Twistors and holography= holomorphy vision

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Abstract

Twistorialization involves several problems. Mention only the identification of the twistor space, the googly problem meaning that only second massless M^4 chirality allows geometrization in this way, the problem that massive fields do not allow twistorialization, and the problem that in general relativity only space-times with vanishing Weyl tensor allow twistor structure.

In the TGD framework, twistorialization should be performed for $H = M^4 \times CP_2$. Now there are no primary bosonic fields since they are represented in terms of the induced spinor connection and metric and also classical color fields are obtained by induction. Twistor lift was based on the replacement of space-time surfaces in $H = M^4 \times CP_2$ with the analogs of their 6-D twistor spaces X^6 as sphere bundles as a surfaces in the twistor space $T(H)$ of H identified as the product $T(M^4) \times T(CP_2)$ of twistor spaces H. In TGD, the replacement of $T(M^4) = CP_3$ with $CP_{2,1}$ having one hypercomplex coordinate is natural. Dimensional reduction for the extremals of 6-D Kähler action and the identification of the fiber spheres CP_1 of $T(M^4)$ and $T(CP_2)$ was needed to product to produce the X^6 as a sphere bundle over X^4.

Holography= holomorphy (H-H) vision in turn allows to solve the field equations for any general coordinate invariant action expressible in terms of the induced geometry allows to solve the field equations, which are extremely nonlinear partial differential equations, exactly by reducing them to purely algebraic local equations. The independence of action means universality. H-H vision conforms with $T(H)$ view but one can ask whether one could twist TGD without the introduction of $T(H)$ by representing the twistor spheres of $T(M^4)$ and $T(CP_2)$ as homologically non-trivial spheres of the causal diamond CD (missing the line connecting its tips) and CP_2. The second condition involved with the H-H principle would represent the identification of the twistor spheres.

In this article various problems of the twistorialization are discussed in the TGD framework and the question whether the H-H principle is enough for twistorialization is discussed.

Contents

1 Introduction

Twistor lift of TGD [K6, K4, K2] relies on the replacement of space-time surfaces in $H = M^4 \times CP_2$ with the analogs of their 6-D twistor spaces X^6 as sphere bundles as surfaces in the twistor space $T(H) = T(M^4) \times T(CP_2)$ of H identified as the product of twistor spaces $T(M^4)$ and $T(CP_2)$.

Dimensional reduction for the extremals of 6-D Kähler action and the identification of the fiber spheres CP_1 of $T(M^4)$ and $T(CP_2)$ is needed to product to produce the X^6 as a sphere bundle over X^4. The dimensionally reduced action is 4-D Kähler action and volume term as in terms of an analog of dynamical, length scale dependent cosmological constant. Holography= holomorphy (H-H) vision [L11, L15] allows us to solve the field equations for the 4-D action exactly.

The structural analogies of the H-H based solutions with the twistor lift led to ask whether the twistor spheres of $T(M^4)$ and $T(CP_2)$ could be represented as surfaces inside space-time surfaces and whether the twistorialization of TGD could be carried out without the introduction of $T(H)$. As a matter of fact, this kind structural analogies should exist since the notion of twistor space is basically deduce from the geometry of M^4 and CP_2 rather than vice versa.

1.1 What twistors are?

The twistor space of M^4 can be defined purely geometrically. Twistor would describe fixing a coordinate frame with origin at a given M^4 point and a fixed quantization axis of spin defined by a direction of light-like momentum characterized by a point of CP_2. The light-like vector also defines a 2-D orthogonal plane. In massless field theories this corresponds to a choice of momentum vector and polarization vector. Light-like geodesics at a given point define the fiber at this point. Fiber is a 2-sphere. The bundle structure is non-trivial. The twistor spheres at points with-like separation have a common point. Not that the twistor sphere would be represented in M^4.

In the twistor Grassmannian approach [B3, B6, B4, B2, B7, B1, B5], the twistor space of M^4 is identified as $CP_3 = SU(4)/SU(3) \times U(1)$. One can end up with this identification in the following way.

1. Single bi-spinor represents a light-like momentum via the correspondence $p^k \rightarrow p^k\sigma_k$, where σ_k are Pauli spin matrices acting on complex bi-spinors. Light-likeness implies that the determinant of this 2×2 matrix vanishes. Determinant is a bilinear function or rows and columns so that the representation so that complex scalings of the bispinor do not affect the condition $p^2 = 0$.

   Twistors thus correspond to pairs of dotted (χ) and undotted (ψ) bi-spinors as conjugate representations of the Lorentz group defining the matrix $p^k\sigma_k$. Dotted and undotted bispinors are related by co-incidence relation $\chi = p^k\sigma_k\psi$: this does not fix ϕ uniquely since $\psi \rightarrow \psi + p^r k\sigma_k\phi$ leaves χ unaffects. χ and ψ span C^2 each so that one has C^4. The invariance of p^k under opposite complex scalings of the bi-spinors suggests that C^4 must be replaced with the projective space $CP_3 = SU(4)/SU(3) \times U(1))$. The problem is that the geometrically identified twistor space is non-compact whereas CP_3 is compact.

2. CP_3 should correspond to S^2 bundle over M^4 with S^2 consisting of light-like geodesics with common origin. Compact CP_3 should correspond to bundles over M^4. This cannot be true since M^4 is not compact. This leads to the proposal that compactification of M^4 is involved. This looks to me questionable.

3. The Minkowskian signature of M^4 leads to ask whether a more appropriate identification of the twistor space could be based on group theory and would be as a non-compact space $CP_{2,1} = SU(3,1)/SU(3) \times U(1)$. It should have one real time-like dimension and 5 space-like real dimensions: one complex coordinate should be hypercomplex and 2 coordinates should be complex. This would fit nicely with the H-H vision in which M^4 has one hypercomplex coordinate and one complex coordinate and a twistor sphere adds one complex coordinate. Note that now the scaling of hypercomplex coordinates with a complex number does not make sense so that the group theoretic view is necessary.

   One key problem of the twistor Grassmannian approach is that the natural signature of the Minkowski space would be (2,2) rather than (1,3). Could one think that for the signature (1,3) the two real time-like coordinates defining complex coordinates are transformed to a hypercomplex coordinate pair $(u = t + z, v = t - z)$. CP_3 naturally associated with the signature $(2, 2)$ would be transformed to $CP_{2,1} \equiv SU(3, 1)/SU(3) \times U(1)$ associated with the signature (1,3).

4. CP_3 ($CP_{2,1}$) is obtained by adding to E^6 the CP_2 ($CP_{1,1} = SU(2, 1)/SU(2) \times U(1)$) at infinity. The set of geodesics directed from the origin of E^6 to infinity is indeed 4-D. CP_3 and $CP_{2,1}$ should allow an interpretation as a bundle with fiber CP_1.

   How could one understand this geometrically? Does the M^4 correspond to the 4-D space of homologically non-trivial 2-spheres in $CP_{2,1}$ as counterparts of twistor spheres? Is this a non-singular manifold? Note that when the points of M^4 as 2-spheres are connected by light-like geodesics, the corresponding 2-spheres must have an intersection point.

The twistor bundle of CP_2 is something completely new from the point of view of field theories. The definition of the twistor space $T(CP_2) = SU(3)/U(1) \times U(1)$ is as the space of choices of quantization axes of color isospin and hypercharge. The fiber is sphere as the set of geodesics directed rom the origin to the infinity, which corresponds to a homologically non-trivial 2-sphere added to E^4.

1.2 About the problems of twistorialization

Twistorialization is also plagued by other difficulties than those already mentioned. Besides the problems associated with the interpretation of CP_3 as twistor space, favoring the (2,2) signature of Minkowski space, there is a problem that the description of massive particles fails in the twistor approach. A heuristic guess is that light-likeness in the 8-D sense holding true for the modes of the second quantized induced spinor fields might help. The classical picture supports this too: for the light-like geodesics in $M^4 \times S^1 \subset M^4 \times CP_2$ M^4 projection of 8-momentum is indeed massive.

There is also the so-called googly problem and the problem that in general relativity only conformally flat space-times allow twistor structure.

1.2.1 Googly problem

Twistorialization means a geometrization of spin in the twistor Grassmannian approach [B3, B6, B4, B2, B7, B1, B5], which indeed allows a very elegant descirption of scattering amplitudes of spinning particles in $\mathcal{N} = 4$ SUSY. This requires massless fields. Spin corresponds to a partial wave in the twistor sphere and there is no need to introduce it as a separate internal degree of freedom. Holomorphy is an essential ingredient and analogous to holomorphy= holography hypothesis of TGD but realized at the level of surfaces rather than fields.

Googly problem means that anti-self-dual massless fields do not allow this geometrization. Only self-dual field configurations allow twistorialization in terms of holomorphic fields in twistor space. Could the fields with opposite chiralities correspond to holomorphic and antiholomorphic fields? Or does anti-holomorphy correspond to antiparticles? Why are both of them not allowed? How would one describe their interaction?

There are several notions involved: the notions of chirality/handedness, helicity and orientability, which is a property of space-time. Reflection P in M^4 changes chirality/helicity whereas charge conjugation changes the helicity. P is not a symmetry in the standard model.

A possible solution of the googly problem in terms of pin structures (see this) has been proposed. Also the reflections in M^4 would be symmetries unlike for spin structure. The two chiralities would be related by a symmetry transforming a left handed glove to a right-handed one if this symmetry is realized geometrically. Spatial reflection P and time reflection T change the orientation of M^4 but PT preserves it. P and T are not representable as transformations generated from identity and this seems to be the case also for PT. Could one somehow extend the Lie-group symmetries (Poincare group) so that PT is generated from identity. To me these proposals look artificial to me.

1.2.2 Conformal flatness is required in GRT

The existence of the twistor structure requires conformal invariance and massless fields in twistor space are indeed holomorphic and self-dual fields. Twistor structure is allowed only by conformally flat space-times. This condition is very strong and implies that the so-called Weyl tensor (see this) vanishes. The vanishing of the Weyl tensor implies that tidal forces describable in terms of geodesic deviation vanish. Also the trace of the energy momentum tensor must vanish as it indeed does for the Yang-Mills action. This condition is violated for typical solutions of Einstein's equations.

2 Twistorialization in TGD

There one can consider two, not mutually exclusive, approaches to twistorialization in TGD [K6, K4, K2]).

1. Twistor lift is based on the twistor space of $T(H)$ identified as the product $T(H) = T(M^4) \times T(CP_2)$ twistor spaces of M^4 and CP_2 is the first approach. It involves the identification of the twistor spheres of $T(M^4) \times T(CP_2)$ and dimensionally reduces the 6-D Kähler action of $T(H)$ to the sum of 4-D Kähler action and a volume term.

2. H-H principle [L11, L15]solves the field equations space-time surfaces and does not exclude $T(H)$ but can be extended to the level of $T(H)$. There are some indications that H-H alone could describe the twistorialization. The twistor spheres indeed have natural representation in $X^4 \subset H$. Since the notion of twistor is realized in terms of the geometry of the space-time, it would be natural that space-time surfaces provide representation of the twistor lift. If not, something might be wrong.

2.1 Some background

The existence of both left and right fermion chiralities are the source of googly problem. Reflection transforming right and left chiralities to each other is therefore closely related to the problem.

In TGD, parity violation is understood. The embedding space $H = M^4 \times CP_2$ is 8-dimensional. Both M^4 chiralities are predicted and parity violation and the strange-looking coupling structure of the Standard Model finds explanation. Spinor connection and second quantized free spinor field from H is induced to the spacetime surface are induced. The baryon and fermion numbers correspond to the two H-chiralities and the couplings to quarks and leptons are obtained correctly. For quarks/leptons, right- and left-handed M^4 chirality correspond to different CP_2 chiralities and the massivation requiring the mixing of M^4 chiralities automatically follows from the mixing of the chiralities for the massless Dirac equation in H.

The counterpart of Googly problem could be however encountered at the H-level if the twistorialization also now requires that only a single H-chirality is allowed. Only quarks or leptons would be fundamental fermions: I have considered both options. The idea about leptons as a bound state of quarks is discussed in [L4] and I have also considered the idea that quarks could be quarks, which have suffered charge fractionation. Now I have become skeptical about both options.

2.2 Twistor lift of TGD

Twistor Grassmannian approach [B3, B6, B4, B2, B7, B1, B5] provides an extremely economical description of scattering amplitudes in $\mathcal{N} = 4$ SUSY and even for more gauge theories. Therefore one can ask whether TGD could have a twistor lift and what would this mean?

1. Around the same time that I started developing TGD, it had been discovered that M^4 (or E^4 or S^4) and CP_2 are in a completely special position with respect to twistorization. Only they allow a twistor space, which has a Kähler structure [A2]. The Kähler structure indeed plays a key ontological role in TGD [L12, L13]. TGD is unique by the requirement that twistor lift exists and would correspond to replacing $M^4 \times CP_2$ with the product $T(H) = T(M^4) \times T(CP_2)$ of the twistor spaces $T(M^4)$ and $T(CP_2)$. This led to the proposal that M^4 has the analog of Kähler structure.

2. The induction procedure for gauge potentials would generalize to the twistor level [K6, K4]. The twistor space for the spacetime surface would be a 6-surface X^6 in $T(H) = T(M^4) \times T(CP_2)$ and therefore an S^2 bundle over the spacetime surface and a twistor structure would be induced on the 6-surface.

2.2.1 About the details of the twistor lift

Consider now this in more detail.

1. The requirement that X^6 is an S^2 bundle, requires a dimensional reduction of the 6-D Kähler action, which reduces it to the sum of the 4-D Kähler action and a volume term that can be interpreted as the emergence of the cosmological constant Λ [K6, K4, K2].

   The cosmological constant Λ is determined as the coefficient of the volume term of 4-D action. It is determined by the sum of Kähler action for the twistor sphere S^2 of X^6, which would depend on the induced metric and Kähler form. Λ would be dynamic and have a spectrum. The Kähler forms of both H and $T(H)$ have both M^4 and CP_2 parts. Their destructive interference can make the induced Kähler and therefore also cosmological constant very small. A natural guess is that it is inversely proportional to the square of the p-adic length scale and approaches zero in long length scales.

2. The induced metric and Kähler form of $T(X^6)$ are obtained in the usual way and in $S^2(X^4)$ the metric and Kähler form are the sum of contributions from $S^2(T(M^4)$ and $S^2(T(CP_2)$. This gives rise to a dynamical cosmological constant determined by the part of the 6-D Kähler action coming from $S^2(X^4)$.

In TGD, only fermions are fundamental particles and all particles, especially bosons, are built from these. Therefore twistor geometrization of fermion spin and isospin might make sense but is not necessary since second quantized free fermion fields in H gives fermionic propagators elegantly.

Color symmetry and rotations represent two exact symmetries realized as isometries and it might be possible to twistorialize the corresponding quantum numbers (spin and color hypercharge and - isospin). Electroweak symmetries are not exact symmetries and are not realized as isometries. Could twistorialization apply also to weak spin and hypercharge?

Is the twistorialization of the fermion spin, electroweak spin, and color quantum numbers possible? The correspondence between twistor spheres of $T(M^4)$ and $T(CP_2)$ poses very strong constraint. One can argue that if fermion spin and color charges can be twistorialized as points of these 2 twistor spheres, spin and color isospin and hypercharge would closely relate to each other. Can this make sense?

1. The first objection is that single quark spin sould correspond to 3 possible color charges. This could be understood in terms of the 3-valued character of the map $S^2(T(M^4))$ to $S^2(CP_2)$. It could allow to assign to a given spin of quark 3 different values of color charges as different space-time surfaces. This would be an analog for the representation of color as partial waves in CP_2 rather than as spin-like quantum number for fermions as in QCD?

2. The second objection is that leptons have no color quantum numbers. Could this correspond to the fact that for leptonic space-time surfaces $S^2(T(X^4)$ correspond a single point of $T(CP_2)$ so that only the twistor sphere of $T(M^4)$ is "activated"?

To sum up, in TGD only fermions are fundamental particles and all particles, especially bosons, are built from these. Therefore twistor geometrization of fermion spin and isospin might make sense but is not necessary since second quantized free fermion fields in H gives fermionic propagators elegantly.

2.2.2 How does twistor lift relate to H-J structure?

How does the induced twistor structure relate to the Hamilton-Jacobi (H-J) structure [L7] required by the existence of H-H structure?

1. H-J structure means the existence of 4 coordinates combining to form hypercomplex coordinate u and its conjugate v and complex coordinate w and its conjugate $\overline{w}$. The coordinate lines of u and v have light-like tangent vectors, which by integrability are proportional to gradients.

2. A point in twistor space corresponds to the choice of an M^4 point and of light-like direction at each point. Making this choice at each point of M^4 defines a section of $T(M^4)$. Could H-J structure correspond to a section of $T(M^4)$.

3. Or could it correspond to a section of $T(H)$. If the light-like coordinate curves for u and v at the space-time surface have CP_2 projections as geodesic lines, twistorialization for massive particles might be possible since M^4 projections could correspond to massive geodesics or more general curves which constant M^4 momentum squared. Induction of the twistor structure of H would generate correlations between twistor structures of M^4 and CP_2 at the space-time surface.

4. Under what conditions two H-J structures are equivalent? Physical intuition suggests that two H-J structures, which are related by a conformal diffeomorphism of H, generating new identical space-time surfaces in H-H vision, are equivalent. Conformal diffeomorphism would reduce to a transformation of hypercomplex coordinate u independent of complex coordinates and an analytic transformation of the 3 complex coordinates with coefficients depending on hypercomplex coordinates. It also looks natural that the conformal diffeomorphisms reduce to products of transformations acting in M^4 and CP_2 degrees of freedom.

5. How many H-J structures do exist? The obstacles come from topology, complex structure and integrability. A physics inspired guess is that they could correspond to self-dual or anti-self-dual solutions of the massless spin 1 field in M^4. One would have a polarization vector and light-like vector at each point. Massless extremals define such sections of $T(M^4)$ in H [K1].

2.3 Could holography= holomorphy vision make possible twistorialization without twistor lift?

H-H vision encourages to ask whether the homologically non-trivial twistor spheres of $T(M^4)$ and $T(CP_2)$ have representations as homologically non-trivial 2-surfaces inside space-time surfaces: this could mean that the introduction $T(H)$ might not be necessary. In particular, cosmological constant would have inherent representation in terms of the solution of field equations according to H-H vision. This does not seem to conform with the idea that $T(H)$ level determines the cosmological constant. On the other hand, number theoretic vision suggests that the couplings appearing in the classical action, including cosmological constant, are determined by the number theoretic expression for the action.

The possible representations of twistor spheres as 2surfaces inside $X^4 \subset H$, should be homologically non-trivial in X^4. One can indeed represent the twistor spheres of M^4 and CP_2 in a natural way at the space-time level.

1. The space-time surface X^4 must contain a homologically non-trivial geodesic sphere $S^2(CP_2)$ in order to allow the representation of CP_2 twistor sphere. Cosmic strings and monopole flux tubes do so but massless extremals do not.

2. The homological non-triviality of a sphere $S^2(M^4)$ embeddable inside the space-time surface X^4 is enough and is possible to realize dynamically. If the space-time surface is analogous to a magnetic monopole in the sense that that $S^2(M^4)$ is mapped to $S^2(CP_2)$, $S^2(M^4)$ cannot be contracted to a point inside X^4.

   For instance, the condition $f_2 = \xi_1 = w = 0$ mapping to each other the twistor spheres $S^2(M^4)$ and $S^2(CP_2)$ identified as homologically non-trivial spheres in X^4 defines also a section of $T(CP_2)$ as the analog of H-J structure of CP_2 [L11, L15].

Note that the homological non-triviality at the level of M^4 for the M^4 sphere $S^2(M^4)$ is not necessary but could be realized inside the CD if the CD has a hole, i.e. does not contain the line connecting its tips. This looks artificial.

2.3.1 H-H vision in more detail

The simplest variant of H-H vision is as follows.

1. A general solution to the field equations is obtained by requiring that the spacetime surfaces correspond to the roots for the pairs $(f_1, f_2) : H \rightarrow C^2$. f_1 and f_2 are analytic functions of the hypercomplex coordinate u and 3-complex coordinates (w, ξ_1, ξ_2) of H. The equations reduce to purely algebraic conditions and the solution is universal and valid for any action that is general coordinate-invariant and based on induced geometry [L15].

2. The surface $(f_1, f_2) = (0, 0)$ would define the intersection of 2 6-D surfaces $f_1 = 0$ and $f_2 = 0$. The functions f_i are analytic functions of a hypercomplex coordinate u and complex coordinate w of M^4 and of complex coordinates ξ_1, ξ_2 for CP_2. The choices of these coordinates correspond to different Hamilton-Jacobi (H-J) structures [L7] which could be identified as sections of the twistor space of M^4.

3. In the simplest situation u and v correspond to light-like coordinates $t + z$ and $t - z$. $w = x + iy$ as a planar coordinate could serve as a local complex coordinate of for the second hemisphere of a homologically non-trivial sphere CP_1 of causal diamond (CD) [L8]. Homological non-triviality means that the time-like axis connecting the vertices of the causal diamond CD defines a hole in the CD. CD with light-like boundaries could be sliced by light-cone boundaries parallel to its second boundary. CP_1 would parametrize the light-like geodesic emanating from the points at the axis of the light-cone boundary.

4. For instance, $f_2 = \xi_2 - w = 0$ gives rise to a 6-surface, which can be interpreted as a bundle-like structure in 2 ways. For the first interpretation, $(u, (v), \xi_1)$ serve as coordinates of the 4-D base space X^4 and w is the local coordinate for the twistor sphere realized as the homologically non-trivial sphere CP_1 of causal diamond (CD) acting as a fiber.

   The second interpretation is that $(u, (v), w)$ serve as coordinates of the 4-D base space and ξ_1 is the coordinate of the homologically non-trivial geodesic sphere CP_1 of CP_2 acting as a fiber. The condition $f_1 = 0$ fixes ξ_1 as a function of w and identifies the two fibers and determines X^4.

This picture is the simplest one and perhaps too simple.

1. The physical picture suggests that there is a dimensional hierarchy of surfaces with dimensions 4, 2, 0 [L15]. The introduction of f_3 would allow us to identify 2-D string world sheets or monopole flux tubes as roots of (f_1, f_2, f_3). The introduction of f_4 would make it possible to identify points of string world sheets as roots of (f_1, f_2, f_3, f_4) having interpretation as fermionic vertices. The analytic maps $g : C^2 \rightarrow C^2$ act as dynamical symmetries for $f = (f_1, f_2) : H \rightarrow C^2$.

   In the ase of $f = (f_1, f_2, f_3) : H \rightarrow C^3$ the analytic local diffeomorphisms of the space-time surfaces for 2-D roots $f = (f_1, f_2, f_3) = 0$ would act as dynamical symmetries.

2. A prediction, made already earlier [L9] is the breaking of extended conformal invariance as a gauge symmetry in the following sense. Various conformal algebras have non-negative conformal weights and have an infinite hierarchy of isomorphic algebras as sub-algebras. The conformal symmetries as gauge symmetries would transform into dynamical symmetries for finite dimensional subalgebra and this conforms with the p-adic mass calculations [L5].

3. One could assign to these sets of these 2-surfaces and points discriminants in the way as to the maps $g : C^2 \rightarrow C^2$ [L15]. This makes sense also for $f = (f_1, f_2, f_3, f_4)$: $H \rightarrow C^4$. The condition that the classical action exponential reduces to the product of exponents of all these 3 discriminants would determine the coupling constant evolution. This would correspond to the assignment of separate action exponentials to these surfaces of the dimensional hierarchy and also this would conform with the physical picture. Note that C^4 defines extended twistor space. Presumably this is a mere accident.

2.3.2 Objections against H-H without $T(H)$

Consider now the objections against H-H without $T(H)$.

1. $T(H)$ option for TGD based view of twistor space of H is very elegant and a rigorous proof that the equivalence with H-H option is lacking.

2. If f_2, appearing in a simple mode, is assumed to be surjective in either direction, all space-time surfaces involved would contain homologically non-trivial 2-spheres of both CD and CP_2. This would exclude for instance massless extremals and cosmic string type extremals [K1]. The problem disappears if the f_2 as a map between the homology spheres can also have winding number 0 in either direction or both directions. This could allow massless extremals, CP_2 type extemals, and cosmic strings and their deformations. Could the value of cosmological constant determined by f_2 vanish for MEs) and have its maximal value for CP_2 type extremals.

   Note that these extremals are possible also for the $T(H)$ option since the twistor sphere for X^6 can be identified with the twistor sphere of only $T(M^4)$ or $T(CP_2)$.

To sum up, HH vision is consistent with the $T(H)$ option. H-H without $T(h)$ would provide twistorialization without twistor lift.

2.4 Is the Googly problem an illusion in the TGD framework?

In the twistor Grassmann approach [B3, B6, B4, B2, B7, B1, B5] twistors are interpreted in terms of Majorana fermions of Weyl fermions of fixed chirality (this in fact is a problem of $\mathcal{N} = 1$ SUSYs). The TGD variant of twistor Grassmannian approach [K4, K7] relies on the assumption that the boundaries of string world sheets at partonic orbits carry quantum numbers.

It must be however emphasized that twistor description of fermions is not necessary in the TGD approach: the propagators for free fermion fields in H play a central role and vertices emerge from exotic smooth structures [L1] possible only in 4-dimensions and allowing the description of fermion pair creation for free fermion fields as a fundamental vertex [L14, L6].

2.4.1 Twistor space $T(H)$ as the space of choices for the quantization axes

It is possible to start from the geometrization of the space of light-like geodesics, where spinors emerge naturally. There is however no need to assume that the twistors correspond to ordinary spinors and have anything to do with fermions. Could the googly problem be an outcome of a wrong interpretation of the twistor space?

1. To get perspective it is better to start from the twistor space $T(CP_2) = SU(3)/U(1) \times U(1)$, which has a natural interpretation as a space for choices of color and isospin quantization axes. The first application was rather unexpected: the honeybee dance. Topologist Barbara Shipman [A1] had discovered that this space appears in the model of the honeybee dance. The idea that quarks could have something to with honeybee dance is of course total nonsense in the framework of standard particle physics but TGD predicts the possibility of quantum in all scales, in particular in biological length scales and this led to a TGD based model [K3] for the finding.

   The wave function in $T(CP_2)$ would correspond to the wave function in the space of choices of quantization axes. The choice of the quantization axes is an essential part of a quantum measurement. It would be very nice if it could correspond to a state function at a higher level, the level of an experimenter. This view would be consistent with the fact that fundamental fermions appear as basic building bricks of all elementary particles in TGD.

2. Could the S^2 part of the M^4 twistor also be interpreted as a choice of the point of M^4 as the origin of the rest frame and the choice of spin quantization axis as a point of S^2. In the case of a massless particle, the spin quantization axis is a direction that is the same as the direction of motion.

2.4.2 Could also the description of elementary particle quantum numbers using twistor wave functions make sense?

The idea about description of elementary particles with spin using wave functions in a twistor sphere is however extremely elegant. Could this make sense? There are two cases to be discussed.

Option I: twistor lift

Consider first the situation for the twistor lift involving $T(H)$. Classically this would mean that a point of a twistor sphere defines not only the direction of quantization axes but also the value of spin. In TGD this could make sense since all fundamental particles are fermions.

1. In M^8, proposed to relate to H by $M^8 - H$ duality as analog of momentum-position duality [L10, L15], momenta as discrete point of M^8 correspond to planewaves in H. This could apply also at the level of twistor space: at the level of H wave functions in twistor sphere would describe fermions spin. At the level of M^8, the point of M^4 twistor sphere would fix the direction of the spin quantization axis and also the spin value. Since the radius of S^2 is fixed, it would fix its magnitude to $s = 1/2$, i.e. the value of the Casimir operator I have built interpretations for twistors based on this observation and the $M^8 - H$ duality.

2. What about CP_2? The point in twistor space $T(CP_2) = SU(3)/U(1) \times U(1)$ would fix the directions of the color isospin and hypercharge quantization axes. Is only quark chirality allowed. Or could it be that for leptons the $S^2(T(X^4))$ is mapped at the level of $T(H)$ to a point of $S^2(T(CP_2))$?

The above arguments suggest abandoning the twistorization of fermion spin. This would not fit well with the twistor-Grassmann approach. On the other hand, despite the undeniable successes of the twistor Grassmann approach, Nima-Arkani Hamed and its other proponents seem to have given it up and the problems for this are obvious.

1. Quantum measurement theory suggests an interpretation of the section of the twistor bundle in terms of choices of quantization axes at a given point of H. The choices of the quantization axes and the spin measurement would mean a localization in the twistor space, in particular S^2. In fact, it would localize in both twistor spheres S^2 since twistor spheres are identified. This conforms with the fact that fermions correspond to isospin and spin doublets.

2. One can of course ask whether the wave functions in the space of choices of quantization axes could also have an interpretation as spin states of fermions.

Option II: H-H principle

One can consider the situation in the framework provided by H-H principle alone, which suggests that the space-time surface, in the case that f_2 is a surjection between the homology spheres, can be seen as a sphere bundle in which the sphere corresponds to the twistor sphere of CP_2 or to the twistor sphere of CD. The wave function in the twistor sphere would correspond to a wave function in either M^4 or CP_2.

But doesn't this allow only the representation of integer spin states and color particles waves with triality $t = 0$? Here the situation is not so simple. The space-time surfaces can have multiple coverings of M^4 and CP_2 and this can lead to the fractionation of quantum numbers for wave functions defined at space-time surfaces but not for this in H. This would be the basic mechanism leading to charge fractionalization and braid statistics, even non-Abelian, in the TGD Universe.

One can construct 4-surfaces for which charge fractionation happens. In M^4, simplest analogs of the 2-sheeted Riemann surface carrying geometric spin 1/2 are associated with $z^{1/2}$. A rotation of 2π along the surface would not lead to the starting point. For more general fractional powers $z^{1/n}$, onehas the spin fractionation occurring for the representations of quantum groups. The same argument applies to various other Cartan charges.

Could the correct conclusion be the following? The space-time wave functions assignable to the twistor spheres of the many-sheeted coverings of M^4 and CP_2, in turn closely related to the notion of effective Planck constants, allow the description of charge fractionation. The fundamental description using spinor modes of H does not however reduce to this kind of description.

2.4.3 The effect of discrete symmetries on H-J structure

Reflection and other discrete symmetries affect the H-J structure defining what hypercomplex and complex coordinates and what the analyticity of (f_1, f_2) means. This would represent these discrete symmetries as geometric transformations of the spacetime surface.

1. The reflection changing left- and right-handed fermions to each other, should also affect the elementary particle-like space-time surfaces associated with them and for the simples H-J structure $(u = t - x, v = t + x, w = x + iy)$ mean the transformation $(u, w \rightarrow (v, -w)$ for the arguments of f_i. Left- and right-handed fermions would live at different elementary particle-like space-time surfaces representing elementary particles and the googly problem, if it is a problem in TGD, could disappear. The nature of holomorphy would not be a fixed but dynamic property and characterize the solution of field equations.

2. Geometrically PT means $(u, w \rightarrow (-u, -w)$. If the functions f_i are odd or even under PT the space-time surface is not affected. If C corresponds to complex conjugation if CP_2 and CPT corresponds to identity, T should induce complex conjugation in CP_2.

3. Baryon and lepton number conservation requiring fixed and opposite H-chiralities for quarks and leptons does not allow independent reflections in M^4 and CP_2 but must be carried out simultaneously. But what is the counterpart of reflection in CP_2? Charge conjugation C is a good guess but does not change orientation: C looks like an analog of PT.

   CPT invariance in geometric sense would require that T is accompanied by complex conjugation in CP_2. CPT would act as $(u, w) \rightarrow (-u, -w)$ and trivially in CP_2. Odd/even property of f with respect to u and w would guarantee the invariance of the space-time surface. Here one must be cautious since in ZEO "big" state function reduction changes the arrow of the geometric time by mapping fermionic vacuum to its dual. This could explain why complex conjugation in CP_2 must be involved. T in this sense is not mere geometric time reflection. This would correspond to a realization at the Hilbert space level as an anti-unitary transformation involving hermitian conjugation analogous to complex conjugation in CP_2.

4. What is interesting is that CP acts as $(u,w) \rightarrow (v,-w)$ in M^4 and as complex conjugation in CP_2. This affects the space-time surfaces so that exact symmetry is not in question. This would conform with the small CP breaking and also with matter antimatter asymmetry, which could be understood if matter and antimatter correspond to different H-J structures so that they must live at different space-time surfaces.

2.4.4 Pin structure and TGD

Ordinary spin structure and also conformal structure require orientable manifolds. Pin structure extending SO(1,3) to O(1,3) containing also P, PT and PT has been discussed as one possible cure of the googly problem. Pin structure is also possible for non-orientable manifolds. P transforms M^4 chiralities of spinors to each other. In the electro-weak gauge transformations respect M^4 chirality but the Dirac equation in H and also the induced Dirac equation couples opposite M^4 chiralities. These couplings are analogous to mass or Higgs couplings.

In TGD, H is orientable so the pin group is not relevant in TGD. In TGD, the 8-D pin structure would mean that there are continuous symmetries that convert quarks into leptons in TGD. This is not possible due to different charges and color quantum numbers as well conservation laws of baryon and lepton number.

How does the possible non-orientability of the space-time surface affect the situation? Certainly non-orientable surfaces are possible but the holography= holomorphism hypothesis does not allow them since complex structure requires orientability.

3 Twistorialization at the level of M^8

M^8-H duality as analog of momentum-position duality for 3-surfaces as particles [L2, L3, L10, L15] is central part of TGD. I have already earlier considered several variants of what the twistor lift at the level of M^8 could mean. There are several questions to be answered.

3.1 Identification of the twistor spaces

What are the twistor spaces of $T(M^8)$ and $T(Y^4)$ for the M^8-H dual Y^4 of the space-time surfaces $X^4 \subset H$?

1. The 12-D space of light-like geodesics in $M(1,7)$ would be the naive guess for the twistor space of M^8. Now however the Minkowski metric of M^8 is number theoretic and given by real part of octonionic product and 14-D G_2, is the number theoretic symmetry group so that the 12-D $G_2/U(1) \times U(1)$ is the natural candidate for the octonionic twistor space of M^8. $U(1) \times U(1)$ has an interpretation as color Cartan algebra.

2. Without further conditions, the twistor sphere defined by light-like rays at a given point of M^8 is a 6-D and the space $S^6 = G_2/SU(3)$ is the natural identification for it. With this identification, the dimension of the total twistor space $T(M^8)$ would be $8+6=14$, the dimension of G_2. This does not conform with the identification as $T(M^8) = G_2$. It is also an open question whether S^6 possesses the twistorially highly desirable Kähler structure.

3. How could one reduce the dimension of the space of light-like rays of M^8 from 6 to 4? Could the condition that the light-rays are associated with a point of M^8-H dual $Y^4 \subset M^8$ are quaternionic, allow to achieve this. M^8-H duality in its recent form indeed requires that the normal space for a given point of $Y^4 \subset M^8$ as M^8-H dual of $X^4 \subset H$ is quaternionic and Minkowskian in number theoretic sense [L15]. This suggests a direct connection between twistorialization and M^8-H duality.

    (a) Could one require that the light-like 8-momentum has vanishing tangential component to Y^4 and is therefore quaternionic? This would replace the twistor sphere with a union of twistor spheres associated with Minkowskian mass shells $p^2=m^2$. The space of light rays would be 3- rather than 4-D and the wistor space of M^8 would be 11-D rather than 12-D. One dimension is missing.

(b) The physical intuition suggests that the light rays do not have a momentum component in the direction of the tangent space of Y^3 defining the 3-D holographic data but that they have a component tangential to Y^4 in a direction normal to Y^3. This would conform with non-point-likeness: by general coordinate invariance, the momentum component tangential to Y^3 would not correspond to anything physical.

The additional condition would be that these light-like vectors are quaternionic. The space of allowed 8-D light-like vectors would be 4-D and the twistor space could be G_2. The associativity of the dynamics at the level of M^8 requires that the normal space is quaternionic and thus Minkowskian and also contains a commutative subspace. Can these two quaternionicity conditions be consistent with each other? If so, 8-D associative light-likeness respecting the 3-dimensality of holographic data implies the desired 4-dimensionality of the analog of the twistor sphere.

4. The section of the twistor bundle assigned to Y^4 assigns to each point of Y^4 a light-like vector. If also quaternionic units are chosen in an integrable way, this would define the M^8 counterpart of the $H-J$ structure which, when mapped to H by M^8-H duality, would provide the H-J structure of H.

   If the selected light-like vectors have a vanishing tangential component in Y^4, the light-like vectors in H are in M^4. If this is not the case, the light-like vectors in M^4 have also CP_2 component. For instance, light-like geodesics in $M^4\times S^1$, $S^1\subset CP_2$ are possible. It therefore seems that the TGD view of twistorialization indeed makes possible the twistor description of massive particles.

The precise identification of the twistor spaces of M^8 is not obvious. The twistor space of M^8 should have 4-D fiber.

1. The condition that the twistor space allows Kähler structure and has $S^2\times S^2$ as a fiber might leave only the product $T(M^8)=T(M^4)\times T(E^4)$, which is consistent with $M^8=M^4\times E^4$. Whether one can identify $T(E^4)$ as CP_3 is quite not clear.

   In this case, the dimensional reduction of 6-D Kähler action to 4-D action involves the identification of the 2 twistor spheres $S^2(T(M^4))$ and $S^2(T(E^4))$. As in the case of $T(H)$, this identification need not and cannot always be 1-1.

   $Y^6=T(Y^4)$ decomposes locally to a Cartesian tensor product $Y^4\times S^2(T(Y^4))$, $Y^4\subset M^4\times E^4$: Y^4 need not correspond to a map $M^4\rightarrow E^4$ or vice versa.

   The twistor spheres of $S^2(T(M^4))$ and $S^2(T(CP_2))$ are mapped to each other. The consistency between the purely geometric and spinorial view of twistorialization requires that $S^2(T(M^4))$ and $S^2(T(CP_2))$ correspond to homologically non-trivial spheres in Y^4, which are therefore mapped to each other. Cosmological constant depends on the winding number of the identification.

2. Very naively, in the spinorial approach the extended twistor space C^4 is replaced with C^8. Division with 2-complex-dimensional planes CP_2 would give Grassmannian $Gr_c(2,8)$ with dimension $2\times(8-2)=12$, which is a complex manifold having the representation $U(8)/U(2)\times U(6)$. Intuition suggests that the fiber is CP_2. Minkowskian signature would suggest that $U(6)$ is replaced with $U(5,1)$ and $U(8)$ with $U(7,1)$.

   The existence of an analog of the previous dimensional reduction of 6-D Kähler action to 4-D action does not seem plausible. CP_2 fiber does not allow $S^2\times S^2$ as a sub-manifold.

3. The number theoretic $G_2/U(1)\times U(1)$ is the third possible identification but it is not clear whether it is consistent with the number theoretic M^4 signature and CP_2 fiber. It is far from clear whether the 4-D fibration exists and whether the fiber is $S^2\times S^2$.

3.2 About the spinorial aspects of M^8 twistorialization

What about the spinorial aspects of M^8 twistorialization? One should generalize a) the map of the points of sphere S^2 to the 2×2 matrices defined by a bi-spinor and its dual, b) the masslessness condition as vanishing of a determinant of the analog of the quaternionic matrix and c) the coincidence relation. One should also understand how the counterparts of the electroweak couplings are represented and solve the Dirac equation in M^8.

1. In the case of M^4, the light-like momenta are mapped to the bispinors providing a matrix representation of quaternions in terms of Pauli sigma matrices. A possible way to achieve this is to introduce octonionic spinor structure [K8, K5, L10] in which massless 8-D momenta correspond to octonions, which should be associative and therefore quaternionic. This would conform with the above identification of light rays.

2. Octonionic spinors, presumably complexified octonionic spinors with $i = \sqrt{-1}$ commuting with the octonionic units, should be also defined. The map of quaternionic massless 8-momenta to the octonionic counterparts of the Pauli spin matrices representing quaternionic basis would define octonionic spinors satisfying the quaternionicity condition. Massless Dirac equation can be solved in the standard way.

3. The matrices defined by bi-spinor pairs associated with M^4 twistors can be regarded as quaternions. The quaternionicity condition means that the octonionic spinor pairs actually reduce to M^4 bi-spinor pairs on a suitable basis, which however depends on the point of Y^4?

   If commutative i is introduced and quaternions are not replaced with their 2×2 matrix representations involving commuting imaginary units, a doubling of degrees of freedom takes place. Does this mean that both M^4 chiralities are obtained? Could this solve the googly problem in M^4?

Also in the case of octonionic spinors complexification would double the degrees of freedom. Does one obtain in this way both spin and electroweak spin?

1. What happens to M^8 spinors as tensor products of Minkowski spinors and electroweak spinors when the octonionic Dirac operator acts on a quaternionic subspace. The electroweak degrees of freedom do not disappear but become passive. One has 8-D complex spinors, which are enough to represent a single H-chirality if the octonionic gamma matrices, which are quaternionic at Y^4, are not represented in terms of Pauli sigma matrices and i is introduced.

2. The electroweak gauge potentials as induced spinor connection represent the geometric view of physics realized at the level of H. Number theoretical vision suggests that the M^8 spinor connection cannot involve sigma matrices, which would be defined as commutators of octonionic units and be octonionic units themselves. Kähler coupling is however possible.

   What could the form of the Kähler gauge potential be? The Kähler form should be apart from a multiplicative imaginary unit i equal to the theoretical flat metric of M^8 so that the Kähler function would represent harmonic oscillator potential.

   The octonionic Dirac equation would have a unique coupling to the Kähler gauge potential with Kähler coupling constant absorbed to it. This would guarantee that the solutions of the modified Dirac equation in M^8 have a finite norm. The solutions can be found by generalizing the procedure to solve Dirac equation in harmonic oscillator potential.

3. The octonionic Dirac operator, which reduces to the quaternionic M^4 Dirac operator and for the local quaternionic M^4 identified as a normal space, the fermions are massless. How to solve this problem? As found, the non-vanishing M^4 mass requires that the light-like M^8 momentum has a component in the direction of Y^4 having a natural interpretation as the analog of the square root of the Higgs field.

4. Complexified octonionic spinors form a complex 8-D space, which corresponds to a single fermion chirality. Do different H chiralities emerge from the mere octonionic picture or must one introduce them in the same way in the case of H? The couplings of quarks and leptons to the induced Kähler form are different and this should be true also at the level of M^8: it seems that both quarks and leptons should be introduced unless on is read consider either leptons or quarks as fundamental fermions.

5. Color $SU(3)$ acts as a number theoretic symmetry of octonions. $SU(3)$ as an automorphism group transforms to each other different quaternionic normal spaces represented as points of CP_2. This representation is realized at the level of H in terms of spinor harmonics. The idea that the low energy and higher energy models for hadron in terms of $SO(4)$ and $SU(3)$ symmetries are dual suggest that fermionic $SO(4)$ harmonics in M^8 could be analogous to the representation of color as spinor harmonics in CP_2.

One should understand the massless octonionic Dirac equation.

1. The octonionic Dirac equation looks like the ordinary Dirac equation but with gamma matrices replaced with octonionic units. The quaternionic Dirac equation involves quaternionic units but it is essential that they are not represented as matrices. This allows the introduction of imaginary uni i commuting with the quaternionic and octonionic units and implies double of the degrees of freedom so that one can have analogs of complex spinors.

   The octonionic units are analogs of Pauli sigma matrices and the first problem is caused by the lacking anticommutativity of the real unit with other octonion units. The Dirac equation however makes sense also in this case.

2. The 8-D masslessness condition must correspond to the condition that the real part of the square of Dirac operator on spinors vanishes. For momentum eigenstates this gives the usual algebraic conditions for masslessness.

3. H spinors have a defined H-chirality guaranteeing separate conservation of quark and lepton numbers. H-chirality ϵ is a product $\epsilon = \epsilon_1\epsilon_2$ of M^4 and CP_2 chiralities. Alls these chiralities should be definable also at the level of M^8. Also the octonionic Dirac equation for H spinors should be consistent with the chirality condition.

   (a) The decomposition of octonion units to quaternion units $\{1, I_k|k = 1, 2, 3\}$ and co-quaternion units $I_4\{1, I_k|k = 1, 2, 3\}$ suggests the identification of the counterpart of Γ_9. The matrix Γ_9 is defined as the product of H gamma matrices satisfies $\Gamma_9^2 = -1$, anticommutes with H gamma matrices. H chirality corresponds to the eigenvalue of $i\Gamma_9$ equal to $\epsilon = \pm 1$. The eigen spinors with chirality ϵ are of the form $(1 + \epsilon i\Gamma_9)\Psi_0$.

   The spinors with fixed H-chirality are tensor products of spinors of fixed M^4 chirality and CP_2 chirality and the product of these chiralities defines H-chirality.

   (b) The operator iI_4, satisfying $(iI_4)^2 = 1$, is a good guess for the counterpart of Γ_9 for the octonionic spinors. The octospinors with a fixed M^8 chirality ϵ should be of the form $(1 + \epsilon iI_4)\Psi_0$. It is easy to check that for an octospinor of form $\Psi_\epsilon = \Psi_a + I_4\Psi_b$ having a fixed chirality ϵ, one obtains

   $$\Psi_b = i\epsilon\Psi_a \quad (3.1)$$

   so that the spinor is determined completely by its quaternionic part. Perhaps this might be regarded as a realization of quaterniocity.

   (c) One can decompose also the quaternionic spinors to two parts corresponding to the decomposition to complex subspace spanned by $\{1, I_1\}$ and co-complex subspace spanned by $\{I_2, I_3\}$. This allows us to define M^4 chirality and its E^4 counterpart.

4. Octonionic Dirac equation for the momentum eigenstates can be decomposed to a sum of quaternionic and co-quaternionic parts

   $$D\Psi = (p_1^k I_k + I_4 p_2^k I_k)\Psi = 0 \ . \quad (3.2)$$

   The real part of D^2 gives the Minkowskian mass shell condition $p_1^2 - p_2^2 = 0$.

   The Dirac equation for the $\Psi_\epsilon = (1 + i\epsilon I_4)\Psi_0$ gives

$$\begin{array}{l} D\Psi_0 = (p_1^k I_k + I_4 p_2^k I_k)\Psi_0 + \tilde{D}\tilde{\Psi}_0 = 0 \ . \\ \tilde{D} = \tilde{p}_1^k I_k + I_4 \tilde{p}_2^k I_k) \ . \end{array} \tag{3.3}$$

Tilde means a conjugation of quaternionic imaginary units. This gives two separate equations? Are they consistent? By multiplying the equation with tildes by $1 = -I_2^2$ from left and transporting the second I_4 through the equation to right, one obtains the equation $-I_2(p_1^k I_k + I_4 p_2^k I_k)\Psi_0 I_5 = 0$. The two equations are therefore consistent.

This picture suggests that 6-D Kähler action as the Kähler function of the twistor space of M^8 could determine surfaces Y^4 as its preferred extremals and that holography= holomorphy principle determines the extremals also now. The 12-D twistor bundle with 4-D fiber should have Kähler structure. This gives very strong condition. One possibility is that it is just the Cartesian product of twistor spaces for M^4 and E^4.

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