

Comparing the S-matrix descriptions of fundamental interactions provided by standard model and TGD

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Abstract

The construction of S-matrix has been a long-standing challenge of TGD and during years I have considered numerous proposals. Holography= holomorphy vision (H-H) allows solving the classical field equations for space-time surfaces exactly.

The Dirac equation in $H = M^4 \times CP_2$ can be solved exactly for M^4 . If M^4 has Kähler structure color confinement can be understood in terms of the H Dirac equation alone since the M^4 Dirac equation allows tachyonic masses and 8-D massless condition allows to construct light states, which must be color singlets. The interaction between space-time surfaces, which by H-H represent particles, is naturally a contact interaction occurring in their intersection consisting of string world sheets if the space-time surfaces have the same Hamilton-Jacobi structure.

The basic structure of QCD generalizes. All external particles are analogous to hadronic phase. Interactions occur in the color deconfined phase governed by the induced/modified Dirac equation. Color- and electroweak interactions can be seen as aspects of the same interaction. At the level of H , color partial waves as representations of the color group $SU(3)$ are analogous to orbital angular momentum eigenstates. Electroweak group $U(2)$ is a subgroup of the color group acting as gauge transformations and color interactions at this level can be identified as electroweak interactions. The value of color coupling strength is predicted correctly.

The basic objection against TGD has been that there are new indications for the new physics from LHC. Quite recently it was however reported there is evidence for anomalies related to the transition to a phase that has been interpreted as quark gluon plasma. Intriguingly, the new physics predicted by TGD indeed relates to this transition.

1 Introduction

The construction of S-matrix has been a long-standing challenge of TGD and during years I have considered numerous proposals.

1. The first naive attempts were based on a naive generalization of path integral but it became clear path integral simply fails to exist. This led to the discovery of the notion "world of classical worlds" (WCW) consisting of 3-surfaces to which one can assign a highly unique space-time surface: this means almost-deterministic holography. Quantum states would be spinor fields in WCW.
2. The geometrization of WCW is highly unique already in the case of loop spaces from the existence of the Riemann connection. The conformal symmetries and Kac-Moody symmetries generalize in the TGD framework to a huge generalization of superconformal symmetries and to super symplectic symmetry and WCW can be seen as a union of analogs of symmetric spaces labelled by zero modes not appearing in the metric.

The condition that the S-matrix is invariant under these huge symmetries gives excellent hopes that the S-matrix is highly unique. The tough problem is to construct it and mere symmetry arguments are not enough to achieve this.

This raised the hope that TGD is unique from its mathematical existence. The existence of the twistor lift of TGD indeed leaves only the option $H = M^4 \times CP_2$ since only M and C_2 allow twistor space with Kähler structure.

3. In TGD only fermions are the fundamental elementary particles and classical fields are obtained by inducing non-dynamic geometric objects to the space-time surfaces. This means a huge simplification since all elementary particles consist of fundamental fermions. The spinor structure at the space-time surface is induced, which means simply restriction of the second quantized free spinor fields of H and identification of induced gamma matrices as projections of H gamma matrices.

This implies that fermionic propagators are simply restrictions of free H -propagators to the space-time surfaces. But How to get various vertices, in particular pair creation vertex, when one only free fermions are available? The idea is that fermion pair creation and scattering takes place in the classical induced field, essentially in the same way as in path integral approach.

There is also a second tough problem. How to get pair creation for free fermions. The solution is simple: creation of a fermion pair means that the fermion turns backwards in time or more generally, the fermion line has an edge. Exotic smooth structures obtained from the standard smooth structure by adding defects is a completely unique feature of 4-dimensional spaces. The proposal is that the vertices as edges of fermion lines correspond to defects of the standard smooth structure. Quantum theory would be possible only for 4-D space-times.

4. Few years ago holography = holomorphy vision (H-H) emerged [L8, L16, L6, L20] and provided a general solution of field equations for space-time surfaces involving only solution of local algebraic equations: TGD is exactly solvable. Space-time surfaces are minimal surfaces and can be seen as analogs of solution of massless field equations. Any general coordinate invariant action principle constructible in terms of induced geometry gives the same space-time surfaces and only the boundary conditions, if even these, depend on the variational principle. Number theoretic vision suggests that the vacuum functional is expressible in terms of number theoretic invariants.
5. H-H led also to an exact general solution of the Dirac equation for the induced spinors in X^4 in terms of holography [L18, L17], very much the same way as in string models. So that Dirac equation can be solved both at the level of H and X^4 . Apart from the right-handed neutrino, all modes of H spinor field have CP_2 mass scale.

However, if M^4 has Kähler structure, one can have modes with negative M^4 mass squared but the total mass squared must vanish for on mass shell states. One can construct many fermion states with vanishing (additive) M^4 mass squared. This requires color singlet property meaning color confinement. All light external particles, both hadrons, leptons, gauge bosons, and gravitons would be in these kinds of states.

These latest advances led to a quite dramatic increase in the understanding of how the TGD view of standard model physics and gravitation differs from the standard view.

1. All interactions reduce to the dynamics of 3-surfaces obeying holography = holomorphy principle [L8, L16, L6, L20] and by general coordinate invariance only 4 H-coordinates define the primary dynamical variables at the fundamental level.
2. The study of the induced Dirac equation led to a generalization of the QCD type description involving deconfinement and hadronization so that it applies quite generally. External states correspond to many particle states constructed from the modes of the H Dirac equation. In interactions, the hadrons transform to a deconfined quark phase at X^4 in which quarks obey the induced Dirac equation and behave like massless fermions. This applies also to leptons, which also can move in color partial waves.
3. Electroweak and strong interactions can be seen as different aspects of the same interaction. Color partial waves in H are analogous to orbital angular momentum states and since the electroweak group can be regarded as a subgroup of $SU(3)$, electroweak quantum numbers can be regarded as color quantum numbers analogous to spin. "Strong-electroweak" is completely analogous to "orbital angular momentum-spin".

$CP_2 = SU(3)/U(2)$ states that also geometrically $U(2)$ acts like a gauge group, whereas $SU(3)$ is not a gauge group. In electroweak degrees of freedom confinement occurs only for $SU(2)$ but not for electromagnetic $U(1)$ and takes place by screening of electroweak isospin but pairs of left and right-handed neutrinos. The estimate for color coupling strength comes out correctly [L17].

4. By H-H, the X^4 spinor modes are analogous to the spin states of a point-like particle. The interaction of two space-time surfaces occurs in the interaction of the particle- like 4-surfaces consisting of string world sheets for identical H-J structures [L19]. This is nothing but the analog for the collision of point-like particles.

2 TGD view of standard model physics and hadron physics in particular

The study of the solution spectrum of Dirac equations in H and X^4 has led to a quite dramatic increase of the understanding of how TGD view of standard model physics and gravitation differs from the standard view. All interactions reduce to the dynamics of 3-surfaces obeying holography = holomorphy principle and by general coordinate invariance only 4 H-coordinates define the primary dynamical variables at the fundamental level.

In particular, in TGD electroweak and strong interactions can be seen as different aspects of the same interaction. Color partial waves in H are analogous to orbital angular momentum states and since the electroweak group can be regarded as a subgroup of $SU(3)$, electroweak quantum numbers can be regarded as color quantum numbers analogous to spin. "Strong-electroweak" is completely analogous "orbital angular momentum-spin" $CP_2 = SU(3)/U(2)$ states that also geometrically $U(2)$ acts like a gauge group, whereas $SU(3)$ is not a gauge group. In electroweak degrees of freedom confinement occurs only for $SU(2)$ but not for electromagnetic $U(1)$ and takes place by screening of electroweak isospin but pairs of left and right-handed neutrinos. The estimate for color coupling strength comes out correctly.

By H-H, the X^4 spinor modes are analogous to the spin states of a point-like particle. The interaction of two space-time surfaces occurs in the interaction of the particle-like 4-surfaces consisting of string world sheets for identical H-J structures. This is nothing but the analog for the collision of point-like particles.

2.1 How could the transitions between hadronic and quark phases occur in the TGD framework?

The solutions of the ordinary Dirac equation in H with M^4 Kähler structure have masses of order CP_2 and light states are color singlets whereas the solutions of the induced/modified Dirac equation for quarks in X^4 are massless. In the case of quarks this suggests an interpretation in terms of hadrons and massless quarks. This picture also applies to leptons.

The QCD description of hadronic reactions is statistical and is in terms of quark and gluon distribution functions characterizing hadrons and fragmentation functions to hadrons for quarks and gluons. What could the TGD counterparts of these functions be and could an analogous description at quantum level be possible? What happens in the transitions hadron phase and free quark phase and how to describe this in TGD?

2.1.1 Hadron phase \leftrightarrow quark phase transition as a transition between phases characterized by $8 - D$ and $4 - D$ masslessness

In the transition to the X^4 phase with free massless quarks, the colored H spinor modes are replaced with holomorphic X^4 spinor modes. The opposite transition takes place in hadronization. $X^4 \rightarrow H$ transition is analogous with the Higgs mechanism in which transition occurs from a massless phase to a massive phase (in M^4 sense). Transition is also between deconfined and confined phases. This description applies also to leptons which also move in H spinor partial waves.

A more general view is based on the breaking of 4-D generalization of conformal symmetry. In hadronization $4 - D$ light-likeness is replaced with 8-D light-likeness in H . Propagation takes place along the space-time surface and the propagator is determined by the induced/modified Dirac operator. What is of crucial importance is that fermionic oscillator operators for the induced spinors fields are expressed in terms of those for the H spinor field.

What about the description of color in the X^4 phase? Does one obtain color triplets in the holomorphic basis? Could the color partial waves $\{\xi_1, \xi_2, 1\}$ proportional form a counterpart of color triplet? Could the color triplet correspond to the 3 coordinate patches for the complex structure of CP_2 as a complex projective space? Why are color triplets special for quarks and color singlets for leptons? Does this relate to generalized conformal invariance, which could make higher partial waves gauge degrees of freedom and imply the analog of gauge invariance of QCD? What about Kac-Moody type gauge invariance? Could fixed H spinor modes as ground states for Kac-Moody representations: this would imply that each scaled variant of hadron physics associated

with hadrons for particle color partial waves for H Dirac equation would define its own TGD counterpart of perturbative QCD or of string model description.

2.1.2 Quantum measurement theory in ZEO as a guideline

Quantum measurement can be seen as a Hilbert space projection. Could this projection be induced by a geometric projection from H to the space-time surface for the spinor modes. Does it take place always in the interaction and how this interaction is realized?

The modes of the X^4 Dirac operator have a fixed M^4 chirality and this is the signature of masslessness. Apart from the covariantly constant right-handed neutrino, H modes have only a fixed H chirality and are therefore massive. Therefore also M^4 chirality would be measured in the transition to the quark phase. Note that also projections to lower dimensional surfaces, such as partonic orbits, string world sheets and fermion lines make sense if this interpretation is correct. In this picture, the overlap between H modes and X^4 modes would characterize the transition from hadrons to quarks and vice versa.

The ZEO based description of any particle reaction involves a pair of BSFRs. In the case of hadronic reactions this would involve the transition of hadrons to quarks in BSFR, time evolution with opposite arrow of time, and second BSFR leading from quark phase to hadron phase.

1. In ZEO, the deconfinement phase transition $H \rightarrow X^4$ from hadron to quark phase would involve a localization from H to X^4 . This also means a localization in the "world of classical worlds" (WCW). In the deconfined state localized to single X^4 , one would have an analog of QFT in a fixed background space-time. Note however that every 3-surface defines its own space-time surface as its Bohr orbit, which is however not quite unique, which in fact forces ZEO. Therefore one has a superposition of scattering amplitudes over the space-time surfaces satisfying holography= holomorphy principle.
2. Hadronization as a transition $X^4 \rightarrow H$ would in turn mean a delocalization in WCW and could be interpreted as a localization in the analog of momentum space for WCW. The observables measured would be quantum numbers of WCW spinor modes. This includes measurement of H quantum numbers but the light states are color singlet many fermion states. Color partial waves have the CP_2 mass scale.

2.1.3 What does the interaction of particles as space-time surfaces mean?

What does the interaction of particles as space-time surfaces obeying holography= holomorphy principle mean? When do the particle interactions lead to the transition to the phase corresponding to a localization in WCW? In strong interactions this kind of interaction requires a high collision energy implying that the interactions occur in a scale smaller than the geometric size scale of the colliding particles so that the internal geometric structure of the particle become visible. In the TGD framework, these details naturally correspond to lower dimensional structure consisting of the light-like parton orbits and string world sheets having their boundaries at the parton orbits. Note that this picture might apply to all interactions. Topological considerations allow to make this picture rather concrete [L19].

1. For topological reasons, the intersection of the generic space-time surfaces is a discrete set of points. The systems should fuse somehow and form a quantum coherent interaction region. If the H-J structures of the space-time surfaces are identical meaning that in the interaction region both space-time surfaces have the same coordinates (u, v, w, ξ_1, ξ_2) , the intersection is 2-D string world sheet, containing point-like fermions as fermion lines at its boundaries assignable to light-like 3-D parton orbit [L20]. This makes possible a string model type description for the interactions of the fundamental fermions. By the hypercomplex holomorphy, the description would be rather simple since the second light-like coordinate of the string world sheet is non-dynamical.
2. Only fermions and their bound states appear as fundamental quantum objects in the TGD framework. If they emerge in the formation of states delocalized in WCW they would correspond to hadrons, leptons and electroweak bosons. In particular, bosons as incoming

and outgoing are identified as bound states of fermions and antifermions. The stringy view of the interactions implies that bosons need not appear at all in the deconfined phase.

If so, there would be no gluons and the fundamental vertex would correspond to a creation or annihilation of a fermion pair from "vacuum" and the classical induced gauge fields would define the vertices. This would take place when string world sheets fuse or split and a pair of fermion lines at separate string world sheets is created or disappears.

3. The notion of exotic smooth structure [A2, A3, A1] possible only in 4-D space-time and reducing to the standard smooth structure apart from defects identifiable as this kind of singularities allows these kinds of edges [L10, L4, L15]. This allows also to consider scattering events in which the fermion line has an edge serving as vertex giving rise to momentum exchange. These edges would correspond to failure of holomorphy at a single point.

2.1.4 The relationship between the oscillator operators of spinor modes in H and X^4

It is possible to express X^4 oscillator operators in terms of H oscillator operators [K7]. Induction means the restriction of the mode expansion of the second quantized H spinor field to the space-time surface X^4 . Similar expansion for X^4 spinor field in terms of conformal modes makes sense. The two representations must be identical. This implies that the oscillator operators at X^4 are expressible as inner products of conformal modes and H spinor field. H oscillator operators are fundamental and no separate second quantization at X^4 is needed.

The inner products between the spinor modes of X^4 and H involve an integration over the space-time surface or a lower-dimensional space-time region. By the 8-D chiral symmetry the matrix element must involve gamma matrices and reduce to an integral of an inner product of the conformal mode of the induced Dirac operator with the fermionic super current over the parton orbit. The 3-D intersection of the space-time surface with the light-like boundary of CD cannot be excluded. The integral over the parton orbit is natural since the transversal hypercomplex coordinate for the associated string world sheet is not dynamical.

Overlap integrals between the c-number valued modes of X^4 spinor field and the second quantized H spinor field give X^4 oscillator operators in terms of H oscillator operators. These integrals characterize the transition between the two phases and its reversal and would replace the parton distribution functions and fragmentation functions in TGD. The conservation of color quantum numbers and corresponding M^4 quantum numbers in holomorphic basis in which X^4 complex coordinates correspond to those of H .

What about propagators in the quark phase? The propagation would be restricted to X^4 rather than occurring in H . X^4 spinor field would be defined as the sum over its conformal modes and the Dirac propagator would be defined as a two point function, which can be calculated because oscillator operators are expressible in terms of H oscillator operators.

2.1.5 Description of hadron reactions in ZEO

As found, zero energy ontology and holomorphy=holography vision suggest a universal description of all particle reactions. The particle reaction involves a temporary time reversal involving two BSFRs.

1. In the first BSFR a projection from the space of hadron states in H to free many-quark states in X^4 would occur. This localization in WCW would also involve a measurement of M^4 chirality by an external observer. The resulting state would consist of free massless quarks in X^4 and evolve by SSFRs backwards in geometric time. The interactions would be mediated by string world sheets having fermion lines at their boundaries and the notion of exotic smooth structure would be essential making possible fermion scattering and pair creation in the absence of fundamental bosonic quantum fields.
2. After that a second BSFR would occur inducing a delocalization in WCW and a hadronic state would emerge and evolve by SSFRs. One can say that the states delocalized in WCW correspond to hadrons (and quite generally color singlet states). WCW observables, which include the observables associated with H , would be measured. Concerning the calculation of the scattering amplitudes, this means that the quark oscillator operators would

be expressed in terms of H oscillator operators and a Hilbert space projection to a state of hadrons would take place.

2.2 A more detailed vision stimulated by the analysis of LLM session

Recently Marko Manninen performed a LLM session using OpenAI's O3 language model (GPT hitherto) using prompts related to the geometric aspects of TGD: the results can be found in the article by Marko and me [L23]. Due to its "education", GPT gave misunderstandings and at the level of detail the model tended to hallucinate in its responses at the level of detail. Included were prompts requesting killer tests and asking whether these kinds of tests were already carried out. The fact that the responses were based on misunderstandings of what TGD is, forced to direct attention to the details of the related areas of TGD landscape and this had a very fruitful outcome.

2.2.1 Three kinds of questions related to the interpretation of TGD

The analysis created three kinds of questions related to the interpretation of TGD.

1. The idea [L20, L17] about the phase transition between phases described in terms of Dirac equation in H *resp.* X^4 as a generalization of the notion of the deconfinement phase transition *resp.* hadronization replaces the QCD type description with a stringy description in which the intersection of the space-time surfaces of colliding particles consisting of 2-D string worlds sheets determines the scattering amplitudes. In ZEO, this phase transition would involve two "big" state function reductions reversing the arrow of time and the time.
2. From the beginning it has been clear that color $SU(3)$ is isometry group rather than gauge group and that its subgroup $U(2)$ identifiable as a holonomy group acting on H spinors corresponds to a gauge group. The very definition of CP_2 as coset space states this geometrically.
 - (a) Could this mean the reduction of color confinement at the level of spinor quantum numbers to $SU(2)_L$ confinement [L21]? Photons would not be confined, or screened by the pairs of right- and left handed neutrinos screening also the color of leptonic color partial waves [L17].
 - (b) Gluons do not appear as couplings of H spinors. Do gluons exist at all and is the identification of classical gluons as projections of Killing vectors wrong? Or do gluons correspond to electroweak gauge potentials in CP_2 spin degrees of freedom and would therefore correspond to electroweak interactions? But is this consistent with the fact that strong interactions are indeed strong?
3. A further stimulus came from the claim of GPT that already the existing data excludes copies of hadron physics labelled by Mersenne primes and their Gaussian variants. Is this really the case and are the earlier indications about bumps [K2, K3] wrong?
 - (a) Under what conditions does the phase transition between M_{107} and M_{89} hadron physics occur with a significant rate?
 - (b) Is quantum criticality, forcing the Compton scales of ordinary hadrons and dark M_{89} hadrons to be identical, necessary? This is indeed assumed in the model for the bumps as M_{89} mesons reported at LHC. If so, the transition from M_{107} H phase to X^4 phase would occur in the first BSFR and the transition from the X^4 phases to X^4 phase to M_{89} H phase would take place in the second BSFR.
Just as in TGD inspired biology, the increase of the h_{eff} by factor 512 would require "metabolic" energy feed increasing the quark energies proportional to $h_{eff}f$ by this factor. This energy would come from the collision energy of colliding heavy nuclei. The decay of M_{89} hadrons to M_{107} hadrons would occur spontaneously. This kind of decay at the surfaces of the Sun is proposed to be responsible for the generation of solar wind and solar energy [L12].
 - (c) Is the assumption about the labelling of scaled variants of hadron physics by nuclear p-adic length scales too restricted since hadrons (say pions) are labelled also by other p-adic length scales than that of nucleon?

- (d) Could the hierarchy of hadron physics correspond to the hierarchy color representations for quarks and leptons in 1-1 correspondence and labelled by single integer k appearing in the solution spectrum of the Dirac equation in H [L18, L17]. If so, hadrons and leptons for a given value of k would correspond to several p-adic primes?

2.2.2 Progress in the understanding TGD view of the relation between electroweak and strong interactions

TGD view predicts at the fundamental level strong correlations between electroweak, strong interactions and gravitational interactions. But the precise understanding of these correlations has developed rather slowly. The writing of the comments to the GPT prompts was a rather exhaustive process but it was not a waste of time. It led to considerable progress in this respect.

Gluon couplings do not appear in Dirac equations and in [L15] the possibility that there are no gluon vertices at the fundamental level was discussed so that somehow electroweak couplings also describe strong interactions. The recent general view of interactions allows to make these considerations much more detailed.

1. Also for X^4 Dirac equation one obtains quark color and it would naturally correspond to conformal modes proportional to $(\xi_1, \xi_2, 1)$ possible for the induced Dirac equation and perhaps having interpretation as reduction of color triplet to $U(2)$ doublet plus singlet. The triplet corresponds to different coordinate patches of CP_2 to which the three Z_3 poles can be assigned. Therefore one obtains annihilation to quark pairs in this sense. Conformal invariance could make higher modes gauge degrees of freedom.
2. As noticed, a long standing puzzle has been the fact that electroweak $U(2)$ has a holonomy group of CP_2 is the maximal compact subgroup of $SU(3)$. Could one see electroweak interactions as an aspect of color interactions or vice versa? Could one say that there is a symmetry breaking reducing isometry group $SU(3)$ to its subgroup $U(2)$ identifiable as holonomy group and an electroweak gauge group? Could $CP_2 = SU(3)/U(2)$ realize the gauge group nature of $U(2)$ geometrically.

Could the proposed electroweak confinement by the pairs of left and right-handed neutrinos [L17] screening the weak isospin correspond to $SU(2)_L \subset SU(3)$ confinement in spin degrees of freedom. There would be no color confinement for photons associated with $U(1)$. Full color confinement would take place for the light states formed from the H spinor modes.

3. Why are strong interactions strong? The annihilation rate to quark pairs by the proposed vertices is sum of three pairs and the rate is 9 times higher than for the annihilation to leptons. The electroweak coupling strength is of order $\alpha_{em} = 1/137$ so that the rate for quark pair production corresponds to $\alpha_s = 9\alpha_{em} \sim .1$. This would give a correct order of magnitude estimate!
4. Old-fashioned hadron physics talked about conserved vector currents (CVC) and partially conserved axial currents (PCAC). These notions emerged from the observations that hadronic reaction rates can be expressed in terms of correlations of electroweak currents. This raises the question whether strong interactions could reduce to electroweak interactions in some sense [K5].
5. What happens to the scaled up variants of hadron and electroweak physics if strong and electroweak physics fuse to whatever one might call it (unified physics)? The only way to understand why the range of strong interactions is given by the hadronic length scale is that strong interactions would correspond to electroweak interactions in p-adic length scales, which correspond to hadrons and possibly also quarks. Weak bosons should correspond to a much longer Compton scale.

Nucleons would correspond to the p-adic length scale $L(107)$ and pions to $M(113)$. The original view of weak bosons was that weak interactions correspond to the scale $L(89)$ corresponding to Mersenne prime. Weak boson mass scales turned out to correspond to $L(91)$

However, the original view is rather attractive and would fit with the view that M_{89} hadron physics fuses with ordinary electroweak physics and several p-adic length scales are involved

with a given copy. The copies of this unified physics in turn could correspond to color partial waves for Dirac equation in H .

Electro-weak bosons would be special kinds of mesons in the sense that they are superpositions of both quark and lepton pairs. Photon would be even more special in that $SU(2) \subset SU(3)$ confinement would not apply to it because $U(1)$ is abelian.

The scaling hypothesis, stating that the mass scales of mesons are scaled by a factor 512 in the transition $M_{107} \rightarrow M_{89}$, is probably too strong but gives testable predictions to start with.

1. One key question concerns the M_{107} counterparts of weak bosons. They would correspond to genus $g = 0$ (u and d quarks). A naive scaling of masses by factor $1/512$ would give a mass scale near 500 MeV. There is no report about the observation of these bosons.

For ρ meson the mass scale without QCD hyperfinite splitting induced by color magnetism is around 500 MeV. Are these weak bosons separate from ρ assumed to involve only quark pairs or do they correspond to ρ ? For the latter option their decays to leptons should reveal this.

2. What about pseudoscalar π accompanying ρ ? Standard model does not predict pseudoscalar electroweak boson. Its counterpart for M_{89} should exist. Evidence is reported for the existence of a pseudoscalar at the intermediate boson mass scale. For $k = 113$, assignable to the Mersenne prime of the nucleus, one obtains the mass estimate 20 MeV. There is strong evidence for X-boson [L1] with mass around 16-17 MeV and I have considered the interpretation as a weak boson. There is also Ytterbium anomaly which could have the same explanation [C4] and Calcium anomaly [C6] related to the difference of atomic energy levels of different isotopes of Ca. These anomalies are discussed from the TGD view point in [L1] [K4].
3. What about M_{107} counterpart of Higgs scalar with mass of 125 GeV? By a naive scaling, it should have mass about 250 MeV. There are many candidates for scalar mesons (see this) but they have masses above the mass 500 MeV of sigma boson whose existence is still not confirmed. σ is a very broad Breit-Wigner type resonance, which does not support interpretation as a scaled down Higgs boson. For $k = 113$ the mass should be around 32 MeV, about twice the mass for X boson.

2.3 Unitarity constraint and the construction of S-matrix in the TGD framework

The recent TGD based view of particle reactions [L17] replaces QCD type approach with its stringy version and allows the construction of S-matrix for arbitrary initial and final states.

1. The construction of S-matrix in elementary particle degrees of freedom reduces to that for fundamental fermions. There are two levels involved. External particles are constructed as bound states of fundamental fermions giving rise to hadrons, leptons, gauge bosons, and gravitons. Number theoretic vision, in particular Galois confinement [L3] plays a key role in the construction of the bound states.

The fundamental fermions correspond to the modes of the Dirac equation in H , being massless in the 8-D sense. If M^4 has hypercomplex Kähler structure the Dirac equation in H allows massless light color singlet states as many-fermion states [L17].

The analog of the quark phase corresponds to modes of the X^4 Dirac operator for fundamental fermions, which are massless in 4-D sense: color triplets can be understood in terms of CP_2 geometry. The oscillator operators for X^4 modes are expressible in terms of those for H modes [L17].

2. The S-matrix is determined by the overlap of these two fermionic state bases and the unitary matrix describing the scattering in the quark phase. Fermion pair creation in the induced classical fields is the basic vertex and reduces to a defect of the standard smooth structure: these defects give rise to an exotic smooth structure [A1, A2, A3].

In the vertex, fermion current fails to be conserved for the standard smooth structure but is proposed to be conserved for the exotic smooth structure [L5, L4, L15, L17]. The non-vanishing divergence at the defect determines various vertices.

3. Besides the fermionic degrees of freedom, also the geometric degrees of freedom of WCW are included. Holography = holomorphy vision (H-H) [L8, L16, L20, L6] implies that the path integral disappears and there is only a functional integral over 3-surfaces X^3 and the sum over the Bohr orbits for each X^3 . Does the role of the functional integral become trivial with respect to unitarity? Locality in WCW suggests that this is the case. Let us assume in the following that this is indeed the case.

Unitarity is a strong constraint in the construction of S-matrix and will be considered in the sequel.

2.3.1 Two T-matrices corresponding to hadronic phase in H and quark phase in X^4

How could the T-matrix for hadronic phase relate to the T-matrix for the quark phase, call it briefly t ?

1. t would be related to scattering in the string phase, where the quarks would be free or rather at the boundary lines of string world sheets at light-like partonic orbits. The phase would consist only of conformally massless quarks and leptons at the fermionic lines. H-H would determine the space-time surfaces X^4 and fermionic modes.
2. We can start from unitarity. In the hadron phase, the scattering amplitude satisfies the condition $T - T^\dagger = TT^\dagger$. Unitarity would also hold for t in the quark phase. In the forward direction, a cut for T in the forward direction essentially gives the total cross section.
3. The scattering would correspond to two "big" state function reductions (BSFRs) changing the arrow of time [L2, L7]. T would be between the hadron phases and T^\dagger between their time reversals. The same applies to t .

This suggests a concrete interpretation of unitarity. T and T^\dagger would correspond to opposite time directions. Analogously, t and t^\dagger would be associated with a sequence of SSFRs in opposite time directions, increasing the size of the CD as a correlate for the geometric time. This would give a concrete geometric meaning for the unitary conditions.

4. T would decompose into a product of three operators. The first one would be the operator O , which would project from the hadron phase to the quark phase. It could, and actually should, be a 1-1 map. The second operator would be t or t^\dagger , which would describe the scattering operator in the quark phase. The third would be the inverse operation of O . It should be possible to identify it uniquely, but if O is not 1-1, then there might be problems.
5. H-H gives strong conditions. t would correspond to a sequence of SSFRs and classical non-determinism would determine t . The creation of quark pairs is the basic process created by t , and here exotic smooth structures would come into play [L5, L4].

2.3.2 Could the unitarity for T reduce to unitarity for t ?

1. O projects the hadronic state into a state consisting of quarks and the latter evolves according to t . After that, the quark state would to a hadronic state and the inverse of O would be included. The reduction $T \rightarrow t$ from the hadronic level to the quark level takes place if an inverse of O exists.
2. If quark states can be mapped in 1-1 way to hadronic states, then the classical non-determinism, which can be interpreted as a cognitive non-determinism, would completely determine t . Everything would be discrete and extremely simple at the quark level. Note however that quark pair production occurs and the defect of the standard smooth structure as a classical correlate.

The transition involves the usual quantum physical non-determinism, which naturally to O and its inverse. O would be completely determined by the overlap of the spinor modes of H and X^4 determined by H-H.

2.3.3 Can the matrix O be invertible?

Can O define an isometry between two different state spaces? The analog of a projection from the hadron phase to the quark phase is in question, and it need not be an isometry. The analog of projection, or rather, the map, of O from the hadron phase to the quark phase is well-defined. Can O have a unique inverse? Light-likeness in H and light-likeness in X^4 are very different notions physically: is a 1-1 correspondence between hadronic and quark states possible?

1. Could the additional degrees of freedom in the quark phase come from the fact that X^4 is not closed like CP_2 and CD is finite? Conformal modes would diverge in H but not in X^4 and increase the number of the fermion modes. The argument does not seem convincing to me.
2. Classical non-determinism [L11] brings in additional degrees of freedom identified as cognitive degrees of freedom. Could this make isometry possible?
3. Could additional degrees of freedom in the quark phase emerge from an improved measurement resolution needed to "see" the quarks. This would correspond to a larger extension to rationals and that in turn to cognitive non-determinism so that this option is equivalent with the third option.

2.3.4 About the role of hyperfinite factors (HFFs)?

1. HFF [K6, K1] [L9] is a fractal and contains hierarchies of subalgebras isomorphic with HFF itself. The number-theoretic vision assigns such hierarchies as hierarchies of algebraic extensions of rationals. Also measurement accuracy can be defined in terms of algebraic complexity.
2. The concept of inclusion is central. A subfactor corresponds to a subalgebra of the factor. Inclusion is not a 1-1 correspondence nor isometry. For a factor, the trace of the unitary operator is $Tr(Id) = 1$ and for a sub-factor, the trace of the projector to it is $Tr(P) = q \leq 1$. q is quantized. There is a close connection with quantum groups and related concepts. The concept of HFF is particularly natural for fermions, so that it nicely fits into TGD.
3. Does the quark phase correspond to a subfactor of the hadron phase? Could classical non-determinism increase the value of q to unity and make the correspondence an isometric embedding of the quark operator algebra to hadronic operator algebra?

2.4 A brief summary of the TGD based view of standard model interactions

The general view of standard model interactions provided by TGD differs dramatically from the QCD view and also from the Standard Model picture and one might hope that the findings could provide convincing support for the TGD view.

1. Space-time at the fundamental level consists of 4-surfaces X^4 in $H = M^4 \times CP_2$ obeying holography= holomorphy principle (H-H), which reduces the field equations to local algebraic conditions. Theory is exactly solvable.
2. Color is not a spin-like quantum number as in QCD but analogous to orbital angular momentum in CP_2 and characterizes both leptons and quarks. Arbitrarily high color partial waves are possible.
3. All particles are bound states of fundamental fermions. Colored fermions as modes of Dirac equation in H have mass of order CP_2 mass ($\sim 10^{-4} M_{Pl}$) but color singlet many quark states and leptons are light and correspond to the particles observed in the laboratory.
4. By H-H, the Dirac equation in X^4 for the induced spinors in induced spinor structure allows massless quarks and leptons. This phase is the analog of the quark-gluon phase: gluons are not however present, just fundamental fermions. The interaction region for the collision

of particles corresponding to 4-D space-time surfaces with the same generalized complex structure is the intersection of the space-time surfaces consisting of string world sheets so that a stringy description of interactions emerges. TGD generalizes the QCD type description of scattering to all interactions.

5. Color and electroweak interactions are very closely related since CP_2 isometries correspond to $SU(3)$ and holonomies of CP_2 correspond to $U(2)$ identifiable as a subgroup of $SU(3)$. One can say that electroweak interactions are color interactions in electroweak spin degrees of freedom and color partial waves are analogous to angular momentum degrees of freedom.

3 Could standard model have anomalies after all?

I heard very interesting news from LHC (see this). The title of the post at Restoration Monk is "CERN Detects First-Ever Quantum Gravity Clues from Proton Collisions".

The official narrative has been that the standard model works too well so that there are no signals serving as guide lines in attempts to extend the standard model. I have had difficulties with swallowing this story since this claim has been in conflict with what I have learned during years.

However, I learned now that over the past 10 years, deviations from both QCD and Standard Model physics, related to the supposed phase transition to quark gluon plasma, have been observed. The reports of these findings are scattered in literature. The article about these findings has been submitted for publication in The European Physical Journal.

The Google summary, which I obtained using the prompt "anomalous energy distributions in quark-gluon plasma events that deviate from predictions of both the Standard Model and supersymmetry" gives the following general data bits.

1. The LHC experiments have observed unusual patterns in the energy distribution of particles within the quark-gluon plasma, a state of matter believed to have existed shortly after the Big Bang.
2. There are deviations from Standard Model and Supersymmetry: These energy distribution patterns don't match predictions from either the Standard Model, which describes fundamental particles and forces, or supersymmetry, a theoretical framework extending the Standard Model.

The proposal mentioned in the popular article is that the observed anomalous effects could relate to quantum gravity. This would require that Newton's constant is renormalized to a very large value and looks to me unrealistic.

3.1 The general TGD based view of standard model interactions

The general view of standard model interactions provided by TGD differs dramatically from the QCD view and also from the Standard Model picture and one might hope that the findings could provide convincing support for the TGD view.

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3.2 TGD inspired guess for the list of deviations

The above summary is very general and does not reveal any details. I have however worked with the problem of understanding the difference between TGD and standard model view for decades and it is relatively easy to fill in the details.

As a matter of fact, the deviations have been observed in what has been interpreted as a transition to quark plasma phase. They are familiar to me and they have emerged during a time period of about 20 years. I have discussed a large number of potential anomalies of the standard model from the TGD point of view [K2], in particular in the section "Still about quark gluon plasma and M_{89} physics". TGD predicts a hierarchy of standard model physics and the ordinary M_{107} hadron physics and M_{89} hadron physics are only two examples of them. These standard model physics correspond to the hierarchy of color partial waves for quarks and leptons [L18, L17, L22].

The first deviations that I have commented on were reported by ALICE collaboration.

1. RHIC had already observed around 2005 in heavy ion collisions that the phase assumed to be quark gluon plasma at quantum criticality for the formation of quark gluon plasma behaved almost like a perfect fluid [C3]. This was surprising. Around 2010 the same observation was made by LHC in proton-proton collisions.
2. The popular article "ALICE collaboration measures the size of the fireball in heavy-ion collisions" [C7] (see this) appeared in CERN Courier 2111. The fireball served as a meson source and had elongated shape in the direction of the collision axes rather than being a spherical object: this suggests that string-like or meson-like object was in question. TGD interpretation was as a meson of M_{89} hadron physics.
3. The second popular article (see this) in CERN COURIER from year 2113 talks about the observation of Alice suggesting an double ridge structure consisting of two peaks in momentum space corresponding to opposite longitudinal momenta [C1]. Also this suggests a string-like or meson-like structure.

The proposed TGD based interpretation was that the phase transition is not from hadron phase to quark gluon plasma but from ordinary M_{107} hadrons to M_{89} hadrons. In TGD, hadrons correspond to stringy objects made from monopole flux tubes and the stringy object could be a meson of M_{89} hadron physics for which the proton mass is 512 the mass of the ordinary proton. The hadrons of this physics would be dark in the sense that they would have $h_{eff}/h = 512$ so that the size of the dark proton would be that of the ordinary proton. This would make possible geometric resonance. Large value of h_{eff}/h would predict small dissipation and this conforms with the ideal fluid behavior. Hydrodynamics boils down to conservation laws and the classical field equations of TGD are indeed conservation laws for isometry charges solved by holography= holomorphy hypothesis.

4. Also bumps were detected that were first interpreted in the framework of then fashionable SUSY. This interpretation failed and the bumps were forgotten. TGD suggested their interpretation as M_{89} mesons and the estimate for their masses using naive scaling by a factor 512 gave encouraging results: see the sections "Scaled Variants of quarks and leptons" and "Scaled variants of hadron physics and electroweak physics" of [K2].

Some time ago I learned from an anomaly related to weak isospin [L21], discussed from the TGD point of view [L13, L14] in [L21]. There are excellent reasons to expect that this anomaly belongs to the list of findings. The production rate for strange mesons is higher than for their charmed counterparts. This implies charge asymmetry, which is very difficult to understand in QCD since electroweak symmetries and color symmetry are completely uncorrelated.

In the TGD framework, leptons and quarks move in color partial waves and the color partial waves are different for different weak isospin values so that the charge asymmetry emerges at the fundamental level for color interactions.

One particular proposed explanation for the findings made during 10 years in terms of gravitons might have some empirical justification. In the TGD framework, a natural counterpart of graviton would be emission of spin=2 meson of M_{89} hadron physics.

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