

General View About Physics in Many-Sheeted Space-Time

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Abstract

This chapter, which is second part of a summary about the recent view about many-sheeted space-time, provides a summary of the developments in TGD that have occurred during last few years (the year I am writing this is 2007). The view is out-of-date in some respects. The most important steps of progress are following ones.

1. Parton level formulation of quantum TGD

The formulation of quantum TGD at partonic level identifying fundamental objects as light-like 3-surfaces having also interpretation as random light-like orbits of 2-D partons having arbitrarily large size. This picture reduces quantum TGD to an almost-topological quantum field theory and leads to a dramatic understanding of S-matrix. A generalization of Feynman diagrams emerges obtained by replacing lines of Feynman diagram with light-like 3-surfaces meeting along their ends at vertices. This picture is different from that of string models and means also a generalization of the view about space-time and 3-surface since these surfaces cannot be assumed to be a smooth manifold anymore.

The condition that the formulation in terms of light-like 3-surfaces is equivalent with that using pairs of space-like 3-surfaces at the ends of causal diamonds leads to strong form of holography stating that partonic 2-surfaces and their tangent space-data code for physics. It has turned out that fermionic string model in 4-D space-time emerges naturally from TGD. This is not yet taken into account in these considerations of the chapter.

2. Zero energy ontology

In zero energy ontology physical states are creatable from vacuum and have vanishing net quantum numbers, in particular energy. Zero energy states can be decomposed to positive and negative energy parts with definite geometro-temporal separation, call it T , and having interpretation in terms of initial and final states of particle reactions. Zero energy ontology is consistent with ordinary positive energy ontology at the limit when the time scale of the perception of observer is much shorter than T .

Zero energy ontology leads to the view about S-matrix as a characterizer of time-like entanglement associated with the zero energy state and a generalization of S-matrix to what might be called M-matrix emerges. M-matrix is complex square root of density matrix expressible as a product of real valued “modulus” and unitary matrix representing phase and can be seen as a matrix valued generalization of Schrödinger amplitude. Also thermodynamics becomes an inherent element of quantum theory in this approach. M-matrices in turn form orthogonal rows of U-matrix which is defined between zero energy states whereas S and M-matrices are defined by entanglement coefficients between positive and negative energy parts of zero energy states.

3. Fusion of real and p-adic physics to single one

The fusion of p-adic physics and real physics to single coherent whole requires generalization of the number concept obtained by gluing reals and various p-adic number fields along common algebraic numbers. This leads to a new vision about how cognition and intentionality make themselves visible in real physics via long range correlations realized via the effective p-adicity of real physics. The success of the p-adic length scale hypothesis and p-adic mass calculations suggest that cognition and intentionality are present already at elementary particle level. This picture leads naturally to an effective discretization of the real physics at the level of S-matrix and relying on the notion of number theoretic braid.

It has turned out that the notion of braid emerges naturally from the localization of spinor modes to 2-D surfaces in the generic case. Braids correspond to the orbits of the strings ends at given space-time sheet.

4. Dark matter hierarchy and hierarchy of Planck constants

Dark matter revolution with levels of the hierarchy labeled by values of Planck constant suggests a further generalization of the notion of imbedding space and thus of space-time - at least as an effective mathematical tool. One can say that imbedding space is a book like structure obtained by gluing together infinite number of copies of the imbedding space like pages of a book: two copies characterized by singular discrete bundle structure are glued together along 4-dimensional set of common points. These points have physical interpretation in terms of quantum criticality. Particle states belonging to different sectors (pages of the book) can interact via field bodies representing space-time sheets which have parts belonging to two pages of this book.

It has turned out that the hierarchy of effective Planck constants $h_{eff} = n \times h$ follows from the quantum criticality implied by the non-determinism of Kähler action and that one can relate it to an infinite hierarchy of breakings of conformal symmetries acting on the orbits of light-like 3-surfaces leaving the space-like ends of space-time surface at boundaries of CD invariant. Hierarchy of conformal algebras corresponds to sub-algebras of conformal algebras with conformal weights coming as multiples of n .

5. Equivalence Principle and evolution of gravitational constant

The views about Equivalence Principle (EP) and GRT limit of TGD have changed quite a lot since 2007 and here the updated view is summarized. Before saying anything about evolution of gravitational constant one must understand whether it is a fundamental constant or prediction of quantum TGD. Also one should understand whether Equivalence Principle holds true and if so, in what sense. Also the identification of gravitational and inertial masses seems to be necessary.

At classical level EP follows from the interpretation of GRT space-time as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrics of space-time sheets from Minkowski metric. Poincare invariance suggests strongly classical EP for the GRT limit in long length scales at least. One can consider also other kinds of limits such as the analog of GRT limit for Euclidian space-time regions assignable to elementary particles. In this case deformations of CP_2 metric define a natural starting point and CP_2 indeed defines a gravitational instanton with very large cosmological constant in Einstein-Maxwell theory. Also gauge potentials of standard model correspond classically to superpositions of induced gauge potentials over space-time sheets.

Gravitational constant, cosmological constant, and various gauge couplings emerge as predictions. Planck length should be related to CP_2 size by a dimensionless numerical factor predicted by the theory. These constants need not be universal constants: cosmological constant is certainly very large for the Euclidian variant of GRT space-time. These constants could also depend on p-adic length scale. p-Adic coupling constant evolution suggests itself as a discretized variant of coupling constant evolution and p-adic scales would relate naturally to the size scales of causal diamonds: perhaps the integer n characterizing the multiple of CP_2 scale giving the distance between the tips of CD has p-adic prime p or its power as a divisor.

At the level of single space-time sheet and CD it is not possible to talk about coupling constant evolution since Kähler action and Kähler-Dirac action contain no coupling constants.

This description however gives rise to p-adic coupling constant evolution since the process of lumping together the sheets of the many-sheeted space-time gives a result which depends on the size scale of CD. If the non-deterministic dynamics of Kähler action for the maxima of Kähler function mimics p-adic non-determinism then one has hopes about p-adic coupling constant evolution. The p-adic prime and therefore also the length scale and coupling constants characterizing the dynamics for given CD would vary wildly as function of integer characterizing CD size scale. This could mean that the CD s whose size scales are related by multiplication of small integer are close to each other. They would be near to each other in logarithmic sense and logarithms indeed appear in running coupling constants. This “prediction” is of course subject to criticism.

6. Renormalization group equations for gauge couplings at space-time level

In classical TGD only Kähler coupling constant appears explicitly but does not affect the classical dynamics. Other gauge couplings do not appear at all in classical dynamics since the definition of classical fields absorbs them as normalization constants. This suggests that the notion of continuous coupling constant evolution at space-time level is not needed in quantum TGD proper and emerges only at the QFT limit when space-time is replaced with general relativistic effective space-time.

For the known extremals of Kähler action gauge couplings are RG invariants inside single space-time sheet, which supports the view that discrete p-adic coupling constant evolution replacing the ordinary continuous coupling constant evolution emerges only when space-time sheets are lumped together to define GRT space-time. This evolution would have as parameters the p-adic length scale characterizing the causal diamond (CD) associated with particle and the phase factors characterizing the algebraic extension of p-adic numbers involved.

The p-adic prime and therefore also the length scale and coupling constants characterizing the dynamics for given CD would vary wildly as function of integer characterizing CD size scale. This could mean that the CD s whose size scales are related by multiplication of small integer are close to each other. They would be near to each other in logarithmic sense and

logarithms indeed appear in running coupling constants. This “prediction” is of course subject to criticism.

7. Quantitative g for the values of coupling constants

All quantitative statements about coupling constants are bound to be guesswork as long as explicit formulas for M-matrix elements are lacking. p-Adic length scale hypothesis provides one guideline for the guesses. Second guideline is provided by number theoretical universality. Third guideline is general physical intuition. What is done can be however seen as exercises perhaps giving some familiarity with the basic notions.

The latest progress in the understanding of p-adic coupling constant evolution comes from a formula for Kähler coupling strength α_K in terms of Dirac determinant of the Kähler-Dirac operator associated with Kähler action.

The formula for α_K fixes its number theoretic anatomy and also that of other coupling strengths. The assumption that simple rationals (p-adicization) are involved can be combined with the input from p-adic mass calculations and with an old conjecture for the formula of gravitational constant allowing to express it in terms of CP_2 length scale and Kähler action of topologically condensed CP_2 type vacuum extremal. The prediction is that α_K is renormalization group invariant and equals to the value of fine structure constant at electron length scale characterized by M_{127} . Although Newton’s constant is proportional to p-adic length scale squared it can be RG invariant thanks to exponential reduction due to the presence of the exponent of Kähler action associated with the two CP_2 type vacuum extremals representing the wormhole contacts associated with graviton. The number theoretic anatomy of R^2/G allows to consider two options. For the first one only M_{127} gravitons are possible number theoretically. For the second option gravitons corresponding to $p \simeq 2^k$ are possible.

A relationship between electromagnetic and color coupling constant evolutions based on the formula $1/\alpha_{em} + 1/\alpha_s = 1/\alpha_K$ is suggested by the induced gauge field concept, and would mean that the otherwise hard-to-calculate evolution of color coupling strength is fixed completely. The predicted value of α_s at intermediate boson length scale is correct.

In this chapter the above topics are discussed in detail. Also the possible role of so called super-symplectic gauge bosons in the understanding of non-perturbative phase of QCD and black-hole physics is discussed.

1 Introduction

In previous chapter “General View About Physics in Many-Sheeted Space-Time” the notion of many-sheeted space-time concept and the understanding of coupling constant evolution at space-time level were discussed without reference to the newest developments in quantum TGD. In this chapter this picture is completed by a summary of the new rather dramatic developments in TGD that have occurred during last few years (the year I am writing this is 2007). The most important steps of progress are following ones.

1.1 Parton Level Formulation Of Quantum TGD

The formulation of quantum TGD at partonic level identifying fundamental objects as light-like 3-surfaces having also interpretation as random light-like orbits of 2-D partons having arbitrarily large size. This picture reduces quantum TGD to an almost-topological quantum field theory and leads to a dramatic understanding of S-matrix. A generalization of Feynman diagrams emerges obtained by replacing lines of Feynman diagram with light-like 3-surfaces meeting along their ends at vertices. This picture is different from that of string models and means also a generalization of the view about space-time and 3-surface since these surfaces cannot be assumed to be a smooth manifold anymore.

Extended super-conformal invariance involving the fusion of ordinary Super-Kac Moody symmetries and so called super-symplectic invariance generalizing the Kac-Moody algebra by replacing the Lie algebra of finite-dimensional Lie group with that for symplectic transformations of $\delta M_{\pm}^4 \times CP_2$ plays a key role in this framework. The help of professionals in this branch of mathematics would be badly needed in order to develop a detailed understanding about the predicted particle spectrum.

1.2 Zero Energy Ontology

The notion of zero energy ontology emerged implicitly in cosmological context from the observation that the imbeddings of Robertson-Walker metrics are always vacuum extremals. In fact, practically all solutions of Einstein's equations have this property very naturally. The explicit formulation emerged with the progress in the formulation of quantum TGD. In zero energy ontology physical states are creatable from vacuum and have vanishing net quantum numbers, in particular energy. Zero energy states can be decomposed to positive and negative energy parts with definite geometro-temporal separation, call it T , and having interpretation in terms of initial and final states of particle reactions. Zero energy ontology is consistent with ordinary positive energy ontology at the limit when the time scale of the perception of observer is much shorter than T . One of the implications is a new view about fermions and bosons allowing to understand Higgs mechanism among other things.

Zero energy ontology leads to the view about S-matrix as a characterizer of time-like entanglement associated with the zero energy state and a generalization of S-matrix to what might be called M-matrix emerges. M-matrix is complex square root of density matrix expressible as a product of real valued "modulus" and unitary matrix representing phase and can be seen as a matrix valued generalization of Schrödinger amplitude. Also thermodynamics becomes an inherent element of quantum theory in this approach.

1.3 Fusion Of Real And P-Adic Physics To Single One

The fusion of p-adic physics and real physics to single coherent whole requires generalization of the number concept obtained by gluing reals and various p-adic number fields along common algebraic numbers. This leads to a completely new vision about how cognition make themselves visible in real physics via long range correlations realized via the effective p-adicity of real physics. The success of p-adic length scale hypothesis and p-adic mass calculations suggest that cognition and intentionality are present already at elementary particle level. This picture leads naturally to an effective discretization of the real physics at the level of S-matrix and relying on the notion of umber theoretic braid.

1.4 Dark Matter Hierarchy And Hierarchy Of Planck Constants

The idea about hierarchy of Planck constants relying on generalization of the imbedding space was inspired both by empirical input (Bohr quantization of planetary orbits and anomalies of biology) and by the mathematics of hyper-finite factors of type II₁ combined with the quantum classical correspondence. Consider first the mathematical structure in question.

1. The Clifford algebra of World of Classical Worlds (WCW) creating many fermion states is a standard example of an algebra expressible as a direct integral of copies of von Neumann algebras known as hyper-finite factor of type II₁ (HFFs). The inclusions of HFFs relate very intimately to the notion of finite measurement resolution. There is a canonical hierarchy of Jones inclusions [A1] labeled by finite subgroups of SU(2) [A11]. Quantum classical correspondence suggests that these inclusions have space-time correlates [K27, K12] and the generalization of imbedding space would provide these correlates.
2. The space $CD \times CP_2$, where $CD \subset M^4$ is so called causal diamond identified as the intersection of future and past directed light-cones defines the basic geometric structure in zero energy ontology. The positive (negative) energy part of the zero energy state is located to the lower (upper) light-like boundaries of $CD \times CP_2$ and has interpretation as the initial (final) state of the physical event in standard positive energy ontology. p-Adic length scale hypothesis follows if one assumes that the temporal distance between the tips of CD comes as an octave of fundamental time scale defined by the size of CP_2 . The "world of classical worlds" (WCW) is union of sub-WCWs associated with spaces $CD \times CP_2$ with different locations in $M^4 \times CP_2$.
3. One can say that causal diamond CD and the space CP_2 appearing as factors in $CD \times CP_2$ forms the basic geometric structure in zero energy ontology, is replaced with a book like

structure obtained by gluing together infinite number of singular coverings and factor spaces of CD *resp.* CP_2 together. The copies are glued together along a common “back” $M^2 \subset M^2$ of the book in the case of CD. In the case of CP_2 the most general option allows two backs corresponding to the two non-isometric geodesic spheres S_i^2 , $i = I, II$, represented as sub-manifolds $\xi^1 = \bar{\xi}^2$ and $\xi^1 = \xi^2$ in complex coordinates transforming linearly under $U(2) \subset SU(3)$. Color rotations in CP_2 produce different choices of this pair.

4. The selection of geodesic spheres S^2 and M^2 is an imbedding space correlate for the fixing of quantization axes and means symmetry breaking at the level of imbedding space geometry. WCW is union over all possible choices of CD and pairs of geodesic spheres so that at the level no symmetry breaking takes place. The points of M^2 and S^2 have a physical interpretation in terms of quantum criticality with respect to the phase transition changing Planck constant (leakage to another page of the book through the back of the book).
5. The pages of the singular coverings are characterized by finite subgroups G_a and G_b of $SU(2)$ and these groups act in covering or leave the points of factor space invariant. The pages are labeled by Planck constants $\hbar(CD) = n_a \hbar_0$ and $\hbar(CP_2) = n_b \hbar_0$, where n_a and n_b are integers characterizing the orders of maximal cyclic subgroups of G_a and G_b . For singular factor spaces one has $\hbar(CD) = \hbar_0/n_a$ and $\hbar(CP_2) = \hbar_0/n_b$. The observed Planck constant corresponds to $\hbar = (\hbar(CD)/\hbar(CP_2)) \times \hbar_0$. What is also important is that $(\hbar/\hbar_0)^2$ appears as a scaling factor of M^4 covariant metric so that Kähler action via its dependence on induced metric codes for radiative corrections coming in powers of ordinary Planck constant: therefore quantum criticality and vanishing of radiative corrections to functional integral over WCW does not mean vanishing of radiative corrections.

The interpretation in terms of dark matter comes as follows.

1. Large values of \hbar make possible macroscopic quantum phase since all quantum scales are scaled upwards by \hbar/\hbar_0 . Anyonic and charge fractionization effects allow to “measure” $\hbar(CD)$ and $\hbar(CP_2)$ rather than only their ratio. $\hbar(CD) = \hbar(CP_2) = \hbar_0$ corresponds to what might be called standard physics without any anyonic effects and visible matter is identified as this phase.
2. Particle states belonging to different pages of the book can interact via classical fields and by exchanging particles, such as photons, which leak between the pages of the book. This leakage means a scaling of frequency and wavelength in such a manner that energy and momentum of photon are conserved. Direct interactions in which particles from different pages appear in the same vertex of generalized Feynman diagram are impossible. This seems to be enough to explain what is known about dark matter. This picture differs in many respects from more conventional models of dark matter making much stronger assumptions and has far reaching implications for quantum biology, which also provides support for this view about dark matter.

1.5 Equivalence Principle And Evolution Of Coupling Constants

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of standard model correspond classically to superpositions of induced gauge potentials over space-time sheets.

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All this is a work in progress and there are many uncertainties involved. Despite this it seems that it is good to sum up the recent view in order to make easier to refer to the new developments in the existing chapters.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at <http://tgdtheory.fi/tgdglossary.pdf> [L2].

2 The New Developments In Quantum TGD

This section summarizes the developments in quantum TGD which have taken place during last few years.

2.1 Reduction Of Quantum TGD To Parton Level

It took surprisingly long time before the realization that quantum TGD can be reduced to parton level in the sense that fundamental objects are light-like 3-surfaces (of arbitrary size). This identification follows from 4-D general coordinate invariance. Light-likeness in turn implies effective 2-dimensionality of the fermionic dynamics. 4-D space-time sheets are identified as preferred extrema of Kähler action. A stronger form of holography is that Kähler-Dirac action and Chern-Simons action for light-like partonic 3-surfaces defined the Kähler action as a logarithm of the fermionic determinant.

2.1.1 Magic properties of 3-D light-like surfaces and generalization of super-conformal symmetries

The very special conformal properties of both boundary δM_{\pm}^4 of 4-D light-cone and of light-like partonic 3-surfaces X^3 imply a generalization and extension of the super-conformal symmetries of super-string models to 3-D context [K8, K7]. Both the Virasoro algebras associated with the light-like coordinate r and to the complex coordinate z transversal to it define super-conformal algebras so that the structure of conformal symmetries is much richer than in string models.

1. The symplectic transformations of $\delta M_{\pm}^4 \times CP_2$ give rise to an infinite-dimensional symplectic/symplectic algebra having naturally a structure of Kac-Moody type algebra with respect to the light-like coordinate of $\delta M_{\pm}^4 = S^2 \times R_+$ and with finite-dimensional Lie group G replaced with the symplectic group. The conformal transformations of S^2 localized with respect to the light like coordinate act as conformal symmetries analogous to those of string models.

The super-symplectic algebra, call it SC, made local with respect to partonic 2-surface can be regarded as a Kac-Moody algebra associated with an infinite-dimensional Lie algebra.

2. The light-likeness of partonic 3-surfaces is respected by conformal transformations of H made local with respect to the partonic 3-surface and gives to a generalization of bosonic Kac-Moody algebra, call it KM, Also now the longitudinal and transversal Virasoro algebras emerge. The commutator $[KM, SC]$ annihilates physical states.
3. Fermionic Kac-Moody algebras act as algebras of left and right handed spinor rotations in M^4 and CP_2 degrees of freedom. Also the Kähler-Dirac operator allows super-conformal symmetries as gauge symmetries of its generalized eigen modes.

2.1.2 Quantum TGD as almost topological quantum field theory at parton level

The original belief was that the light-like character of basic dynamical objects X_l^3 at which the signature of the induced metric changes implies that Chern-Simons action for the induced Kähler gauge potential of CP_2 determines the classical dynamics of partonic 3-surfaces [K28]. This turned out to be a wrong guess: Kähler action and corresponding Kähler-Dirac action is enough.

1. Number theoretical compactification and the properties of known extremals of Kähler action suggests strongly the slicing of space-time surface by 3-D light-like surfaces Y_l^3 parallel to X_l^3 . The surfaces Y_l^3 behave as independent dynamical units in the sense that conserved currents flow along them so that quantum holography is realized. Number theoretic compactification allows also dual slicings of $X^4(X_l^3)$ by string world sheets Y^2 and partonic 2-surfaces X^2 .
2. The Kähler-Dirac action obtained as the super-symmetric counterpart Kähler action fixes the dynamics of the second quantized free fermionic fields in terms of which WCW gamma matrices and WCW spinor s can be constructed. The essential difference to the ordinary massless Dirac action is that induced gamma matrices are replaced by the contractions of the symplectic momentum densities Kähler action with imbedding space gamma matrices. Therefore the effective metric defined by the Kähler-Dirac gamma matrices replaces ordinary gamma matrices and the corresponding effective metric can be non-singular even when induced metric is degenerate. Effective 3-dimensionality means that the modes of the induced spinor field are constant with respect to the light-like coordinate labeling the slices Y_l^3 .
3. Kähler-Dirac action is consistent with the symmetries of Kähler action provided its first variation with respect to H coordinates vanishes - or equivalently- the second variation of Kähler action varies. This would realize quantum criticality at space-time level. The second variation vanishes only for those deformations which correspond to conserved currents.

The natural expectation is that the number of critical deformations is infinite and corresponds to conformal symmetries naturally assignable to criticality. The number n of conformal equivalence classes of the deformations can be finite and n would naturally relate to the hierarchy of Planck constants $h_{eff} = n \times h$ (see **Fig.** <http://tgdtheory.fi/appfigures/planckhierarchy.jpg> or **Fig. ??** in the appendix of this book).

4. Kähler-Dirac operator decomposes as $D_K = D_K(Y^2) + D_K(X^2)$ and its zero modes for effectively 3-D solutions can be chosen to be generalized eigenmodes of $D_K(X^2)$. The product of the generalized eigenvalues of $D_K(X^2)$ defines the exponent of Kähler function conjectured to reduce to Kähler action for the preferred extremal.

Fermionic statistics is geometrized in terms of spinor geometry of WCW since gamma matrices are linear combinations of fermionic oscillator operators identifiable also as super-symplectic generators [K28]. Only the light-likeness property involving the notion of induced metric breaks the topological QFT property of the theory so that the theory is as close to a physically trivial theory as it can be.

The resulting generalization of $N = 4$ super-conformal symmetry [A9] involves super-symplectic algebra (SC) and super Kac-Moody algebra (SKM) [K7] There are considerable differences as compared to string models. Super generators carry fermion number, no sparticles are predicted (at least super Poincare invariance is not obtained), SKM algebra and corresponding Virasoro algebra

associated with light-like coordinates of X^3 and δM_{\pm}^4 do not annihilate physical states which justifies p-adic thermodynamics used in p-adic mass calculations, four-momentum does not appear in Virasoro generators so that there are no problems with Lorentz invariance, and mass squared is p-adic thermal expectation of conformal weight.

2.2 Quantum Measurement Theory With Finite Measurement Resolution

Infinite-dimensional Clifford algebra of CH can be regarded as a canonical example of a von Neumann algebra known as a hyper-finite factor of type II_1 [A7, A11] (shortly HFF) characterized by the defining condition that the trace of infinite-dimensional unit matrix equals to unity: $Tr(Id) = 1$. In TGD framework the most obvious implication is the absence of fermionic normal ordering infinities whereas the absence of bosonic divergences is guaranteed by the basic properties of WCW Kähler geometry, in particular the non-locality of the Kähler function as a functional of 3-surface.

The special properties of this algebra, which are very closely related to braid and knot invariants [A6, A5], quantum groups [A11], non-commutative geometry [A3], spin chains, integrable models [B4], topological quantum field theories [A10], conformal field theories, and at the level of concrete physics to anyons [D3], generate several new insights and ideas about the structure of quantum TGD.

Jones inclusions $\mathcal{N} \subset \mathcal{M}$ [A1, A11] of these algebras lead to quantum measurement theory with a finite measurement resolution characterized by \mathcal{N} [K27, K12]. Quantum Clifford algebra \mathcal{M}/\mathcal{N} interpreted as \mathcal{N} -module creates physical states modulo measurement resolution. Complex rays of the state space resulting in the ordinary state function reduction are replaced by \mathcal{N} -rays and the notions of unitarity, hermiticity, and eigenvalue generalize [K6, K12].

The notion of entanglement generalizes so that entanglement coefficients are N -valued. Generalized eigenvalues are in turn N -valued hermitian operators. S- and U-matrices become N valued and probabilities are obtained from N -valued probabilities as traces.

Non-commutative physics would be interpreted in terms of a finite measurement resolution rather than something emerging below Planck length scale. An important implication is that a finite measurement sequence can never completely reduce quantum entanglement so that entire universe would necessarily be an organic whole. Topologically condensed space-time sheets could be seen as correlates for sub-factors which correspond to degrees of freedom below measurement resolution. Topological condensation in turn corresponds to the inclusion $\mathcal{N} \subset \mathcal{M}$. This is however not the only possible interpretation.

2.3 Hierarchy Of Planck Constants

The idea about hierarchy of Planck constants relying on generalization of the imbedding space was inspired both by empirical input (Bohr quantization of planetary orbits) and by the mathematics of hyper-finite factors of type II_1 combined with the quantum classical correspondence.

2.3.1 The generalization of imbedding space concept and hierarchy of Planck constants

Quantum classical correspondence suggests that Jones inclusions [A1] have space-time correlates [K27, K12]. There is a canonical hierarchy of Jones inclusions labeled by finite subgroups of $SU(2)$ [A11] This leads to a generalization of the imbedding space obtained by gluing an infinite number of copies of H regarded as singular bundles over $H/G_a \times G_b$, where $G_a \times G_b$ is a subgroup of $SU(2) \times SU(2) \subset SL(2, C) \times SU(3)$. Gluing occurs along a factor for which the group is same. The generalized imbedding space has clearly a book like structure with pages of books intersecting along 4-D sub-manifold $M^2 \times S^2$, S^2 a geodesic sphere of CP_2 characterizing the choice of quantization axes. Entire configuration space is union over “books” corresponding to various choices of this sub-manifold.

The groups in question define in a natural manner the direction of quantization axes for various isometry charges and this hierarchy seems to be an essential element of quantum measurement theory. Ordinary Planck constant, as opposed to Planck constants $\hbar_a = n_a \hbar_0$ and $\hbar_b = n_b \hbar_0$

appearing in the commutation relations of symmetry algebras assignable to M^4 and CP_2 , is naturally quantized as $\hbar = (n_a/n_b)\hbar_0$, where n_i is the order of maximal cyclic subgroup of G_i . The hierarchy of Planck constants is interpreted in terms of dark matter hierarchy [K12]. What is also important is that $(n_a/n_b)^2$ appear as a scaling factor of M^4 metric so that Kähler action via its dependence on induced metric codes for radiative corrections coming in powers of ordinary Planck constant: therefore quantum criticality and vanishing of radiative corrections to functional integral over WCW does not mean vanishing of radiative corrections.

G_a would correspond directly to the observed symmetries of visible matter induced by the underlying dark matter [K12]. For instance, in living matter molecules with 5- and 6-cycles could directly reflect the fact that free electron pairs associated with these cycles correspond to $n_a = 5$ and $n_a = 6$ dark matter possibly responsible for anomalous conductivity of DNA [K12, K3] and recently reported strange properties of graphene [D2]. Also the tetrahedral and icosahedral symmetries of water molecule clusters could have similar interpretation [K10], [D4].

A further fascinating possibility is that the observed indications for Bohr orbit quantization of planetary orbits [E1] could have interpretation in terms of gigantic Planck constant for underlying dark matter [K22] so that macroscopic and -temporal quantum coherence would be possible in astrophysical length scales manifesting itself in many manners: say as preferred directions of quantization axis (perhaps related to the CMB anomaly) or as anomalously low dissipation rates.

Since the gravitational Planck constant is proportional to the product of the gravitational masses of interacting systems, it must be assigned to the field body of the two systems and characterizes the interaction between systems rather than systems themselves. This observation applies quite generally and each field body of the system (em, weak, color, gravitational) is characterized by its own Planck constant.

In the gravitational case the order of G_a is gigantic and at least GM_1m/v_0 , $v_0 = 2^{-11}$ the favored value. The natural interpretation is as a discrete rotational symmetry of the gravitational field body of the system having both gravimagnetic and gravi-electric parts. The subgroups of G_a for which order is a divisor of the order of G_a define broken symmetries at the lower levels of dark matter hierarchy, in particular symmetries of visible matter.

The number theoretically simple ruler-and-compass integers having as factors only first powers of Fermat primes and power of 2 would define a physically preferred sub-hierarchy of quantum criticality for which subsequent levels would correspond to powers of 2: a connection with p-adic length scale hypothesis suggests itself. Ruler and compass hypothesis implies that besides p-adic length scales also their 3- and 5- multiples should be important. Note that in the structure of chromosomes p-adic length scale $L(151) \simeq 10$ characterizes beads-on-string structure of DNA whereas the length scale $3L(151)$ appears in the coiling of this structure.

2.3.2 Implications of dark matter hierarchy

The basic implication of dark matter hierarchy is hierarchy of macroscopic quantum coherent systems covering all length scales. The presence of this hierarchy is visible as exact discrete symmetries of field bodies reflecting at the level of visible matter as broken symmetries. In case of gravitational interaction these symmetries are highest and also the scale of quantum coherence is astrophysical. Together with ruler-and-compass hypothesis and p-adic length scale hypothesis this leads to very powerful predictions and p-adic length scale hypothesis might reduce to the ruler-and-compass hypothesis.

At the level of condensed matter one application is nuclear string model explaining also the selection rules of cold fusion and predicting that dark copy of weak physics with atomic scale defining the range of weak interaction is involved. Note that cold fusion has recently gained considerable support. High T_c super-conductivity is second application of dark matter hierarchy.

The 5- and 6-fold symmetries of the sugar backbone of DNA suggest that corresponding cyclic groups or cyclic groups having these groups as factors are symmetries of dark matter part of DNA presumably consisting of what is called as free electron pairs assignable to 5- and 6-cycles. The model allows to understand the observed high conductivity of DNA not consistent with the insulator property of DNA at the level of visible matter.

2.3.3 Dark matter and bio-control

The hierarchy of dark matters provides rather concrete realization for the vision about living matter as quantum critical system. This vision will be discussed in more detail later.

The large Planck constants characterize various field bodies of physical system. This gives justification to the notion of (magnetic) field body which plays key role in TGD inspired model of living matter serving as intentional agent controlling the behavior of field body. For instance, the model of EEG relies and of bio-control relies on this notion. The large value of the Planck constant is absolutely essential since for a given low frequency it allows to have gauge boson energy above thermal threshold. Large value of Planck constant is essential for time mirror mechanism which is behind the models of metabolism, long term memory, and intentional action.

The huge values of gravitational Planck constant supports the vision of Penrose [J2] about the special role of quantum gravitation in living matter. In TGD framework the proposal of Penrose and Hameroff for the emergence of consciousness known as Orch-Or (Orchestrated Objective Reduction [J3]) is however too restricted since it gives a very special role to micro-tubules.

A reasonable guess - based on the hypothesis that transition to dark matter phase occurs when perturbation theory for standard value of Planck constant fails - is that $GMm > 1$ is the criterion for the transition to dark phase for the gravitational field body characterizing the interaction between the two masses so that Planck mass becomes the critical mass for this transition. For the density of water this means size scale of .1 mm, the size of large neuron.

2.4 Zero Energy Ontology

Zero energy ontology has roots in TGD inspired cosmology [K23]. The problem has been that the imbeddings of Robertson-Walker cosmologies have vanishing densities of Poincare momenta identified as inertial momenta whereas gravitational energy density is non-vanishing. This led to the conclusion that one must allow space-time sheets with both time orientations such that the signs of Poincare energies are different for them and total density of inertial energy vanishes. Gravitational momenta can be identified as difference of the Poincare momenta and need not be conserved.

2.4.1 Construction of S-matrix and zero energy ontology

The construction of S-matrix allows to formulate this picture more sharply. Zero energy states have positive and negative energy parts located in geometric past and future and S-matrix can be identified as time-like entanglement coefficients between these states. Positive energy ontology is a good approximation in time scales shorter than the temporal distance between positive and negative energy states. This picture leads also to a generalization of Feynman graphs obtained by gluing light-like partonic 3-surfaces together along their ends at vertices. These Feynman cobordisms become a basic element of quantum TGD having interpretation as almost topological QFT and category theoretical formulation of quantum TGD emerges.

2.4.2 Elementary particles and zero energy ontology

At the level of elementary particles zero energy ontology means that fermionic quantum numbers are located at the light-like throats of wormhole contacts connecting CP_2 type extremals with Euclidian signature of induced metric to space-time sheets with Minkowskian signature of induced metric. Gauge bosons in turn correspond to pieces of CP_2 type extremals connecting positive and negative energy space-time sheets with fermion and anti-fermion quantum numbers at the throats of the wormhole contact. Depending on the sign of net energy one has ordinary boson or its phase conjugate. Gravitons correspond to pairs of fermion or gauge boson pair with particle and antiparticle connected by flux tube. This string picture emerges automatically if one assumes that the fermions of the conformal field theory associated with partonic 3-surface are free. It is also possible to have gauge bosons corresponding to single wormhole throat: these particles correspond to bosonic generators of super-symplectic algebra and excitations which correspond to genuine WCW degrees of freedom so that description in terms of quantum field theory in fixed background space-time need not work.

2.5 U- And S-Matrices

In quite early stage physical arguments led to the conclusion that the universal U-matrix associated with quantum jump must be distinguished from the S-matrix characterizing the rates of particle reactions. The notion of zero energy ontology was however needed before it became possible to characterize the difference between these matrices in a more precise manner.

2.5.1 Some distinctions between U- and S-matrices

The distinctions between U- and S-matrices discussed in more detail in [K29] have become rather clear.

1. U-matrix is the universal unitary matrix assignable to quantum jump between zero energy states whereas S-matrix can be identified assigned with the square root of density matrix expressible as its hermitian square root multiplied with a unitary S-matrix, which is universal. M-matrices form in ZEO an orthonormal basis of hermitian matrices so that the choice of the density matrix is not arbitrary. M-matrix defines time-like entanglement coefficients between positive and negative energy parts of the zero energy state. S-matrix characterizes zero energy states and actually codes the physics unless one is interested in consciousness.
2. State function reduction for S-matrix elements reduces the entanglement between positive and negative energy parts of a given zero energy state and is completely analogous to ordinary quantum measurement reducing entanglement between systems having space-like separation. It can take place at either boundary of CD and the sequence of repeated state function reduction at passive boundary give rise to self as conscious entity which dies and re-incarnates when the first reduction to the opposite boundary of CD takes place.
3. U-matrix is unitary as also S-matrix. For HFFs of type II_1 M-matrix can be taken to be S-matrix since the trace of the unit matrix equals to one. In the most general case S-matrix can be regarded as a “square” root of the density matrix assignable to time like entanglement: this hypothesis would unify the notions of S-matrix and density matrix and one could regard quantum states as matrix analogs of Schrödinger amplitudes expressible as products of its modulus (square root of probability density replaced with square root of density matrix) and phase (possibly universal unitary S-matrix). Thermal S-matrices define an important special case and thermodynamics becomes an integral part of quantum theory in zero energy ontology.

2.5.2 What can one say about the general structure of U-, M-, and S-matrices?

In Zero Energy Ontology (ZEO) S-matrix must be replaced with the triplet U-matrix, M-matrix, and S-matrix. U-matrix realizes unitary time evolution in the space for zero energy states realized geometrically as dispersion in the moduli space of causal diamonds (CDs) leaving second boundary (passive boundary) of CD and states at it fixed.

This process can be seen as the TGD counterpart of repeated state function reductions leaving the states at passive boundary unaffected and affecting only the member of state pair at active boundary (Zeno effect). In TGD inspired theory of consciousness self corresponds to the sequence these state function reductions. M-matrix describes the entanglement between positive and negative energy parts of zero energy states and is expressible as a hermitian square root H of density matrix multiplied by a unitary matrix S, which corresponds to ordinary S-matrix, which is universal and depends only the size scale n of CD through the formula $S(n) = S^n$. M-matrices and H-matrices defined by hermitian square roots of density matrices form an orthonormal basis at given CD and H-matrices would naturally correspond to the generators of super-symplectic algebra.

The first state function reduction to the opposite boundary corresponds to what happens in quantum physics experiments. The relationship between U- and S-matrices has remained poorly understood. In [K29] this relationship is analyzed by starting from basic principles. One ends up to formulas allowing to understand the architecture of U-matrix and to reduce its construction to that for S-matrix having interpretation as exponential of the generator L_{-1} of the Virasoro algebra associated with the super-symplectic algebra.

The first state function reduction to the opposite boundary corresponds to what happens in quantum physics experiments. The relationship between U- and S-matrices has remained poorly understood.

The original view about the relationship was a purely formal guess: M -matrices would define the orthonormal rows of U -matrix. This guess is not correct physically and one must consider in detail what U-matrix really means.

1. First about the geometry of CD [K29]. The boundaries of CD will be called passive and active: passive boundary correspond to the boundary at which repeated state function reductions take place and give rise to a sequence of unitary time evolutions U followed by localization in the moduli of CD each. Active boundary corresponds to the boundary for which U induces delocalization and modifies the states at it.

The moduli space for the CDs consists of a discrete subgroup of scalings for the size of CD characterized by the proper time distance between the tips and the sub-group of Lorentz boosts leaving passive boundary and its tip invariant and acting on the active boundary only. This group is assumed to be represented unitarily by matrices Λ forming the same group for all values of n .

The proper time distance between the tips of CDs is quantized as integer multiples of the minimal distance defined by CP_2 time: $T = nT_0$. Also in quantum jump in which the size scale n of CD increases the increase corresponds to integer multiple of T_0 . Using the logarithm of proper time, one can interpret this in terms of a scaling parametrized by an integer. The possibility to interpret proper time translation as a scaling is essential for having a manifest Lorentz invariance: the ordinary definition of S-matrix introduces preferred rest system.

2. The physical interpretation would be roughly as follows. M-matrix for a given CD codes for the physics as we usually understand it. M-matrix is product of square root of density matrix and S-matrix depending on the size scale of CD and is the analog of thermal S-matrix. State function at the opposite boundary of CD corresponds to what happens in the state function reduction in particle physics experiments. The repeated state function reductions at same boundary of CD correspond to TGD version of Zeno effect crucial for understanding consciousness. Unitary U-matrix describes the time evolution zero energy states due to the increase of the size scale of CD (at least in statistical sense). This process is dispersion in the moduli space of CDs: all possible scalings are allowed and localization in the space of moduli of CD localizes the active boundary of CD after each unitary evolution.

From this picture one ends up to general formulas [K29] allowing to understand the architecture of U-matrix and to reduce its construction to that for S-matrix having interpretation as exponential of the generator L_1 of the Virasoro algebra associated with the super-symplectic algebra.

2.5.3 Number theoretic universality and S-matrix

The fact that zero energy states are created by p-adic to-real transitions and must be number theoretically universal suggests strongly that the data about partonic 2-surfaces contributing to S-matrix elements come from the intersection of real partonic 2-surface and its p-adic counterpart satisfying same algebraic equations. The intersection consists of algebraic points and contains as subset number theoretic braids central for the proposed construction of S-matrix.

The question is whether also states for which S-matrix receives data from non-algebraic points should be allowed or whether the data can come even from continuous string like structures at partonic 2-surfaces as standard conformal field theory picture would suggest. If also S-matrix is algebraic, one can wonder whether there is any difference between p-adic and real physics at all. The latter option would mean that intentional action is followed by a unitarity process U analogous to a dispersion of completely localized particle implied by Schrödinger equation.

The algebraic universality of S-matrix could mean that S-matrix is obtained as an algebraic continuation of an S-matrix in algebraic extension of rationals by replacing incoming momenta and other continuous quantum numbers with real ones. Similar continuation should make sense in p-adic sector. S-matrix and U-matrix in a given algebraic extension of rationals or p-adics are not in general diagonalizable. Thus number theory would allow to avoid the paradoxical conclusion that S-matrix is always diagonal in a suitable basis.

Later progress in understanding of quantum TGD allows to refine and simplify this view dramatically. In adelic vision real and p-adic numbers are aspects of existence in all length scales and mean that cognition is present at all levels rather than emerging. Intentions have interpretation in terms of state function reductions in ZEO and there is no need to identify p-adic space-time sheets as their correlates.

2.6 Number Theoretic Ideas

p-Adic physics emerged roughly at the same time via p-adic mass calculations. The interpretation of p-adic physics as physics of cognition emerged.

Cognition would be present already at elementary particle level and p-adic fractality would be the experimental signature of it making itself visible in elementary particle mass spectrum among other things. The success of p-adic mass calculations provides strong support for the hypothesis.

This led gradually to the vision about physics as generalized number theory. It involves three separate aspects.

1. The p-adic approach led eventually to the program of fusing real physics and various p-adic physics to a single coherent whole by generalizing the number concept by gluing reals and various p-adics to a larger structure along common rationals and algebraics (see **Fig.** <http://tgdtheory.fi/appfigures/book.jpg> or **Fig. ??** in the appendix of this book). This inspired the notion of algebraic universality stating that for instance S-matrix should result by algebraic continuation from rational or at most algebraic valued S-matrix.

The notion of number theoretic braid belonging to the algebraic intersection of real and p-adic partonic 2-surface obeying same algebraic equations emerged also and gives a further connection with topological QFT: s. The perturbation theoretic definition of S-matrix is definitely excluded in this approach and TGD indeed leads to the understanding of coupling constant evolution at the level of “free” theory as a discrete p-adic coupling constant evolution so that radiative corrections are not needed for this purpose.

2. Also the classical number fields relate closely to TGD and the vision is that imbedding space $M^4 \times CP_2$ emerges from the physics based on hyper-octonionic 8-space with associativity as the fundamental dynamical principle both at classical and quantum level. Hyper-octonion space M^8 with space-time surface identified as hyper-quaternionic sub-manifolds or their duals and $M^4 \times CP_2$ would provide in this framework dual manners to describe physics and this duality would provide TGD counterpart for compactification.
3. The construction of infinite primes is analogous to repeated second quantization of supersymmetric arithmetic quantum field theory. This notion implies a further generalization of real and p-adic numbers allowing space-time points to have infinitely complex number theoretic structure not visible at the level of real physics. The idea is that space-time points define the Platonia able to represent in its structure arbitrarily complex mathematical structures and that space-time points could be seen as evolving structures becoming quantum jump by quantum jump increasingly complex number theoretically. Even the world of classical worlds (light-like 3-surfaces) and quantum states of Universe might be represented in terms of the number theoretic anatomy of space-time points (number theoretic Brahman=Atman and algebraic holography).

2.6.1 S-matrix as a functor and the groupoid structure formed by S-matrices

In zero energy ontology S-matrix can be seen as a functor from the category of Feynman cobordisms to the category of operators. S-matrix can be identified as a “square root” of the positive energy density matrix $S = \rho_+^{1/2} S_0$, where S_0 is a unitary matrix and ρ_+ is the density matrix for positive energy part of the zero energy state. Obviously one has $SS^\dagger = \rho_+$. $S^\dagger S = \rho_-$ gives the density matrix for negative energy part of zero energy state. Clearly, S-matrix can be seen as matrix valued generalization of Schrödinger amplitude. Note that the “indices” of the S-matrices correspond to WCW spinor s (fermions and their bound states giving rise to gauge bosons and gravitons) and to WCW degrees of freedom (world of classical worlds). For hyper-finite factor of II_1 it is not strictly

speaking possible to speak about indices since the matrix elements are traces of the S-matrix multiplied by projection operators to infinite-dimensional subspaces from right and left.

The functor property of S-matrices implies that they form a multiplicative structure analogous but not identical to groupoid [A2]. Recall that groupoid has associative product and there exist always right and left inverses and identity in the sense that ff^{-1} and $f^{-1}f$ are always defined but not identical and one has $fgg^{-1} = f$ and $f^{-1}fg = g$.

The reason for the groupoid like property is that S-matrix is a map between state spaces associated with initial and final sets of partonic surfaces and these state spaces are different so that inverse must be replaced with right and left inverse. The defining conditions for groupoid are replaced with more general ones. Also now associativity holds but the role of inverse is taken by hermitian conjugate. Thus one has the conditions $fgg^\dagger = f\rho_{g,+}$ and $f^\dagger fg = \rho_{f,-}g$, and the conditions $ff^\dagger = \rho_+$ and $f^\dagger f = \rho_-$ are satisfied. Here ρ_\pm is density matrix associated with positive/negative energy parts of zero energy state. If the inverses of the density matrices exist, groupoid axioms hold true since $f_L^{-1} = f^\dagger \rho_{f,+}^{-1}$ satisfies $ff_L^{-1} = Id_+$ and $f_R^{-1} = \rho_{f,-}^{-1} f^\dagger$ satisfies $f_R^{-1}f = Id_-$.

There are good reasons to believe that also tensor product of its appropriate generalization to the analog of co-product makes sense with non-triviality characterizing the interaction between the systems of the tensor product. If so, the S-matrices would form very beautiful mathematical structure bringing in mind the corresponding structures for 2-tangles and N-tangles. Knowing how incredibly powerful the group like structures have been in physics, one has good reasons to hope that groupoid like structure might help to deduce a lot of information about the quantum dynamics of TGD.

A word about nomenclature is in order. S has strong associations to unitarity and it might be appropriate to replace S with some other letter. The interpretation of S-matrix as a generalized Schrödinger amplitude would suggest Ψ -matrix. Since the interaction with Kea's M-theory blog (with M denoting Monad or Motif in this context) was crucial for the realization of the connection with density matrix, also M -matrix might work. S-matrix as a functor from the category of Feynman cobordisms in turn suggests C or F. Or could just Matrix denoted by M in formulas be enough?

2.6.2 Number theoretic braids

The notion of number theoretic braid has gradually evolved to a fundamental notion in quantum TGD and both number theoretical universality (p-adicization), TGD as almost-TQFT, and the notion of finite measurement resolution lead to this notion. The decisive proof of the notion came from the observation that the special properties of Kähler action imply this concept. In the quantization of induced spinor fields the number of fermionic oscillators is finite so that anti-commutation relations can hold true only for a finite point set defining the points of the number theoretic braid. The natural identification of the number theoretic braid is as the intersection of M^4 (CP_2) projection of X_1^3 with the back M^2 of M^4 book (back S_i^2 , $i = I, II$, of CP_2 book) so that the points of braid would be always quantum critical. Both homologically trivial ($i = I$) and non-trivial geodesic sphere ($i = II$) can be considered in the case of CP_2 so that there would be three possibly equivalent braidings defining kind of holy trinity.

The notion of number theoretic braid is especially interesting from the point of view of quantum biology. Generalized Feynman diagrams obtained by gluing light-like partonic 3-surfaces (whose sizes can be arbitrarily large) along their ends and define what might be called Feynman cobordisms. The first expectation was that number theoretic braids replicate in the vertices identifiable as partonic 2-surfaces at which the incoming and outgoing lines of generalized Feynman diagram meet. This would be nice but is not the case since by the lacking anti-commutativity of the incoming and outgoing oscillator operators the lines need not meet in this manner. This suggested an attractive information theoretic interpretation of generalized Feynman diagrams. Incoming and outgoing "lines" would give rise to topological quantum computations characterized by corresponding M-matrices, vertices would represent the replication of number theoretic braids analogous to DNA replication, and internal lines would be analogous to quantum communications. One could generalize this simple view about computation by allowing creation of new strands instead of mere replication.

Number theoretic braids are associated with light-like 3-surfaces and can be said to have both

dynamical and static characteristics. Partonic 2-surfaces as sub-manifolds of space-like 3-surface can also become linked and knotted and would naturally define space-like counterparts of tangles. Number theoretic braids could define dynamical topological quantum computation like operations whereas partonic 2-surfaces associated with say RNA could define as their space-like counterparts tangles and in the special case braids analogous to printed quantum programs so that there is duality between space-like and light-like braids [K11]. In terms of dance metaphor the dynamical braiding defined by the light like braid points interpreted as dancers has as a dual space-like braiding resulting as the threads connecting the feet of the dancers get tangled. An interesting question is how light-like and space-like braidings are transformed to each other: could this process correspond to a conscious reading like process and how closely DNA relates to language so that reading and writing would be fundamental processes appearing in all scales.

It came as a pleasant surprise that the idea about duality of space-like and light-like braidings inspired by DNA as topological quantum computer [J1] [K11] is realized at the level of basic quantum TGD [K28]. The dual slicings of $X^4(X_i^3)$ to string world sheets Y^2 and partonic 2-surfaces X^2 generalize the original picture in the sense that one can speak either about partons or string world sheets as basic objects. The strings connecting points of braid strands in X_i^3 would define space-like braidings whereas time like braidings are associated with X_i^3 . The light-like braiding at X_i^3 induces the space-like braiding of strings connecting the points of the strands to the strands of other braids.

2.6.3 Dark matter hierarchy and hierarchy of quantum critical systems in modular degrees of freedom

Dark matter hierarchy corresponds to a hierarchy of conformal symmetries Z_n of partonic 2-surfaces with genus $g \geq 1$ such that factors of n define subgroups of conformal symmetries of Z_n . By the decomposition $Z_n = \prod_{p|n} Z_p$, where $p|n$ tells that p divides n , this hierarchy corresponds to an hierarchy of increasingly quantum critical systems in modular degrees of freedom. For a given prime p one has a sub-hierarchy $Z_p, Z_{p^2} = Z_p \times Z_p$, etc... such that the moduli at $n+1$: th level are contained by n : th level. In the similar manner the moduli of Z_n are sub-moduli for each prime factor of n . This mapping of integers to quantum critical systems conforms nicely with the general vision that biological evolution corresponds to the increase of quantum criticality as Planck constant increases. This hierarchy would also define a hierarchy of conscious entities and could relate directly to mathematical cognition.

The group of conformal symmetries could be also non-commutative discrete group having Z_n as a subgroup. This inspires a very short-lived conjecture that only the discrete subgroups of $SU(2)$ allowed by Jones inclusions are possible as conformal symmetries of Riemann surfaces having $g \geq 1$. Besides Z_n one could have tetrahedral and icosahedral groups plus cyclic group Z_{2n} with reflection added but not Z_{2n+1} nor the symmetry group of cube. The conjecture is wrong. Consider the orbit of the subgroup of rotational group on standard sphere of E^3 , put a handle at one of the orbits such that it is invariant under rotations around the axis going through the point, and apply the elements of subgroup. You obtain a Riemann surface having the subgroup as its isometries. Hence all discrete subgroups of $SU(2)$ can act even as isometries for some value of g .

The number theoretically simple ruler-and-compass integers having as factors only first powers of Fermat primes and power of 2 would define a physically preferred sub-hierarchy of quantum criticality for which subsequent levels would correspond to powers of 2: a connection with p -adic length scale hypothesis suggests itself.

Spherical topology is exceptional since in this case the space of conformal moduli is trivial and conformal symmetries correspond to the entire $SL(2, C)$. This would suggest that only the fermions of lowest generation corresponding to the spherical topology are maximally quantum critical. This brings in mind Jones inclusions for which the defining subgroup equals to $SU(2)$ and Jones index equals to $M/N = 4$. In this case all discrete subgroups of $SU(2)$ label the inclusions. These inclusions would correspond to fiber space $CP_2 \rightarrow CP_2/U(2)$ consisting of geodesic spheres of CP_2 . In this case the discrete subgroup might correspond to a selection of a subgroup of $SU(2) \subset SU(3)$ acting non-trivially on the geodesic sphere. Cosmic strings $X^2 \times Y^2 \subset M^4 \times CP_2$ having geodesic spheres of CP_2 as their ends could correspond to this phase dominating the very early cosmology.

3 Identification Of Elementary Particles And The Role Of Higgs In Particle Massivation

The development of the recent view about the identification of elementary particles and particle massivation has taken fifteen years since the discovery of p-adic thermodynamics around 1993. p-Adic thermodynamics worked excellently from the beginning for fermions. Only the understanding of gauge boson masses turned out to be problematic and group theoretical arguments led to the proposal that Higgs boson should be present and give the dominating contribution to the masses of gauge bosons whereas the contribution to fermion masses should be small and even negligible. The detailed understanding of quantum TGD at partonic level eventually led to the realization that the coupling to Higgs is not needed after all. The deviation Δh of the ground state conformal weight from negative integer has interpretation as effective Higgs contribution since Higgs vacuum expectation is naturally proportional to Δh but the coupling to Higgs does not cause massivation. In the following I summarize the basic identification of elementary particles and massivation. A more detailed discussion can be found in [K14].

3.1 Identification Of Elementary Particles

The developments in the formulation of quantum TGD which have taken place during the period 2005-2007 [K7, K6] suggest dramatic simplifications of the general picture discussed in the earlier version of this chapter. p-Adic mass calculations [K16, K19, K18] leave a lot of freedom concerning the detailed identification of elementary particles.

3.1.1 Elementary fermions and bosons

The basic open question is whether the *theory is on some sense free at parton level* as suggested by the recent view about the construction of S-matrix (actually its generalization M-matrix) and by the almost topological QFT property of quantum TGD at parton level [K6]. If partonic 2-surfaces at elementary particle level carry only free many-fermion states, no bi-local composites of second quantized induced spinor field would be needed in the construction of the quantum states and this would simplify the theory enormously.

If this is the case, the basic conclusion would be that light-like 3-surfaces - in particular the ones at which the signature of induced metric changes from Minkowskian to Euclidian - are carriers of fermionic quantum numbers. These regions are associated naturally with CP_2 type vacuum extremals identifiable as correlates for elementary fermions if only fermion number ± 1 is allowed for the stable states. The question however arises about the identification of elementary bosons.

Wormhole contacts with two light-like wormhole throats carrying fermion and anti-fermion quantum numbers are the first thing that comes in mind. The wormhole contact connects two space-time sheets with induced metric having Minkowski signature. Wormhole contact itself has an Euclidian metric signature so that there are two wormhole throats which are light-like 3-surfaces and would carry fermion and anti-fermion number. In this case a delicate question is whether the space-time sheets connected by wormhole contacts have opposite time orientations or not. If this the case the two fermions would correspond to positive and negative energy particles.

I considered first the identification of only Higgs as a wormhole contact but there is no reason why this identification should not apply also to gauge bosons (certainly not to graviton). This identification would imply quite a dramatic simplification since the theory would be free at single parton level and the only stable parton states would be fermions and anti-fermions.

This picture allows to understand the difference between fermions and gauge bosons and Higgs particle. For fermions topological explanation of family replication predicts three fermionic generations [K5] corresponding to handle numbers $g = 0, 1, 2$ for the partonic 2-surface. In the case of gauge bosons and Higgs this replication is not visible. This could be due to the fact that gauge bosons form singlet and octet representation of the dynamical $SU(3)$ group associated with the handle number $g = 0, 1, 2$ since bosons correspond to pairs of handles. If octet representation is heavy the experimental absence of family replication for bosons can be understood.

3.1.2 Graviton and other stringy states

Fermion and anti-fermion can give rise to only single unit of spin since it is impossible to assign angular momentum with the relative motion of wormhole throats. Hence the identification of graviton as single wormhole contact is not possible. The only conclusion is that graviton must be a superposition of fermion-anti-fermion pairs and boson-anti-boson pairs with coefficients determined by the coupling of the parton to graviton. Graviton-graviton pairs might emerge in higher orders. Fermion and anti-fermion would reside at the same space-time sheet and would have a non-vanishing relative angular momentum. Also bosons could have non-vanishing relative angular momentum and Higgs bosons must indeed possess it.

Gravitons are stable if the throats of wormhole contacts carry non-vanishing gauge fluxes so that the throats of wormhole contacts are connected by flux tubes carrying the gauge flux. The mechanism producing gravitons would be the splitting of partonic 2-surfaces via the basic vertex. A connection with string picture emerges with the counterpart of string identified as the flux tube connecting the wormhole throats. Gravitational constant would relate directly to the value of the string tension.

The development of the understanding of gravitational coupling has had many twists and it is perhaps to summarize the basic misunderstandings.

1. CP_2 length scale R , which is roughly $10^{3.5}$ times larger than Planck length $l_P = \sqrt{\hbar G}$, defines a fundamental length scale in TGD. The challenge is to predict the value of Planck length $\sqrt{\hbar G}$. The outcome was an identification of a formula for $R^2/\hbar G$ predicting that the magnitude of Kähler coupling strength α_K is near to fine structure constant in electron length scale (for ordinary value of Planck constant should be added here).
2. The emergence of the parton level formulation of TGD finally demonstrated that G actually appears in the fundamental parton level formulation of TGD as a fundamental constant characterizing the M^4 part of CP_2 Kähler gauge potential [K28, K20]. This part is pure gauge in the sense of standard gauge theory but necessary to guarantee that the theory does not reduce to topological QFT. Quantum criticality requires that G remains invariant under p-adic coupling constant evolution and is therefore predictable in principle at least.
3. The TGD view about coupling constant evolution [K33] predicts the proportionality $G \propto L_p^2$, where L_p is p-adic length scale. Together with input from p-adic mass calculations one ends up to two conclusions. The correct conclusion was that Kähler coupling strength is equal to the fine structure constant in the p-adic length scale associated with Mersenne prime $p = M_{127} = 2^{127} - 1$ assignable to electron [K33]. I have considered also the possibility that α_K would be equal to electro-weak $U(1)$ coupling in this scale.
4. The additional - wrong- conclusion was that gravitons must always correspond to the p-adic prime M_{127} since G would otherwise vary as function of p-adic length scale. As a matter fact, the question was for years whether it is G or g_K^2 which remains invariant under p-adic coupling constant evolution. I found both options unsatisfactory until I realized that RG invariance is possible for both g_K^2 and G ! The point is that the exponent of the Kähler action associated with the piece of CP_2 type vacuum extremal assignable with the elementary particle is exponentially sensitive to the volume of this piece and logarithmic dependence on the volume fraction is enough to compensate the $L_p^2 \propto p$ proportionality of G and thus guarantee the constancy of G .

The explanation for the small value of the gravitational coupling strength serves as a test for the proposed picture. The exchange of ordinary gauge boson involves the exchange of single CP_2 type extremal giving the exponent of Kähler action compensated by state normalization. In the case of graviton exchange two wormhole contacts are exchanged and this gives second power for the exponent of Kähler action which is not compensated. It would be this additional exponent that would give rise to the huge reduction of gravitational coupling strength from the naive estimate $G \sim L_p^2$.

Gravitons are obviously not the only stringy states. For instance, one obtains spin 1 states when the ends of string correspond to gauge boson and Higgs. Also non-vanishing electro-weak and color quantum numbers are possible and stringy states couple to elementary partons via

standard couplings in this case. TGD based model for nuclei as nuclear strings having length of order $L(127)$ [K24] suggests that the strings with light M_{127} quark and anti-quark at their ends identifiable as companions of the ordinary graviton are responsible for the strong nuclear force instead of exchanges of ordinary mesons or color van der Waals forces.

Also the TGD based model of high T_c super-conductivity involves stringy states connecting the space-time sheets associated with the electrons of the exotic Cooper pair [K3, K4]. Thus stringy states would play a key role in nuclear and condensed matter physics, which means a profound departure from stringy wisdom, and breakdown of the standard reductionistic picture.

3.1.3 Spectrum of non-stringy states

The 1-throat character of fermions is consistent with the generation-genus correspondence. The 2-throat character of bosons predicts that bosons are characterized by the genera (g_1, g_2) of the wormhole throats. Note that the interpretation of fundamental fermions as wormhole contacts with second throat identified as a Fock vacuum is excluded.

The general bosonic wave-function would be expressible as a matrix M_{g_1, g_2} and ordinary gauge bosons would correspond to a diagonal matrix $M_{g_1, g_2} = \delta_{g_1, g_2}$ as required by the absence of neutral flavor changing currents (say gluons transforming quark genera to each other). 8 new gauge bosons are predicted if one allows all 3×3 matrices with complex entries orthonormalized with respect to trace meaning additional dynamical $SU(3)$ symmetry. Ordinary gauge bosons would be $SU(3)$ singlets in this sense. The existing bounds on flavor changing neutral currents give bounds on the masses of the boson octet. The 2-throat character of bosons should relate to the low value $T = 1/n \ll 1$ for the p-adic temperature of gauge bosons as contrasted to $T = 1$ for fermions.

If one forgets the complications due to the stringy states (including graviton), the spectrum of elementary fermions and bosons is amazingly simple and almost reduces to the spectrum of standard model. In the fermionic sector one would have fermions of standard model. By simple counting leptonic wormhole throat could carry $2^3 = 8$ states corresponding to 2 polarization states, 2 charge states, and sign of lepton number giving $8+8=16$ states altogether. Taking into account phase conjugates gives $16+16=32$ states.

In the non-stringy boson sector one would have bound states of fermions and phase conjugate fermions. Since only two polarization states are allowed for massless states, one obtains $(2+1) \times (3+1) = 12$ states plus phase conjugates giving $12+12=24$ states. The addition of color singlet states for quarks gives 48 gauge bosons with vanishing fermion number and color quantum numbers. Besides 12 electro-weak bosons and their 12 phase conjugates there are 12 exotic bosons and their 12 phase conjugates. For the exotic bosons the couplings to quarks and leptons are determined by the orthogonality of the coupling matrices of ordinary and boson states. For exotic counterparts of W bosons and Higgs the sign of the coupling to quarks is opposite. For photon and Z^0 also the relative magnitudes of the couplings to quarks must change. Altogether this makes $48+16+16=80$ states. Gluons would result as color octet states. Family replication would extend each elementary boson state into $SU(3)$ octet and singlet and elementary fermion states into $SU(3)$ triplets.

3.1.4 What about light-like boundaries and macroscopic wormhole contacts?

Light-like boundaries of the space-time sheet as also wormhole throats can have macroscopic size and can carry free many-fermion states but not elementary bosons. Number theoretic braids and anyons might be assignable to these structures. Deformations of cosmic strings to magnetic flux tubes with a light-like outer boundary are especially interesting in this respect.

If the ends of a string like object move with light velocity as implied by the usual stringy boundary conditions they indeed define light-like 3-surfaces. Many-fermion states could be assigned at the ends of string. One could also connect in pairwise manner the ends of two time-like strings having opposite time orientation using two space-like strings so that the analog of boson state consisting of two wormhole contacts and analogous to graviton would result. "Wormhole throats" could have arbitrarily long distance in M^4 .

Wormhole contacts can be regarded as slightly deformed CP_2 type extremals only if the size of M^4 projection is not larger than CP_2 size. The natural question is whether one can construct macroscopic wormhole contacts at all.

1. The throats of wormhole contacts cannot belong to vacuum extremals. One might however hope that small deformations of macroscopic vacuum extremals could yield non-vacuum wormhole contacts of macroscopic size.
2. A large class of macroscopic wormhole contacts which are vacuum extremals consists of surfaces of form $X_1^2 \times X_2^2 \subset (M^1 \times Y^2) \times E^3$, where Y^2 is Lagrangian manifold of CP_2 (induced Kähler form vanishes) and $M^4 = M^1 \times E^3$ represents decomposition of M^1 to time-like and space-like sub-spaces. X_2^2 is a stationary surface of E^3 . Both $X_1^2 \subset M^1 \times CP_2$ and X_2^2 have an Euclidian signature of metric except at light-like boundaries $X_a^1 \times X_2^2$ and $X_b^1 \times X_2^2$ defined by ends of X_1^2 defining the throats of the wormhole contact.
3. This kind of vacuum extremals could define an extremely general class of macroscopic wormhole contacts as their deformations. These wormhole contacts describe an interaction of wormhole throats regarded as closed strings as is clear from the fact that X^2 can be visualized as an analog of closed string world sheet X_1^2 in $M^1 \times Y^2$ describing a reaction leading from a state with a given number of incoming closed strings to a state with a given number of outgoing closed strings which correspond to wormhole throats at the two space-time sheets involved.

If one accepts the hierarchy of Planck constants [K12] leading to the generalization of the notion of imbedding space, the identification of anyonic phases in terms of macroscopic light-like surfaces emerges naturally. In this kind of states large fermion numbers are possible. Dark matter would correspond to this kind of phases and “partonic” 2-surfaces could have even astrophysical size. Also black holes can be identified as dark matter at light-like 3-surfaces analogous to black hole horizons and possessing gigantic value of Planck constant [K20].

3.2 New View About The Role Of Higgs Boson In Massivation

The proposed identifications challenge the standard model view about particle massivation.

1. The standard model inspired interpretation would be that Higgs vacuum expectation associated with the coherent state of neutral Higgs wormhole contacts generates gauge boson mass. The TGD counterpart of Higgs would be however not H -scalar but complex CP_2 tangent vector. There are no covariantly constant vector fields in CP_2 so that the idea about Higgs vacuum expectation is not mathematically feasible. This led to the original exaggerated conclusion that TGD does not allow Higgs: it is however only Higgs vacuum expectation which does not look plausible. Fermionic mass would be solely due to p-adic thermodynamics. Also in the case gauge boson masses one encounters a problem: the natural guess for the p-adic prime as M_{89} represents too small gauge boson masses, and it is very difficult to understand Weinberg angle, which is essentially group theoretical notion.
2. The Kähler-Dirac equation plus well-definedness of em charge requires that the spinor modes are restricted to stringy curves connecting the throats of two wormhole contacts associated with the elementary particles and carrying monopole fluxes. One can say that the wormhole throats are connected by flux tube behaving like string. The obvious idea is that the flux tube gives additional contribution to the mass squared, which can be interpreted as a contribution to the conformal weight of the ground state. If the string tension is proportional to gauge coupling strength for W and Z and to the counterpart of Higgs self coupling λ for Higgs one can explain the mass ratios of gauge bosons.
3. Besides the thermodynamical contribution to the particle mass there would be a small contribution from the ground state conformal weight unless this weight is not negative integer. Gauge boson mass would correspond to the ground state conformal weight present in both fermionic and bosonic states and in the case of gauge bosons this contribution would dominate due to the small value of p-adic temperature. For fermions p-adic thermodynamics for super Virasoro algebra would give the dominating contribution to the mass.
4. The remaining problem is to understand how the negative value of the ground state conformal weight emerges. This negative conformal weight compensated by the action of Super Virasoro

generators is necessary for the success of p-adic mass calculations. The intuitive expectation is that the solution of this problem must relate to the Euclidian signature of the regions representing lines of generalized Feynman diagrams.

- (a) Kähler-Dirac action gives for the solutions of Dirac action a boundary term which is essentially contraction of the normal component of the vector defined by Kähler-Dirac gamma matrices. In absence of measurement interaction terms the boundary condition for K-D equation states $\Gamma^n \Psi = 0$ at the stringy curves at the space-like ends of space-time surface. Γ^n must be lightlike and the assumption is that the spinor modes are generalized eigenmodes of Γ^n : $\Gamma^n \Psi = p^k \gamma_k \Psi = 0$ where p^k is constant lightlike four-momentum. This conforms with the idea that all fermions are massless and massive states of super-conformal representations emerges as bound states of fermions at wormhole throats. Elementary particles would correspond to pair of wormholes with magnetic flux flowing between the throats at the two space-time sheets involved. Massivation would be many-sheeted phenomenon. The string like objects would have string tension explaining the masses of weak bosons at microscopic level.

Very naively, $\Gamma^n \Psi = 0$ is possible only in the regions of space-like 3-surface which belong to Minkowskian space-time regions. Since Kähler-Dirac gamma matrices are in question it can however happen that the effective metric of string world sheet defined by Γ^α is degenerate. If CP_2 projection is 4-D as it is for CP_2 type extremals, one however expects that Γ^α is not degenerate inside wormhole contacts, and one can even question the localization of the spinor modes to 2-D string world sheets in these regions. The TGD based variant of stringy diagrammatics would indeed involve massless fermionic propagators only in the Minkowskian regions. The interaction of fermions at opposite throats of wormhole contacts would be described by stringy propagator $1/L_0$ or its non-local generalization to the product $(1/G)(1) \times (1/G)^\dagger(2)$ with supergenerators $G(i)$ assigned with the opposite wormhole throats.

- (b) One can add to the Kähler action measurement interaction term fixing the space-time surfaces to have conserved classical identical to their quantum counterparts belonging to Cartan algebra of symmetries. This can be achieved by adding Lagrange multiplier terms. These terms contribute to the Kähler-Dirac action a term at space-like ends of 3-surface and this term modifies the TGD counterpart of massless Dirac equation. The original generalized massless generalized eigenvalue spectrum associated with $p^k \gamma_k \Psi = 0$ of Γ^n is modified to massive spectrum given by the condition

$$\Gamma^n \Psi = - \sum_i \lambda_i \Gamma_{Q_i}^\alpha D_\alpha \Psi = p^k \gamma_k \Psi \quad ,$$

where Q_i refers to i : th conserved charge. Fermions are not massless anymore. This description is certainly over-simplified since several wormhole throats are involved. It is also only a formal description for the values of quantum numbers Q_i . One might say that $(\Gamma^n)^2$ serves as the analog of Higgs field vacuum expectation defined at the string curve.

- (c) It is not clear whether the tachyonic value of mass squared for ground state of superconformal representations can emerge from this kind of description. This might be possible inside wormhole contacts which have Euclidian signature of induced metric and define the lines of generalized Feynman diagrams.

3.3 General Mass Formulas

In the following general view about p-adic mass formulas and related problems is discussed.

3.3.1 Mass squared as a thermal expectation of super Kac-Moody conformal weight

The general view about particle massivation is based on the generalized coset construction allowing to understand the p-adic thermal contribution to mass squared as a thermal expectation value of the conformal weight for super Kac-Moody Virasoro algebra ($SKMV$) or equivalently super-symplectic

Virasoro algebra (SSV). Conformal invariance holds true only for the generators of the differences of $SKMV$ and SSV generators. In the case of SSV and $SKMV$ only the generators L_n , $n > 0$, annihilate the physical states. Obviously the actions of super-symplectic Virasoro (SSV) generators and Super Kac-Moody Virasoro generators on physical states are identical. The interpretation is in terms of Equivalence Principle. p-Adic mass expectation value is same irrespective of whether it is calculated for the excitations created by SSV or $KKMV$ generators and p-adic mass calculations are consistent with super-conformal invariance.

1. Super-Kac Moody conformal weights must be negative for elementary fermions and this can be understood if the ground state conformal weight corresponds to the square of the imaginary eigenvalue of the modified Dirac operator having dimensions of mass. If the value of ground state conformal weight is not negative integer, a contribution to mass squared analogous to Higgs expectation is obtained.
2. Massless state is thermalized with respect to $SKMV$ (or SSV) with thermal excitations created by generators L_n , $n > 0$.

3.3.2 Under what conditions conformal weight is additive

The question whether four- momentum or conformal weight is additive in p-adic mass calculations becomes acute in hadronic mass calculations. Only the detailed understanding of quantum TGD at partonic level allowed to understand the situation. One can consider three options.

1. Conformal weight and thus mass squared is additive only inside the regions of X_l^3 , which correspond to non-vanishing of induced Kähler magnetic field since these behave effectively as separate 3-surfaces as far as eigenmodes of the Kähler-Dirac operator are considered. The spectrum of the ground state conformal weights is indeed different for these regions in the general case. The four-momenta associated with different regions would be additive. This makes sense since the tangent space of $X^4(X_l^3)$ contains at each point of X_l^3 a subspace $M^2(x) \subset M^4$ defining the plane of non-physical polarizations and the natural interpretation is that four-momentum is in this plane. Hence the problem of original mass calculations forcing to assign all partonic four-momenta to a fixed plane M^2 is avoided.
2. If assigns independent translational degrees of freedom only to disjoint partonic 2-surfaces, a separate mass formula for each X_i^2 would result and four-momenta would be additive:

$$M_i^2 = \sum_i L_{0i}(SKM) . \quad (3.1)$$

Here $L_{0i}(SKM)$ contains a CP_2 cm term giving the CP_2 contribution to the mass squared known once the spinorial partial waves associated with super generators used to construct the state are known. Also vacuum conformal weight is included.

3. At the other extreme one has the option is based on the assignment of the mass squared with the total cm. This option looked the only reasonable one for 15 years ago. This would give

$$M^2 = \left(\sum_i p_i \right)^2 = \sum_i M_i^2 + 2 \sum_{i \neq j} p_i \cdot p_j = - \sum_i L_{0i}(SKM) . \quad (3.2)$$

The additivity of mass squared is strong condition and p-adic mass calculations for hadrons suggest that it holds true for quarks of low lying hadrons. For this option the decomposition of the net four momentum to a sum of individual momenta can be regarded as subjective unless there is a manner to measure the individual masses.

3.3.3 Mass formula for bound states of partons

The coefficient of proportionality between mass squared and conformal weight can be deduced from the observation that the mass squared values for CP_2 Dirac operator correspond to definite values of conformal weight in p-adic mass calculations. It is indeed possible to assign to partonic 2-surface $X^2 CP_2$ partial waves correlating strongly with the net electro-weak quantum numbers of the parton so that the assignment of ground state conformal weight to CP_2 partial waves makes sense. In the case of M^4 degrees of freedom it is not possible to talk about momentum eigen states since translations take parton out of δH_+ so that momentum must be assigned with the tip of the light-cone containing the particle.

The additivity of conformal weight means additivity of mass squared at parton level and this has been indeed used in p-adic mass calculations. This implies the conditions

$$\left(\sum_i p_i\right)^2 = \sum_i m_i^2 \quad (3.3)$$

The assumption $p_i^2 = m_i^2$ makes sense only for massless partons moving collinearly. In the QCD based model of hadrons only longitudinal momenta and transverse momentum squared are used as labels of parton states, which would suggest that one has

$$\begin{aligned} p_{i,\parallel}^2 &= m_i^2 , \\ -\sum_i p_{i,\perp}^2 + 2\sum_{i,j} p_i \cdot p_j &= 0 . \end{aligned} \quad (3.4)$$

The masses would be reduced in bound states: $m_i^2 \rightarrow m_i^2 - (p_T^2)_i$. This could explain why massive quarks can behave as nearly massless quarks inside hadrons.

4 Super-Symplectic Degrees Of Freedom

4.1 What Could Happen In The Transition To Non-Perturbative QCD?

What happens mathematically in the transition to non-perturbative QCD has remained more or less a mystery. The number theoretical considerations of [K26] inspired the idea that Planck constant is dynamical and has a spectrum given as $\hbar(n) = n\hbar_0$, where n characterizes the quantum phase $q = \exp(i2\pi/n)$ associated with Jones inclusion. The strange finding that the orbits of planets seem to obey Bohr quantization rules with a gigantic value of Planck constant inspired the hypothesis that the increase of Planck constant provides a unique mechanism allowing strongly interacting system to stay in perturbative phase [K22, K12]. The resulting model allows to understand dark matter as a macroscopic quantum phase in astrophysical length and time scales, and strongly suggest a connection with dark matter and biology.

The phase transition increasing Planck constant could provide a model for the transition to confining phase in QCD. When combined with the recent ideas about value spectrum of Kähler coupling strength one ends up with a rather explicit model about non-perturbative aspects of hadron physics already successfully applied in hadron mass calculations [K19]. Mersenne primes seem to define the p-adic length scales of gauge bosons and of hadronic space-time sheets. The quantization of Planck constant provides additional insight to p-adic length scales hypothesis and to the preferred role of Mersenne primes.

4.1.1 Super-symplectic gluons and non-perturbative aspects of hadron physics

According to the model of hadron masses [K19], in the case of light pseudo-scalar mesons the contribution of quark masses to the mass squared of meson dominates whereas spin 1 mesons contain a large contribution identified as color interaction conformal weight (color magnetic spin-spin interaction conformal weight and color Coulombic conformal weight). This conformal weight cannot however correspond to the ordinary color interactions alone and is negative for pseudo-scalars and compensated by some unknown contribution in the case of pion in order to avoid

tachyonic mass. Quite generally this realizes the idea about light pseudo-scalar mesons as Goldstone bosons. Analogous mass formulas hold for baryons but in this case the additional contribution which dominates.

The unknown contribution can be assigned to the $k = 107$ hadronic space-time sheet and must correspond to the non-perturbative aspects of QCD and the failure of the quantum field theory approach at low energies. In TGD the failure of QFT picture corresponds to the presence of WCW degrees of freedom (“world of classical worlds”) in which super-symplectic algebra acts. The failure of the approximation assuming single fixed background space-time is in question.

The purely bosonic generators carry color and spin quantum numbers: spin has however the character of orbital angular momentum. The only electro-weak quantum numbers of super-generators are those of right-handed neutrino. If the super-generators degrees carry the quark spin at high energies, a solution of proton spin puzzle emerges.

The presence of these degrees of freedom means that there are two contributions to color interaction energies corresponding to the ordinary gluon exchanges and exchanges of super-symplectic gluons. It turns out the model assuming same topological mixing of super-symplectic bosons identical to that experienced by U type quarks leads to excellent understanding of hadron masses assuming that hadron spin correlates with the super-symplectic particle content of the hadronic space-time sheet.

According to the argument already discussed, at the hadronic $k = 107$ space electro-weak interactions would be absent and classical $U(1)$ action should vanish. This is guaranteed if $\alpha_{U(1)}$ diverges. This would give

$$\alpha_s = \alpha_K = \frac{1}{4} .$$

This would give also a quantitative articulation for the statement that strong interactions are charge independent.

This α_s would correspond to the interaction via super-symplectic colored gluons and would lead to the failure of perturbation theory. By the general criterion stating that the failure of perturbation theory leads to a phase transition increasing the value of Planck constant one expects that the value of \hbar increases [K12]. The value leaving the value of α_K invariant would be $\hbar \rightarrow 26\hbar$ and would mean that p-adic length scale L_{107} is replaced with length scale $26L_{107} = 46$ fm, the size of large nucleus so that also the basic length scale nuclear physics would be implicitly coded into the structure of hadrons.

4.1.2 Why Mersenne primes should label a fractal hierarchy of physics?

There are motivations for the working hypothesis stating that there is fractal hierarchy of copies of standard model physics, and that Mersenne primes label both hadronic space-time sheets and gauge bosons. The reason for this is not yet well understood and I have considered several speculative explanations.

1. First picture

The first thing to come in mind is that Mersenne primes correspond to fixed points of the discrete p-adic coupling constant evolution, most naturally to the maxima of the color coupling constant strength. This would mean that gluons are emitted with higher probability than in other p-adic length scales.

There is however an objection against this idea. If one accepts the new vision about non-perturbative aspects of QCD, it would seem that super-symplectic bosons or the interaction between super-symplectic bosons and quarks for some reason favors Mersenne primes. However, if color coupling strength corresponds to $\alpha_K = \alpha_s = 1/4$ scaled down by the increase of the Planck constant, the evolution of super-symplectic color coupling strength does not seem to play any role. What becomes large should be a geometric “form factor”, when the boson in the vertex corresponds to Mersenne prime rather than “bare” coupling.

The resolution of the problem could be that boson emission vertices $g(p_1, p_2, p_3)$ are functions of p-adic primes labeling the particles of the vertices so that actually three p-adic length scales are involved instead of single length scale as in the ordinary coupling constant evolution. Hence one can imagine that the interaction between particles corresponding to primes near powers of 2

and Mersenne primes is especially strong and analogous to a resonant interaction. The geometric resonance due to the fact that the length scales involved are related by a fractal scaling by a power of 2 would make the form factors $F(p_1 \simeq 2^{k_1}, p_2 \simeq 2^{k_2}, M_n)$ large. The selection of primes near powers of two and Mersenne bosons would be analogous to evolutionary selection of a population consisting of species able to interact strongly.

Since $k = 113$ quarks are possible for $k = 107$ hadron physics, it seems that quarks can have flux tubes directed to M_n space-times with $n < k$. This suggests that neighboring Mersenne primes compete for flux tubes of quarks. For instance, when the p-adic length scale characterizing quark of M_{107} hadron physics begins to approach M_{89} quarks tend to feed their gauge flux to M_{89} space-time sheet and M_{89} hadron physics takes over and color coupling strength begins to increase. This would be the space-time correlate for the loss of asymptotic freedom.

2. Second picture

Preferred values of Planck constants could play a key role in the selection of Mersenne primes. Ruler-and-compass hypothesis predicts that Planck constants, which correspond to ratios of ruler and compass integers proportional to a product of distinct Fermat primes (four of them are known) and any power of two are favored. As a special case one obtains ruler and compass integers. As a consequence, p-adic length scales have satellites obtained by multiplying them with ruler-and-compass integers, and entire fractal hierarchy of power-of-two multiples of a given p-adic length scale results.

Mersenne length scales would be special since their satellites would form a subset of satellites of shorter Mersenne length scales. The copies of standard model physics associated with Mersenne primes would define a kind of resonating subset of physics since corresponding wavelengths and frequencies would coincide. This would also explain why fermions labeled by primes near power of two couple strongly with Mersenne primes.

4.2 Super-Symplectic Bosons As A Particular Kind Of Dark Matter

4.2.1 Super-symplectic bosons

TGD predicts also exotic bosons which are analogous to fermion in the sense that they correspond to single wormhole throat associated with CP_2 type vacuum extremal whereas ordinary gauge bosons corresponds to a pair of wormhole contacts assignable to wormhole contact connecting positive and negative energy space-time sheets. These bosons have super-conformal partners with quantum numbers of right handed neutrino and thus having no electro-weak couplings. The bosons are created by the purely bosonic part of super-symplectic algebra [K8, K28], whose generators belong to the representations of the color group and 3-D rotation group but have vanishing electro-weak quantum numbers. Their spin is analogous to orbital angular momentum whereas the spin of ordinary gauge bosons reduces to fermionic spin. Recall that super-symplectic algebra is crucial for the construction of WCW Kähler geometry. If one assumes that super-symplectic gluons suffer topological mixing identical with that suffered by say U type quarks, the conformal weights would be (5, 6, 58) for the three lowest generations. The application of super-symplectic bosons in TGD based model of hadron masses is discussed in [K19] and here only a brief summary is given.

As explained in [K19], the assignment of these bosons to hadronic space-time sheet is an attractive idea.

1. Quarks explain only a small fraction of the baryon mass and that there is an additional contribution which in a good approximation does not depend on baryon. This contribution should correspond to the non-perturbative aspects of QCD. A possible identification of this contribution is in terms of super-symplectic gluons. Baryonic space-time sheet with $k = 107$ would contain a many-particle state of super-symplectic gluons with net conformal weight of 16 units. This leads to a model of baryons masses in which masses are predicted with an accuracy better than 1 per cent.
2. Hadronic string model provides a phenomenological description of non-perturbative aspects of QCD and a connection with the hadronic string model indeed emerges. Hadronic string tension is predicted correctly from the additivity of mass squared for $J = 2$ bound states of super-symplectic quanta. If the topological mixing for super-symplectic bosons is equal

to that for U type quarks then a 3-particle state formed by 2 super-symplectic quanta from the first generation and 1 quantum from the second generation would define baryonic ground state with 16 units of conformal weight. A very precise prediction for hadron masses results by assuming that the spin of hadron correlates with its super-symplectic particle content.

3. Also the baryonic spin puzzle caused by the fact that quarks give only a small contribution to the spin of baryons, could find a natural solution since these bosons could give to the spin of baryon an angular momentum like contribution having nothing to do with the angular momentum of quarks.
4. Super-symplectic bosons suggest a solution to several other anomalies related to hadron physics. The events observed for a couple of years ago in RHIC [C1] suggest a creation of a black-hole like state in the collision of heavy nuclei and inspire the notion of color glass condensate of gluons, whose natural identification in TGD framework would be in terms of a fusion of hadronic space-time sheets containing super-symplectic matter materialized also from the collision energy. In the collision, valence quarks connected together by color bonds to form separate units would evaporate from their hadronic space-time sheets in the collision, and would define TGD counterpart of Pomeron, which experienced a reincarnation for few years ago [C3]. The strange features of the events related to the collisions of high energy cosmic rays with hadrons of atmosphere (the particles in question are hadron like but the penetration length is anomalously long and the rate for the production of hadrons increases as one approaches surface of Earth) could be also understood in terms of the same general mechanism.

4.2.2 Topological evaporation, quark gluon plasma and Pomeron

Topological evaporation of elementary particles means nothing if CP_2 type vacuum extremal evaporates so that one must assume that it is quark space-time sheet or join along boundaries block of quark space-time sheets which evaporates. Second new element is the identification of valence quarks as dark matter in the sense of having large \hbar : $\hbar_s \simeq (n/v_0)\hbar$, $v_0 \simeq 2^{-11}$, $n = 1$ so that Compton length is scaled by the same factor. Quark gluon plasma would correspond to a phase with ordinary value \hbar and possibly also sea partons can be regarded as this kind of phase. Color bonds between partons are possible also in this phase.

Concerning the evaporation there are two options.

1. The space-time sheets of sea partons are condensed at much larger space-time sheets defined by the space-time sheets of valence quarks connected by color bonds. Topological evaporation of the parton sea would correspond to the splitting of $\#$ contacts connecting sea partons space-time sheets to valence quark space-time sheets.
2. Sea partons condensed at a larger space-time sheet which in turn condenses at the space-time sheet of valence quarks. In this case topological evaporation occurs for the entire sea parton space-time sheet.

One can consider two possible scenarios for topological evaporation of quarks and gluons.

1. Color gauge charge is not identified as gauge flux and single secondarily condensed quark space-time sheet can suffer topological evaporation. In this case quark gluon plasma could be identified as vapor phase state for quarks and gluons.
2. Color gauge charge is identified as gauge flux and only join along boundaries blocks formed from quarks can evaporate. Join along boundaries contacts are naturally identified as color flux tubes between quarks. These tubes need not be static. Quark gluon plasma corresponds to condensed state in which the flux tubes between quark like 3-surfaces are broken. The evaporation of single quark is possible but as a consequence a compensating color charge develops on the interior of the outer boundary of the evaporated quark and the process probably can be interpreted as an emission of meson from hadron. The production of hadrons in hadron collision could be interpreted as a topological evaporation process for sea and valence quarks.

The problematic feature of scenario 1) is the understanding of color confinement. In scenario 2) color confinement of the vapor phase particles is an automatic consequence of the assumption that color charge corresponds to gauge flux classically (gauge field is $H^A J_{\alpha\beta}$, H^A being the Hamiltonian of the color isometry). This does not however exclude the possibility that hadron might feed part of its color isospin or hypercharge gauge flux to surrounding condensate. The concept of anomalous hypercharge introduced in earlier work as proportional to electromagnetic charge indeed suggests this kind of possibility. It should be noticed that for the vacuum extremals of Kähler action induced Kähler field and thus also color fields vanish identically.

The alternatives a) and b) have an additional nice feature that they lead to elegant description for the mysterious concept of Pomeron originally introduced to describe hadronic diffractive scattering as the exchange of Pomeron Regge trajectory [C5]. No hadrons belonging to Pomeron trajectory were however found and via the advent of QCD Pomeron was almost forgotten. Pomeron has recently experienced reincarnation [C3, C2, C4]. In Hera [C3] $e-p$ collisions, in which proton scatters essentially elastically whereas jets in the direction of incoming virtual photon emitted by electron are observed. These events can be understood by assuming that proton emits color singlet particle carrying a small fraction of proton's momentum. This particle in turn collides with the virtual photon (antiproton) whereas proton scatters essentially elastically.

The identification of the color singlet particle as Pomeron looks natural since Pomeron emission describes nicely the diffractive scattering of hadrons. Analogous hard diffractive scattering events in pX diffractive scattering with $X = \bar{p}$ [C2] or $X = p$ [C4] have also been observed. What happens is that proton scatters essentially elastically and the emitted Pomeron collides with X and suffers hard scattering so that large rapidity gap jets in the direction of X are observed. These results suggest that Pomeron is real and consists of ordinary partons.

The TGD identification of Pomeron is as sea partons in vapor phase. In TGD inspired phenomenology events involving Pomeron correspond to pX collisions, where incoming X collides with proton, when sea quarks have suffered coherent simultaneous (by color confinement) evaporation into vapor phase. System X sees only the sea left behind in the evaporation and scatters from it whereas dark valence quarks continue without noticing X and condense later to form quasi-elastically scattered proton. If X suffers hard scattering from the sea, the peculiar hard diffractive scattering events are observed. The fraction of these events is equal to the fraction f of time spent by sea quarks in vapor phase.

Dimensional arguments suggest a rough order of magnitude estimate for $f \sim \alpha_K \sim 1/137 \sim 10^{-2}$ for f . The fraction of the peculiar deep inelastic scattering events at Hera is about 5 percent, which suggest that f is about 6.8 times larger and of same order of magnitude as QCD α_s . The time spent in condensate is by dimensional arguments of the order of the p-adic length scale $L(M_{107})$, not far from proton Compton length. Time dilation effects at high collision energies guarantee that valence quarks indeed stay in vapor phase during the collision. The identification of Pomeron as sea explains also why Pomeron Regge trajectory does not correspond to actual on mass shell particles.

The existing detailed knowledge about the properties of sea structure functions provides a stringent test for the TGD based scenario. According to [C2] Pomeron structure function seems to consist of soft $((1-x)^5)$, hard $((1-x))$ and super-hard component (delta function like component at $x = 1$). The peculiar super hard component finds explanation in TGD based picture. The structure function $q_P(x, z)$ of parton in Pomeron contains the longitudinal momentum fraction z of the Pomeron as a parameter and $q_P(x, z)$ is obtained by scaling from the sea structure function $q(x)$ for proton $q_P(x, z) = q(zx)$. The value of structure function at $x = 1$ is non-vanishing: $q_P(x = 1, z) = q(z)$ and this explains the necessity to introduce super hard delta function component in the fit of [C2].

4.2.3 Simulating big bang in laboratory

An important steps in the development of ideas were stimulated by the findings made during period 2002-2005 in Relativist Heavy Ion Collider (RHIC) in Brookhaven compared with the finding of America and for full reason.

1. The first was finding of longitudinal Lorentz invariance at single particle level suggesting a collective behavior. This was around 2002.

2. The collective behavior which was later interpreted in terms of color glass condensate meaning the presence of a blob of liquid like phase decaying later to quark gluon plasma since it was found that the density of what was expected to be quark gluon plasma was about ten times higher than expected.
3. The last finding is that this object seems to absorb partons like black hole and behaves like evaporating black hole.

In my personal Theory Universe the history went as follows.

1. I proposed 2002 a model for Gold-Gold collision as a mini big bang identified as a scaled down variant of TGD inspired cosmology. This makes sense because in TGD based critical cosmology the initial state has vanishing mass per comoving volume instead of being infinite as in radiation dominated cosmology. Any phase transition involving a generation of a new space-time sheet might proceed in this universal manner.
2. Cosmic string soup in the primordial stage is replaced by a tangle of color flux tubes containing the color glass condensate. Flux tubes correspond to flow lines of incompressible liquid flow and non-perturbative macroscopic quantum phase with a very large \hbar is in question. Gravitational constant is replaced by strong gravitational constant defined by the relevant p-adic length scale squared since color flux tubes are analogs of hadronic strings. Presumably L_p , $p = M_{107} = 2^{107} - 1$, is the p-adic length scale since Mersenne prime M_{107} labels the space-time sheet at which partons feed their color gauge fluxes. Temperature during this phase could correspond to Hagedorn temperature for strings and is determined by string tension. Density would be maximal.
3. Next phase is critical phase in which the notion of space-time in ordinary sense makes sense and 3-space is flat since there is no length scale in critical system (so that curvature vanishes). During this critical phase a transition to quark gluon plasma occurs. The duration of this phase fixes all relevant parameters such as temperature (which is the analog of Hagedorn temperature corresponding since critical density is maximal density of gravitational mass in TGD Universe).
4. The next phase is radiation dominated quark gluon plasma phase and then follows hadronization to matter dominated phase provided cosmological picture still applies.

Since black hole formation and evaporation is very much like formation big crunch followed by big bang, the picture is more or less equivalent with the picture in which black hole like object consisting of string like objects (mass is determined by string length just as it is determined by the radius for black holes) is formed and then evaporates. Black hole temperature corresponds to Hagedorn temperature and to the duration of critical period of the mini cosmology.

4.2.4 Are ordinary black-holes replaced with super-symplectic black-holes in TGD Universe?

Some variants of super string model predict the production of small black-holes at LHC. I have never taken this idea seriously but in a well-defined sense TGD predicts black-holes associated with super-symplectic gravitons with strong gravitational constant defined by the hadronic string tension. The proposal is that super-symplectic black-holes have been already seen in Hera, RHIC, and the strange cosmic ray events.

Baryonic super-symplectic black-holes of the ordinary M_{107} hadron physics would have mass 934.2 MeV, very near to proton mass. The mass of their M_{89} counterparts would be 512 times higher, about 478 GeV if quark masses scale also by this factor. This need not be the case: if one has $k = 113 \rightarrow 103$ instead of 105 one has 434 GeV mass. "Ionization energy" for Pomeron, the structure formed by valence quarks connected by color bonds separating from the space-time sheet of super-symplectic black-hole in the production process, corresponds to the total quark mass and is about 170 MeV for ordinary proton and 87 GeV for M_{89} proton. This kind of picture about black-hole formation expected to occur in LHC differs from the stringy picture since a fusion of the hadronic mini black-holes to a larger black-hole is in question.

An interesting question is whether the ultrahigh energy cosmic rays having energies larger than the GZK cut-off of 5×10^{10} GeV are baryons, which have lost their valence quarks in a collision with hadron and therefore have no interactions with the microwave background so that they are able to propagate through long distances.

In neutron stars the hadronic space-time sheets could form a gigantic super-symplectic black-hole and ordinary black-holes would be naturally replaced with super-symplectic black-holes in TGD framework (only a small part of black-hole interior metric is representable as an induced metric). This obviously means a profound difference between TGD and string models.

1. Hawking-Bekenstein black-hole entropy would be replaced with its p-adic counterpart given by

$$S_p = \left(\frac{M}{m(CP_2)} \right)^2 \times \log(p) , \quad (4.1)$$

where $m(CP_2)$ is CP_2 mass, which is roughly 10^{-4} times Planck mass. M is the contribution of p-adic thermodynamics to the mass. This contribution is extremely small for gauge bosons but for fermions and super-symplectic particles it gives the entire mass.

2. If p-adic length scale hypothesis $p \simeq 2^k$ holds true, one obtains

$$S_p = k \log(2) \times \left(\frac{M}{m(CP_2)} \right)^2 , \quad (4.2)$$

$m(CP_2) = \hbar/R$, R the “radius” of CP_2 , corresponds to the standard value of \hbar_0 for all values of \hbar .

3. Hawking-Bekenstein area law gives in the case of Schwarzschild black-hole

$$S = \frac{A}{4G} \times \hbar = \pi GM^2 \times \hbar . \quad (4.3)$$

For the p-adic variant of the law Planck mass is replaced with CP_2 mass and $k \log(2) \simeq \log(p)$ appears as an additional factor. Area law is obtained in the case of elementary particles if k is prime and wormhole throats have M^4 radius given by p-adic length scale $L_k = \sqrt{k}R$ which is exponentially smaller than L_p . For macroscopic super-symplectic black-holes modified area law results if the radius of the large wormhole throat equals to Schwarzschild radius. Schwarzschild radius is indeed natural: in [K25] I have shown that a simple deformation of the Schwarzschild exterior metric to a metric representing rotating star transforms Schwarzschild horizon to a light-like 3-surface at which the signature of the induced metric is transformed from Minkowskian to Euclidian.

4. The formula for the gravitational Planck constant appearing in the Bohr quantization of planetary orbits and characterizing the gravitational field body mediating gravitational interaction between masses M and m [K22] reads as

$$\hbar_{gr} = \frac{GMm}{v_0} \hbar_0 .$$

$v_0 = 2^{-11}$ is the preferred value of v_0 . One could argue that the value of gravitational Planck constant is such that the Compton length \hbar_{gr}/M of the black-hole equals to its Schwarzschild radius. This would give

$$\hbar_{gr} = \frac{GM^2}{v_0} \hbar_0 , \quad v_0 = 1/2 . \quad (4.4)$$

The requirement that \hbar_{gr} is a ratio of ruler-and-compass integers expressible as a product of distinct Fermat primes (only four of them are known) and power of 2 would quantize the mass spectrum of black hole [K22]. Even without this constraint M^2 is integer valued using p-adic mass squared unit and if p-adic length scale hypothesis holds true this unit is in an excellent approximation power of two.

5. The gravitational collapse of a star would correspond to a process in which the initial value of v_0 , say $v_0 = 2^{-11}$, increases in a stepwise manner to some value $v_0 \leq 1/2$. For a supernova with solar mass with radius of 9 km the final value of v_0 would be $v_0 = 1/6$. The star could have an onion like structure with largest values of v_0 at the core as suggested by the model of planetary system. Powers of two would be favored values of v_0 . If the formula holds true also for Sun one obtains $1/v_0 = 3 \times 17 \times 2^{13}$ with 10 per cent error.
6. Black-hole evaporation could be seen as means for the super-symplectic black-hole to get rid of its electro-weak charges and fermion numbers (except right handed neutrino number) as the antiparticles of the emitted particles annihilate with the particles inside super-symplectic black-hole. This kind of minimally interacting state is a natural final state of star. Ideal super-symplectic black-hole would have only angular momentum and right handed neutrino number.
7. In TGD light-like partonic 3-surfaces are the fundamental objects and space-time interior defines only the classical correlates of quantum physics. The space-time sheet containing the highly entangled cosmic string might be separated from environment by a wormhole contact with size of black-hole horizon.

This looks the most plausible option but one can of course ask whether the large partonic 3-surface defining the horizon of the black-hole actually contains all super-symplectic particles so that super-symplectic black-hole would be single gigantic super-symplectic parton. The interior of super-symplectic black-hole would be a space-like region of space-time, perhaps resulting as a large deformation of CP_2 type vacuum extremal. Black-hole sized wormhole contact would define a gauge boson like variant of the black-hole connecting two space-time sheets and getting its mass through Higgs mechanism. A good guess is that these states are extremely light.

5 Number Theoretic Compactification And $M^8 - H$ Duality

This section summarizes the basic vision about number theoretic compactification reducing the classical dynamics to associativity or co-associativity. Originally $M^8 - H$ duality was introduced as a number theoretic explanation for $H = M^4 \times CP_2$. Much later it turned out that the completely exceptional twistorial properties of M^4 and CP_2 are enough to justify $X^4 \subset H$ hypothesis. Skeptic could therefore criticize the introduction of M^8 (actually its complexification) as an un-necessary mathematical complication producing only unproven conjectures and bundle of new statements to be formulated precisely. However, if quaternionicity can be realized in terms of M_c^8 using O_c -real analytic functions and if quaternionicity is equivalent with preferred extremal property, a huge simplification results and one can say that field equations are exactly solvable.

One can question the feasibility of $M^8 - H$ duality if the dynamics is purely number theoretic at the level of M^8 and determined by Kähler action at the level of H . Situation becomes more democratic if Kähler action defines the dynamics in both M^8 and H : this might mean that associativity could imply field equations for preferred extremals or vice versa or there might be equivalence between two. This means the introduction Kähler structure at the level of M^8 , and motivates also the coupling of Kähler gauge potential to M^8 spinors characterized by Kähler charge or em charge. One could call this form of duality strong form of $M^8 - H$ duality.

The strong form $M^8 - H$ duality boils down to the assumption that space-time surfaces can be regarded either as 4-surfaces of H or as surfaces of M^8 or even M_c^8 composed of associative and co-associative regions identifiable as regions of space-time possessing Minkowskian *resp.* Euclidian signature of the induced metric. They have the same induced metric and Kähler form and WCW associated with H should be essentially the same as that associated with M^8 . Associativity

corresponds to hyper-quaternionicity at the level of tangent space and co-associativity to co-hyper-quaternionicity - that is associativity/hyper-quaternionicity of the normal space. Both are needed to cope with known extremals. Since in Minkowskian context precise language would force to introduce clumsy terms like hyper-quaternionicity and co-hyper-quaternionicity, it is better to speak just about associativity or co-associativity.

Remark: The original assumption was that space-times could be regarded as surfaces in M^8 rather than in its complexification M_c^8 identifiable as complexified octonions. This assumption is un-necessarily strong and if one assumes that octonion-real analytic functions characterize these surfaces M_c^8 must be assumed.

For the octonionic spinor fields the octonionic analogs of electroweak couplings reduce to mere Kähler or electromagnetic coupling and the solutions reduce to those for spinor d'Alembertian in 4-D harmonic potential breaking $SO(4)$ symmetry. Due to the enhanced symmetry of harmonic oscillator, one expects that partial waves are classified by $SU(4)$ and by reduction to $SU(3) \times U(1)$ by em charge and color quantum numbers just as for CP_2 - at least formally.

Harmonic oscillator potential defined by self-dual em field splits M^8 to $M^4 \times E^4$ and implies Gaussian localization of the spinor modes near origin so that E^4 effectively compactifies. The resulting physics brings strongly in mind low energy physics, where only electromagnetic interaction is visible directly, and one cannot avoid associations with low energy hadron physics. These are some of the reasons for considering $M^8 - H$ duality as something more than a mere mathematical curiosity.

Remark: The Minkowskian signatures of M^8 and M^4 produce technical nuisance. One could overcome them by Wick rotation, which is however somewhat questionable trick. $M_c^8 = O_c$ provides the proper formulation.

1. The proper formulation is in terms of complexified octonions and quaternions involving the introduction of commuting imaginary unit j .
2. Hyper-quaternions/octonions define as subspace of complexified quaternions/octonions spanned by real unit and jI_k , where I_k are quaternionic units. These spaces are obviously not closed under multiplication. One can however define the notion of associativity for the subspace of M^8 by requiring that the products and sums of the tangent space vectors generate complexified quaternions.
3. Ordinary quaternions Q are expressible as $q = q_0 + q^k I_k$. Hyper-quaternions are expressible as $q = q_0 + jq^k I_k$ and form a subspace of complexified quaternions $Q_c = Q \oplus jQ$. Similar formula applies to octonions and their hyper counterparts which can be regarded as subspaces of complexified octonions $O \oplus jO$. Tangent space vectors of H correspond hyper-quaternions $q_H = q_0 + jq^k I_k + ji q_2$ defining a subspace of doubly complexified quaternions: note the appearance of two imaginary units.

The recent definitions of associativity and M^8 duality has evolved slowly from in-accurate characterizations and there are still open questions.

1. Kähler form for M^8 non-trivial only in $E^4 \subset M^8$ implies unique decomposition $M^8 = M^4 \times E^4$ needed to define $M^8 - H$ duality uniquely. This applies also to M_c^8 . This forces to introduce also Kähler action, induced metric and induced Kähler form. Could strong form of duality meant that the space-time surfaces in M^8 and H have same induced metric and induced Kähler form? Could the WCW s associated with M^8 and H be identical with this assumption so that duality would provide different interpretations for the same physics?
2. One can formulate associativity in M^8 (or M_c^8) by introducing octonionic structure in tangent spaces or in terms of the octonionic representation for the induced gamma matrices. Does the notion have counterpart at the level of H as one might expect if Kähler action is involved in both cases? The analog of this formulation in H might be as quaternionic "reality" since tangent space of H corresponds to complexified quaternions: I have however found no acceptable definition for this notion.

The earlier formulation is in terms of octonionic flat space gamma matrices replacing the ordinary gamma matrices so that the formulation reduces to that in M^8 tangent space.

This formulation is enough to define what associativity means although one can protest. Somehow H is already complex quaternionic and thus associative. Perhaps this just what is needed since dynamics has two levels: *imbedding space level* and *space-time level*. One must have imbedding space spinor harmonics assignable to the ground states of super-conformal representations and quaternionicity and octonionicity of H tangent space would make sense at the level of space-time surfaces.

3. Whether the associativity using induced gamma matrices works is not clear for massless extremals (MEs) and vacuum extremals with the dimension of CP_2 projection not larger than 2.
4. What makes this notion of associativity so fascinating is that it would allow to iterate duality as a sequence $M^8 \rightarrow H \rightarrow H \dots$ by mapping the space-time surface to $M^4 \times CP_2$ by the same recipe as in case of M^8 . This brings in mind the functional composition of O_c -real analytic functions (O_c denotes complexified octonions: complexification is forced by Minkowskian signature) suggested to produce associative or co-associative surfaces. The associative (co-associative) surfaces in M^8 would correspond to loci for vanishing of imaginary (real) part of octonion-real-analytic function.

It might be possible to define associativity in H also in terms of Kähler-Dirac gamma matrices defined by Kähler action (certainly not M^8).

1. All known extremals are associative or co-associative in H in this sense. This would also give direct correlation with the variational principle. For the known preferred extremals this variant is successful partially because the Kähler-Dirac gamma matrices need not span the entire tangent space. The space spanned by the Kähler-Dirac gammas is not necessarily tangent space. For instance for CP_2 type vacuum extremals the Kähler-Dirac gamma matrices are CP_2 gamma matrices plus an additional light-like component from M^4 gamma matrices. If the space spanned by Kähler-Dirac gammas has dimension D smaller than 3 co-associativity is automatic. If the dimension of this space is $D = 3$ it can happen that the triplet of gammas spans by multiplication entire octonionic algebra. For $D = 4$ the situation is of course non-trivial.
2. For Kähler-Dirac gamma matrices the notion of co-associativity can produce problems since Kähler-Dirac gamma matrices do not in general span the tangent space. What does co-associativity mean now? Should one replace normal space with orthogonal complement of the space spanned by Kähler-Dirac gamma matrices? Co-associativity option must be considered for $D = 4$ only. CP_2 type vacuum extremals provide a good example. In this case the Kähler-Dirac gamma matrices reduce to sums of ordinary CP_2 gamma matrices and light-like M^4 contribution. The orthogonal complement for the Kähler-Dirac gamma matrices consists of dual light-like gamma matrix and two gammas orthogonal to it: this space is subspace of M^4 and trivially associative.

5.1 Basic Idea Behind $M^8 - M^4 \times CP_2$ Duality

If four-surfaces $X^4 \subset M^8$ under some conditions define 4-surfaces in $M^4 \times CP_2$ indirectly, the spontaneous compactification of super string models would correspond in TGD to two different manners to interpret the space-time surface. This correspondence could be called number theoretical compactification or $M^8 - H$ duality.

The hard mathematical facts behind the notion of number theoretical compactification are following.

1. One must assume that M^8 has unique decomposition $M^8 = M^4 \times E^4$. This decomposition generalizes also to the case of M_c^8 . This would be most naturally due to Kähler structure in E^4 defined by a self-dual Kähler form defining parallel constant electric and magnetic fields in Euclidian sense. Besides Kähler form there is vector field coupling to sigma matrix representing the analog of strong isospin: the corresponding octonionic sigma matrix however is imaginary unit times gamma matrix - say ie_1 in M^4 - defining a preferred plane M^2 in

M^4 . Here it is essential that the gamma matrices of E^4 defined in terms of octonion units commute to gamma matrices in M^4 . What is involved becomes clear from the Fano triangle illustrating octonionic multiplication table.

2. The space of hyper-complex structures of the hyper-octonion space - they correspond to the choices of plane $M^2 \subset M^8$ - is parameterized by 6-sphere $S^6 = G^2/SU(3)$. The subgroup $SU(3)$ of the full automorphism group G_2 respects the a priori selected complex structure and thus leaves invariant one octonionic imaginary unit, call it e_1 . Fixed complex structure therefore corresponds to a point of S^6 .
3. Quaternionic sub-algebras of M^8 (and M_c^8) are parametrized by $G_2/U(2)$. The quaternionic sub-algebras of octonions with fixed complex structure (that is complex sub-space defined by real and preferred imaginary unit and parametrized by a point of S^6) are parameterized by $SU(3)/U(2) = CP_2$ just as the complex planes of quaternion space are parameterized by $CP_1 = S^2$. Same applies to hyper-quaternionic sub-spaces of hyper-octonions. $SU(3)$ would thus have an interpretation as the isometry group of CP_2 , as the automorphism sub-group of octonions, and as color group. Thus the space of quaternionic structures can be parametrized by the 10-dimensional space $G_2/U(2)$ decomposing as $S^6 \times CP_2$ locally.
4. The basic result behind number theoretic compactification and $M^8 - H$ duality is that associative sub-spaces $M^4 \subset M^8$ containing a fixed commutative sub-space $M^2 \subset M^8$ are parameterized by CP_2 . The choices of a fixed hyper-quaternionic basis $1, e_1, e_2, e_3$ with a fixed complex sub-space (choice of e_1) are labeled by $U(2) \subset SU(3)$. The choice of e_2 and e_3 amounts to fixing $e_2 \pm \sqrt{-1}e_3$, which selects the $U(2) = SU(2) \times U(1)$ subgroup of $SU(3)$. $U(1)$ leaves 1 invariant and induced a phase multiplication of e_1 and $e_2 \pm e_3$. $SU(2)$ induces rotations of the spinor having e_2 and e_3 components. Hence all possible completions of $1, e_1$ by adding e_2, e_3 doublet are labeled by $SU(3)/U(2) = CP_2$.

Consider now the formulation of $M^8 - H$ duality.

1. The idea of the standard formulation is that associative manifold $X^4 \subset M^8$ has at its each point associative tangent plane. That is X^4 corresponds to an integrable distribution of $M^2(x) \subset M^8$ parametrized 4-D coordinate x that is map $x \rightarrow S^6$ such that the 4-D tangent plane is hyper-quaternionic for each x .
2. Since the Kähler structure of M^8 implies unique decomposition $M^8 = M^4 \times E^4$, this surface in turn defines a surface in $M^4 \times CP_2$ obtained by assigning to the point of 4-surface point $(m, s) \in H = M^4 \times CP_2$: $m \in M^4$ is obtained as *projection* $M^8 \rightarrow M^4$ (this is modification to the earlier definition) and $s \in CP_2$ parametrizes the quaternionic tangent plane as point of CP_2 . Here the local decomposition $G_2/U(2) = S^6 \times CP_2$ is essential for achieving uniqueness.
3. One could also map the associative surface in M^8 to surface in 10-dimensional $S^6 \times CP_2$. In this case the metric of the image surface cannot have Minkowskian signature and one cannot assume that the induced metrics are identical. It is not known whether S^6 allows genuine complex structure and Kähler structure which is essential for TGD formulation.
4. Does duality imply the analog of associativity for $X^4 \subset H$? The tangent space of H can be seen as a sub-space of doubly complexified quaternions. Could one think that quaternionic sub-space is replaced with sub-space analogous to that spanned by real parts of complexified quaternions? The attempts to define this notion do not however look promising. One can however define associativity and co-associativity for the tangent space M^8 of H using octonionization and can formulate it also terms of induced gamma matrices.
5. The associativity defined in terms of induced gamma matrices in both in M^8 and H has the interesting feature that one can assign to the associative surface in H a new associative surface in H by assigning to each point of the space-time surface its M^4 projection and point of CP_2 characterizing its associative tangent space or co-associative normal space. It seems that one continue this series ad infinitum and generate new solutions of field equations! This brings in mind iteration which is standard manner to generate fractals as limiting sets. This certainly makes the heart of mathematician beat.

6. Kähler structure in $E^4 \subset M^8$ guarantees natural $M^4 \times E^4$ decomposition. Does associativity imply preferred extremal property or vice versa, or are the two notions equivalent or only consistent with each other for preferred extremals?

A couple of comments are in order.

1. This definition generalizes to the case of M_c^8 : all that matters is that tangent space is complexified quaternionic and there is a unique identification $M^4 \subset M_c^8$: this allows to assign the point of 4-surfaces a point of $M^4 \times CP_2$. The generalization is needed if one wants to formulate the hypothesis about O_c real-analyticity as a manner to build quaternionic space-time surfaces properly.
2. This definition differs from the first proposal for years ago stating that each point of X^4 contains a *fixed* $M^2 \subset M^4$ rather than $M_2(x) \subset M^8$ and also from the proposal assuming integrable distribution of $M^2(x) \subset M^4$. The older proposals are not consistent with the properties of massless extremals and string like objects for which the counterpart of M^2 depends on space-time point and is not restricted to M^4 . The earlier definition $M^2(x) \subset M^4$ was problematic in the co-associative case since for the Euclidian signature is not clear what the counterpart of $M^2(x)$ could be.
3. The new definition is consistent with the existence of Hamilton-Jacobi structure meaning slicing of space-time surface by string world sheets and partonic 2-surfaces with points of partonic 2-surfaces labeling the string world sheets [K2]. This structure has been proposed to characterize preferred extremals in Minkowskian space-time regions at least.
4. Co-associative Euclidian 4-surfaces, say CP_2 type vacuum extremal do not contain integrable distribution of $M^2(x)$. It is normal space which contains $M^2(x)$. Does this have some physical meaning? Or does the surface defined by $M^2(x)$ have Euclidian analog?

A possible identification of the analog would be as string world sheet at which W boson field is pure gauge so that the modes of the modified Dirac operator [K28] restricted to the string world sheet have well-defined em charge. This condition appears in the construction of solutions of Kähler-Dirac operator.

For octonionic spinor structure the W coupling is however absent so that the condition does not make sense in M^8 . The number theoretic condition would be as commutative or co-commutative surface for which imaginary units in tangent space transform to real and imaginary unit by a multiplication with a fixed imaginary unit! One can also formulate co-associativity as a condition that tangent space becomes associative by a multiplication with a fixed imaginary unit.

There is also another justification for the distribution of Euclidian tangent planes. The idea about associativity as a fundamental dynamical principle can be strengthened to the statement that space-time surface allows slicing by hyper-complex or complex 2-surfaces, which are commutative or co-commutative inside space-time surface. The physical interpretation would be as Minkowskian or Euclidian string world sheets carrying spinor modes. This would give a connection with string model and also with the conjecture about the general structure of preferred extremals.

5. Minimalist could argue that the minimal definition requires octonionic structure and associativity *only* in M^8 . There is no need to introduce the counterpart of Kähler action in M^8 since the dynamics would be based on associativity or co-associativity alone. The objection is that one must assume the decomposition $M^8 = M^4 \times E^4$ without any justification.

The map of space-time surfaces to those of $H = M^4 \times CP_2$ implies that the space-time surfaces in H are in well-defined sense quaternionic. As a matter of fact, the standard spinor structure of H can be regarded as quaternionic in the sense that gamma matrices are essentially tensor products of quaternionic gamma matrices and reduce in matrix representation for quaternions to ordinary gamma matrices. Therefore the idea that one should introduce octonionic gamma matrices in H is questionable. If all goes as in dreams, the mere associativity or co-associativity would code for the preferred extremal property of Kähler action in

H . One could at least hope that associativity/co-associativity in H is consistent with the preferred extremal property.

6. One can also consider a variant of associativity based on modified gamma matrices - but only in H . This notion does not make sense in M^8 since the very existence of quaternionic tangent plane makes it possible to define $M^8 - H$ duality map. The associativity for modified gamma matrices is however consistent with what is known about extremals of Kähler action. The associativity based on induced gamma matrices would correspond to the use of the space-time volume as action. Note however that gamma matrices are *not* necessary in the definition.

5.2 Hyper-Octonionic Pauli “Matrices” And The Definition Of Associativity

Octonionic Pauli matrices suggest an interesting possibility to define precisely what associativity means at the level of M^8 using gamma matrices (for background see [K32, K31]).

1. According to the standard definition space-time surface $X^4 \subset M^8$ is associative if the tangent space at each point of X^4 in $X^4 \subset M^8$ picture is associative. The definition can be given also in terms of octonionic gamma matrices whose definition is completely straightforward.
2. Could/should one define the analog of associativity at the level of H ? One can identify the tangent space of H as M^8 and can define octonionic structure in the tangent space and this allows to define associativity locally. One can replace gamma matrices with their octonionic variants and formulate associativity in terms of them locally and this should be enough.

Skeptic however reminds M^4 allows hyper-quaternionic structure and CP_2 quaternionic structure so that complexified quaternionic structure would look more natural for H . The tangent space would decompose as $M^8 = HQ + ijQ$, where j is commuting imaginary unit and HQ is spanned by real unit and by units iI_k , where i second commuting imaginary unit and I_k denotes quaternionic imaginary units. There is no need to make anything associative.

There is however far from obvious that octonionic spinor structure can be (or need to be!) defined globally. The lift of the CP_2 spinor connection to its octonionic variant has questionable features: in particular vanishing of the charged part and reduction of neutral part to photon. Therefore it is unclear whether associativity condition makes sense for $X^4 \subset M^4 \times CP_2$. What makes it so fascinating is that it would allow to iterate duality as a sequences $M^8 \rightarrow H \rightarrow H \dots$. This brings in mind the functional composition of octonion real-analytic functions suggested to produce associative or co-associative surfaces.

I have not been able to settle the situation. What seems the working option is associativity in both M^8 and H and Kähler-Dirac gamma matrices defined by appropriate Kähler action and correlation between associativity and preferred extremal property.

5.3 Are Kähler And Spinor Structures Necessary In M^8 ?

If one introduces M^8 as dual of H , one cannot avoid the idea that hyper-quaternionic surfaces obtained as images of the preferred extremals of Kähler action in H are also extremals of M^8 Kähler action with same value of Kähler action defining Kähler function. As found, this leads to the conclusion that the $M^8 - H$ duality is Kähler isometry. Coupling of spinors to Kähler potential is the next step and this in turn leads to the introduction of spinor structure so that quantum TGD in H should have full M^8 dual.

5.3.1 Are also the 4-surfaces in M^8 preferred extremals of Kähler action?

It would be a mathematical miracle if associative and co-associative surfaces in M^8 would be in 1-1 correspondence with preferred extremals of Kähler action. This motivates the question whether Kähler action make sense also in M^8 . This does not exclude the possibility that associativity implies or is equivalent with the preferred extremal property.

One expects a close correspondence between preferred extremals: also now vacuum degeneracy is obtained, one obtains massless extremals, string like objects, and counterparts of CP_2 type

vacuum extremals. All known extremals would be associative or co-associative if modified gamma matrices define the notion (possible only in the case of H).

The strongest form of duality would be that the space-time surfaces in M^8 and H have same induced metric same induced Kähler form. The basic difference would be that the spinor connection for surfaces in M^8 would be however neutral and have no left handed components and only em gauge potential. A possible interpretation is that M^8 picture defines a theory in the phase in which electroweak symmetry breaking has happened and only photon belongs to the spectrum.

The question is whether one can define WCW also for M^8 . Certainly it should be equivalent with WCW for H : otherwise an inflation of poorly defined notions follows. Certainly the general formulation of the WCW geometry generalizes from H to M^8 . Since the matrix elements of symplectic super-Hamiltonians defining WCW gamma matrices are well defined as matrix elements involve spinor modes with Gaussian harmonic oscillator behavior, the non-compactness of E^4 does not pose any technical problems.

5.3.2 Spinor connection of M^8

There are strong physical constraints on M^8 dual and they could kill the hypothesis. The basic constraint to the spinor structure of M^8 is that it reproduces basic facts about electro-weak interactions. This includes neutral electro-weak couplings to quarks and leptons identified as different H -chiralities and parity breaking.

1. By the flatness of the metric of E^4 its spinor connection is trivial. E^4 however allows full S^2 of covariantly constant Kähler forms so that one can accommodate free independent Abelian gauge fields assuming that the independent gauge fields are orthogonal to each other when interpreted as realizations of quaternionic imaginary units. This is possible but perhaps a more natural option is the introduction of just single Kähler form as in the case of CP_2 .
2. One should be able to distinguish between quarks and leptons also in M^8 , which suggests that one introduce spinor structure and Kähler structure in E^4 . The Kähler structure of E^4 is unique apart from $SO(3)$ rotation since all three quaternionic imaginary units and the unit vectors formed from them allow a representation as an antisymmetric tensor. Hence one must select one preferred Kähler structure, that is fix a point of S^2 representing the selected imaginary unit. It is natural to assume different couplings of the Kähler gauge potential to spinor chiralities representing quarks and leptons: these couplings can be assumed to be same as in case of H .
3. Electro-weak gauge potential has vectorial and axial parts. Em part is vectorial involving coupling to Kähler form and Z^0 contains both axial and vector parts. The naive replacement of sigma matrices appearing in the coupling of electroweak gauge fields takes the left handed parts of these fields to zero so that only neutral part remains. Further, gauge fields correspond to curvature of CP_2 which vanishes for E^4 so that only Kähler form form remains. Kähler form couples to 3L and q so that the basic asymmetry between leptons and quarks remains. The resulting field could be seen as analog of photon.
4. The absence of weak parts of classical electro-weak gauge fields would conform with the standard thinking that classical weak fields are not important in long scales. A further prediction is that this distinction becomes visible only in situations, where H picture is necessary. This is the case at high energies, where the description of quarks in terms of $SU(3)$ color is convenient whereas $SO(4)$ QCD would require large number of E^4 partial waves. At low energies large number of $SU(3)$ color partial waves are needed and the convenient description would be in terms of $SO(4)$ QCD. Proton spin crisis might relate to this.

5.3.3 Dirac equation for leptons and quarks in M^8

Kähler gauge potential would also couple to octonionic spinors and explain the distinction between quarks and leptons.

1. The complexified octonions representing H spinors decompose to $1 + 1 + 3 + \bar{3}$ under $SU(3)$ representing color automorphisms but the interpretation in terms of QCD color does not

make sense. Rather, the triplet and single combine to two weak isospin doublets and quarks and leptons corresponds to “spin” states of octonion valued 2-spinor. The conservation of quark and lepton numbers follows from the absence of coupling between these states.

2. One could modify the coupling so that coupling is on electric charge by coupling it to electromagnetic charge which as a combination of unit matrix and sigma matrix is proportional to $1 + kI_1$, where I_1 is octonionic imaginary unit in $M^2 \subset M^4$. The complexified octonionic units can be chosen to be eigenstates of Q_{em} so that Laplace equation reduces to ordinary scalar Laplacian with coupling to self-dual em field.
3. One expects harmonic oscillator like behavior for the modes of the Dirac operator of M^8 since the gauge potential is linear in E^4 coordinates. One possibility is Cartesian coordinates is $A(A_x, A_y, A_z, A_t) = k(-y, x, t, -z)$. The coupling would make E^4 effectively a compact space.
4. The square of Dirac operator gives potential term proportional to $r^2 = x^2 + y^2 + z^2 + t^2$ so that the spectrum of 4-D harmonic oscillator operator and $SO(4)$ harmonics localized near origin are expected. For harmonic oscillator the symmetry enhances to $SU(4)$.

If one replaces Kähler coupling with em charge symmetry breaking of $SO(4)$ to vectorial $SO(3)$ is expected since the coupling is proportional to $1 + ike_1$ defining electromagnetic charge. Since the basis of complexified quaternions can be chosen to be eigenstates of e_1 under multiplication, octonionic spinors are eigenstates of em charge and one obtains two color singlets $1 \pm e_1$ and color triplet and antitriplet. The color triplets cannot be however interpreted in terms of quark color.

Harmonic oscillator potential is expected to enhance $SO(3)$ to $SU(3)$. This suggests the reduction of the symmetry to $SU(3) \times U(1)$ corresponding to color symmetry and em charge so that one would have same basic quantum numbers as to CP_2 harmonics. An interesting question is how the spectrum and mass squared eigenvalues of harmonics differ from those for CP_2 .

5. In the square of Dirac equation $J^{kl}\Sigma_{kl}$ term distinguishes between different em charges (Σ_{kl} reduces by self duality and by special properties of octonionic sigma matrices to a term proportional to iI_1 and complexified octonionic units can be chosen to be its eigenstates with eigen value ± 1). The vacuum mass squared analogous to the vacuum energy of harmonic oscillator is also present and this contribution are expected to cancel themselves for neutrinos so that they are massless whereas charged leptons and quarks are massive. It remains to be checked that quarks and leptons can be classified to triality $T = \pm 1$ and $t = 0$ representations of dynamical $SU(3)$ respectively.

5.3.4 What about the analog of Kähler Dirac equation

Only the octonionic structure in $T(M^8)$ is needed to formulate quaternionicity of space-time surfaces: the reduction to O_c -real-analyticity would be extremely nice but not necessary (O_c denotes complexified octonions needed to cope with Minkowskian signature). Most importantly, there might be no need to introduce Kähler action (and Kähler form) in M^8 . Even the octonionic representation of gamma matrices is un-necessary. Neither there is any absolute need to define octonionic Dirac equation and octonionic Kähler Dirac equation nor octonionic analog of its solutions nor the octonionic variants of imbedding space harmonics.

It would be of course nice if the general formulas for solutions of the Kähler Dirac equation in H could have counterparts for octonionic spinors satisfying quaternionicity condition. One can indeed wonder whether the restriction of the modes of induced spinor field to string world sheets defined by integrable distributions of hyper-complex spaces $M^2(x)$ could be interpreted in terms of commutativity of fermionic physics in M^8 . $M^8 - H$ correspondence could map the octonionic spinor fields at string world sheets to their quaternionic counterparts in H . The fact that only holomorphy is involved with the definition of modes could make this map possible.

5.4 How Could One Solve Associativity/Co-Associativity Conditions?

The natural question is whether and how one could solve the associativity/-co-associativity conditions explicitly. One can imagine two approaches besides $M^8 \rightarrow H \rightarrow H\dots$ iteration generating new solutions from existing ones.

5.4.1 Could octonion-real analyticity be equivalent with associativity/co-associativity?

Analytic functions provide solutions to 2-D Laplace equations and one might hope that also the field equations could be solved in terms of octonion-real-analyticity at the level of M^8 perhaps also at the level of H . Signature however causes problems - at least technical. Also the compactness of CP_2 causes technical difficulties but they need not be insurmountable.

For E^8 the tangent space would be genuinely octonionic and one can define the notion octonion-real analytic map as a generalization of real-analytic function of complex variables (the coefficients of Laurent series are real to guarantee associativity of the series). The argument is complexified octonion in $O \oplus iO$ forming an algebra but not a field. The norm square is Minkowskian as difference of two Euclidian octonionic norms: $N(o_1 + io_2) = N(o_1) - N(o_2)$ and vanishes at 15-D light cone boundary. Obviously, differential calculus is possible outside the light-cone boundary. Rational analytic functions have however poles at the light-cone boundary. One can wonder whether the poles at M^4 light-cone boundary, which is subset of 15-D light-cone boundary could have physical significance and relevant for the role of causal diamonds in ZEO.

The candidates for associative surfaces defined by O_c -real-analytic functions (I use O_c for complexified octonions) have Minkowskian signature of metric and are 4-surfaces at which the projection of $f(o_1 + io_2)$ to $Im(O_1)$, $iIm(O_2)$, and $iRe(Q_2) \oplus Im(Q_1)$ vanish so that only the projection to hyper-quaternionic Minkowskian sub-space $M^4 = Re(Q_1) + iIm(Q_2)$ with signature $(1, -1, -, 1-)$ is non-vanishing. The inverse image need not belong to M^8 and in general it belongs to M_c^8 but this is not a problem: all that is needed that the tangent space of inverse image is complexified quaternionic. If this is the case then $M^8 - H$ duality maps the tangent space of the inverse image to CP_2 point and image itself defines the point of M^4 so that a point of H is obtained. Co-associative surfaces would be surfaces for which the projections of image to $Re(O_1)$, $iRe(O_2)$, and to $Im(O_1)$ vanish so that only the projection to $iIm(O_2)$ with signature $(-1, -1, -1, -1)$ is non-vanishing.

The inverse images as 4-D sub-manifolds of M_c^8 (not M^8 !) are excellent candidates for associative and co-associative 4-surfaces since $M^8 - H$ duality assigns to them a 4-surface in $M^4 \times CP_2$ if the tangent space at given point is complexified quaternionic. This is true if one believes on the analytic continuation of the intuition from complex analysis (the image of real axes under the map defined by O_c -real-analytic function is real axes in the new coordinates defined by the map: the intuition results by replacing "real" by "complexified quaternionic"). The possibility to solve field equations in this manner would be of enormous significance since besides basic arithmetic operations also the functional decomposition of O_c -real-analytic functions produces similar functions. One could speak of the algebra of space-time surfaces.

What is remarkable that the complexified octonion real analytic functions are obtained by analytic continuation from single real valued function of real argument. The real functions form naturally a hierarchy of polynomials (maybe also rational functions) and number theoretic vision suggests that there coefficients are rationals or algebraic numbers. Already for rational coefficients hierarchy of algebraic extensions of rationals results as one solves the vanishing conditions. There is a temptation to regard this hierarchy coding for space-time sheets as an analog of DNA.

Note that in the recent formulation there is no need to pose separately the condition about integrable distribution of $M^2(x) \subset M^4$.

5.4.2 Quaternionicity condition for space-time surfaces

Quaternionicity actually has a surprisingly simple formulation at the level of space-time surfaces. The following discussion applies to both M^8 and H with minor modifications if one accepts that also H can allow octonionic tangent space structure, which does not require gamma matrices.

1. Quaternionicity is equivalent with associativity guaranteed by the vanishing of the associator $A(a, b, c) = a(bc) - (ab)c$ for any triplet of imaginary tangent vectors in the tangent space of

the space-time surface. The condition must hold true for purely imaginary combinations of tangent vectors.

2. If one is able to choose the coordinates in such a manner that one of the tangent vectors corresponds to real unit (in the imbedding map imbedding space M^4 coordinate depends only on the time coordinate of space-time surface), the condition reduces to the vanishing of the octonionic product of remaining three induced gamma matrices interpreted as octonionic gamma matrices. This condition looks very simple - perhaps too simple!- since it involves only first derivatives of the imbedding space vectors.

One can of course whether quaternionicity conditions replace field equations or only select preferred extremals. In the latter case, one should be able to prove that quaternionicity conditions are consistent with the field equations.

3. Field equations would reduce to tri-linear equations in in the gradients of imbedding space coordinates (rather than involving imbedding space coordinates quadratically). Sum of analogs of 3×3 determinants deriving from $a \times (b \times b)$ for different octonion units is involved.
4. Written explicitly field equations give in terms of vielbein projections e_α^A , vielbein vectors e_k^A , coordinate gradients $\partial_\alpha h^k$ and octonionic structure constants f_{ABC} the following conditions stating that the projections of the octonionic associator tensor to the space-time surface vanishes:

$$\begin{aligned}
 e_\alpha^A e_\beta^B e_\gamma^C A_{ABC}^E &= 0 \ , \\
 A_{ABC}^E &= f_{AD}^E f_{BC}^D - f_{AB}^D f_{DC}^E \ , \\
 e_\alpha^A &= \partial_\alpha h^k e_k^A \ , \\
 \Gamma_k &= e_k^A \gamma_A \ .
 \end{aligned}
 \tag{5.1}$$

The very naive idea would be that the field equations are indeed integrable in the sense that they reduce to these tri-linear equations. Tri-linearity in derivatives is highly non-trivial outcome simplifying the situation further. These equations can be formulated as the as purely algebraic equations written above plus integrability conditions

$$F_{\alpha\beta}^A = D_\alpha e_\beta^A - D_\beta e_\alpha^A = 0 \ . \tag{5.2}$$

One could say that vielbein projections define an analog of a trivial gauge potential. Note however that the covariant derivative is defined by spinor connection rather than this effective gauge potential which reduces to that in $SU(2)$. Similar formulation holds true for field equations and one should be able to see whether the field equations formulated in terms of derivatives of vielbein projections commute with the associativity conditions.

5. The quaternionicity conditions can be formulated as vanishing of generalization of Cayley's hyperdeterminant for "hypermatrix" a_{ijk} with 2-valued indexed (see <http://tinyurl.com/ya7h3n9z>). Now one has 8 hyper-matrices with 3 8-valued indices associated with the vanishing $A_{BCD}^E x^B y^C z^D = 0$ of trilinear forms defined by the associators. The conditions say something only about the octonion structure constants and since octonionic space allow quaternionic sub-spaces these conditions must be satisfied.

The inspection of the Fano triangle [A8] (see **Fig. 1**) expressing the multiplication table for octonionic imaginary units reveals that give any two imaginary octonion units e_1 and e_2 their product $e_1 e_2$ (or equivalently commutator) is imaginary octonion unit (2 times octonion unit) and the three units span together with real unit quaternionic sub-algebra. There it seems that one can generate local quaternionic sub-space from two imaginary units plus real unit. This generalizes to the vielbein components of tangent vectors of space-time surface and one can build the solutions

to the quaternionicity conditions from vielbein projections e_1, e_2 , their product $e_3 = k(x)e_1e_2$ and real fourth “time-like” vielbein component which must be expressible as a combination of real unit and imaginary units:

$$e_0 = a \times 1 + b^i e_i$$

For static solutions this condition is trivial. Here summation over i is understood in the latter term. Besides these conditions one has integrability conditions and field equations for Kähler action. This formulation suggests that quaternionicity is additional - perhaps defining - property of preferred extremals.

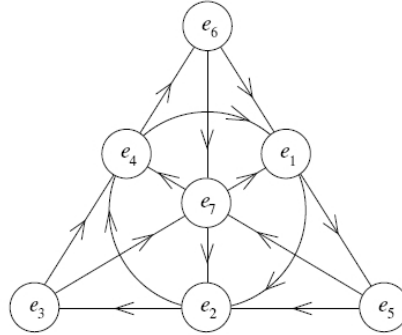


Figure 1: Octonionic triangle: the six lines and one circle containing three vertices define the seven associative triplets for which the multiplication rules of the ordinary quaternion imaginary units hold true. The arrow defines the orientation for each associative triplet. Note that the product for the units of each associative triplets equals to real unit apart from sign factor.

5.5 Quaternionicity At The Level Of Imbedding Space Quantum Numbers

From the multiplication table of octonions as illustrated by Fano triangle [A8] one finds that all edges of the triangle, the middle circle and the three the lines connecting vertices to the midpoints of opposite side define triplets of quaternionic units. This means that by taking real unit and any imaginary unit in quaternionic M^4 algebra spanning $M^2 \subset M^4$ and two imaginary units in the complement representing CP_2 tangent space one obtains quaternionic algebra. This suggests an explanation for the preferred M^2 contained in tangent space of space-time surface (the M^2 : s could form an integrable distribution). Four-momentum restricted to M^2 and I_3 and Y interpreted as tangent vectors in CP_2 tangent space defined quaterionic sub-algebra. This could give content for the idea that quantum numbers are quaternionic.

I have indeed proposed that the four-momentum belongs to M^2 . If $M^2(x)$ form a distribution as the proposal for the preferred extremals suggests this could reflect momentum exchanges between different points of the space-time surface such that total momentum is conserved or momentum exchange between two sheets connected by wormhole contacts.

5.6 Questions

In following some questions related to $M^8 - H$ duality are represented.

5.6.1 Could associativity condition be formulated using modified gamma matrices?

Skeptic can criticize the minimal form of $M^8 - H$ duality involving no Kähler action in M^8 is unrealistic. Why just Kähler action? What makes it so special? The only defense that I can

imagine is that Kähler action is in many respects unique choice.

An alternative approach would replace induced gamma matrices with the modified ones to get the correlation. In the case of M^8 this option cannot work. One cannot exclude it for H .

1. For Kähler action the Kähler-Dirac gamma matrices $\Gamma^\alpha = \frac{\partial L_K}{\partial h_\alpha^k} \Gamma^k$, $\Gamma_k = e_k^A \gamma_A$, assign to a given point of X^4 a 4-D space which need not be tangent space anymore or even its sub-space. The reason is that canonical momentum current contains besides the gravitational contribution coming from the induced metric also the “Maxwell contribution” from the induced Kähler form not parallel to space-time surface. In the case of M^8 the duality map to H is therefore lost.
2. The space spanned by the Kähler-Dirac gamma matrices need not be 4-dimensional. For vacuum extremals with at most 2-D CP_2 projection Kähler-Dirac gamma matrices vanish identically. For massless extremals they span 1-D light-like subspace. For CP_2 vacuum extremals the modified gamma matrices reduces to ordinary gamma matrices for CP_2 and the situation reduces to the quaternionicity of CP_2 . Also for string like objects the conditions are satisfied since the gamma matrices define associative sub-space as tangent space of $M^2 \times S^2 \subset M^4 \times CP_2$. It seems that associativity is satisfied by all known extremals. Hence Kähler-Dirac gamma matrices are flexible enough to realize associativity in H .
3. Kähler-Dirac gamma matrices in Dirac equation are required by super conformal symmetry for the extremals of action and they also guarantee that vacuum extremals defined by surfaces in $M^4 \times Y^2$, Y^2 a Lagrange sub-manifold of CP_2 , are trivially hyper-quaternionic surfaces. The modified definition of associativity in H does not affect in any manner $M^8 - H$ duality necessarily based on induced gamma matrices in M^8 allowing purely number theoretic interpretation of standard model symmetries. One can however argue that the most natural definition of associativity is in terms of induced gamma matrices in both M^8 and H .

Remark: A side comment not strictly related to associativity is in order. The anti-commutators of the Kähler-Dirac gamma matrices define an effective Riemann metric and one can assign to it the counterparts of Riemann connection, curvature tensor, geodesic line, volume, etc... One would have two different metrics associated with the space-time surface. Only if the action defining space-time surface is identified as the volume in the ordinary metric, these metrics are equivalent. The index raising for the effective metric could be defined also by the induced metric and it is not clear whether one can define Riemann connection also in this case. Could this effective metric have concrete physical significance and play a deeper role in quantum TGD? For instance, AdS-CFT duality leads to ask whether interactions be coded in terms of the gravitation associated with the effective metric.

Now skeptic can ask why should one demand $M^8 - H$ correspondence if one in any case is forced to introduced Kähler also at the level of M^8 ? Does $M^8 - H$ correspondence help to construct preferred extremals or does it only bring in a long list of conjectures? I can repeat the questions of the skeptic.

5.6.2 Minkowskian-Euclidian \leftrightarrow associative-co-associative?

The 8-dimensionality of M^8 allows to consider both associativity of the tangent space and associativity of the normal space- let us call this co-associativity of tangent space- as alternative options. Both options are needed as has been already found. Since space-time surface decomposes into regions whose induced metric possesses either Minkowskian or Euclidian signature, there is a strong temptation to propose that Minkowskian regions correspond to associative and Euclidian regions to co-associative regions so that space-time itself would provide both the description and its dual.

The proposed interpretation of conjectured associative-co-associative duality relates in an interesting manner to p-adic length scale hypothesis selecting the primes $p \simeq 2^k$, k positive integer as preferred p-adic length scales. $L_p \propto \sqrt{p}$ corresponds to the p-adic length scale defining the size of the space-time sheet at which elementary particle represented as CP_2 type extremal is topologically condensed and is of order Compton length. $L_k \propto \sqrt{k}$ represents the p-adic length scale of the wormhole contacts associated with the CP_2 type extremal and CP_2 size is the natural length

unit now. Obviously the quantitative formulation for associative-co-associative duality would be in terms $p \rightarrow k$ duality.

5.6.3 Can $M^8 - H$ duality be useful?

Skeptic could of course argue that $M^8 - H$ duality generates only an inflation of unproven conjectures. This might be the case. In the following I will however try to defend the conjecture. One can however find good motivations for $M^8 - H$ duality: both theoretical and physical.

1. If $M^8 - H$ duality makes sense for induced gamma matrices also in H , one obtains infinite sequence if dualities allowing to construct preferred extremals iteratively. This might relate to octonionic real-analyticity and composition of octonion-real-analytic functions.
2. $M^8 - H$ duality could provide much simpler description of preferred extremals of Kähler action as hyper-quaternionic surfaces. Unfortunately, it is not clear whether one should introduce the counterpart of Kähler action in M^8 and the coupling of M^8 spinors to Kähler form. Note that the Kähler form in E^4 would be self dual and have constant components: essentially parallel electric and magnetic field of same constant magnitude.
3. $M^8 - H$ duality provides insights to low energy physics, in particular low energy hadron physics. M^8 description might work when H -description fails. For instance, perturbative QCD which corresponds to H -description fails at low energies whereas M^8 description might become perturbative description at this limit. Strong $SO(4) = SU(2)_L \times SU(2)_R$ invariance is the basic symmetry of the phenomenological low energy hadron models based on conserved vector current hypothesis (CVC) and partially conserved axial current hypothesis (PCAC). Strong $SO(4) = SU(2)_L \times SU(2)_R$ relates closely also to electro-weak gauge group $SU(2)_L \times U(1)$ and this connection is not well understood in QCD description. $M^8 - H$ duality could provide this connection. Strong $SO(4)$ symmetry would emerge as a low energy dual of the color symmetry. Orbital $SO(4)$ would correspond to strong $SU(2)_L \times SU(2)_R$ and by flatness of E^4 spin like $SO(4)$ would correspond to electro-weak group $SU(2)_L \times U(1)_R \subset SO(4)$. Note that the inclusion of coupling to Kähler gauge potential is necessary to achieve respectable spinor structure in CP_2 . One could say that the orbital angular momentum in $SO(4)$ corresponds to strong isospin and spin part of angular momentum to the weak isospin.

This argument does not seem to be consistent with $SU(3) \times U(1) \subset SU(4)$ symmetry for Mx Dirac equation. One can however argue that $SU(4)$ symmetry combines $SO(4)$ multiplets together. Furthermore, $SO(4)$ represents the isometries leaving Kähler form invariant.

5.6.4 $M^8 - H$ duality in low energy physics and low energy hadron physics

$M^8 - H$ can be applied to gain a view about color confinement. The basic idea would be that $SO(4)$ and $SU(3)$ provide provide dual descriptions of quarks using E^4 and CP_2 partial waves and low energy hadron physics corresponds to a situation in which M^8 picture provides the perturbative approach whereas H picture works at high energies.

A possible interpretation is that the space-time surfaces vary so slowly in CP_2 degrees of freedom that can approximate CP_2 with a small region of its tangent space E^4 . One could also say that color interactions mask completely electroweak interactions so that the spinor connection of CP_2 can be neglected and one has effectively E^4 . The basic prediction is that $SO(4)$ should appear as dynamical symmetry group of low energy hadron physics and this is indeed the case.

Consider color confinement at the long length scale limit in terms of $M^8 - H$ duality.

1. At high energy limit only lowest color triplet color partial waves for quarks dominate so that QCD description becomes appropriate whereas very higher color partial waves for quarks and gluons are expected to appear at the confinement limit. Since WCW degrees of freedom begin to dominate, color confinement limit transcends the descriptive power of QCD.
2. The success of $SO(4)$ sigma model in the description of low lying hadrons would directly relate to the fact that this group labels also the E^4 Hamiltonians in M^8 picture. Strong $SO(4)$ quantum numbers can be identified as orbital counterparts of right and left handed electro-weak isospin coinciding with strong isospin for lowest quarks. In sigma model pion

and sigma boson form the components of E^4 valued vector field or equivalently collection of four E^4 Hamiltonians corresponding to spherical E^4 coordinates. Pion corresponds to S^3 valued unit vector field with charge states of pion identifiable as three Hamiltonians defined by the coordinate components. Sigma is mapped to the Hamiltonian defined by the E^4 radial coordinate. Excited mesons corresponding to more complex Hamiltonians are predicted.

3. The generalization of sigma model would assign to quarks E^4 partial waves belonging to the representations of $SO(4)$. The model would involve also 6 $SO(4)$ gluons and their $SO(4)$ partial waves. At the low energy limit only lowest representations would be important whereas at higher energies higher partial waves would be excited and the description based on CP_2 partial waves would become more appropriate.
4. The low energy quark model would rely on quarks moving $SO(4)$ color partial waves. Left *resp.* right handed quarks could correspond to $SU(2)_L$ *resp.* $SU(2)_R$ triplets so that spin statistics problem would be solved in the same manner as in the standard quark model.
5. Family replication phenomenon is described in TGD framework the same manner in both cases so that quantum numbers like strangeness and charm are not fundamental. Indeed, p-adic mass calculations allowing fractally scaled up versions of various quarks allow to replace Gell-Mann mass formula with highly successful predictions for hadron masses [K19].

To my opinion these observations are intriguing enough to motivate a concrete attempt to construct low energy hadron physics in terms of $SO(4)$ gauge theory.

5.7 Summary

The overall conclusion is that the most convincing scenario relies on the associativity/co-associativity of space-time surfaces define by induced gamma matrices and applying both for M^8 and H . The fact that the duality can be continued to an iterated sequence of duality maps $M^8 \rightarrow H \rightarrow H \dots$ is what makes the proposal so fascinating and suggests connection with fractality.

The introduction of Kähler action and coupling of spinors to Kähler gauge potentials is highly natural. One can also consider the idea that the space-time surfaces in M^8 and H have same induced metric and Kähler form: for iterated duality map this would mean that the steps in the map produce space-time surfaces which identical metric and Kähler form so that the sequence might stop. M_H^8 duality might provide two descriptions of same underlying dynamics: M^8 description would apply in long length scales and H description in short length scales.

6 Weak Form Electric-Magnetic Duality And Its Implications

The notion of electric-magnetic duality [B2] was proposed first by Olive and Montonen and is central in $\mathcal{N} = 4$ supersymmetric gauge theories. It states that magnetic monopoles and ordinary particles are two different phases of theory and that the description in terms of monopoles can be applied at the limit when the running gauge coupling constant becomes very large and perturbation theory fails to converge. The notion of electric-magnetic self-duality is more natural since for CP_2 geometry Kähler form is self-dual and Kähler magnetic monopoles are also Kähler electric monopoles and Kähler coupling strength is by quantum criticality renormalization group invariant rather than running coupling constant. The notion of electric-magnetic (self-)duality emerged already two decades ago in the attempts to formulate the Kähler geometric of world of classical worlds. Quite recently a considerable step of progress took place in the understanding of this notion [K8]. What seems to be essential is that one adopts a weaker form of the self-duality applying at partonic 2-surfaces. What this means will be discussed in the sequel.

Every new idea must be of course taken with a grain of salt but the good sign is that this concept leads to precise predictions. The point is that elementary particles do not generate monopole fields in macroscopic length scales: at least when one considers visible matter. The first question is whether elementary particles could have vanishing magnetic charges: this turns out to be impossible. The next question is how the screening of the magnetic charges could take place and leads to

an identification of the physical particles as string like objects identified as pairs magnetic charged wormhole throats connected by magnetic flux tubes.

1. The first implication is a new view about electro-weak massivation reducing it to weak confinement in TGD framework. The second end of the string contains particle having electroweak isospin neutralizing that of elementary fermion and the size scale of the string is electro-weak scale would be in question. Hence the screening of electro-weak force takes place via weak confinement realized in terms of magnetic confinement.
2. This picture generalizes to the case of color confinement. Also quarks correspond to pairs of magnetic monopoles but the charges need not vanish now. Rather, valence quarks would be connected by flux tubes of length of order hadron size such that magnetic charges sum up to zero. For instance, for baryonic valence quarks these charges could be $(2, -1, -1)$ and could be proportional to color hyper charge.
3. The highly non-trivial prediction making more precise the earlier stringy vision is that elementary particles are string like objects: this could become manifest at LHC energies.
4. The weak form electric-magnetic duality together with Beltrami flow property of Kähler leads to the reduction of Kähler action to Chern-Simons action so that TGD reduces to almost topological QFT and that Kähler function is explicitly calculable. This has enormous impact concerning practical calculability of the theory.
5. One ends up also to a general solution ansatz for field equations from the condition that the theory reduces to almost topological QFT. The solution ansatz is inspired by the idea that all isometry currents are proportional to Kähler current which is integrable in the sense that the flow parameter associated with its flow lines defines a global coordinate. The proposed solution ansatz would describe a hydrodynamical flow with the property that isometry charges are conserved along the flow lines (Beltrami flow). A general ansatz satisfying the integrability conditions is found.

The strongest form of the solution ansatz states that various classical and quantum currents flow along flow lines of the Beltrami flow defined by Kähler current (Kähler magnetic field associated with Chern-Simons action). Intuitively this picture is attractive. A more general ansatz would allow several Beltrami flows meaning multi-hydrodynamics. The integrability conditions boil down to two scalar functions: the first one satisfies massless d'Alembert equation in the induced metric and the gradients of the scalar functions are orthogonal. The interpretation in terms of momentum and polarization directions is natural.

6.1 Could A Weak Form Of Electric-Magnetic Duality Hold True?

Holography means that the initial data at the partonic 2-surfaces should fix the WCW metric. A weak form of this condition allows only the partonic 2-surfaces defined by the wormhole throats at which the signature of the induced metric changes. A stronger condition allows all partonic 2-surfaces in the slicing of space-time sheet to partonic 2-surfaces and string world sheets. Number theoretical vision suggests that hyper-quaternionicity *resp.* co-hyperquaternionicity constraint could be enough to fix the initial values of time derivatives of the imbedding space coordinates in the space-time regions with Minkowskian *resp.* Euclidian signature of the induced metric. This is a condition on modified gamma matrices and hyper-quaternionicity states that they span a hyper-quaternionic sub-space.

6.1.1 Definition of the weak form of electric-magnetic duality

One can also consider alternative conditions possibly equivalent with this condition. The argument goes as follows.

1. The expression of the matrix elements of the metric and Kähler form of WCW in terms of the Kähler fluxes weighted by Hamiltonians of δM_{\pm}^4 at the partonic 2-surface X^2 looks very attractive. These expressions however carry no information about the 4-D tangent space of the partonic 2-surfaces so that the theory would reduce to a genuinely 2-dimensional theory,

which cannot hold true. One would like to code to the WCW metric also information about the electric part of the induced Kähler form assignable to the complement of the tangent space of $X^2 \subset X^4$.

2. Electric-magnetic duality of the theory looks a highly attractive symmetry. The trivial manner to get electric magnetic duality at the level of the full theory would be via the identification of the flux Hamiltonians as sums of of the magnetic and electric fluxes. The presence of the induced metric is however troublesome since the presence of the induced metric means that the simple transformation properties of flux Hamiltonians under symplectic transformations -in particular color rotations- are lost.
3. A less trivial formulation of electric-magnetic duality would be as an initial condition which eliminates the induced metric from the electric flux. In the Euclidian version of 4-D YM theory this duality allows to solve field equations exactly in terms of instantons. This approach involves also quaternions. These arguments suggest that the duality in some form might work. The full electric magnetic duality is certainly too strong and implies that space-time surface at the partonic 2-surface corresponds to piece of CP_2 type vacuum extremal and can hold only in the deep interior of the region with Euclidian signature. In the region surrounding wormhole throat at both sides the condition must be replaced with a weaker condition.
4. To formulate a weaker form of the condition let us introduce coordinates (x^0, x^3, x^1, x^2) such (x^1, x^2) define coordinates for the partonic 2-surface and (x^0, x^3) define coordinates labeling partonic 2-surfaces in the slicing of the space-time surface by partonic 2-surfaces and string world sheets making sense in the regions of space-time sheet with Minkowskian signature. The assumption about the slicing allows to preserve general coordinate invariance. The weakest condition is that the generalized Kähler electric fluxes are apart from constant proportional to Kähler magnetic fluxes. This requires the condition

$$J^{03}\sqrt{g_4} = KJ_{12} . \quad (6.1)$$

A more general form of this duality is suggested by the considerations of [K15] reducing the hierarchy of Planck constants to basic quantum TGD and also reducing Kähler function for preferred extremals to Chern-Simons terms [B1] at the boundaries of CD and at light-like wormhole throats. This form is following

$$J^{n\beta}\sqrt{g_4} = K\epsilon \times e^{n\beta\gamma\delta} J_{\gamma\delta}\sqrt{g_4} . \quad (6.2)$$

Here the index n refers to a normal coordinate for the space-like 3-surface at either boundary of CD or for light-like wormhole throat. ϵ is a sign factor which is opposite for the two ends of CD. It could be also opposite of opposite at the opposite sides of the wormhole throat. Note that the dependence on induced metric disappears at the right hand side and this condition eliminates the potentials singularity due to the reduction of the rank of the induced metric at wormhole throat.

5. Information about the tangent space of the space-time surface can be coded to the WCW metric with loosing the nice transformation properties of the magnetic flux Hamiltonians if Kähler electric fluxes or sum of magnetic flux and electric flux satisfying this condition are used and K is symplectic invariant. Using the sum

$$J_e + J_m = (1 + K)J_{12} , \quad (6.3)$$

where J denotes the Kähler magnetic flux, , makes it possible to have a non-trivial WCW metric even for $K = 0$, which could correspond to the ends of a cosmic string like solution carrying only Kähler magnetic fields. This condition suggests that it can depend only on

Kähler magnetic flux and other symplectic invariants. Whether local symplectic coordinate invariants are possible at all is far from obvious, If the slicing itself is symplectic invariant then K could be a non-constant function of X^2 depending on string world sheet coordinates. The light-like radial coordinate of the light-cone boundary indeed defines a symplectically invariant slicing and this slicing could be shifted along the time axis defined by the tips of CD.

6.1.2 *Electric-magnetic duality physically*

What could the weak duality condition mean physically? For instance, what constraints are obtained if one assumes that the quantization of electro-weak charges reduces to this condition at classical level?

1. The first thing to notice is that the flux of J over the partonic 2-surface is analogous to magnetic flux

$$Q_m = \frac{e}{\hbar} \oint B dS = n .$$

n is non-vanishing only if the surface is homologically non-trivial and gives the homology charge of the partonic 2-surface.

2. The expressions of classical electromagnetic and Z^0 fields in terms of Kähler form [L1] , [L1] read as

$$\begin{aligned} \gamma &= \frac{eF_{em}}{\hbar} = 3J - \sin^2(\theta_W)R_{03} , \\ Z^0 &= \frac{g_Z F_Z}{\hbar} = 2R_{03} . \end{aligned} \quad (6.4)$$

Here R_{03} is one of the components of the curvature tensor in vielbein representation and F_{em} and F_Z correspond to the standard field tensors. From this expression one can deduce

$$J = \frac{e}{3\hbar} F_{em} + \sin^2(\theta_W) \frac{g_Z}{6\hbar} F_Z . \quad (6.5)$$

3. The weak duality condition when integrated over X^2 implies

$$\begin{aligned} \frac{e^2}{3\hbar} Q_{em} + \frac{g_Z^2 p}{6} Q_{Z,V} &= K \oint J = Kn , \\ Q_{Z,V} &= \frac{I_V^3}{2} - Q_{em} , \quad p = \sin^2(\theta_W) . \end{aligned} \quad (6.6)$$

Here the vectorial part of the Z^0 charge rather than as full Z^0 charge $Q_Z = I_L^3 + \sin^2(\theta_W)Q_{em}$ appears. The reason is that only the vectorial isospin is same for left and right handed components of fermion which are in general mixed for the massive states.

The coefficients are dimensionless and expressible in terms of the gauge coupling strengths and using $\hbar = r\hbar_0$ one can write

$$\begin{aligned} \alpha_{em} Q_{em} + p \frac{\alpha_Z}{2} Q_{Z,V} &= \frac{3}{4\pi} \times rnK , \\ \alpha_{em} &= \frac{e^2}{4\pi\hbar_0} , \quad \alpha_Z = \frac{g_Z^2}{4\pi\hbar_0} = \frac{\alpha_{em}}{p(1-p)} . \end{aligned} \quad (6.7)$$

4. There is a great temptation to assume that the values of Q_{em} and Q_Z correspond to their quantized values and therefore depend on the quantum state assigned to the partonic 2-surface. The linear coupling of the Kähler-Dirac operator to conserved charges implies correlation between the geometry of space-time sheet and quantum numbers assigned to the partonic 2-surface. The assumption of standard quantized values for Q_{em} and Q_Z would be also seen as the identification of the fine structure constants α_{em} and α_Z . This however requires weak isospin invariance.

6.1.3 The value of K from classical quantization of Kähler electric charge

The value of K can be deduced by requiring classical quantization of Kähler electric charge.

1. The condition that the flux of $F^{03} = (\hbar/g_K)J^{03}$ defining the counterpart of Kähler electric field equals to the Kähler charge g_K would give the condition $K = g_K^2/\hbar$, where g_K is Kähler coupling constant which should be invariant under coupling constant evolution by quantum criticality. Within experimental uncertainties one has $\alpha_K = g_K^2/4\pi\hbar_0 = \alpha_{em} \simeq 1/137$, where α_{em} is fine structure constant in electron length scale and \hbar_0 is the standard value of Planck constant.
2. The quantization of Planck constants makes the condition highly non-trivial. The most general quantization of r is as rationals but there are good arguments favoring the quantization as integers corresponding to the allowance of only singular coverings of CD and CP_2 . The point is that in this case a given value of Planck constant corresponds to a finite number of pages of the “Big Book”. The quantization of the Planck constant implies a further quantization of K and would suggest that K scales as $1/r$ unless the spectrum of values of Q_{em} and Q_Z allowed by the quantization condition scales as r . This is quite possible and the interpretation would be that each of the r sheets of the covering carries (possibly same) elementary charge. Kind of discrete variant of a full Fermi sphere would be in question. The interpretation in terms of anyonic phases [K20] supports this interpretation.
3. The identification of J as a counterpart of eB/\hbar means that Kähler action and thus also Kähler function is proportional to $1/\alpha_K$ and therefore to \hbar . This implies that for large values of \hbar Kähler coupling strength $g_K^2/4\pi$ becomes very small and large fluctuations are suppressed in the functional integral. The basic motivation for introducing the hierarchy of Planck constants was indeed that the scaling $\alpha \rightarrow \alpha/r$ allows to achieve the convergence of perturbation theory: Nature itself would solve the problems of the theoretician. This of course does not mean that the physical states would remain as such and the replacement of single particles with anyonic states in order to satisfy the condition for K would realize this concretely.
4. The condition $K = g_K^2/\hbar$ implies that the Kähler magnetic charge is always accompanied by Kähler electric charge. A more general condition would read as

$$K = n \times \frac{g_K^2}{\hbar}, n \in Z . \quad (6.8)$$

This would apply in the case of cosmic strings and would allow vanishing Kähler charge possible when the partonic 2-surface has opposite fermion and anti-fermion numbers (for both leptons and quarks) so that Kähler electric charge should vanish. For instance, for neutrinos the vanishing of electric charge strongly suggests $n = 0$ besides the condition that abelian Z^0 flux contributing to em charge vanishes.

It took a year to realize that this value of K is natural at the Minkowskian side of the wormhole throat. At the Euclidian side much more natural condition is

$$K = \frac{1}{\hbar \text{bar}} . \quad (6.9)$$

In fact, the self-duality of CP_2 Kähler form favours this boundary condition at the Euclidian side of the wormhole throat. Also the fact that one cannot distinguish between electric and magnetic charges in Euclidian region since all charges are magnetic can be used to argue in favor of this form. The same constraint arises from the condition that the action for CP_2 type vacuum extremal has the value required by the argument leading to a prediction for gravitational constant in terms of the square of CP_2 radius and α_K the effective replacement $g_K^2 \rightarrow 1$ would spoil the argument.

The boundary condition $J_E = J_B$ for the electric and magnetic parts of Kähler form at the Euclidian side of the wormhole throat inspires the question whether all Euclidian regions could be self-dual so that the density of Kähler action would be just the instanton density. Self-duality follows if the deformation of the metric induced by the deformation of the canonically imbedded CP_2 is such that in CP_2 coordinates for the Euclidian region the tensor $(g^{\alpha\beta}g^{\mu\nu} - g^{\alpha\nu}g^{\mu\beta})/\sqrt{g}$ remains invariant. This is certainly the case for CP_2 type vacuum extremals since by the light-likeness of M^4 projection the metric remains invariant. Also conformal scalings of the induced metric would satisfy this condition. Conformal scaling is not consistent with the degeneracy of the 4-metric at the wormhole.

6.1.4 Reduction of the quantization of Kähler electric charge to that of electromagnetic charge

The best manner to learn more is to challenge the form of the weak electric-magnetic duality based on the induced Kähler form.

1. Physically it would seem more sensible to pose the duality on electromagnetic charge rather than Kähler charge. This would replace induced Kähler form with electromagnetic field, which is a linear combination of induced Kähler field and classical Z^0 field

$$\begin{aligned}\gamma &= 3J - \sin^2\theta_W R_{03} \ , \\ Z^0 &= 2R_{03} \ .\end{aligned}\tag{6.10}$$

Here $Z_0 = 2R_{03}$ is the appropriate component of CP_2 curvature form [L1]. For a vanishing Weinberg angle the condition reduces to that for Kähler form.

2. For the Euclidian space-time regions having interpretation as lines of generalized Feynman diagrams Weinberg angle should be non-vanishing. In Minkowskian regions Weinberg angle could however vanish. If so, the condition guaranteeing that electromagnetic charge of the partonic 2-surfaces equals to the above condition stating that the em charge assignable to the fermion content of the partonic 2-surfaces reduces to the classical Kähler electric flux at the Minkowskian side of the wormhole throat. One can argue that Weinberg angle must increase smoothly from a vanishing value at both sides of wormhole throat to its value in the deep interior of the Euclidian region.
3. The vanishing of the Weinberg angle in Minkowskian regions conforms with the physical intuition. Above elementary particle length scales one sees only the classical electric field reducing to the induced Kähler form and classical Z^0 fields and color gauge fields are effectively absent. Only in phases with a large value of Planck constant classical Z^0 field and other classical weak fields and color gauge field could make themselves visible. Cell membrane could be one such system [K21]. This conforms with the general picture about color confinement and weak massivation.

The GRT limit of TGD suggests a further reason for why Weinberg angle should vanish in Minkowskian regions.

1. The value of the Kähler coupling strength must be very near to the value of the fine structure constant in electron length scale and these constants can be assumed to be equal.
2. GRT limit of TGD with space-time surfaces replaced with abstract 4-geometries would naturally correspond to Einstein-Maxwell theory with cosmological constant which is non-vanishing only in Euclidian regions of space-time so that both Reissner-Nordström metric and

CP_2 are allowed as simplest possible solutions of field equations [K25]. The extremely small value of the observed cosmological constant needed in GRT type cosmology could be equal to the large cosmological constant associated with CP_2 metric multiplied with the 3-volume fraction of Euclidian regions.

3. Also at GRT limit quantum theory would reduce to almost topological QFT since Einstein-Maxwell action reduces to 3-D term by field equations implying the vanishing of the Maxwell current and of the curvature scalar in Minkowskian regions and curvature scalar + cosmological constant term in Euclidian regions. The weak form of electric-magnetic duality would guarantee also now the preferred extremal property and prevent the reduction to a mere topological QFT.
4. GRT limit would make sense only for a vanishing Weinberg angle in Minkowskian regions. A non-vanishing Weinberg angle would make sense in the deep interior of the Euclidian regions where the approximation as a small deformation of CP_2 makes sense.

The weak form of electric-magnetic duality has surprisingly strong implications for the basic view about quantum TGD as following considerations show.

6.2 Magnetic Confinement, The Short Range Of Weak Forces, And Color Confinement

The weak form of electric-magnetic duality has surprisingly strong implications if one combines it with some very general empirical facts such as the non-existence of magnetic monopole fields in macroscopic length scales.

6.2.1 How can one avoid macroscopic magnetic monopole fields?

Monopole fields are experimentally absent in length scales above order weak boson length scale and one should have a mechanism neutralizing the monopole charge. How electroweak interactions become short ranged in TGD framework is still a poorly understood problem. What suggests itself is the neutralization of the weak isospin above the intermediate gauge boson Compton length by neutral Higgs bosons. Could the two neutralization mechanisms be combined to single one?

1. In the case of fermions and their super partners the opposite magnetic monopole would be a wormhole throat. If the magnetically charged wormhole contact is electromagnetically neutral but has vectorial weak isospin neutralizing the weak vectorial isospin of the fermion only the electromagnetic charge of the fermion is visible on longer length scales. The distance of this wormhole throat from the fermionic one should be of the order weak boson Compton length. An interpretation as a bound state of fermion and a wormhole throat state with the quantum numbers of a neutral Higgs boson would therefore make sense. The neutralizing throat would have quantum numbers of $X_{-1/2} = \nu_L \bar{\nu}_R$ or $X_{1/2} = \bar{\nu}_L \nu_R$. $\nu_L \bar{\nu}_R$ would not be neutral Higgs boson (which should correspond to a wormhole contact) but a super-partner of left-handed neutrino obtained by adding a right handed neutrino. This mechanism would apply separately to the fermionic and anti-fermionic throats of the gauge bosons and corresponding space-time sheets and leave only electromagnetic interaction as a long ranged interaction.
2. One can of course wonder what is the situation situation for the bosonic wormhole throats feeding gauge fluxes between space-time sheets. It would seem that these wormhole throats must always appear as pairs such that for the second member of the pair monopole charges and I_V^3 cancel each other at both space-time sheets involved so that one obtains at both space-time sheets magnetic dipoles of size of weak boson Compton length. The proposed magnetic character of fundamental particles should become visible at TeV energies so that LHC might have surprises in store!

6.2.2 Well-definedness of electromagnetic charge implies stringiness

Well-definedness of electromagnetic charged at string world sheets carrying spinor modes is very natural constraint and not trivially satisfied because classical W boson fields are present. As a matter fact, all weak fields should be effectively absent above weak scale. How this is possible classical weak fields identified as induced gauge fields are certainly present.

The condition that em charge is well defined for spinor modes implies that the space-time region in which spinor mode is non-vanishing has 2-D CP_2 projection such that the induced W boson fields are vanishing. The vanishing of classical Z^0 field can be poses as additional condition - at least in scales above weak scale. In the generic case this requires that the spinor mode is restricted to 2-D surface: string world sheet or possibly also partonic 2-surface. This implies that TGD reduces to string model in fermionic sector. Even for preferred extremals with 2-D projecting the modes are expected to allow restriction to 2-surfaces. This localization is possible only for Kähler-Dirac action.

A word of warning is however in order. The GRT limit or rather limit of TGD as Einstein Yang-Mills theory replaces the sheets of many-sheeted space-time with Minkowski space with effective metric obtained by summing to Minkowski metric the deviations of the induced metrics of space-time sheets from Minkowski metric. For gauge potentials a similar identification applies. YM-Einstein equations coupled with matter and with non-vanishing cosmological constant are expected on basis of Poincare invariance. One cannot exclude the possibility that the sums of weak gauge potentials from different space-time sheet tend to vanish above weak scale and that well-definedness of em charge at classical level follows from the effective absence of classical weak gauge fields.

6.2.3 Magnetic confinement and color confinement

Magnetic confinement generalizes also to the case of color interactions. One can consider also the situation in which the magnetic charges of quarks (more generally, of color excited leptons and quarks) do not vanish and they form color and magnetic singlets in the hadronic length scale. This would mean that magnetic charges of the state $q_{\pm 1/2} - X_{\mp 1/2}$ representing the physical quark would not vanish and magnetic confinement would accompany also color confinement. This would explain why free quarks are not observed. To how degree then quark confinement corresponds to magnetic confinement is an interesting question.

For quark and antiquark of meson the magnetic charges of quark and antiquark would be opposite and meson would correspond to a Kähler magnetic flux so that a stringy view about meson emerges. For valence quarks of baryon the vanishing of the net magnetic charge takes place provided that the magnetic net charges are $(\pm 2, \mp 1, \mp 1)$. This brings in mind the spectrum of color hyper charges coming as $(\pm 2, \mp 1, \mp 1)/3$ and one can indeed ask whether color hypercharge correlates with the Kähler magnetic charge. The geometric picture would be three strings connected to single vertex. Amusingly, the idea that color hypercharge could be proportional to color hyper charge popped up during the first year of TGD when I had not yet discovered CP_2 and believed on $M^4 \times S^2$.

p-Adic length scale hypothesis and hierarchy of Planck constants defining a hierarchy of dark variants of particles suggest the existence of scaled up copies of QCD type physics and weak physics. For p-adically scaled up variants the mass scales would be scaled by a power of $\sqrt{2}$ in the most general case. The dark variants of the particle would have the same mass as the original one. In particular, Mersenne primes $M_k = 2^k - 1$ and Gaussian Mersennes $M_{G,k} = (1 + i)^k - 1$ has been proposed to define zoomed copies of these physics. At the level of magnetic confinement this would mean hierarchy of length scales for the magnetic confinement.

One particular proposal is that the Mersenne prime M_{89} should define a scaled up variant of the ordinary hadron physics with mass scaled up roughly by a factor $2^{(107-89)/2} = 512$. The size scale of color confinement for this physics would be same as the weal length scale. It would look more natural that the weak confinement for the quarks of M_{89} physics takes place in some shorter scale and M_{61} is the first Mersenne prime to be considered. The mass scale of M_{61} weak bosons would be by a factor $2^{(89-61)/2} = 2^{14}$ higher and about 1.6×10^4 TeV. M_{89} quarks would have virtually no weak interactions but would possess color interactions with weak confinement length scale reflecting themselves as new kind of jets at collisions above TeV energies.

In the biologically especially important length scale range 10 nm -2500 nm there are as many as four scaled up electron Compton lengths $L_e(k) = \sqrt{5}L(k)$: they are associated with Gaussian Mersennes $M_{G,k}$, $k = 151, 157, 163, 167$. This would suggest that the existence of scaled up scales of magnetic-, weak- and color confinement. An especially interesting possibly testable prediction is the existence of magnetic monopole pairs with the size scale in this range. There are recent claims about experimental evidence for magnetic monopole pairs [D1] .

6.2.4 Magnetic confinement and stringy picture in TGD sense

The connection between magnetic confinement and weak confinement is rather natural if one recalls that electric-magnetic duality in super-symmetric quantum field theories means that the descriptions in terms of particles and monopoles are in some sense dual descriptions. Fermions would be replaced by string like objects defined by the magnetic flux tubes and bosons as pairs of wormhole contacts would correspond to pairs of the flux tubes. Therefore the sharp distinction between gravitons and physical particles would disappear.

The reason why gravitons are necessarily stringy objects formed by a pair of wormhole contacts is that one cannot construct spin two objects using only single fermion states at wormhole throats. Of course, also super partners of these states with higher spin obtained by adding fermions and anti-fermions at the wormhole throat but these do not give rise to graviton like states [K13] . The upper and lower wormhole throat pairs would be quantum superpositions of fermion anti-fermion pairs with sum over all fermions. The reason is that otherwise one cannot realize graviton emission in terms of joining of the ends of light-like 3-surfaces together. Also now magnetic monopole charges are necessary but now there is no need to assign the entities X_{\pm} with gravitons.

Graviton string is characterized by some p-adic length scale and one can argue that below this length scale the charges of the fermions become visible. Mersenne hypothesis suggests that some Mersenne prime is in question. One proposal is that gravitonic size scale is given by electronic Mersenne prime M_{127} . It is however difficult to test whether graviton has a structure visible below this length scale.

What happens to the generalized Feynman diagrams is an interesting question. It is not at all clear how closely they relate to ordinary Feynman diagrams. All depends on what one is ready to assume about what happens in the vertices. One could of course hope that zero energy ontology could allow some very simple description allowing perhaps to get rid of the problematic aspects of Feynman diagrams.

1. Consider first the recent view about generalized Feynman diagrams which relies ZEO. A highly attractive assumption is that the particles appearing at wormhole throats are on mass shell particles. For incoming and outgoing elementary bosons and their super partners they would be positive it resp. negative energy states with parallel on mass shell momenta. For virtual bosons they the wormhole throats would have opposite sign of energy and the sum of on mass shell states would give virtual net momenta. This would make possible twistor description of virtual particles allowing only massless particles (in 4-D sense usually and in 8-D sense in TGD framework). The notion of virtual fermion makes sense only if one assumes in the interaction region a topological condensation creating another wormhole throat having no fermionic quantum numbers.
2. The addition of the particles X^{\pm} replaces generalized Feynman diagrams with the analogs of stringy diagrams with lines replaced by pairs of lines corresponding to fermion and $X_{\pm 1/2}$. The members of these pairs would correspond to 3-D light-like surfaces glued together at the vertices of generalized Feynman diagrams. The analog of 3-vertex would not be splitting of the string to form shorter strings but the replication of the entire string to form two strings with same length or fusion of two strings to single string along all their points rather than along ends to form a longer string. It is not clear whether the duality symmetry of stringy diagrams can hold true for the TGD variants of stringy diagrams.
3. How should one describe the bound state formed by the fermion and X^{\pm} ? Should one describe the state as superposition of non-parallel on mass shell states so that the composite state would be automatically massive? The description as superposition of on mass shell states does not conform with the idea that bound state formation requires binding energy.

In TGD framework the notion of negentropic entanglement has been suggested to make possible the analogs of bound states consisting of on mass shell states so that the binding energy is zero [K17]. If this kind of states are in question the description of virtual states in terms of on mass shell states is not lost. Of course, one cannot exclude the possibility that there is infinite number of this kind of states serving as analogs for the excitations of string like object.

4. What happens to the states formed by fermions and $X_{\pm 1/2}$ in the internal lines of the Feynman diagram? Twistor philosophy suggests that only the higher on mass shell excitations are possible. If this picture is correct, the situation would not change in an essential manner from the earlier one.

The highly non-trivial prediction of the magnetic confinement is that elementary particles should have stringy character in electro-weak length scales and could behaving to become manifest at LHC energies. This adds one further item to the list of non-trivial predictions of TGD about physics at LHC energies [K18].

6.3 Could Quantum TGD Reduce To Almost Topological QFT?

There seems to be a profound connection with the earlier unrealistic proposal that TGD reduces to almost topological quantum theory in the sense that the counterpart of Chern-Simons action assigned with the wormhole throats somehow dictates the dynamics. This proposal can be formulated also for the Kähler-Dirac action action. I gave up this proposal but the following argument shows that Kähler action with weak form of electric-magnetic duality effectively reduces to Chern-Simons action plus Coulomb term.

1. Kähler action density can be written as a 4-dimensional integral of the Coulomb term $j_K^\alpha A_\alpha$ plus and integral of the boundary term $J^{n\beta} A_\beta \sqrt{g_4}$ over the wormhole throats and of the quantity $J^{0\beta} A_\beta \sqrt{g_4}$ over the ends of the 3-surface.
2. If the self-duality conditions generalize to $J^{n\beta} = 4\pi\alpha_K \epsilon^{n\beta\gamma\delta} J_{\gamma\delta}$ at throats and to $J^{0\beta} = 4\pi\alpha_K \epsilon^{0\beta\gamma\delta} J_{\gamma\delta}$ at the ends, the Kähler function reduces to the counterpart of Chern-Simons action evaluated at the ends and throats. It would have same value for each branch and the replacement $h \rightarrow n \times h$ would effectively describe this. Boundary conditions would however give $1/n$ factor so that \hbar would disappear from the Kähler function! It is somewhat surprising that Kähler action gives Chern-Simons action in the vacuum sector defined as sector for which Kähler current is light-like or vanishes.

Holography encourages to ask whether also the Coulomb interaction terms could vanish. This kind of dimensional reduction would mean an enormous simplification since TGD would reduce to an almost topological QFT. The attribute “almost” would come from the fact that one has non-vanishing classical Noether charges defined by Kähler action and non-trivial quantum dynamics in M^4 degrees of freedom. One could also assign to space-time surfaces conserved four-momenta which is not possible in topological QFTs. For this reason the conditions guaranteeing the vanishing of Coulomb interaction term deserve a detailed analysis.

1. For the known extremals j_K^α either vanishes or is light-like (“massless extremals” for which weak self-duality condition does not make sense [K2]) so that the Coulomb term vanishes identically in the gauge used. The addition of a gradient to A induces terms located at the ends and wormhole throats of the space-time surface but this term must be cancelled by the other boundary terms by gauge invariance of Kähler action. This implies that the M^4 part of WCW metric vanishes in this case. Therefore massless extremals as such are not physically realistic: wormhole throats representing particles are needed.
2. The original naive conclusion was that since Chern-Simons action depends on CP_2 coordinates only, its variation with respect to Minkowski coordinates must vanish so that the WCW metric would be trivial in M^4 degrees of freedom. This conclusion is in conflict with quantum classical correspondence and was indeed too hasty. The point is that the allowed variations of Kähler function must respect the weak electro-magnetic duality which relates

Kähler electric field depending on the induced 4-metric at 3-surface to the Kähler magnetic field. Therefore the dependence on M^4 coordinates creeps via a Lagrange multiplier term

$$\int \Lambda_\alpha (J^{n\alpha} - K \epsilon^{n\alpha\beta\gamma} J_{\beta \text{ gamma}}) \sqrt{g_4} d^3x . \quad (6.11)$$

The (1,1) part of second variation contributing to M^4 metric comes from this term.

3. This erratic conclusion about the vanishing of M^4 part WCW metric raised the question about how to achieve a non-trivial metric in M^4 degrees of freedom. The proposal was a modification of the weak form of electric-magnetic duality. Besides CP_2 Kähler form there would be the Kähler form assignable to the light-cone boundary reducing to that for $r_M = \text{constant}$ sphere - call it J^1 . The generalization of the weak form of self-duality would be $J^{n\beta} = \epsilon^{n\beta\gamma\delta} K (J_{\gamma\delta} + \epsilon J_{\gamma\delta}^1)$. This form implies that the boundary term gives a non-trivial contribution to the M^4 part of the WCW metric even without the constraint from electric-magnetic duality. Kähler charge is not affected unless the partonic 2-surface contains the tip of CD in its interior. In this case the value of Kähler charge is shifted by a topological contribution. Whether this term can survive depends on whether the resulting vacuum extremals are consistent with the basic facts about classical gravitation.
4. The Coulombic interaction term is not invariant under gauge transformations. The good news is that this might allow to find a gauge in which the Coulomb term vanishes. The vanishing condition fixing the gauge transformation ϕ is

$$j_K^\alpha \partial_\alpha \phi = -j^\alpha A_\alpha . \quad (6.12)$$

This differential equation can be reduced to an ordinary differential equation along the flow lines j_K by using $dx^\alpha/dt = j_K^\alpha$. Global solution is obtained only if one can combine the flow parameter t with three other coordinates- say those at the either end of CD to form space-time coordinates. The condition is that the parameter defining the coordinate differential is proportional to the covariant form of Kähler current: $dt = \phi j_K$. This condition in turn implies $d^2t = d(\phi j_K) = d\phi \wedge j_K + \phi dj_K = 0$ implying $j_K \wedge dj_K = 0$ or more concretely,

$$\epsilon^{\alpha\beta\gamma\delta} j_\beta^K \partial_\gamma j_{\delta}^K = 0 . \quad (6.13)$$

j_K is a four-dimensional counterpart of Beltrami field [B3] and could be called generalized Beltrami field.

The integrability conditions follow also from the construction of the extremals of Kähler action [K2]. The conjecture was that for the extremals the 4-dimensional Lorentz force vanishes (no dissipation): this requires $j_K \wedge J = 0$. One manner to guarantee this is the topologization of the Kähler current meaning that it is proportional to the instanton current: $j_K = \phi j_I$, where $j_I = *(J \wedge A)$ is the instanton current, which is not conserved for 4-D CP_2 projection. The conservation of j_K implies the condition $j_I^\alpha \partial_\alpha \phi = \partial_\alpha j^\alpha \phi$ and from this ϕ can be integrated if the integrability condition $j_I \wedge dj_I = 0$ holds true implying the same condition for j_K . By introducing at least 3 or CP_2 coordinates as space-time coordinates, one finds that the contravariant form of j_I is purely topological so that the integrability condition fixes the dependence on M^4 coordinates and this selection is coded into the scalar function ϕ . These functions define families of conserved currents $j_K^\alpha \phi$ and $j_I^\alpha \phi$ and could be also interpreted as conserved currents associated with the critical deformations of the space-time surface.

5. There are gauge transformations respecting the vanishing of the Coulomb term. The vanishing condition for the Coulomb term is gauge invariant only under the gauge transformations $A \rightarrow A + \nabla \phi$ for which the scalar function the integral $\int j_K^\alpha \partial_\alpha \phi$ reduces to a total divergence

a giving an integral over various 3-surfaces at the ends of CD and at throats vanishes. This is satisfied if the allowed gauge transformations define conserved currents

$$D_\alpha(j^\alpha\phi) = 0 . \quad (6.14)$$

As a consequence Coulomb term reduces to a difference of the conserved charges $Q_\phi^e = \int j^0\phi\sqrt{g_4}d^3x$ at the ends of the CD vanishing identically. The change of the Chern-Simons type term is trivial if the total weighted Kähler magnetic flux $Q_\phi^m = \sum \int J\phi dA$ over wormhole throats is conserved. The existence of an infinite number of conserved weighted magnetic fluxes is in accordance with the electric-magnetic duality. How these fluxes relate to the flux Hamiltonians central for WCW geometry is not quite clear.

6. The gauge transformations respecting the reduction to almost topological QFT should have some special physical meaning. The measurement interaction term in the Kähler-Dirac interaction corresponds to a critical deformation of the space-time sheet and is realized as an addition of a gauge part to the Kähler gauge potential of CP_2 . It would be natural to identify this gauge transformation giving rise to a conserved charge so that the conserved charges would provide a representation for the charges associated with the infinitesimal critical deformations not affecting Kähler action. The gauge transformed Kähler gauge potential couples to the Kähler-Dirac equation and its effect could be visible in the value of Kähler function and therefore also in the properties of the preferred extremal. The effect on WCW metric would however vanish since K would transform only by an addition of a real part of a holomorphic function.
7. A first guess for the explicit realization of the quantum classical correspondence between quantum numbers and space-time geometry is that the deformation of the preferred extremal due to the addition of the measurement interaction term is induced by a $U(1)$ gauge transformation induced by a transformation of $\delta CD \times CP_2$ generating the gauge transformation represented by ϕ . This interpretation makes sense if the fluxes defined by Q_ϕ^m and corresponding Hamiltonians affect only zero modes rather than quantum fluctuating degrees of freedom.
8. Later a simpler proposal assuming Kähler action with Chern-Simons term at partonic orbits and Kähler-Dirac action with Chern-Simons Dirac term at partonic orbits emerged. Measurement interaction terms would correspond to Lagrange multiplier terms at the ends of space-time surface fixing the values of classical conserved charges to their quantum values. Super-symmetry requires the assignment of this kind of term also to Kähler-Dirac action as boundary term.

Kähler-Dirac equation gives rise to a boundary condition at space-like ends of the space-time surface stating that the action of the Kähler-Dirac gamma matrix in normal direction annihilates the spinor modes. The normal vector would be light-like and the value of the incoming on mass shell four-momentum would be coded to the geometry of the space-time surface and string world sheet.

One can assign to partonic orbits Chern-Simons Dirac action and now the condition would be that the action of C-S-D operator equals to that of massless M^4 Dirac operator. C-S-D Dirac action would give rise to massless Dirac propagator. Twistor Grassmann approach suggests that also the virtual fermions reduce effectively to massless on-shell states but have non-physical helicity.

7 How To Define Generalized Feynman Diagrams?

S-matrix codes to a high degree the predictions of quantum theories. The longstanding challenge of TGD has been to construct or at least demonstrate the mathematical existence of S-matrix- or actually M-matrix which generalizes this notion in ZEO (ZEO) [K30] . This work has led to the notion of generalized Feynman diagram and the challenge is to give a precise mathematical meaning

for this object. The attempt to understand the counterpart of twistors in TGD framework [K32] has inspired several key ideas in this respect but it turned out that twistors themselves need not be absolutely necessary in TGD framework.

1. The notion of generalized Feynman diagram defined by replacing lines of ordinary Feynman diagram with light-like 3-surfaces (elementary particle sized wormhole contacts with throats carrying quantum numbers) and vertices identified as their 2-D ends - I call them partonic 2-surfaces is central. Speaking somewhat loosely, generalized Feynman diagrams (plus background space-time sheets) define the “world of classical worlds” (WCW). These diagrams involve the analogs of stringy diagrams but the interpretation is different: the analogs of stringy loop diagrams have interpretation in terms of particle propagating via two different routes simultaneously (as in the classical double slit experiment) rather than as a decay of particle to two particles. For stringy diagrams the counterparts of vertices are singular as manifolds whereas the entire diagrams are smooth. For generalized Feynman diagrams vertices are smooth but entire diagrams represent singular manifolds just like ordinary Feynman diagrams do. String like objects however emerge in TGD and even ordinary elementary particles are predicted to be magnetic flux tubes of length of order weak gauge boson Compton length with monopoles at their ends as shown in accompanying article. This stringy character should become visible at LHC energies.
2. ZEO (ZEO) and causal diamonds (intersections of future and past directed light-cones) define second key ingredient. The crucial observation is that in ZEO it is possible to identify off mass shell particles as pairs of on mass shell fermions at throats of wormhole contact since both positive and negative signs of energy are possible and one obtains also space-like total momenta for wormhole contact behaving as a boson. The localization of fermions to string world sheets and the fact that super-conformal generator G carries fermion number combined with twistorial consideration support the view that the propagators at fermionic lines are of form $(1/G)ip^k\gamma_k(1/G^\dagger + h.c.)$ and thus hermitian. In strong models $1/G$ would serve as a propagator and this requires Majorana condition fixing the dimension of the target space to 10 or 11.
3. A powerful constraint is number theoretic universality requiring the existence of Feynman amplitudes in all number fields when one allows suitable algebraic extensions: roots of unity are certainly required in order to realize p-adic counterparts of plane waves. Also imbedding space, partonic 2-surfaces and WCW must exist in all number fields and their extensions. These constraints are enormously powerful and the attempts to realize this vision have dominated quantum TGD for last two decades.
4. Representation of 8-D gamma matrices in terms of octonionic units and 2-D sigma matrices is a further important element as far as twistors are considered [K32]. Kähler-Dirac gamma matrices at space-time surfaces are quaternionic/associative and allow a genuine matrix representation. As a matter of fact, TGD and WCW could be formulated as study of associative local sub-algebras of the local Clifford algebra of 8-D imbedding space parameterized by quaternionic space-time surfaces.
5. A central conjecture has been that associative (co-associative) 4-surfaces correspond to preferred extremals of Kähler action [K28]. It took long time to realize that in ZEO the notion of preferred extremal might be un-necessary! The reason is that 3-surfaces are now pairs of 3-surfaces at boundaries of causal diamonds and for deterministic dynamics the space-time surface connecting them is expected to be more or less unique. Now the action principle is non-deterministic but the non-determinism would give rise to additional discrete dynamical degrees of freedom naturally assignable to the hierarchy of Planck constants $h_{eff} = n \times h$, n the number of space-time surface with same fixed ends at boundaries of CD and with same values of Kähler action and of conserved quantities. One must be however cautious: this leaves the possibility that there is a gauge symmetry present so that the n sheets correspond to gauge equivalence classes of sheets. Conformal invariance is associated with criticality and is expected to be present also now.

One can of course also ask whether one can assume that the pairs of 3-surfaces at the ends of CD are totally un-correlated. If this assumption is not made then preferred extremal property

would make sense also in ZEO and imply additional correlation between the members of these pairs. This kind of correlations would correspond to the Bohr orbit property, which is very attractive space-time correlate for quantum states. This kind of correlates are also expected as space-time counterpart for the correlations between initial and final state in quantum dynamics.

6. A further conjecture has been that preferred extremals are in some sense critical (second variation of Kähler action could vanish for infinite number of deformations defining a super-conformal algebra). The non-determinism of Kähler action implies this property for $n > 0$ in $h_{eff} = nh$. If the criticality is present, it could correspond to conformal gauge invariance defined by sub-algebras of conformal algebra with conformal weights coming as multiples of n and isomorphic to the conformal algebra itself.
7. As far as twistors are considered, the first key element is the reduction of the octonionic twistor structure to quaternionic one at space-time surfaces and giving effectively 4-D spinor and twistor structure for quaternionic surfaces.

Quite recently quite a dramatic progress took place in this approach [K28, K32] .

1. The progress was stimulated by the simple observation that on mass shell property puts enormously strong kinematic restrictions on the loop integrations. With mild restrictions on the number of parallel fermion lines appearing in vertices (there can be several since fermionic oscillator operator algebra defining SUSY algebra generates the parton states)- all loops are manifestly finite and if particles has always mass -say small p-adic thermal mass also in case of massless particles and due to IR cutoff due to the presence largest CD- the number of diagrams is finite. Unitarity reduces to Cutkosky rules [B5] automatically satisfied as in the case of ordinary Feynman diagrams.
2. Ironically, twistors which stimulated all these development do not seem to be absolutely necessary in this approach although they are of course possible. Situation changes if one does not assume small p-adically thermal mass due to the presence of massless particles and one must sum infinite number of diagrams. Here a potential problem is whether the infinite sum respects the algebraic extension in question.

This is about fermionic and momentum space aspects of Feynman diagrams but not yet about the functional (not path-) integral over small deformations of the partonic 2-surfaces. The basic challenges are following.

1. One should perform the functional integral over WCW degrees of freedom for fixed values of on mass shell momenta appearing in the internal lines. After this one must perform integral or summation over loop momenta. Note that the order is important since the space-time surface assigned to the line carries information about the quantum numbers associated with the line by quantum classical correspondence realized in terms of Kähler-Dirac operator.
2. One must define the functional integral also in the p-adic context. p-Adic Fourier analysis relying on algebraic continuation raises hopes in this respect. p-Adicity suggests strongly that the loop momenta are discretized and ZEO predicts this kind of discretization naturally.

It indeed seems that the functional integrals over WCW could be carried out at general level both in real and p-adic context. This is due to the symmetric space property (maximal number of isometries) of WCW required by the mere mathematical existence of Kähler geometry [K15] in infinite-dimensional context already in the case of much simpler loop spaces [A4] .

1. The p-adic generalization of Fourier analysis allows to algebraize integration- the horrible looking technical challenge of p-adic physics- for symmetric spaces for functions allowing the analog of discrete Fourier decomposition. Symmetric space property is indeed essential also for the existence of Kähler geometry for infinite-D spaces as was learned already from the case of loop spaces. Plane waves and exponential functions expressible as roots of unity and powers of p multiplied by the direct analogs of corresponding exponent functions are the basic

building bricks and key functions in harmonic analysis in symmetric spaces. The physically unavoidable finite measurement resolution corresponds to algebraically unavoidable finite algebraic dimension of algebraic extension of p-adics (at least some roots of unity are needed). The cutoff in roots of unity is very reminiscent to that occurring for the representations of quantum groups and is certainly very closely related to these as also to the inclusions of hyper-finite factors of type II_1 defining the finite measurement resolution.

2. WCW geometrization reduces to that for a single line of the generalized Feynman diagram defining the basic building brick for WCW. Kähler function decomposes to a sum of “kinetic” terms associated with its ends and interaction term associated with the line itself. p-Adicization boils down to the condition that Kähler function, matrix elements of Kähler form, WCW Hamiltonians and their super counterparts, are rational functions of complex WCW coordinates just as they are for those symmetric spaces that I know of. This would allow a continuation to p-adic context.

In the following this vision about generalized Feynman diagrams is discussed in more detail.

7.1 Questions

The goal is a proposal for how to perform the integral over WCW for generalized Feynman digrams and the best manner to proceed to to this goal is by making questions.

7.1.1 What does finite measurement resolution mean?

The first question is what finite measurement resolution means.

1. One expects that the algebraic continuation makes sense only for a finite measurement resolution in which case one obtains only finite sums of what one might hope to be algebraic functions. The finiteness of the algebraic extension would be in fact equivalent with the finite measurement resolution.
2. Finite measurement resolution means a discretization in terms of number theoretic braids. p-Adicization condition suggests that that one must allow only the number theoretic braids. For these the ends of braid at boundary of CD are algebraic points of the imbedding space. This would be true at least in the intersection of real and p-adic worlds.
3. The question is whether one can localize the points of the braid. The necessity to use momentum eigenstates to achieve quantum classical correspondence in the Kähler-Dirac action [K28] suggests however a de-localization of braid points, that is wave function in space of braid points. In real context one could allow all possible choices for braid points but in p-adic context only algebraic points are possible if one wants to replace integrals with sums. This implies finite measurement resolution analogous to that in lattice. This is also the only possibility in the intersection of real and p-adic worlds.

A non-trivial prediction giving a strong correlation between the geometry of the partonic 2-surface and quantum numbers is that the total number $n_F + n_{\bar{F}}$ of fermions and anti-fermions is bounded above by the number n_{alg} of algebraic points for a given partonic 2-surface: $n_F + n_{\bar{F}} \leq n_{alg}$. Outside the intersection of real and p-adic worlds the problematic aspect of this definition is that small deformations of the partonic 2-surface can radically change the number of algebraic points unless one assumes that the finite measurement resolution means restriction of WCW to a sub-space of algebraic partonic surfaces.

4. Braids defining propagator lines for fundamental fermions (to be distinguished from observer particles) emerges naturally. Braid strands correspond to the boundaries of string world sheets at which the modes of induced spinor fields are localized from the condition that em charge is well-defined: induced W field and above weak scale also Z^0 field vanish at them.

In order to obtain non-trivial fermion propagator one must add to Dirac action 1-D Dirac action in induced metric with the boundaries of string world sheets at the light-like parton orbits. Its bosonic counterpart is line-length in induced metric. Field equations imply that

the boundaries are light-like geodesics and fermion has light-like 8-momentum. This suggests strongly a connection with quantum field theory and an 8-D generalization of twistor Grassmannian approach. By field equations the bosonic part of this action does not contribute to the Kähler action. The light-like 8-momenta p^k have same M^4 and CP_2 mass squared and latter correspond to the the eigenvalues of the CP_2 spinor d'Alembertian by quantum-classical correspondence.

5. One has also discretization of the relative position of the second tip of CD at the hyperboloid isometric with mass shell. Only the number of braid points and their momenta would matter, not their positions.
6. The quantum numbers characterizing positive and negative energy parts of zero energy states couple directly to space-time geometry via the measurement interaction terms in Kähler action expressing the equality of classical conserved charges in Cartan algebra with their quantal counterparts for space-time surfaces in quantum superposition. This makes sense if classical charges parametrize zero modes. The localization in zero modes in state function reduction would be the WCW counterpart of state function collapse.

7.1.2 How to define integration in WCW degrees of freedom?

The basic question is how to define the integration over WCW degrees of freedom.

1. What comes mind first is Gaussian perturbation theory around the maxima of Kähler function. Gaussian and metric determinants cancel each other and only algebraic expressions remain. Finiteness is not a problem since the Kähler function is non-local functional of 3-surface so that no local interaction vertices are present. One should however assume the vanishing of loops required also by algebraic universality and this assumption look unrealistic when one considers more general functional integrals than that of vacuum functional since free field theory is not in question. The construction of the inverse of the WCW metric defining the propagator is also a very difficult challenge. Duistermaat-Hecke theorem states that something like this known as localization might be possible and one can also argue that something analogous to localization results from a generalization of mean value theorem.
2. Symmetric space property is more promising since it might reduce the integrations to group theory using the generalization of Fourier analysis for group representations so that there would be no need for perturbation theory in the proposed sense. In finite measurement resolution the symmetric spaces involved would be finite-dimensional. Symmetric space structure of WCW could also allow to define p-adic integration in terms of p-adic Fourier analysis for symmetric spaces. Essentially algebraic continuation of the integration from the real case would be in question with additional constraints coming from the fact that only phase factors corresponding to finite algebraic extensions of rationals are used. Cutoff would emerge automatically from the cutoff for the dimension of the algebraic extension.

7.1.3 How to define generalized Feynman diagrams?

Integration in symmetric spaces could serve as a model at the level of WCW and allow both the understanding of WCW integration and p-adicization as algebraic continuation. In order to get a more realistic view about the problem one must define more precisely what the calculation of the generalized Feynman diagrams means.

1. WCW integration must be carried out separately for all values of the momenta associated with the internal lines. The reason is that the spectrum of eigenvalues λ_i of the Kähler-Dirac operator D depends on the momentum of line and momentum conservation in vertices translates to a correlation of the spectra of D at internal lines.
2. For tree diagrams algebraic continuation to the p-adic context if the expression involves only the replacement of the generalized eigenvalues of D as functions of momenta with their p-adic counterparts besides vertices. If these functions are algebraically universal and expressible in terms of harmonics of symmetric space, there should be no problems.

3. If loops are involved, one must integrate/sum over loop momenta. In p-adic context difficulties are encountered if the spectrum of the momenta is continuous. The integration over on mass shell loop momenta is analogous to the integration over sub-CDs, which suggests that internal line corresponds to a *sub-CD* in which it is at rest. There are excellent reasons to believe that the moduli space for the positions of the upper tip is a discrete subset of hyperboloid of future light-cone. If this is the case, the loop integration indeed reduces to a sum over discrete positions of the tip. p-Adicization would thus give a further good reason why for ZEO.
4. Propagator is expressible in terms of the inverse of generalized eigenvalue and there is a sum over these for each propagator line. At vertices one has products of WCW harmonics assignable to the incoming lines. The product must have vanishing quantum numbers associated with the phase angle variables of WCW. Non-trivial quantum numbers of the WCW harmonic correspond to WCW quantum numbers assignable to excitations of ordinary elementary particles. WCW harmonics are products of functions depending on the “radial” coordinates and phase factors and the integral over the angles leaves the product of the first ones analogous to Legendre polynomials $P_{l,m}$. These functions are expected to be rational functions or at least algebraic functions involving only square roots.
5. In ordinary QFT incoming and outgoing lines correspond to propagator poles. In the recent case this would mean that incoming stringy lines at the ends of CD correspond to fermions satisfying the stringy mass formula serving as a generalization of masslessness condition.

7.2 Generalized Feynman Diagrams At Fermionic And Momentum SpaceLevel

Negative energy ontology has already led to the idea of interpreting the virtual particles as pairs of positive and negative energy wormhole throats. Hitherto I have taken it as granted that ordinary Feynman diagrammatics generalizes more or less as such. It is however far from clear what really happens in the verties of the generalized Feynman diagrams. The safest approach relies on the requirement that unitarity realized in terms of Cutkosky rules in ordinary Feynman diagrammatics allows a generalization. This requires loop diagrams. In particular, photon-photon scattering can take place only via a fermionic square loop so that it seems that loops must be present at least in the topological sense.

One must be however ready for the possibility that something unexpectedly simple might emerge. For instance, the vision about algebraic physics allows naturally only finite sums for diagrams and does not favor infinite perturbative expansions. Hence the true believer on algebraic physics might dream about finite number of diagrams for a given reaction type. For simplicity generalized Feynman diagrams without the complications brought by the magnetic confinement since by the previous arguments the generalization need not bring in anything essentially new.

The basic idea of duality in early hadronic models was that the lines of the dual diagram representing particles are only re-arranged in the vertices. This however does not allow to get rid of off mass shell momenta. ZEO encourages to consider a stronger form of this principle in the sense that the virtual momenta of particles could correspond to pairs of on mass shell momenta of particles. If also interacting fermions are pairs of positive and negative energy throats in the interaction region the idea about reducing the construction of Feynman diagrams to some kind of lego rules might work.

7.2.1 *Virtual particles as pairs of on mass shell particles in ZEO*

The first thing is to try to define more precisely what generalized Feynman diagrams are. The direct generalization of Feynman diagrams implies that both wormhole throats and wormhole contacts join at vertices.

1. A simple intuitive picture about what happens is provided by diagrams obtained by replacing the points of Feynman diagrams (wormhole contacts) with short lines and imagining that the throats correspond to the ends of the line. At vertices where the lines meet the incoming on mass shell quantum numbers would sum up to zero. This approach leads to a straightforward generalization of Feynman diagrams with virtual particles replaced with pairs of on mass shell

throat states of type $++$, $--$, and $+-$. Incoming lines correspond to $++$ type lines and outgoing ones to $--$ type lines. The first two line pairs allow only time like net momenta whereas $+-$ line pairs allow also space-like virtual momenta. The sign assigned to a given throat is dictated by the sign of the on mass shell momentum on the line. The condition that Cutkosky rules generalize as such requires $++$ and $--$ type virtual lines since the cut of the diagram in Cutkosky rules corresponds to on mass shell outgoing or incoming states and must therefore correspond to $++$ or $--$ type lines.

2. The basic difference as compared to the ordinary Feynman diagrammatics is that loop integrals are integrals over mass shell momenta and that all throats carry on mass shell momenta. In each vertex of the loop mass incoming on mass shell momenta must sum up to on mass shell momentum. These constraints improve the behavior of loop integrals dramatically and give excellent hopes about finiteness. It does not however seem that only a finite number of diagrams contribute to the scattering amplitude besides tree diagrams. The point is that if a the reactions $N_1 \rightarrow N_2$ and $N_2 \rightarrow N_3$, where N_i denote particle numbers, are possible in a common kinematical region for N_2 -particle states then also the diagrams $N_1 \rightarrow N_2 \rightarrow N_2 \rightarrow N_3$ are possible. The virtual states N_2 include all all states in the intersection of kinematically allow regions for $N_1 \rightarrow N_2$ and $N_2 \rightarrow N_3$. Hence the dream about finite number possible diagrams is not fulfilled if one allows massless particles. If all particles are massive then the particle number N_2 for given N_1 is limited from above and the dream is realized.
3. For instance, loops are not possible in the massless case or are highly singular (bringing in mind twistor diagrams) since the conservation laws at vertices imply that the momenta are parallel. In the massive case and allowing mass spectrum the situation is not so simple. As a first example one can consider a loop with three vertices and thus three internal lines. Three on mass shell conditions are present so that the four-momentum can vary in 1-D subspace only. For a loop involving four vertices there are four internal lines and four mass shell conditions so that loop integrals would reduce to discrete sums. Loops involving more than four vertices are expected to be impossible.
4. The proposed replacement of the elementary fermions with bound states of elementary fermions and monopoles X_{\pm} brings in the analog of stringy diagrammatics. The 2-particle wave functions in the momentum degrees of freedom of fermion and X_{\pm} might allow more flexibility and allow more loops. Note however that there are excellent hopes about the finiteness of the theory also in this case.

7.2.2 Loop integrals are manifestly finite

One can make also more detailed observations about loops.

1. The simplest situation is obtained if only 3-vertices are allowed. In this case conservation of momentum however allows only collinear momenta although the signs of energy need not be the same. Particle creation and annihilation is possible and momentum exchange is possible but is always light-like in the massless case. The scattering matrices of supersymmetric YM theories would suggest something less trivial and this raises the question whether something is missing. Magnetic monopoles are an essential element of also these theories as also massivation and symmetry breaking and this encourages to think that the formation of massive states as fermion X_{\pm} pairs is needed. Of course, in TGD framework one has also high mass excitations of the massless states making the scattering matrix non-trivial.
2. In YM theories on mass shell lines would be singular. In TGD framework this is not the case since the propagator is defined as the inverse of the 3-D dimensional reduction of the Kähler-Dirac operator D containing also coupling to four-momentum (this is required by quantum classical correspondence and guarantees stringy propagators),

$$\begin{aligned} D &= i\hat{\Gamma}^{\alpha}p_{\alpha} + \hat{\Gamma}^{\alpha}D_{\alpha} \ , \\ p_{\alpha} &= p_k\partial_{\alpha}h^k \ . \end{aligned} \tag{7.1}$$

The propagator does not diverge for on mass shell massless momenta and the propagator lines are well-defined. This is of course of essential importance also in general case. Only for the incoming lines one can consider the possibility that 3-D Dirac operator annihilates the induced spinor fields. All lines correspond to generalized eigenstates of the propagator in the sense that one has $D_3\Psi = \lambda\gamma\Psi$, where γ is Kähler-Dirac gamma matrix in the direction of the stringy coordinate emanating from light-like surface and D_3 is the 3-dimensional dimensional reduction of the 4-D Kähler-Dirac operator. The eigenvalue λ is analogous to energy. Note that the eigenvalue spectrum depends on 4-momentum as a parameter.

3. Massless incoming momenta can decay to massless momenta with both signs of energy. The integration measure $d^2k/2E$ reduces to dx/x where $x \geq 0$ is the scaling factor of massless momentum. Only light-like momentum exchanges are however possible and scattering matrix is essentially trivial. The loop integrals are finite apart from the possible delicacies related to poles since the loop integrands for given massless wormhole contact are proportional to dx/x^3 for large values of x .
4. Irrespective of whether the particles are massless or not, the divergences are obtained only if one allows too high vertices as self energy loops for which the number of momentum degrees of freedom is $3N - 4$ for N -vertex. The construction of SUSY limit of TGD in [K13] led to the conclusion that the parallelly propagating N fermions for given wormhole throat correspond to a product of N fermion propagators with same four-momentum so that for fermions and ordinary bosons one has the standard behavior but for $N > 2$ non-standard so that these excitations are not seen as ordinary particles. Higher vertices are finite only if the total number N_F of fermions propagating in the loop satisfies $N_F > 3N - 4$. For instance, a 4-vertex from which $N = 2$ states emanate is finite.

7.2.3 Taking into account magnetic confinement

What has been said above is not quite enough. The weak form of electric-magnetic duality [B2] leads to the picture about elementary particles as pairs of magnetic monopoles inspiring the notions of weak confinement based on magnetic monopole force. Also color confinement would have magnetic counterpart. This means that elementary particles would behave like string like objects in weak boson length scale. Therefore one must also consider the stringy case with wormhole throats replaced with fermion- X_{\pm} pairs (X_{\pm} is electromagnetically neutral and \pm refers to the sign of the weak isospin opposite to that of fermion) and their super partners.

1. The simplest assumption in the stringy case is that fermion- X_{\pm} pairs behave as coherent objects, that is scatter elastically. In more general case only their higher excitations identifiable in terms of stringy degrees of freedom would be created in vertices. The massivation of these states makes possible non-collinear vertices. An open question is how the massivation fermion- X_{\pm} pairs relates to the existing TGD based description of massivation in terms of Higgs mechanism and Kähler-Dirac operator.
2. Mass renormalization could come from self energy loops with negative energy lines as also vertex normalization. By very general arguments supersymmetry implies the cancellation of the self energy loops but would allow non-trivial vertex renormalization [K13] .
3. If only 3-vertices are allowed, the loops containing only positive energy lines are possible if on mass shell fermion- X_{\pm} pair (or its superpartner) can decay to a pair of positive energy pair particles of same kind. Whether this is possible depends on the masses involved. For ordinary particles these decays are not kinematically possible below intermediate boson mass scale (the decays $F_1 \rightarrow F_2 + \gamma$ are forbidden kinematically or by the absence of flavor changing neutral currents whereas intermediate gauge bosons can decay to on mass shell fermion-anti-fermion pair).
4. The introduction of IR cutoff for 3-momentum in the rest system associated with the largest CD (causal diamond) looks natural as scale parameter of coupling constant evolution and p-adic length scale hypothesis favors the inverse of the size scale of CD coming in powers of two. This parameter would define the momentum resolution as a discrete parameter of the

p-adic coupling constant evolution. This scale does not have any counterpart in standard physics. For electron, d quark, and u quark the proper time distance between the tips of CD corresponds to frequency of 10 Hz, 1280 Hz, and 160 Hz: all these frequencies define fundamental bio-rhythms [K9] .

These considerations have left completely untouched one important aspect of generalized Feynman diagrams: the necessity to perform a functional integral over the deformations of the partonic 2-surfaces at the ends of the lines- that is integration over WCW. Number theoretical universality requires that WCW and these integrals make sense also p-adically and in the following these aspects of generalized Feynman diagrams are discussed.

7.3 Harmonic Analysis In WCW As A Manner To Calculate WCW-Functional Integrals

Previous examples suggest that symmetric space property, Kähler and symplectic structure and the use of symplectic coordinates consisting of canonically conjugate pairs of phase angles and corresponding “radial” coordinates are essential for WCW integration and p-adicization. Kähler function, the components of the metric, and therefore also metric determinant and Kähler function depend on the “radial” coordinates only and the possible generalization involves the identification the counterparts of the “radial” coordinates in the case of WCW.

7.3.1 *Conditions guaranteeing the reduction to harmonic analysis*

The basic idea is that harmonic analysis in symmetric space allows to calculate the functional integral over WCW.

1. Each propagator line corresponds to a symmetric space defined as a coset space G/H of the symplectic group and Kac-Moody group and one might hope that the proposed p-adicization works for it- at least when one considers the hierarchy of measurement resolutions forced by the finiteness of algebraic extensions. This coset space is as a manifold Cartesian product $(G/H) \times (G/H)$ of symmetric spaces G/H associated with ends of the line. Kähler metric contains also an interaction term between the factors of the Cartesian product so that Kähler function can be said to reduce to a sum of “kinetic” terms and interaction term.
2. Effective 2-dimensionality and ZEO allow to treat the ends of the propagator line independently. This means an enormous simplification. Each line contributes besides propagator a piece to the exponent of Kähler action identifiable as interaction term in action and depending on the propagator momentum. This contribution should be expressible in terms of generalized spherical harmonics. Essentially a sum over the products of pairs of harmonics associated with the ends of the line multiplied by coefficients analogous to $1/(p^2 - m^2)$ in the case of the ordinary propagator would be in question. The optimal situation is that the pairs are harmonics and their conjugates appear so that one has invariance under G analogous to momentum conservation for the lines of ordinary Feynman diagrams.
3. Momentum conservation correlates the eigenvalue spectra of the Kähler-Dirac operator D at propagator lines [K28] . G -invariance at vertex dictates the vertex as the singlet part of the product of WCW harmonics associated with the vertex and one sums over the harmonics for each internal line. p-Adicization means only the algebraic continuation to real formulas to p-adic context.
4. The exponent of Kähler function depends on both ends of the line and this means that the geometries at the ends are correlated in the sense that that Kähler form contains interaction terms between the line ends. It is however not quite clear whether it contains separate “kinetic” or self interaction terms assignable to the line ends. For Kähler function the kinetic and interaction terms should have the following general expressions as functions of complex WCW coordinates:

$$\begin{aligned}
 K_{kin,i} &= \sum_n f_{i,n}(Z_i) \overline{f_{i,n}(Z_i)} + c.c. , \\
 K_{int} &= \sum_n g_{1,n}(Z_1) \overline{g_{2,n}(Z_2)} + c.c. , i = 1, 2 .
 \end{aligned}
 \tag{7.2}$$

Here $K_{kin,i}$ define ‘‘kinetic’’ terms and K_{int} defines interaction term. One would have what might be called holomorphic factorization suggesting a connection with conformal field theories.

Symmetric space property -that is isometry invariance- suggests that one has

$$f_{i,n} = f_{2,n} \equiv f_n , \quad g_{1,n} = g_{2,n} \equiv g_n \tag{7.3}$$

such that the products are invariant under the group H appearing in G/H and therefore have opposite H quantum numbers. The exponent of Kähler function does not factorize although the terms in its Taylor expansion factorize to products whose factors are products of holomorphic and antiholomorphic functions.

5. If one assumes that the exponent of Kähler function reduces to a product of eigenvalues of the Kähler-Dirac operator eigenvalues must have the decomposition

$$\lambda_k = \prod_{i=1,2} \exp \left[\sum_n c_{k,n} g_n(Z_i) \overline{g_n(Z_i)} + c.c. \right] \times \exp \left[\sum_n d_{k,n} g_n(Z_1) \overline{g_n(Z_2)} + c.c. \right] \tag{7.4}$$

Hence also the eigenvalues coming from the Dirac propagators have also expansion in terms of G/H harmonics so that in principle WCW integration would reduce to Fourier analysis in symmetric space.

7.3.2 Generalization of WCW Hamiltonians

This picture requires a generalization of the view about configuration space Hamiltonians since also the interaction term between the ends of the line is present not taken into account in the previous approach.

1. The proposed representation of WCW Hamiltonians as flux Hamiltonians [K8, K28]

$$\begin{aligned}
 Q(H_A) &= \int H_A (1 + K) J d^2 x , \\
 J &= \epsilon^{\alpha\beta} J_{\alpha\beta} , \quad J^{03} \sqrt{g_4} = K J_{12} .
 \end{aligned}
 \tag{7.5}$$

works for the kinetic terms only since J cannot be the same at the ends of the line. The formula defining K assumes weak form of self-duality (03 refers to the coordinates in the complement of X^2 tangent plane in the 4-D tangent plane). K is assumed to be symplectic invariant and constant for given X^2 . The condition that the flux of $F^{03} = (\hbar/g_K) J^{03}$ defining the counterpart of Kähler electric field equals to the Kähler charge g_K gives the condition $K = g_K^2/\hbar$, where g_K is Kähler coupling constant. Within experimental uncertainties one has $\alpha_K = g_K^2/4\pi\hbar_0 = \alpha_{em} \simeq 1/137$, where α_{em} is finite structure constant in electron length scale and \hbar_0 is the standard value of Planck constant.

The assumption that Poisson bracket of WCW Hamiltonians reduces to the level of imbedding space - in other words $\{Q(H_A), Q(H_B)\} = Q(\{H_A, H_B\})$ - can be justified. One starts from the representation in terms of say flux Hamiltonians $Q(H_A)$ and defines $J_{A,B}$ as

$J_{A,B} \equiv Q(\{H_A, H_B\})$. One has $\partial H_A / \partial t_B = \{H_B, H_A\}$, where t_B is the parameter associated with the exponentiation of H_B . The inverse J^{AB} of $J_{A,B} = \partial H_B / \partial t_A$ is expressible as $J^{A,B} = \partial t_A / \partial H_B$. From these formulas one can deduce by using chain rule that the bracket $\{Q(H_A), Q(H_B)\} = \partial t_C Q(H_A) J^{CD} \partial t_D Q(H_B)$ of flux Hamiltonians equals to the flux Hamiltonian $Q(\{H_A, H_B\})$.

2. One should be able to assign to WCW Hamiltonians also a part corresponding to the interaction term. The symplectic conjugation associated with the interaction term permutes the WCW coordinates assignable to the ends of the line. One should reduce this apparently non-local symplectic conjugation (if one thinks the ends of line as separate objects) to a non-local symplectic conjugation for $\delta CD \times CP_2$ by identifying the points of lower and upper end of CD related by time reflection and assuming that conjugation corresponds to time reflection. Formally this gives a well defined generalization of the local Poisson brackets between time reflected points at the boundaries of CD. The connection of Hermitian conjugation and time reflection in quantum field theories is in accordance with this picture.
3. The only manner to proceed is to assign to the flux Hamiltonian also a part obtained by the replacement of the flux integral over X^2 with an integral over the projection of X^2 to a sphere S^2 assignable to the light-cone boundary or to a geodesic sphere of CP_2 , which come as two varieties corresponding to homologically trivial and non-trivial spheres. The projection is defined as by the geodesic line orthogonal to S^2 and going through the point of X^2 . The hierarchy of Planck constants assigns to CD a preferred geodesic sphere of CP_2 as well as a unique sphere S^2 as a sphere for which the radial coordinate r_M or the light-cone boundary defined uniquely is constant: this radial coordinate corresponds to spherical coordinate in the rest system defined by the time-like vector connecting the tips of CD. Either spheres or possibly both of them could be relevant.

Recall that also the construction of number theoretic braids and symplectic QFT [K6] led to the proposal that braid diagrams and symplectic triangulations could be defined in terms of projections of braid strands to one of these spheres. One could also consider a weakening for the condition that the points of the number theoretic braid are algebraic by requiring only that the S^2 coordinates of the projection are algebraic and that these coordinates correspond to the discretization of S^2 in terms of the phase angles associated with θ and ϕ .

This gives for the corresponding contribution of the WCW Hamiltonian the expression

$$Q(H_A)_{int} = \int_{S^2_{\pm}} H_A X \delta^2(s_+, s_-) d^2 s_{\pm} = \int_{P(X^2_{\pm}) \cap P(X^2)} \frac{\partial(s^1, s^2)}{\partial(x^1_{\pm}, x^2_{\pm})} d^2 x_{\pm} . \quad (7.6)$$

Here the Poisson brackets between ends of the line using the rules involve delta function $\delta^2(s_+, s_-)$ at S^2 and the resulting Hamiltonians can be expressed as a similar integral of $H_{[A,B]}$ over the upper or lower end since the integral is over the intersection of S^2 projections. The expression must vanish when the induced Kähler form vanishes for either end. This is achieved by identifying the scalar X in the following manner:

$$\begin{aligned} X &= J_+^{kl} J_{kl}^- , \\ J_{\pm}^{kl} &= (1 + K_{\pm}) \partial_{\alpha} s^k \partial_{\beta} s^l J_{\pm}^{\alpha\beta} . \end{aligned} \quad (7.7)$$

The tensors are lifts of the induced Kähler form of X^2_{\pm} to S^2 (not CP_2).

4. One could of course ask why these Hamiltonians could not contribute also to the kinetic terms and why the brackets with flux Hamiltonians should vanish. This relate to how one defines the Kähler form. It was shown above that in case of flux Hamiltonians the definition of Kähler form as brackets gives the basic formula $\{Q(H_A), Q(H_B)\} = Q(\{H_A, H_B\})$ and same should hold true now. In the recent case $J_{A,B}$ would contain an interaction term defined in terms of flux Hamiltonians and the previous argument should go through also now by identifying Hamiltonians as sums of two contributions and by introducing the doubling of the coordinates t_A .

5. The quantization of the Kähler-Dirac operator must be reconsidered. It would seem that one must add to the super-Hamiltonian completely analogous term obtained by replacing $(1+K)J$ with $X\partial(s^1, s^2)/\partial(x_{\pm}^1, x_{\pm}^2)$. Besides the anti-commutation relations defining correct anti-commutators to flux Hamiltonians, one should pose anti-commutation relations consistent with the anti-commutation relations of super Hamiltonians. In these anti-commutation relations $(1+K)J\delta^2(x, y)$ would be replaced with $X\delta^2(s^+, s^-)$. This would guarantee that the oscillator operators at the ends of the line are not independent and that the resulting Hamiltonian reduces to integral over either end for $H_{[A,B]}$.
6. In the case of CP_2 the Hamiltonians generating isometries are rational functions. This should hold true also now so that p-adic variants of Hamiltonians as functions in WCW would make sense. This in turn would imply that the components of the WCW Kähler form are rational functions. Also the exponentiation of Hamiltonians make sense p-adically if one allows the exponents of group parameters to be functions $Exp_p(t)$.

7.3.3 Does the expansion in terms of partial harmonics converge?

The individual terms in the partial wave expansion seem to be finite but it is not at all clear whether the expansion in powers of K actually converges.

1. In the proposed scenario one performs the expansion of the vacuum functional $exp(K)$ in powers of K and therefore in negative powers of α_K . In principle an infinite number of terms can be present. This is analogous to the perturbative expansion based on using magnetic monopoles as basic objects whereas the expansion using the contravariant Kähler metric as a propagator would be in positive powers of α_K and analogous to the expansion in terms of magnetically bound states of wormhole throats with vanishing net value of magnetic charge. At this moment one can only suggest various approaches to how one could understand the situation.
2. Weak form of self-duality and magnetic confinement could change the situation. Performing the perturbation around magnetic flux tubes together with the assumed slicing of the space-time sheet by stringy world sheets and partonic 2-surfaces could mean that the perturbation corresponds to the action assignable to the electric part of Kähler form proportional to α_K by the weak self-duality. Hence by $K = 4\pi\alpha_K$ relating Kähler electric field to Kähler magnetic field the expansion would come in powers of a term containing sum of terms proportional to α_K^0 and α_K . This would leave to the scattering amplitudes the exponents of Kähler function at the maximum of Kähler function so that the non-analytic dependence on α_K would not disappear.

A further reason to be worried about is that the expansion containing infinite number of terms proportional to α_K^0 could fail to converge.

1. This could be also seen as a reason for why magnetic singlets are unavoidable except perhaps for $\hbar < \hbar_0$. By the holomorphic factorization the powers of the interaction part of Kähler action in powers of $1/\alpha_K$ would naturally correspond to increasing and opposite net values of the quantum numbers assignable to the WCW phase coordinates at the ends of the propagator line. The magnetic bound states could have similar expansion in powers of α_K as pairs of states with arbitrarily high but opposite values of quantum numbers. In the functional integral these quantum numbers would compensate each other. The functional integral would leave only an expansion containing powers of α_K starting from some finite possibly negative (unless one assumes the weak form of self-duality) power. Various gauge coupling strengths are expected to be proportional to α_K and these expansions should reduce to those in powers of α_K .
2. Since the number of terms in the fermionic propagator expansion is finite, one might hope on basis of super-symmetry that the same is true in the case of the functional integral expansion. By the holomorphic factorization the expansion in powers of K means the appearance of terms with increasingly higher quantum numbers. Quantum number conservation at vertices would leave only a finite number of terms to tree diagrams. In the case of loop diagrams pairs of

particles with opposite and arbitrarily high values of quantum numbers could be generated at the vertex and magnetic confinement might be necessary to guarantee the convergence. Also super-symmetry could imply cancellations in loops.

7.3.4 *Could one do without flux Hamiltonians?*

The fact that the Kähler functions associated with the propagator lines can be regarded as interaction terms inspires the question whether the Kähler function could contain only the interaction terms so that Kähler form and Kähler metric would have components only between the ends of the lines.

1. The basic objection is that flux Hamiltonians too beautiful objects to be left without any role in the theory. One could also argue that the WCW metric would not be positive definite if only the non-diagonal interaction term is present. The simplest example is Hermitian 2×2 -matrix with vanishing diagonal for which eigenvalues are real but of opposite sign.
2. One could of course argue that the expansions of $\exp(K)$ and λ_k give in the general powers $(f_n \bar{f}_n)^m$ analogous to diverging tadpole diagrams of quantum field theories due to local interaction vertices. These terms do not produce divergences now but the possibility that the exponential series of this kind of terms could diverge cannot be excluded. The absence of the kinetic terms would allow to get rid of these terms and might be argued to be the symmetric space counterpart for the vanishing of loops in WCW integral.
3. In ZEO this idea does not look completely non-sensical since physical states are pairs of positive and negative energy states. Note also that in quantum theory only creation operators are used to create positive energy states. The manifest non-locality of the interaction terms and absence of the counterparts of kinetic terms would provide a trivial manner to get rid of infinities due to the presence of local interactions. The safest option is however to keep both terms.

7.3.5 *Summary*

The discussion suggests that one must treat the entire Feynman graph as single geometric object with Kähler geometry in which the symmetric space is defined as product of what could be regarded as analogs of symmetric spaces with interaction terms of the metric coming from the propagator lines. The exponent of Kähler function would be the product of exponents associated with all lines and contributions to lines depend on quantum numbers (momentum and color quantum numbers) propagating in line via the coupling to the Kähler-Dirac operator. The conformal factorization would allow the reduction of integrations to Fourier analysis in symmetric space. What is of decisive importance is that the entire Feynman diagrammatics at WCW level would reduce to the construction of WCW geometry for a single propagator line as a function of quantum numbers propagating on the line.

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