# Holography= holomorphy vision and a more precise view of partonic orbits

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**Abstract**

In this article a more precise view about the 3-D light-like trajectories of 2-dimensional parton surfaces is developed on the basis of holograpy= holomorphy hypothesis (H-H). Partonic orbits are identified as light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian so that the metric determinant vanishes.

The so-called $CP_2$ type extremals, having 4-D CP$_2 projection 1-D light-like curve as$M$^4$ projection, and their deformations have been known from the very beginning and satisfy H-H. However, all attempts to obtain them from $(f_1, f_2) = (0, 0)$ ansatz failed since $CP_2$ projection was 3-D and this spoiled the holomorphy property.

In Minkowskian regions one has two kinds of solutions for which hypercomplex coordinate $u$ or $v$ appears in $f_i$. If the space-time surface is invariant under generalized conjugation, these should correspond to a single solution and the only way is to consider their union. The two regions in question have a natural identification as Minkowskian space-time sheets connected by a wormhole contact with an Euclidean signature of the induced metric and identifiable as a deformation of a $CP_2$ type extremal.

One must give up $(f_1, f_2) = (0, 0)$ ansatz and construct the Euclidean regions as regions with $v = v_0$ $(u = u_0)$ by replacing $v$ $(u)$ with a real $CP_2$ coordinate $s$ which is function of light-like coordinate $v$ $(u)$ of $M^4$. The two sheets meet at the 3-surface $u_0 = v_0$ and there is an edge at which $u$ and $v$ coordinate lines meet. The interpretation is as a defect of the standard smooth structure making it an exotic smooth structure. Physically the edge can be assigned with fermion scattering or creation of a fermion pair. Note that two $M^4$ time coordinates, with time coordinate included, are constant for the vertex.

The light-likeness condition for the partonic orbits generalizes the Virasoro conditions for 1-dimensional light-like curves to the 3-dimensional light-like partonic orbits. Also an explicit procedure for finding the partonic orbits is discussed.

# Contents

# 1 Introduction

3-D light-like partonic orbits are a central piece of the TGD view of elementary particles. In the sequel a more precise identification of these surfaces is considered.

It is however useful to start by clarifying some basic aspects of dynamics in the TGD Universe.

1. There are two kinds of degrees of freedom in TGD: geometric, i.e. degrees of freedom of the space-time surface, and fermionic. All elementary particles are made up of fermions and antifermions: bosons emerge. There are no bosonic primary quantum fields.

2. The basic result from the solution of the Dirac equation for $H$ spinor fields, assuming that $M^4$ has a non-trivial Kähler structure [L5], is that the mass scale of colored partial waves of fermions is given by $CP_2$ mass scale and there are no free massless gluons or quarks. However, massless color singlets for which the difference in the numbers of quarks and antiquarks is a multiple of three, are possible. This gives baryons and mesons. p-Adic thermodynamics gives small thermal masses for the massless modes states appearing as ground states for the generalization of super-conformal representations.

   Here comes a crucial difference between QCD and TGD. In lattice QCD there would be no $g-2$ anomaly whereas the approach based on the information given by physical hadrons imply the anomaly (see this). In TGD, color singlets, in particular hadrons, are indeed the fundamental objects. The anomaly would be real and the new physics implied by TGD predicts it [L5]. For example, copies of hadron physics at larger mass scales are predicted. Also the color singlets formed from higher color partial waves of quarks and leptons give rise to an infinite number of new hadrons and also leptons: I have called them lepto hadrons and there is evidence for them [K3]. This could not be farther from the notion of the desert assumed in GUTs. It will be exciting to see whether QCD or TGD is right.

3. The arguments of the n-point functions of the second quantized free fermion fields of $H$ (scattering amplitudes) are points of the spacetime surface so that the dynamics of the spacetime surface affects the scattering amplitudes. Effectively, the spacetime surface defines the classical background in terms of the induced fields: induced metric, spinor connection, etc... Free fermion field do not allow pair creation in ordinary QFTs. The possibillity of exotic smooth structures for 4-D space-times comes in rescue here [L4] [**?**] The exotic smooth structure can be seen as the ordinary smooth structure with defects. Defects define analogs of vertices for the creation of fermion pair interpreted as turning of a fermion line in time direction. Since bosons correspond bound states of fermions and antifermions rather than primary quantum fields, all interaction vertices reduce to this vertex.

A particle can be seen in two ways:

1. Particle as a 3-surface and its Bohr orbit as a four-surface $X^4$.

2. Particle as a fermion and its orbit, the fermion line, is a light-like curve, maybe even a light-like geodesic line in $M^4 \times CP_2$ or $M^4$.

The spacetime surface $X^4$ has a rich anatomy and this leads to a more detailed view of what particles are.

1. $X^4$ has internal structure and the 3-D partonic orbits define light-like surfaces $X^3$ at which the Minkoski signature of the surface becomes Euclidean so that the metric determinant vanishes.

2. A fermion line would be an intersection of 2-D string world sheet and a 3-D light-like partonic orbit. The proposal is that string world sheet can be obtained as the intersection of two spacetime surfaces $X^4$ and $Y^4$ if they have the same Hamilton-Jacobi structure at the level of $H$ [L2], i.e. allow the same generalized complex $H$ coordinates $u$, $w$, $\xi_1$, $\xi_2$ and their conjugates ($\overline{u} = v$). The corrected view of generalized analyticity however forces to challenges this assumption although it is physically very attractive.

   One can ask whether the mutual interactions of particles as space-time surfaces occur only when they have the same Hamilton-Jacobi (H-J) structure. If so, the interactions can be described in terms of their intersections consisting of string world sheets and fermion lines at their boundaries. If so, a strong analogy with string models would emerge. The second option is that the intersections are discrete. Also now fermionic n-point functions are well-defined.

   Also the self-interactions could be described by considering infinitesimal deformation of the space-time surface preserving H-J structure and finding the string world sheets in this case.

3. In TGD, the genus of the parton surface is an important topological quantum number [K2]. The genera $g = 0, 1, 2$ corresponds to the observed fermion generations. $g = 2$ allows a bound state for the 2 handles of the sphere that are like particles. This is because $g \leq 2$ allows global conformal symmetry. In the $g \geq 2$ topology, $g$ handles are like particles in a multiparticle state, and the mass spectrum of the states is continuous, unlike for elementary particles.

   Also the homological charge of the partonic 2-surface, identifiable as Kähler magnetic charge of the space-time surface is an important topological quantum number.

This article was born as an attempt to develop a more precise view about the 3-D light-like trajectories of 2-dimensional parton surfaces on the basis of holograpy= holomorphy hypothesis (H-H).

1. $CP_2$ type extremals [K1] have a 1-D light-like curve as $M^4$ projection. Their complex deformations satisfy H-H. The projection corresponds to a coordinate curve of the hypercomplex coordinate $u$ or $v$. The generalization of the $(f_1, f_2) = (0, 0)$ hypothesis to them turned out to be impossible: $CP_2$ projection turned out to be 3-dimensional and failed to satisfy H-H.

   The assumption that the space-time surface $X^4$ is invariant under generalized conjugation taking $u$ to $v$ and vice versa, implies that $X^4$ must have two sheets with $v = v_0$ or $u = u_0$ permuted by the generalized conjugation. They meet at the 3-surface $X^3$ $u = v$. This implies $u_0 = v_0$ implying that two $M^4$ coordinates $u, v$ are constant and one has 3-D surface of $CP_2$ invariant under complex conjugation of complex $H$ coordinates. At this surface, the light-like curves for the two sheets meet at an edge, which has an interpretation in terms of an exotic smooth structure in turn having interpretation in terms of a vertex for a creation of a fermion pair.

2. Partonic orbits can be identified as light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian so that the metric determinant vanishes and the induced 4-metric degenerates to an effectively 2-D metric.

3. The light-like $u$ or $v$ coordinate lines can have edges at the partonic orbits. This has led to a proposal for how exotic smooth structures necessary for defining fermion pair creation vertices emerge via partonic orbits as defects of the standard smooth structure [L4, L1]. Fermion pair as a fermion returning backwards in time would correspond to the edge of $u$ (or $v$) coordinate line. These conditions generalize the Virasoro conditions for 1-dimensional light-like curves to the 3-dimensional light-like partonic orbits.

4. The light-likeness of the coordinate lines generalizes the Virasoro conditions for 1-dimensional light-like curves to the 3-dimensional light-like partonic orbits and one obtains a set of 1-D Virasoro conditions parametrized by the points of the partonic 2-surface. In fact, the 1-D Virasoro conditions emerged first for $CP_2$ type extremals [K1] and led to the realization that the generalization of conformal invariance in some sense must be a fundamental symmetry of TGD: the discovery of holography= holomorphy principle finally led to a detailed understanding of this symmetry [L3]. Also an explicit procedure for finding the partonic orbits is discussed.

# 2 The identification of the partonic orbits

It took a considerable time to realize that holography= holomorphy vision has delicate technical problems and the recent view was found by trial and error.

## 2.1 Definition of hypercomplex conjugation

What does one mean with the generalization of the complex conjugation when applied to the argument of $f$? Could it correspond a) to $(u, w, \xi_1, \xi_2) \to (u, \overline{w}, \overline{\xi_1}, \overline{\xi_2})$ so that there is no hypercomplex conjugation or b) to $(v, w, \xi_1, \xi_2) \to (u, \overline{w}, \overline{\xi_1}, \overline{\xi_2})$ so that there is hypercomplex conjugation.

1. For option a), the roots of $f$ and $\overline{f}$ represent the same surface. For the roots of $f$ the contribution of complex coordinates to $g_{uv}$ and $g_{vw}$ is vanishing but the components $g_{u\overline{w}}$ and there is only the contribution of $M^4$ metric to $g_{uv}$. Partonic orbits are not possible.

2. For option b), the roots of the conjugate $\overline{f}$ do not coincide with the roots of $f$ unless symmetries exist. If the space-time surface is invariant under the generalized conjugation (in analogy with complex plane), it must be a union of the u-type and v-type regions defines the space-time surface. Hypercomplex conjugation would be a non-local symmetry transforming to each other two parts of the space-time surface. The 3-surface $u = v$ would be a 3-dimensional surface along which the two space-time regions would be glued together.

Consider the option b) in more detail.

1. How to identify the $u$- and $v$-type regions? In the model for elementary particles, Euclidian regions as deformations of $CP_2$ extremals connect two Minkowskian space-time sheets, which are extremely near to each other having a distance of order $CP_2$ radius. Could the two Minkowskian space-time sheets correspond to $u$- and $v$-type regions and could generalized complex conjugation $(u, w, \xi_1, \xi_2) \leftrightarrow (v, \overline{w}, \overline{\xi_1}, \overline{\xi_2})$ transform then to each other.

2. Could the 3-surface $X^3$ at which the sheets intersect so that both $u$ and $v$ coordinates associated with the sheets are identical, define the 2-surface $X^2$ along which the sheets are glued together? Could this surface be identifiable as the light-like partonic orbit.

   The wormhole contact identified in this way has 3-D $CP_2$ projection and does not correspond to the $CP_2$ type extremal. It is not clear whether this is a problem or not.

3. Presumably, there would be discontinuity associated with the derivatives of the embedding space coordinates at $X^3$, where the $u$- and $v$-type time evolutions at the two sheets would be glued together.

4. Could $X^3$ be interpreted in terms of an exotic smooth structure [A2, A3, A1] allowing an interpretation as the standard smooth structure with defects? Could the $u$- lines transform to $v$-lines at $X^3$ and give rise to edges violating the standard smoothness.

   Also the partonic orbits could define analogous defects since the $u$- *resp.* $v$-lines could have an edge. The identification of fermion lines as these kinds of lines allow the interpretation of defects as vertices for the creation of fermion-antifermion pair as turning of fermion line backwards in time [L4, L1]?

## 2.2 Technical problems of the holography= holomorphy vision

Consider first the technical problems related to the finding of the roots of $(f_1, f_2)$ appearing in the Euclidean space-time regions. Note that this is only an ansatz, which is less general than H-H and need not work for wormhole contacts as deformations of $CP_2$ type extremals [K1].

1. The first problem is that in Minkowskian regions defining the parallel space-time sheets one has two kinds of solutions for which hypercomplex coordinate $u$ *resp.* its conjugate $v$ appears in $f_i$ *resp.* its conjugate. These should correspond to a single solution and the only way is to consider their union. The two regions in question have a natural identification

as Minkowskian space-time sheets connected by a wormhole contact with an Euclidean signature of the induced metric.

At the surface, where the two sheets are glued, $f_i$ must be invariant under conjugation, which for real coefficients of $f_i$ requires $u = v$ and reality of various complex coordinates or at least that the surface in question is invariant under complex conjugation.

2. In Euclidean regions, the realization of H-H, using $(f_1, f_2) = (0, 0)$ ansatz assuming that either hypercomplex coordinate $u$ or $v$ is a dynamical variable, leads to a problem. Either $u$ or $v$ is a complex analytic function $f$ of $CP_2$ coordinates and its reality implies $Im(f) = 0$ so that $CP_2$ projection is 3-dimensional, which means the failure of the holomorphy with respect to the $CP_2$ coordinates. For a moment I thought that Wick rotation might help but this was not the case.

3. This forces to give up $(f_1, f_2) = (0, 0)$ ansatz and assume only H-H. The original vision was that the Euclidean region as a wormhole contact corresponds to a deformation of a canonically embedded $CP_2$ so that it has a light-like coordinate curve of $u$ or $v$ as $M^4$ projection. These space-time surfaces are holomorphic so that field equations are satisfied.

   The gluing condition implies constancy condition $v = v_0$ *resp.* $u = u_0$ and $v$ *resp.* $u$ is replaced with a real $CP_2$ coordinate $s(u)$ *resp.* $s(v)$. $M^4$ complex coordinate $w$ can be a function of $CP_2$ coordinates.

4. The gluing condition for the two sheets requires $u_0 = v_0$ which for $u = m^0 + m^3$ and $v = m^0 - m^3$ gives $m^0 = 2u_0$ and $m^3 = 0$. At the points of this 3-surface) there is an edge at which the coordinate curves for $u$ and $v$ meet: the interpretation could be in terms of an exotic smooth structure [A2, A3, A1] as standard smooth structure with a defect to which fermion pair creation or fermion scattering vertex can be assigned. The two sheets are glued together along a 3-surface $X^3$ with 3-D $CP_2$ projection invariant under complex conjugation. The $CP_2$ projection $X^3$ must contain a homologically non-trivial 2-surface since the wormhole contact must carry a monopole flux between the space-time sheets.

This tentative picture would relate several key ideas of TGD: H-H involving hypercomplex numbers, the notion of light-like partonic orbit, the idea that exotic smooth structures make possible non-trivial scattering theory in 4 dimensional space-time. One can compare this picture with the intuitive phenomenological picture.

## 2.3 The 3-D light-like orbits of partonic 2-surfaces

The trajectories of partonic 2-surfaces are singularities at which the Euclidean induced 4-geometry transforms into Minkowskian. The light-like dimension implies $\sqrt{|det(g_4)|} = 0$. The challenge is to derive the partonic orbits from this.

1. H-J structure defines Kähler structure $M^4 \subset H$ inducing that of $X^4$ and is independent of holography= holomorphy hypothesis. The induced Kähler structure of $X^4$ is defined by the projection of the sum of $M^4$ and $CP_2$ Kähler forms and need not be the same as that of $M^4$. If the proposal holds true, these structures differ only at the partonic orbits. The generalized complex coordinates of $X^4$ (hypercomplex coordinate $u$ (or $v$) and complex coordinate $w$) are a subset of the generalized complex coordinates of $H$, which also include 2 complex coordinates of $CP_2$.

   The induced Kähler structure of $X^4$, which is more or less equivalent with Hamilton-Jacobi structure, defines a slicing of $X^4$ by light-like 3-surfaces with one light-like curves, which can be taken to correspond to the hypercomplex coordinate $u$, which is constant along the lines $u = u_0$. Also its dual slicing, assignable to the $v$-surface is well-defined.

   The 4-metric is hermitian and is a tensor of type (1,1) having only 4 independent components. The only non-vanishing component of the induced 3-metric $g^3$ at $X^3$ defined by the projection of the 4-metric is $g_{w\overline{w}}$ so that the slice is metrically 2-dimensional. Light-cone boundary provides a simple example of this.

2. The space-time surface $X^4$ is defined by the conditions $(f_1, f_2) = (0, 0)$, where $f_1$ and $f_2$ are analytic functions $H = M^4 \times CP_2 \rightarrow C^2$ depending only on the hypercomplex coordinate $u$ with light-like coordinate curves and complex coordinates $w$, $\xi_1$ and $\xi_2$ of $H$ but not on the coordinate $v$ as hypercomplex conjugate $u$ and the conjugates $\overline{w}, \overline{\xi_1}, \overline{\xi_2}$. The surfaces are same.

   As a special case, $f_i$ are polynomials or rational functions. Additional restrictions can be posed on the coefficients of the polynomials. The conditions $(f_1, f_2) = (0, 0)$ have been studied in some cases [L6].

3. $\sqrt{det(g_4)} = 0$ gives an additional condition and gives a 3-D light-like partonic orbit $X^3$.

## 2.4 $det(g_4) = 0$ condition as a generalization of Virasoro conditions

The $g^4_{uv} = 0$ condition has an interpretation as a generalization of the Virasoro conditions of string models to the 4-D context.

1. If the situation were 2-dimensional instead of 4-D, the $det(g_4) = 0$ condition would give a light-like curve and the light-likeness would give rise to the Virasoro conditions. This was actually one of the first observations as I discovered $CP_2$ extremals, whose $M^4$ projection is a light-like curve for the Kähler action [K1]. For the action defined by the sum of Kähler action and volume term the light-like curves are replaced with light-like geodesics of $M^4$ and possibly of $H$. The conditions as such are not Virasoro conditions. It is the derivative of the conditions with respect to the curve parameter, which gives the Virasoro conditions. By taking Fourier transform one obtains the standard form of the Virasoro conditions.

   The Virasoro conditions can fail at discrete points and these singularities have an interpretation as vertices and also as points at which the generalized holomorphy fails. The poles and zeros of the ordinary analytic function are analogs for this.

2. By holomorphy= holography vision alone implies that the space-time surface is sliced by light-like curves. These curves satisfy Virasoro conditions so that one has a generalization of Virasoro conditions to a bundle of conditions parameterized by points of a 3-D section of the space-time surface. Space-time surface itself does not define a light-like orbit of the 3-surface.

3. For the 4-D generalization, the light-like curve is replaced by a 3-D light-like parton trajectory identifiable as a 2-D bundle of light-like curves so that 1-D Virasoro conditions are true for each curve. The analogs of Virasoro conditions are indeed very natural also now because 2-D conformal invariance is generalized to 4-dimensional one. The Virasoro conditions have one integer, the conformal weight. Now the Fourier transform with respect to the coordinates of $X^4$, say $u$ and $w$ gives conditions labelled by two integers having interpretation as conformal weights.

   This suggests that conditions can be seen as analogs of Virasoro conditions. Their generalization gives rise to analogs of the corresponding gauge conditions for the Kac-Moody algebra, just like in the string model. A lot of physics would be involved.

4. A new element brought by TGD is that algebras would have non-negative conformal weights meaning that an entire fractal hierarchy of isomorphic algebras is predicted such that sub-algebra and its commutator with the entire algebra annihilate the physical states [L3]. This makes possible a hierarchy of gauge symmetry breakings in which a subspace of the entire algebra transforms from a gauge algebra to a dynamical algebra.

# A How to find the partonic orbits?

In the sequel, the partonic orbit refers to the light-like boundary at which the signature of the induced metric changes from Minkowskian to Euclidian. In the Minkowskian region $(f_1, f_2) = (0, 0)$ ansatz works and,depending on which sheet one considers, the passive coordinate $v$ or $u$ becomes constant at the boundary.

One must solve the induced metric for a given solution $f = (f_1, f_2) = (0, 0)$ in Minkowskian region and find what happens to it at the boundary. This means moving from mere algebraic geometry to differential geometry because the induced metric depends on the partial derivatives of the imbedding coordinates. The complexity of the task depends on how strong assumptions one makes.

## A.1 Two alternative identifications of partonic orbits

One can consider two alternative identifications of partonic orbits.

1. One could start from a completely general solution in Minkowskian region and consider only the $det(g_4) = 0$ condition without any additional assumptions such as the Hamilton-Jacobi structure.

2. If one assumes holography= holomorphy principle, 3-surfaces with $g^4_{uv} = 0$ implying $det(g_4) = 0$ condition, are good candidates candidates for partonic orbits, which must be metrically 2-dimensional. Since the signature transforms to Euclidian, the induced metric must receive a $CP_2$ contribution, which implis the conditions $det(g_4) = 0$ and $g^4_{uv} = 0$ implying metric 2-dimensionality.

Simple physical considerations help to understand what the partonic orbits look like. The simplest surface to consider is deformed $M^4$ for which $CP_2$ projection is a geodesic line: $\Phi = \omega t$. The induced metric is $g_{tt} = 1 - R^2\omega^2, g_{ij} = -\delta_{ij}$, where $R$ is $CP_2$ length scale. For $R^2\omega^2 = 1$, the time-like direction becomes light-like. Something analogous happens also in the general case. The rapid time variation of the $\xi_i(w, u)$ and $\xi_i(\overline{w}, v)$ is what can change the sign of $det(g_4)$. Some partial derivatives $\partial_u\xi_i(u, 2)$ and $\partial\overline{\xi_i}(v, \overline{w})$ must have order of magnitude $1/R$. Therefore the numerical calculation must start from a situation in which these time derivatives are large.

To find the partonic orbits defined in the way already discussed, it is useful to find a region of space-time surface whether the gradients of $CP_2$ coordinates as functions of $u$ coordinate are of order $1/R$ so that $g^4_{uv}$ can be near zero.

## A.2 $det(g_4) = 0$ condition as a possible definition of the parton orbit

This section gives some idea about how concrete calculations might proceed. The condition $det(g_4) = 0$ is a natural guess for the precise definition of the partonic orbit as light-like 3-surfaces at which 4-metric degenerates to 2-dimensional metric.

The condition $det(g_4) = 0$ is a natural guess for the precise definition of the partonic orbit as light-like 3-surfaces at which 4-metric degenerates to 2-dimensional metric. Consider in more detail the $det(g_4 = 0)$ option for partonic surfaces using H-J coordinates but without assuming H-H vision. The following also describes how to calculate the induced metric.

For $X^4$ Kähler form is obtained by inducing the sum of Kähler forms of $M^4$ and $CP_2$ and is in general different from that $M^4$. The H-J coordinates are however the same. If the coordinates of $X^4$ are not H-J coordinates one must $det(g_4) = 0$ condition without hermiticity conditions on the induced metric. This requires an additional computational effort.

For H-J coordinates for $X^4$, the $det(g_4) = 0$ is equivalent with the $g^4_{uv} = 0$ condition and the situation simplifies dramatically and one must find the 3-surfaces with $g^4_{uv} = 0$.

1. The general form of the induced metric is

$$g_{\alpha\beta} = h_{kl}\partial_\alpha h^k \partial_\beta h^l \quad . \tag{A.1}$$

   For H-J coordinates, $\alpha$ and $\beta$ refer to $u, v, w, \overline{w}$ and $k$ and $l$ refer to $u, v, w, \overline{w}, \xi_1, \xi_2$. The metric of $H$ in these coordinates can be written easily. From this, we one can calculate the induced metric.

2. For the generalized complex coordinates, not necessarily consistent with the H-J structure, the rows of the induced metric $g$ can be written as a matrix in the general case in the form

$$\begin{pmatrix} g_{uu} & g^4_{uv} & g_{uw} & g_{u\overline{w}} \\ g_{vu} & g_{vv} & g_{vw} & g_{v\overline{w}} \\ g_{wu} & g_{wv} & g_{ww} & g_{w\overline{w}} \\ g_{\overline{w}u} & g_{\overline{w}v} & g_{\overline{w}w} & g_{\overline{w}\overline{w}} \end{pmatrix} \quad (A.2)$$

All components of the metric are in general non-vanishing.

3. Holomorphy, implying that the embedding space metric and induced metric are tensors of type (1,1), implies the vanishing of a large fraction of elements of $g^4$. This gives

$$\begin{pmatrix} 0 & g^4_{uv} & 0 & g_{u\overline{w}} \\ g_{vu} & 0 & g_{vw} & 0 \\ 0 & g_{wv} & 0 & g_{w\overline{w}} \\ g_{\overline{w}u} & 0 & g_{\overline{w}w} & 0 \end{pmatrix} \quad (A.3)$$

The symmetry $g_{\alpha\beta} = g_{\beta\alpha}$ leaves only 4 independent matrix elements. $g^4_{uv}, g_{u\overline{w}}, g_{vw}, g_{w\overline{w}}$. The determinant of this metric vanishes if a partonic orbit is in question.

This is the expression of the induced metric in the Euclidean regions. Partonic orbit corresponds to the interfaces at which $g_{uv} = 0$ is true. The field equations ($f_1 = 0, f_2 = 0$) in the Euclidean region must be solved using Wick rotation.

4. In the Minkowskian regions, where $u$ (or $v$) serves as a parameter, $g^4_{uv}$ reduces to its Minkowskian contribution and components $g_{vw}$ and $g_{u\overline{w}}$ vanish. Partonic orbits are not possible in these regions. The induced 3-metric $g_3$ at light-like $u$ coordinate lines in Minkowskian regions reduces to

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & g_{w\overline{w}} \\ 0 & g_{\overline{w}w} & 0 \end{pmatrix} \quad (A.4)$$

The situation is metrically 2-dimensional. Also $g_4$ is metrically 2-dimensional if the metric changes from Minkowskian to Euclidean so that $g^4_{uv}$ vanishes.

5. If one has $f_2 = \xi_2 - w$ and $f_1(\xi_1, w, h)$ is a polynomial of degree $n < 5$ with respect to $w$, analytic expressions for $\xi_i(h, w)$ are possible and the analytic calculation of the partial derivatives can be considered. Otherwise, we have to use numerical methods. One could hope that a symbolic program for calculating partial derivatives could be found .

6. If the reduction of the condition $det(g_4) = 0$ to the condition $g^4_{uv} = 0$ indeed takes place, the key variable is

$$g^4_{uv} = \partial_u h^k \partial_v h^l = g^0_{uv} + s_{k\bar{l}} \partial_u s^k \partial_v \overline{s}^l \quad . \quad (A.5)$$

Here $g^0_{uv}$ denotes the $M^4$ contribution to the induced metric. For $det(g_4) = 0$ the $M^4$ and $CP_2$ contributions cancel each other and one has

$$g^0_{uv} = -s_{k\bar{l}} \partial s^k \overline{s}^l \quad . \quad (A.6)$$

A generalization of a light-like geodesic of $H$ to a bundle of light-like curves parameterized by the points of the partonic 2-surface is in question.

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