Is $M^8 - H$ duality consistent with Fourier analysis at the level of $M^4 \times CP_2$?

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Matti Pitkänen

Email: matpitka6@gmail.com.
http://tgdtheory.com/public_html/.
Recent postal address: Rinnekatu 2-4 A 8, 03620, Karkkila, Finland.

Abstract

$M^8 - H$ duality predicts that space-time surfaces as algebraic surfaces in complexified M^8 (complexified octonions) determined by polynomials can be mapped to $H = M^4 timesCP_2$. The polynomials do *not* involve periodic functions typically associated with the minimal space-time surfaces in H. Since M^8 is analogous to momentum space, the periodicity is not needed.

However, the representation of the space-time surfaces in H obey dynamics and the H-images of $X^4 \subset M^8$ should involve periodic functions and Fourier analysis for CP_2 coordinates as functions of M^4 coordinates.

Neper number, and therefore trigonometric and exponential functions are p-adically very special. As a consequence, Fourier analysis extended to allow exponential functions in the case of hyperbolic signature is a number theoretically universal concept making sense also for p-adic number fields.

The proposal is that by its non-locality, the map of the tangent space of the space-time surface $X^4 \subset M^8$ to CP_2 point brings in dynamics and therefore requires the expansion of CP_2 coordinates as exponential and trigonometric functions of M^4 coordinates that is Fourier analysis.The connection with hierarchy of effective Planck constants and the implications for the quantum model of cognitive process are considered.

1 Introduction

$M^8 - H$ duality predicts that space-time surfaces as algebraic surfaces in complexified M^8 (complexified octonions) determined by polynomials can be mapped to $H = M^4 timesCP_2$.

In [L6] it was proposed that the strong form of $M^8 - H$ duality in M^4 degrees of freedom is realized by the inversion map $p^k \in M^4 \rightarrow \hbar_{eff} p^k/p^2$. This conforms with the Uncertainty Principle. However, the polynomials do *not* involve periodic functions typically associated with the minimal space-time surfaces in H. Since M^8 is analogous to momentum space, the periodicity is not needed. In contrast to this, the representation of the space-time surfaces in H obey dynamics and the H-images of $X^4 \subset M^8$ should involve periodic functions and Fourier analysis for CP_2 coordinates as functions of M^4 coordinates.

Neper number, and therefore trigonometric and exponential functions are p-adically very special. In particular, e^p is a p-adic number so that roots of e define finite-D extensions of p-adic numbers. As a consequence, Fourier analysis extended to allow exponential functions required in the case of Minkowskian signatures is a number theoretically universal concept making sense also for p-adic number fields.

The map of the tangent space of the space-time surface $X^4 \subset M^8$ to CP_2 involves the analog velocity missing at the level of M^8 and brings in the dynamics of minimal surfaces. Therefore the expectation is that the expansion of CP_2 coordinates as exponential and trigonometric functions of M^4 coordinates emerges naturally.

The possible physical interpretation of this picture is considered. The proposal is that the dimension of extension of rationals (EQ) *resp.* the dimension of the transcendental extension defined by roots of Neper number correspond to relatively small values of h_{eff} assignable to gauge interactions *resp.* to very large value of gravitational Planck constant $\hbar_{gr}$ [L1, K2] originally introduced by Nottale [E1].

Also the connections with the quantum model for cognitive processes as cascades of cognitive measurements in the group algebra of Galois group [L7, L8] and its counterpart for the transcendental extension defined by the root of e are considered. The geometrical picture suggests the interpretation of cognitive process as an analog of particle reaction emerges.

2 How to achieve periodic dynamics at the level of $M^4 \times CP_2$?

Assuming $M^8 - H$ duality, how could one achieve typical periodic dynamics at the level of H - at least effectively?

It seems that one cannot have an "easy" solution to the problem?

1. Irreducible polynomials which are products of monomials corresponding to roots r_n which are in good approximation evenly spaced $r_n = r_0 + nr_1\Delta r_n$ would give "very special moments in the life of self" as values of M^4 time which are evenly spaced [L3, L2]. This could give rise to an effective periodicity but it would be at the level of M^8, not H, where it is required.

2. Is it enough that the periodic functions are *only* associated with the spinor harmonics of H involved with the construction of scattering amplitudes in H [L4]? For the modified Dirac equation [K1] the periodic behavior is possible. Note also that the induced spinors defining ground states of super-symplectic representations are restrictions of second quantized spinors of H proportional to plane waves in M^4. These solutions do not guarantee quantum classical correspondence.

2.1 The unique aspects of Neper number and number theoretical universality of Fourier analysis

Could one assume more general functions than polynomials at the level of H? Discrete Fourier basis is certainly an excellent candidate in this respect but does it allow number theoretical universality?

1. Discrete Fourier analysis involves in the Euclidian geometry periodic functions $exp(2\pi x)$, n integer and in hyperbolic geometry exponential functions $exp(kx)$.

   Roots of unity $exp(i2\pi/n)$ allow to generalize Fourier analysis. The p-adic variants of $exp(ix)$ exist for rational values of $x = k2\pi/n$ for $n = K$ if $exp(i2\pi/K)$ belongs to the extension of rationals. $x = k = 2pi/n$ does not exist as a p-adic number but $exp(x) = exp(i2\pi/n)$ can exist as phase replacing x as coordinate in extension of p-adics. One can therefore define Fourier basis $\{exp(inx)|n \in Z\}$ which exist at discrete set of rational points $x = k/n$

   Neper number e is also p-adically exceptional in that e^p exists as a p-adic number for all primes p. One has a hierarchy of finite-D extensions of p-adic numbers spanned by the roots $e^{1/n}$. Finiteness of cognition might allow them. Hyperbolic functions $exp(nx)$, $n = 1, 2...$ would have values in extension of p-adic number field containing $exp(1/N)$ in a discrete set of points $\{x = k/N|k \in Z\}$.

2. (Complex) rationality guarantees number theoretical universality and is natural since CP_2 geometry is complex. This would correspond to the replacement $x \rightarrow exp(ix)$ or $x \rightarrow exp(x)$ for powers x^n. The change of the signature by replacing real coordinate x with ix would automatically induce this change.

3. Exponential functions are in a preferred position also group theoretically. Exponential map maps $g \rightarrow exp(itg)$ the points of Lie algebra to the points of the Lie group so that the tangent space of the Lie algebra defines local coordinates for the Lie group. One can say that tangent space is mapped to space itself. M^4 defines an Abelian group and the exponential map would mean replacing of the M^4 coordinates with their exponential, which are p-adically more natural. Ordinary Minkowski coordinates have both signs so that they would correspond to the Lie algebra level.

4. CP_2 is a coset space and its points are obtained as selected points of $SU(3)$ using exponentiation of a commutative subalgebra t in the decomposition $g = h + t + \bar{t}$ in the Lie-algebra of $SU(3)$. One could interpret the CP_2 points as exponentials and the emergence of exponential basis as a basis satisfying number theoretical universality.

2.2 Are CP_2 coordinates as functions of M^4 coordinates expressible as Fourier expansion

Exponential basis is not natural at the level of M^8. Exponential functions belong to dynamics, not algebraic geometry, and the level H represents dynamics.

It is the dependence of CP_2 coordinates on M^4 coordinates, where the periodicity is needed. The map of the tangent spaces of $X^4 \subset M^8$ to points of CP_2 is slightly local since it depends on the first derivatives crucial for dynamics. Could this bring in dynamics and exponential functions at the level of H?

These observations inspire the working hypothesis that CP_2 points as functions of M^4 coordinates are expressible as polynomials of hyperbolic and trigonometric exponentials of M^4 coordinates.

Consider now the situation in more detail.

1. The basis for roots of e would be characterized by integer K in $e^{1/K}$. This brings in a new parameter characterizing the extension of rationals inducing finite extensions of p-adic numbers. K is analogous to the dimension of extension of rationals: the p-adic extension has dimension $d = Kp$ depending on the p-adic prime explicitly.

2. If CD size T is given, $e^{-T/K}$ defines temporal and spatial resolution in H. K or possibly Kp could naturally correspond to the gravitational Planck constant [L1] [K2] [E1] $K = n_{gr} = \hbar_{gr}/h_0$.

3. In [L5] many-sheetedness with respect to CP_2 was proposed to correspond to flux tubebundles in M^4 forming quantum coherent structures. A given CP_2 point corresponds to several M^4 points with the same tangent space and their number would correspond to the number of the flux tubes in the bundle.

   Does the number of these points relate to K or Kp? p-Adic extension would have finite dimension $d = Kp$. Could $d = Kp$ be analogous to a degree of polynomial defining the dimension of extension of rationals? Could this be true in p-adic length scale resolution $O(p^2) = 0$ The number of points would be Kp and very large. For electron one has $p = M_{127} = 2^{127} - 1$.

4. The dimension n_A Abelian extension associated with EQ would naturally satisfy $n_A = K$ since the trigonometric and hyperbolic exponentials are obtained from each other by replacing a real coordinate with an imaginary one.

5. There would be two effective Planck constants. $h_{eff} = nh_0$ would be defined by the degree n of the polynomial P defining $X^4 \subset M^8$. $\hbar_{gr} = n_{gr}h_0$ would define infra-red cutoff in M^4 as the size scale of CD in $H = M^4 \times CP_2$. n *resp.* $n_{gr} = Kp$ would characterize many-sheetedness in M^4 *resp.* CP_2 degrees of freedom.

2.3 Connection with cognitive measurements as analogs of particle reactions

There is an interesting connection to the notion of cognitive measurement [L5, L7, L8].

1. The dimension n of the extension of rationals as the degree of the polynomial $P = P_{n_1} \circ P_{n_2} \circ ...$ is the product of degrees of degrees n_i: $n = \prod_i n_i$ and one has a hierarchy of Galois groups G_i associated with $P_{n_i} \circ ....$ G_{i+1} is a normal subgroup of G_i so that the coset space $H_i = G_i/G_{i+1}$ is a group of order n_i. The groups H_i are simple and do not have this kind of decomposition: simple finite groups appearing as building bricks of finite groups are classified. Simple groups are primes for finite groups.

   Remark: Any finite group can appear as a Galois group of Galois extension of some field but the inverse Galois problem concerning whether any finite group can appear as a Galois group of an extension of rationals in unsolved (`https://cutt.ly/KmimfQi`). David Hilbert has shown the following. If K is any extension of Q, on which G acts as an automorphism group and the invariant field K^G is rational over Q then G is realizable over Q. That the field K^{Gal} invariant under Gal is rational over Q means that it has a basis, whose elements are algebraically (not linearly!) independent over K, which generated K^{Gal}. If G is a subgroup of Gal, K^G is larger than K^{Gal}, and contains additional elements algebraically dependent over K. An interesting question is what happens if Q as the coefficient field of polynomials is replaced with EQ. Could any finite group appear as a Galois group of an extension of some EQ?

2. The wave function in group algebra $L(G)$ of Galois group G of P has a representation as an entangled state in the product of simple group algebras $L(H_i)$. Since the Galois groups act on the space-time surfaces in M^8 they do so also in H. One obtains wave functions in the space of space-time surfaces. G has decomposition to a product (not Cartesian in general) of simple groups. In the same manner, $L(G)$ has a representation of entangled states assignable to $L(H_i)$ [L5, L8].

This picture leads to a model of analysis as a cognitive process identified as a cascade of "small state function reductions" (SSFRs) analogous to "weak" measurements.

1. Cognitive measurement would reduce the entanglement between $L(H_1)$ and $L(H_2)$, the between $L(H_2)$ and $L(H_3)$ and so on. The outcome would be an unentangled product of wave functions in $L(H_i)$ in the product $L(H_1) \times L(H_2) \times ....$ This cascade of cognitive measurements has an interpretation as a quantum correlate for analysis as factorization of a Galois group to its prime factors. Similar interpretation applies in M^4 degrees of freedom.

2. This decomposition could correspond to a replacement of P with a product $\prod_i P_i$ of polynomials with degrees $n = n_1 n_2 ...$, which is irreducible and defines a union of separate surfaces without any correlations. This process is indeed analogous to analysis.

3. The analysis cannot occur for simple Galois groups associated with extensions having no decomposition to simpler extensions. They could be regarded as correlates for irreducible primal ideas. In Eastern philosophies the notion of state empty of thoughts could correspondto these cognitive states in which SSFRs cannot occur.

4. An analogous process should make sense also in the gravitational sector and would mean the splitting of $K = n_A$ appearing as a factor $n_{gr} = Kp$ to prime factors so that the sizes of CDs involved with the resulting structure would be reduced. This process would reduce to a simultaneous measurement cascade in hyperbolic and trigonometric Abelian extensions. The IR cutoffs having interpretation as coherence lengths would decrease in the process as expected. Nature would be performing ordinary prime factorization in the gravitational degrees of freedom.

Cognitive process would also have a geometric description.

1. For the algebraic EQs, the geometric description would be as a decay of n-sheeted 4-surface with respect to M^4 to a union of n_i-sheeted 4-surfaces by SSFRs. This would take place for flux tubes mediating all kinds of interactions.

   In gravitational degrees of freedom, that is for trascendental EQs, the states with $n_{gr} = Kp$ having bundles of Kp flux tubes would deca to flux tubes bundles of $n_{gr,i} = K_i p$, where K_i is a prime dividing K. The quantity $log(K)$ would be conserved in the process and is analogous to the corresponding conserved quantity in arithmetic quantum field theories (QFTs) and relates to the notion of infinite prime inspired by TGD [K3].

2. This picture leads to ask whether one could speak of cognitive analogs of particle reactions representing interactions of "thought bubbles" i.e. space-time surfaces as correlates of cognition. The incoming and outgoing states would correspond to a Cartesian product of simple subgroups: $G = \prod_i^{\times} H_i$. In this composition the order of factors does not matter and the situation is analogous to a many particle system without interactions. The non-commutativity in general case leads to ask whether quantum groups might provide a natural description of the situation.

3. Interestingly, Equivalence Principle is consistent with the splitting of gravitational flux tube structures to smaller ones since gravitational binding energies given by Bohr model in $1/r$ gravitational potential do not depend on the value of $\hbar_{gr}$ if given by Nottale formula $\hbar_{gr} = GMm/v_0$ [L9]. The interpretation would be in terms of spontaneous quantum decoherence taking place as a decay of gravitational flux tube bundles as the distance from the source increases.

2.4 Still some questions about $M^8 - H$ duality

There are still on questions to be answered.

1. The map $p^k \rightarrow m^k = \hbar_{eff} p^k/p\cdot p$ defining $M^8 - H$ duality is consistent with Uncertainty Principle but this is not quite enough. Momenta in M^8 should correspond to plane waves in H.

   Should one demand that the momentum eigenstate as a point of cognitive representation associated with $X^4 \subset M^8$ carrying quark number should correspond to a plane wave with momentum at the level of $H = M^4 \times CP_2$? This does not make sense since $X^4 \subset CD$ contains a large number of momenta assignable to fundamental fermions and one does not know which of them to select.

2. One can however weaken the condition by assigning to CD a 4-momentum, call it P. Could one identify P as

   (a) the total momentum assignable to either half-cone of CD

   (b) or the sum of the total momenta assignable to the half-cones?

The first option does not seem to be realistic. The problem with the latter option is that the sum of total momenta is assumed to vanish in ZEO. One would have automatically zero momentum planewave. What goes wrong?

1. Momentum conservation for a single CD is an ad hoc assumption in conflict with Uncertainty Principle, and does not follow from Poincare invariance. However, the sum of momenta vanishes for non-vanishing planewave when defined in the entire M^4 as in QFT, not for planewaves inside finite CDs. Number theoretic discretization allows vanishing in finite volumes but this involves finite measurement resolution.

2. Zero energy states represent scattering amplitudes and at the limit of infinite size for the large CD zero energy state is proportional to momentum conserving delta function just as S-matrix elements are in QFT. If the planewave is restricted within a large CD defining the measurement volume of observer, four-momentum is conserved in resolution defined by the large CD in accordance with Uncertainty Principle.

3. Note that the momenta of fundamental fermions inside half-cones of CD in H should be determined at the level of H by the state of a super-symplectic representation as a sum of the momenta of fundamental fermions assignable to discrete images of momenta in $X^4 \subset H$.

2.4.1 $M^8 - H$-duality as a generalized Fourier transform

This picture provides an interpretation for M^8-H duality as a generalization of Fourier transform.

1. The map would be essentially Fourier transform mapping momenta of zero energy as points of $X^4 \subset CD \subset M^8$ to plane waves in H with position interpreted as position of CD in H. CD and the superposition of space-time surfaces inside it would generalize the ordinary Fourier transform . A wave function localized to a point would be replaced with a superposition of space-time surfaces inside the CD having interpretation as a perceptive field of a conscious entity.

2. $M^8 - H$ duality would realize momentum-position duality of wave mechanics. In QFT this duality is lost since space-time coordinates become parameters and quantum fields replace position and momentum as fundamental observables. Momentum-position duality would have much deeper content than believed since its realization in TGD would bring number theory to physics.

2.4.2 How to describe interactions of CDs?

Any quantum coherent system corresponds to a CD. How can one describe the interactions of CDs? The overlap of CDs is a natural candidate for the interaction region.

1. CD represents the perceptive field of a conscious entity and CDs form a kind of conscious atlas for M^8 and H. CDs can have CDs within CDs and CDs can also intersect. CDs can have shared sub-CDs identifiable as shared mental images.

2. The intuitive guess is that the interactions occur only when the CDs intersect. A milder assumption is that interactions are observed only when CDs intersect.

3. How to describe the interactions between overlapping CDs? The fact the quark fields are induced from second quantized spinor fields in in H *resp.* M^8 solves this problem. At the level of H, the propagators between the points of space-time surfaces belonging to different CDs are well defined and the systems associated with overlapping CDs have well-defined quark interactions in the intersection region. At the level of M^8 the momenta as discrete quark carrying points in the intersection of CDs can interact.

2.4.3 Zero energy states as scattering amplitudes and subjective time evolution as sequence of SSFRs

This is not yet the whole story. Zero energy states code for the ordinary time evolution in the QFT sense described by the S-matrix. What about subjective time evolution defined by a sequence of "small" state function reductions (SSFRs) as analogs of "weak" measurements followed now and then by BSFRs? How does the subjective time evolution fit with the QFT picture in which single particle zero energy states are planewaves associated with a fixed CD.

1. The size of CD increases at least in statistical sense during the sequence of SSFRs. This increase cannot correspond to M^4 time translation in the sense of QFTs. Single unitary step followed by SSFR can be identified as a scaling of CD leaving the passive boundary of the CD invariant. One can assume a formation of an intermediate state which is quantum superposition over different size scales of CD: SSFR means localization selecting single size for CD. The subjective time evolution would correspond to a sequence of scalings of CD.

2. The view about subjective time evolution conforms with the picture of string models in which the Lorentz invariant scaling generator L_0 takes the role of Hamiltonian identifiable in terms of mass squared operator allowing to overcome the problems with Poincare invariance. This view about subjective time evolution also conforms with super-symplectic and Kac-Moody symmetries of TGD.

   One could perhaps say that the Minkowski time T as distance between the tips of CDs corresponds to exponentiated scaling: $T = exp(L_0 t)$. If t has constant ticks, the ticks of T increase exponentially.

The precise dynamics of the unitary time evolutions preceding SSFRs has remained open.

1. The intuitive picture that the scalings of CDs gradually reveal the entire 4-surface determined by polynomial P in M^8: the roots of P as "very special moments in the life of self" would correspond to the values of time coordinate for which SSFRs occur as one new root emerges. These moments as roots of the polynomial defining the space-time surface would correspond to scalings of the size of both half-cones for which the space-time surfaces are mirror images. Only the upper half-cone would be dynamical in the sense that mental images as sub-CDs appear at "geometric now" and drift to the geometric future.

2. The scaling for the size of CD does *not* affect the momenta associated with fermions at the points of cognitive representation in $X^4 \subset M^8$ so that the scaling is not a genuine scaling of M^4 coordinates which does not commute with momenta. Also the fact that L_0 for super symplectic representations corresponds to mass squared operator means that it commutes with Poincare algebra so that M^4 scaling cannot be in question.

3. The Hamiltonian defining the time evolution preceding SSFR could correspond to an exponentiation of the sum of the generators L_0 for super-symplectic and super-Kac Moody representations and the parameter t in exponential corresponds to the scaling of CD assignable to the replaced of root r_n with root r_{n+1} as value of M^4 linear time (or energy in M^8). L_0 has a natural representation at light cone boundaries of CD as scalings of light-like radial coordinate.

4. Does the unitary evolution create a superposition over all over all scalings of CD and does SSFR measure the scale parameter and select just a single CD?

   Or does the time evolution correspond to scaling? Is it perhaps determined by the increase of CD from the size determined by the root r_n as "geometric now" to the root r_{n+1} so that one would have a complete analogy with Hamiltonian evolution? The scaling would be the ratio r_{n+1}/r_n which is an algebraic number.

   Hamiltonian time evolution is certainly the simplest option and predicts a fixed arrow of time during SSFR sequence. L_0 identifiable essentially as a mass squared operator acts like conjugate for the logarithm of the logarithm of light-cone proper time for a given half-cone.

   One can assume that L_0 as the sum of generators associated with upper and lower half-cones if the fixed state at the lower half-cone is eigenstate of L_0.

How does this picture relate to p-adic thermodynamics in which thermodynamics isdetermined by partition function which would in real sector be regarded as a vacuum expectation value of an exponential $exp(iL_0t)$ of a Hamiltonian for imaginary time $t = i\beta$ $\beta = 1/T$ defined by temperature. L_0 is proportional to mass squared operator.

1. In p-adic thermodynamics temperature T is dimensionless parameter and $\beta = 1/T$ is integer valued. The partition function as exponential $exp(-H/T)$ is replaced with $p^{\beta L_0)}$, $\beta = n$, which has the desired behavior if L_0 has integer spectrum. The exponential form $e^{L_0/T_R)}$, $\beta_R = nlog(p)$ equivalent in the real sector does not make sense p-adically since the p-adic exponential function has p-adic norm 1 if it exists p-adically.

2. The time evolution operator $exp(-iL_0t)$ for SSFRs (t would be the scaling parameter) makes sense for the extensions of p-adic numbers if the phase factors for eigenstates are roots of unity belonging to the extension. $t = 2\pi k/n$ since L_0 has integer spectrum. SSFRs would define a clock. The scaling $exp(t) = exp(2\pi k/n)$ is however not consistent with the scaling by r_{n-1}/r_n.

   Both the temperature and scaling parameter for time evolution by SSFRs would be quantized by number theoretical universality. p-Adic thermodynamics could have its origins in the subjective time evolution by SSFRs.

3. In the standard thermodynamics it is possible to unify temperature and time by introducing a complex time variable $\tau = t + i\beta$, where $\beta = 1/T$ is inverse temperature. For the space-time surface in complexified M^8, M^4 time is complex and the real projection defines the

4-surface mapped to H. Could thermodynamics correspond to the imaginary part of the time coordinate?

Could one unify thermodynamics and quantum theory as I have indeed proposed: this proposal states that quantum TGD can be seen as a "complex square root" of thermodynamics. The exponentials $U = exp(\tau L_0/2)$ would define this complex square root and thermo-dynamical partition function would be given by $UU^\dagger = exp(-\beta L_0)$.

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