About Hydrodynamical and Thermodynamical Interpretations of TGD

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Abstract

This chapter is collected from the material related to the relationship between TGD and hydrodynamics one one hand and TGD and thermodynamics on the one hand. What I have called hydrodynamics ansatz is a proposal for what the preferred extremals of Kähler action might be. The basic vision behind the ansatz is the reduction of quantum TGD to almost topological QFT. The basic condition is the vanishing of the contraction of the conserved Kähler current j with the induced Kähler gauge potential A implying the reduction of the Kähler action to 3-D contributions coming from the boundaries between space-time regions of Minkowskian and Euclidian signature.

Hydrodynamical interpretation demands that the flow parameters associated with the flow lines of isometry currents and Kähler current extend to global coordinates. Otherwise the flow line would resemble those for a gas of particles moving randomly. This leads to integrability conditions implying generalized Beltrami flow and Kähler action for the preferred extremals reduces to Chern-Simons action when the weak electro-weak duality is applied as boundary conditions. This allows also a definition of non-constant quantal order parameters depending on the spatial coordinates transversal to the flow lines.

Kiehn and others have studied Beltrami flows as integrable flows for which the flow lines define coordinate lines. In D=3 this requires that the rotor of the flow vector is parallel to the flow vector stating that Lorentz force vanishes. In D=4 the condition states that Lorentz 4-force vanishes so that also dissipation is absent. This kind of extremals are of special interest as asymptotic self-organization patterns: in fact all preferred extremals might satisfy these conditions. 3-D Beltrami flows are highly interesting topologically since the flow lines can get knotted. Their 4-D counterparts would have flow lines replaced with world sheets which can develop 2-knots. String world sheets carrying induced spinor fields are fundamental objects in TGD framework and they could indeed get knotted.

Kiehn has worked with both Betrami flows developed what he calls topological thermodynamics (TTD). This work is rather interesting from TGD point of view and the relationship between TTD and TGD is discussed in this chapter.

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What I have called hydrodynamics ansatz is a proposal for what the preferred extremals of Kähler action might be. The basic vision behind the ansatz is the reduction of quantum TGD to almost topological QFT. The basic condition is the vanishing of the contraction of the conserved Kähler current j with the induced Kähler gauge potential A implying the reduction of the Kähler action to 3-D contributions coming from the boundaries between space-time regions of Minkowskian and Euclidian signature.

The latter ones do not appear in standard physics but in TGD they serve as 4-D space-time correlates for lines of generalized Feynman graphs: the concrete identification is as wormhole contacts connecting 2 space-time sheets and carrying magnetic flux. Wormhole contacts appear necessarily as pairs due to the presence of the magnetic monopole flux and elementary particles correspond to this kind of pairs. Weak form of electric-magnetic duality reduces the 3-D contributions to Chern-Simons terms. The interior of wormhole contact can in principle contain additional 3-D contribution besides the "boundary" contribution.

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The strongest form of the hydrodynamical interpretation requires that all conserved currents are parallel to Kähler current. In the more general case one would have several hydrodynamic flows. Also the braidings (several of them for the most general ansatz) assigned with the light-like 3-surfaces are naturally defined by the flow lines of conserved currents. The independent behavior of particles at different flow lines can be seen as a realization of the complete integrability of the theory. In free quantum field theories on mass shell Fourier components are in a similar role but the geometric interpretation in terms of flow is of course lacking. This picture should generalize also to the solution of the Kähler-Dirac equation.

The localization of the induced spinor fields to string world sheets with massless Dirac action as 1-D boundary term forcing the fermion lines to be imbedding space geodesics makes this picture very concrete. One could even say that the light-like orbits of partonic 2-surfaces containing possibly several parallel fermions define discrete bundles of flow lines.

TGD relies on Zero Energy Ontology (ZEO). In ZEO quantum theory can be at least formally seen as a "square root" of thermodynamics. The rows of unitary U-matrix between zero energy states are identified as M-matrices. M-matrix defining entanglement coefficients between positive and negative energy parts of zero energy state is identified as a square root of density matrix reducing to a product of real and positive square root of the density matrix and unitary S-matrix.

Kiehn [B6] and others have studied Beltrami flows [B3] as integrable flows for which the flow lines define coordinate lines. In D=3 this requires that the rotor of the flow vector is parallel to the flow vector stating that Lorentz force vanishes. In D=4 the condition states that Lorentz 4-force vanishes so that also dissipation is absent. This kind of extremals are of special interest as asymptotic self-organization patterns and in fact all preferred extremals might satisfy these conditions. 3-D Beltrami flows are highly interesting topologically since the flow lines can get knotted. Their 4-D counterparts would have flow lines replaced with world sheets which can develop 2-knots. String world sheets carrying induced spinor fields are fundamental objects in TGD framework and they could indeed get knotted.

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The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [?].

1 Hydrodynamic Interpretation Of Extremals

1.1 Possible Role Of Beltrami Flows And Symplectic Invariance In The Description Of Gauge And Gravitational Interactions

One of the most recent observations made by people working with twistors is the finding of Monteiro and O'Connell described in the preprint "The Kinematic Algebra From the Self-Dual Sector" (see `http://tinyurl.com/ya2oa8na`) [B5]. The claim is that one can obtain supergravity amplitudes by replacing the color factors with kinematic factors which obey formally 2-D symplectic algebra defined by the plane defined by light-like momentum direction and complexified variable in the plane defined by polarizations. One could say that momentum and polarization dependent kinematic factors are in exactly the same role as the factors coming from Yang-Mills couplings. Unfortunately, the symplectic algebra looks rather formal object since the first coordinate is light-like coordinate and second coordinate complex transverse coordinate. It could make sense only in the complexification of Minkowski space.

In any case, this would suggest that the gravitational gauge group (to be distinguished from diffeomorphisms) is symplectic group of some kind having enormous representative power as we know from the fact that the symmetries of practically any physical system are realized in terms of symplectic transformations. According to the authors of [B5] one can identify the Lie algebra of symplectic group of sphere with that of $SU(N)$ at large N limit in suitable basis. What makes this interesting is that at large N limit non-planar diagrams which are the problem of twistor Grassmann approach vanish: this is old result of t'Hooft, which initiated the developments leading to AdS/CFT correspondence.

The symplectic group of $\delta M^4_\pm \times CP_2$ is the isometry algebra of WCW and I have proposed that the effective replacement of gauge group with this group implies the vanishing of non-planar diagrams [K10]. The extension of SYM to a theory of also gravitation in TGD framework could make Yangian symmetry exact, resolve the infrared divergences, and the problems caused by non-planar diagrams. It would also imply stringy picture in finite measurement resolution. Also the construction of the non-commutative homology and cohomology in TGD framework led to the lifting of Galois group algebras to their braided variants realized as symplectic flows [K9] and to the conjecture that in finite measurement resolution the cohomology obtained in this manner represents WCW ("world of classical worlds") spinor fields (or at least something very essential about them).

It is however difficult to understand how one could generalize the symplectic structure so that also symplectic transformations involving light-like coordinate and complex coordinate of the partonic 2-surface would make sense in some sense. In fact, a more natural interpretation for the kinematic algebra would in terms of volume preserving flows which are also Beltrami flows [B3, B4]. This gives a connection with quantum TGD since Beltrami flows define a basic dynamical symmetry for the preferred extremals of Kähler action which might be called Maxwellian phase.

1. Classical TGD is defined by Kähler action which is the analog of Maxwell action with Maxwell field expressed as the projection of CP_2 Kähler form. The field equations are extremely non-linear and only the second topological half of Maxwell equations is satisfied. The remaining equations state conservation laws for various isometry currents. Actually much more general conservation laws are obtained.

2. As a special case one obtains solutions analogous to those for Maxwell equations but there are also other objects such as CP_2 type vacuum extremals providing correlates for elementary particles and string like objects: for these solutions it does not make sense to speak about QFT in Minkowski space-time. For the Maxwell like solutions linear superposition is lost but a superposition holds true for solutions with the same local direction of polarization and massless four-momentum. This is a very quantal outcome (in accordance with quantum classical correspondence) since also in quantum measurement one obtains final state with fixed polarization and momentum. So called massless extremals (topological light rays) analogous to wave guides containing laser beam and its phase conjugate are solutions of this kind. The solutions are very interesting since no dispersion occurs so that wave packet preserves its form and the radiation is precisely targeted.

3. Maxwellian preferred extremals decompose in Minkowskian space-time regions to regions that can be regarded as classical space-time correlates for massless particles. Massless particles are characterized by polarization direction and light-like momentum direction. Now these directions can depend on position and are characterized by gradients of two scalar functions Φ and Ψ. Φ defines light-like momentum direction and the square of the gradient of Φ in Minkowski metric must vanish. Ψ defines polarization direction and its gradient is orthogonal to the gradient of Φ since polarization is orthogonal to momentum.

4. The flow has the additional property that the coordinate associated with the flow lines integrates to a global coordinate. Beltrami flow is the term used by mathematicians. Beltrami property means that the condition $j \wedge dj$ =0 is satisfied. In other words, tjhe current is in the plane defined by its exterior derivative. The above representation obviously guarantees this. Beltrami property allows to assign order parameter to the flow depending only the parameter varying along flow line.

   This is essential for the hydrodynamical interpretation of the preferred extremals which relies on the idea that varies conservation laws hold along flow lines. For instance, super-conducting phase requires this kind of flow and velocity along flow line is gradient of the order parameter. The breakdown of super-conductivity would mean topologically the loss of the Beltrami flow property. One might say that the space-time sheets in TGD Universe represent analogs of supra flow and this property is spoiled only by the finite size of the sheets. This strongly suggests that the space-time sheets correspond to perfect fluid flows with very low viscosity to entropy ratio and one application is to the observed perfect flow behavior of quark gluon plasma.

5. The current $J = \Phi\nabla\Psi$ has vanishing divergence if besides the orthogonality of the gradients the functions Ψ and Φ satisfy massless d'Alembert equation. This is natural for massless field modes and when these functions represent constant wave vector and polarization also d'Alembert equations are satisfied. One can actually add to $\nabla\Psi$ a gradient of an arbitrary function of Φ this corresponds to U(1) gauge invariance and the addition to the polarization vector a vector parallel to light-like four-momentum. One can replace Φ by any function of Φ so that one has Abelian Lie algebra analogous to $U(1)$ gauge algebra restricted to functions depending on Φ only.

The general Beltrami flow gives as a special case the kinetic flow associated by Monteiro and O'Connell with plane waves. For ordinary plane wave with constant direction of momentum vector and polarization vector one could take $\Phi = cos(\phi)$, $\phi = k \cdot m$ and $\Psi = \epsilon \cdot m$. This would give a real flow. The kinematical factor in SYM diagrams corresponds to a complexified flow $\Phi = exp(i\phi)$ and $\Psi = \phi + w$, where w is complex coordinate for polarization plane or more naturally, complexificaton of the coordinate in polarization direction. The flow is not unique since gauge invariance allows to modify ϕ term. The complexified flow is volume preserving only in the formal algebraic sense and satisfies the analog of Beltrami condition only in Dolbeault cohomology where d is identified as complex exterior derivative ($df = df/dz\ dz$ for holomorphic functions). In ordinary cohomology it fails. This formal complex flow of course does not define a real diffeomorphism at space-time level: one should replace Minkowski space with its complexification to get a genuine flow.

The finding of Monteiro and O'Connel encourages to think that the proposed more general Abelian algebra pops up also in non-Abelian YM theories. Discretization by braids would actually select single polarization and momentum direction. If the volume preserving Beltrami flows characterize the basic building bricks of radiation solutions of both general relativity and YM theories, it would not be surprising if the kinematic Lie algebra generators would appear in the vertices of YM theory and replace color factors in the transition from YM theory to general relativity. In TGD framework the construction of vertices at partonic two-surfaces would define local kinematic factors as effectively constant ones.

1.2 A General Solution Ansatz Based On Almost Topological QFT Property

The basic vision behind the ansatz is the reduction of quantum TGD to almost topological QFT. This requires that the flow parameters associated with the flow lines of isometry currents and Kähler current extend to global coordinates. This leads to integrability conditions implying generalized Beltrami flow and Kähler action for the preferred extremals reduces to Chern-Simons action when weak electro-weak duality is applied as boundary conditions. The strongest form of the hydrodynamical interpretation requires that all conserved currents are parallel to Kähler current. In the more general case one would have several hydrodynamic flows. Also the braidings (several of them for the most general ansatz) assigned with the light-like 3-surfaces are naturally defined by the flow lines of conserved currents. The independent behavior of particles at different flow lines can be seen as a realization of the complete integrability of the theory. In free quantum field theories on mass shell Fourier components are in a similar role but the geometric interpretation in terms of flow is of course lacking. This picture should generalize also to the solution of the Kähler-Dirac equation.

1.2.1 *Basic field equations*

Consider first the equations at general level.

1. The breaking of the Poincare symmetry due to the presence of monopole field occurs and leads to the isometry group $T \times SO(3) \times SU(3)$ corresponding to time translations, rotations, and color group. The Cartan algebra is four-dimensional and field equations reduce to the conservation laws of energy E, angular momentum J, color isospin I_3, and color hypercharge Y.

2. Quite generally, one can write the field equations as conservation laws for I, J, I_3, and Y.

$$D_\alpha \left[D_\beta (J^{\alpha\beta} H_A) - j_K^\alpha H^A + T^{\alpha\beta} j_A^l h_{kl} \partial_\beta h^l \right] = 0 \ . \quad (1.1)$$

The first term gives a contraction of the symmetric Ricci tensor with antisymmetric Kähler form and vanishes so that one has

$$D_\alpha \left[j_K^\alpha H^A - T^{\alpha\beta} j_A^k h_{kl} \partial_\beta h^l \right] = 0 \ . \quad (1.2)$$

For energy one has $H_A = 1$ and energy current associated with the flow lines is proportional to the Kähler current. Its divergence vanishes identically.

3. One can express the divergence of the term involving energy momentum tensor as as sum of terms involving $j_K^\alpha J_{\alpha\beta}$ and contraction of second fundamental form with energy momentum tensor so that one obtains

$$j_K^\alpha D_\alpha H^A = j_K^\alpha J_\alpha{}^\beta j_\beta^A + T^{\alpha\beta} H^k_{\alpha\beta} j_k^A \ . \quad (1.3)$$

1.2.2 *Hydrodynamical solution ansatz*

The characteristic feature of the solution ansatz would be the reduction of the dynamics to hydrodynamics analogous to that for a continuous distribution of particles initially at the end of X^3 of the light-like 3-surface moving along flow lines defined by currents j_A satisfying the integrability condition $j_A \wedge dj_A = 0$. Field theory would reduce effectively to particle mechanics along flow lines with conserved charges defined by various isometry currents. The strongest condition is that all isometry currents j_A and also Kähler current j_K are proportional to the same current j. The more general option corresponds to multi-hydrodynamics.

Conserved currents are analogous to hydrodynamical currents in the sense that the flow parameter along flow lines extends to a global space-time coordinate. The conserved current is proportional to the gradient $\nabla\Phi$ of the coordinate varying along the flow lines: $J = \Psi\nabla\Phi$ and by a proper choice of Ψ one can allow to have conservation. The initial values of Ψ and Φ can be selected freely along the flow lines beginning from either the end of the space-time surface or from wormhole throats.

If one requires hydrodynamics also for Chern-Simons action (effective 2-dimensionality is required for preferred extremals), the initial values of scalar functions can be chosen freely only at the partonic 2-surfaces. The freedom to chose the initial values of the charges conserved along flow lines at the partonic 2-surfaces means the existence of an infinite number of conserved charges so that the theory would be integrable and even in two different coordinate directions. The basic difference as compared to ordinary conservation laws is that the conserved currents are parallel and their flow parameter extends to a global coordinate.

1. The most general assumption is that the conserved isometry currents

$$J_A^\alpha = j_K^\alpha H^A - T^{\alpha\beta} j_A^k h_{kl} \partial_\beta h^l \quad (1.4)$$

and Kähler current are integrable in the sense that $J_A \wedge J_A = 0$ and $j_K \wedge j_K = 0$ hold true. One could imagine the possibility that the currents are not parallel.

2. The integrability condition $dJ_A \wedge J_A = 0$ is satisfied if one one has

$$J_A = \Psi_A d\Phi_A \ . \quad (1.5)$$

The conservation of J_A gives

$$d * (\Psi_A d\Phi_A) = 0 \ . \quad (1.6)$$

This would mean separate hydrodynamics for each of the currents involved. In principle there is not need to assume any further conditions and one can imagine infinite basis of scalar function pairs (Ψ_A, Φ_A) since criticality implies infinite number deformations implying conserved Noether currents.

3. The conservation condition reduces to d'Alembert equation in the induced metric if one assumes that $\nabla\Psi_A$ is orthogonal with every $d\Phi_A$.

$$d*d\Phi_A = 0 \ , \ d\Psi_A\cdot d\Phi_A = 0 \ . \tag{1.7}$$

Taking $x = \Phi_A$ as a coordinate the orthogonality condition states $g^{xj}\partial_j\Psi_A = 0$ and in the general case one cannot solve the condition by simply assuming that Ψ_A depends on the coordinates transversal to Φ_A only. These conditions bring in mind $p\cdot p = 0$ and $p\cdot e$ condition for massless modes of Maxwell field having fixed momentum and polarization. $d\Phi_A$ would correspond to p and $d\Psi_A$ to polarization. The condition that each isometry current corresponds its own pair (Ψ_A, Φ_A) would mean that each isometry current corresponds to independent light-like momentum and polarization. Ordinary free quantum field theory would support this view whereas hydrodynamics and QFT limit of TGD would support single flow.

These are the most general hydrodynamical conditions that one can assume. One can consider also more restricted scenarios.

1. The strongest ansatz is inspired by the hydrodynamical picture in which all conserved isometry charges flow along same flow lines so that one would have

$$J_A = \Psi_A d\Phi \ . \tag{1.8}$$

In this case same Φ would satisfy simultaneously the d'Alembert type equations.

$$d*d\Phi = 0 \ , \ d\Psi_A\cdot d\Phi = 0. \tag{1.9}$$

This would mean that the massless modes associated with isometry currents move in parallel manner but can have different polarizations. The spinor modes associated with light-light like 3-surfaces carry parallel four-momenta, which suggest that this option is correct. This allows a very general family of solutions and one can have a complete 3-dimensional basis of functions Ψ_A with gradient orthogonal to $d\Phi$.

2. Isometry invariance under $T\times SO(3)\times SU(3)$ allows to consider the possibility that one has

$$J_A = k_A\Psi_A d\Phi_{G(A)} \ , \ d*(d\Phi_G(A)) = 0 \ , \ d\Psi_A\cdot d\Phi_G(A)) = 0 \ . \tag{1.10}$$

where $G(A)$ is T for energy current, $SO(3)$ for angular momentum currents and $SU(3)$ for color currents. Energy would thus flow along its own flux lines, angular momentum along its own flow lines, and color quantum numbers along their own flow lines. For instance, color currents would differ from each other only by a numerical constant. The replacement of Ψ_A with $\Psi_{G(A)}$ would be too strong a condition since Killing vector fields are not related by a constant factor.

To sum up, the most general option is that each conserved current J_A defines its own integrable flow lines defined by the scalar function pair (Ψ_A, Φ_A). A complete basis of scalar functions satisfying the d'Alembert type equation guaranteeing current conservation could be imagined with restrictions coming from the effective 2-dimensionality reducing the scalar function basis effectively to the partonic 2-surface. The diametrically opposite option corresponds to the basis obtained by assuming that only single Φ is involved.

The proposed solution ansatz can be compared to the earlier ansatz [K4] stating that Kähler current is topologized in the sense that for $D(CP_2) = 3$ it is proportional to the identically conserved instanton current (so that 4-D Lorentz force vanishes) and vanishes for $D(CP_2) = 4$

(Maxwell phase). This hypothesis requires that instanton current is Beltrami field for $D(CP_2) = 3$. In the recent case the assumption that also instanton current satisfies the Beltrami hypothesis in strong sense (single function Φ) generalizes the topologization hypothesis for $D(CP_2) = 3$. As a matter fact, the topologization hypothesis applies to isometry currents also for $D(CP_2) = 4$ although instanton current is not conserved anymore.

1.2.3 *Can one require the extremal property in the case of Chern-Simons action?*

Effective 2-dimensionality is achieved if the ends and wormhole throats are extremals of Chern-Simons action. The strongest condition would be that space-time surfaces allow orthogonal slicings by 3-surfaces which are extremals of Chern-Simons action.

Also in this case one can require that the flow parameter associated with the flow lines of the isometry currents extends to a global coordinate. Kähler magnetic field $B = *J$ defines a conserved current so that all conserved currents would flow along the field lines of B and one would have 3-D Beltrami flow. Note that in magnetohydrodynamics the standard assumption is that currents flow along the field lines of the magnetic field.

For wormhole throats light-likeness causes some complications since the induced metric is degenerate and the contravariant metric must be restricted to the complement of the light-like direction. This means that d'Alembert equation reduces to 2-dimensional Laplace equation. For space-like 3-surfaces one obtains the counterpart of Laplace equation with partonic 2-surfaces serving as sources. The interpretation in terms of analogs of Coulomb potentials created by 2-D charge distributions would be natural.

Almost perfect fluids seems to be abundant in Nature. For instance, QCD plasma was originally thought to behave like gas and therefore have a rather high viscosity to entropy density ratio $x = \eta/s$. Already RHIC found that it however behaves like almost perfect fluid with x near to the minimum predicted by AdS/CFT. The findings from LHC gave additional conform the discovery [?]. Also Fermi gas is predicted on basis of experimental observations to have at low temperatures a low viscosity roughly 5-6 times the minimal value [D2] . In the following the argument that the preferred extremals of Kähler action are perfect fluids apart from the symmetry breaking to space-time sheets is developed. The argument requires some basic formulas summarized first.

The detailed definition of the viscous part of the stress energy tensor linear in velocity (oddness in velocity relates directly to second law) can be found in [D1] .

1. The symmetric part of the gradient of velocity gives the viscous part of the stress-energy tensor as a tensor linear in velocity. Velocity gradient decomposes to a term traceless tensor term and a term reducing to scalar.

$$\partial_i v_j + \partial_j v_i = \frac{2}{3}\partial_k v^k g_{ij} + (\partial_i v_j + \partial_j v_i - \frac{2}{3}\partial_k v^k g_{ij}) \ . \tag{1.11}$$

   The viscous contribution to stress tensor is given in terms of this decomposition as

$$\sigma_{visc;ij} = \zeta \partial_k v^k g_{ij} + \eta(\partial_i v_j + \partial_j v_i - \frac{2}{3}\partial_k v^k g_{ij}) \ . \tag{1.12}$$

   From $dF^i = T^{ij}S_j$ it is clear that bulk viscosity ζ gives to energy momentum tensor a pressure like contribution having interpretation in terms of friction opposing. Shear viscosity η corresponds to the traceless part of the velocity gradient often called just viscosity. This contribution to the stress tensor is non-diagonal and corresponds to momentum transfer in directions not parallel to momentum and makes the flow rotational. This term is essential for the thermal conduction and thermal conductivity vanishes for ideal fluids.

2. The 3-D total stress tensor can be written as

$$\sigma_{ij} = \rho v_i v_j - p g_{ij} + \sigma_{visc;ij} \ . \tag{1.13}$$

The generalization to a 4-D relativistic situation is simple. One just adds terms corresponding to energy density and energy flow to obtain

$$T^{\alpha\beta} = (\rho - p)u^\alpha u^\beta + pg^{\alpha\beta} - \sigma^{\alpha\beta}_{visc} \ . \tag{1.14}$$

Here u^α denotes the local four-velocity satisfying $u^\alpha u_\alpha = 1$. The sign factors relate to the concentrations in the definition of Minkowski metric ($(1, -1, -1, -1)$).

3. If the flow is such that the flow parameters associated with the flow lines integrate to a global flow parameter one can identify new time coordinate t as this flow parameter. This means a transition to a coordinate system in which fluid is at rest everywhere (comoving coordinates in cosmology) so that energy momentum tensor reduces to a diagonal term plus viscous term.

$$T^{\alpha\beta} = (\rho - p)g^{tt}\delta^\alpha_t \delta^\beta_t + pg^{\alpha\beta} - \sigma^{\alpha\beta}_{visc} \ . \tag{1.15}$$

In this case the vanishing of the viscous term means that one has perfect fluid in strong sense.

The existence of a global flow parameter means that one has

$$v_i = \Psi \partial_i \Phi \ . \tag{1.16}$$

Ψ and Φ depend on space-time point. The proportionality to a gradient of scalar Φ implies that Φ can be taken as a global time coordinate. If this condition is not satisfied, the perfect fluid property makes sense only locally.

AdS/CFT correspondence allows to deduce a lower limit for the coefficient of shear viscosity as

$$x = \frac{\eta}{s} \geq \frac{\hbar}{4\pi} \ . \tag{1.17}$$

This formula holds true in units in which one has $k_B = 1$ so that temperature has unit of energy.

What makes this interesting from TGD view is that in TGD framework perfect fluid property in appropriately generalized sense indeed characterizes locally the preferred extremals of Kähler action defining space-time surface.

1. Kähler action is Maxwell action with U(1) gauge field replaced with the projection of CP_2 Kähler form so that the four CP_2 coordinates become the dynamical variables at QFT limit. This means enormous reduction in the number of degrees of freedom as compared to the ordinary unifications. The field equations for Kähler action define the dynamics of space-time surfaces and this dynamics reduces to conservation laws for the currents assignable to isometries. This means that the system has a hydrodynamic interpretation. This is a considerable difference to ordinary Maxwell equations. Notice however that the "topological" half of Maxwell's equations (Faraday's induction law and the statement that no non-topological magnetic are possible) is satisfied.

2. Even more, the resulting hydrodynamical system allows an interpretation in terms of a perfect fluid. The general ansatz for the preferred extremals of field equations assumes that various conserved currents are proportional to a vector field characterized by so called Beltrami property. The coefficient of proportionality depends on space-time point and the conserved current in question. Beltrami fields by definition is a vector field such that the time parameters assignable to its flow lines integrate to single global coordinate. This is highly non-trivial and one of the implications is almost topological QFT property due to the fact that Kähler action reduces to a boundary term assignable to wormhole throats which are light-like 3-surfaces at the boundaries of regions of space-time with Euclidian and Minkowskian signatures. The

Euclidian regions (or wormhole throats, depends on one's tastes) define what I identify as generalized Feynman diagrams.

Beltrami property means that if the time coordinate for a space-time sheet is chosen to be this global flow parameter, all conserved currents have only time component. In TGD framework energy momentum tensor is replaced with a collection of conserved currents assignable to various isometries and the analog of energy momentum tensor complex constructed in this manner has no counterparts of non-diagonal components. Hence the preferred extremals allow an interpretation in terms of perfect fluid without any viscosity.

This argument justifies the expectation that TGD Universe is characterized by the presence of low-viscosity fluids. Real fluids of course have a non-vanishing albeit small value of x. What causes the failure of the exact perfect fluid property?

1. Many-sheetedness of the space-time is the underlying reason. Space-time surface decomposes into finite-sized space-time sheets containing topologically condensed smaller space-time sheets containing.... Only within given sheet perfect fluid property holds true and fails at wormhole contacts and because the sheet has a finite size. As a consequence, the global flow parameter exists only in given length and time scale. At imbedding space level and in zero energy ontology the phrasing of the same would be in terms of hierarchy of causal diamonds (CDs).

2. The so called eddy viscosity is caused by eddies (vortices) of the flow. The space-time sheets glued to a larger one are indeed analogous to eddies so that the reduction of viscosity to eddy viscosity could make sense quite generally. Also the phase slippage phenomenon of super-conductivity meaning that the total phase increment of the super-conducting order parameter is reduced by a multiple of 2π in phase slippage so that the average velocity proportional to the increment of the phase along the channel divided by the length of the channel is reduced by a quantized amount.

   The standard arrangement for measuring viscosity involves a lipid layer flowing along plane. The velocity of flow with respect to the surface increases from $v = 0$ at the lower boundary to v_{upper} at the upper boundary of the layer: this situation can be regarded as outcome of the dissipation process and prevails as long as energy is fed into the system. The reduction of the velocity in direction orthogonal to the layer means that the flow becomes rotational during dissipation leading to this stationary situation.

   This suggests that the elementary building block of dissipation process corresponds to a generation of vortex identifiable as cylindrical space-time sheets parallel to the plane of the flow and orthogonal to the velocity of flow and carrying quantized angular momentum. One expects that vortices have a spectrum labelled by quantum numbers like energy and angular momentum so that dissipation takes in discrete steps by the generation of vortices which transfer the energy and angular momentum to environment and in this manner generate the velocity gradient.

3. The quantization of the parameter x is suggestive in this framework. If entropy density and viscosity are both proportional to the density n of the eddies, the value of x would equal to the ratio of the quanta of entropy and kinematic viscosity η/n for single eddy if all eddies are identical. The quantum would be $\hbar/4\pi$ in the units used and the suggestive interpretation is in terms of the quantization of angular momentum. One of course expects a spectrum of eddies so that this simple prediction should hold true only at temperatures for which the excitation energies of vortices are above the thermal energy. The increase of the temperature would suggest that gradually more and more vortices come into play and that the ratio increases in a stepwise manner bringing in mind quantum Hall effect. In TGD Universe the value of h_{eff} can be large in some situations so that the quantal character of dissipation could become visible even macroscopically. Whether this a situation with large h_{eff} is encountered even in the case of QCD plasma is an interesting question.

The following poor man's argument tries to make the idea about quantization a little bit more concrete.

1. The vortices transfer momentum parallel to the plane from the flow. Therefore they must have momentum parallel to the flow given by the total cm momentum of the vortex. Before continuing some notations are needed. Let the densities of vortices and absorbed vortices be n and n_{abs} respectively. Denote by $v_{\|}$ *resp.* $v_{\perp}$ the components of cm momenta parallel to the main flow *resp.* perpendicular to the plane boundary plane. Let m be the mass of the vortex. Denote by S are parallel to the boundary plane.

2. The flow of momentum component parallel to the main flow due to the absorbed at S is

$$n_{abs} m v_{\|} v_{\perp} S \quad . \tag{1.18}$$

   This momentum flow must be equal to the viscous force

$$F_{visc} = \eta \frac{v_{\|}}{d} \times S \quad . \tag{1.19}$$

   From this one obtains

$$\eta = n_{abs} m v_{\perp} d \quad . \tag{1.20}$$

   If the entropy density is due to the vortices, it equals apart from possible numerical factors to

$$s = n$$

   so that one has

$$\frac{\eta}{s} = m v_{\perp} d \quad . \tag{1.21}$$

   This quantity should have lower bound $x = \hbar/4\pi$ and perhaps even quantized in multiples of x, Angular momentum quantization suggests strongly itself as origin of the quantization.

3. Local momentum conservation requires that the comoving vortices are created in pairs with opposite momenta and thus propagating with opposite velocities $v_{\perp}$. Only one half of vortices is absorbed so that one has $n_{abs} = n/2$. Vortex has quantized angular momentum associated with its internal rotation. Angular momentum is generated to the flow since the vortices flowing downwards are absorbed at the boundary surface.

   Suppose that the distance of their center of mass lines parallel to plane is $D = \epsilon d$, ϵ a numerical constant not too far from unity. The vortices of the pair moving in opposite direction have same angular momentum $mv\ \ D/2$ relative to their center of mass line between them. Angular momentum conservation requires that the sum these relative angular momenta cancels the sum of the angular momenta associated with the vortices themselves. Quantization for the total angular momentum for the pair of vortices gives

$$\frac{\eta}{s} = \frac{n\hbar}{\epsilon} \tag{1.22}$$

Quantization condition would give

$$\epsilon = 4\pi \quad . \tag{1.23}$$

One should understand why $D = 4\pi d$ - four times the circumference for the largest circle contained by the boundary layer- should define the minimal distance between the vortices of the pair. This distance is larger than the distance d for maximally sized vortices of radius $d/2$ just touching. This distance obviously increases as the thickness of the boundary layer increases suggesting that also the radius of the vortices scales like d.

4. One cannot of course take this detailed model too literally. What is however remarkable that quantization of angular momentum and dissipation mechanism based on vortices identified as space-time sheets indeed could explain why the lower bound for the ratio η/s is so small.

1.3 Hydrodynamic Picture In Fermionic Sector

Super-symmetry inspires the conjecture that the hydrodynamical picture applies also to the solutions of the Kähler-Dirac equation. This would mean that the solutions of Dirac equation can be localized to lower-dimensional surface or even flow lines.

1.3.1 *Basic objection*

The obvious objection against the localization to sub-manifolds is that it is not consistent with uncertainty principle in transversal degrees of freedom. More concretely, the assumption that the mode is localized to a lower-dimensional surface of X^4 implies that the action of the transversal part of Dirac operator in question acts on delta function and gives something singular.

The situation changes if the Dirac operator in question has vanishing transversal part at the lower-dimensional surface. This is not possible for the Dirac operator defined by the induced metric but is quite possible in the case of Kähler-Dirac operator. For instance, in the case of massless extremals Kähler-Dirac gamma matrices are non-vanishing in single direction only and the solution modes could be one-dimensional. For more general preferred extremals such as cosmic strings this is not the case.

In fact, there is a strong physical argument in favor of the localization of spinor modes at 2-D string world sheets so that hydrodynamical picture would result but with flow lines replaced with fermionic string world sheets.

1. Well-definedness of electromagnetic charged at string world sheets carrying spinor modes is very natural constraint and not trivially satisfied because classical W boson fields are present. As a matter fact, all weak fields should be effectively absent above weak scale. How this is possible classical weak fields identified as induced gauge fields are certainly present.

2. The condition that em charge is well defined for spinor modes implies that the space-time region in which spinor mode is non-vanishing has 2-D CP_2 projection such that the induced W boson fields are vanishing. The vanishing of classical Z^0 field can be poses as additional condition - at least in scales above weak scale. In the generic case this requires that the spinor mode is restricted to 2-D surface: string world sheet or possibly also partonic 2-surface. This implies that TGD reduces to string model in fermionic sector. Even for preferred extremals with 2-D projecting the modes are expected to allow restriction to 2-surfaces. This localization is possible only for Kähler-Dirac action and requires that the part of the Kähler-Dirac operator transversal to 2-surface vanishes.

3. This localization does not hold for cosmic string solutions which however have 2-D CP_2 projection which should have vanishing weak fields so that 4-D spinor modes with well-defined em charge are possible.

4. A word of warning is however in order. The GRT limit or rather limit of TGD as Einstein Yang-Mills theory replaces the sheets of many-sheeted space-time with Minkowski space with effective metric obtained by summing to Minkowski metric the deviations of the induced metrics of space-time sheets from Minkowski metric. For gauge potentials a similar identification applies. YM-Einstein equations coupled with matter and with non-vanishing cosmological constant are expected on basis of Poincare invariance. One cannot exclude the possibility that the sums of weak gauge potentials from different space-time sheet tend to vanish above weak scale and that well-definedness of em charge at classical level follows from the effective absence of classical weak gauge fields.

1.3.2 *4-dimensional Kähler-Dirac equation and hydrodynamical picture*

In following consideration is restricted to preferred extremals for which one has decomposition to regions characterized by local light-like vector and polarization direction. In this case one has good hopes that the modes can be restricted to 1-D light-like geodesics.

Consider first the solutions of of the induced spinor field in the interior of space-time surface.

1. The local inner products of the modes of the induced spinor fields define conserved currents

$$\begin{aligned} D_\alpha J^\alpha_{mn} &= 0 \ , \\ J^\alpha_{mn} &= \overline{u}_m \hat{\Gamma}^\alpha u_n \ , \\ \hat{\Gamma}^\alpha &= \frac{\partial L_K}{\partial(\partial_\alpha h^k)} \Gamma_k \ . \end{aligned} \tag{1.24}$$

   The conjecture is that the flow parameters of also these currents extend to a global coordinate so that one would have in the completely general case the condition

$$\begin{aligned} J^\alpha_{mn} &= \Phi_{mn} d\Psi_{mn} \ , \\ d*(d\Phi_{mn}) &= 0 \ , \ \nabla\Psi_{mn}\cdot\Phi_{mn} = 0 \ . \end{aligned} \tag{1.25}$$

   The condition $\Phi_{mn} = \Phi$ would mean that the massless modes propagate in parallel manner and along the flow lines of Kähler current. The conservation condition along the flow line implies tht the current component J_{mn} is constant along it. Everything would reduce to initial values at the ends of the space-time sheet boundaries of CD and 3-D Kähler-Dirac equation would reduce everything to initial values at partonic 2-surfaces.

2. One might hope that the conservation of these super currents for all modes is equivalent with the Kähler-Dirac equation. The modes u_n appearing in Ψ in quantized theory would be kind of "square roots" of the basis Φ_{mn} and the challenge would be to deduce the modes from the conservation laws.

3. The quantization of the induced spinor field in 4-D sense would be fixed by those at 3-D space-like ends by the fact that the oscillator operators are carried along the flow lines as such so that the anti-commutator of the induced spinor field at the opposite ends of the flow lines at the light-like boundaries of CD is in principle fixed by the anti-commutations at the either end. The anti-commutations at 3-D surfaces cannot be fixed freely since one has 3-D Chern-Simons flow reducing the anti-commutations to those at partonic 2-surfaces.

The following argument suggests that induced spinor fields are in a suitable gauge simply constant along the flow lines of the Kähler current just as massless spinor modes are constant along the geodesic in the direction of momentum.

1. The Kähler-Dirac gamma matrices are of form $T^\alpha_k \Gamma^k$, $T^\alpha_k = \partial L_K/\partial(\partial_\alpha h^k)$. The H-vectors T^α_k can be expressed as linear combinations of a subset of Killing vector fields j^k_A spanning the tangent space of H. For CP_2 the natural choice are the 4 Lie-algebra generators in the

complement of $U(2)$ sub-algebra. For CD one can used generator time translation and three generators of rotation group SO(3). The completeness of the basis defined by the subset of Killing vector fields gives completeness relation $h_l^k = j^{Ak}j_{Ak}$. This implies $T^{\alpha k} = T^{\alpha k}j_k^A j_A^k = T^{\alpha A}j_A^k$. One can defined gamma matrices Γ_A as $\Gamma_k j_A^k$ to get $T_k^\alpha \Gamma^k = T^{\alpha A}\Gamma_A$.

2. This together with the condition that all isometry currents are proportional to the Kähler current (or if this vanishes to same conserved current- say energy current) satisfying Beltrami flow property implies that one can reduce the Kähler-Dirac equation to an ordinary differential equation along flow lines. The quantities T^{tA} are constant along the flow lines and one obtains

$$T^{tA}j_A D_t\Psi \quad = \quad 0 \ . \tag{1.26}$$

By choosing the gauge suitably the spinors are just constant along flow lines so that the spinor basis reduces by effective 2-dimensionality to a complete spinor basis at partonic 2-surfaces.

2 Does Thermodynamics Have A Representation At The Level Of Space-Time Geometry?

R. Kiehn has proposed what he calls Topological Thermodynamics (TTD) (see `http://tinyurl.com/y9z3jtre`) [B9] as a new formulation of thermodynamics. The basic vision is that thermodynamical equations could be translated to differential geometric statements using the notions of differential forms and Pfaffian system (see `http://tinyurl.com/6ks8jkc`) [A1]. That TTD differs from TGD by a single letter is not enough to ask whether some relationship between them might exist. Quantum TGD can however in a well-defined sense be regarded as a square root of thermodynamics in zero energy ontology (ZEO) and this leads leads to ask seriously whether TTD might help to understand TGD at deeper level. The thermodynamical interpretation of space-time dynamics would obviously generalize black hole thermodynamics to TGD framework and already earlier some concrete proposals have been made in this direction.

One can raise several questions. Could the preferred extremals of Kähler action code for the square root of thermodynamics? Could induced Kähler gauge potential and Kähler form (essentially Maxwell field) have formal thermodynamic interpretation? The vacuum degeneracy of Kähler action implies 4-D spin glass degeneracy and strongly suggests the failure of strict determinism for the dynamics of Kähler action for non-vacuum extremals too. Could thermodynamical irreversibility and preferred arrow of time allow to characterize the notion of preferred extremal more sharply?

It indeed turns out that one can translate Kiehn's notions to TGD framework rather straightforwardly.

1. Kiehn's work 1- form corresponds to induced Kähler gauge potential implying that the vanishing of instanton density for Kähler form becomes a criterion of reversibility and irreversibility is localized on the (4-D) "lines" of generalized Feyman diagrams, which correspond to space-like signature of the induced metric. The localization of heat production to generalized Feynman diagrams conforms nicely with the kinetic equations of thermodynamics based on reaction rates deduced from quantum mechanics. It also conforms with Kiehn's vision that dissipation involves topology change.

2. Heat produced in a given generalized Feynman diagram is just the integral of instanton density and the condition that the arrow of geometric time has definite sign classically fixes the sign of produced heat to be positive. In this picture the preferred extremals of Kähler action would allow a trinity of interpretations as non-linear Maxwellian dynamics, thermodynamics, and integrable hydrodynamics.

3. The 4-D spin glass degeneracy of TGD breaking of ergodicity suggests that the notion of global thermal equilibrium is too naive. The hierarchies of Planck constants and of p-adic length scales suggests a hierarchical structure based on CDs withing CDs at imbedding space

level and space-time sheets topologically condensed at larger space-time sheets at space-time level. The arrow of geometric time for quantum states could vary for sub-CDs and would have thermodynamical space-time correlates realized in terms of distributions of arrows of geometric time for sub-CDs, sub-sub-CDs, etc...

The hydrodynamical character of classical field equations of TGD means that field equations reduce to local conservation laws for isometry currents and Kähler gauge current. This requires the extension of Kiehn's formalism to include besides forms and exterior derivative also induced metric, index raising operation transforming 1-forms to vector fields, duality operation transforming k- forms to n-k forms, and divergence which vanishes for conserved currents.

2.1 Motivations And Background

It is good to begin by discussing the motivations for the geometrization of thermodynamics and by introducing the existing mathematical framework identifying space-time surfaces as preferred extremals of Kähler action.

2.1.1 ZEO and the need for the space-time correlates for square root of thermodynamics

Quantum classical correspondence is basic guiding principle of quantum TGD. In ZEO TGD can be regarded as a complex square root of thermodynamics so that the thermodynamics should have correlates at the level of the geometry of space-time.

1. Zero energy states consist of pairs of positive and negative energy states assignable to opposite boundaries of a causal diamond (CD). There is entire hierarchy of CDs characterized by their scale coming as an integer multiple of a basic scale (also their Poincare transforms are allowed).

2. In ZEO zero energy states are automatically time-irreversible in the sense that either end of the causal diamond (CD) corresponds to a state consisting of single particle states with well-defined quantum numbers. In other words, this end of CD carries a prepared state. The other end corresponds to a superposition of states which can have even different particle numbers: this is the case in particle physics experiment typically. State function reduction reduces the second end of CD to a prepared state. This process repeats itself. This suggests that the arrow of time or rather, its geometric counterpart which we experience, alternates. This need not however be the case if quantum classical correspondence holds true.

3. To illustrate what I have in mind consider a path towel, which has been been folded forth and back. Assume that the direction in which folding is carried is time direction. Suppose that the inhabitant of bath towel Universe is like the habitant of the famous Flatland and therefore not able to detect the folding of the towel. If the classical dynamics of towel is time irreversible (time corresponds to the direction in which the folding takes place), the inhabitant sees ever lasting irreversible time evolution with single arrow of geometric time identified as time coordinate for the towel: no changes in the arrow of geometric time. If the inhabitant is able to make measurements about 3-D space the situation he or she might be able to see that his time evolution takes place forth and back with respect to the time coordinate of higher-dimensional imbedding space.

4. One might understand the arrow of time - albeit differently as in normal view about the situation - if classical time evolution for the preferred extremals of Kähler action defines a geometric correlate for quantum irreversibility of zero energy states. There are of course other space-time sheets and other CDs present an it might be possible to detect the alternation of the arrow of geometric time at imbedding space level by making measurements giving information about their geometric arrows of time [K1].

By quantum classical correspondence one expects that the geometric arrow of time - irreversibility - for zero energy states should have classical counterparts at the level of the dynamics of preferred extremals of Kähler action. What could be this counterpart? Thermodynamical evolution by quantum jumps does not obey ordinary variational principle that would make it deterministic: Negentropy Maximization Principle (NMP) [K5] for state function reductions of system is analogous to Second Law for an ensemble of copies of system and actually implies it. Could one mimic irreversibility by single classical evolution defined by a preferred extremal? Note that the dynamics of preferred extremals is not actually strictly deterministic in the ordinary sense of the word: the reason is the enormous vacuum degeneracy implying 4-D spin glass degeneracy. This makes it possible to mimic not only quantum states but also sequences of quantum jumps by piece-wise deterministic evolution.

2.1.2 Preferred extremals of Kähler action

In Quantum TGD the basic arena of quantum dynamics is "world of classical worlds" (WCW [K7]), see `http://tinyurl.com/ycqyk49f`). Purely classical spinor fields in this infinite-dimensional space define quantum states of the Universe. General Coordinate Invariance (GCI) implies that classical worlds can be regarded as either 3-surfaces or 4-D space-time surfaces analogous to Bohr orbits. Strong form of GCI implies in ZEO strong form of holography in the sense that the points of WCW effectively correspond to collections of partonic 2-surfaces belonging to both ends of causal diamonds (CDs) plus their 4-D tangent space-time data.

Kähler geometry reduces to the notion of Kähler function [K4] and by quantum classical correspondence a good guess is that Kähler function corresponds to so called Kähler action for Euclidian space-time regions. Minkowskian space-time regions give a purely imaginary to Kähler action (square root of metric determinant is imaginary) and this contribution plays the role of Morse function for WCW . Stationary phase approximation imples that in first the approximation the extremals of the Kähler *function* (to be distinguished from preferred extremals of Kähler *action* !) select one particular 3-surface and corresponding classical space-time surface (Bohr orbit) as that defining "classical physics".

GCI implies holography and holography suggests that action reduces to 3-D terms. This is true if one has $j^\mu A_\mu = 0$ in the interior of space-time. If one assumes so called weak form of electric-magnetic duality [K8] at the real and effective boundaries of space-time surface (3-D surfaces at the ends of CDs and the light-like 3-surfaces at which the signature of induced 4-metric changes so that 4-metric is degenerate), one obtains a reduction of Kähler action to Chern-Simons terms at the boundaries. TGD reduces to almost topological QFT. "Almost" means that the induced metric does not disappear completely from the theory since it appears in the conditions expressing weak form of electric magnetic duality and in the condition $j^\mu A_\mu = 0$.

The strong form of holography implies effective 2-dimensionality and this suggests the reduction of Chern-Simons terms to 2-dimensional areas of string world sheets and possible of partonic 2-surfaces. This would mean almost reduction to string theory like theory with string tension becoming a dynamic quantity.

Under additional rather general conditions the contributions from Minkowskian and Euclidian regions of space-time surface are apart from the value of coefficient identical at light-like 3-surfaces. At space-like 3-surfaces at the ends of space-time surface they need not be identical.

Quantum classical correspondence suggests that space-time surfaces provide a representation for the square root of thermodynamics and therefore also for thermodynamics. In general relativity black hole thermodynamics suggests the same. This idea is not new in TGD framework. For instance, Hawking-Bekenstein formula (see `http://tinyurl.com/3ytg9m6`) for blackbody entropy [B1] allows a p-adic generalization (see `http://tinyurl.com/y9e22rr6`) in terms of area of partonic 2-surfaces [K6]. The challenge is to deduce precise form of this correspondence and here Kiehn's topological thermodynamics might help in this task.

2.2 Kiehn's Topological Thermodynamics (TTD)

The basic in the work of Kiehn is that thermodynamics allows a topological formulation in terms of differential geometry.

1. Kiehn introduces also the notions of Pfaff system (see `http://tinyurl.com/6ks8jkc`) and Pfaff dimension as the number of non-vanishing forms in the sequence for given 1-form such as W or Q: W, dW, $W \wedge dW$, $dW \wedge dW$. Pfaff dimension $D \leq 4$ tells that one can describe

W as sum $W = \sum W_k dx^k$ of gradients of D variables. $D = 4$ corresponds to open system, $D = 3$ to a closed system and $W \wedge dW \neq 0$ defines what can be regarded as a chirality. For $D = 2$ chirality vanishes no spontaneous parity breaking.

2. Kiehn's king idea that Pfaffian systems provide a universal description of thermodynamical reversibility. Kiehn introduces heat 1- form Q. System is thermodynamically reversible if Q is integrable. In other words, the condition $Q \wedge dQ = 0$ holds true which implies that one can write $Q = TdS$: Q allows an integrable factor T and is expressible in terms of the gradient of entropy. $Q = TdS$ condition implies that Q correspond to a global flow defined by the coordinate lines of S. This in turn implies that it is possible define phase factors depending on S along the flow line: this relates to macroscopic quantum coherence for macroscopic quantum phases.

3. The first law expressing the work 1-form W as $W = Q - dU = TdS - dU$ for reversible processes. This gives $dW \wedge dW = 0$. The condition $dW \wedge dW \neq 0$ therefore characterizes irreversible processes.

4. Symplectic transformations are natural in Kiehn's framework but not absolutely essential.

Reader is encouraged to get familiar with Kiehn's examples [B9] about the description of various simple thermodynamical systems in this conceptual framework. Kiehn has also worked with the differential topology of electrodynamics and discussed concepts like integrable flows known as Beltrami flows. These flows generalized to TGD framework and are in key role in the construction of proposals for preferred extremals of Kähler action: the basic idea would be that various conserved isometry currents define Beltrami flows so that theit flow lines can be associated with coordinate lines.

2.3 Attempt To Identify TTD In TGD Framework

Let us now try to identify TTD or its complex square root in TGD framework.

2.3.1 The role of symplectic transformations

Symplectic transformations are important in Kiehn's approach although they are not a necessary ingredient of it and actually impossible to realize in Minkowski space-time.

1. Symplectic symmetries of WCW induced by synmplectic symmetries of CP_2 and light-like boundary of CD are important also in TGD framework [K2] and define the isometries of WCW . As a matter fact, symplectic group parameterizes the quantum fluctuating degrees of freedom and zero modes defining classical variables are symplectic invariants. One cannot assign to entire space-time surfaces symplectic structure although this is possible for partonic 2-surfaces.

2. The symplectic transformations of CP_2 act on the Kähler gauge potential as $U(1)$ gauge transformations formally but modify the shape of the space-time surface. These symplectic transformations are symmetries of Kähler action only in the vacuum sector which as such does not belong to WCW whereas small deformations of vacua belong. Therefore genuine gauge symmetries are not in question. One can of course formally assign to Kähler gauge potential a separate $U(1)$ gauge invariance.

3. Vacuum extremals with at most 2-D CP_2 projection (Lagrangian sub-manifold) form an infinite-dimensional space. Both M^4 diffeomorphisms and symplectic transformations of CP_2 produce new vacuum extremals, whose small deformations are expected to correspond preferred extremals. This gives rise to 4-D spin glass degeneracy (see `http://tinyurl.com/y9e22rr6`) [K6] to be distinguished from 4-D gauge degeneracy.

2.3.2 Identification of basic 1-forms of TTD in TGD framework

Consider nex the identification of the basic variables which are forms of various degrees in TTD.

1. Kähler gauge potential is analogous to work 1-form W. In classical electrodynamics vector potential indeed has this interpretation. $dW \wedge dW$ is replaced with $J \wedge J$ defining instanton density ($E_K \cdot B_K$ in physicist's notation) for Kähler form and its non-vanishing - or equivalently 4-dimensionality of CP_2 projection of space-time surface - would be the signature of irreversibility. $dJ = 0$ holds true only locally and one can have magnetic monopoles since CP_2 has non-trivial homology. Therefore the non-trivial topology of CP_2 implying that the counterpart of W is not globablly defined, brings in non-trivial new element to Kiehn's theory.

2. Chirality $C - S = A \wedge J$ is essentially Chern-Simons 3-form and in ordinary QFT non-vanishing of $C - S$- if present in action - means parity breaking in ordinary quantum field theories. Now one must be very cautious since parity is a symmetry of the imbedding space rather than that of space-time sheet.

3. Pfaff dimension equals to the dimension of CP_2 projection and has been used to classify existing preferred extremals. I have called the extremals with 4-D CP_2 projection chaotic and so called CP_2 vacuum extremals with 4-D CP_2 projection correspond to such extremals. Massless extremals or topological light rays correspond to $D = 2$ as do also cosmic strings. In Euclidian regions preferred extremals with $D = 4$ are are possible but not in Minkowskian regions if one accepts effective 3-dimensionality. Here one must keep mind open.

Irreversibility identified as a non-vanishing of the instanton density $J \wedge J$ has a purely geometrical and topological description in TGD Universe if one accepts effective 3-dimensionality.

1. The effective 3-dimensionality for space-time sheets (holography implied by general coordinate invariance) implies that Kähler action reduces to Chern-Simons terms so that the Pfaff dimension is at most $D = 3$ for Minkowskian regions of space-time surface so that they are are thermodynamically reversible.

2. For Euclidian regions (say deformations of CP_2 type vacuum extremals) representing orbits of elementary particles and lines of generalized Feynman diagrams $D = 4$ is possible. Therefore Euclidian space-like regions of space-time would be solely responsible for the irreversibility. This is quite strong conclusion but conforms with the standard quantum view about thermodynamics according to which various particle reaction rates deduced from quantum theory appear in kinetic equations giving rise to irreversible dynamics at the level of ensembles. The presence of Morse function coming from Minkowskian regions is natural since square root of thermodynamics is in question. Morse function is analogous to the action in QFTs whereas Kähler function is analogous to Hamiltonian in thermodynamics. Also this conforms with the square root of TTD interpretation.

2.3.3 Instanton current, instanton density, and irreversibility

Classical TGD has the structure of hydrodynamics in the sense that field equations are conservation laws for isometry currents and Kähler current. These are vector fields although induced metric allows to transform them to forms. This aspect should be visible also in thermodynamic interpretation and forces to add to the Kiehn's formulation involving only forms and exterior derivative also induced metric transforming 1-forms to vector fields, the duality mapping 4-k forms and k-forms to each other, and divergence operation.

It was already found that irreversibility and dissipation corresponds locally to a non-vanishing instanton density $J \wedge J$. This form can be regarded as exterior derivative of Chern-Simons 3-form or equivalently as divergence of instanton current.

1. The dual of C-S 3-form given by $*(A \wedge J)$ defines what I have called instanton current. This current is not conserved in general and the interpretation as a heat current would be natural. The exterior derivative of C-S gives instanton density $J \wedge J$. Equivalently, the divergence

of instanton currengives the dual of $J \wedge J$ and the integral of instanton density gives the analog of instanton number analogous to the heat generated in a given space-time volume. Note that in Minkowskian regions one can multiply instanton current with a function of CP_2 coordinates without losing closedness property so that infinite number similar conserved currents is possible.

The heat 3-form is expressible in terms of Chern-Simons 3-form and for preferred extremals it would be proportional to the weight sum of Kähler actions from Minkowskian and Euclidian regions (coeffecients are purely imaginary and real in these two regions). Instead of single real quantity one would have complex quantity characterizing irreversibility. Complexity would conform with the idea that quantum TGD is complex square root of thermodynamics.

2. The integral of heat 3-form over effective boundaries associated with a given space-time region define the net heat flow from that region. Only the regions defining the lines of generalized Feynman diagrams give rise to non-vanishing heat fluxes. Second law states that one has $\Delta Q \geq 0$. Generalized second law means at the level of quantum classical correspondence would mean that depending on the arrow of geometric time for zero energy state ΔQ is defined as difference between upper and lower or lower and upper boundaries of CD. This condition applied to CD and sub-CD: s would generalize the conditions familiar from hydrodynamics (stating for instance that for shock waves the branch of bifurcation for which the entropy increases is selected). Note that the field equations of TGD are hydrodynamical in the sense that they express conservation of various isometry currents. The naive picture about irreversibility is that classical dynamics generates CP_2 type vacuum extremals so that the number of outgoing lines of generalized Feynman diagram is higher than that of incoming ones. Therefore that the number of space-like 3-surfaces giving rise to Chern-Simons contribution is larger at the end of CD corresponding to the final (negative energy) state.

3. A more precise characterization of the irreversible states involves several non-trivial questions.

   (a) By the failure of strict classical determinism the condition that for a given CD the number outgoing lines is not smaller than incoming lines need not provide a unique manner to fix the preferred extremal when partonic 2-surfaces at the ends are fixed. Could the arrow of geometric time depend on sub-CD as the model for living matter suggests (recall also phase conjugate light rays)?

   In ordinary quantum mechanical approach to kinetic equations also the reactions, which decrease entropy are allowed but their weight is smaller in thermal equilibrium. Could this fact be described as a probability distribution for the arrow of time associated for the sub-CDs, sub-sub-CDs, etc... ? Space-time correlates for quantal thermodynamics would be probability distributions for space-time sheets and hierarchy of sub-CDs.

   (b) 4-D spin glass degeneracy suggests breaking of ergodic hypothesis: could this mean that one does not have thermodynamical equilibrium but very large number of spin glass states caused by the frustation for which induced Kähler form provides a representation? Could these states correspond to a varying arrow of geometric time for sub-CDs? Or could different deformed vacuum extremals correspond to different space-time sheets in thermal equilibrium with different thermal parameters.

2.3.4 Also Kähler current and isometry currents are needed

The conservation Kähler current and of isometry currents imply the hydrodynamical character of TGD.

1. The conserved Kähler current j_K is defined as 3-form $j_K = *(d * J)$, where $d * J$ is closed 3-form and defines the counterpart of d^*dW. Field equations for preferred extremals require $*j_K \wedge A = 0$ satsfied if one Kähler current is proportional to instanton current: $*j_K \propto A \wedge J$. As a consequence Kähler action reduces to 3-dimensional Chern-Simons terms (classical holography) and Minkowskian space-time regions have at most 3-D CP_2 projection (Pfaff dimension $D \leq 3$) so that one has $J \wedge J = 0$ and reversibility. This condition holds true for

preferred extremals representing macroscopically the propagation of massless quanta but not Euclidian regions representing quanta themselves and identifiable as basic building bricks of wormhole contacts between Minkowskian space-time sheets.

2. A more general proposal is that all conserved currents transformed to 1-forms using the induced metric (classical gravitation comes into play!) are integrable: in other words, on has $j \wedge dj = 0$ for both isometry currents and Kähler current. This would mean that they are analogous to heat 1-forms in the reversible case and therefore have a representation analogous to $Q = TdS$, $W = PdV$, μdN and the coordinate along flowline defines the analog of S, V, or N (note however that dS, dV, dN would more naturally correspond to 3-forms than 1-forms, see below) A stronger form corresponds to the analog of hydrodynamics for one particle species: all one-forms are proportional (by scalar function) to single 1-form which is $A \wedge J$ (all quantum number flows are parallel to each other).

2.3.5 Questions

There are several questions to be answered.

1. In Darboux coordinates in which one has $A = P_1 dQ^1 + P_2 dQ_2$. The identification of A as counterpart for $W = PdV - \mu dN$ comes first in mind. For thermodynamical equilibria one would have $TdS = dU + W$ translating to $TdS = dU + A$ so that Q for reversible processes would be apart from $U(1)$ gauge transformation equal to the Kähler gauge potential. Symplectic transformations of CP_2 generate $U(1)$ gauge transformations and dU might have interpretation in terms of energy flow induced by this kind of transformation. Recall however that symplectic transformations are not symmetries of space-time surfaces but only of the WCW metric and act on partonic 2-surfaces and their tangent space data as such.

2. Does the conserved Kähler current j_K have any thermodynamical interpretation? Clearly the counterparts of conserved (and also non-conserved quantities) in Kiehn's formulation would be 3-forms with vanishing curl $d(*j_K) = 0$ in conserved case. Therefore it seems impossible to reduce them to 1-forms unless one introduces divergence besides exterior derivative as a basic differential operation.

   The hypothesis that the flow lines of these 1-forms associated with j_K vector field are integrable implies that they are gradients apart from the presence of integrating factor. Reduction to a gradient ($j = dU$) means that U satisfies massless d'Alembert equation $d * dU = 0$. Note that local polarization and light-like momentum are gradients of scalar functions which satisfy massless d'Alembert equation for the Minkowskian space-time regions representing propagating of massless quanta.

3. In genuinely 3-dimensional context S, V, N are integrals of 3-forms over 3-surfaces for some current defining 3-form. This is in conflict with Kiehn's description where they are 0-forms. One can imagine three cures and first two ones look

   (a) The integrability of the flows allows to see them as superposition of independent 1-dimensional flows. This picture would make it natural to regard the TGD counterparts of S, V, N as 0-forms rather than 2-forms. This would also allow to deduce $J \wedge J = 0$ as a reversibility condition using Kiehn's argument.

   (b) Unless one requires integrable flows, one must consider the replacement of $Q = TdS$ *resp.* $W = PdV$ *resp.* μdN $Q = TdS$ *resp.* $W = PdV$ *resp.* μdN where W, Q, dS,dV, anddN with 3-forms. So that S, V, N would be 2-forms and the 3-integrals of dS, dV, dN over 3-surfaces would reduce to integrals over partonic 2-surfaces, which is of course highly non-trivial but physically natural implication of the effective 2-dimensionality. First law should now read as $*W = T*dS - *dU$ and would give $d*W = dT \wedge *dS + Td*dS + d*dU$. If S and U as 2-forms satisfy massless d'Alembert equation, one obtains $d*W = dT \wedge *dS$ giving $d * W \wedge d * W = 0$ as the reversibility condition. If one replaces $W \leftrightarrow A$ correspondence with $*W \leftrightarrow A$ correspondence, one obtains the vanishing of instanton density as a condition for reversibility. For the preferred extremals having interpretation as massless modes the massless d'Alembert equations are satisfied and it might that this option makes sense and be equivalent with the first option.

(c) In accordance with the idea that finite measurement resolution is realized at the level of Kähler-Dirac equation, its solutions at light-like 3-surfaces reduces to solutions restricted to lines connecting partonic 2-surfaces. Could one regard W, Q, dS, dV, and dN as singular one-forms restricted to these lines? The vanishing of instanton density would be obtained as a condition for reversibility only at the braid strands, and one could keep the original view of Kiehn. Note however that the instanton density could be non-vanishing elsewhere unless one develops a separate argument for its vanishing. For instance, the condition that isometries of imbedding space say translations produce braid ends points for which instanton density also vanishes for the reversible situation might be enough.

To sum up, it seems that TTD allows to develop considerable insights about how classical space-time surfaces could code for classical thermodynamics. An essential ingredient seems to be the reduction of the hydrodynamical flows for isometry currents to what might be called perfect flows decomposing to 1-dimensional flows with conservation laws holding true for individual flow lines. An interesting challenge is to find expressions for total heat in terms of temperature and entropy. Blackhole-elementary particle analogy suggest the reduction as well as effective 2-dimensionality suggest the reduction of the integrals of Chern-Simons terms defining total heat flux to two 2-D volume integrals over string world sheets and/or partonic 2-surfaces and this would be quite near to Hawking-Bekenstein formula.

3 Robert Kiehn's Ideas About Falaco Solitons And Generation Of Turbulent Wake From TGD Perspective

I have been reading two highly interesting articles by Robert Kiehn. The first article has the title "Hydrodynamics wakes and minimal surfaces with fractal boundaries" (see `http://tinyurl.com/y8emhmt7`) [B7]. Second article is titled "Instability patterns, wakes and topological limit sets" (see `http://tinyurl.com/y8v4e3xr`) [B8]. There are very many contacts on TGD inspired vision and its open interpretational problems.

The notion of Falaco soliton has surprisingly close resemblance with Kähler magnetic flux tubes defining fundamental structures in TGD Universe. Fermionic strings are also fundamental structures of TGD accompanying magnetic flux tubes and this supports the vision that these string like objects could allow reduction of various condensed matter phenomena such as sound waves -usually regarded as emergent phenomena allowing only highly phenomenological description - to the fundamental microscopic level in TGD framework. This can be seen as the basic outcome of this article.

Kiehn proposed a new description for the generation of various instability patterns of hydrodynamics flows (Kelvin-Helmholtz and Rayleigh-Taylor instabilities) in terms of hyperbolic dynamics so that a connection with wave phenomena like interference and diffraction would emerge. The role of characteristic surfaces as surfaces of tangential and also normal discontinuities is central for the approach. In TGD framework the characteristic surfaces have as analogs light-like wormhole throats at which the signature of the induced 4-metric changes and these surfaces indeed define boundaries of two phases and of material objects in general. This inspires a more detailed comparison of Kiehn's approach with TGD.

3.1 Falaco Solitons And TGD

In the first article [B7] Kiehn tells about his basic motivations. The first motivating observations were related to so called Falaco solitons. Second observation was related to the so called mushroom pattern associated with RayleighTaylor instability (see `http://tinyurl.com/brypvgm`) or fingering instability [B2], which appears in very many contexts, the most familiar being perhaps the mushroom shaped cloud created by a nuclear explosion. The idea was that both structures whose stability is not easy to understand in standard hydrodynamics, could have topological description.

Falaco solitons are very fascinating objects. Kiehn describes in detail the formation and properties in [B7]: anyone possessing swimming pool can repeat these elegant and simple experiments. The vortex string connecting the end singularities - dimpled indentations at the surface of water - is the basic notion. Kiehn asks whether there migh be a deeper connection with a model of mesons in which strings connecting quark and antiquark appear. The formation of spiral structures around the end gaps in the initial formative states of Falaco soliton is emphasized and compared to the structure of spiral galaxies. The suggestion is that galaxies could appear as pairs connected by strings.

Kähler magnetic tubes carrying monopole flux are central in TGD and have several interesting resemblances with Falaco solutions.

1. In TGD framework so called cosmic strings fundamental primordial objects (see `http://tinyurl.com/yblk638z`). They have 2-D Minkowski space projection and 2-D CP_2 projection so that one can say that there is no space-time in ordinary sense present during the primordial phase. During cosmic evolution their time= constant M^4 projection gradually thickens from ideal string to a magnetic flux tube. Among other things this explains the presence of magnetic fields in all cosmic scale not easy to understand in standard view. The decay of cosmic strings generates visible and dark matter much in the same manner as the decay of inflaton field does in inflationary scenario. One however avoids the many problems of inflationary scenario.

   Cosmic strings would contain ordinary matter and dark matter around them like necklace contains pearls along it. Cosmic strings carry Kähler magnetic monopole flux which stabilizes them. The magnetic field energy explains dark energy. Magnetic tension explains the negative "pressure" explaining accelerated expansion. The linear distribution of field energy along cosmic strings gives rise to logarithmic gravitational potential, which explains the constant velocity spectrum of distant stars around galaxy and therefore galactic dark matter.

2. Magnetic flux tubes form a fractal structure and the notion of Falaco soliton has also an analogy in TGD based description of elementary particles. In TGD framework the ends caps of vortices correspond to pairs of wormhole throats connected by short wormhole contact and there is a magnetic flux tube carrying monopole flux at both space-time sheets.

   So called Kähler-Dirac equation assigns with this flux tube 1-D closed string and to it string world sheets, which might be 2-D minimal surface of space-time surface [K8]. Rather surprisingly, string model in 4-D space-time emerges naturally in TGD framework and has also very special properties due to the knotting of strings as 1-knots and knotting of string world sheets as 2-knots. Braiding and linking of strings is also involved and make dimension D=4 for space-time completely unique.

   Both elementary particles and hadron like state are describable in terms of these string like objects. Wormhole throats are the basic building brick of particles which are in the simplest situation two-sheeted structure with wormhole contact structures connecting the sheets and giving rise to one or more closed flux tubes accompanied by closed strings.

3.2 Stringy Description Of Condensed Matter Physics And Chemistry?

What is important that magnetic flux tubes and associated string world sheets can also connect wormhole throats associated with different elementary particles in the sense that their boundaries are along light-like wormhole throats assignable to different elementary particles. These string worlds sheets therefore mediate interactions between elementary particles.

1. What these interactions are? Could *string world sheets could provide a microscopic first principle description of condensed matter phenomena* - in particular of sound waves and various waves analogs of sound waves usually regarded as emergent phenomena requiring phenomenological models of condensed matter?

   The hypothesis that this is the case would allow to test basic assumptions of quantum TGD at the level of condensed matter physics. String model in 4-D space-time could describe concrete experimental everyday reality rather than esoteric Planck length scale physics! The phenomena of condensed matter physics often thought to be high level emergent phenomena would have first principle microscopic description at the level of space-time geometry.

2. The idea about stringy reductionism extends also to chemistry. One of the poorly understanding basic notions of molecular chemistry is the formation of valence bond as pairing of

two valence electrons belonging to different atoms. Could this pairing correspond to a formation of a closed Kähler magnetic flux tube with two wormhole contacts carrying quantum numbers of electron? Could also Cooper pairs be regarded as this kind of structure with long connecting pair of flux tubes between electron carrying wormhole contacts as has been suggested already earlier?

3. The proposal indeed is that TGD inspired biochemistry and neuroscience indeed has magnetic flux tubes and flux sheets as a key element. For instance, the notion of magnetic body plays a key role in TGD inspired view about EEG and magnetic flux tubes represent braid strands in the model for DNA-cell membrane system as topological quantum computer [K3].

One can argue that this is not a totally new idea: basically one particular variant of holography[1] is in question and follows in TGD framework from general coordinate invariance alone: the geometry of world of classical worlds must assign to a given 3-surface a unique space-time surface.

1. The fashionable manner to realize holography is by replacing 4-D space-time with 10-D one. String world sheets in 10-D space-time $AdS_5 - S_5$ connecting the points of 4+5-D boundary of $AdS_5 - S_5$ are hoped to provide a dual description of even condensed matter phenomena in the case that the system is described by a theory enjoying conformal invariance in 4-D sense.

2. In TGD framework holography is much more concrete: 3-D light-like 3-surfaces (giving rise to generalized conformal invariance by their metric 2-dimensionality) are enough. One has actually a strong form of holography stating that 2-D partonic 2-surfaces plus their 4-D tangent space data are enough. Partonic 2-surfaces define the ends of light-like 3-surfaces at the ends of space-time surface at the light-like 7-D boundaries of causal diamonds. 10-D space is replaced with the familiar 4-D space-time and 4+5-D boundary with end 2-D ends of 3-D light-like wormhole orbits (plus 4-D tangent space data). These partonic 2-surfaces are highly analogous to the 2-D sections of your characteristic surfaces.

Consider now how sound waves as and various oscillations of this kind could be understood in terms of string word sheets. String world sheets have both geometric and fermionic degrees of freedom.

1. A good first guess is that string world sheet is minimal surface in space-time - this does not mean minimal surface property in imbedding space and the non-vanishing second fundamental form- in particular its CP_2 part should have physical meaning - maybe the parameter that would be called Higgs vacuum expectation in QFT limit of TGD could relate to it.

2. Another possibility that I have proposed is that a minimal surface of imbedding space (not the minimal surface is geometric analog for a solution of massless wave equation) but in the effective metric defined by the anti-commutators of modified gamma matrices defined by the canonical momentum densities of Kähler action is in question: in this case one might even dream about the possibility that the analog of light-velocity defined by the effective metric has interpretation as sound velocity.

For string world sheets as minimal surfaces of X^4 (the first option) oscillations would propagate with light-velocity but as one adds massive particle momenta at wormhole throats defining their ends the situation changes due to the additional inertia making impossible propagation with light-velocity. Consideration of the situation for ordinary non-relativistic condensed matter string with masses at ends as a simple example, the velocity of propagation is in the first naive estimate just square root of the ratio of the magnetic energy of string portion to its total energy which also concludes the mass at its ends. Kähler magnetic energy is given by string tension which has a spectrum determined by p-adic length scale hypothesis so that one ends up with a rough quantitative picture and coil understand the dependence of the sound velocity on temperature.

[1] The equivalent of holography emerged from the construction of the Kähler geometry of "world of classical worlds" as an implication of general coordinate invariance around 1990, about five years before it was introduced by t'Hooft and Susskind.

In TGD framework massless quanta moving in different directions correspond to different space-time sheets: linear superposition for fields is replaced with a set theoretic union and effects superpose instead of fields. This would hold true also for sound waves which would always be restricted at stringy world sheets: superposition can make sense only for wave moving in exactly the same direction. This of course conforms with the properties of phonons so that Bohr orbitology would be realized for sound waves and ordinary description of sound waves would be only an approximation. The fundamental difference between light and sound defining fundamental qualia would be the dimension of the quanta as geometric structures.

3.3 New Manner To Understand The Generation Of Turbulent Wake

Kiehn proposes a new manner to understand the generation of turbulent wake (see `http://tinyurl.com/y8v4e3xr`) [B8]. The dynamics generating it would be that of hyperbolic wave equation rather than diffusive parabolic or elliptic dynamics. The decay of the turbulence would however obey the diffusive parabolic dynamics. Therefore sound velocity and supersonic velocities would be involved with the generation of the turbulence.

Kiehn considers Landau's nonlinear model for a scalar potential of velocity in the case of 2-D compressible isentropic fluid as an example. The wave equation is given by

$$(c^2-\Phi_x^2)\Phi_{xx}+(c^2-\Phi_y^2)\Phi_{yy}-2\Phi_x\Phi_u\Phi_{xy} = 0 \ . \tag{3.1}$$

Here c denotes sound velocity and velocity is given by $v=\nabla\Phi$. 3-D generalization is obvious. This partial differential equation for the velocity potential is quasi-linear equation of the form

$$A\Phi_{\eta\eta}+2B\Phi_{\eta\xi}+C\Phi_{\xi\xi} = 0 \ . \tag{3.2}$$

The characteristic surfaces contain imbedded curves which are given by solutions to ordinary differentia equations

$$\frac{d\eta}{d\xi} = \frac{B\pm(B^2AC)^{1/2}}{C} \ . \tag{3.3}$$

Real solutions are possible when the argument of the square root is positive. This is true when the local velocity exceeds the local characteristic speed c. These characteristic lines combine to form characteristic surfaces.

Velocity field would be compressible ($\nabla\cdot v\neq 0$) but irrotational ($\nabla\times v=0$) in this approach whereas in standard approach velocity field would be incompressible ($\nabla\cdot v=0$) but irrotational ($\nabla\times v\neq 0$). There would be two phases in which these two different options would be realized and at the boundary the dynamics would be both in-compressible and irrotational and these boundaries would correspond to characteristic surfaces which are minimal surfaces which evolve with time somehow. The presence of scalar function satisfying Laplace equation ($\nabla^2\Phi=0$) would serve as a signature of this.

The emergence of this hyperbolic dynamics would explain the sharpness and long-lived character of the singular structures. Kiehn also proposes that the formation of wake could have analogies with diffraction and interference - basic aspects of wave motion. This picture does not conform with standard view which assumes diffusive parabolic or elliptic dynamics as the origin of the wake turbulence.

3.3.1 Characteristic surfaces and light-like wormhole throat orbits

Characteristic surface is key notion in Kiehn's approach and he suggests that the creation of wakes relies on hyperbolic dynamics (see `http://tinyurl.com/y8v4e3xr`) in restricted regions [B8]. If I have understood correctly, the boundaries of vortices created in the process could be seen as this kind of characteristic surfaces: some physical quantities would have tangential discontinuities at them since a boundary between different phases (fluid and air) would be in question.

Another situation corresponds to a shock wave in which case there is a flow of matter through the characteristic surface. Also boundary patterns associated with Kelvin-Helmholtz instability (see http://tinyurl.com/p92zx) (formation of waves due to wind and their breaking) and Rayleigh-Taylor instability (see http://tinyurl.com/brypvgm) (the formation of mushroom like fingers of heavier substance resting above lighter one).

The proposal of Kiehn is that the characteristic minimal surfaces have the following general form:

$$\begin{array}{ll} u=\frac{d\eta}{ds}=A(\rho)\times sin(Q(s)) \ , & v=\frac{d\eta}{ds}=-A(\rho)\times cos(Q(s)) \ , \\ w=F(u,v)=Q(u/v=s) & Q(s)=arctan(s) \ . \end{array} \tag{3.4}$$

If $F(u,v)$ satisfies the equation

$$(1+F_v^2)F_{uu}+(1+F_u^2)F_{vv}-2F_uF_vF_{uv} \ = \ 0 \ . \tag{3.5}$$

This expresses the vanishing of the trace of the second fundamental form, actually the component corresponding to the coordinate w. The minimal surface in question is known as right helicoid.

In TGD framework light-like 3-surfaces defined by wormhole throats are the counterparts of characteristic surfaces.

1. By their light-likeness the light-like wormhole throats are analogous to characteristic surfaces (In TGD context light-velocity of course replace local sound velocity). Since the signature of the metric changes at wormhole throats, the 4-D tangent space reduces to 3-D in metric sense at them so that they indeed are singular in a unique sense. Gravitational effects imply that they need not look expanding in Minkowski coordinates. The light-velocity in the induced metric is in general smaller than maximal signal velocity in Minkowski space and can be arbitrarily small.

2. In TGD framework light-like 3-surfaces would be naturally associated with phase boundaries defining boundaries of physical objects. They would be light-like metrically degenerate 3-surfaces in space-time along which the space-time sheet assignable to fluid flow meets the space-time sheet assignable to say air. The generation of wake turbulence would in TGD framework mean the decay of a large 3-surface representing a laminar flow to sheet of separate cylindrical 3-surfaces representing vortex sheet. Also the amalgamation of vortices can be considered as a reverse process.

3. Interesting question related to the time evolution of these 2-D boundaries. In TGD framework it should give rise to 3-D light-like surface. The simulations for the evolution of Kelvin-Helmholtz instability and Rayleigh-Taylor mushroom pattern (see http://tinyurl.com/brypvgm) in Wikipedia and its seems that at the initial stages there is period of growth bringing in mind expanding light-front: the velocity of expansion is not its value in Minkowski space but corresponds to that assignable to the induced metric and can be much smaller. Recall also that in TGD framework gravitational effects are large near the singularity so that growth is not with the light-velocity in vacuum.

The proposal of Kiehn that very special minimal surfaces (right helicoids) are in question would in TGD framework correspond to a light-like 3-surfaces representing light-like orbits of these minimal surfaces presumably expanding at least in the beginning of the time evolution.

3.3.2 Minkowskian hydrodynamics/Maxwellian dynamics as hyperbolic dynamics and Euclidian hydrodynamics as elliptic dynamics

In Kiehn's proposal both the hyperbolic wave dynamics (about which Maxwell's equations provide a simple linear example) and diffusive elliptic or parabolic dynamics are present. In TGD framework both aspects are present at the level of field equations and correspond to the hyperbolic dynamics in Minkowskian space-time regions and elliptic dynamics in Euclidian space-time regions.

The dynamics of preferred extremals can be seen in two manners. Either as hydrodynamics or as Maxwellian dynamics with Bohr rules expressing the decomposition of the field to quanta-magnetic flux quanta or massless radiation quanta.

1. Maxwellian hydrodynamics involves a considerable restriction: superposition of modes moving in different directions is not allowed: one has just left-movers or right-movers in given direction, not both. Preferred extremals are "Bohr orbit like" and resemble outcomes of state function reduction measuring polarization and wave vector. The linear superposition of fields is replaced with the superposition of effects. The test particle topologically condenses to several space-time sheets simultaneously and experiences the sum of the forces of classical fields associated with the space-time sheets. Therefore one avoids the worst objection against TGD that I have been able to invent. Only four primary field like variables would replace the multitude of primary fields encountered in a typical unification. Besides this one has second quantized induced spinor fields.

2. Field equations are hydrodynamical in the sense that the field equations state classical conservation laws of four-momentum and color charges. In fermionic sector conservation of electromagnetic charge (in quantum sense so that different charge states for spinor mode do not mix) requires the localization of solutions to 2-D string world sheets for all states except right-handed neutrino. This leads to 2-D conformal invariance. A possible identification of string world sheet is as 2-D minimal surface of space-time (rather than that of imbedding space).

   What is remarkable that in Minkowskian space-time regions most preferred extremals (magnetic flux tube structures define an exception to this) are locally analogous to the modes of massless field with polarization direction and light-like momentum direction which in the general case can depend on position so that one has curvilinear light-like curve as analog of light-ray. The curvilinear light-like orbits results when two parallel preferred extremals with constant light-like direction form bound states via the formation of magnetically charged wormhole contact structures identifiable as elementary particles. Total momentum is conserved and is time-like for this kind of states, and the hypothesis is that the values of mass squared are given by p-adic thermodynamics. The conservation of Kähler current holds true as also its integrability in the sense of Frobenius giving $j = \Psi\nabla\Phi$. Besides this massless wave equations hold true for both Ψ and Φ. This looks like 4-D generalization of your equations at the characteristic defined by phase boundary.

3. In Euclidian regions one has naturally elliptic "hydrodynamics". Euclidian regions correspond for 4-D CP_2 projection to the 4-D "lines" of generalized Feynman diagrams. Their M^4 projections can be arbitrary large and the proposal is that the space-time sheet characterizing the macroscopic objects is actually Euclidian. In $AdS_5 - S^5$correspondence the corresponding idea is that macroscopic object is described as a blackhole in 10-D space. Now blackhole interiors have Euclidian signature as lines of generalized Feynman diagrams and blackhole interior does not differ from the interior of any system in any dramatical manner. Whether the Euclidian and Minkowskian dynamics are dual of each other or whether both are necessary is an open question.

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