.

Electromagnetic fields in Maxwell's theory and in Topological Geometrodynamics

M. Pitkänen *
Email: matpitka6@gmail.com

June 20, 2019

This article considers the differences between the notions of electromagnetic field according to Maxwell's electrodynamics describing interactions between matter and electromagnetic fields and Topological Geometrodynamics (TGD) (see my homepage `http://www.tgdtheory.fi` and the article `http://tinyurl.com/zrx5mdz`). The discussion actually applies also to electroweak (electromagnetic and weak interactions), color (strong interactions), and gravitational fields. All these fields emerge from much more simpler dynamics (only 4 primary dynamical variables rather than all fields of standard model and general relativity) of space-time surfaces in certain 1+7-dimensional space-time (1 time dimension and 7 spatial dimensions) uniquely fixed to be $H = M^4 \times CP_2$. Here 1+3-D M^4 is Minkowski space-time of Special Relativity (**SRT**) and 4-D CP_2 so called complex projective space replacing points of M^4. H is obtained by replacing the points of M^4 by 4-D CP_2 with very small size. Like taking a line and replacing every point by a disk to get a cylinder.

For illustrations of future light-cone of M^4, CP_2 and $H = M^4 \times CP_2$ see `http://tinyurl.com/zr9qt8z`, `http://tinyurl.com/hrbnalq`, and `http://tinyurl.com/zlrateg` For space-time as surface in H see `http://tinyurl.com/hacfax7`.

Note: Some shorthand notations are in order. Special Relativity Theory ↔ **SRT**; General Relativity Theory ↔ **GRT**; Relativity Principle ↔ **RP**; General Coordinate Invariance ↔ **GCI**; Equivalence Principle ↔ **EP**; Quantum Field Theory ↔ **QFT**.

Consider first Maxwell's theory.

1. Maxwell's equations consists of two pairs of equations. The first pair does not involve currents and charge densities. The first equation states that there are no magnetic charges. Local form says that the magnetic field has vanishing divergence: $\nabla \cdot B = 0$. Second equation formulates Faraday's law stating that time dependent magnetic field is accompanied by an electric field not expressible as gradient of potential. Local form states $\partial_t B = -\nabla \times E$ ($c = 1$).

   Second pair says that charge density and current serve as sources of electromagnetic fields: $\nabla \cdot E = -\rho$ and $\nabla \times B = j - \partial_t E$. Unlike the first pair, these equations require a model for charged matter.

   Maxwell's equations are simple linear wave equations derivable from minimization of Maxwell action and allowing in absence of currents radiation fields as a general solution. In presence of currents and charge densities also static magnetic and electric fields such as Coulomb field of charged particle and magnetic field created by inductance coil are possible.

2. Maxwell's equations have two exceedingly important symmetries. The first symmetry is Poincare invariance, which led to the discovery of special relativity theory by Einstein. Translations and Lorentz transformations leaving the 4-dimensional distance function $s^2 = t^2 - x^2 - y^2 - z^2$ unchanged (generalization of law of Pythagoras) leave light velocity invariant as maximal signal velocity. These symmetries form the basics of entire particle physics: all particle experiments are analyzed assuming relativistic kinematics: conservation laws and transformation formulas for quantities like four-momentum.

   Second symmetry - gauge invariance - guarantees that magnetic charges vanish and electric charge is conserved. Gauge invari-

*Matti Pitknen is a theoretical physics working as a free researcher in Finland. He has developed Topological Geometrodynamics (TGD) during the last 39 years (PhD 1982). The material related to TGD can be found at homepage (`http://tgdtheory.fi`), in blog (`http://matpitka.blogspot.fi`), in Research Gate, and as various published articles.

ance allows to express electric and magnetic fields in terms of a scalar potential Φ and vector potential A (the expression is not unique): $(E = -\nabla\Phi, B = \nabla \times A)$. Gauge symmetry is the starting point of the generalizations of Maxwell's field to non-Abelian gauge fields representing electroweak and color gauge fields of standard model unifying the theories for electromagnetic, weak, and strong interactions.

3. In the relativistic formulation of special relativity (**SRT**) in the 4-D Minkowski space M^4 with linear coordinates (t, x, y, z) and endowed with above mentioned distance function electric and magnetic fields combine to single antisymmetric tensor $F \leftrightarrow (E, B)$ expressible in terms of 4-vector gauge potential $A \leftrightarrow (\Phi, A)$. Charge density and current are combined to 4-dimensional current. The relativistic formulation unifies magnetic and electric fields to single entity and demonstrates the amazingly simple basic structure of Maxwell's theory.

   In more advanced formulation gauge potentials define what is known as gauge connection telling what happens to charged particle in parallel translation generalizing that on curved surface or in the space-time according to general relativity. This boils down to the notion of covariant derivative $D_\mu = \partial_\mu + igA_\mu$ and $\partial_\mu \to D_\mu$ defines the basic substitution rule allowing to construct gauge theories from free field theories. This is a partial geometrization of gauge fields very much analogous to that for gravitational fields in General Relativity and implies automatically the counterparts of Maxwell's equations. The outcome is standard model with gauge group $G = SU(2)_L \times U(1) \times SU(3)$ coding for electroweak and color interactions. The geometrization is not complete since the parallel translation defined by G has no obvious meaning in terms of space-time geometry.

TGD starts from the dream of Einstein - the geometrization of both gravitational and electromagnetic interactions.

1. Einstein constructed two theories: Special Relativity (**SRT**) and General Relativity (**GRT**) reducing gravitation to dynamical space-time geometry in accordance with geometrization of physics program. **SRT** relies on the Relativity Principle (**RP**) and **GRT** on General Coordinate Invariance (**GCI**) and Equivalence Principle (**EP**). RP says that time is not absolute: Lorentz transformations mixing space and time coordinates replace Galilean transformations as transformations between frames moving with constant velocity with respect to each other. **GCI** says that physics must have formulation independent of the coordinate system used. Equivalence Principle says that locally gravitational force is not genuine force but can be eliminated in the rest system of freely falling system and implies that gravitational and inertial mass coincide. Both theories have been extremely successful. Einstein wanted to extend his successful geometrization of gravitation to electromagnetism but failed.

   To proceed one must identify the problem. The basic problem of **GRT** relates to the notions of energy and momentum well-defined in **SRT** but not in **GRT**. By Noether's theorem these conservation laws follow from the symmetries of space-time geometry (Poincare invariance) but since translational and rotational symmetries of **SRT** are lost in the curved space-time of **GRT** (see `http://tinyurl.com/juhqxdu`), also conservation laws are lost. Could one have a variant of **GRT** for which Poincare invariance and thus conservation laws are not lost?

2. One can! One must only generalize super string models, which modelled particles as strings having 2-D string world sheets as orbits and defining 2-D space-times as surfaces in 10-D space. The problem was that space-time is 4-D rather than 2-D and this eventually led to the failure of string models as physical theories. Poincare symmetry requires 4-D Minkowski space M^4. What if space-time X^4 is a 4-D *surface* in some higher-dimensional space $M^4 \times S$, S some internal space (actually CP_2)? Geometric symmetries would not be symmetries of space-time surface X^4 but those of empty Minkowski space M^4! The symmetries would not shift or rotate points along curves X^4 but the entire X^4 along curves in H, behaving like 4-D rigid body! Point like particles would be replaced with 3-D surfaces and world lines with X^4.

How to geometrize gauge fields and gravitational fields in this framework, in particular electromagnetic fields?

1. $H = M^4 \times S$ has Riemannian metric defining length measurement by giving generalization of the law of Pythagoras for infinitesimal distances ($ds^2 = g_{\mu\nu} dx^\mu dx\nu$) and one can induce this metric ($g_{mu\nu}$) to X^4: one just measures distances at X^4 using the metric of H and performs parallel translation using the connection of H. This is exactly what one does for 2-D surfaces in 3-D space.

   Classical gravitational fields emerge from the dynamics of space-time surface X^4 determined by some action principle. By general coordinate invariance (**GCI**) telling that 4 suitably chosen coordinates of H serve as primary dynamical field variables determining the dynamics of classical fields - a huge simplification.

2. How to geometrize electroweak and color fields? The choice $S = CP_2$ allows to achieve this. Classical electroweak gauge potentials would be induced from the spinor connection of CP_2 and would have correct coupling structure: this means projection to space-time surface (see `http://tinyurl.com/jrk69qs`). One performs parallel translation using the spinor connection of H. One can identify color gauge potentials as projections of so called Killing vectors of CP_2 representing infinitesimal isometries. One obtains the counterparts of Faraday's law and law stating the vanishing of magnetic charges whereas the 2 other equation are replaced by equations expressing conservation of classical four-momentum and color charges.

   One can also induce the generalized spinor structure of $H = M^4 \times CP_2$. This explains standard model quantum numbers and predicts correctly the coupling structure. Even more, $M^4 \times CP_2$ is the only possible option in the following sense. The existence of twistor lift of TGD lifting space-time surfaces to their 6-D twistor spaces requires that the 6-D twistor spaces of M^4 and CP_2 allow Kähler structure (imaginary unit represented geometrically as a tensor): this is indeed possible for M^4 and CP_2 and only for these 4-manifolds.

Electromagnetic fields (all classical fields) are geometrized and inherit the dynamics of space-time as 4-surface X^4 in H. In classical physics variational principle defines the dynamics: solutions of field equations - extremals - minimize the action or at least make it stationary with respect to variations. In standard quantum physics classical dynamics is only an approximation whereas in TGD it is an exact part of quantum dynamics.

The first guess for the action principle was so called Kähler action S_K, a non-linear geometric analog of Maxwell action. It took long time to realize that S_K is not quite enough to achieve a realistic theory. Also 4-D volume term V forced by so called twistor lift of TGD and interpretable in terms of cosmological constant is also needed.

All the known non-vacuum extremals of S_K are also minimal surfaces extremizing the volume term V. This is expected to be true generally (for 2-D minimal surfaces see `http://tinyurl.com/zqlv322`). Minimal surface property guarantees that non-linear variant of massless d'Alembert equations with gravitational self-coupling and generalizing Laplacian equation for Newtonian gravity are satisfied. The field equations have dual nature: they express conservation laws on one hand and massless propagation on the other hand. Extremal property for Kähler action defines the analogs for the Maxwell's equations. Gravitational dynamics and Maxwell dynamics decouple but only apparently.

One can study space-time surfaces as solutions of the field equations.

1. 4 primary field like variables is certainly not enough to describe the physics as we know it. One can however use space-time sheets as building bricks to engineer more complex space-time surfaces - many-sheeted space-time with fractal hierarchical structure (see `http://tinyurl.com/jg9le7h`). Given sheet of many-sheeted space-time carries smaller sheets glued to it by topological sum contacts (wormhole contacts) and is in turn glued to larger sheet: particles consisting of particles! We interpret these sheets as matter in background space-time: the wild topology of many-sheeted space-time is directly visible as "matter" but we do not realize this!

2. 3-surfaces are quite generally bounded. Either 3-surface develops an outer boundary - or more plausibly, is a covering space obtained by gluing two 3-surfaces along their outer boundaries to form a single 3-surface without boundary. The reason is that Maxwell gauge potentials defining a linear field are effectively replaced with the 4 coordinates of compact space CP_2. By compactness global imbeddings of arbitrary

gauge field fail. Space-time surface decomposes to topological field quanta.

Topological quantization occurs for both sources (particles - these we “see”) and fields and distinguishes form Maxwell's electrodynamics. Elementary particles have regions of space-time with Euclidian signature of induced metric (something new!) as building bricks; the TGD counterpart of classical radiation field decomposes to topological light-rays analogous to laser beams; magnetic field decomposes to flux quanta - flux tubes and flux sheets, and so on (see `http://tinyurl.com/jcmpq2t`).

The reason for topological field quantization is that the non-compact 4-D field space of gauge potentials in Maxwell's theory is replaced with compact (finite-sized) 4-D space CP_2. For instance, when one tries to realize constant magnetic field as a surface one obtains only flux quantum. At the boundaries of flux quantum real CP_2 coordinates “try” to become complex or go out of the range of definition.

3. The notions of field body and magnetic body emerge. Every system creates classical fields giving rise to field body - field identity of system. This is not possible in Maxwell's theory, where the fields of all systems interfere. The notion of magnetic body (MB) has become central in TGD inspired quantum biology. MB can be seen as intentional agent using biological body as a motor instrument and sensory receptor. This also allows to define the notion of coherence: only fields at the same sheet can interfere.

4. There is objection against this picture. The linear superposition of Maxwell's theory and classical field theories is lost and applies only for modes of topological light rays representing radiation moving in fixed direction. This is not a catastrophe. The physical motivation for the linear superposition is that the forces caused by different systems on particle sum up in a good approximation. One introduces fields and expresses forces in terms of them and thus reduces superposition of forces to that for fields.

   In TGD framework one can look what happens for a particle - small 3-surface in many-sheeted space-time. Space-time sheets can be envisaged as 4-D analogs of slightly deformed planes extremely near to each other (the distance cannot be larger than CP_2 size scale). Particle like 3-surfaces (say electrons) necessarily touches these sheets and experiences the sum of forces caused by the induced fields at sheets. Superposition for classical fields is replaced with set theoretic union of corresponding space-time sheets implying superposition of their effects.

   This allows to understand **GRT**-QFT limit of TGD. In long length scales the many-sheeted space-time is replaced with single slightly curved region of M^4. Classical gravitational field as deviation of the metric from flat M^4 metric is identified as sum of corresponding deviations of the induced metrics for space-time sheets. Gauge potentials are identified as sums of induced gauge potentials for space-time sheets. Since the number of space-time sheets can be very large, the complexity of **GRT**-QFT emerges. The classical dynamics of single sheet is extremely simple.

Many-sheetedness and the notion of field body imply deviations from **GRT-QFT** picture. One end up to a concrete topological model for elementary particles as a region of space-time surface with Euclidian signature of induced metric. Radiation fields in TGD are extremely simple in TGD framework at the level of single space-time sheet - behaving very **quantally** - and linear superposition emerges only at the QFT limit. The construction space-time surfaces reduces to a kind of engineering by gluing simpler minimal surfaces - legos - to larger structures. In quantum biology the notion of magnetic body as intentional agent using biological body as a sensory receptor and motor instrument adds third level to the organism-environment duality. At GRT limit the neglect of many-sheetedness implies anomalies since the signal velocities along different space-time sheets are different (two Hubble constants, and two neutrino bursts from SN1987A and gamma ray bursts with different arrival times). The GRT view about blackholes is modified. I have discussed these anomalies in various books and online books about TGD.