The results of a project related to the holography= holomorphy vision

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Matti Pitkänen[1], Tuomas Sorakivi[2] and Marko T. Manninen[3]

[1] **email**: matpitka6@gmail.com,
url: http://tgdtheory.com/public_html/,
[2] **email**: tuomas@gigabrain.io,
[3] **email**: elonmedia@gmail.com.

Abstract

This article is about a collaboration inspired by our Zoom group and included Tuomas Sorakivi and Marko Manninen besides me. Holography = holomorphy vision, reduces field equations to algebraic equations and predicts that space-times are 4-dimensional minimal surfaces in $H = M^4 \times CP_2$ identifiable as roots for a pair $f = (f_1, f_2)$ of analytic maps $f_i : H \to C^2$ depending on single hypercomplex and 3 complex coordinates of H. The analytic maps $g = (g_1, g_2) : C^2 \to C^2$ act as dynamical symmetries. The simplest choices of f_i and g_i are as polynomials. The graphical visualization of the space-time surfaces with the assistance of a large language model (LLM) was one part of the project. In the case studied, the model reduced to a visualization of elliptic surfaces as surfaces in C^2 determined by analytically solvable third order polynomial.

The origin of the p-adic primes as characterizers of elementary particles and p-adic length scale hypothesis are two key problems of TGD. The working hypothesis has been that the p-adic primes correspond to ramified primes characterizing polynomials, say as $g = (g_1 = P_1, g_2 = Id)$. There is also a second option. One can define the notion of primeness for f *resp.* g as maps not decomposing as $f = g \circ h$ *resp.* $g = g_1 \circ g_2$. These prime maps are polynomials with degree which is prime p and universal in the sense that only the degree as a counterpart of ordinary p-adic prime p matters.

Holography = holomorphy vision allows to sharpen these two hypotheses and ask could p-adic primes p near power of 2 be associated with iterates P_2^k for a suitable P_2. The concrete LLM assisted test of ramified prime hypothesis involved the calculation of the roots of the iterates $P_2^{\circ k}$ for $P_2 = x(x-1)$. The results were not encouraging. This led to the realization that there seems to be no way to guess what the correct polynomial might be and why it would be physically special.

The concrete study of the alternative hypothesis led to what might be regarded as a solution of the two key problems, perhaps even as breakthrough. One ends up to a generalization of the arithmetics of maps g based on ordinary product $\times$ and sum $+$ to an arithmetics based on functional composition $\circ$ replacing $\times$ and $\times$ replacing sum. The condition that the polynomial coefficients a_k in $a_k \circ P_p^{\circ k}$ appearing in the pinary expansion of a functional p-adic number, with a_k and P_p being analogs of quantum mechanical observables, commute with P_p and also mutually, implies that functional p-adic numbers have a natural morphism to ordinary p-adic numbers.

The functional p-adic counterparts of ordinary primes can be identified, the p-adic length scale hypothesis can be understood, and the origin of p-adic physics can be identified. It is also possible to understand how the predicted slight failure of classical determinism corresponds to p-adic non-determinism, how quantum criticality is realized, and how the hierarchy of effective Planck constants emerges.

Contents

1 Introduction

In TGD, geometric and number theoretic visions of physics are complementary. This complementarity is analogous to momentum position duality of quantum theory and implied by the replacement of a point-like particle with 3-surface, whose 4-dimensional Bohr orbit defines space-time surface.

1. Space-time surfaces are identified as roots of $f = (f_1, f_2)$ is a pair of analytic functions $H = M^4 \times CP_2 \rightarrow C^2$. Analyticity means that they depend only on the second hypercomplex (real) coordinate of the pair (u, v), say u, complex coordinates w of M^4, and complex coordinates (ξ_1, ξ_2) of CP_2 but not on their complex conjugates. Light-like coordinates $(u, v) = (t+z, t-z)$, where (t, z) are Minkowski coordinates for $M^2 \subset M^4 = M^2 \times E^2$ and complex coordinate $w = x + iy$ of the plane E^2 represents the basic example.

2. The functional compositions $g \circ f$, and $g = (g_1, g_2) : C^2 \rightarrow C^2$ is a pair analytic functions of 2 complex variables, define a spectrum generating algebra. The interpretation is as a cognitive hierarchy of function of functions of .... and the pairs (f_1, f_2) which do not allow a composition of form $f = g \circ h$ correspond to elementary function and to the lowest levels of this hierarchy, kind of elementary particles of cognition. Also the pairs g can be expressed as composites of elementary functions.

   If g_1 and g_2 are polynomials with coefficients in field E identified as an extension of rationals, one can assign to $g \circ f$ root a set of pairs (r_1, r_2) as roots $f_1, f_2) = (r_1, r_2)$ and r_i are algebraic numbers defining disjoint space-time surfaces. One can assign to the set of root pairs the analog of the Galois group as automorphisms of the algebraic extension of the field E appearing as the coefficient field of (f_1, f_2) and (g_1, g_2). This hierarchy leads to the idea that physics could be seen as an analog of a formal system appearing in Gödel's theorems and that the hierarchy of functional composites could correspond to a hierarchy of meta levels in mathematical cognition.

3. The quantum generalization of integers, rationals and algebraic numbers to their functional counterparts is possible for maps $g : C^2 \to C^2$. The counterpart of the ordinary product is functional composition $\circ$ for maps g. Degree is multiplicative in $\circ$. In sum, call it $+_e$, the degree should be additive, which leads to the identification of the sum $+_e$ as an element-wise product. The neutral element $1_\circ$ of $\circ$ is $1_\circ = Id$ and the neutral element 0_e of $+_e$ is the ordinary unit $0_e = 1$.

   The inverses correspond to g^{-1} for $\circ$, which in general is a many-valued algebraic function and to $1/g$ for *times*. The maps g, which do not allow decomposition $g = h \circ i$, can be identified as functional primes and have prime degree. $f : H \to C^2$ is prime if it does not allow composition $f = g \circ h$. Functional integers are products of functional primes.

   The non-commutativity of $\circ$ could be seen as a problem. The fact that the maps g act like operators suggest that the functional primes g_p in the product commute. Functional integers/rationals can be mapped to ordinary by a morphism mapping their degree to integer/rational.

4. One can define functional polynomials $P(X)$, quantum polynomials, using these operations. In $P(X)$, the terms $p_n \circ X^n$, p_n and X should commute. The sum $\sum_e p_n X^n$ corresponds to $+_e$. The zeros of functional polynomials satisfy the condition $P(X) = 0_e = 1$ and give as solutions roots X_k as functional algebraic numbers. The fundamental theorem of algebra generalizes at least formally if X_k and X commute. The roots have representation as a space-time surface. One can also define functional discriminant D as the $\circ$ product of root differences $X_k -_e X_l$, with $-_e$ identified as element-wise division and the functional primes dividing it have space-time surface as a representation.

5. The iteration of functional rimes g_p defines analogs for the powers of p-adic primes and one can define a functional generalization of p-adic numbers as quantum p-adics. The coefficients of g_p^k are now polynomials with degree smaller than p. The generalization of

   Witt polynomials as a representation of p-adic numbers relies on the same arithmetics as the definition of integers and makes it possible to realize the functional p-adic numbers as space-time surfaces. The space-time surfaces as roots of Witt polynomials are characterized by ramified primes. The iterates of prime polynomials g_p might allow us to understand the p-adic length scale hypothesis.

   Large powers of prime appearing in p-adic numbers must approach 0_e with respect to the p-adic norm so that g_p^n must effectively approach Id with respect to $\circ$. Intuitively, a large n in g_P^n corresponds to a long p-adic length scale. For large n, g_p^n cannot be realized as a space-time surface in a fixed CD. This would prevent their representation and they would correspond to 0_e and Id. During the sequence of SSFRs the size of CD increases and for some critical SSFRs a new power can emerge to the quantum p-adic.

6. One can consider also the analogs of multi-p p-adic numbers with functional prime g_p replaced with a functional integer $g_n = g_{p_1} \circ g_{p_2} ... \circ g_{p_m}$, $n = p_1 \times p_2 ... \times p_m$. A functional multi-p-adic number would be a product of factors $a_k \circ g_n^{\circ k}$. The coefficients a_k as polynomials with degree smaller than n would $\circ$-commute with g_n and with each other. It is not clear whether g_{p_i} should $\circ$-commute with each other. These numbers would form functional adeles.

7. In the TGD framework, one can consider a potential connection to biology and genetic code. TGD associates to genes what I call dark genes [L2, L5]. They consist of dark proton triplets at monopole flux tubes associated with the DNA. Dark genes would be dynamical units and superpositions of different dark genes would be analogous to bit sequences and could be involved with quantum computation-like operations [L7] so that the dark genome would be a

kind of R& D department of the genome. The ordinary gene would corresponds to the most probable configuration in a given superposition as a minimum energy state.

If cognition and data processing reduce to the g-sector, also genetic code could do so. Genetic code involves 61 active codons plus 3 stop codons. Could the 64 DNA codons correspond to the 64 roots of $g_2^{\circ 6}$ and could the functional prime $g_{61} = g_2^{\circ 6}/g_3$ correspond to the 61 codons coding for amino acids to the 3 stop codons.

Since only the degree of the prime polynomial matters, the factors $g_{2,i}$ in the power need not be identical and therefore need not be $\circ$-commutative. Could this serve as a correlate for entanglement of the protons of dark letters and codons? The DNA letters are bit pairs rather than bits. Could the 4 letters correspond to pairs $g_{2,1} \circ g_{2,2}$ in which $g_{2,1}$ and $g_{2,2}$ do not $\circ$ -commute and therefore form an indivisible whole. Could genes correspond to $\circ$-products of codons and could quantum entanglement for codons mean non-commutativity of codons with respect to $\circ$?

There are many open questions.

1. Could the transitions $f \to g \circ f$ correspond to the classical non-determinism in which one root of g is selected? If so, the p-adic non-determinism would correspond to classical non-determinism. Quantum superposition of the roots would make it possible to realize the quantum notion of concept.

2. What is the interpretation of the maps g^{-1} which in general are many-valued algebraic functions if g is rational function? g increases the complexity but g^{-1} preserves or even reduces it so that its action is entropic. Could selection between g and g_{-1} relate to a conscious choice between good and evil?

3. Could one understand the p-adic length scale hypothesis in terms of functional primes. The counter for functional Mersenne prime would be g_2^n/g_1, where division is with respect to elementwise product defining $+_e$? For g_2 and g_3 and also their iterates the roots allow analytic expression. Could primes near powers of g_2 and g_3 be cognitively very special?

This article summarizes results of a collaboration inspired by our Zoom group and included Tuomas Sorakivi and Marko Manninen besides me who provided the IT knowhow that I do not have. Holography = holomorphy vision reduces field equations to algebraic equations and predicts that space-times are 4-dimensional minimal surfaces in $H = M^4 \times CP_2$ identifiable as roots for a pair $f = (f_1, f_2)$ of analytic maps $f_i : H \to C^2$ depending on single hypercomplex and 3 complex coordinates of H. The analytic maps $g = (g_1, g_2) : C^2 \to C^2$ act as dynamical symmetries. The simplest choices of f_i and g_i are as polynomials.

What is especially nice is that the dynamics associated with g and f separate. The roots of $g \circ f = 0$ are roots of g independently of f. This has an analogy in computer science. f is analogous to the substrate and g to the program. The assignment of correlates of cognition to the hierarchies of functional compositions of g is analogous to this principle but does not mean that conscious experience is substrate independent.

The graphical visualization of the space-time surfaces with the assistance of a large language model (LLM) was one part of the project. In the case studied, the model is reduced to a visualization of elliptic surfaces as surfaces in C^2 determined by analytically solvable third order polynomials.

The origin of the p-adic primes as characterizers of elementary particles and p-adic length scale hypothesis are two key problems of TGD. The working hypothesis has been that the p-adic primes correspond to ramified primes characterizing polynomials, say as $g = (g_1 = P_1, g_2 = Id)$. There is also a second option. One can define the notion of primeness for f *resp.* g as maps not decomposing as $f = g \circ h$ *resp.* $g = g_1 \circ g_2$. These prime maps are polynomials with degree which is prime p and universal in the sense that only the degree as a counterpart of ordinary p-adic prime p matters.

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The functional p-adic counterparts of ordinary primes can be identified, the p-adic length scale hypothesis can be understood, and the origin of p-adic physics can be identified. It is also possible to understand how the predicted slight failure of classical determinism corresponds to p-adic non-determinism, how quantum criticality is realized, and how the hierarchy of effective Planck constants emerges.

2 Holography= holomorphy vision and space-time as a 4-dimensional surface in the 8-dimensional space $H = M^4 \times CP_2$

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In this section the holography= holomorphy vision in its recent form is described at the end of the section the LLM assisted study of graphical illustrations of space-time surfaces is briefly discussed and the details related to the elliptic surfaces are left to Appendix A.

2.1 Space-time surfaces according to holography= holomorphy vision

Holography= holomorphy vision allows to solve the classical field equations explicitly by reducing field equations to local algebraic equations. The solutions are universal in the sense that minimal surfaces are obtained irrespective of the action as long as it is general coordinate invariant and depends only on the induced geometry.

1. The generalized complex structure on the space $H = M^4 \times CP_2$ means following.

   (a) M^4 is associated with a complex coordinate w and a hypercomplex coordinate pair (u, v). u and v are real and v is the hypercomplex conjugate of u. Both can act as time coordinates. The simplest example of u and v is $(u = t - z, v = t + z)$. These are light-like coordinates.

   (b) CP_2 is associated with two complex coordinates ξ_1 and ξ_2.

2. The space-time surface is complex in the sense that it is a root for the function pair (f_1, f_2) i.e. $(f_1, f_2) = (0, 0)$. (f_1, f_2) is an analytical map $H \to C^2$ i.e. f_1 and f_2 depend only on the coordinates (u, w, ξ_1, ξ_2) but not on their conjugate coordinates $v, \overline{w}, \overline{\xi_1}, \overline{\xi_2}$.

   This guarantees that the classical field equations are automatically valid. The space-time surface is a minimal surface regardless of the action principle except for lower-dimensional singularities.

   The conditions $f_1 = 0$ and $f_2 = 0$ give 2 conditions each and they thus determine a 4-dimensional surface, which corresponds to the space-time surface. It is natural to speak of the regions of the space-time surface as roots of $f = (f_1, f_2)$. At each point we obtain algebraic equations that are analogous to the equations determining the roots of polynomials. Extremely nonlinear partial differential equations reduce to algebraic equations and the classical theory is exactly solvable.

3. We restrict ourselves to studying the important special case: $(f_1, f_2) = (P_1, P_2)$, where $P_1(u, w, \xi_1, \xi_2)$ and $P_2(u, w, \xi_1, \xi_2)$ are polynomials. When the roots of the polynomials P_1 and P_2 are solved in the space-time surface. This gives a physically important special case where $P_2 = w - \xi_1$ (or more generally $w^n - \xi_1^2$). In this case, $\xi_1 = w$ can be solved and it can be placed in P_1 and we obtain the condition $P_1 = (u, \xi_1, w, \xi_2 = w) = 0$.

This polynomial equation has several roots and they correspond to different regions of the space-time surface.

From this we can solve ξ_1 as a function $\xi_1(w, u)$ of u and w or also as a function $w = w(\xi_1, u)$ of w, u and ξ_1 .

2.2 Restriction of (f_1, f_2) to polynomial pairs (P_1, P_2)

In the simplest case, the polynomial P_1 (P_2) has n_1 (n_2) roots if its degree is n_1 (n_2). In total, we get $n_1 n_2$ roots and in the simplest case they correspond to different regions of a uniform space-time surface. In the more general case, we get a set of separate space-time surfaces that are distributed in the same way.

The maps $f \to g \circ (f_1, f_2)$, where $g : C^2 \to C^2$ is an analytic map, produce new solutions/space-time surfaces. If the surface map $f = (f_1, f_2) : H \to C^2$ cannot be represented in the form $f = g \circ h$, then it is a space-time surface: f is the analogy of a prime number and a uniform surface.

On the other hand, if $f = g \circ h$ holds, then the roots correspond to the surfaces $(P_1, P_2 = (r_1, r_2)$, where (r_1, r_2) is the root of $g = (g_1, g_2)$. These surfaces have no common points. We obtain a set of distinct space-time surfaces. There are reasons to expect that only $g = (g_1, Id)$ need to be considered.

The space-time regions corresponding to the two roots r_1, r_2 meet when the roots for P_1 (and P_2) are the same: $r_1 = r_2$. In the case under consideration, this means that the two roots $\xi_1 = r_1(w, u)$ and $\xi_2 = r_2(2, u)$ of P_1 coincide. The result is a 3-dimensional surface, an "interface" between the 4-dimensional regions. These regions correspond to criticality and are of particular interest physically.

The cusp catastrophe is a useful analogy (found on Wikipedia). In the space of a surface (x,a,b) determine the real roots x_1, x_2, x_3 of the 3-degree polynomial $P_3(x, a, b)$. In the cusp region, 3 roots are obtained. Two of them meet at the edges of the cusp, which are folds. The projection of the cusp on the a,b plane is a V-shaped curve. On the sides of v (folds), the real roots x_1 and x_2 (x_2 and x_3) meet. At the vertex of V, where the folds meet, $x_1 = x_2 = x_3$.

In the 4-dimensional case, the folds are 3-D surfaces.

2.3 The maps $f \to g \circ$f as dynamical symmetries

The maps $f = (f_1, f_2) \to g \circ (f_1, f_2)$, where $f : H \to C^2$ are dynamic symmetries and produce new solutions. As a special case, we obtain the iterations $f \to g \circ f \to g \circ g \circ f$..., which produce analogies of Mandelbrot fractals and Julia sets.

When g is a polynomial this map increases algebraic complexity because the degree $g \circ f$ is the product of the degrees g and f. Complexity increases exponentially in iteration. In biology this would correspond to evolution. From the point of view of the cognitive model $f \to g \circ$f corresponds to abstraction, the emergence of a new level of reflection. The system becomes aware of itself. At the lowest level, where f cannot be represented in the form $f = g \circ h$, there is no reflective awareness.

2.4 Graphical representations in the case of elliptic surfaces

The illustration of space-time surface in 8-D embedding space as such is impossible. The holography holomorphy hypothesis however makes it possible to illustrate the essential information for time= constant snapshots so that animation is obtained.

1. In the graphical representation u corresponds to time and the time evolution can be described as an animation, i.e. a series of snapshots $\xi_1 = \xi_1(w, u_n)$ or $w = w(\xi_1, u_n)$. These functions can be multi-valued. It is useful to print both to get a better idea of the structure of the surface which is complex because it is multi-leafed.

2. When the time coordinate is fixed ($u = u_n$) then depending on whether $\xi_1 = \xi_1(w, u_n)$ or $w = w(\xi_1, u_n)$ is chosen, the mapping $w \to \xi_1$ or $x_1 \to w$ must be presented. That is, the mapping of the complex plane w to the complex plane ξ_1 or vice versa. 4 dimensions would be needed and the mapping would be a 2-dimensional surface in a 4-dimensional space.

The problem is solved when a representation in a 3-dimensional space (x, y, z) is used. In this case, for example, $w = x + iy$ corresponds to the (x, y) plane and $Re(\xi_1)(x, y)$ corresponds to the z-coordinate and the value of $Im(\xi_1)(x, y)$ corresponds to the color.

Graphical representation are examined in the situations in which P_1 is a third-degree polynomial in either ξ_1 or w. The reason is that in this case the roots for ξ_1 or w can be solved analytically. This dramatically simplifies the calculation and reduces errors. Even if P_1 is of degree four, an analytical solution is possible. For higher degrees it is not possible in general case. Situation changes for iterates of polynomials whose degree is smaller than 5.

When degree is 3, with certain additional assumptions, we are dealing with an elliptic 2-surfaces in 4-D space. These surface have 2-dimensional discrete translations as symmetries and the graphical results should make this visible. It is precisely elliptic surfaces that have been considered. The details related to the mathematics of elliptic surfaces are discussed in Appendix A.

Tuomas Sorakivi has carried out together with me a large language model (LLM) assisted experimentation with the graphical representation of the space-time surfaces satisfying holography= holomorphy hypothesis.

1. A brief summary TGD based model can be found at `https://gitlab.com/topological-geometrodynamics/tgd-model`https://gitlab.com/topological-geometrodynamics/tgd-model.

2. The equations for the space-time surface associated with a third order polynomial $P_3(\xi_1, w, u)$ can be solved analytically. In 2-dimensional algebraic geometry, these surfaces correspond to elliptic surfaces in 2-dimensional complex space with ξ_1 and w as coordinates. Space-time surface can be represented as an animation for the time evolution of an elliptic surface. A graphical representation is constructed (see this).

3. The roots of the polynomial defining space-time surface have as interfaces 3-dimensional surfaces at which two roots are identical. For the illustration of the interfaces see this.

4. Weierstrass elliptic functions (see this) are particular case of elliptic functions, being meromorphic and doubly periodic. For a visualization in this case see this.

3 Do ramified primes correspond to p-adic primes and how to understand p-adic length scale hypothesis

The surprisingly successful p-adic mass calculations [K2, K1] [L4] led to the conclusion that elementary particles and also more general systems are characterized by p-adic primes, which assign to these systems p-adic length scale. The identification of the p-adic primes remained the problem

The original hypothesis was that p-adic primes correspond to ramified primes appearing as divisors of the discriminant of a polynomial defined as the product of root differences. Assuming holography= holomorphy vision, the identification of the polynomial of a single variable in question is not trivial but is possible. The p-adic length scale hypothesis was that iterates of a suitable second-degree polynomial P_2 could produce ramified primes close to powers of two. Tuomas Sorakivi helped with LLM assisted calculation to study this hypothesis for the iterates of the chosen polynomial $P_2 = x(x - 1)$ did not support this hypothesis and I became skeptical.

This inspired the question whether the p-adic prime p correspond to a functional prime that is a polynomial P_p of degree p, which is therefore a prime in the sense that it cannot be written as functional composite of lower-degree polynomials. The concept of a prime would become much more general but these polynomials could be mapped to ordinary primes and this is in spirit with the notion of morphism in category theory.

This led to a burst of several ideas allowing to unify loosely related ideas of holography=holomorphy vision.

3.1 Functional primes and connection to quantum measurement theory

Could functional p-adic numbers correspond to "sums" of powers of the initial polynomial P_p multiplied by polynomials Q of lower degree than p. This is possible, but it must be assumed that the product is replaced by the function composition $\circ$ and the usual sum by the product of polynomials. In the $g = (g_1, g_2)$ and $h = (h_1, h_2)$ the analytic functions $g_i : C^2 \to C^2$ and h_i are multiplied and in the physically interesting special case the product reduces to the product of g_1 and h_1.

The non-commutativity for $\circ$ is a problem. In functional composition $f \to g \circ f$ the effect of g is analogous to the effect of an operator on quantum state in quantum mechanics and functions are like quantum mechanical observables represented as operators. In quantum mechanics, only mutually commuting observables can be measured simultaneously. The equivalent of this would be that when P_p is fixed, only the coefficients Q (lower degree polynomials) to powers of P_p are such that $Q \circ P_p = P_p \circ Q$ and also the Qs commute with respect to $\circ$. One can talk about quantum-padic numbers or functional p-adic numbers.

p-adic primes correspond to functional primes that can be described by ordinary primes: this is easy to understand if you think in category theoretical terms. All prime polynomials of degree p correspond to the same ordinary prime p. One can talk about universality. Number-theoretic physics, just like topological field theory, is the same for all surfaces that a polynomial of degree p corresponds to.

Electron, characterized by Mersenne prime $p = M_127 = 2^127 - 1$, would correspond to an extremely large number of space-time surfaces as far p-adic mass calculations are considered.

3.2 The arithmetic of functional polynomials is not conventional

Functional polynomials are polynomials of polynomials. This notion emerges also in the construction of infinite primes [K3]. Their roots are not algebraic numbers but algebraic functions as inverses of polynomials. They can be represented in terms of their roots which are space-time surfaces. In TGD, all numbers can be represented as spacetime surfaces. Mathematical thought bubbles are, at the basic level, spacetime surfaces (actually 4-D soap bubbles as minimal surfaces!).

For functional polynomials product and division are replaced with $\circ$. + and - operations are replaced with product and division of polynomials. Also rational functions R= P/Q must be allowed and this leads to the generalization of complex analysis from dimension D=2 to dimension D=4. This is an old dream that was now realized in a precise sense.

1. The non-conventional arithmetic of functional polynomials makes it possible to understand the p-adic length-scale hypothesis. Functional polynomials lead to an explicit formula for the functional analogs of Mersenne primes and more generally for primes close to powers of two, and even more generally primes near powers of small primes. The functional Mersenne prime is $P_2^(\circ n)/P_1$ and any P_2 will do!

2. The same p-adic prime p corresponds to all polynomials P_p of degree p. p-Adic primes are universal and depend very little on the space-time surfaces associated with them: this is very important concerning p-adic mass calculations. The problem with the ramified prime option was that they depend strongly on the space-time surface determined as root of (f_1, f_2): the effect of (g_1, Id) giving $(g_1 \circ f_1, f_2)$ does not have particle mass at all.

3.3 Inverse functions of polynomials are also needed

The inverse element with respect to $\circ$ corresponds to the inverse function of the polynomial, which is an n-valued algebraic function for an n-degree polynomial. They must also be allowed. Operating the polynomial g_1 on f increases the degree and complexity. Operating with the inverse function preserves the number of roots or even reduces it if g_1 operates on g_1 iterated. The complexity can decrease. Complexity can be considered as a kind of universal IQ and evolution would correspond to the increase in complexity in statistical sense. Inverse polynomials can reduce it by dismantling algebraic structures.

In TGD inspired theory of consciousness I have associated ethics with the number theoretic evolution as increase of algebraic complexity. A good deed increases potential conscious information, i.e. algebraic complexity, and this is indeed what happens in a statistical sense. Could conscious and intentional evil deed correspond to these inverse operations? Evil deed would make good deed undone. If so, it is easy to see that negentropy still increases in a statistical sense. This however would mean that evil deed can be regarded as a genuine choice.

3.4 How do quantum criticality, classical non-determinism and p-adic non-determinism to each other?

3.4.1 What criticality means mathematically?

The simplest representation of criticality is by means of a monomial x^n. It has n identical roots at x=0 and extremely small perturbation can transform them to separate roots. Mathematicians consider them as separate, if there were on n copies of the root $x = 0$ on top of each other. $g_1 = f_1^n$ as the equivalent of this gives n identical space-time surfaces as roots on top of each other. Are they the same surface or separate? A mathematician would say that they are separate. If the polynomial is slightly perturbed, there are n separate roots. This would be the classical equivalent of quantum criticality.

At quantum criticality, the functional polynomials would have $g_1 = f_1^n$ at quantum criticality. The corresponding spacetime surface would be susceptible to breaking up into separate spacetime surfaces when the monomial f_1^n becomes a more general polynomial and n roots are obtained as separate space-time surfaces.

There is a fascinating connection with cell replication. In TGD it would be controlled by the field body and one can ask whether $f_1^2 = P_2^2$ as a critical polynomial representing the field body is perturbed and leads to two field bodies which become controllers of separate cells. One can ask whether in a cell replication sequence $P_2^{\circ 2n}$ becomes less critical step by step so that eventually there are 2^n separate field bodies and cells.

In zero energy ontology (ZEO) one can also ask whether the creation of a critical space-time surface characterized by f_1^n could give rise to n space-time surfaces when criticality is lost. Zero energy ontology understood in the Eastern sense would allow this without conflict with conservation laws.

3.4.2 Mother Nature likes her theorists

If the critical surface is considered as a single surface, the classical action associated with it is n-fold compared to the surface corresponding to one root. This means that the Kähler coupling strength α_K is smaller by a factor of $1/n$ after the splitting. This was the basic idea in the hypothesis that I formulated by saying that Mother Nature likes theorists.

When the perturbation theory ceases to converge (a catastrophe for the theorist), criticality arises, the polynomial takes the form $P_p = f_1^p$. Deformation and splitting of the surface into a p discrete surface follows, the coupling strength decreases by a factor of $1/p$ and the perturbation theory converges again. The theorist is happy again.

3.4.3 Classical non-determinism corresponds to p-adic non-determinism

Criticality is associated with non-determinism. In classical time evolution, mild non-determinism corresponds to such a criticality. In these phase transitions, a choice is made between p alternatives in the "small" state function reduction (SSFR). The essential thing is that this series of phase transitions can be realized as a classical time evolution. Without criticality, this would not be possible.

The fact that a choice is made between p alternatives corresponds to the fact that the dynamics is effectively p-adic. So that classical non-determinism corresponds to p-adic non-determinism.

3.4.4 Connection to the p-adic length-scale hypothesis

What is particularly interesting is that if $p = 2$ or 3, the roots of the polynomial P_p can be solved analytically. The same applies to the iterates of P_p. Therefore these cases are cognitively special as every mathematician knows from her own experience! The p-adic length-scale hypothesis says that the rather large p-adic primes p are close to powers of small prime $q = 2, 3, ...$ (denoted by p above). Intriguingly, there is empirical evidence for the hypothesis in the cases $q = 2$ and $q = 3$!

In the cusp catastrophe, the Mother of all catastrophes, which is 2-dimensional surface in the space (x, a, b) defined by the real roots x_i, $i = 1, 2, 3$ of $P_3(x, a, b)$, $q = 2$ and $q = 3$ occur. The projection of the cusp to the (a,b) plane is V-shaped. At the tip of V, the polynomial $P_3(x, a, b)$

determining the cusp is proportional to x^3, i.e. 3 roots concide. At the two folds, whose projections on (a,b) plane define the sides of V, two roots co-incide .

To sum up, it seems that finally the basic ideas of TGD have found each other and form a coherent whole. I managed also to clarify the relationship of $M^8 - H$ duality to the holography=holomorphism hypothesis.

4 Conclusions

The project contained two parts: graphical illustrations of space-time surfaces in $H = M^4 \times CP_2$ and the study of ramified primes associated with the iterates of polynomials of order 2 motivated by the p-adic length scale hypothesis.

The study of possible visualization of the space-time surfaces in $H = M^4 \times CP_2$. As far as information content is considered, the holomorphy= holography principle allows to cast the problem to a visualization of time evolution 2-D surfaces in C^2 which can be represented as an animation.

The study of the ramified primes associated with the iterates of second order polynomials $P_2(x) = x(x-1)$ demonstrated that the identification of ramified primes as p-adic primes assignable to these iterates is not plausible. This led to the realization of what is in hindsight obvious: the ramified find prime hypothesis is in practice non-testable: there are two many polynomials to be tested.

The assignment of p-adic primes to the prime degrees of prime polynomials g_1 however turned out to be plausible. This led to various theoretical developments based on the notion of a functional counterpart of ordinary arithmetics of polynomials replacing ordinary arithmetic operations $\times$ and with functional composition $\circ$ and product $\times$. One could also speak of quantum arithmetics and quantum p-adics. The outcome was the identification of the origin of p-adic physics and understanding of p-adic length scale hypothesis and the preferred cognitive role of powers of primes 2 and 3, the natural identification of classical and p-adic non-determinism, the identification of the fundamental geometric mechanism behind quantum criticality implying the hierarchy of effective Planck constants, and justification of the "Mother Nature likes her theoreticians" principle.

A Holography = holomorphy vision and elliptic functions and curves in TGD framework

Holography = holography principle [L9, L3, L6, L8] leads to an explicit construction of the solutions of field equations by reducing the field equations from extremely nonlinear partial differential equations to algebraic equations. In this article, elliptic curves and functions are considered as an application.

A.1 Holography=holomorphy as the basic principle

Holography=holomorphy principle allows to solve the field equations for the space-time surfaces exactly by reducing them to algebraic equations.

1. Two functions f_1 and f_2 that depend on the generalized complex coordinates of $H = M^4 \times CP_2$ are needed to solve the field equations. These functions depend on the two complex coordinates ξ_1 and ξ_2 of CP_2 and the complex coordinate w of M^4 and the hypercomplex coordinate u for which the coordinate curves are light-like. If the functions are polynomials, denote them $f_1 \equiv P_1$ and $f_2 \equiv P_2$.

   Assume that the Taylor coefficients of these functions are rational or in the expansion of rational numbers, although this is not necessary either.

2. The condition $f_1 = 0$ defines a 6-D surface in H and so does $f_2 = 0$. This is because the condition gives two conditions (both real and imaginary parts for f_i vanish). These 6-D surfaces are interpreted as analogs of the twistor bundles corresponding to M^4 and CP_2. They have fiber which is 2-sphere. This is the physically motivated assumption, which might

require an additional condition stating that ξ_1 and ξ_2 are functions of w as analogs of the twistor bundles corresponding to M^4 and CP_2. This would define the map mapping the twistor sphere of the twistor space of M^4 to the twistor sphere of the twistor space of CP_2 or vice versa. The map need not be a bijection but would be single valued.

The conditions $f_1 = 0$ and $f_2 = 0$ give a 4-D spacetime surface as the intersection of these surfaces, identifiable as the base space of both twistor bundle analogies.

3. The equations obtained in this way are algebraic equations rather than partial differential equations. Solving them numerically is child's play because they are completely local. TGD is solvable both analytically and numerically. The importance of this property cannot be overstated.

4. However, a discretization is needed, which can be number-theoretic and defined by the expansion of rationals. This is however not necessary if one is interested only in geometry and forgets the aspects related to algebraic geometry and number theory.

5. Once these algebraic equations have been solved at the discretization points, a discretization for the spacetime surface has been obtained.

   The task is to assign a spacetime surface to this discretization as a differentiable surface. Standard methods can be found here. A method that produces a surface for which the second partial derivatives exist because they appear in the curvature tensor.

   An analogy is the graph of a function for which the (y, x) pairs are known in a discrete set. One can connect these points, for example, with straight line segments to obtain a continuous curve. Polynomial fit gives rise to a smooth curve.

6. It is good to start with, for example, second-degree polynomials P_1 and P_2 of the generalized complex coordinates of H.

A.2 How could the solution be constructed in practice?

For simplicity, let's assume that $f_1 \equiv P_1$ and $f_2 \equiv P_2$ are polynomials.

1. First, one can solve for instance the equation $P_2(u, w, \xi_1, \xi_2) = 0$ giving for example $\xi_2(u, w, \xi_1)$ as its root. Any complex coordinates w, ξ_1 or ξ_2 is a possible choice and these choices can correspond to different roots as space-time regions and all must be considered to get the full picture. A completely local ordinary algebraic equation is in question so that the situation is infinitely simpler than for second order partial differential equations. This miracle is a consequence of holomorphy.

2. Substitute $\xi_2(u, w, \xi_1)$ in P_1 to obtain the algebraic function $P_1(u, w, \xi_1, \xi_2(u, w, \xi_1)) = Q_1(u, w, \xi_1)$.

3. Solve ξ_1 from the condition $Q_1 = 0$. Now we are dealing with the root of the algebraic function, but the standard numerical solution is still infinitely easier than for partial differential equations.

   After this, the discretization must be completed to get a space-time surface using some method that produces a surface for which the second partial derivatives are continuous.

Very interesting special cases are polynomials with order not larger than 4 since for these the roots can be solved explicitly. I have proposed that P_2 characterizes the cosmological constant as a correspondence between the twistor spheres of M^4 and CP_2 and is characterized by the winding number. In standard cosmology Λ is a constant of Nature but in TGD it is predicted to have a hierarchy of values. The simplest relationship would be $P_2 = \xi_2 - w^n$, n integer. In this case, one can solve $\xi_2(w)$ and substitute it to P_1 to obtain the condition

$$P_1(\xi_1, \xi_2(w), w, u) = 0 \quad . \tag{A.1}$$

If P_1 as a polynomial of ξ_1 has order lower than 5, the roots of ξ_1 can be solved explicitly. Elliptic curves satisfy the condition

$$\xi_1^2 - w^3 + aw + b = 0 \ . \tag{A.2}$$

The projections of the w-plane are doubly periodic curves and therefore of special interest. For $P_2 = \xi_2 - w^2$ and $P_1 = \xi_1^2 - w\xi_2 + aw + b$, the space-time surface would be a 4-D analog of an elliptic curve. If a and b depend on u, the 3-surface becomes dynamical.

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A.3 Elliptic curves as an application

One can test whether the numerical method works when the equation giving ξ_1 in terms of w can be solved analytically. For elliptic curves $\xi_1 = \xi_1(w)$, which I have discussed already earlier [L1, L3], this is the case.

A.3.1 Elliptic curves

The third order polynomial characterizing the elliptic curve (see this) can be be expressed in terms of the root of a third order polynomial $P_3(w)$ as

$$E : \xi_1^2 = 4(w - e_1)(w - e_2)(w - e_3) \ , \tag{A.3}$$

One can choose the complex w in such a manner that the equation contains no term proportional to w^2. This is guaranteed if the condition $e_1 + e_2 + e_3 = 0$ holds true. In this case one obtains the form

$$\begin{aligned} &E : \xi_1^2 = 4w^3 - g_2 w - g_3 \ , \qquad\qquad , \\ &g_2 = -4(e_1e_2 + e_2e_3 + e_3e_1) \ , \quad g_3 = 4e_1e_2e_3 \ , \quad e_1 + e_2 + e_3 = 0 \ . \end{aligned} \tag{A.4}$$

A.3.2 Connection with Weierstrass elliptic functions

There is a connection with Weierstrass elliptic functions, which satisfy the differential equation

$$\wp'(z)^2 = 4\wp(z)^3 - g_2\wp(z) - g_3 \ . \tag{A.5}$$

Clearly, By using z as a complex coordinate instead of w, $\xi_1(w)$ and w for the elliptic curve can be expressed in terms of Weierstrass elliptic function, which is a solution of this differential equation

$$\xi_1(w) = \wp'(z) \ , \quad w(z) = \wp(z) \ . \tag{A.6}$$

Elliptic functions are doubly periodic and using $z = \wp^{-1}(w)$ as a complex coordinate instead of w, this periodicity becomes manifest. The solution possesses a discrete conformal symmetry consisting of a discrete subgroup of 2-D translations and this gives rise to a lattice structure. This conforms with the fact that the elliptic curve, as a compact 2-D surface in the space spanned by coordinates (ξ_1, w) has the topology of a torus and therefore can allow translations as conformal symmetries. This is the case for the elliptic curves considered.

One can represent torus in a complex plane with coordinate z in terms of Weierstrass elliptic function $\wp$ having a double periodicity in z-plane as conformal symmetries. The torus corresponds to the fundamental domain (2-D lattice cell) obtained by identifying the opposite boundaries of the lattice cell. The periods ω_1 and ω_2 define non-orthogonal directions and their ratio $\tau = \omega_1/\omega_2$ is conformal invariant.

One can solve the fundamental periods ω_1 and ω_2 in the following way. Define the auxiliary quantities

$$a_0 = \sqrt{e_1 - e_3} , \qquad b_0 = \sqrt{e_1 - e_2} , \qquad c_0 = \sqrt{e_2 - e_3} , \ . \tag{A.7}$$

The condition $e_1 + e_2 + e_3 = 0$ allows to eliminate e_3 so that one has

$$a_0 = \sqrt{-e_2} , \qquad b_0 = \sqrt{e_1 - e_2} , \qquad c_0 = \sqrt{-e_1} , \ . \tag{A.8}$$

The fundamental periods ω_1 and ω_2 for the elliptic curve can be calculated very rapidly by

$$\omega_1 = \frac{\pi}{\mathrm{M}(a_0, b_0)} , \qquad \omega_2 = \frac{\pi}{\mathrm{M}(c_0, ib_0)} \tag{A.9}$$

Or more explicitly

$$\omega_1 = \frac{\pi}{\mathrm{M}(\sqrt{-e_2}, \sqrt{e_1 - e_2})} \ , \ \omega_2 = \frac{\pi}{\mathrm{M}(\sqrt{-e_1}, i\sqrt{e_1 - e_2})} \ . \tag{A.10}$$

Here $\mathrm{M}(x, y)$ is defined as arithmo-geometric mean of x and y by a geometric iteration (see this). Assuming $x \geq y \geq 0$ one has

$$a_0 = x, g_0 = y \ , \ a_{n+1} = (a_n + g_n)/2, g_{n+1} = \sqrt{(a_n g_n)} \ . \tag{A.11}$$

At the limit $n \to \infty$ one has a_{n+1}
$simeqa_n \to a$ and $g_{n+1} \simeq g_n \to g$ and one has $a = (a + g)/2$ and $g = \sqrt{ag}$ implying $a = g$ so that arithmetic and geometric means are identical. Care is required to take the correct sign of square root at each step of iteration (positive in the case considered). The iteration generalizes to the complex case and there probably exist tested programs performing the iteration.

B Could p-adic primes correspond to ramified primes for the iterates $g_p^{\circ n}$?

p-Adic length scale hypothesis [K2, K1] [L4] states that physically interesting p-adic primes p are near to powers of $q = 2$ and possibly also $q = 3$. Could p-adic length scale hypothesis relate to the iteration of polynomials P_q? A second conjecture, or rather question, is whether p-adic primes p assigned to elementary particles in p-adic mass calculations correspond to ramified primes for a suitably identified polynomial. The following argument does not support this conjecture.

One can ask whether the analogs of ramified primes polynomials assignable abstraction hierarchies $g \circ g \circ ... \circ f$ and powers $g^{\circ n}$. The physically interesting special case corresponds to $g = (g_p, Id)$ for which the degrees of the iterates $g_p^{\circ n}$ are $n \times p$, p the prime assignable to prime polynomial g_p.

1. The ramified primes for $g_p \circ g_p \circ ... g_p \circ f$ and g_p^n define analogs of powers p^n of p-adic numbers. Note that the roots of $g_p \circ g_p \circ ... g_p \circ f$ are a property of $g_p \circ g_p \circ ... g_p$ and do not depend on f in case that they exist as surfaces inside the CD.

2. There is hope that even the p-adic length scale hypothesis could be understood as a ramified primes assignable to some functional prime. The large values of p-adic primes require that very large ramified primes for the functional primes (f_1, f_2). This would suggest that the iterate $g \circ ..... \circ g \circ f$ acting on prime f is involved. For $p \simeq q^k$, k^{th} power of g characterized by prime g is the first guess.

Generalized p-adic numbers as such are a very large structure and the systems satisfying the p-adic length scale hypothesis should be physically and mathematically special. Consider the following assumptions.

1. Consider generalized p-adic primes associated restricted to the case when f_2 is not affected in the iteration so that one has $g = (g_1, Id)$ and $g_1 = g_1(f_1)$ is true. This would conform with the hypothesis that f_2 defines the analog of a slowly varying cosmological constant. If one assumes that the small prime corresponds to $q = 2$, the iteration reduces to the iteration appearing in the construction of Mandelbrot fractals and Julia sets. If one assumes $g_1 = g_1(f_1, f_2)$, f_2 defines the analog of the complex parameter appearing in the definition of Mandelbrot fractals. The values of f_2 for which the iteration converges to zero would correspond to the Mandelbrot set having a boundary, which is fractal.

2. For the generalized p-adic numbers one can restrict the consideration to mere powers g_1^n as analogs of powers p^n. This would be a sequence of iterates as analogs of abstractions. This would suggest $g_1(0) = 0$.

3. The physically interesting polynomials g_1 should have special properties. One possibility is that for $q = 2$ the coefficients of the simplest polynomials make sense in finite field F_2 so that the polynomials are $P_2(z \equiv f_1, \epsilon) = z^2 + \epsilon z = z(z + \epsilon)$, $\epsilon = \pm 1$ are of special interest. For $q > 2$ the coefficients could be analogous to the elements of the finite field F_q represented as phases $exp(i2\pi k/3)$.

One can see what these premises imply. Here Tuomas Sorakivi helped to do the calculations using the assistance of a large language model.

1. Quite generally, the roots of $P^{\circ n}(g_1)$ are given $R(n) = P^{\circ -n}(0)$. $P(0) = 0$ implies that the set R_n of roots at the level n are obtained as $R_n = R_n(new) \cup R_{n-1}$, where $R_n(new)$ consist of q new roots emerging at level n. Each step gives q^{n-1} roots at the previous level and q^{n-1} new roots.

2. It is possible to analytically solve the roots for the iterates of polynomials with degree 2 or 3. Hence for $q = 2$ and 3 (there is evidence for the 3-adic length scale hypothesis) the inverse of g_1 can be solved analytically. The roots at level n are obtained by solving the equation $P(r_n) = r_{n-1,k}$ for all roots $r_{n-1,k}$ at level $n - 1$. The roots in $R_{n-1}(new)$ give q^{n-1} new roots in $R_n(new)$.

3. For $q = 2$, the iteration would proceed as follows:

$$0 \to \{0, r_1\} \to \{0, r_1\} \cup \{r_{21}, r_{22}\} \to \{0, r_1\} \cup \{r_{21}, r_{22}\} \cup \{r_{121}, r_{221}, r_{122}, r_{222}\} \to ... \ .$$

4. The expression for the discriminant D of $g_1^{\circ n}$ can be deduced from the structure of the root set. D satisfies the recursion formula $D(n) = D(n, new) \times D(n-1) \times D(n, new; n-1)$. Here $D(n, new)$ is the product

$$\prod_{r_i, r_j \in \in D(n,new)} (r_i - r_j)^2$$

and

$D(n, new; n - 1)$ is the product

$$\prod_{r_i \in D(n,new), r_j \in D(n-1)} (r_i - r_j)^2 \ .$$

5. At the limit $n \to \infty$, the set $R_n(new)$ approaches the boundary of the Fatou set defining the Julia set.

As an example one can study discriminant D and ramified primes for the iterates of $g_1(z) = z(z - \epsilon)$. Does it produce Mersenne primes or primes near a power of 2 as ramified primes as the p-adic length scale hypothesis predicts? Of course, there exists an endless variety of these kinds of polynomials but one might hope that the chosen polynomial might be special because of its 2-adic features. The study was carried out with Tuomas Sorakivi (see this).

1. The roots of $z(z-\epsilon)=0$ are $\{0,r_1\}=\{0,\epsilon\}$. At second level, the new roots satisfy $z(z-\epsilon)=r_1=\epsilon$ given by $\{(\epsilon/2)(1\pm\sqrt{1+4r_1}\}$. At the third level the new roots satisfy $z(z-\epsilon)=r_2$ and given by $\{(\epsilon/2)(1\pm\sqrt{1+4r_2}\}$.

2. The points $z=0$ and $z=\epsilon$ are fixed points. Assume $\epsilon=1$ for definiteness. The image points $w(z)=z(z-\epsilon)$ satisfy the condition $|w(z)/z|=|z-1|$. For the disk $D(1,1):|z-1|\leq 1$ the image points therefore satisfy $|w|\leq|z|\leq 2$ and belong to the disk $D(0,2):|z|\leq 2$.

   For the points in $D(0,2)\backslash D(1,1)$ the image point satisfies $|w|=|z-1||z|$ giving $|z|-1|\leq |w|\leq|z|+1$. Inside $D(0,2)\backslash D(1,1)$ this gives $0\leq|w|\leq 3$. Therefore w can be inside $D(2,0)$ including $D(1,1)$ also inside disk $D(0,3)$.

   For the points z outside $D(2,0)$ $|w|=|z-1||z|\geq 2$. So that the iteration leads to infinity here.

3. For the inverse of the iteration relevant for finding the roots of $f^{\circ(-n)}$ leads from the exterior of $D(2,0)$ to its interior but cannot lead from interior to the exterior since in this case f would lead to exterior to interior. Hence the values of the roots w_n in $\cup_n f^{\circ(-n)}(0)$ belong to the disc $D(2,0)$.

4. One can look at the asymptotic situation for very large values of n. At n^{th} step 2^{n-1} new roots emerge by doubling and one has $r_{n+1,\pm}=(1/2)(1\pm\sqrt{1+4r_{n,\pm}})$. For $r_{n,\pm}<-1/4$ the root pair becomes complex and could stay complex at the next steps. This happens already at the step from $r_2=1/2(1\pm\sqrt{\sqrt{5}})\rightarrow r_3$. If the iteration gives at some step a double real root, its further iterations could approach a fixed point at this limit. This root $r_n\rightarrow r$ would satisfy $r=(1/2)(1\pm\sqrt{1+4r})$ giving $r^2-2r=0$ with root $r_1=2$ and $r_1=0$ these are the intersections of the disk $D(0,2)$ with real axis. Note that $r_1=2$ is not a fixed point of $z(z-1)$.

   There should exist a root r_n, which at the real axes in the range $(0,2)$. This would require that $1+4r_n=0$ giving a double root $r_n=-1/4$. The next steps would give $r_{n+1}=+1/2\pm\sqrt{3}\rightarrow r_{n+2}=1/2(1\pm\sqrt{2\pm\sqrt{3}})$. Second root would be complex. The positive real roots are $r_{n+1,+}\simeq 1.366$ and $r_{n+2,+}=1.7708$. This suggests that the convergence to $r=2$ takes place for the positive roots. If this is the case the D discriminant contains the product of the differences for these positive roots approaching zero. There is however no guarantee that the double root $r_n=1/2$ emerges in the iteration.

   The prime decompositions of D for $k=1,2,...,7$ are $\{1:1\},\{5:1\},\{5:3,11:1\},\{5:7,11:3,311:1\},\{2:48,3:3,43:1,7^3:1,6577:1,5521801:1,-1:1\}$, $\{2:209,59:2,3117269:1,356831:1\}$, $\{2:596,2358900226164371:1,-1:1\}$, where $p:m$ denotes the prime and its multiplicity. $-1:1$ tells that the discriminant is negative.

The conjecture was that the discriminant D for the iterate has Mersenne primes as factors for primes n defining Mersenne primes $M_n=2^n-1$ and that also for other values of n D contains as a factor ramified primes near to 2^n. The above calculation shows that this is not the case for the polynomial $P_2(x)=x(x-1)$ for small n. There are an infinite number of polynomials of order two but the idea of starting to search all of them does not look attractive. This result challenged the proposal that p-adic primes could correspond to ramified primes and led to the realization that functional primes as polynomials with prime degree provide a more natural explanation of p-adic length scale hypothesis. The p-adic length scale hypothesis however looks attractive.

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