Electron as a trefoil or something more general?

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Abstract

There have been suggestions that elementary particle could be braided structure and that standard model quantum numbers could be reduced to topology. In TGD framework this option does not look plausible. The braiding at the level of wormhole throat orbits is however in principle possible but need not be significant for the known elementary particles. In TGD framework elementary particle is identified as a closed Kähler magnetic flux tube carrying monopole magnetic field. This flux tube is accompanied by a closed string representing the end of string world sheet carrying induced spinor field. This string can be homologically non-trivial curve and can also get knotted. Bosons even braiding becomes possible and in the general case knotting, braiding, and non-trivial homology are possible. Therefore an extremely rich topological structure is predicted, which might corresponds to relatively low energy scale. Topological sum for knots and reconnection are the basic topological reactions for these strings.

1 Introduction

The possibility that electron, and also other elementary particles could correspond to knot is very interesting. The video model [B1] was so fascinating (I admire the skills of the programmers) that I started to question my belief that all related to knots and braids represents new physics (say anyons), [K2] and that it is hopeless to try to reduce standard model quantum numbers with purely group theoretical explanation (except family replication) to topological quantum numbers.

Electroweak and color quantum numbers should by quantum classical correspondence have geometric correlates in space-time geometry. Could these correlates be topological? As a matter of fact, the correlates existing if the present understanding of the situation is correct but they are not topological.

Despite this, I played with various options and found that in TGD Universe knot invariants do not provide plausible space-time correlates for electroweak quantum numbers. The knot invariants and many other topological invariants are however present and mean new physics. As following arguments try to show, elementary particles in TGD Universe are characterized by extremely rich spectrum of topological quantum numbers, in particular those associated with knotting and linking: this is basically due to the 3-dimensionality of 3-space.

For a representation of trefoil knot by R.W. Gray see
http://www.rwgrayprojects.com/Lynn/Presentation20070926/p008.html. The homepage of Louis Kauffman [A2] is a treasure trove for anyone interested in ideas related to possible applications of knots to physics. One particular knotty idea is discussed in the article Emergent Braided Matter of Quantum Geometry by Bilson-Thompson, Hackett, and Kauffman [B2].

2 Space-time as 4-surface and the basic argument

Space-time as a 4-surface in $M^4 \times CP_2$ is the key postulate. The dynamics of space-time surfaces is determined by so called Kähler action - essentially Maxwell action for the Kähler form of CP_2 induced to X^4 in induced metric. Only so called preferred extremals are accepted and one can in very loose sense say that general coordinate invariance is realized by assigning to a given 3-surface a unique 4-surface as a preferred extremal analogous to Bohr orbit for a particle identified as 3-D surface rather than point-like object.

One ends up with a radical generalization of space-time concept to what I call many-sheeted space-time. The sheets of many-sheeted space-time are at distance of CP_2 size scale (10^4 Planck lengths as it turns out) and can touch each other which means formation of wormhole contact with wormhole throats as its ends. At throats the signature of the induced metric changes from Minkowskian to Euclidian. Euclidian regions are identified as 4-D analogs of lines of generalized Feynman diagrams and the M^4 projection of wormhole contact can be arbitrarily large: macroscopic, even astrophysical. Macroscopic object as particle like entity means that it is accompanied by Euclidian region of its size.

Elementary particles are identified as wormhole contacts. The wormhole contacts born in mere touching are not expected to be stable. The situation changes if there is a monopole magnetic flux (CP_2 carries self dual purely homological monopole Kähler form defining Maxwell field, this is not Dirac monopole) since one cannot split the contact. The lines of the Kähler magnetic field must be closed, and this requires that there is another wormhole contact nearby. The magnetic flux from the upper throat of contact A travels to the upper throat of contact B along "upper space-time sheet", goes to "lower" space-time sheet along contact B and returns back to the wormhole contact A so that closed loop results.

In principle, wormhole throat can have arbitrary orientable topology characterized by the number g of handles attached to sphere and known as genus. The closed flux tube corresponds to topology $X^2_g \times S^1$, g=0,1,2,... Genus-generation correspondence [K1] states that electron, muon, and tau lepton and similarly quark generations correspond to $g = 0, 1, 2$ in TGD Universe and CKM mixing is induced by topological mixing.

Suppose that one can assign to this flux tube a closed string: this is indeed possible but I will not bother reader with details yet. What one can say about the topology of this string?

1. X^2_g has homology Z^{2g} and S^1 homology S^1. The entire homology is Z^{2g+1} so that there are $2g + 1$ additional integer valued topological quantum numbers besides genus. Z^{2g+1} obviously breaks topologically universality stating that fermion generations are exact copies of each other apart from mass. This would be new physics. If the size of the flux loop is of order Compton length, the topological excitations need not be too heavy. One should however know how to excite them.

2. The circle S^1 is imbedded in 3-surface and can get knotted. This means that all possible knots characterize the topological states of the the fermion. Also this means extremely rich spectrum of new physics.

3 What is the origin of strings going around the magnetic flux tube?

What is then the origin of these knotted strings? The study of the modified Dirac equation [K3] determining the dynamics of induced spinor fields at space-time surface led to a considerable insight here. This requires however additional notions such as zero energy ontology (ZEO), and causal diamond (CD) defined as intersection of future and past directed light-cones (double 4-pyramid is the M^4 projection. Note that CD has CP_2 as Cartesian factor and is analogous to Penrose diagram.

1. ZEO means the assumption that space-time surfaces for a particular sub-WCW ("world of classical worlds") are contained inside given CD identifiable as a the correlate for the "spotlight of consciousness" in TGD inspired theory of consciousness. The space-time surface has ends at the upper and lower light-like boundaries of CD. The 3-surfaces at the the ends define space-time correlates for the initial and final states in positive energy ordinary ontology. In ZEO they carry opposite total quantum numbers.

2. General coordinate invariance (GCI) requires that once the 3-D ends are known, space-time surface connecting the ends is fixed (there is not path integral since it simply fails). This reduces ordinary holography to GCI and makes classical physics defined by preferred extremals an exact part of quantum theory, actually a key element in the definition of Kähler geometry of WCW.

   Strong form of GCI is also possible. One can require that 3-D light-like orbits of wormhole throats at which the induced metric changes its signature, and space-like 3-surfaces at the ends of CD givee equivalent descriptions. This implies that quantum physics is coded by the their intersections which I call partonic 2-surfaces - wormhole throats - plus the 4-D tangent spaces of X^4 associated with them. One has strong form of holography. Physics is almost 2-D but not quite: 4-D tangent space data is needed.

3. The study of the modified Dirac equation [K3] leads to further results. The mere conservation of electromagnetic charge defined group theoretically for the induced spinors of $M^4 \times CP_2$ carrying spin and electroweak quantum numbers implies that for all other fermion states except right handed neutrino (, which does not couple at all all to electroweak fields), are localized at 2-D string world sheets and partonic 2-surfaces.

   String world sheets intersect the light-like orbits of wormhole throats along 1-D curves having interpretation as timelike braid strands (a convenient metaphor: braiding in time direction si created by dancers in the parquette).

   One can say that dynamics automatically implies effective discretization: the ends of time like braid strands at partonic 2-surfaces at the ends of CD define a collection of discrete points to each of which one can assign fermionic quantum numbers.

4. Both throats of the wormhole contact can carry many fermion state and known fermions correspond to states for which either throat carries single braid strand. Known bosons correspond to states for which throats carry fermion and antifermion number.

5. Partonic 2-surface is replaced with discrete set of points effectively. The interpretation is in terms of a space-time correlate for finite measurement resolution. Quantum correlate would be the inclusion of hyperfinite factors of type II_1.

This interpretation brings in even more topology!

1. String world sheets - present both in Euclidian and Minkowskian regions - intersect the 3-surfaces at the ends of CD along curves - one could speak of strings. These strings give rise to the closed curves that I discussed above. These strings can be homologically non-trivial - in string models this corresponds to wrapping of branes.

2. For known bosons one has two closed loop but these loops could fuse to single. Space-like 2-braiding (including linking) becomes possible besides knotting.

3. When the partonic 2-surface contains several fermionic braid ends one obtains even more complex situation than above when one has only single braid end. The loops associated with the braid ends and going around the monopole flux tube can form space-like N-braids. The states containing several braid ends at either throat correspond to exotic particles not identifiable as ordinary elementary particles.

4 How elementary particles interact as knots?

Elementary particles could reveal their knotted and even braided character via the topological interactions of knots. There are two basic interactions.

1. The basic interaction for single string is by self-touching and this can give to a local connected sum or a reconnection. In both cases the knot invariants can change and it is possible to achieve knotting or unknotting of the string by this mechanism. String can also split into two pieces but this might well be excluded in the recent case.

The space-time dynamics for these interactions is that of closed string model with 4-D target space. The first guess would be topological string model describing only the dynamics of knots. Note that string world sheets define 2-knots and braids.

2. The basic interaction vertex for generalized Feynman diagrams (lines are 4-D space-time regions with Euclidian signature) is join along 3-D boundaries for the three particles involved: this is just like ordinary 3-vertex for Feynman diagrams and is not encountered in string models. The ends of lines must have same genus g. In this interaction vertex the homology charges in Z^{2g+1} is conserved so that these charges are analogous to U(1) gauge charges. The strings associated with the two particles can touch each other and connected sum or reconnection is the outcome.

Consider now in more detail connected sum and reconnection vertices responsible for knotting and un-knotting.

1. The first interaction is connected sum of knots [A1]. A little mental exercise demonstrates that a local connected sum for the pieces of knot for which planar projections cross, can lead to a change in knotted-ness. Local connected sum is actually used to un-knot the knot in the construction of knot invariants.

   In dimension 3 knots form a module with respect to the connected sum. One can identify unique prime knots and construct all knots as products of prime knots with product defined as a connected sum of knots. In particular, one cannot have a situation on which a product of two non-trivial knots is un-knot so that one could speak about the inverse of a knot (indeed, the inverse of ordinary prime is not an integer!). For higher-dimensional knots the situation changes (string world sheets at space-time surface could form 2-knots but instead of linking they intersect at discrete points).

   Connected sum in the vertex of generalized Feynman graph (as described above) can lead to a decay of particle to two particles, which correspond to the summands in the connected sum as knots. Could one consider a situation in which un-knotted particle decomposes via the time inverse of the connected sum to a pair of knotted particles such that the knots are inverses of each other? This is not possible since knots do not have inverse.

2. Touching knots can also reconnect. For braids the strands $A \rightarrow B$ and $C \rightarrow D$ touch and one obtains strands $A \rightarrow D$ and $C \rightarrow B$. If this reaction takes place for strands whose planar projections cross, it can also change the character of the knot. One one can transform knot to un-knot by repeatedly applying connected sum and reconnection for crossing strands (the Alexandrian way).

3. In the evolution of knots as string world sheets these two vertices corresponds to closed string vertices. These vertices can lead to topological mixing of knots leading to a quantum superposition of different knots for a given elementary particle. This mixing would be analogous to CKM mixing understood to result from the topological mixing of fermion genera in TGD framework. It could also imply that knotted particles decay rapidly to un-knots and make the un-knot the only long-lived state.

   A naive application of Uncertainty Principle suggests that the size scale of string determines the life time of particular knot configuration. The dependence on the length scale would however suggest that purely topological string theory cannot be in question. Zero energy ontology suggests that the size scale of the causal diamond assignable to elementary particle determines the time scale for the rates as secondary p-adic time scale: in the case of electron the time scale would be .1 seconds corresponding to Mersenne prime $M_{127} = 2^{127} - 1$ so that knotting and unknotting would be very slow processes. For electron the estimate for the scale of mass differences between different knotted states would be about $10^{-19} m_e$: electron mass is known for certain for 9 decimals so that there is no hope of detecting these mass differences. The pessimistic estimate generalizes to all other elementary particles: for weak bosons characterized by M_{89} the mass difference would be of order $10^{-13} m_W$.

4. A natural guess is that p-adic thermodynamics can be applied to the knotting. In p-adic thermodynamics Boltzmann weights in are of form $p^{H/T}$ (p-adic number) and the allowed values of the Hamiltonian H are non-negative integer powers of p. Clearly, H representing a contribution

to p-adic valued mass squared must be a non-negative integer valued invariant additive under connected sum. This guarantees extremely rapid convergence of the partition function and mass squared expectation value as the number of prime knots in the decomposition increases.

An example of an knot invariant [A4] additive under connected sum is knot genus [A3] defined as the minimal genus of 2-surface having the knot as boundary (Seifert surface). For trefoil and figure eight knot one has $g = 1$. For torus knot $(p, q) \equiv (q, p)$ one has $g = (p-1)(q-1)/2$. Genus vanishes for un-knot so that it gives the dominating contribution to the partition function but a vanishing contribution to the p-adic mass squared.

p-Adic mass scale could be assumed to correspond to the primary p-adic mass scale just as in the ordinary p-adic mass calculations. If the p-adic temperature is $T = 1$ in natural units (highest possible), and if one has $H = 2g$, the lowest order contribution corresponds to the value $H = 2$ of the knot Hamiltonian, and is obtained for trefoil and figure eight knot so that the lowest order contribution to the mass would indeed be about $10^{-19}m_e$ for electron. An equivalent interpretation is that $H = g$ and $T = 1/2$ as assumed for gauge bosons in p-adic mass calculations.

There is a slight technical complication involved. When the string has a non-trivial homology in $X^2_g \times S^1$ (it always has by construction), it does not allow Seifert surface in the ordinary sense. One can however modify the definition of Seifert surface so that it isolates knottedness from homology. One can express the string as connected sum of homologically non-trivial un-knot carrying all the homology and of homologically trivial knot carrying all knottedness and in accordance with the additivity of genus define the genus of the original knot as that for the homologically trivial knot.

5. If the knots assigned with the elementary particles have large enough size, both connected sum and reconnection could take place for the knots associated with different elementary particles and make the many particle system a single connected structure. TGD based model for quantum biology is indeed based on this kind of picture. In this case the braid strands are magnetic flux tubes and connect bio-molecules to single coherent whole. Could electrons form this kind of stable connected structures in condensed matter systems? Could this relate to super-conductivity and Cooper pairs somehow? If one takes p-adic thermodynamics for knots seriously then knotted and braided magnetic flux tubes are more attractive alternative in this respect.

What if the thermalization of knot degrees of freedom does not take place? One can also consider the possibility that knotting contributes only to the vacuum conformal weight and thus to the mass squared but that no thermalization of ground states takes place. If the increment Δm of inertial mass squared associated with knotting is of from kgp^2, where k is positive integer and g the above described knot genus, one would have $\Delta m/m \simeq 1/p$. This is of order $M_{127}^{-1} \simeq 10^{-38}$ for electron.

Could the knotting and linking of elementary particles allow topological quantum computation at elementary particle level? The huge number of different knottings would give electron a huge ground state degeneracy making possible negentropic entanglement. For negentropic entanglement probabilities must belong to an algebraic extension of rationals: this would be the case in the intersection of p-adic and real worlds and there is a temptation to assign living matter to this intersection. Negentropy Maximization Principle could stabilize negentropic entanglement and therefore allow to circumvent the problems due to the fact that the energies involved are extremely tiny and far below thus thermal energy. In this situation bit would generalize to "nit" corresponding to N different ground states of particle differing by knotting.

A very naive dimensional analysis using Uncertainty Principle would suggest that the number changes of electron state identifiable as quantum computation acting on q-nits is of order $1/\Delta t = \Delta m/hbar$. More concretely, the minimum duration of the quantum computation would be of order $\Delta t = \hbar/\Delta m$. Single quantum computation would take an immense amount time: for electron single operation would take time of order 10^{17} s, which is of the order of the recent age of the Universe. Therefore this quantum computation would be of rather limited practical value!

Mathematics

[A1] Connected sum. http://en.wikipedia.org/wiki/Connected_sum.

[A2] Homepage of Louis Kauffman. http://homepages.math.uic.edu/~kauffman/.

[A3] Knot genus. http://en.wikipedia.org/wiki/Knot_genus.

[A4] Knot invariants. http://en.wikipedia.org/wiki/Knot_invariants.

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[B2] L. Kauffman S. Bilson-Thompson, J. Hackett. Emergent Braided Matter of Quantum Geometry. http://arxiv.org/pdf/1109.0080v2.pdf, 2011.

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[K3] M. Pitkänen. The Recent Vision About Preferred Extremals and Solutions of the Modified Dirac Equation. In *Quantum Physics as Infinite-Dimensional Geometry*. Onlinebook. http://tgdtheory.com/public_html/tgdgeom/tgdgeom.html#dirasvira, 2012.