmetries are not isometries of the space-time itself but those of imbedding space. Instead of moving a point along space-time surface isometries move the entire space-time surface in imbedding space.

There are also symmetries assignable to the factor S for which my first guess was that it is just 2-D sphere S^2 with rotation group $SO(3)$ as symmetries. First I tried to interpret $SO(3)$ as a counterpart of color group with quarks identified as 3-D representation of SO(3). This was of course wrong.

1.2 First things to learn

What I had to do after this first step.

1. I had to learn what Riemannian geometry and sub-manifold geometry are. Here I found the "Riemannian Geometry" of Einsenhart published 1964 very useful. It was an old-fashioned mathematics book written with a non-Bourbakian purpose that some-one in this planet could indeed read it.

   The book discussed the notion of induced metric and deduced expressions for the curvature tensor, Ricci tensor, and curvature scalar in terms of the induced metric. This is all still standard Riemann geometry: it does not matter whether the surface is imbedded to a higher-dimensional space or not. Surfaces have however additional geometry: the shape as seen by the inhabitant of the higher dimensional space. This makes the sub-manifolds and this is something which must distinguish TGD from GRT. The books discusses also some of these aspects.

   Unfortunately Eisenhart's book is not enough. There are also other aspects, such as induction of the spinor structure of the imbedding space ("square root" of metric structure), which are overall important but about which I have not found mention in literature. For some mysterious reason string model builders favor monsters like heterotic string and refuse to realize the importance of the purely geometric notion of induced spinor structure although it follows by straightforwardly applying bundle theoretic thinking: if you have a bundle structure (say spinor structure) in higher-D space, you can induced it to sub-manifold. By the way, I made my graduate work in mathematics about the induction of spinor structure to Lounesto, who is a well-known spinor guru.

2. Second fundamental form $H^k_{\alpha\beta} = D_\beta \partial_\alpha h^k$ consisting of covariant derivatives of the tangent space vectors of sub-manifold is the basic characterizer of local "external aspects" of sub-manifold geometry.

   (a) Second fundamental form is space-time tensor and vector of imbedding space. $H^k_{\alpha\beta}$ depends on second derivatives of the imbedding space coordinates.

   (b) The trace $H^k \equiv ga^{\alpha\beta} H^k_{\alpha\beta}$ of the second fundamental form defines a space-time scalar and imbedding space vector field restricted to space-time surface. For curves of higher dimensional space $H^k \equiv ga^{\alpha\beta} H^k_{\alpha\beta}$ has interpretation as a generalization of acceleration vector vanishing for geodesic lines. This interpretation makes sense also for higher-dimensional surfaces. Another interpretation for $H^k = 0$ is as a non-linear generalization of massless d'Alembert equation. Both particle and wave aspects are involved. In 2-D case this states that the sum of *extermal curvatures* assignable to two orthogonal lines vanishes. The surface looks like saddle locally.

   (c) Geodesic sub-manifolds are also a central notion. For them one has $H^k_{\alpha\beta} = 0$: tangent vectors defined by the partial derivatives of imbedding space coordinates are covariantly constants. This states that sub-manifold is the analog of geodesic line in strong sense. All geodesics of geodesic sub-manifold are geodesics of imbedding space.

   In the case of $M^4 \times CP_2$ the geodesic spheres S^2 of CP_2 are of special importance since they define sub-theory in the sense that one can assume that space-time surface belongs to $M^4 \times S^2$: field equations respect this restriction. CP_2 has two different kinds of geodesic spheres: those which are homologically trivial and those which are not.

3. Another important aspect of the induced geometry are isometries of the imbedding space. They are indeed enormously important in TGD framework. I do not remember how much attention Eisenhart devoted to this aspect.

4. Bringing in the notion of symmetries means also bringing in variational principles. Minimization of the surface volume in the induced metric is the simplest variational principle and Eisenhart must have discussed it. Once one assumes general coordinate invariant (GCI) variational principle and decomposition $H = M^4 \times S^2$ one obtains conservation laws (Poincare symmetry for M^4 and $SO(3)$ for S^2. Field equations reduce to hydrodynamical equations in the sense that they state conservation laws. On the other hand, for minimal surfaces these equations reduce to non-linear wave equations.

5. Being sub-manifold has also its *global* aspects. Seeing space-time from imbedding space level eventually led to the notion of many-sheeted space-time, the identification of physical objects as

space-time sheets glued to larger ones by topological condensation, to the notion of wormhole contact identified as Euclidian region of space-time in induced metric, the notion of wormhole throat, tgeneralized Feynman diagram, magnetic/field body, etc.. The implications for the world view are really revolutionary and form the basics of all applications of TGD. Without the understanding of these simple concepts TGD remains incomprehensible.

For instance, the recent picture about elementary particle is as a monopole flux tube obtained from parallel flux tubes at parallel space-time sheets by connecting their ends by wormhole contacts relies on these "external aspects" of sub-manifold geometry and is essential for understanding particle massivation in p-adic thermodynamics framework and its relationship to Higgs mechanism providing QFT mimicry of this description. I ended up to thi concept by a stepwise real and error procedure.

What one should learn to proceed further.

1. Eisenhart or some other old-fashioned book emphasziing the geometric aspects of geometry would be extremely helpful for a physicist wanting to understand what is involved. There is a fantastic Gentle Giant by Spivak called "Differential Geometry I,II,III,IV" published in 1970 Publish or Perish. This tells everything that one might be able to imagine about differential geometry and also about sub-manifold geometry. The cover pages of the books are also entertaining.

2. One should also learn Noether's theorem and learn to apply it in the case of sub-manifold geometry. The deduction of conserved currents is a completely mechanical procedure but one must have it in spine to do anything practical. Any text book in physics gives the needed skills. Deduction of the minimal surface equations of string model could be a good starting exercise. Everything boils dow to the identification of basic dynamical field variables, expression of them in terms of imbedding space coordinates and their gradients, application of chain rule to obtain the variation of action, and transformation total gradients to boundary terms to obtain field equations. Noether's theorem follows in a similar manner.

3. One should also learn the basic ideas about Lie groups and their realization as isometries of manifolds (now imbedding space) inducing the conservation laws identifiable as field equations when imbedding space has the highly symmetric form $H = M^4 \times S$, S symmetric space. This aspect is present also in string models but after Polyakov introduced the approach in which the 2-metric at string world sheet is independent dynamical quantity and co-incides with induced metric only for the minimal surfaces, it was forgotten, which is a great pity. I would be surprised if Eisenhart's and Spivak's book would not tell anything about Killing vectors. In any case, some text book about Lie groups with strong emphasis on geometric and physics applications would be very helpful.

1.3 What could be the classical variational principle determining space-time dynamics?

The first thing to do was to study various classical variational principles. General coordinate invariance (GCI) was the obvious condition on them. The natural first guess was that they involve only induced geometry in Eisenhart's sense.

1. Space-time volume would be the naivest guess and string model builder would make it routinely. The problem is however that the volume tends to be infinite if one assumes infinitely large space-time surfaces. Also vacuum functional - one expects something like exponent of action is expected to exclude all but small enough space-time surfaces - by exponential supression in Euclidian context and by destructive interference in Minkowskian context. A further problem is that there is no connection to classical gauge fields defined by induced spinor connection.

2. At this period I did not have yet the alternative view idea TGD as a generalization of string model: I wanted Einstein's equations and conservation laws! Curvature scalar was therefore the obvious first guess for the action density.

(a) The idea was that this gives generalization of Einstein's equations by replacing Einstein tensor with a collection of conserved vector currents. Einstein's equations would be given by Noether's theorem: the conserved four-momentum currents would be given by

$$T^{\alpha}_{i)} = kG^{\alpha\beta}j_{i)\beta} \ \ , \ \ j_{i)\beta} = h^k_{\beta}j^l_i h_{kl} \ \ .$$

Here interpretation is obvious: $j_{i)\beta}$ is the projection of the Killing vector to the space-time surface. This space-time vector is contracted with Einstein tensor to get the conserved current. Field equations read as

$$G^{\alpha\beta}H^k_{\alpha\beta} = 0 \ \ .$$

Clearly, G replaces g in the trace of the second fundamental form. What is amusing that the recently discovered solution ansatz for the preferred extremals of Kähler action actually implies these equations so that field equations were correct but the action was wrong! Also minimal surface equations follow although also now 4-volume would define wrong action! The solution ansatz leads to a generalization of minimal surfaces equations and state generalization of conformal invariance to 4-D context. This generalized conformal invariance is behind this "action independence" of extremals.

(b) It was surprisingly easy to discover solutions of field equations. This art must be just learned: there are no text books teaching it.

For instance, assume objects of form $X^4 = M^2 \times X^2 \subset M^2 times E^2 \times S$. Field equations are trivially satisfied since Einstein tensor reduces to that of X^2 and vanishes because it does so identically in 2-D case. There is however a serious problem. The rest energy is not positive definite! By the static character of the solution the rest energy is proportional to the integral of curvature scalar over X^2 and is integer in suitable units by Gauss-Bonnet theorem. For sphere the sign of energy is correct for torus one obtains vanishing energy, and for higher genera the sign is negative. The conclusion was that the simplest guess fails despite its conceptual beauty!

(c) The surprise was that the simplest extremals of any GCI action principle actually described orbits of simple particle like objects and this led to an overall important realization: TGD can be also seen as a generalization of string models obtained by replacing string world sheet by space-time surface. Particle becomes background space-time for smaller 4-surfaces topologically condensed at it. Einstein's vision generalizes: particles reduce classically to topological inhomogenities of the space-time surface. Much later came the identification of basic building bricks of particles defining the lines of generalized Feynman diagrams as wormhole throats at which the signature of the induced metric changes. This identification is also revolutionary. For instance, it leads to a totally new view about black-hole interior (as Euclidian space-time region) and also to the conclusion that any material object involves an Euclidian space-time sheet whose size and shape define those of the object.

3. The next candidate was the YM action for the electroweak gauge fields with an additional term describing color gauge interaction, which actually reduces to Kähler action. I soon realized that this was quite too complex and inelegant and realized that Kähler action defined as a Maxwell action for the induced Kähler form of CP_2 is the uniquecjoice for the action.

(a) I believe that I realized the special role of Kähler action around about decade after the first idea about TGD: for this it was quite essential to have the idea about physics as physics of classical spinor fields in WCW ("world of classical worlds"). The identification of Kähler action for preferred extremal as Kähler function for WCW led to the idea that uniqueness of TGD is achieved if Kähler coupling strength as the analog of temperature corresponds to critical temperature. This took place around 1988. Without the discovery of WCW I would have continued to plod in the swamp of effective actions producing semiclassical arguments based on mathematically non-existing path integral over all 4-surfaces. This is indeed what became the sad fate of super string theories.

(b) The completely unique feature of Kähler action is its huge vacuum degeneracy: any 6-D sub-theory defined in $M^4 \times Y^2$, Y^2 a Lagrangian sub-manifold with vanishing Kähler form is vacuum. This observation is fundamental in attempts to understand the dynamics of quantum TGD small deformations of vacuum extremals are expected to define preferred extremals of Kähler action. Preferred extremal property roughly means that space-time surface is like Bohr orbit: this property is required in the construction of quantum TGD in "world of classical worlds" approach. 3-surface is the fundamental objects and 4-D GCI requires that the definition of the Kähler metric of WCW assigns to a 3-surface X^3 a unique four-surface $X^4(X^3)$. Quantum classical correspondence requires that this surface is a preferred extremal of some action and Kähler action is the unique choice.

(c) Kähler action involves involves besides induced metric also the induced Kähler form. They are not primary fields but expressible in terms of imbedding space coordinates and their gradients so that by GCI effectively eliminating for imbedding space coordinates only 4 classical fields are present as primary dynamical quantities. Skeptic can argue that this huge reduction in degrees of freedom is a catastrophe. Among other things one loses the superposition of classical fields. Many-sheeted space-time however solves the problems. It is not classical fields but their effects which must superpose to satisfy the experimental facts. This is true if particle topological condensed to several space-time sheets simultaneously! Also this is quite recent discovery. The intuition transforms quite slowly to precise understanding.

(d) What does "preferred extremal " really mean? The first guess was "absolute minimum of Kähler action". This however led to problems in the p-adicization of the theory since p-adic numbers are not well-ordered and one cannot speak about absolute minimum. The definition of "preferred" must be purely algebraic and cannot involve topological notion such as minimization. About 35 years later (this year!) I discovered that if the preferred extremals of Kähler action obey generalization of field equations for string world sheets guaranteeing generalization of conformal structure to 4-D case and that of conformal invariance, Einstein's equations follow as a consistency condition implying that the energy momentum tensor for Kähler action has a vanishing covariant divergence! Einstein's original argument for Einstein's equations as equations guaranteeing local variant of conservation laws is transformed to a consistency condition with the same implication.

Another implication is that space-time surfaces satisfy also minimal surface equations although variational principle is different. Minimal surface equations mean that solutions of non-linear massless wave equations are in question and also that classical one can regarded space-time surfaces as 4-D generalization of geodesics (generalization of lines of Feynman diagram!). The lesson is that both Einstein's equations and minimal surface equations are something much more general than equations derivable from some action principle: they are manifestations of generalized conformal invariance.

To sum up, what I had learned hitherto was following.

1. TGD can be seen as sub-manifold gravity or as a generalization of string models. I soon realized that the conformal invariance of string models must have TGD counterpart and the challenge was to discover it. This took quite a lot of time. But already at this stage it was clear that it would be good to learn the basic ideas about conformal symmetry. Here again articles or books written by physicists can help the novice.

2. Particle can be seen as a topological inhomogenuity of space time.Much later I was to learn that it can be seen also as a singularity of induced metric, namely as a locus of signature change and that the dogma about everywhere Minkowskian signature of space-time must be given up.

3. I must develop a more general view about induced geometry and I must do this by myself.

1.4 Generalizing the notion of induced geometry

The next step after the failure of the approach based on curvature scalar required something more that text books could provide. I had to generalize the notion of induced geometry. I wanted also spinors as spinors of the imbedding space. I had to induce also spinor connection of $M^2 \times S$ reducing to that of S. The components of the induced spinor connection would have interpretation as gauge potentials and yhr projections of spinor curvature would give gauge fields at space-time surface. Therefore a geometrization of gauge field concept would result automatically. This was a total surprise and very positive such. I realized that sub-manifold gravity as generalization of string models could do what Einstein tried for the rest of his life in vain.

As I already said the induction procedure is completely standard procedure in bundle theory used routinely but for some reason physicists have not realized its fundamental importance. Here some text book about bundle theory should be enough: the needed basic wisdom can be probably found from first 30 pages or so. The procedure as such is boringly simple: they key world is project. If you have a tensor at imbedding space level, project it to a tensor at space-time level by contracting it with the tangent vectors defined by the gradients of imbedding space coordinates.

You can project also gamma matrices of H and this gives rise to a generalization of spinor structure for which gamma matrices are not covariantly constant anymore. Internal consistency however requires that gamma matrices define a vector which is divergenceless ($D_\mu \Gamma^\mu = 0$). This means minimal surface property. In the case of Kähler action, one must use modified gamma matrices obtained by contracting the partial derivatives $\partial L/\partial(\partial_\alpha h^k)$ of action density with imbedding space gamma matrices Γ_k. This unexpected connection between fermionic and bosonic variational principles reflects supersymmetry in the sense that infinite number of conserved super currents exists.

These considerations led to the realization that $M^4 \times S^2$ is not enough. For S^2 the spinor connection is Abelian. Electromagnetic field is certainly not enough: at least electroweak gauge fields are needed. The fact that there is no spontaneous symmetry breaking in QCD suggests that the origin of classical gluons relates to the isometries of the factor S giving rise to exact conservation laws. The natural guess inspired by the Kaluza-Klein approach was that the projections of Killing vector fields of CP_2 define gluon gauge potentials. This lead to a unique candidate for S. $S = CP_2$ is the only symmetric space with Kähler structure that can be considered. It indeed gives classical induced gauge potentials with the couplings of standard model and also classical gluon fields.

The first surprise was that spinor structure of CP_2 is not well-defined unless one couples spinors to the Kähler gauge potential defining Kähler form of CP_2. Both Kähler structure and the failure of ordinary spinor structure is necessary for obtaining correct standard model couplings via Spin_c structure. This delicacy of CP_2 spinor structure was observed already by Hawking and others while studying gravitational instantons and it might be an excellent idea to read these articles. If they had realized the connection with the standard model, the landscape of particle physics would look totally different today. The development of science is not a rational process but involves stagnation periods which can last half a century.

A coupling to odd integer multiple of the Kähler gauge potential is needed to get acceptable spinor structure. It can be different for the two chiralities o f$M^4 \times CP_2$ spinors if the generalization of Dirac action does not contain mass term. This guarantees chiral invariance characterizing gauge theories and implies separate conservation of B and L. By choosing the integer multiple to be n=1 for the first chirality and n=3 for the second chirality one obtains standard model couplings for electroweak gauge potentials.

My skills in differential geometry at that time (and maybe also now;-) ere as limited as my calculational skills. My luck was that CP_2 had been discovered as gravitational instanton and I could utiize this understanding. Below are some references.

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1.5 Understanding the differences between TGD and other TOEs

I gradually understood that TGD differs from standard model, GUTs and strings in several important aspects. To understand these differences one must of course know the basics of elementary particle phenomenology. It is also good to have a very critical attitude to the prevailing GUT wisdom about elementary particles: this is just a bundle of beliefs which have not been tested experimentally, and LHC is now mercilessly killing these beliefs. Also the refusal of proton to decay (at least in the GUT-like manner) has tried for decades to tell us that GUT paradigm might be badly wrong.

1. The basic difference is that point like particle is replaced with a 3-surface (as a matter partonic 2-surface and its tangent space data by strong form of GCI). The shape and size of the particle bring in new degrees of freedom making it possible to eventually understand the massivation of particles. The reason is utterly simple: the size of the particle as 3-surface brings in length scale which in turn dictates mass scale by Uncertainty Principle. In QFT context particles are point-like and mass spectrum can be only mimicked, not predicted. Also cm degrees of freedom are present and lead to the notion of color partial wave and quite different interpretation of color symmetry meaning deviations from the QCD picture. The latest experimentally discovered deviation is the observation of correlated pairs of charged particles in ultra high energy p-p collisions at LHC strongly suggesting string like objects assignable to meson like entities. These strings cannot be however objects of low energy QCD so that the only possibly conclusion seems to be in terms of scaled up variant of hadron physics: one of the many new physics predictions of TGD.

2. A further basic difference is induced spinor structure leading to an explanation of the electroweak quantum numbers in terms of geometry of $M^4 \times CP_2$. Also classical electroweak gauge fields are geometrized.

3. The separate conservation of B and L also implies that the possibly existing TGD variant of SUSY cannot be $\mathcal{N} = 1$ SUSY requiring Majorana spinors and mixing of spinor chiralities. The recent futile search for $\mathcal{N} = 1$ SUSY has demonstrated that something is probably badly wrong in standard SUSY. Quite remarkably, Mikhail Shifman - the discoverer of SUSY symmetry - strongly emphasizes in his article in Scientific American that something is really wrong and theoreticians should finally take the message of Nature seriously. It is nice to see that research ethics has not totally disappeared during the era of unashamed science hyping. TGD indeed leads to very different realization of SUSY as a badly broken symmetry but this came much later.

4. Standard model does not explain family replication phenomenon of fermions? In TGD there is no need to force all elementary fermions to single multiplet since family replication phenomenon can be explained topologically: the genus of partonic 2-surface defines topological quantum number distinguishing between different fermion families. If this is accepted, then $M^4 \times CP_2$ is all that is needed and there is no need for GUTs.

    (a) An infinite number of fermion genera is predicted and the challenge was to understand why only the three lowest ones are observed in physics at recently available collision energies. A possible explanation in terms of so called hyper-ellipticity came much later. All 2-surfaces with genus $g \leq 2$ have Z^2 as a conformal symmetry (are hyperelliptic) but for higher genera this is not usually the case. This additional global conformal symmetry could make lower genera stable and light.

    (b) Does one obtain also bosonic genera and in what sense? The answer depends on how one identifies bosons and the probably correct interpretation of bosons in terms of fermion anti-fermion pairs assignable to the throats of wormhole contacts came much later.

    (c) Why elementary particles should be assignable to 2-D surfaces and can one choose this assignment uniquely? If this were the case one would have something which one could call strong form of holography. The progress was stepwise.

        i. My first guess was that these 2-D surface corresponds to the boundary of 3-surface assignable to particle. I was wrong.

        ii. Much later I realized that strong form of general coordinate invariance (GCI) implies a strong form of holography. If one allows both Minkowskian and Euclidian space-time regions one cannot avoid the 3-D dimensional surfaces at which the signature of the induced metric changes. These 3-surfaces are light-like.

Another obvious identification for particle like 3-surface is as a space-like 3-surface. Suppose that this space-like 3-surface can be fixed uniquely. If GCI means that both choices produce same physics, the physics must reduce to their 2-D intersection - I call it partonic 2-surface - and 4-D tangent space data at this intersection. This defines strong form of holography. The uniqueness of intersection means that one can assign to partonic 2-surface unique conformal equivalence class with turns to be central for understanding the contribution of the conformal modular degrees of freedom to particle mass. Wormhole throat as particle has wave function in the space of conformal moduli characterized the conformal equivalence classes of induced 2-metrics.

But how to fix the space-like 3-surface uniquely? Here zero energy ontology (ZEO) and causal diamonds (CD) come in rescue. Also this fails to be text book knowledge. I do not repeat what I have said about them so many times already.

(d) This was not the end of the story. The discovery of bosonic emergence made it clear that wormhole contact is required to describe bosons consisting of fermion antifermion pair with members located at the throats of the wormhole contact. The stability of the contact against splitting is guaranteed if it carries monopole magnetic flux. This flux must however form a loop so that there must second wormhole contact nearby (also second particle could in principle be in question). Particle as a monopole flux loop connecting two parallel space-time sheets is the recent view and modified Dirac equation assigns to this flux loop a closed string in X^4.

(e) Classical gluons do not couple to the induced spinor fields unlike electroweak gauge potentials and anyone used to think about gluons in terms of standard model and GUTs would say that this is catastrophe.

Here the physical intuition suggests that it is the color-rotational degrees of freedom analogous to rigid body degrees of freedom which give rise to color quantum numbers. Particles have color partial waves in center of mass degrees of freedom. Color looks only in long length scales spin like degree of freedom. What was also obvious that the color particles waves for spinors of CP_2 characterized the color quantum numbers of particle: p-adic mass calculations force to develop this picture in detail. An infinite number of color excitations are possible but their mass scale could quite well be of order CP_2 mass scale so that they need not cause worries.

The challenge was to translate this intuition to mathematics. This took some time;-). The recent view is that color partial partial waves correspond to ground states for supersymplectic representations assignable to symplectic group of $\delta M^4_\pm \times CP_2$. It is ZEO which makes the light-cone boundary naturals. These degrees of freedom are imbedding space degrees of freedom analogous to those of Kaluza-Klein type QFT in $M^4 \times CP_2$: partonic 2-surfaces have position in imbedding space.

All this requires additional learning besides understanding of the basic particle phenomenology. One must learn the notions of super-conformal symmetry, Kac-Moody symmetry, symplectic invariance and its generalization to $\delta M^4_\pm \times CP_2$. Here one cannot proceed anymore just by reading text books. One must develop mathematical concepts which do not yet exist as rigorously formulated mathematics. Luckily, the physical intuition helps here enormously.