Surface area as geometric representation of entanglement entropy?

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June 20, 2019

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1 Introduction

In Thinking Allowed Original there was a link to a talk by James Sully and having the title Geometry of Compression. I must admit that I understood very little about the talk. My not so educated guess is however that information is compressed: UV or IR cutoff eliminating entanglement in short length scales and describing its presence in terms of density matrix - that is thermodynamically - is another manner to say it. The TGD inspired proposal for the interpretation of the inclusions of hyper-finite factors of type II_1 (HFFs) [K4] is in spirit with this.

The space-time counterpart for the compression would be in TGD framework discretization. Discretizations using rational points (or points in algebraic extensions of rationals) make sense also p-adically and thus satisfy number theoretic universality. Discretization would be defined in terms of intersection (rational or in algebraic extension of rationals) of real and p-adic surfaces. At the level of "world of classical worlds" the discretization would correspond to - say - surfaces defined in terms of polynomials, whose coefficients are rational or in some algebraic extension of rationals. Pinary UV and IR cutoffs are involved too. The notion of p-adic manifold allows to interpret the p-adic variants of space-time surfaces as cognitive representations of real space-time surfaces.

Finite measurement resolution does not allow state function reduction reducing entanglement totally. In TGD framework also negentropic entanglement stable under Negentropy Maximixation Principle (NMP) is possible [K2]. For HFFs the projection into single ray of Hilbert space is indeed impossible: the reduction takes always to infinite-D sub-space.

The visit to the URL was however not in vain. There was a link to an article [B1] discussing the geometrization of entanglement entropy inspired by the AdS/CFT hypothesis.

Quantum classical correspondence is basic guiding principle of TGD and suggests that entanglement entropy should indeed have space-time correlate, which would be the analog of Hawking-Bekenstein entropy.

2 Generalization of AdS/CFT to TGD context

AdS/CFT generalizes to TGD context in non-trivial manner. There are two alternative interpretations, which both could make sense. These interpretations are not mutually exclusive. The first interpretation makes sense at the level of "world of classical worlds" (WCW) with symplectic algebra and extended conformal algebra associated with $\delta M^4_\pm$ replacing ordinary conformal and Kac-Moody algebras. Second interpretation at the level of space-time surface with the extended conformal algebras of the light-likes orbits of partonic 2-surfaces replacing the conformal algebra of boundary of AdS^n.

2.1 First interpretation

For the first interpretation 2-D conformal invariance is generalised to 4-D conformal invariance relying crucially on the 4-dimensionality of space-time surfaces and Minkowski space.

1. One has an extension of the conformal invariance provided by the symplectic transformations of $\delta CD \times CP_2$ for which Lie algebra has the structure of conformal algebra with radial light-like coordinate of δM^4_+ replacing complex coordinate z.

2. One could see the counterpart of AdS_n as imbedding space $H = M^4 \times CP_2$ completely unique by twistorial considerations and from the condition that standard model symmetries are obtained and its causal diamonds defined as sub-sets $CD \times CP_2$, where CD is an intersection of future and past directed light-cones. I will use the shorthand CD for $CD \times CP_2$. Strings in $AdS_5 \times S^5$ are replaced with space-time surfaces inside 8-D CD.

3. For this interpretation 8-D CD replaces the 10-D space-time $AdS_5 \times S^5$. 7-D light-like boundaries of CD correspond to the boundary of say AdS_5, which is 4-D Minkowski space so that zero energy ontology (ZEO) allows rather natural formulation of the generalization of AdS/CFT correspondence since

the positive and negative energy parts of zero energy states are localized at the boundaries of CD.

2.2 Second interpretation

For the second interpretation relies on the observation that string world sheets as carriers of induced spinor fields emerge in TGD framework from the condition that electromagnetic charge is well-defined for the modes of induced spinor field.

1. One could see the 4-D space-time surfaces X^4 as counterparts of AdS_4. The boundary of AdS_4 is replaced in this picture with 3-surfaces at the ends of space-time surface at opposite boundaries of CD and by strong form of holography the union of partonic 2-surfaces defining the intersections of the 3-D boundaries between Euclidian and Minkowskian regions of space-time surface with the boundaries of CD. Strong form of holography in TGD is very much like ordinary holography.

2. Note that one has a dimensional hierarchy: the ends of the boundaries of string world sheets at boundaries of CD as pointlike partices, boundaries as fermion number carrying lines, string world sheets, light-like orbits of partonic 2-surfaces, 4-surfaces, imbedding space $M^4 \times CP_2$. Clearly the situation is more complex than for AdS/CFT correspondence.

3. One can restrict the consideration to 3-D sub-manifolds X^3 at either boundary of causal diamond (CD): the ends of space-time surface. In fact, the position of the other boundary is not well-defined since one has superposition of CDs with only one boundary fixed to be piece of light-cone boundary. The delocalization of the other boundary is essential for the understanding of the arrow of time. The state function reductions at fixed boundary leave positive energy part (say) of the zero energy state at that boundary invariant (in positive energy ontology entire state would remain unchanged) but affect the states associated with opposite boundaries forming a superposition which also changes between reduction: this is analog for unitary time evolution. The average for the distance between tips of CDs in the superposition increases and gives rise to the flow of time.

4. One wants an expression for the entanglement entropy between X^3 and its partner. Bekenstein area law allows to guess the general expression for the entanglement entropy: for the proposal discussed in the article the entropy would be the area of the boundary of X^3 divided by gravitational constant: $S = A/4G$. In TGD framework gravitational constant might be replaced by the square of CP_2 radius apart from numerical constant. How gravitational constant emerges in TGD framework is not completely understood although one can deduce for it an estimate using dimensional analyses. In any case, gravitational constant is a parameter which characterizes GRT limit of TGD in which many-sheeted space-time is in long scales replaced with a piece of Minkowski space such that the classical gravitational fields and gauge potentials for sheets are summed. The physics behind this relies on the generaliza-

tion of linear superposition of fields: the effects of different space-time sheets particle touching them sum up rather than fields.

5. The counterpart for the boundary of X^3 appearing in the proposal for the geometrization of the entanglement entropy naturally corresponds to partonic 2-surface or a collection of them if strong form of holography holds true.

2.3 With what kind of systems 3-surfaces can entangle?

With what system X^3 is entangled/can entangle? There are several options to consider and they could correspond to the two TGD variants for the AdS/CFT correspondence.

1. X^3 could correspond to a wormhole contact with Euclidian signature of induced metric. The entanglement would be between it and the exterior region with Minkowskian signature of the induced metric.

2. X^3 could correspond to single sheet of space-time surface connected by wormhole contacts to a larger space-time sheet defining its environment. More precisely, X^3 and its complement would be obtained by throwing away the wormhole contacts with Euclidian signature of induce metric. Entanglement would be between these regions. In the generalization of the formula

$$S = \frac{A}{4\hbar G}$$

area A would be replaced by the total area of partonic 2-surfaces and G perhaps with CP_2 length scale squared.

3. In ZEO the entanglement could also correspond to time-like entanglement between the 3-D ends of the space-time surface at opposite light-like boundaries of CD. M-matrix, which can be seen as the analog of thermal S-matrix, decomposes to a product of hermitian square root of density matrix and unitary S-matrix and this hermitian matrix could also define p-adic thermodynamics. Note that in ZEO quantum theory can be regarded as square root of thermodynamics.

2.4 Minimal surface property is not favored in TGD framework

Minimal surface property for the 3-surfaces X^3 at the ends of space-time surface looks at first glance strange but a proper generalization of this condition makes sense if one assumes strong form of holography. Strong form of holography realizes General Coordinate Invariance (GCI) in strong sense meaning that light-like parton orbits and space-like 3-surfaces at the ends of space-time surfaces are equivalent physically. As a consequence, partonic 2-surfaces and their 4-D tangent space data must code for the quantum dynamics.

The mathematical realization is in terms of conformal symmetries accompanying the symplectic symmetries of $\delta M^4_\pm \times CP_2$ and conformal transformations of the light-like partonic orbit [?]. The generalizations of ordinary conformal algebras correspond to conformal algebra, Kac-Moody algebra at the light-like parton orbits and to symplectic transformations $\delta M^4 \times CP_2$ acting as isometries of WCW and having conformal structure with respect to the light-like radial coordinate plus conformal transformations of δM^4_+, which is metrically 2-dimensional and allows extended conformal symmetries.

1. If the conformal realization of the strong form of holography works, conformal transformations act at quantum level as gauge symmetries in the sense that generators with no-vanishing conformal weight are zero or generate zero norm states. Conformal degeneracy can be eliminated by fixing the gauge somehow. Classical conformal gauge conditions analogous to Virasoro and Kac-Moody conditions satisfied by the 3-surfaces at the ends of CD are natural in this respect. Similar conditions would hold true for the light-like partonic orbits at which the signature of the induced metric changes.

2. What is also completely new is the hierarchy of conformal symmetry breakings associated with the hierarchy of Planck constants $h_{eff}/h = n$ [K1]. The deformations of the 3-surfaces which correspond to non-vanishing conformal weight in algebra or any sub-algebra with conformal weights vanishing modulo n give rise to vanishing classical charges and thus do not affect the value of the Kähler action [?].

   The inclusion hierarchies of conformal sub-algebras are assumed to correspond to those for hyper-finite factors. There is obviously a precise analogy with quantal conformal invariance conditions for Virasoro algebra and Kac-Moody algebra. There is also hierarchy of inclusions which corresponds to hierarchy of measurement resolutions. An attractive interpretation is that singular conformal transformations relate to each other the states for broken conformal symmetry. Infinitesimal transformations for symmetry broken phase would carry fractional conformal weights coming as multiples of $1/n$.

3. Conformal gauge conditions need not reduce to minimal surface conditions holding true for all variations.

4. Note that Kähler action reduces to Chern-Simons term at the ends of CD if weak form of electric magnetic duality holds true. The conformal charges at the ends of CD cannot however reduce to Chern-Simons charges by this condition since only the charges associated with CP_2 degrees of freedom would be non-trivial.

2.5 Technicalities

The generalisation of the conjecture about surface area proportionality of entropy to TGD context looks rather straightforward but is physically highly non-trivial. There are however some technicalities involved.

1. In TGD framework it is not quite clear whether

   (a) G still appears in the formula or

(b) whether G should be replaced with the square R^2 of CP_2 radius to give

$$S = \frac{A}{4\pi R^2}$$

apart from numerical constant.

For option a) one must include Planck constant explicitly to the formula to give $S = A/4h_{eff}G$: the entropy would decrease as $h_{eff} = n \times h$ increases. The condition $h_{eff} = h_{gr} = GM^2/v_0$ would give $S = v_0/c < 1$. The entropy using b) would be by a factor of order 10^{-5} smaller and would not depend on the value of h_{eff} at all. It will be found that p-adic mass calculations lead to entropy allowing to circumvent these problems.

2. There is also the question about the identification of the area A. For blackhole A would be determined by Schwartschild radius $r_S = 2GM$ depending on mass only. In TGD framework one has several candidates.

   (a) The area of partonic 2-surface is an obvious first guess. One cannot however expect that the area of partonic 2-surface is constant. Could conformal gauge fixing fixes the 3-surfaces highly uniquely. Ordinary conformal invariance for partonic 2-surface does not however seem to be consistent with the fixing of the area of partonic 2-surface since conformal transformations do not preserve area.

   (b) Could the area of partonic 2-surface be replaced with the area of the boundary of space-time sheet at which particle is topologically condensed and has size scale of order Compton length? This option looks the most feasible one on basis of p-adic mass calculations as will be found.

2.6 p-Adic variant of Bekenstein-Hawking law

When the 3-surface corresponds to elementary particle, a direct connection with p-adic thermodynamics suggests itself and allows to answer the questions above. p-Adic thermodynamics could be interpreted as a description of the entanglement with environment. In ZEO the entanglement could also correspond to time-like entanglement between the 3-D ends of the space-time surface at opposite light-like boundaries of CD. M-matrix, which can be seen as the analog of thermal S-matrix, decomposes to a product of hermitian square root of density matrix and unitary S-matrix and this hermitian matrix could also define p-adic thermodynamics.

1. p-Adic thermodynamics [K5] would not be for energy but for mass squared (or scaling generator L_0) would describe the entanglement of the particle with environment defined by the larger space-time sheet. Conformal weights would comes as positive powers of integers (p_0^L would replace $exp(-H/T)$ to guarantee the number theoretical existence and convergence of the Boltzmann weight: note that conformal invariance that is integer spectrum of L_0 is also essential).

2. The interactions with environment would excite very massive CP_2 mass scale excitations (mass scale is about 10^{-4} times Planck mass) of the particle and give it thermal mass squared identifiable as the observed mass squared. The Boltzmann weights would be extremely small having p-adic norm about $1/p^n$, p the p-adic prime: $M_{127} = 2^{127} - 1$ for electron.

3. One of the first ideas inspired by p-adic vision was that p-adic entropy could be seen as a p-adic counterpart of Bekenstein-Hawking entropy [K3]. $S = (R^2/\hbar^2) \times M^2$ holds true identically apart from numerical constant. Note that one could interpret $R^2M/\hbar$ as the counterpart of Schwartschild radius. Note that this radius is proportional to $1/\sqrt{p}$ so that the area A would correspond to the area defined by Compton length. This is in accordance with the third option.

2.7 What is the space-time correlate for negentropic entanglement?

The new element brought in by TGD framework is that number theoretic entanglement entropy is negative for negentropic entanglement assignable to unitary entanglement and NMP states that this negentropy increases [K2]. Since entropy is essentially number of energy degenerate states, a good guess is that the number $n = h_{eff}/h$ of space-time sheets associated with h_{eff} defines the negentropy. An attractive space-time correlate for the negentropic entanglement is braiding. Braiding defines unitary S-matrix between the states at the ends of braid and this entanglement is negentropic. This entanglement gives also rise to topological quantum computation.

REFERENCES

Theoretical Physics

[B1] Takayanagi T Ryu S. Holographic derivation of entanglement entropy from ads/cft. arXiv:1401.4118/[hep-th].Available at: http://arxiv.org/pdf/hep-th/0603001v2.pdf, 2008.

Books related to TGD

[K1] Pitkänen M. Does TGD Predict the Spectrum of Planck Constants? In *Hyper-finite Factors and Dark Matter Hierarchy*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/neuplanck.html#Planck, 2006.

[K2] Pitkänen M. Negentropy Maximization Principle. In *TGD Inspired Theory of Consciousness*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/tgdconsc.html#nmpc, 2006.

[K3] Pitkänen M. p-Adic Physics: Physical Ideas. In *TGD as a Generalized Number Theory*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/tgdnumber.html#phblocks, 2006.

[K4] Pitkänen M. Was von Neumann Right After All? In *Hyper-finite Factors and Dark Matter Hierarchy*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/padphys.html#vNeumann, 2006.

[K5] Pitkänen M. *p-Adic length Scale Hypothesis*. Online book. Available at: http://www.tgdtheory.fi/tgdhtml/padphys.html, 2013.