$M^8 - H$ duality *viz.* Hubble law, and gravitational Planck constant *viz.* Allais effect and warping

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Matti Pitkänen

orcid:0000-0002-8051-4364.
email: matpitka6@gmail.com,
url: `http://tgdtheory.com/public_html/`,
address: Valtatie 8, as. 2, 03600, Karkkila, Finland.

Abstract

In this article two developments in TGD are discussed. These developments emerged in a "wrong context".

The first discovery was the realization that M^8-H duality in zero energy ontology implies a fractal generalization of Hubble's law. It emerged in a work related to TGD inspired quantum biology, where gravitational Planck constant $h_{gr} = GMm/\beta_0$ (M and m are masses and β_0 is velocity parameter) introduced by Nottale plays a key role. This already also led to a partial understanding of Hubble tension: the value of β in cosmic scales is very near to 1 but its values differ slightly in long and short cosmic scales.

The second discovery emerged while developing a TGD based model for Allais effect, which has no explanation in the context of general relativity or Newtonian gravitation. The work led to the question concerning the interpretation of the velocity parameter β_0 appearing in the formula of the gravitational Planck constant h_{gr} introduced already by Nottale. The gravitational field bodies of the Earth and structures of cosmological size scale are characterized by $\beta_0 = v_0/c \simeq 1$ and cannot correspond to a velocity of matter for a matter flow. Solar system characterized by $\beta_0 \simeq 2^{-11}$, which cannot correspond to light-velocity in the general relativistic framework.

The solution of the problem came from a prediction of TGD, which emerged already 47 years ago during the first year of TGD. TGD predicts warped space-time surfaces which are flat like Minkowski space M^4 (no gravitation) and have vanishing gauge fields. They have Poincare symmetry but have reduced light-velocity due to the warping. Light-like geodesics of M^4 are replaced by light-like geodesics of $M^4 \times S^1 \subset H = M^4 \times CP_2$. Warped space-time surface can explain the Allais effect, in particular the dramatic reduction of the parabolic oscillator frequency in terms of reduced light velocity. The notion of warping generalizes: one can speak of warped Hamilton-Jacobi (H-J) structures for which the hypercomplex structure of M^4 and complex structure of CP_2 are mixed in the sense that M^4 H-J structure is replaced with that of a warped space-time surface.

Contents

1 Introduction

In this article two developments in TGD are discussed. These developments emerged in a "wrong context" as side products and are discussed in separate articles [L22, L21]. Since these discoveries are very relevant for the general theoretical framework of TGD, it is natural to publish them in a separate article.

The first discovery was the realization that $M^8 - H$ duality in zero energy ontology implies a fractal generalization of Hubble's law. It emerged in a work [L22] related to TGD inspired quantum biology, where gravitational Planck constant $h_{gr} = GMm/\beta_0$ (M and m are masses and β_0 is velocity parameter) introduced by Nottale [E2] plays a key role [L22]. This already also led to a partial understanding of Hubble tension: the value of β in cosmic scales is very near to 1 but its values differ slightly in long and short cosmic scales.

The second discovery emerged while developing a TGD based model for Allais effect, which has no explanation in the context of general relativity or Newtonian gravitation. The work led to the question concerning the interpretation of the velocity parameter β_0 appearing in the formula of the gravitational Planck constant h_{gr} introduced already by Nottale [E2]. The gravitational field bodies of the Earth and structures of cosmological size scale are characterized by $\beta_0 = v_0/c \simeq 1$ and cannot correspond to a velocity of matter for a matter flow. Solar system characterized by $\beta_0 \simeq 2^{-11}$, which cannot correspond to light-velocity in the general relativistic framework.

The solution of the problem came from a prediction of TGD, which emerged already 47 years ago during the first year of TGD. TGD predicts warped space-time surfaces which are flat like Minkowski space M^4 (no gravitation) and have vanishing gauge fields. They have Poincare symmetry but have reduced light-velocity due to the warping. Light-like geodesics of M^4 are replaced by light-like geodesics of $M^4 \times S^1 \subset H = M^4 \times CP_2$. Warped space-time surface can explain the Allais effect, in particular the dramatic reduction of the parabolic oscillator frequency in terms of reduced light velocity. The notion of warping generalizes: one can speak of warped Hamilton-Jacobi (H-J) structures for which the hypercomplex structure of M^4 and complex structure of CP_2 are mixed in the sense that M^4 H-J structure is replaced with that of a warped space-time surface.

1.1 $M^8 - H$ duality implies Hubble's law

An attempt to understand the highly suggestive connection of icosa tetrahedral tessellation [L8, L19] as a model of genetic code and icosahedral superclusters as hydrogen bonded clusters in water led to to completely unexpected developments suggesting deep connections between fundamental physics ($M^8 - H$ duality [L20] and the notions of gravitational Planck constant [E2] [L6] and electric Planck constant [L9] as implications of number theoretic vision), physics of water (hydrogen bonded water clusters), consciousness theory (field body as controller of biological body forming sensory representations of biological body), biology (ITT view of the genetic code) and cosmology (generalization of Hubble's law to all scales).

In particular, a prediction for the ordinary Planck Hubble constant implies a predictions for the mass density of the Universe and prediction is consistent with the observed mass density including contributions of dark matter and energy if the value of the velocity parameter $\beta_0 \leq 1$ appearing in the formula for the gravitational Planck constant is equal to $\beta_0 \simeq 1$ and the very large system containing observable Universe as quantum coherence region with Hubble radius L_H is a blackhole-like object with Schwartschild radius $r_s = 2\beta_0 L_H \simeq 2L_H$ for $\beta_0 \simeq 1$. Hubble tension means that the value of the Hubble length $L_H = 1/H_0$ in short scales is roughly 10 percent smaller than its value in long length scales. Since L_H is proportional to $1/\beta_0$ this can be understood if β_0 in the early universe (long scales) is roughly 10 per cent larger than in the long scales. This would imply that $\beta_0 \simeq .92$ in long length scales $\beta_0 \simeq 1$ in short scales. The predictions for the fraction of baryons would be about 5 percent in short length scales and 7.4 percent in long length scales.

1.2 Allais effect

The Allais effect [E1, E7] (see this and this) was first reported by Maurice Allais in 1954. It involves an abrupt change in the azimuth of a paraconical pendulum's oscillation plane during the solar eclipse, totaling up to 13.5 degrees.

1.2.1 Empirical findings

Consider first a brief summary of the findings of Allais and others [E7].

1. Paraconical pendulum consists of a rigid rod of ~ 1 meter and a metal ball. The bob, that is the weight at the bottom, has lense like shape. Paraconical pendulum differs from the conical pendulum in that the suspension point of the pendulum is not fixed but is a metal sphere able to roll without sliding in plane. Therefore it has 2 degrees of freedom rather than only one: both swinging and rotation around the vertical axis are possible.

2. In the absence of any other forces than the gravitation of Earth) paraconical pendulum can behave much like a conical or Foucault pendulum. The oscillation plane of the paraconical pendulum turned by 13,5 degrees during 14 minutes (see https://plus.maths.org/mathematical-mysteries-foucaults-pendulum-and-eclipse). It is difficult to see how the gravitational fields of the Sun and Moon could explain this behaviour by changing the effective value of the Earth's gravitational acceleration.

3. Allais concludes from his experimental studies that the orbital plane approach always asymptotically to a limiting plane and the effect is only particularly spectacular during the eclipse. During solar eclipse the limiting plane contains the line connecting Earth, Moon, and Sun. Allais explains this in terms of what he calls the anisotropy of space.

4. Some experiments carried out during eclipse have reproduced the findings of Allais, some experiments not. In the experiment carried out by Jeverdan and collaborators in Romania it was found that the period of oscillation of the pendulum decreases by $\Delta f/f \simeq 5 \times 10^{-4}$ [E1, E6] which happens to correspond to the constant $\beta_0 = 2^{-11}$ appearing in the formula of the gravitational Planck constant for the Sun. It must be however emphasized that the overall magnitude of $\Delta f/f$ varies by five orders of magnitude. Even the sign of $\Delta f/f$ varies from experiment to experiment.

5. There is also the finding by Popescu and Olenici, which they interpret as a quantization of the plane of oscillation of paraconical pendulum during solar eclipse [E8].

6. There is also evidence that the effect is present also before and after the full eclipse. The time scale is 1 hour. Allais emphasized that the effect is a dynamic, not static, phenomenon, connected to the variation of weight or inertia in the space swept by the pendulum during the eclipse. The 10% excessive bending of light is reported during some eclipses (the "residual arc") is also reported.

While many attempts to confirm it have met with varied or ambiguous results, several observations indicated anomalous behavior that cannot be easily explained by general relativity (GR) or standard Newtonian mechanics.

1.2.2 The TGD view of Allais effect briefly

The TGD view of Allais effect involves quantum physics based on TGD based quantum ontology. One can consider several different levels for how quantum physics appears in the description.

1. One can start from a harmonic oscillator model for the gravitational pendulum and perform a quantization using the gravitational Planck constants of the Sun or the Earth. The huge values of these Planck constants imply that small values of the harmonic oscillator quantum number are involved. The changes of this quantum number could explain the fluctuations at quantum criticality assignable to the transition to the eclipse. The effect would not be gravitational but quantum mechanical and due to the large value of $\hbar_{gr}$.

2. During a full eclipse, the screening of the solar gravitational field might explain the Allais effect. There is however evidence that the Allais effect appears also outside the regions of full eclipse and therefore in the scale of the Earth. This suggests that a description involving interference and diffraction effects besides screening is needed.

   In the TGD framework, models involving classical long range gravitational or Z^0/Kähler fields cannot be excluded. These models do not however look promising: the standard physics based expectation is that the effects are quite too small.

3. The description in terms of wave functions identified as spinor fields of the "world of classical worlds" (WCW) is more promising. The argument of the WCW spinor field would be the space-time surface as analog of Bohr orbit for a particle as 3-surface. One would have essentially wave mechanics in WCW. Instead of a Bohr orbit one would have a wave in the space of Bohr orbits (WCW).

   The Moon would act as an obstacle giving rise to quantum diffraction, which reduces to screening immediately behind the Moon. The diffraction would not be caused by the classical gravitational interaction but would be analogous to the diffraction of electrons in a double screen and a genuine quantum effect.

4. The observed reduction $r = \Delta f/f \simeq 2^{-11}$ of the oscillation frequency of the pendulum is several orders of magnitude larger than the prediction and happens to be near to the velocity parameter β_0 appearing in the solar gravitational Planck constant. r is also near to the electron proton mass ratio m_e/m_p. Which intepretation is correct?

   The pondering of this question led to a solution of a longstanding problem concerning the interpretation of the velocity parameter β_0 appearing in the Notale's hypothesis. Field equations allow as solutions warped space-time surfaces, which are flat just like Minkowski space but have reduced light velocity $c_\# < c$. $\beta_0 = c_\#$ identification is natural. Warping as a universal quantum critical phenomenon distinguishing between TGD and GRT, allows to identify a mechanism for the reduction of the oscillator frequency in the Allais effect: $\Delta f/f \sim m_e/mp$ would be the correct interpretation.

The cautious conclusion would be that the Allais effect does not tell so much about new gravitational physics than about the new quantum ontology predicting the notion of WCW realizing holography = holomorphy vision, the hierarchy of Planck constants, and ZEO.

2 $M^8 - H$ duality implies generalized Hubble's law

The attempt to understand how the icosa tetrahedral tessellation (ITT) of H^3 [L8, L19] has a vertex figure the third shell of the icosahedral supercluster (ISC) [I1]. The problem was to understand how the complement of the vertex figure of ITT, which should be outside it, can correspond to the first and second shell of the ISC which are below the third shell.

The obvious guess is that the ITT realized at the field body of the ISC is related by inversion to ISC. $M^8 - H$ duality, as the TGD counterpart of the momentum position duality, involves inversion in $M^4 \subset M^8$, having interpretation as momentum space, mapping it to $M^4 \times CP_2$. Is $M^8 - H$ duality involved?

This question led to surprising developments suggesting deep connections between fundamental physics ($M^8 - H$ duality and the notions of gravitational and electric Planck constant as implications of number theoretic vision), physics of water (hydrogen bonded water clusters), consciousness theory (field body as controller of biological body forming sensory representations of biological body), biology (ITT view of the genetic code) and cosmology (generalization of Hubble's law to all scales).

2.1 $M^8 - H$ duality

In TGD, geometric and number theoretic visions of physics are complementary [L12, L17]: $M^8 - H$ duality [L20, L18] in which M^8 is analogous to 8-D momentum space associated with 8-D $H = M^4 \times CP_2$ is a formulation for this duality and makes Galois groups and their generalizations dynamic symmetries in the TGD framework. This complementarity is analogous to momentum position duality of quantum theory and implied by the replacement of a point-like particle with 3-surface, whose Bohr orbit defines space-time surface.

The points of $M^4 \subset M^8$ having interpretation as quaternions with number theoretic inner product identified as the real part of the square of the quaternion. The points p^k of $M^4 \subset M^8$ having interpretation as 4-momenta are mapped to points m^k of $M^4 \subset H = M^4 \times CP_2$ having interpretation as points of Minkowski space by inversion

$$p^k \rightarrow m^k = h_{eff}\frac{p^k}{p^2} \quad .$$

Here h_{eff} is the effective Planck constant having a number theoretic interpretation. The proposal is that h_{eff} is a multiple of its minimal value h_0: $\hbar_{eff} = n\hbar_0$, $\hbar = n_0 h_0$. The assumption that CP_2 size scale $R(CP_2)$ is actually scaled up up Planck length $R(CP_2) = (\hbar/\hbar_0)l_P$, $l_P = \sqrt{\hbar G}$, gives the estimate $n_0 = (7!)^2$. This predicts also values $h_{eff} < h$ and there exists some evidence for them [D1] [L1].

There are at least two possible choices for h_{eff} if one assigns it to a field body mediating a particular type of interaction. The are at least gravitational [L7, L6] and electric [L9] Planck constants $\hbar_{gr}$ and $\hbar_{em}$. The gravitational Planck constant is given by

$$\hbar_{gr} = \frac{GMm}{\beta_0} = \frac{r_S m}{2\beta_0} \quad . \tag{2.1}$$

Here M and $m$are two masses, typically m is much smaller than M. This is something new a possible interpretation is in terms of a generalization of ordinary Lie symmetry algebras to multi-local symmetry algebras known as Yangians. The bilocal Planck constant could take the role of the ordinary Planck constant for Yangians [B1] [L2].

$\beta_0 = v_0/c \leq 1$ is a velocity parameter. Number theoretical arguments [L3] leads to ask whether β_0 is inverse integer valued but also rational values are possible. This would imply that Lorentz boosts generated by β_0 define a discrete subgroup of the Lorentz group. The formula for $\hbar_{gr}$ generalizes to massless particles by replacing the rest mass m with the energy E of the particle.

One can assign to $\hbar_{gr}$ gravitational Compton length

$$L_H = \Lambda_{gr} = \frac{r_S}{2\beta_0} \quad .$$

which by Equivalence Principle is independent of the small mass m.

A natural identification of the gravitational Compton length $\Lambda_{gr}(M) = GM/\beta_0(M) = r_S/2\beta_0(M)$ is as the size scale of gravitationally quantum coherent regions associated with a pair of systems with mass M and m. Note that by Equivalence Principle there is no dependence on mass m. The large mass M would correspond to the mass associated with a system with mass M and typically with a size scale considerably larger than its Schwartschild radius r_S.

A good guess is that the size of the CD associated with a gravitational field body of mass M corresponds to $\Lambda_{gr}(M) = r_s(M)/2\beta_0$ for a larger system with mass M creating the gravitational field in which m moves. This poses a condition on the parameter $\beta_0(M)$. For the Earth $\Lambda_{gr} = r_s/2 \simeq .5$ cm is the size scale of a snowflake. For the Sun with $\beta_0 \simeq 2^{-11}$ (, which happens to be electron-proton mass ratio) $\Lambda_{gr} \simeq R_E/2$ is considerably smaller than the solar radius. That the value is one half of the Earth's radius is probably not an accident.

The notion of the gravitational Planck constant generalizes to the electric case [L9] by the replacement $GMm \rightarrow Qqe^2$ where Q and q are interacting charges expressed using elementary charged e as unit.

2.2 $M^8 - H$ duality generalizes Hubble law

TGD predicts a Russian doll hierarchy defined by CDs. This leads to a generalization of the Hubble law for both gravitational and electric Planck constants. A natural proposal is that the gravitational Compton length $\Lambda_{gr} = GM(CD)/\beta_0$ is identifiable as the size scale $L(CD)$ of the CD. Here $M(CD)$ has an interpretation as mass of the system defining the gravitational field in which the system corresponds to quantum coherence regions as a space-time sheet. CD would contain the quantum coherence regions whose M^4 projection is light-cone proper time= constant hyperboloid H^3.

Since the half-cones of CD are analogous to empty cosmologies, another natural proposal is that the size scale $L(CD)$ of CD is identifiable as the Hubble radius $L_H(CD) = c/H_0(CD)$ of CD and that the Hubble law holds true inside the sub-cosmology associated with the CD and is apart from small corrections a consequence of the hyperbolic geometry of light-cone proper time = constant surface H^3.

Gravitational $M^8 - H$ duality predicts $m^k = \hbar_{gr} p^k/p^2$. If applied to particle with mass m appearing in $\hbar_{gr} = GMm/\beta_0$, this gives

$$m^k = \frac{r_S}{2}\frac{\beta^k}{\beta_0} \quad ,$$

where β^k is four-velocity of the particle. Four-velocity has unit length which fixes the value the light-cone proper time a as

$$a = \frac{r_S}{2\beta_0} \quad .$$

The value of 3-dimensional M^4 distance r_3 is bounded by the condition

$$r_3 \leq L_H = \frac{r_S}{2\beta_0} \quad ,$$

stating that the perceptive field of the system is bounded by causal diamond with radius L_H.

If the particle is allowed to have mass $m_1 \neq m$, where m defines $\hbar gr$, the outcome is different and summarized by the formulas

$$\begin{array}{l} m^k = \frac{r_S}{2}\frac{m}{m_1}\frac{\beta^k}{\beta_0} \quad , \\ a = \frac{r_S}{2\beta_0}\frac{m}{m_1} \quad , \\ r_3 \leq \frac{r_S}{2\beta_0}\frac{m}{m_1} \quad . \end{array}$$

Particles with different masses would correspond to different values of light-cone proper time and Hubble constant would be inversely proportional to the particle mass. This would suggest a violation of the Equivalence Principle so that this option looks implausible.

The formula

$$\beta_3 = H_0(CD) r_3$$

looks like a generalization of the Hubble law with Hubble length $L_H(CD) = c/H_0(CD)$ given by the gravitational quantum coherence length

$$L_H = \Lambda_{gr} = \frac{r_S}{2\beta_0} \quad .$$

For the Earth L_H would have the value of $\Lambda_{gr} = .5$ cm for $\beta_0 = 1$. For the Sun with $\beta_0 \simeq 2^{-11}$, L_H would in a good approximation equal $R_E/2$, where R_E is the Earth radius.

Hubble's law generalizes also in the case of electric Planck constant $\hbar_{em} = Q_1 q_2 e^2/\beta_0$. Electric Compton length identifiable as Hubble length $L_{H,em}$ is

$$\Lambda_{em} = L_{H,em} = \frac{Q_1 q_2 e^2}{\beta_0 m} = \frac{4\pi\alpha_{em}}{\beta_0} Q_1 q_2 \times \lambda_C \quad .$$

Q_1 and q_2 are charges using e as a unit. Now the Hubble length is proportional to Compton length.

In both electric and gravitational cases the predicted redshift is rather large and it is interesting to see whether this is a testable effect. The redshift reflects the fact that the Lorentz invariant light-cone proper time is the natural time coordinate for the CD. Its presence could be seen as a support for the notion of CD and the notion of many-sheeted space-time. In the gravitational case all particles would have the same time coordinate. In the electric case the time coordinate would be inversely proportional to the particle mass.

It is interesting to look the situation from the point of view of zero energy ontology (ZEO) [K7] [L4, L11]. The value of the light-cone proper time for cd$\subset$ CD=cd$\times$ CP_2 is quantized and equal to Λ_{em} *resp.* Λ_{gr} in the electric *resp.* gravitational case. ZEO predicts that CD is identifiable as a perceptive field of a conscious entity, self. The subjective time evolution corresponds to a sequence of "small" state function reductions (SSFRs) defining the TGD counterpart of the Zeno effect. The size of the cd increases in a statistical sense. The simplest assumption is that during this sequence the boundary of cd is only scaled (this induces a conformal transformation leaving the 3-D quantum state at the boundary invariant). The value of the geometric time, for instance identifiable as the temporal distance between the tips, of cd increases.

The problem is that the identification $a = L_H$ as subjective time means that subjective time would not increase. This is not consistent with the TGD view of memories. The interpretation of subjective memories [L13] in terms of classical non-determinism would assign them to definite values of a in the geometric past characterizing the loci of non-determinism. If only a single value of β_0 is assumed, this is not consistent with $a = L_H$. Could the moments associated with loci of memories correspond to different values of β_0? Smaller values of β_0 would correspond to earlier memories. The higher the evolutionary level, the larger the number of values of β_0 would be. The spectrum of β_0 would be fixed by the non-determinism of the space-time surface.

2.3 Ordinary Hubble law follows from $M^8 - H$ duality

One can test the generalization of Hubble's law on cosmological scales. The proposal implies that the ordinary Hubble radius $L_H = c/H_0 = 1.44 \times 10^9$ ly can be identified as gravitational Compton length:

$$L_H = \frac{GM(CD)}{\beta_0(CD)} = \frac{r_S(CD)}{2\beta_0(CD)} \quad .$$

where r_S is the Schwartschild radius associated with the mass inside the cosmic CD containing the quantum coherence regions with size $L(CD)$ as the application to the cases of the Earth and Sun forces to conclude. This gives $r_S(CD) = 2\beta_0(CD)L_H$.

It is important to notice that the mass M is the mass inside CD_1 for which the visible Universe corresponds to the quantum coherence region with radius L_H. From the examples of the Earth and Sun, M is considerably larger than the mass within radius L_H. The examples provided by the Earth and Sun raise the question whether there is a hierarchy of CD sizes corresponding to planets, stars, galaxies and whether the spectrum of β_0 reflects this hierarchy and serves as measure for the complexity of space-time surfaces and associated failure of strict classical determinism.

2.4 Estimate for the density of the Universe

The proposed picture predicts that the mass of the visible Universe inside CD using solar mass as a unit is

$$\frac{M(CD_1)}{M_{Sun}} = \frac{2\beta_0(CD)L_H(CD)}{r_S(Sun)} .$$

Here $r_S(Sun)$ equals 3 km. Assume that the radius of CD_1 is given by $L_H(CD_1) = xL_H(CD)$ so that one has $V(CD_1) = x^3V(CD)$.

This gives the estimate for

$$M(CD_1) \sim 2 \times 10^{22} \times x^{-3}\beta_0^4(CD)M_{Sun} .$$

Here the mass $M(CD_1)$ corresponds to the mass within CD_1. $M_{Sun} \sim 1.88 \times 10^{57}m_p$, where m_p is proton mass. This predicts the average density

$$\rho \sim \beta_0^4 x^{-3} \times 12 \times 10^2 m_p/m^3 .$$

The density of baryons is estimated to be $5.9 - 6$ protons per cubic meter (see this). The density ρ_B of ordinary (baryonic) matter is believed to be about $p = 1/20$ that is 5 percent of the total density: $\rho \sim \rho_B/p = 20\rho_B \simeq 120m_p/m^3$. This gives $\beta_0^4 x^{-3} \sim 1/10$.

p-Adic length scales are good candidates for the size scales of CDs and seem to correspond to octaves $p \simeq 2^{2k}$ so that minimal scaling relating the sizes of CD and CD_1 containing CD should correspond to $x = 2$. For $\beta_0 = 1$ the Universe would be a blackhole-like object with $L_H = r_s/2\beta_0 = r_s/2$. For $(p = 1/20, x = 2)$ would predict $\beta_0 \simeq .95$. $(\beta_0 = 1, x = 2)$ would predict $p \simeq 6.1$ per cent.

Hubble tension means that the Hubble length in short scales is 5-10 percent shorter than in long scales. This requires that in short scales β_0 is 5-10 per cent smaller than in long scales. By $\beta_0 \leq 1$ $\beta_0 = 1$ cannot be true in long scales $(\beta_0 = 1, x = 2)$ could be true in short scales (the rough estimate for $\hbar_{gr,E}$ gives $\beta_0 \simeq 1$) and $(\beta_0 = .95, x = 2)$ in long scales would predict difference 7.5 per cent $\Delta H_0/H_0$ and resolve the Hubble tension.

$\beta_0 = 1$ in short scales as opposed to $\beta_0 = .95$ in long length scales would require the scaling of baryon fraction from 5 percent in short scales to 6.1 percent in long scales. One would have $L_H = r_s/2$ and the Universe could be seen as a blackhole-like system for which the quantum coherence region would have radius $L_H = r_s/2$. This would give a p-adic fractal hierarchy of blackhole-like objects, which are quantum coherence regions of blackhole-like objects.

Why should the fraction of baryons be smaller in short scales than in long scales? A possible explanation is the transfer of baryons to dark baryons at monopole flux tubes, reducing the fraction of baryons in short scales (recent universe) from 6.1 percent to 5 percent. The cosmic evolution as an unavoidable increase of algebraic complexity would generate large h_{eff} phases and would also manifest as the formation of gravitational bound states such as galaxies, stars and planets.

2.5 Some implications of the generalized Hubble law

The generalized Hubble law gives for the 4-velocity $\beta_4^k = (m^k/H_0(CD))$. $\beta_4 = 1$ fixes the value light-cone proper time to $a = H_0(CD)$ and also the size of the CD. For 3-velocity β_3 Hubble law gives $\beta_3 = L_H r_3 \leq 1$, which implies

$$r_3 \leq L_H(CD) = \frac{r_S(CD)}{2\beta_0} = \frac{GM(CD_1)}{\beta_0} .$$

The interpretation is in terms of a maximal value for the size of the CD coming from the mass $M(CD)$ and $\beta_0(CD)$. Note that the mass $M(CD_1)$ is for CD_1 for the object containing CD in its gravitational field.

In the case of the Sun with $\beta_0 \simeq 2^{-11}$, one has $L_H(Sun) \simeq 6,000$ km to be compared with the radius of the solar core about 139,000 km, which is roughly twice the value of $L_H(Sun)$. Note also that $L_H(Sun)$ is not far from the radius of Earth and also of the maximal radius of a solar spot: this plays a key role in the TGD based model of the Sun [L14].

In the case of Earth, L_H is of order .5 cm, much smaller than the size of Earth. From this one can conclude that the mass $M(CD)$ cannot be the mass assignable to the CD but to a larger system: for instance, the mass of the Sun or Earth. The maximal size of the CD is fixed by the gravitational field of a larger object.

$L_H(CD)$ as an analog of the event horizon defines an upper boundary of some object in the gravitational field of an object with mass M. One can consider two options.

1. L_H is a bound on the size of a space-time sheet interpreted as the size of quantum coherence regions.

2. L_H is a bound on the size of a CD defining the perceptive field of some object.

Consider now the interpretation as the size of the quantum coherence region.

1. Does L_H give an upper bound for the size of the Sun or of the quantum coherent objects inside the Sun and even outside it. Note that the rocky planets (Mercury, Venus, Earth and Mars) satisfy the condition $R \leq R_E$, whereas the outer planets fail to satisfy it. One can also consider the interpretation of space-time sheets as analogs of blackhole-like entities. The bound on M^4 radius r_3 would state that radiation cannot escape from this region.

2. The original model of Nottale for the planetary orbits as Bohr orbits assumes that the value of the velocity parameter β_0 for the outer planets is by factor 1/5 smaller than for the inner planets. This would increase the value of L_H as upper bound for their size to $5R_H$. For Jupiter, Saturn , Uranus and Neptune the values of R/R_E are 11.2, 9.5,4.0, and 3.88. For Jupiter and Saturn, which are gas planets, this proposal fails.

3. It is also possible that the value of β_0 is the same for the entire solar system: in this case only the principal quantum number n determining the Bohr radius for outer planets would be 5 times larger than for the first option. For this option Jupiter and Saturn should have an inner rocky core, which is not larger than Earth. This conforms with their character as gas planets.

The simplest interpretation is that the Sun and solar system decompose into space-time sheets with a size not exceeding $L_H(Sun)$. The notion of the many-sheeted space-time however allows us to ask whether there is a space-time sheet corresponding to the solar core topologically condensed at a larger connected space-time sheet identifiable as a quantum coherence region corresponding to the solar exterior or some other system, such as the field body of the Sun?

How to increase the value of L_H for the larger space-time sheet?

1. One could increase the mass M. In the case of the solar exterior, the increase of M would be small. Second option is that the value of $\beta_0 = 1/n$ is reduced. This would predict a spectrum of H_0 as integer multiples of a minimal value.

   Could the value of n label the onion-like layers of the field bodies with different values of the light-cone proper time a? Could they correspond to a hierarchy of values of $H_0(CD)$ coming as multiples of n? Note that some special values of n such as powers of 2 inspired by the p-adic length scale hypothesis can be considered.

2. Could the variation of n also resolve the Hubble tension: could the different values of the Hubble constant would correspond to different values of β_0. values of Hubble constant (10 per cent different) in long and short scales can be understood in terms of different values of β_0 but now one one must assume that $\beta_0 = 1/n$, with $n \sim 10$ or assume quantization as rational numbers.

3. There is evidence that the values of the cosmic recession velocities for distant astrophysical objects along lines of sight, originally discovered by Halton Arp, are quantized as multiples of n [E5, E11]. $\beta_0 = 1/n$ would predict subharmonics of the standard redshift rather than harmonics. Hyperbolic tessellations of H^3 could explain these mysterious 'God's fingers' as sequences of identical look stars or galaxies of hyperbolic tessellations along the line of sight [L5] [K5].

4. The notion of gravitational Planck constant makes sense only if one has $\hbar_{gr} \geq \hbar$. Could one assume that the upper layers of the solar surface have $h_{gr} = h$? This does not conform with the TGD based model for the surface layer of the Sun [L14], which assumes that the quantum coherence regions are of size of $L_H(Sun)$.

3 The TGD view of the Allais effect

Appendix gives a brief summary of the basic assumptions and implications of TGD necessary for the understanding of the TGD view of the Allais effect. The view of TGD as it was 2024 can be found in [L15, L16].

3.1 A TGD inspired quantum model for gravitational pendulum

It is natural to simplify the situation by considering a conical pendulum, that is gravitational pendulum for which the oscillation plane is free, instead of the paraconical pendulum.

To build a qualitative view, one can model the gravitational pendulum idealized as a harmonic oscillator. If h_{gr} is important then one must build a quantum model for the oscillator. One could first consider the simplest gravitational pendulum with one angle as a degree of freedom so that no attempt is made to explain the change in the oscillation plane. A more realistic case would be as a conical pendulum with two degrees of freedom corresponding to swinging and rotation.

The conservative option is that the Allais effect can be understood in terms of clasical forces. Second option is that the effect is purely quantum mechanical. Consider first the forces involved.

1. Newtonian gravity of the Earth, Moon and Sun is characterized by gravitational potentials. Each of these produces a gradient force. During an eclipse of the longitudinal effect, the Sun's gravitational force could weaken and disappear during a total eclipse if screening occurs. If the classical Z^0 force of Kähler force is involved, it could interfere with the gravitational force and contribute to the effect.

   The change of the gravitational force in vertical direction is reported to be consistent Newtonian predictions. One should understand why the effect is large in the transversal directions. The Sun-Moon pair and the Earth's vertical are in different directions. What is the direction of the Sun relative to the Earth's gravitational field in the studied cases? The shadowing effect depends on this direction.

2. Could the gravitomagnetism predicted by general relativity play a role?In the Maxwellian approximation, the gravitomagnetic field of the Sun is extremely weak: there are however anomalies, which challenge this assumption [E10, E3, E4] but might have different explanation in the TGD framework.

3. In the TGD framework, electroweak magnetism associated with induced classical Z^0 or Kähler field could be involved. For $\hbar_{gr}$ these fields could be important, at least in the scale $\Lambda_{gr} = GM/\beta_0 = r_S/2\beta_0$, which for the Sun is about $R_E/2$ and for the Earth about 5 mm. In the TGD picture, monopole flux tubes carrying Kähler monopole fluxes could give rise to a Lorentz force. Could this relate to the transversality of the Allais effect?

   What do the flux tubes related to electroweak magnetism or gravito-magnetism look like? The proposal is that gravitational interaction is mediated by narrow square-like radial monopole loops with a magnetic field parallel to the flux tube.

4. Could the large value of h_{gr} characterizing large scale quantum coherence amplify the effect? Could the effect be proportional to square N^2 of the number of flux tubes meeting the pendulum rather than N in the quantum coherence region? Or are the energies of the gravitons (say) as $E = h_{gr} f$ much larger than for the ordinary Planck constant and therefore amplify the effect? A possible interpretation for the large value of h_{gr} is as the replacement of a single flux tube with a bundle of $n = h_{gr}/h$ flux tubes defining an analog of Bose-Einstein condensate. For this interpretation the two alternatives might be equivalent.

Two standard physics views can be considered.

1. In the first case, there is no shadowing of gravitation by the Moon and the effects of the Sun and the Moon are additive. It is difficult to see how this picture could explain the Allais effect.

2. In the second case, the Moon partially shadows the Sun and only the Moon would contribute to the gravitational force during complete shadowing. This does not conform with the belief that gravitational interaction is not screened.

TGD allows two options.

1. In TGD inspired phenomenology, gravitational interactions would be mediated by the monopole flux tubes which are 3-D generalizations of strings but can have an astrophysical length scale. The gravitational flux proportional to the monopole flux would be analogous to an incompressible fluid flow and the Moon could act as an obstacle. The gravitational flux would not be lost but would go past the Moon.

   Contact between the pendulum and the flux tubes would occur. Wormhole contacts define contacts and also the 2-D string world sheets as intersections of the space-time sheets are involved.

2. TGD also allows the purely quantal view based on quantum diffraction in WCW. Monopole flux tubes would be analogous to quantum particles and the Moon would serve as an obstacle inducing quantum diffraction in the same way as in double slit experiment. The absence of screening would correspond to unitarity and probability conservations.

3.2 Could the quantum harmonic oscillator model predict genuine quantum effects caused by the large values of $\hbar_{gr}$?

One can start from a harmonic oscillator model for the conical pendulum.

1. For a harmonic oscillator, the frequency is independent of the amplitude of the oscillation and one has $E = n\hbar\omega$ in the standard quantum mechanics. At the limit of large values of oscillator number n, the system behaves classically. The replacement $\hbar \rightarrow \hbar_{gr}$ implies $E = n\hbar_{gr}\omega$. This can lead to a situation in which the value of n_{gr} is small. This can lead to large effects as the value of n_{gr} changes.

2. What is the situation for a non-linear oscillator such as a gravitational pendulum? In this case the frequency depends on amplitude classically. In the quantum case the frequency could be identified in terms of the eigenvalues of energy as $\omega = E/\hbar$. The energy eigenvalues are however expected to depend on $\hbar$. Hydrogen atoms are a good example: in this case the energy eigenvalues are proportional to α_{em}^2 and therefore to $1/\hbar^2$.

3. Can one imagine that the scaling $\hbar \rightarrow \hbar_{gr}$ could lead during the eclipse to a reduction of f, which is by 3 orders of magnitude larger than predicted by the classical model? For the classical oscillator the reduction of frequency is at maximum about 14 per cent. The classical amplitude of the oscillator relates by quantum-classical correspondence to the oscillator number n as $A \propto \sqrt{n}$.

   Very large value of $\hbar_{eff}$ reduces the value of n to $n_{gr} = (\hbar/\hbar_{gr})n$ and it can become small. Amplitude would be by $(\hbar/\hbar_{gr})^{1/2}$. Does this mean that the harmonic oscillator approximation should improve? If so, even the non-linear quantum model is unable to explain the large reduction of f.

   Oscillator quantum number as an analog for the number of quanta and oscillator amplitudes are canonically conjugate variables and Uncertainty Principle states that if the amplitude is precisely defined, the value of n is uncertain.

Could the large value of $\hbar_{gr}$ make the oscillator genuinely quantal?

1. It is good to take the values of $\hbar_{gr}$ for proton as a standard example. For the general mass $m = Nm_p$ one has $\hbar_{gr}(M,m) = N\hbar_{gr}(M,m_p)$. For $N = 1$ one has $\hbar_{gr}(M_E,m_p) = GM_Em_p/\beta_0 = r_s/2L_p \simeq 2.377 \times 10^{13}$. For the Sun one has $\hbar_{gr}(M_S,m_p) = GM_Sm_p/2\beta_0 = r_S 2^{11}/2L_p \simeq 1.426 \times 10^{22}$.

2. The energy of the classical oscillator is $E_{cl} = m\omega^2 A^2/2$. Quantum-classical correspondence $E_{cl} = E_{qu} = n\hbar\omega$ gives the estimate $n = mAR^1\omega/2\hbar$, with $\hbar = 1.054 \times 10^{-34}$ J. One can consider a simple numerical example. Assume $\omega = 2\pi$ s^{-1}, mass $m = 1$ kg and amplitude $A = 1$ m. In this case one obtains $n \simeq .47 \times 10^{34}$.

3. Assume that the oscillator has a size of quantum coherence region with radius $\Lambda_{gr}(E) = r_S(E)/2 \simeq .5$ cm for $\beta_0(E) \simeq 1$. Suppose that a cube sized region with this size has the density of water $\rho \sim 10^{30} m_p/\mathrm{m}^3$. In this case one would have $N \sim 1.25 \times 10^{23}$.

   This gives $n/n_{gr}(E) = 2.3 \times 10^{36}$. Using the estimate for n this gives $n_{gr}(E) = n/2.31 \times 10^{36} \sim .21 \times 10^{-2}$. This does not make sense since $n_{gr}(E)$ should be larger than 1. The volume should be larger by at least by a factor of about 500. The size of the quantum coherence regions should be at least 5 cm, 10 times larger.

4. Assume that the oscillator has a mass of 1 kg and has in the case of water density a volume of 10^{-3} m^3. One can argue that $\Lambda_{gr}(E)$ gives only a lower bound to the size of the quantum coherence region. If the scale up volume corresponds to a gravitational quantum coherence region, the size would be 10 cm so that n_{gr} would be equal to 17.6 and larger than 1.

5. In the case of the Sun the estimate gives $n/n_{gr} = 7.7 \times 10^{44}$. This is by factor $(3.3 \times 10^8$ larger than for the Earth and the problem becomes even worse. The scaling of about factor 10^3 for the size of the quantum coherence region to about 100 m would be required.

   The size scale of the quantum coherence region in the solar gravitational field is $R_E/2$ and in this case one would have the limit with a large value of n_{gr}: this situation cannot of course correspond to the recent case.

3.3 Geometric optics is not enough: is diffraction is needed

The Allais effect occurs during the entire period that it takes for a full shadow to travel over the Earth and also in the region where the full shadow is not present and the effects measured at different loci correlate [E9]. Therefore the effect of the screening by the Moon is not local like in the geometric optics picture and diffractive effects are suggestive.

1. A hole in the screen generates a characteristic diffraction pattern (see) expressible in terms of a Bessel function, whose argument is proportional to the ratio R/λ of the radius R hole and and the wavelength λ of the incoming light.

2. Assuming linearity of the underlying field equations, the Moon could be formally regarded as an "antihole". If a wave falls on a screen, a diffraction pattern is created. When a hole in the screen is replaced with an obstacle such as Moon, this pattern must be subtracted from the pattern in the absence of the hole.

What is diffracted?

1. Could classical waves be gravitational waves or possibly classical Z^0 waves generated by the Sun? In the general relativity framework, gravitational waves from the Sun are predicted to be extremely weak. Could the diffracting waves correspond to Z^0 cyclotron radiation generated at the flux tubes carrying dark electrons?

   Since the classical Z^0 field must be weaker than the gravitational field, also this option looks implausible unless constructive interference due to the long range quantum coherence is in question. The large value of h_{gr} would give them a large amount of energy in the energy scale of visible light. However, the very idea of large h_{eff} is that lowest order contribution given as a classical contribution having no dependence on h_{eff} dominates.

2. Could a diffraction analogous to the diffraction of quantum particles identified as flux tubes emanating from the Sun be in question? The flux tubes as analogs of strings would be analogs of quantum particles and would be described by an analog of Schrödinger amplitude identifiable as a spinor field in the "world of classical worlds" (WCW) consisting of space-time surfaces satisfying holography= holomorphy principle.

The flux tubes arriving from the Sun would be like a beam of particles described by a wave function and interference and diffraction for wave functions would take place. Moon could act as an obstacle producing the opposite of the effect produced by a hole in the screen. The effect has little do with classical or quantum gravitational fields.

Diffraction is characterized by two scales: the radius R of the hole in the screen (the radius of the obstacle in the dual situation) and the wavelength λ of the diffracting wave.

1. The natural identification for R is as the radius of the Moon.
2. What about the wavelength λ? In the [E9] it is proposed that the Sun is emitting particles with 44 m spatial periodicity. This scale could correspond to the wavelength λ and to the frequency $f = 6.9$ MHz if the waves are associated with a massless field.

   Electron has a cyclotron frequency of .56 MHz in the endogenous magnetic field of .2 Gauss assigned with magnetic monopole flux tubes in the TGD inspired model of living matter. There is a difference of order of magnitude. However, the TGD inspired model for music harmony suggests also the octaves of this field strength.

The parameters R and λ should have meaning at the level of both space-time surface and WCW.

3.4 Monopole flux tubes as mediators of the gravitational and other interactions

TGD suggests that monopole flux tubes serve as mediators of gravitational and also other interactions.

1. Magnetic monopole flux tubes are thickened strings but their lengths can be astrophysical and they carry a conserved monopole flux. This means a connection with string models. The difference is however that the flux tubes as string like objects could have astrophysical length scales. Could one regard flux tubes as particles and assign to them wave function as WCW spinor field so that one would have quantum diffraction at the level of WCW?
2. Interesting questions relate to the possible role of reconnections possible for the flux tubes returning to the Sun. The conservation of monopole flux allows U-shaped flux tubes turning back at the Moon and returning to the Sun. This would be an analog of reflection. But doesn't this mean that these flux tubes cause no long range gravitational effect in this case? Also reconnections creating closed flux tubes propagating past the Moon are possible but can they contribute to the static part of the gravitational force?

Can one make any guesses about the dimensions of the monopole flux tubes? The proposal of [E9] is that the Sun is emitting particles with $\lambda \sim 44$ m spatial periodicity. Could these particles correspond to monopole flux tubes? Could one think that flux tubes from the are emitted as flux tube bundles of radius which corresponds to the quantum gravitational coherence scale $\Lambda_{gr}(S) \simeq 3000$ km $\sim R_E/2$ of the Sun. Could the separation between these flux tubes be $\lambda \sim 44$ m? If so, a single flux tube bundle would contain $N \sim (\Lambda_{gr}/\lambda)^2 \simeq .68 \times 10^{10}$ flux tubes.

3.5 How to describe the effects of the Moon?

One can consider several models for the effects of the Moon.

1. The first option is screening so that the Moon would absorb the gravitational flux tubes or force them to turn backwards. This does not conform with the Newtonian view of gravitation and with the conservation of gravitational flux which would correspond to the monopole flux. Also the findings about the presence of Allais effect also outside the eclipse region are in conflict with this idea.

2. The situation could be also seen as an incompressible hydrodynamic flow past an obstacle. The gravitational flux tubes would be analogous to hydrodynamical flow lines. The incompressibility of the flow would correspond to the conservation of the magnetic and gravitational flux. This would give screening behind the Moon but conservation of flux would force the flux past the Moon.

3. One could try to describe the situation as diffraction of classical gravitational and Z^0 fields. In this case the Moon would serve as an "anti-source" generating the negative of the diffraction pattern generated by a hole in the screen. If the fields are identified as magnetic fields quantized to flux tubes in TGD, the flux conservation is obtained and there is an analogy with hydrodynamics of incompressible fluid.

4. A further, and perhaps the most realistic, view is in terms of quantum diffraction at the level of the WCW for the monopole flux tubes as basic objects as 3-surfaces generalizing the point-like particles. The WCW spinor field would correspond to the analog of Schrödinger amplitude."

What happens during an eclipse if the interference and diffraction play a role? One must distinguish between short and long wavelengths.

1. Concerning short wavelengths, it could be imagined that the contribution from the Sun is shaded during the eclipse and only the lunar contribution is received. For short wavelengths there would be no diffraction during the total eclipse and geometric optics would work.

2. The large value of the gravitational Planck constants h_{gr} implies a long gravitational Compton wavelength of order $R_E/2$ for $\beta_0(Sun) \simeq 2^{-11}$ and would indicate that diffraction and interference are important for long wavelengths. An analogy is, for example, the interference pattern that occurs around a boat dock in waves.

3. Screening occurs also in diffraction by an obstacle immediately behind the obstacle. One should clarify what happens at the level of geometry in the screening. Could the flux tube turn back and return to its source in the screening, say by the Moon? Could the flux tube reconnect and give rise to flux tube loops. Also the Earth causes similar screening and the Allais effect would occur in the Moon when shadowed by the Earth.

3.6 Could the induced Z^0 and Kähler fields have a role in the Allais effect?

The interpretation of Allais effect in terms of diffraction in WCW is rather attractive but one can also consider the possibility that the diffraction of gravitational and Z^0 fields is involved and perhaps even provide an alternative description. This also relates to the fundamental description of quantum gravitation. Quantization of metric does not work in general relativity: could the description of quantum gravitation be in terms of WCW spinor fields having as arguments Bohr orbit like space-time surfaces rather than of the second quantized metric tensor.

1. TGD predicts the possibility of long range Z^0/Kähler forces and I have also considered classical Z^0 force as a source of the Allais effect [K6]. Both electromagnetic and Z^0 field involve Kähler field as a part and the latter could be the long range field which is not subject to the weak screening.

2. Could Z^0 eclipse give a non-gravitational contribution to the Allais effect?. Z^0 field and maybe also classical W fields would be massless below Λ_{gr} due to the large value of h_{gr}. Z^0 Coulombic force directed along the flux tubes would be screened and also a Z_0 magnetic torque would be present. One could say that besides em radiation also Z^0 readiation represented as waves from the Sun is screened.

3.7 The reduction of the frequency of the pendulum as evidence for ZEO?

The reduction of the oscillator frequency is 3 orders of magnitude too large. It corresponds also to the parameter $\beta_0(S)$ appearing in $\hbar_{gr}(S)$. Suppose that $\Delta f/f \simeq \beta_0(S) \simeq 2^{-11}$ is not an accident. Could β_0 correspond to a real velocity?

1. Intriguingly, the formula for redshift for this velocity gives in $\Delta f/f = \sqrt{(1-\beta_0)/(1+\beta_0)} \simeq -\beta_0$. Could it be that the oscillation frequency is in some sense redshifted during the eclipse.

   The magnitude of $v_0 = c\beta_0 \simeq 150$ km/s is huge as compared to the velocity of the oscillator having order of magnitude 1 m/s. Therefore the redshift cannot be caused by a real motion of the pendulum relative to the observer.

2. Could one imagine any physical interpretation for $\beta_0(S)$? The solar system rotates around the galactic center with a velocity of 200–230 km/s. Could $\beta_0(S)$ relate to this velocity? Note that Earth moves relative to the Sun with velocity 29.78 km/s.

What comes to mind as an explanation for $\Delta f/f$ in the TGD framework is zero energy ontology (ZEO) already briefly summarized.

1. Could the screening of the solar gravitational field by the Moon mean that cd(S), the solar CD as a candidate for the cd of the observer, separate from the cd(P), the CD of the pendulum as sub-CD of cd(S)? The splitting of the U-shaped monopole flux tube contacts coming from the Sun by a reconnection as a correlate for the screening by the Moon could correspond to this splitting at the space-time level.

2. Quantum criticality is expected to prevail in the beginning of the solar eclipse. Large length scale fluctuations should take place. Could the large fluctuations observed in the beginning and end of the eclipse correspond to fluctuations of $\hbar_{gr}$ and thus in the values of the velocity parameter β_0 as a moduli parameter for CD?

   The causal diamond cd(P), associated with the quantum coherence region U of the pendulum, is a sub-cd of cd(Sun). Could the velocity parameter β_0 appearing in $\hbar_{gr}$ parameterize a Lorentz boost for cd relating it to a standard cd. A possible problem with this interpretation and the $\beta_0 \leq 1$ as a good approximation in the case of the Earth and cosmology [L22] would correspond to a boost with light velocity.

3. Before SSFR a delocalization in the moduli space $M(P)$ of cd(P):s takes place and generates a quantum superposition in $M(P)$ involving different size scales of CD, different values of $\beta_0(P)$ and possibly also different directions of the Lorentz boost.

   In the SSFR a localization in the moduli space for cd(P) occurs: in particular, the size of the cd(P) and the value $\beta_0(P)$ are fixed. The size of the cd is bound to increase in a statistical sense and this corresponds to the increase of the geometric identifiable as the temporal distance between the tips of cd.

   The new value of $\beta_0(P)$ serves as a moduli parameter for cd. cd(Sun) and cd(P) (or rather their active boundaries) are related by Lorentz boost but it is not clear whether one can say that they move relative to each other. This analog for relative motion of CDs would occur with a maximal velocity $\beta_0 = \beta_0(Sun)$ in which case $\beta_0(P) = 0$ would be true.

4. The analog of cosmic redshift would take place. The observed frequencies of all periodic processes for P are reshifted by $\beta_0 \leq \beta_0(Sun) \simeq 2^{-11}$. The Allais effect could be seen as evidence for the prediction that at the fundamental level the space-time is not connected but consists of separate space-times sheets within CDs forming a scale hierarchy.

It seems that although this view has some truth in it, it is not realistic as such.

3.8 About the interpretation of the parameter β_0 and the reduction of the oscillator frequency in Allais effect

The problems related to the physical interpretation of the parameter β_0 led to a rather detailed understanding of the frequency reduction occurring in the Allais effect.

3.8.1 Problems related to the interpretation of the velocity parameter β_0

There are several longstanding questions related to the parameter β_0 appearing in the formula $\hbar_{gr} = GMm/\beta_0$ introduced originally by Nottale [E2].

1. Is the interpretation of β_0 as a velocity parameter necessary? The gravitational Compton length $\Lambda_{gr} = r_s/2\beta_0$ has no dependence on the small mass m, which conforms with the Equivalence Principle. Also the cyclotron frequencies at the monopole flux tubes of the gravitational field body are independent of m.

2. There are two preferred values for β_0: $\beta_0 \simeq 1$ assigned with the Earth's gravitational field body an and $\beta_0 \simeq 2^{-11}$ assigned with the field body of the Sun.

3. The velocity of the solar system with respect to the galaxy is of the same magnitude as $\beta_0 \simeq 2^{-11}$ assigned with the inner planets, which supports the interpretation as velocity. The interpretation of $\beta_0 = v_0/c \simeq 1$ assigned with the Earth as a velocity of a massive object does not however look sensible. The realization that $M^8 - H$ duality implies Hubble's law lead to the conclusion that the Hubble tension finds a solution if one has two slightly different values of β_0 near unity applying in short and long scales.

There might be a very simple solution to these interpretational problems, which I have failed to notice.

1. In the standard quantum theory two quantum lengths characterize a massive particle. The Compton length $\lambda_c = h/m$ and the de-Broglie wavelength $\lambda_{de-B} = h/m\beta_0$, where $\beta_0 = v_0/c$ is the velocity of particle using light velocity as a unit.

2. Could the gravitational Planck constant $\hbar_{gr}(S)$ assigned to Sun and also planets in the Bohr model for planetary orbits corresponds to de-Broglie wave length and could β_0 correspond to a velocity $220 - 230$ km/s giving $\beta \in [(.73, .77) \times 10^{-3}]$ of the solar system with respect to galactic center. The error is about 20 per cent. If the gravitational Planck constant assigned with the Earth would correspond to the gravitational Compton length Λ_{gr}, the problem with $\beta_0 = 1$ would disappear.

There are however objections against this proposal.

1. The problem is that the Bohr orbit quantization of the planetary system [K4] does not make sense for this interpretation. The quantum input in the quantization is the quantization of angular momentum and it would say that L_z/m equals to a multiple of the gravitational de-Broglie wavelength. This does not make sense in the framework of standard QM. This suggests that β_0 cannot have an interpretation as physical velocity of a massive object. Could it correspond to an analog of light velocity? Neither can the value $\beta_0(E) \simeq 1$ for the Earth for cosmological scales be identified as a velocity for a massive object.

2. $M^8 - H$ duality for the gravitational Planck constant leads to a fractal generalization of Hubble's law suggesting that Hubble tension might relate to two slightly different values of $\beta_0 \simeq 1$ in short and long length scales differing by 5-6 percent [L22]. This interpretation is not consistent with the interpretation of Λ_{gr} for $\beta_0 = 1$ as gravitational Compton length.

   The problem disappears if one can interpret $v_0 \leq c$ as light velocity with $c_\# = \sqrt{g_{tt}}c \leq c$ along the space-time surface in the formula for the gravitational Compton length.

3. This interpretation would have non-trivial consequences. In the case of the Sun, the disappearance of the $1/\beta_0(S) \simeq 2^{11}$ from the formula h_{gr} reduces the gravitational Compton length and gives $\Lambda_{gr}(S) = 3 \times 10^5 \Lambda_{gr}(E)$ rather than $\Lambda_{gr}(S) \simeq 2^{11} \times 3 \times 10^5 \times \Lambda_{gr}(E)$. The energy $E = h_{gr}(S)f$ for a given frequency would be also reduced by $\beta_0(S) \simeq 2^{-11}$. And as noticed, the Bohr quantization of the planetary system would not make sense anymore.

4. It seems that the only solution to the problem is that β_0 is quite generally identifiable as reduced light velocity $c_\#$. The reduction of $c_\# = \sqrt{g_{tt}}$ to say $c_\# \simeq 2^{-11}$ would however require huge gravitational fields: this does not make sense in general relativistic framework.

3.8.2 Warping of the space-time surfaces as a solution of the problems

A possible solution of the problem comes from a basic distinction between TGD and General Relativity noticed already during the first year of TGD.

1. TGD allows solutions of field equations, which are gravitational vacua in the sense of GRT and also gauge theory vacua for induced gauge fields. The solutions however allow warping possible only for surfaces. A thin metal plate or a sheet of paper are good examples of a system unstable against warping and therefore critical systems.

2. TGD indeed allows minimal surface solutions with a 1-D CP_2 projection belonging to geodesic circle $S^1 \subset CP_2$ for which M^4 time coordinate in the rest system of the causal diamond CD is of form $m^0 = t - \Phi/\omega$. The induced metric of X^4 given by $ds^2 = (1-R^2\omega^2) - dz^2 - dwd\overline{w}$ is flat and has a deformation of the Poinca group as isometries. The interpretation $c_\# = \sqrt{1-R^2\omega^2}$ as a reduced light velocity is natural: the path around a warped space-time surface is longer than along a non-warped one. There would be no gravitational force but the vacuum would be warped. This warping makes sense also for monopole flux tubes obtained as deformations of the Cartesian product $M^2 \subset Y^2 \subset M^4 \times CP_2$. M^2 would be completely analogous to a metal plate and could be warped.

3. The warping can occur also at the level of the embedding space $H = M^4 \times CP_2$ for the Hamilton-Jacobi structure [L10]. Now $M^2 \subset M^2$ and CP_2 degrees would mix. An analogy is provided by a cylinder surface for which the coordinates (z, Φ) are replaced with coordinates $z - k\phi, z + k\phi$ for which coordinate lines are dual helices.

   The hypercomplex coordinates $(u, v) \to (t - z, t + z)$ would be replaced with $(u = T - z, v = T + z)$ where T is defined as $T = t - \phi/\omega$. The canonical embedding of $M^2 \subset M^4$ with constant CP_2 coordinates would be tilted towards the direction of $S^1 \subset CP_2$. CP_2 complex coordinates would suffer a time dependent $U(1)$ rotation $\Phi \to \Phi - \omega t$, which is holomorphic transformation and gives rise to a twisted Hamilton-Jacobi structure.

4. Even more general twisted Hamilton-Jacobi structures can be imagined [L10]. The TGD based model for the honeybee dance [K3] led to the proposal that there are preferred extremals as sphere bundles, which assign to a given point of the space-time surface a geodesic sphere, whose position in CP_2 depends on 2 M^4 coordinates so that one speak of local $SU(3)$ rotation of the geodesic sphere depending on two M^4 coordinates. Could also these kinds of twistings define exotic Hamilton-Jacobi structures? Could also twistings depending on time coordinate and complex coordinate w define exotic exotic Hamilton-Jacobi structures?

5. The twisted Hamilton-Jacobi structures could be associated with monopole flux tubes serving as body parts of field bodies. This would give connection with $\hbar_{gr}$. Also space-time surfaces representable as graps $M^4 \times CP_2$ could have a twisted Hamilton-Jacobi structure and the Hubble tension [L22] could be understood if the Hamilton structures differ by a small twist in long and short cosmological scales.

   In the planetary system there are two options for the Bohr quantization. $\beta_0 \simeq 2^{-11}$ would be true for the inner planets. For outer planets there are two options. Either $\beta_0 \simeq 2^{-11}$ is true but the principal quantum number n comes as multiples of 5 or $\beta_0 = 2^{-11}/5$ is true and Earth corresponds to the principal quantum number $n = 1$ for outer planets or $n = 5$ for the inner planets. For the second option $c_\# = \beta_0$ would be different at the gravitational monopole flux tubes.

3.8.3 A connection with the frequency reduction in Allais effect

There would be a connection with the model explaining the Allais effect [L21].

1. There is a surprisingly large reduction of the value of the oscillation frequency having upper bound $\Delta f/f \leq 2^{-11}$. This brings in mind $\beta_0(S)$ and the proposal was that the quantum critical transitions involve fluctuations reducing the oscillator frequency satisfying the formula $E = h_{gr}(E)f$: now the mass of the pendulum would be in the role of the small mass.

The modification $\Delta c_\#/c_\# \simeq 2^{-11}$ would be needed. The gravitational fluctuations $\Delta c_\#/c_\#$ required to produce the effect would be quite too large as compared to the reduction of the value of c from its maximal value by $GM_S/AU = r_s(S)/2AU \sim 10^{-9}$ and $GM_E/R_E = r_s(E)/2R_E \sim 10^{-9}$.

2. The modification of the inherently quantum critical Hamilton-Jacobi structure makes a large change $\Delta c_\#/c_\# \simeq 2^{-11}$ possible. It could occur at the level of the ordinary space-time surface or at the level of the field body. In the case of a gravitational pendulum, the reduction of oscillation frequency $\omega_1 \propto \sqrt{g/l} = \sqrt{GM/lR^2} = c_{\#,1} r_s/2R^2$ to $\omega_2 = c_{\#,2} r_s/R^2$ would be needed. That the velocity of the solar system with respect to the galactic center is near to $\beta_0 = c_\# \simeq 2^{-11}$ could follow from the warping in this framework. One could say that the solar system moves with a reduced light-velocity! One can wonder how general this phemenon is.

The physical mechanism causing this modification should be identified and explain the large value of $\Delta c_\#/c_\#$.

1. Warping is a critical phenomenon. Space-time warping as a fundamental quantum critical phenomenon could accompany and even induce many kinds of quantum critical phenomena, in particular Allais effect.

2. The model for the Allais effect proposes that diffraction-like effect for the gravitational flux tubes meaning a deviation of the monopole flux tubes, analogous to the deviation of flow lines of a hydrodynamic flow past solid object, could produce reduction of the effective gravitational flux. This would reduce the effective gravitational mass M_S experienced by the pendulum but the reduction is expected to be extremely small.

3. Could gravitational Planck constant and $c_\#$ of the gravitational field body change? $\Delta f/f \leq 2^{-11}$ is not far from the electron-proton mass ratio $m_e/m_p \simeq 1/1880$: the deviation is 9 per cent. If the field body contains hydrogen atoms, their ionization to protons and electrons transforming to ordinary electrons would reduce h_{gr} by the required amount.

   The hydrogen atoms should be Rydberg atoms with a very small binding energy and therefore with very large size: this is indeed possible at the field body. The dropped electrons should have smaller energy compensating for the energy needed for the energy needed for ionization. The transition could take place by tunnelling and therefore involve a pair of "big" state function reductions (BSFRs).

   This kind of phase transition should occur at quantum criticality assigned with the beginning of the solar eclipse? Why the turning of the monopole flux tubes meeting the Moon should induce a phase transition leading to the transformation of dark electrons to ordinary electrons? Are the electrons so near to ionization state the turning ionizes them?

3.8.4 How to test the proposal?

How could the proposal $\hbar_{gr} = GMm/c_\#$ implying the formulas for the gravitational Compton length be tested?

1. For the dark cyclotron the transitions at the magnetic body, the dependendence of cyclotron energy on m disappears. For other frequencies this is not the case and one would have $E = h_{gr} f = (GMm/2\pi c_\#) \times f$. A possible test is to look whether the energies for slightly different masses m differ. The second possibility is that $c_\#$ varies for critical phenomena.

2. Examples would be proton and hydrogen atom with a relative mass difference of order 2^{-11} and proton and neutron with mass difference of .14 per cent. One can imagine an entire spectroscopy allowing to test the notion of gravitational Planck constant by using the effects caused by the transformation of gravitationally dark photons to ordinary ones. Biophotons could emerge as an outcome of this transformation [K1].

3.8.5 How do quantum criticality and the criticality of warping relate?

The flat warped space-time surfaces are characterized by the reduced light-velocity $\beta_0 = c_{\#}/c \leq 1$. There is a criticality with respect to the variations of $c_{\#}$ (instability of metal plates illustrates this). Also the twisted Hamilton-Jacobi structures would be characterized by $c_{\#}$.

The criticality of the warping could induce or accompany various kinds of quantum criticalities. In the case of the Allais effect, this kind of quantum criticality would explain the variation of the pendulum frequency cannot be explained in terms of gravitation.

Quite generally, one can write $f_{\#} = c_{\#}/\lambda = f/n$, where $n = c/c_{\#}$ is analogous to the refractive index appearing in electrodynamics in presence of matter. In Maxwellian electrodynamics, refractive index relates to the relative dielectric constant ϵ_r via the formula $n = c/c_{\#} = \sqrt{\epsilon_r}$. Could reflective index and dielectric properties have a geometric description in terms of the warping of the space-time surface? If so, the warping of the space-time surface could be seen directly via the reflection of light!

Refractive index depends on frequency. This can be understood in terms of quantum criticality implying the value of $c_{\#}$ associated with the massless extremal assignable to the photons depends on frequency. At resonance, at which ϵ_r diverges, the value $c_{\#}$ would in the ideal case vanish: there would be no propagation of signals. The standard interpretation would be in terms of absorption of the signal by atoms, which contribute to the resonance frequencies.

How the criticality of warping could manifest itself in critical systems?

1. For a harmonic oscillator, the frequency is given in terms of force constant and mass as $\omega = \sqrt{k/m}$. A reasonable dimensional guess is that the force constant k characterizing the electromagnetic force is proportional to $(c_{\#}/c)^2$. For instance, cyclotron frequency would be proportional to $c_{\#}$. More generally, the Coulomb force in a dielectric is scaled from its vacuum value by $1/\epsilon_r = (c_{\#}/c)^2$. Also capaticance of a capacitor would be propoertional to $(c_{\#}/c)^2$. The variation of $c_{\#}$ at quantum criticality would make it possible to change the contribution of the electromagnetic force.

2. Gravitational masses have always the same sign so that the notion of dielectret does not make sense and $c_{\#}$ is not expected to play any role: this conforms with the character of warping. For instance, the gravitational force created by a constant mass density ρ corresponds to potential energy proportional to $Gm\rho r^2$, which is harmonic oscillator potential energy. The force constant $k \propto Gm\rho$ does not depend on $c_{\#}$.

3. If the system is in an equilibrium involving electromagnetic and gravitational forces, the variation of $c_{\#}$ appearing in the electromagnetic component of force could make possible the loss of equilibrium. The tuning of $c_{\#}$ could allow the field body to change the equilibrium point of a physical system and even destroy or create the equilibrium. In biology the generation of nerve pulse, the splitting of DNA double strand preceding transcription and replication could serve as examples of this.

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