The realization of topological qubits in many-sheeted space-time

March 15, 2025

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Abstract

Microsoft announced that it has created the first topological quantum computer utilizing topological qubits realized as condensed matter Majorana fermions. The condensed matter Majorana fermions are superpositions of fermions and holes: this breaks fermion number conservation or at least, the superselection rule for fermion number. The hole should correspond to a fermion "somewhere else". The many-sheeted space-time of TGD allows us to understand "somewhere else" as a second space-time sheet, a magnetic monopole flux tube. This leads to a model in which the Majorana Dirac equation is replaced with a description which respects fermion number conservation and super selection rule. TGD also predicts that the hierarchy of Planck constant makes topological superconductivity possible at physiological temperatures: biology would be the basic example.

TGD also leads to a generalization of the description in terms of Majorana fermions. It is based on the theoretical vision of TGD. The Galois group would serve as a generalization of the group Z_2 defining the parity of Majorana fermion. TGD predicts a 4-D variant of Galois group representing the transfers of fermions between different regions of the space-time surfaces identified in holography= holomorphy vision as roots $(f_1, f_2) = (0, 0)$ for an function pairs $H = M^4 \times CP_2 \to C^2$ analytic with respect to Hamilton-Jacobi coordinates generalizing complex coordinates. The Galois group is realized as analytic flows analogous to braidings mapping the roots to each other. The second Galois group is associated with dynamical complex analytic symmetries $g : C \to C$ $(f_1, f_2) \to (g \circ f_1, f_2)$. One can talk of number theoretic/topological n-ary digits for n-sheeted space-time surfaces. Pinary digits (n is prime) are in a well-defined sense fundamental.

Contents

1 Introduction

Microsoft has unveiled Majorana 1 (thanks to Marko Manninen for sending the link), claimed to be the world's first quantum processor powered by topological qubits [D1, D2] (see also the popular article at this).

1.1 How could one stabilize computations and qubits?

The basic problem is how to realize computations in a stable way and how to make stable enough qubits? Concerning computation, topology comes to rescue.

1. Topological quantum computations (see this) can be represented as braidings, which are topologically stable under small deformations. Each braid strand represent a unitary evolution of a particle representing a unitary evolution if a qubit and the braiding operation would represent the computation. Braiding can be either time-like dynamical operation for point-like particles in plane or space-like for a braid connects two planes.

2. Since the 2-D plane containing particles as punctures, the homotopy group is non-abelian. This means that the rotation of a puncture around a second puncture of say bound state can transform the state such that transformation is not a mere phase factor but is a rotation which change the directions of the spins of the particles involved. Therefore the exchange of particles which can be seen as basic braiding operation changing the braid strands can induce an operation, which can be used as a basic building brick for a topological quantum computation.

How could one obtain stable qubits? Qubit represented as a spin is not thermodynamically stable and extremely low temperatures are required. This is the case also for the proposed topological quantum computation: the reason is now that superconductivity is required and this is possible only at temperatures of order milli Kelvins. In any case, the notion of qubit should be topologized. How to achieve this? Here Majorana bound bound states have been proposed as an answer (see this).

1. Non-Abelian braid statistics, which means that their exchange realized as a 2-D rotational flow generated by braiding induces, instead of change of a sign in Fermi statistics, a non-Abelian unitary transformation of the state. It could be used to change the directions of their spins and affect the anyons.

   2π rotation would induce a non-Abelian rotation instead of a mere sign change or phase factor in brain statistics. This is only possible in dimension 2 where the homotopy group can be non-abelian if there are punctures in the plane that the braids would represent. Similarly, swapping two Majorana fermions in braid produces a SU(2) rotation and can flip the spins and thus the qubits. This swap would be an essential operation in quantum computing. In order to have non-trivial topological quantum computation, one must have non-Abelian braid statistics characterized by a Lie group. Rotation group SO(2) or its covering SU(2) are the minimal options

2. The bound state of two Majorana fermions associated with planar punctures, anyons, would thus obey non-Abelian braid stastistics. It is also possible to affect the second fermion of Majorana bound state by rotating a puncture containing a fermion around the second fermion. Braidings could therefore represent unitary transformations having an interpretation as topological quantum computations.

   Wikipedia article mentions several realizations of Majorana bound states in superconductors. Quantum vortices in super conductors can provide this kind of states. The ends of the superconducting wire or of line defects can contain the Majorana fermions. Also fractional Hall effect can provide this kind of states. The realization studied by Microsoft has the fermions of the Majorana fermion at the ends of a superconducting wire.

3. As I understand it, a condensed matter Majorana fermion would correspond formally to a superposition of an electron and a hole. The statistics would no longer be normal but non-Abelian Fermi statistic but would be that of a non-abelian anion.

   The weird sounding property of this statistics is that the the creation operator is equal to annihilation operator. One obtains two creation operators corresponding to two spin statse and square the creation operator of is unit operator: for fermions it vanishes. This implies that Majorana fermion number is defined only modulo 2 and only the number of fermions modulo 2 matters. Also the anticommutator of two creation operators at different points is equal to unit operator so that the system is highly nonlocal.

4. How the braiding could be realized? One can consider two options. Dance metaphor allows to understand the situation. Imagine that particles are dancers at the parquette. The dance would give rise to a time like braiding. If the feet of the dancers are tied to a wall of the dancing house by threads, also a space-like braiding is induced since the threads get tangled.

5. In the TGD framework, dancers would correspond to particle-like 3-surfaces moving in the plane and the dance would define the dancing pattern as a time-like braiding. This classical view is actually exact in the TGD framework since classical physics is an exact part of quantum physics in TGD. If thee particles are connected to the wall by threads realized as monopole flux tubes, a space-like braiding is induced.

6. These threads bring in mind the wires connecting superconductor and another object and containing Majorana fermions at its ends. Now the second end would be fixed and second would correspond to a moving particle. Majorana bound states would correspond to the ends of the thread and the superconducting flow of the second end would correspond to the dynamical braiding.

1.2 Algebraic description of Majorana fermions

The dissertation of Aran Sivagure contains a nice description of Majorana fermions (see this). Majorana fermions would be quasiparticles possible in a many-fermion state. They would create from a fermion state with N fermions a superposition of states with fermion numbers $N+1$ and $N-1$. They would be created by hermitian operators $\gamma_{n,1} = a_n^\dagger + a_n$ and $\gamma_{n,2} = i(a_n^\dagger - a_n)$ formed from the fermionic oscillator operators satisfying the standard anticommutation relations $\{a_m^\dagger, a_n\} = \delta_{m,n}$. Note that one consider also more general Hermitian operators $\gamma_{n,1} = exp(i\phi)a_n^\dagger + exp(-i\phi)a_n$ and $\gamma_{n,2} = i(exp(i\phi)a_n^\dagger - exp(-i\phi)a_n)$.

One can also form analogs of plane waves as superpositions of these operators $\gamma_{k,1} = \sum_n [exp(ikx_n)a_n^\dagger + exp(-ikx_n)a_n]/\sqrt{N}$ and $\gamma_{k,2} = i\sum_n [exp(ikx_n)a_n^\dagger - exp(-ikx_n)a_n]\sqrt{N}$. Here N is the number of lattice points and discrete Fourier analysis is used.

The anticommutations would be $\{\gamma_{i,k_1}, \gamma_{i,k_2}\} = 2 \times Id\delta_{k_1,k_2}$, $i = 1,2$ where Id denotes the unit operator. For different points $i \neq j$ the anticommutativity implies that the anticommutators vanish. Therefore the statistics are not the ordinary Bose- or Fermi statistics and non-Abelian statistics. The anticommutation relations reflect the fact that the application of the creation operators twice does not change the physical states so that the number of Majorana fermions is determined only modulo 2.

1.3 TGD description of the situation

The condensed matter Majorana fermions are superpositions of electrons and holes: this breaks fermion number conservation or at least, the superselection rule for fermion number. The hole should correspond to a fermion "somewhere else". In condensed matter, "elsewhere" could correspond to a conduction band in momentum space. The many-sheeted space-time of TGD allows us to understand "somewhere else" as a second space-time sheet, a magnetic monopole flux tube. This leads to a model in which the Majorana Dirac equation is replaced with a description which respects fermion number conservation and super selection rule. TGD also predicts that the hierarchy of Planck constant makes topological superconductivity possible at physiological temperatures: biology would be the basic example.

TGD leads to a generalization of the description in terms of Majorana fermions based on the number theoretical vision of TGD [L3, L4]. The Galois group would serve as a generalization of the group Z_2 defining the parity of Majorana fermion. Two Galois groups are possible: the internal and external Galois group.

1. TGD predicts a 4-D variant of Galois group, the internal Galois group, representing the transfers of fermions between different regions of the space-time surfaces identified in holography= holomorphy vision as roots $(f_1, f_2) = (0, 0)$ for function pairs $H = M^4 \times CP_2 \to C^2$ analytic with respect to Hamilton-Jacobi coordinates generalizing complex coordinates. The internal Galois group is realized as analytic flows analogous to braidings mapping the roots $(f_1, f_2) = (0, 0)$ to each other and having as interfaces the regions at which two or more roots co-incide.

2. The simpler version of the external Galois group, is associated with dynamical complex analytic symmetries $g : C \to C$: $(f_1, f_2) \to (g \circ (f_1, f_2)$. In this case, the Galois group relates to each other disjoint space-time surfaces. When g reduces to a map $g = (g_1, Id)C \to C$, where g_1 has no parametric dependence on f_2, one can assign to it an ordinary Galois group relating to each other the disjoint roots of $g_1 \circ f_1$, which are algebraic numbers.

   The notion of external Galois group generalizes. For the general case $g = (g_1, g_2)$, the roots of $g \circ f$ are disjoint space-time surfaces representing pairs of algebraic numbers $(f_1, f_2) = (r_{i,1}, r_{i,2})$. It is possible to assign to the roots the analog of the Galois group. This group should act as a group of automorphisms of some algebraic structure. This structure cannot be a field but algebra structure is enough. The arithmetic operations would be component-wise sum $(a, b) + (c + d) = (a + c, b + d)$ and componentwise multiplication $(a, b) * (c, d) = (ac, bd)$. The basic algebra would correspond to the points of $(x, y) \in E^2$ or rationals and the extension would be generated by the pairs $(f_1, f_2) = (r_{i,1}, r_{i,2})$. This structure has an automorphism group and would serve as a Galois group. The dimension of the extension of E^2 could define the value of the effective Planck constant.

3. In [L4] the idea that space-time surfaces can be regarded as numbers was discussed. For a given g, one can indeed construct polynomials having any for algebraic numbers in the extension F of E defined by g. g itself can be represented in terms of its n roots $r_i = (r_{i,1}, r_{i,2})$, $i = 1, n$ represented as space-time surfaces as a product $\prod_i (f_1 - r_{i,1}, f_2 - r_{i,2})$ of pairs of monomials. One can generalize this construction by replacing the pairs $(r_{i,1}, r_{i,2})$ with any pair of algebraic numbers in F. Therefore all algebraic numbers in F can be represented as space-time surfaces. Also the sets formed by numbers in F can be represented as unions of the corresponding space-time surfaces.

2 Could many-sheeted spacetime allow a more fundamental description of Majorana like states?

The problematic aspect of the notion of Majorana fermion as a fundamental particle is that the many-fermion states in this kind of situation do not in general have a well-defined fermion number. Physically, fermion number conservation is a superselection rule so that the superposition of fermion and hole must physically correspond to a superposition of fermion states, where the hole corresponds to a fermion which is outside the system. Condensed matter Majoranas avoid this problem but the assumption of ill-defined fermion number seems phenological: holes must correspond to fermions which are somewhere else.

2.1 Could Majorana fermions corresponds to excitations for which fermions are transferred between different space-time sheets

In TGD, the notion of many-sheeted space-time however suggests an elegant solution to the problem at the fundamental level and also suggests that the analogs of Majorana fermions and the associated superconductivity are possible at room temperatures.

1. In condensed matter physics Majorana fermions could be assigned with the vortices of superconductors. In the TGD Universe, these vortices could correspond to monopole flux tubes as body parts of the field body. The states created by γ_i would be superpositions of states in which the fermion is at the monopole flux tube or at the normal space-time sheet representing the part of the condensed matter system that we see. The Majorana description would be only an effective description.

2. The Majorana creation operators γ_i would be replaced with operators which shift the fermion from ordinary space-time sheet to the monopole flux tube and vice versa. From the geometric interpretation it is clear that this operation must be idempotent. This operation must be representable in terms of annihilation and creation operators. The operators γ_i would be expressible products of creation and annihilation operators acting at the space-time sheets 1 and 2.

   One can consider either commutation or anticommutation relations for these operators. Since the operation does not change the total fermion number, the interpretations as a bosonic operator can be argued to be natural so that commutation relations look more plausible.

3. Neglecting for a moment the indices labelling positions and spins and denoting the oscillator operators associated with the two space-time sheets a and b a rather general expression for the hermitian operators γ_1 and γ_2 would be

   $$\gamma_1 = b^\dagger a + a^\dagger b \ \ , \quad \gamma_2 = i(b^\dagger a - a^\dagger b) \ \ .$$

   Suppose fermionic anticommutations are satisfied. Only cross terms contribute to anticommutators (and also commutators).

4. Anticommutators are given by

   $$2\gamma_1^2 = 2\gamma_2^2 = b^\dagger a a^\dagger b + a^\dagger b b^\dagger a = a^\dagger a - b^\dagger b = N(a) + N(b) - 2N(a)N(b) \ \ .$$

   $$\{\gamma_1, \gamma_2\} = 0 \ \ .$$

   The eigenvalues of $N(a) + N(b) - 2N(a)N(b)$ vanish for $(N_a, N_b) \in \{(1,1),(0,0)\}$ and are equal to 1 for $(N_a, N_b) \in \{(1,0),(0,1)\}$. The result implies that the squares of the operators γ_i act like an identity operator, which conforms with the Majorana property. The two operators would anticommute.

5. One can also consider the commutator, which could be argued to be more natural on the basis of the physical interpretation as a hermitian observables. In this case one has trivially $[\gamma_i, gamma_i] = 0$ and $[\gamma_1, gamma_2] = N(a) - N(b)$. The commutator would vanish only for $N(a) = N(b)$ and the physical states could be eigenstates of only γ_1 or γ_2 as an observable. In any case, the Majorana-like property would hold true.

One can also form analogs of plane waves as superpositions of these operators

$$\gamma_{k,1} = \sum_n [exp(ikx_n) b_n^\dagger a_n + exp(-ikx_n) a_n^\dagger b_n]/\sqrt{N} \ \ ,$$

$$\gamma_{k,2} = i\sum_n [exp(ikx_n) b_n^\dagger a_n - exp(-ikx_n) a_n^\dagger b_n]/\sqrt{N} \ \ .$$

Here N is the number of lattice points and discrete Fourier analysis is used. The commutators and anticommutators vanish for different points. Assume that the occupations numbers $N(a,n)$ and $N(b,n)$ do not depend on n so that one $N(a,n) = N(a)$ and $N(b,n) = N(b)$.

1. The anticommutators are given

$$\{\gamma_{k_1,1}, \gamma_{k_2,1}\} = \{\gamma_{k_1,2}, \gamma_{k_2,2}\} = (N(a) + N(b) - 2N(a)N(b))\delta_{k_1+k_2}/N \ .$$

$$\{\gamma_{k_1,1}, \gamma_{k_2,2}\} = 0 \ .$$

   The analog of the Majorana property is true and reflects the fact the transfer operator is classically idempotent.

2. The non-trivial commutators are

$$\{\gamma_{k_1,1}, \gamma_{k_2,2}\} == (N(a) - N(b))\delta_{k_1,k_2}/N \ .$$

   $\gamma_{k_1,1}$ and $\gamma_{k_2,2}$ can be regarded as non-commuting observables.

2.2 $OH - OH^- + p$ as topological qubit?

While writing this, I noticed that the $OH-OH^-+p$ qubits, where p is a dark proton ag monopole flux tubes, that I proposed earlier to play fundamental role in biology and perhaps even make quantum counterparts of ordinary computes possible [L5], are to some degree analogous to Majorana fermions. The extremely nice feature of these qubits would be that superconductivity, in particular bio-superconductivity, would be possible at room temperature. This is would be possible by the new physics predicted by TGD both at the space-time level and at the level of quantum theory.

1. In TGD space-times are surfaces in $H = M^4 \times CP_2$ and many-sheetedness is the basic prediction. Another related prediction is the notion of field body (magnetic/electric) body. Number theoretica view of TGD predicts a hierarchy of effective Planck constants making possible quantum coherence in arbitrarily long length scales. Second new element is zero energy ontology modifyin profoundly quantum measurement theory and solving its basic problem.

2. $OH - OH^- + p$ qubit qubit means that one considers protons but also electrons can be considered. Now the proton is either in the OH group associated with water molecule in the simplest situation in which Pollack effect occurs or the proton is a dark proton at a monopole flux tube. A proton in OH would be analog of non-hole state and the dark proton in the flux tube be the anaog of hole state.

3. What is new is that the proton being on/off the spacetime surface would represent a bit. For Majorana fermions, the situation is rather similar: the hole state corresponds to the electron being "somewhere else", which could also correspond to being on a monopole flux tube as I have suggested. In standard quantum computation, a qubit would correspond to a spin.

4. If the energies for OH and $OH-OH^-+p$ bits are close to each other, the situation is quantum critical and the qubits can be flipped and a process similar to quantum computation becomes possible. Also superconductivity becomes possible at the magnetic flux tubes analogous to magnetic vortices appearing in superconductivity and in fractional Quantum Hall effect.

   These are truly topological qubits also because the topologies of the spacetime surface for different bit values are different. However, the energy difference must be larger than the thermal energy, otherwise the qubits become unstable. With the help of electric fields, qubits can be sensitized to quantum criticality and their inversion becomes possible.

5. The above argument suggests that a non-abelian statistics could be understood for $OH - OH^- + p$ qubits. The anticommutation/commutation relations for the operators trasferring protons to the magnetic body would not be identical to those for Majorana oscillator operators the squares of these operators would be proportional to unit operator which is essentially the Majorana property.

I have proposed a possible realization for this in a more general case. The exchange of dark protons/qubits would be induced by reconnection of monopole flux tubes: it would therefore be a purely topological process. Nothing would be done to the dark electrons, but the flux tubes would be reconnected. Strands AB and CD would become strands AD and BC. At the same time, the unilluminated protons would become associated with different O^-. In this exchange, could the final result be represented as an SU(2) rotation for the entire space.

6. The transfer of proton from OH to magnetic monopole flux tube would correspond to the Majorana like quasiparticle. In zero energy ontology (ZEO) [L1], point-like particles are replaced with 3-surfaces and holography forces to replace them with their 4-D Bohr orbits. The Majorana quasiparticle would classically correspond to a Bohr orbit leading from proton in H to dark proton at the monopole flux tube.

3 A more detailed view of topological qubit in the TGD framework

The Zoom discussion with Tuomas Sorakivi about Microsoft's claimed realization of a topological qubit was very inspiring and led to a generalization of the notion of Majorana qubit characterized by Z_2 group acting as reflection so that one an assign parity to Majora qubit. In TGD Z_2 os replaced by a generalization of the Galois group and this leads to a discrete group bringing in mind anyons with a larger number of internal states. This also involves the notion of Galois confinement discussed earlier. What would be achieved would be a dual interpretation as topological qubit or as number theoretic qubit. This conforms with the notion of geometric Langlands duality realized in the TGD framework as $M^8 - H$ duality [L3, L4].

3.1 Background

The basic idea is that the Majorana fermions of condensed matter are assumed to define a qubit. A Majorana fermion would be a superposition of an electron and a hole. The idea is not pretty because it violates the superselection rule for fermions and the conservation of the fermion number is also questionable. It has also been found that the existence of the Majorana fermion claimed by the Microsoft research group and the superconductivity it requires have not been demonstrated.

A hole must physically correspond to the electron being "somewhere else". In the case of an insulating band, it could be in the conduction band, or in the case of a conduction band, in another conduction band: this description would hold in wave vector space.

In TGD, the electron corresponding to a hole could be in another space-time plane. The equivalent of a Majorana fermion would be a superposition of states where the fermion would be on two space-time sheets. It would be a topological qubit because small deformations of the space-time surfaces do not cause contact between the surfaces. Of course, one can argue that the energies must be the same on different sheets. In the case of condensed matter, this would correspond to the branches of the Fermi surface touching each other.

This idea can be realized concretely: a transfer is an operation that, when repeated, produces the original state, i.e. acts like a unitary operator. The square of the Majorana fermion creation operator is correspondingly a unitary operator. This leads to a concrete model [L6] and the idea that OH-O^-+p qubits could realize topological qubits, at least in biology.

Yesterday's discussion led to a review of holography=holomorphic vision.

3.2 About Galois groups and their TGD counterparts

How to define a Galois group when we are in dimension 4 and not in the complex plane? Is it possible to define a generalization of the concept of ramified primes: these would give a generalization of p-adic primes that label elementary particles in TGD?

3.2.1 Space-time surfaces as solutions of the equations $(f_1, f_2) = (0, 0)$

Holography= holomorphy vision leads to the following picture.

1. Space-time surfaces are roots $(f_1, f_2) = (0,0)$ of two complex values functionsf_i defining an analytic map from $H = M^4 \times CP_2$ to C^2. f_i, $i = 1,2$ is an analytic function of 3 complex coordinates of $H = M^4 \times CP_2$ and one hypercomplex coordinate of M^4. The Taylor coefficients of f_i are in an extension E of rationals. A very important special case corresponds to a situation in which f_i are polynomials. There are good physical reasons to believe that f_2 is the same for a very large class of space-time surfaces and its roots actually define a slowly varying analog of cosmological constant.

   The roots $(f_1, f_2) = (0,0)$ correspond to space-time sheets, which are algebraic surfaces. The space-time surface need not be connected. The Hamilton-Jacobi coordinates [L2] serve the coordinates of H: there is one hypercomplex coordinate u and its dual v and 3 complex coordinates w for M^4 and ξ_1 and ξ_2 for CP_2. The coordinate curves for u and v of M^4 have light-like tangent vectors.

2. Dimensional reduction occur because the hypercomplex coordinates are separated from the dynamics and take role of parameters appearing as coefficients of f_i interpreted as functions of $w, \xi_1\xi_2$ so that only three complex coordinates ξ_1,ξ_2 and w would effectively remain dynamical. For partonic orbits as the interfaces between Minkowskian regions and CP_2-like regions with Euclidean signature of the induced metric, u= constant would be a natural condition. At these 3-surfaces, the dimensional reduction would be complete: the roots would not depend on u. In the interior of CP_2 like region u would be also constant and Minkowskian contribution to the induced metric would vanish as for CP_2 type extremals.

3. If f_i is polynomial P_i with coefficients in the rational expansion E, analytic flows as analogs of homotopies that take roots as regions of the space-time surface to each other would correspond to a 4-D version of the Galois group. The definition of the Galois group operation would be as a flow rather than an automorphism of an algebraic extension leaving E unaffected as usual. Definition as flow is used in braid representations of groups.

   This is new mathematics for me and perhaps for mathematicians as well. It would be a generalization of the 2-D Galois group.

4. The 4-surfaces corresponding to different roots would have lower-dimensional surfaces interfaces. The hypercomplex sector effectively decouples and this gives 2 conditions in 4-D space stating that the complex coordinates, say w, are identical at the boundary so that interfaces are string world sheets. This fixes $w(u)$ at the interface.

   (a) The roots as 4-surfaces could correspond to branches of a fold taking place along a string world sheet. This suggests a complexification of a cusp catastrophe. For cusp catastrophe, the catastrophe curve is a V-shaped curve along which two real roots of a polynomial of degree 3 depending on a real coordinate x and real parameters a, b co-incide. Now x is replaced with a complex coordinate w which at the string world sheet depends on the coordinate hypercomplex coordinate u. One can say that the 1-D boundary of V is replaced with string world sheets. What happens in the vertex of V is an interesting question. The boundaries of V having coinciding root pairs as analogs co-incide. Does this mean that two string world sheets fuse. Could this be regarded as a reaction in which strings fuse along their full length?

   (b) Could the space-time regions defined by the roots genuinely intersect along a string world sheet? This kind of intersection would be analogous to a self-intersection of a 1-dimensional curve. The basic example is the curve $x^2 - y^2 = 0$ splitting to the curves $x - y = 0$ and $x + y = 0$.

   If for instance, $f_1 = P_1$ fails to be irreducible and decomposes to a product $P_1 = Q_1Q_2$ of two polynomials Q_i, the roots $Q_1 = 0$ and $Q_2 = 0$ intersect at the common root $Q_1 = Q_2 = 0$. These kinds of intersections are excluded if one allows only irreducible polynomials. The irreducibility can fail for some values of the coefficients of the polynomials.

   The space-time surface would decompose to a union of 2 surfaces represented as roots of Q_1 and Q_2 and do not interact unless they intersect along a string world sheet. The dimensional reduction due to the same Hamilton-Jacobi structure implies that 2

2-surfaces intersect in 6-dimensional space. This does not happen in the generic case. Hence this option does not seem possible.

Analytical flows take the points corresponding to the roots from one sheet to another through string world sheets: here cusp catastrophe helps to visualize. String world sheets correspond to the common values of ξ_1, ξ_2, w. For instance w can serve as coordinate and at the intersection w the value is fixed.

The ends of the strings correspond to complex numbers that depend on the time parameter u: the complex number, say w, would represent the intersection of the space-time sheets as a root. The complex roots depend on u through polynomial coefficients. If one has $u = constant$ at the parton trajectories at which the signature of induced metric changes, the u-dependence disappears at the paths of the string ends at which fermions are attached in the physical picture about the situation. Under very mild assumption about the polynomials $P_i(w, \xi_2, \xi_2, u = 0)$, the roots can be algebraic numbers in an extension of E and would characterize the intersections of the roots of the equation $(P_1, P_2) = (0, 0)$.

These complex numbers are considered a generalization of complex roots and would be related to quantum criticality, i.e., the fact that the two roots are the same and the system is at the interface between space-time regions. The criticality would correspond to a fold of the cusp catastrophe.

If it is possible to attach a Galois group to the set of string world sheets transforming them to each other, it would transform different string world sheets into each other. Could this group serve as an algebraization for the generalized Galois group represented as a geometric flow?

What about the counterparts of p-adic primes? The product of the differences of the roots defines the discriminant D. Can it be decomposed into the product of powers of algebraic primes of the extension E? If so , this would generalize the concept of a p-adic prime. The intersections of the sheets of the space-time surface, or rather their intersections with partonic 2-surfaces, could be associated with p-adic primes. This has just been a physical picture.

3.2.2 The analogs of Galois group associated with dynamic symmetries

The descriptions $g : C^2 \to C^2$ define dynamic symmetries $f = (f_1, f_2) \to g(f)$, which produce new space-time surfaces of higher complexity.

1. What happens in the operation $(f : H \to C^2) \to (g \circ f : H \to C^2)$, $fH \to C^2$ and $g : C^2 \to C^2$? The surface $g(f) = 0$ would correspond to the surface $(g_1(f_1, f_2), g_2(f_1, f_2)) = (0, 0)$.

   The intuitive picture is that complexity increases the in these dynamical symmetries. For example, in the case of C, iterations produce fractals. These descriptions would provide a geometric model for the abstraction and can be combined and iterated.

2. If $g(0, 0) = (0, 0)$ then $(f_1, f_2) = (0, 0)$ remains a root and in the "Gödelian" view of the classical dynamics of the space-time surfaces produces analogies to theorems (see Gtgd). Other roots represent more complex space-time surfaces: the non-trivial action of g brings in the meta-level and makes the composition with g provides statements about statements represented by$(f_1, f_2) = (0, 0)$. "Simple" spacetime sheets, which do not allow a decomposition to $f = g \circ h$, would represent lowest level statements. The associated magnetic bodies could correspond to the surfaces $(g_1(f_1, f_2), g_2(f_1, f_2)) = (0, 0)$. Entire hierarchies of meta-levels are possible.

   Magnetic bodies indeed represent a higher level in the number theoretic hierarchies and correspond to larger values of the effective Planck constant as dimension of extension associated with E. In the TGD inspired quantum biology, the magnetic body serves as a controller of the biological body.

Can the concept of Galois group be generalized in this case?

1. The regions of the surface $(g_1(f_1, f_2), g_2(f_1, f_2) = (0, 0)$ correspond to roots. 2+2 conditions fix the roots $f_1 = a$ and $f_2 = b$ are 6-surfaces, and their intersection is a 4-surface.

If the consideration is restricted to the surface $u = constant$, assumed to correspond to a partonic orbit, then the roots do not depend on u and can be algebraic numbers and perhaps a generalization of the Galois group could be defined.

The condition $g_2(f_1, f_2) = 0$ gives $f_1 = h(f_2)$, where h is an algebraic function. The condition $g_1(f_1, h(f_1)) = 0$ gives $f_1 = a$ and $f_2 = b$, where a and b are algebraic numbers. They correspond to 6-surfaces: the space-time surface is the intersection of two algebraic 6-surfaces. If (a, b) and (c, d) are not identical, then the corresponding surfaces are disjoint.

2. Is it possible to define a Galois group using the algebraic extension of E defined by the roots? The Galois group would permute the surfaces $(f_1 = a, f_2 = b)$, which would correspond to pairs of complex numbers and would be disjoint.

   Now the element of the Galois group would not correspond to a flow permuting the pairs (a, b). It should act as an automorphism of $E \times E$. Is this possible? One cannot provide $E \times E$ with the structure of a number field. It is however enough to have algebra structure involving component-wise sum $(a, b) + (c, d) = (a + c, b + d)$ and product $(a, b) * (c, d) = (ac, bd)$. The algebraic extension of E^2 defined by the roots of $g \circ f$ as pairs $(r_{i,1}, r_{i,2})$ would have an automorphism group identifiable as the Galois group. Also discriminant $D = (D_1, D_2)$ could be defined using the component-wise product for the differences of the root pairs. It would have two components and one can ask whether D_1 and D_2 could be decomposed to products of algebra primes of E.

3. Is it possible to generalize the concept of ramified prime? They would define generalized p-adic primes. The discriminant can be defined as the product of the differences of the roots, which would factor into the product of algebraic primes in the extension E. The roots (a, b) would be in $E \times E$ so that the structure of the number field would be required. For quaternions the lack of commutativity implies that the product of the root differences depends on their order.

It was already noticed that there are good physical motivations for decomposing WCW to sub-WCWs for which f_2 is fixed. The counterpart of the ordinary Galois group is obtained in the sub-WCWs: $g = (g_1, I)$ reduces to a map $g_1 : C \to C$. The roots of $g_1(f_1) = 0$ are surfaces $(g_1(f_1), f_2) = (0, 0)$. g_1 has n surfaces as roots. The transitions between these disjoint surfaces would generate the analog of the ordinary Galois group acting as a number-theoretic dynamical symmetry group. Also ramified primes as primes of algebraic extension of E are obtained.

1. Representations of the Galois group transfer fermions between space-time regions corresponding to different roots of g_1. The Galois group is generally non-Abelian and its elements could appear in topological quantum computation as basic operations for the topological qubits. The analogs of anyons would be irreducible representations of the Galois group.

2. If the degree n is prime, g is a prime polynomial. It cannot be represented as a composite of polynomials, whose degree is a product of smaller integers.

   Remark: If P is irreducible then it cannot be a product, in which case the degree would be the sum of their degrees. Therefore one has two kinds of primeness.

3. The surfaces corresponding to different roots of g_1 are disjoint. If the roots are the same then the surfaces are the same. If $g(0, 0) = 0$ then $(f_1, f_2) = (0, 0)$ is a root. As two roots approach each other. the two separate surfaces merge into one. What does this mean physically? Should one regard the identical copies of the surface as different surfaces, members of a double, and carrying different many-fermion states? In any case, the order of the Galois group is reduced in this case.

3.3 About intersections of 4-surfaces

There are several options to consider.

1. The 2 4-surfaces X^4 and Y^4 correspond to different pairs (f_1, f_2). If the Hamilton-Jacobi structures are different so that the hypercomplex coordinates (u, v) are different, the intersection $X^4 \cap Y^4$ is a discrete set of points. Field theory suggests itself as a natural description of fermions assigned with the interaction points.

   If the Hamilton-Jacobi structures are the same, the dimensional reduction occurs and one has effective intersection of 2 complex surfaces in 6-D complex space. In the generic case the intersection is empty.

2. One can also consider the analogs of self-intersections as interfaces for 2 4-D roots for the same pair (f_1, f_2). The intersection consists of string world sheets. As found, genuine self-intersection is exclude so that only the analogy of a complexified cusp catastrophe remains.

   String model is a natural description of the interactions of 4-surfaces and the self-interaction of 4-surfaces in the fermionic sector. Fermion propagators can be calculated because the induced spinor field is a restriction of the corresponding H.

The analogy of TGD based physics with formal systems discussed in [**?**] led to ask whether the interaction of space-time surfaces involves the fusion of the 3-surfaces with different Hamilton-Jacobi structures to a single connected 3-surface with a common Hamilton-Jacobi structure for the components. Physically the tusion could mean a generation of monopole flux tube contacts between the 3-surfaces.

In the Gödelian framework, this interaction would have an interpretation as a morphism realized as an action of the composite space-time surfaces on each other. In the connected intermediate state, string model type description might apply in the fermionic degrees of freedom. Even stronger condition would be that fermions reside at the string ends at partonic orbits.

3.4 Galois group as group of possible transfer operations for fermions and a generalization of the Majorana qubit

3.4.1 Roots for the condition $(f_1, f_2) = (0, 0)$ as space-time sheets

Generalization of the Galois group. Galois generalizes Z_2 to Majorana fermions. Classical equivalent of the transfer operation between space-time sheets. A particle is transported through a string word sheet corresponding to a common root pair to another sheet.

Topological/number-theoretic qubit. Transfer through a string world sheet. What is the physical interpretation. String 1-D object in 3-space. Could the Riemann surface for $z^{1/n}$ serve as an analogy. Anyons and braid statistics. Since hypercomplex coordinates are passive, we get effective 2-dimensionality and braid statistics.

3.4.2 Roots in the special case $g = (g_1, Id)$

Ordinary roots of a polynomial represented as 4-surfaces. Disjoint or identical. However, the representation of the Galois group of g_1 is non-trivial. These would correspond to abstractions. Fermion transfer between disjoint surfaces Galois group operation represented using oscillator operators.

When does this?

1. This happens only if f_1 allows the decomposition $f_1 = g_1 \circ h_1$. When could this be possible? In the case of polynomials, this means that the degree of f_1 for a given H complex coordinate ξ_1, ξ_2, or w polynomial is the product of the degrees of $n_1 \times n_2 \times n_3$ for the lower degree polynomials n_1, n_2, n_3.

2. If the degrees of the polynomial for different coordinates are primes, then the decomposition is not possible. These would be "prime polynomials". The 3 prime numbers p_1, p_2, p_3 characterize these. If it is a homogeneous polynomial, then one prime number p is enough. These polynomials would be in a special position physically. They would correspond to "elementary particles". The tetrahedra associated with them would be uniform.

3.4.3 Concrete realization of topological/number-theoretic qubit and generalization of qubits

The TGD based view leads to generalization of bit to n-ary digit or pinary digit, where n or p corresponds to a degree of a polynomial g_1 in $g = (f_1, Id)$ defining a dynamical symmetry and associated Galois group whose elements would correspond to transfers of fermions between different branches of the space-time surface.

1. Roots as regions of an n-sheeted space-time surface correspond to roots $(f_1, f_2) = (0, 0)$ and would correspond to different values of an n-ary digit. They are glued together along string world sheets as analogs of folds.

   The functional composition $f \to g(f)$ gives rise to hierarchies of Galois groups. The Galois group, represented as analytic flows, replaces the group Z_2 of the Majorana case. Analytic flows define braiding operations, which define the 4-D Galois group.

2. Also the dynamical symmetries g give rise to an analog of a Galois group. The non-vanishing roots of g are disjoint. It seems that the Galois group can be defined only if one has $g = (g_1, I)$.

   For OH-O$^-$+p qubits [L5, L6] they could correspond to different pairs because h_{eff} would be of different magnitude.

3.4.4 Generalization of a bit to n-ary digit and pinary-digit

The replacement of bit with *n-ary digit* would take place when the degree d of the polynomial P_1 (or g_1 in $g = (g_1, Id)$) is $d = n$ and bit $\to$ pinary digit when the d is a prime: $d = p$. Polynomials for which the degrees with respect to complex coordinates of H are primes are primes with respect to the functional composition and could physically correspond to fundamental objects appearing at the bottom of the hierarchy obtained by a functional composition with maps g. This picture generalizes also to more general dynamical symmetries $(g_1, g_2) = (P_1, P_2)$.

These primes should not be confused with ramified primes. One can of course ask whether the p-adic primes appearing in p-adic mass calculations could actually correspond to these primes.

This allows us to consider a possible definition for a topological/number-theoretic qubit. For $g(0) = 0$, the original surface is included in the set of $g \circ f = 0$ surfaces. In the case of OH-O$^-$+p qubits, the magnet monopole flux tubes could correspond to the non-vanishing root $f \neq 0$ of g. In this case the Galois group of g would be Z_2 and correspond to the parity of Majorana fermions. In the general case more complex Galois groups are possible.

3.4.5 A more precise connection to the Majorana qubit of condensed matter

The definition of a Majorana qubit involves the observation that when two branches of the Fermi surface that correspond to an insulator and to a conduction band touch each other, the gap energy disappears. In superconductivity, this gap energy is very small but non-vanishing. If this energy vanishes, Majorana type excitation becomes possible and is interpreted as a quantum superposition of an electron and a hole.

What could this situation correspond to or how could it generalize in TGD?

1. $M^8 - H$ duality [L3] strongly suggests that Fermi surfaces determined as an energy constant surface in momentum space have space-time counterparts.

2. The group Z_2 defining the parity of Majorana qubit would be generalized to Galois group and and one can consider two options corresponding 1) to the 4-D Galois group realized as analytic flows assignable to a connected 4-surface (f_1, f_2) and 2) to the Galois group assignable to $g = (g_1, Id)$ acting as a dynamical symmetry. The notions of Galois group, discriminant and ramified primes generalize to the case of (g_1, g_2) using component-wise product and sum for the pairs (g_1, g_2) since algebra structure is enough to identify Galois group as automorphisms for an extension of E^2.

Consider option 1) first.

1. The Galois group would relate string world sheets to each other. The branches of the Fermi surface could at the space-time level correspond to 2-D string world sheets at which the roots associated with the different space-time surface sheets $(f_1, f_2) = (0, 0)$ coincide . One could move from one branch of the space-time sheet to another through the string world sheets. Each string world sheet would correspond to a discrete complex point (ξ_1, ξ_2, w).

2. The E^3 projection of the string world sheet would be a string, which would have apparent ends at the "boundary" of the 3-surface. The 2-D "boundaries" of the 3-surfaces are surfaces, where the 3-surface has a fold, i.e. the normal M^4 coordinate has a maximum value. One can say that the string effectively ends at these surfaces although it actually has a fold.

   String would sheet would also have an end at the partonic orbit, where the signature of the space-time metric changes. Since the coordinate u would be constant inside the CP_2 type extremals, the 2-D string world sheet reduces to a 1-D light-like curve inside it.

   In the case of topological qubits, the superconducting wire could correspond to the string identifiable as the superconducting wire whose ends correspond to the points of the Fermi surface at which the branches of the Fermi surface touch. The ends of the wire, assumed to carry Majorana fermions, would correspond to the real ends of the string at partonic orbits to which fermions are assigned or to an apparent end at the fold.

3. The situation would correspond to quantum criticality, since even a small perturbation will move the particle to one of the branches.

For option 2), the space-time surfaces related by the Galois group for $g = (g_1, Id)$ would be disjoint. This does not conform with the assumption that Fermi surfaces touch at a point. This picture could however work for OH-H^- topological qubits for which the two surfaces related by Z_2 Galois group for $g = (g_1, g_2)$ would have different "internal" Galois groups represented as flows leaving the space-time surface invariant.

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