Knots and TGD

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Abstract

Khovanov homology generalizes the Jones polynomial as knot invariant. The challenge is to find a quantum physical construction of Khovanov homology analous to the topological QFT defined by Chern-Simons action allowing to interpret Jones polynomial as vacuum expectation value of Wilson loop in non-Abelian gauge theory.

Witten's approach to Khovanov homology relies on fivebranes as is natural if one tries to define 2-knot invariants in terms of their cobordisms involving violent un-knottings. Despite the difference in approaches it is very useful to try to find the counterparts of this approach in quantum TGD since this would allow to gain new insights to quantum TGD itself as almost topological QFT identified as symplectic theory for 2-knots, braids and braid cobordisms. This comparison turns out to be extremely useful from TGD point of view.

1. Key question concerns the identification of string world sheets. A possible identification of string world sheets and therefore also of the braids whose ends carry quantum numbers of many particle states at partonic 2-surfaces emerges if one identifies the string word sheets as singular surfaces in the same manner as is done in Witten's approach.

   In TGD framework the localization of the modes of the induced spinor fields at 2-D surfaces carrying vanishing induced W boson fields guaranteeing that the em charge of spinor modes is well-defined for a generic preferred extremal is natural. Besides string world sheets partonic 2-surfaces are good candidates for this kind of surfaces. It is not clear whether one can have continuous slicing of this kind by string world sheets and partonic 2-surfaces orthogonal to them or whether only discrete set of these surfaces is possible.

2. Also a physical interpretation of the operators Q, F, and P of Khovanov homology emerges. P would correspond to instanton number and F to the fermion number assignable to right handed neutrinos. The breaking of M^4 chiral invariance makes possible to realize Q physically. The finding that the generalizations of Wilson loops can be identified in terms of the gerbe fluxes $\int H_A J$ supports the conjecture that TGD as almost topological QFT corresponds essentially to a symplectic theory for braids and 2-knots.

The basic challenge of quantum TGD is to give a precise content to the notion of generalized Feynman diagram and the reduction to braids of some kind is very attractive possibility inspired by zero energy ontology. The point is that no $n > 2$-vertices at the level of braid strands are needed if bosonic emergence holds true.

1. For this purpose the notion of algebraic knot is introduce and the possibility that it could be applied to generalized Feynman diagrams is discussed. The algebraic structures kei, quandle, rack, and biquandle and their algebraic modifications as such are not enough. The lines of Feynman graphs are replaced by braids and in vertices braid strands redistribute. This poses several challenges: the crossing associated with braiding and crossing occurring in non-planar Feynman diagrams should be integrated to a more general notion; braids are replaced with sub-manifold braids; braids of braids ....of braids are possible; the redistribution of braid strands in vertices should be algebraized. In the following I try to abstract the basic operations which should be algebraized in the case of generalized Feynman diagrams.

2. One should be also able to concretely identify braids and 2-braids (string world sheets) as well as partonic 2-surfaces and I have discussed several identifications during last years. Legendrian braids turn out to be very natural candidates for braids and their duals for the partonic 2-surfaces. String world sheets in turn could correspond to the analogs of Lagrangian sub-manifolds or two minimal surfaces of space-time surface satisfying the weak form of electric-magnetic duality. The latter opion turns out to be more plausible. This identification - if correct - would solve quantum TGD explicitly at string world sheet level which corresponds to finite measurement resolution.

3. Also a brief summary of generalized Feynman rules in zero energy ontology is proposed. This requires the identification of vertices, propagators, and prescription for integrating over al 3-surfaces. It turns out that the basic building blocks of generalized Feynman diagrams are well-defined.

4. The notion of generalized Feynman diagram leads to a beautiful duality between the descriptions of hadronic reactions in terms of hadrons and partons analogous to gauge-gravity duality and AdS/CFT duality but requiring no additional assumptions. The model of quark gluon plasma as s strongly interacting phase is proposed. Color magnetic

flux tubes are responsible for the long range correlations making the plasma phase more like a very large hadron rather than a gas of partons. One also ends up with a simple estimate for the viscosity/entropy ratio using black-hole analogy.

1 Introduction

Witten has highly inspiring popular lecture about knots and quantum physics [A11] mentioning also his recent work with knots related to an attempt to understand Khovanov homology. Witten manages to explain in rather comprehensible way both the construction recipe of Jones polynomial and the idea about how Jones polynomial emerges from topological quantum field theory as a vacuum expectation of so called Wilson loop defined by path integral with weighting coming from Chern-Simons action [A15]. Witten also tells that during the last year he has been working with an attempt to understand in terms of quantum theory the so called Khovanov polynomial associated with a much more abstract link invariant whose interpretation and real understanding remains still open. In particular, he mentions the approach of Gukov, Schwartz, and Vafa [A17, A17] as an attempt to understand Khovanov polynomial.

This kind of talks are extremely inspiring and lead to a series of questions unavoidably culminating to the frustrating "Why I do not have the brain of Witten making perhaps possible to answer these questions?". This one must just accept. In the following I summarize some thoughts inspired by the associations of the talk of Witten with quantum TGD and with the model of DNA as topological quantum computer. In my own childish way I dare believe that these associations are interesting and dare also hope that some more brainy individual might take them seriously.

An idea inspired by TGD approach which also main streamer might find interesting is that the Jones invariant defined as vacuum expectation for a Wilson loop in 2+1-D space-time generalizes to a vacuum expectation for a collection of Wilson loops in 2+2-D space-time and could define an invariant for 2-D knots and for cobordisms of braids analogous to Jones polynomial. As a matter fact, it turns out that a generalization of gauge field known as gerbe is needed and that in TGD framework classical color gauge fields defined the gauge potentials of this field. Also topological string theory in 4-D space-time could define this kind of invariants. Of course, it might well be that this kind of ideas have been already discussed in literature.

Khovanov homology generalizes the Jones polynomial as knot invariant. The challenge is to find a quantum physical construction of Khovanov homology analous to the topological QFT defined by Chern-Simons action allowing to interpret Jones polynomial as vacuum expectation value of Wilson loop in non-Abelian gauge theory.

Witten's approach to Khovanov homology relies on fivebranes as is natural if one tries to define 2-knot invariants in terms of their cobordisms involving violent un-knottings. Despite the difference in approaches it is very useful to try to find the counterparts of this approach in quantum TGD since this would allow to gain new insights to quantum TGD itself as almost topological QFT identified as symplectic theory for 2-knots, braids and braid cobordisms. This comparison turns out to be extremely useful from TGD point of view.

1. A highly unique identification of string world sheets and therefore also of the braids whose ends carry quantum numbers of many particle states at partonic 2-surfaces emerges if one identifies the string word sheets as singular surfaces in the same ways as is done in Witten's approach.

   This identification need not of course be correct and in TGD framework the localization of the modes of the induced spinor fields at 2-D surfaces carrying vanishing induced W boson fields guaranteeing that the em charge of spinor modes is well-defined for a generic preferred extremal is natural. Besides string world sheets partonic 2-surfaces are good candidates for this kind of surfaces. It is not clear whether one can have continuous slicing of this kind by string world sheets and partonic 2-surfaces orthogonal to them or whether only discrete set of these surfaces is possible.

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gerbe fluxes $\int H_A J$ supports the conjecture that TGD as almost topological QFT corresponds essentially to a symplectic theory for braids and 2-knots.

The basic challenge of quantum TGD is to give a precise content to the notion of generalization Feynman diagram and the reduction to braids of some kind is very attractive possibility inspired by zero energy ontology. The point is that no $n > 2$-vertices at the level of braid strands are needed if bosonic emergence holds true.

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The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://tgdtheory.fi/cmaphtml.html` [L10].

2 Some TGD Background

What makes quantum TGD [L2, L3, L6, L7, L4, L1, L5, L8] interesting concerning the description of braids and braid cobordisms is that braids and braid cobordisms emerge both at the level of generalized Feynman diagrams and in the model of DNA as a topological quantum computer [K1].

2.1 Time-Like And Space-Like Braidings For Generalized Feynman Diagrams

1. In TGD framework space-times are 4-D surfaces in 8-D embedding space. Basic objects are partonic 2-surfaces at the two ends of causal diamonds CD (intersections of future and past directed light-cones of 4-D Minkowski space with each point replaced with CP_2). The light-like orbits of partonic 2-surfaces define 3-D light-like 3-surfaces identifiable as lines of generalized Feynman diagrams. At the vertices of generalized Feynman diagrams incoming and outgoing light-like 3-surfaces meet. These diagrams are not direct generalizations of string diagrams since they are singular as 4-D manifolds just like the ordinary Feynman diagrams.

   By strong form of holography one can assign to the partonic 2-surfaces and their tangent space data space-time surfaces as preferred extremals of Kähler action. This guarantees also general coordinate invariance and allows to interpret the extremals as generalized Bohr orbits.

2. One can assign to the partonic 2-surfaces discrete sets of points carrying quantum numbers. These sets of points emerge from the solutions of of the Kähler-Dirac equation, which are localized at 2-D surfaces - string world sheets and possibly also partonic 2-surfaces - carrying vanishing induced W fields and also Z^0 fields above weak scale. These points and their orbits identifiable as boundaries of string world sheets define braid strands at the light-like orbits of partonic 2-surfaces. In the generic case the strands get tangled in time direction and one has linking and knotting giving rise to a time-like braiding. String world sheets and also partonic surfaces define 2-braids and 2-knots at 4-D space-time surface so that knot theory generalizes.

3. Also space-like braidings are possible. One can imagine that the partonic 2-surfaces are connected by space-like curves defining TGD counterparts for strings and that in the initial state these curves define space-like braids whose ends belong to different partonic 2-surfaces. Quite generally, the basic conjecture is that the preferred extremals define orbits of string-like objects with their ends at the partonic 2-surfaces. One would have slicing of space-time surfaces by string world sheets one one hand and by partonic 2-surface on one hand. This string model is very special due to the fact that the string orbits define what could be called braid cobordisms representing which could represent unknotting of braids. String orbits in higher dimensional space-times do not allow this topological interpretation.

2.2 Dance Metaphor

Time like braidings induces space-like braidings and one can speak of time-like or dynamical braiding and even duality of time-like and space-like braiding. What happens can be understood in terms of dance metaphor.

1. One can imagine that the points carrying quantum numbers are like dancers at parquettes defined by partonic 2-surfaces. These parquettes are somewhat special in that it is moving and changing its shape.

2. Space-like braidings means that the feet of the dancers at different parquettes are connected by threads. As the dance continues, the threads connecting the feet of different dancers at different parquettes get tangled so that the dance is coded to the braiding of the threads. Time-like braiding induce space-like braiding. One has what might be called a cobordism for space-like braiding transforming it to a new one.

2.3 DNA As Topological Quantum Computer

The model for topological quantum computation is based on the idea that time-like braidings defining topological quantum computer programs. These programs are robust since the topology of braiding is not affected by small deformations.

1. The first key idea in the model of DNA as topological quantum computer is based on the observation that the lipids of cell membrane form a 2-D liquid whose flow defines the dance in which dancers are lipids which define a flow pattern defining a topological quantum computation. Lipid layers assignable to cellular and nuclear membranes are the parquettes. This 2-D flow pattern can be induced by the liquid flow near the cell membrane or in case of nerve pulse transmission by the nerve pulses flowing along the axon. This alone defines topological quantum computation.

2. In DNA as topological quantum computer model one however makes a stronger assumption motivated by the vision that DNA is the brain of cell and that information must be communicated to DNA level wherefrom it is communicated to what I call magnetic body. It is assumed that the lipids of the cell membrane are connected to DNA nucleotides by magnetic flux tubes defining a space-like braiding. It is also possible to connect lipids of cell membrane to the lipids of other cell membranes, to the tubulins at the surfaces of microtubules, and also to the aminoadics of proteins. The spectrum of possibilities is really wide.

   The space-like braid strands would correspond to magnetic flux tubes connecting DNA nucleotides to lipids of nuclear or cell membrane. The running of the topological quantum computer program defined by the time-like braiding induced by the lipid flow would be coded to a space-like braiding of the magnetic flux tubes. The braiding of the flux tubes would define a universal memory storage mechanism and combined with 4-D view about memory provides a very simple view about how memories are stored and how they are recalled.

3 Could Braid Cobordisms Define More General Braid Invariants?

Witten says that one should somehow generalize the notion of knot invariant. The above described framework indeed suggests a very natural generalization of braid invariants to those of braid cobordisms reducing to braid invariants when the braid at the other end is trivial. This description is especially natural in TGD but allows a generalization in which Wilson loops in 4-D sense describe invariants of braid cobordisms.

3.1 Difference Between Knotting And Linking

Before my modest proposal of a more general invariant some comments about knotting and linking are in order.

1. One must distinguish between internal knotting of each braid strand and linking of 2 strands. They look the same in the 3-D case but in higher dimensions knotting and linking are not the same thing. Codimension 2 surfaces get knotted in the generic case, in particular the 2-D orbits of the braid strands can get knotted so that this gives additional topological flavor to the theory of strings in 4-D space-time. Linking occurs for two surfaces whose dimension d_1 and d_2 satisfying $d_1 + d_2 = D - 1$, where D is the dimension of the embedding space.
2. 2-D orbits of strings do not link in 4-D space-time but do something more radical since the sum of their dimensions is $D = 4$ rather than only $D - 1 = 3$. They intersect and it is impossible to eliminate the intersection without a change of topology of the stringy 2-surfaces: a hole is generated in either string world sheet. With a slight deformation intersection can be made to occur generically at discrete points.

3.2 Topological Strings In 4-D Space-Time Define Knot Cobordisms

What makes the 4-D braid cobordisms interesting is following.

1. The opening of knot by using brute force by forcing the strands to go through each other induces this kind of intersection point for the corresponding 2-surfaces. From 3-D perspective this looks like a temporary cutting of second string, drawing the string ends to some distance and bringing them back and gluing together as one approaches the moment when the strings would go through each other. This surgical operation for either string produces a pair of non-intersecting 2-surfaces with the price that the second string world sheet becomes topologically non-trivial carrying a hole in the region were intersection would occur. This operation relates a given crossing of braid strands to its dual crossing in the construction of Jones polynomial in given step (string 1 above string 2 is transformed to string 2 above string 1).
2. One can also cut both strings temporarily and glue them back together in such a way that end a/b of string 1 is glued to the end c/d of string 2. This gives two possibilities corresponding to two kinds of reconnections. Reconnections appears as the second operation in the construction of Jones invariant besides the operation putting the string above the second one below it or vice versa. Jones polynomial (see `http://tinyurl.com/2jctzy`) relates in a simple manner to Kauffman bracket (see `http://tinyurl.com/yc2wu47x`) allowing a recursive construction. At a given step a crossing is replaced with a weighted sum of the two reconnected terms [A1, A7]. Reconnection represents the analog of trouser vertex for closed strings replaced with braid strands.

3. These observations suggest that stringy diagrams describe the braid cobordisms and a kind of topological open string model in 4-D space-time could be used to construct invariants of braid cobordisms. The dynamics of strand ends at the partonic 2-surfaces would partially induce the dynamics of the space-like braiding. This dynamics need not induce the un-knotting of space-like braids and simple string diagrams for open strings are enough to define a cobordism leading to un-knotting. The holes needed to realize the crossover for braid strands would contribute to the Wilson loop an additional factor corresponding to the rotation of the gauge potential around the boundary of the hole (non-integrable phase factor). In abelian case this gives simple commuting phase factor.

Note that braids are actually much more closer to the real world than knots since a useful strand of knotted structure must end somewhere. The abstract closed loops of mathematician floating in empty space are not very useful in real life albeit mathematically very convenient as Witten notices. Also the braid cobordisms with ends of a collection of space-like braids at the ends of causal diamond are more practical than 2-knots in 4-D space. Mathematician would see these objects as analogous to surfaces in relative homology allowed to have boundaries if they located at fixed sub-manifolds. Homology for curves with ends fixed to be on some surfaces is a good example of this. Now these fixed sub-manifolds would correspond to space-like 3-surfaces at the ends CDs and light-like wormhole throats at which the signature of the induced metric changes and which are carriers of elementary particle quantum numbers.

4 Invariants 2-Knots As Vacuum Expectations Of Wilson Loops In 4-D Space-Time?

The interpretation of string world sheets in terms of Wilson loops in 4-dimensional space-time is very natural. This raises the question whether Witten's a original identification of the Jones polynomial as vacuum expectation for a Wilson loop in 2+1-D space might be replaced with a vacuum expectation for a collection of Wilson loops in 3+1-D space-time and would characterize in the general case (multi-)braid cobordism rather than braid. If the braid at the lower or upped boundary is trivial, braid invariant is obtained. The intersections of the Wilson loops would correspond to the violent un-knotting operations and the boundaries of the resulting holes give an additional Wilson loop. An alternative interpretation would be as the analog of Jones polynomial for 2-D knots in 4-D space-time generalizing Witten's theory. This description looks completely general and does not require TGD at all.

The following considerations suggest that Wilson loops are not enough for the description of general 2-knots and that Wilson loops must be replaced with 2-D fluxes. This requires a generalization of gauge field concept so that it corresponds to a 3-form instead of 2-form is needed. In TGD framework this kind of generalized gauge fields exist and their gauge potentials correspond to classical color gauge fields.

4.1 What 2-Knottedness Means Concretely?

It is easy to imagine what ordinary knottedness means. One has circle imbedded in 3-space. One projects it in some plane and looks for crossings. If there are no crossings one knows that un-knot is in question. One can modify a given crossing by forcing the strands to go through each other and this either generates or removes knottedness. One can also destroy crossing by reconnection and this always reduces knottedness. Since knotting reduces to linking in 3-D case, one can find a simple interpretation for knottedness in terms of linking of two circles. For 2-knots linking is not what gives rise to knotting.

One might hope to find something similar in the case of 2-knots. Can one imagine some simple local operations which either increase of reduce 2-knottedness?

1. To proceed let us consider as simple situation as possible. Put sphere in 3-D time= constant section E^3 of 4-space. Add a another sphere to the same section E^3 such that the corresponding balls do not intersect. How could one build from these two spheres a knotted 2-sphere?

2. From two spheres one can build a single sphere in topological sense by connecting them with a small cylindrical tube connecting the boundaries of disks (circles) removed from the two spheres. If this is done in E^3, a trivial 2-knot results. One can however do the gluing of the cylinder in a more exotic manner by going temporarily to "hyper-space", in other words making a time travel. Let the cylinder leave the second sphere from the outer surface, let it go to future or past and return back to recent but through the interior. This is a good candidate for a knotted sphere since the attempts to deform it to self-non-intersecting sphere in E^3 are expected to fail since the cylinder starting from interior necessarily goes through the surface of sphere if wants to the exterior of the sphere.

3. One has actually 2×2 ways to perform the connected sum of 2-spheres depending on whether the cylinders leave the spheres through exterior or interior. At least one of them (exterior-exterior) gives an un-knotted sphere and intuition suggests that all the three remaining options requiring getting out from the interior of sphere give a knotted 2-sphere. One can add to the resulting knotted sphere new spheres in the same manner and obtain an infinite number of them. As a matter fact, the proposed 1+3 possibilities correspond to different versions of connected sum and one could speak of knotting and non-knotting connected sums. If the addition of knotted spheres is performed by non-knotting connected sum, one obtains composites of already existing 2-knots. Connected sum composition is analogous to the composition of integer to a product of primes. One indeed speaks of prime knots and the number of prime knots is infinite. Of course, it is far from clear whether the connected sum operation is enough to build all knots. For instance it might well be that cobordisms of 1-braids produces knots not producible in this manner. In particular, the effects of time-like braiding induce braiding of space-like strands and this looks totally different from local knotting.

4.2 Are All Possible 2-Knots Possible For Stringy WorldSheets?

Whether all possible 2-knots are allowed for stringy world sheets, is not clear. In particular, if they are dynamically determined it might happen that many possibilities are not realized. For instance, the condition that the signature of the induced metric is Minkowskian could be an effective killer of 2-knottedness not reducing to braid cobordism.

1. One must start from string world sheets with Minkowskian signature of the induced metric. In other words, in the previous construction one must E^3 with 3-dimensional Minkowski space M^3 with metric signature 1+2 containing the spheres used in the construction. Time travel is replaced with a travel in space-like hyper dimension. This is not a problem as such. The spheres however have at least one two special points corresponding to extrema at which the time coordinate has a local minimum or maximum. At these points the induced metric is necessarily degenerate meaning that its determinant vanishes. If one allows this kind of singular points one can have elementary knotted spheres. This liberal attitude is encouraged by the fact that the light-like 3-surfaces defining the basic dynamical objects of quantum TGD correspond to surfaces at which 4-D induced metric is degenerate. Otherwise 2-knotting reduces to that induced by cobordisms of 1-braids. If one allows only the 2-knots assignable to the slicings of the space-time surface by string world sheets and even restricts the consideration to those suggested by the duality of 2-D generalization of Wilson loops for string world sheets and partonic 2-surfaces, it could happen that the string world sheets reduce to braidings.

2. The time=constant intersections define a representation of 2-knots as a continuous sequence of 1-braids. For critical times the character of the 1-braids changes. In the case of braiding this corresponds to the basic operations for 1-knots having interpretation as string diagrams (reconnection and analog of trouser vertex). The possibility of genuine 2-knottedness brings in also the possibility that strings pop up from vacuum as points, expand to closed strings, are fused to stringy words sheet temporarily by the analog of trouser vertex, and eventually return to the vacuum. Essentially trouser diagram but second string open and second string closed and beginning from vacuum and ending to it is in question. Vacuum bubble interacting

with open string would be in question. The believer in string model might be eager to accept this picture but one must be cautious.

4.3 Are Wilson Loops Enough For 2-Knots?

Suppose that the space-like braid strands connecting partonic 2-surfaces at given boundary of CD and light-like braids connecting partonic 2-surfaces belonging to opposite boundaries of CD form connected closed strands. The collection of closed loops can be identified as boundaries of Wilson loops and the expectation value is defined as the product of traces assignable to the loops. The definition is exactly the same as in 2+1-D case [A15].

Is this generalization of Wilson loops enough to describe 2-knots? In the spirit of the proposed philosophy one could ask whether there exist two-knots not reducible to cobordisms of 1-knots whose knot invariants require cobordisms of 2-knots and therefore 2-braids in 5-D space-time. Could it be that dimension $D = 4$ is somehow very special so that there is no need to go to $D = 5$? This might be the case since for ordinary knots Jones polynomial is very faithful invariant.

Innocent novice could try to answer the question in the following manner. Let us study what happens locally as the 2-D closed surface in 4-D space gets knotted.

1. In 1-D case knotting reduces to linking and means that the first homotopy group of the knot complement is changed so that the embedding of first circle implies that the there exists embedding of the second circle that cannot be transformed to each other without cutting the first circle temporarily. This phenomenon occurs also for single circle as the connected sum operation for two linked circles producing single knotted circle demonstrates.

2. In 2-D case the complement of knotted 2-sphere has a non-trivial second homotopy group so that 2-balls have homotopically non- equivalent embeddings, which cannot be transformed to each other without intersection of the 2-balls taking place during the process. Therefore the description of 2-knotting in the proposed manner would require cobordisms of 2-knots and thus 5-D space-time surfaces. However, since 3-D description for ordinary knots works so well, one could hope that the generalization the notion of Wilson loop could allow to avoid 5-D description altogether. The generalized Wilson loops would be assigned to 2-D surfaces and gauge potential A would be replaced with 2-gauge potential B defining a three-form $F = dB$ as the analog of gauge field.

3. This generalization of bundle structure known as gerbe structure has been introduced in algebraic geometry [A5, A21] and studied also in theoretical physics [A18]. 3-forms appear as analogs of gauge fields also in the QFT limit of string model. Algebraic geometer would see gerbe as a generalization of bundle structure in which gauge group is replaced with a gauge groupoid. Essentially a structure of structures seems to be in question. For instance, the principal bundles with given structure group for given space defines a gerbe. In the recent case the space of gauge fields in space-time could be seen as a gerbe. Gerbes have been also assigned to loop spaces and WCW can be seen as a generalization of loop space. Lie groups define a much more mundane example about gerbe. The 3-form F is given by $F(X, Y, Z) = B(X, [Y, Z])$, where B is Killing form and for $U(n)$ reduces to $(g^{-1}dg)^3$. It will be found that classical color gauge fields define gerbe gauge potentials in TGD framework in a natural manner.

5 TGD Inspired Theory Of Braid Cobordisms And 2-Knots

In the sequel the considerations are restricted to TGD and to a comparison of Witten's ideas with those emerging in TGD framework.

5.1 Weak Form Of Electric-Magnetic Duality And Duality Of Space-Like And Time-Like Braidings

Witten notices that much of his work in physics relies on the assumption that magnetic charges exist and that rather frustratingly, cosmic inflation implies that all traces of them disappear. In TGD

Universe the non-trivial topology of CP_2 makes possible Kähler magnetic charge and inflation is replaced with quantum criticality. The recent view about elementary particles is that they correspond to string like objects with length of order electro-weak scale with Kähler magnetically charged wormhole throats at their ends. Therefore magnetic charges would be there and LHC might be able to detect their signatures if LHC would get the idea of trying to do this.

Witten mentions also electric-magnetic duality. If I understood correctly, Witten believes that it might provide interesting new insights to the knot invariants. In TGD framework one speaks about weak form of electric magnetic duality. This duality states that Kähler electric fluxes at space-like ends of the space-time sheets inside CDs and at wormhole throats are proportional to Kähler magneic fluxes so that the quantization of Kähler electric charge quantization reduces to purely homological quantization of Kähler magnetic charge.

The weak form of electric-magnetic duality fixes the boundary conditions of field equations at the light-like and space-like 3-surfaces. Together with the conjecture that the Kähler current is proportional to the corresponding instanton current this implies that Kähler action for the preferred extremal sof Kähler action reduces to 3-D Chern-Simons term so that TGD reduces to almost topological QFT. This means an enormous mathematical simplification of the theory and gives hopes about the solvability of the theory. Since knot invariants are defined in terms of Abelian Chern-Simons action for induced Kähler gauge potential, one might hope that TGD could as a by-product define invariants of braid cobordisms in terms of the unitary U-matrix of the theory between zero energy states. The detailed construction of U-matrix is discussed in [K10].

Electric magnetic duality is 4-D phenomenon as is also the duality between space-like and time like braidings essential also for the model of topological quantum computation. Also this suggests that some kind of topological string theory for the space-time sheets inside CDs could allow to define the braid cobordism invariants.

5.2 Could Kähler Magnetic Fluxes Define Invariants Of Braid Cobordisms?

Can one imagine of defining knot invariants or more generally, invariants of knot cobordism in this framework? As a matter fact, also Jones polynomial describes the process of unknotting and the replacement of unknotting with a general cobordism would define a more general invariant. Whether the Khovanov invariants might be understood in this more general framework is an interesting question.

1. One can assign to the 2-dimensional stringy surfaces defined by the orbits of space-like braid strands Kähler magnetic fluxes as flux integrals over these surfaces and these integrals depend only on the end points of the space-like strands so that one deform the space-like strands in an arbitrarily manner. One can in fact assign these kind of invariants to pairs of knots and these invariants define the dancing operation transforming these knots to each other. In the special case that the second knot is un-knot one obtains a knot-invariant (or link- or braid-invariant).

2. The objection is that these invariants depend on the orbits of the end points of the space-like braid strands. Does this mean that one should perform an averaging over the ends with the condition that space-like braid is not affected topologically by the allowed deformations for the positions of the end points?

3. Under what conditions on deformation the magnetic fluxes are not affect in the deformation of the braid strands at 3-D surfaces? The change of the Kähler magnetic flux is magnetic flux over the closed 2-surface defined by initial non-deformed and deformed stringy two-surfaces minus flux over the 2-surfaces defined by the original time-like and space-like braid strands connected by a thin 2-surface to their small deformations. This is the case if the deformation corresponds to a U(1) gauge transformation for a Kähler flux. That is diffeomorphism of M^4 and symplectic transformation of CP_2 inducing the U(1) gauge transformation.

   Hence a natural equivalence for braids is defined by these transformations. This is quite not a topological equivalence but quite a general one. Symplectic transformations of CP_2 at light-like and space-like 3-surfaces define isometries of the world of classical worlds so that also in this sense the equivalence is natural. Note that the deformations of space-time surfaces correspond to this kind of transformations only at space-like 3-surfaces at the ends of CDs and at the light-like wormhole throats where the signature of the induced metric changes. In fact, in quantum TGD the sub-spaces of world of classical worlds with constant values of zero modes (non-quantum fluctuating degrees of freedom) correspond to orbits of 3-surfaces under symplectic transformations so that the symplectic restriction looks rather natural also from the point of view of quantum dynamics and the vacuum expectation defined by Kähler function be defined for physical states.

4. A further possibility is that the light-like and space-like 3-surfaces carry vanishing induced Kähler fields and represent surfaces in $M^4 \times Y^2$, where Y^2 is Lagrangian sub-manifold of CP_2 carrying vanishing Kähler form. The interior of space-time surface could in principle carry a non-vanishing Kähler form. In this case weak form of self-duality cannot hold true. This however implies that the Kähler magnetic fluxes vanish identically as circulations of Kähler gauge potential. The non-integrable phase factors defined by electroweak gauge potentials would however define non-trivial classical Wilson loops. Also electromagnetic field would do so. It would be therefore possible to imagine vacuum expectation value of Wilson loop for given quantum state. Exponent of Kähler action would define for non-vacuum extremals the weighting. For 4-D vacuum extremals this exponent is trivial and one might imagine of using imaginary exponent of electroweak Chern-Simons action. Whether the restriction to vacuum extremals in the definition of vacuum expectations of electroweak Wilson loops could define general enough invariants for braid cobordisms remains an open question.

5. The quantum expectation values for Wilson loops are non-Abelian generalizations of exponentials for the expectation values of Kähler magnetic fluxes. The classical color field is proportional to the induced Kähler form and its holonomy is Abelian which raises the question whether the non-Abelian Wilson loops for classical color gauge field could be expressible in terms of Kähler magnetic fluxes.

5.3 Classical Color Gauge Fields And Their Generalizations Define Gerbe Gauge Potentials Allowing To Replace Wilson Loops With Wilson Sheets

As already noticed, the description of 2-knots seems to necessitate the generalization of gauge field to 3-form and the introduction of a gerbe structure. This seems to be possible in TGD framework.

1. Classical color gauge fields are proportional to the products $B_A = H_A J$ of the Hamiltonians of color isometries and of Kähler form and the closed 3-form $F_A = dB_A = dH_A \wedge J$ could serve as a colored 3-form defining the analog of U(1) gauge field. What would be interesting that color would make F non-vanishing. The "circulation" $h_A = \oint H_A J$ over a closed partonic 2-surface transforms covariantly under symplectic transformations of CP_2, whose Hamiltonians can be assigned to irreps of SU(3): just the commutator of Hamiltonians defined by Poisson bracket appears in the infinitesimal transformation. One could hope that the expectation values for the exponents of the fluxes of B_A over 2-knots could define the covariants able to catch 2-knotted-ness in TGD framework. The exponent defining Wilson loop would be replaced with $exp(iQ^A h_A)$, where Q^A denote color charges acting as operators on particles involved.

2. Since the symplectic group acting on partonic 2-surfaces at the boundary of CD replaces color group as a gauge group in TGD, one can ask whether symplectic SU(3) should be actually replaced with the entire symplectic group of $\cup_{\pm} \delta M^4_{\pm} \times CP_2$ with Hamiltonians carrying both spin and color quantum numbers. The symplectic fluxes $\oint H_A J$ are indeed used in the construction of both quantum states and of WCW geometry. This generalization is indeed possible for the gauge potentials $B_A J$ so that one would have infinite number of classical gauge fields having also interpretation as gerbe gauge potentials.

3. The objection is that symplectic transformations are not symmetries of Kähler action. Therefore the action of symplectic transformation induced on the space-time surface reduces to a

symplectic transformation only at the partonic 2-surfaces. This spoils the covariant transformation law for the 2-fluxes over stringy world sheets unless there exist preferred stringy world sheets for which the action is covariant. The proposed duality between the descriptions based on partonic 2-surfaces and stringy world sheets realized in terms of slicings of space-time surface by string world sheets and partonic 2-surfaces suggests that this might be the case.

This would mean that one can attach to a given partonic 2-surface a unique collection string world sheets. The duality suggests even stronger condition stating that the total exponents $exp(iQ^A h_A)$ of fluxes are the same irrespective whether h_A evaluated for partonic 2-surfaces or for string world sheets defining the analog of 2-knot. This would mean an immense calculational simplification! This duality would correspond very closely to the weak form of electric magnetic duality whose various forms I have pondered as a must for the geometry of WCW . Partonic 2-surfaces indeed correspond to magnetic monopoles at least for elementary particles and stringy world sheets to surfaces carrying electric flux (note that in the exponent magnetic charges do not make themselves visible so that the identity can make sense also for $H_A = 1$).

4. Quantum expectation means in TGD framework a functional integral over the symplectic orbits of partonic 2-surfaces plus 4-D tangent space data assigned to the upper and lower boundaries of CD. Suppose that holography fixes the space-like 3-surfaces at the ends of CD and light-like orbits of partonic 2-surfaces. In completely general case the braids and the stringy space-time sheets could be fixed using a representation in terms of space-time coordinates so that the representation would be always the same but the embedding varies as also the values of the exponent of Kähler function, of the Wilson loop, and of its 2-D generalization. The functional integral over symplectic transforms of 3-surfaces implies that Wilson loop and its 2-D generalization varies.

   The proposed duality however suggests that both Wilson loop and its 2-D generalization are actually fixed by the dynamics of quantum TGD. One can ask whether the presence of 2-D analog of Wilson loop has a direct physical meaning bringing into almost topological stringy dynamics associated with color quantum numbers and coding explicit information about space-time interior and topology of field lines so that color dynamics would also have interpretation as a theory of 2-knots. If the proposed duality suggested by holography holds true, only the data at partonic 2-surfaces would be needed to calculate the generalized Wilson loops.

   In TGD framework the localization of the modes of the induced spinor fields at 2-D surfaces carrying vanishing induced W boson fields guaranteeing that the em charge of spinor modes is well-defined for a generic preferred extremal is natural [K15]. Besides string world sheets partonic 2-surfaces are good candidates for this kind of surfaces. It is not clear whether one can have a continuous slicing of this kind by string world sheets and partonic 2-surfaces orthogonal to them or whether only discrete set of these surfaces is possible.

This picture is very speculative and sounds too good to be true but follows if one consistently applies holography.

5.4 Summing Sup The Basic Ideas

Let us summarize the ideas discussed above.

1. Instead of knots, links, and braids one could study knot and link cobordisms, that is their dynamical evolutions concretizable in terms of dance metaphor and in terms of interacting string world sheets. Each space-like braid strand can have purely internal knotting and braid strands can be linked. TGD could allow to identify uniquely both space-like and time-like braid strands and thus also the stringy world sheets more or less uniquely and it could be that the dynamics induces automatically the temporary cutting of braid strands when knot is opened violently so that a hole is generated. Gerbe gauge potentials defined by classical color gauge fields could make also possible to characterize 2-knottedness in symplectic invariant manner in terms of color gauge fluxes over 2-surfaces.

The weak form of electric-magnetic duality would reduce the situation to almost topological QFT in general case with topological invariance replaced with symplectic one which corresponds to the fixing of the values of non-quantum fluctuating zero modes in quantum TGD. In the vacuum sector it would be possible to have the counterparts of Wilson loops weighted by 3-D electroweak Chern-Simons action defined by the induced spinor connection.

2. One could also leave TGD framework and define invariants of braid cobordisms and 2-D analogs of braids as vacuum expectations of Wilson loops using Chern-Simons action assigned to 3-surfaces at which space-like and time-like braid strands end. The presence of light-like and space-like 3-surfaces assignable to causal diamonds could be assumed also now.

I checked whether the article of Gukov, Scwhartz, and Vafa entitled "Khovanov-Rozansky Homology and Topological Strings" [A17, A17] relies on the primitive topological observations made above. This does not seem to be the case. The topological strings in this case are strings in 6-D space rather than 4-D space-time.

There is also an article by Dror Bar-Natan with title "Khovanov's homology for tangles and cobordisms" [A13]. The article states that the Khovanov homology theory for knots and links generalizes to tangles, cobordisms and 2-knots. The article does not say anything explicit about Wilson loops but talks about topological QFTs.

An article of Witten about his physical approach to Khovanov homology has appeared in arXiv [A16]. The article is more or less abracadabra for anyone not working with M-theory but the basic idea is simple. Witten reformulates 3-D Chern-Simons theory as a path integral for $\mathcal{N}=4$ SYM in the 4-D half space $W\times;R$. This allows him to use dualities and bring in the machinery of M-theory and 6-branes. The basic structure of TGD forces a highly analogous approach: replace 3-surfaces with 4-surfaces, consider knot cobordisms and also 2-knots, introduce gerbes, and be happy with symplectic instead of topological QFT, which might more or less be synonymous with TGD as almost topological QFT. Symplectic QFT would obviously make possible much more refined description of knots.

6 Witten's Approach To Khovanov Homology From TGD Point Of View

Witten's approach to Khovanov comohology [A16] relies on fivebranes as is natural if one tries to define 2-knot invariants in terms of their cobordisms involving violent un-knottings. Despite the difference in approaches it is very useful to try to find the counterparts of this approach in quantum TGD since this would allow to gain new insights to quantum TGD itself as almost topological QFT identified as symplectic theory for 2-knots, braids and braid cobordisms.

An essentially unique identification of string world sheets and therefore also of the braids whose ends carry quantum numbers of many particle states at partonic 2-surfaces emerges if one identifies the string word sheets as singular surfaces in the same manner as is done in Witten's approach [A16].

Also a physical interpretation of the operators Q, F, and P of Khovanov homology emerges. P would correspond to instanton number and F to the fermion number assignable to right handed neutrinos. The breaking of M^4 chiral invariance makes possible to realize Q physically. The finding that the generalizations of Wilson loops can be identified in terms of the gerbe fluxes $\int H_A J$ supports the conjecture that TGD as almost topological QFT corresponds essentially to a symplectic theory for braids and 2-knots.

6.1 Intersection Form And Space-Time Topology

The violent unknotting corresponds to a sequence of steps in which braid or knot becomes trivial and this very process defines braid invariants in TGD approach in nice concordance with the basic recipe for the construction of Jones polynomial. The topological invariant characterizing this process as a dynamics of 2-D string like objects defined by braid strands becomes knot invariant or more generally, invariant depending on the initial and final braids.

The process is describable in terms of string interaction vertices and also involves crossings of braid strands identifiable as self-intersections of the string world sheet. Hence the intersection form for the 2-surfaces defining braid strand orbits becomes a braid invariant. This intersection form is also a central invariant of 4-D manifolds and Donaldson's theorem [A4] says that for this invariant characterizes simply connected smooth 4-manifold completely. Rank, signature, and parity of this form in the basis defined by the generators of 2-homology (excluding torsion elements) characterize smooth closed and orientable 4-manifold. It is possible to diagonalize this form for smoothable 4-surfaces. Although the situation in the recent case differs from that in Donaldson theory in that the 4-surfaces have boundary and even fail to be manifolds, there are reasons to believe that the theory of braid cobordisms and 2-knots becomes part of the theory of topological invariants of 4-surfaces just as knot theory becomes part of the theory of 3-manifolds. The representation of 4-manifolds as space-time surfaces might also bring in physical insights.

This picture leads to ideas about string theory in 4-D space-time as a topological QFT. The string world sheets define the generators of second relative homology group. "Relative" means that closed surfaces are replaced with surfaces with boundaries at wormhole throats and ends of CD. These string world sheets, if one can fix them uniquely, would define a natural basis for homology group defining the intersection form in terms of violent unbraiding operations (note that also reconnections are involved).

Quantum classical correspondence encourages to ask whether also physical states must be restricted in such a way that only this minimum number of strings carrying quantum numbers at their ends ending to wormhole throats should be allowed. One might hope that there exists a unique identification of the topological strings implying the same for braids and allowing to identify various symplectic invariants as Hamiltonian fluxes for the string world sheets.

6.2 Framing Anomaly

In 3-D approach to knot theory the framing of links and knots represents an unavoidable technical problem [A16]. Framing means a slight shift of the link so that one can define self-linking number as a linking number for the link and its shift. The problem is that this framing of the link - or trivialization of its normal bundle in more technical terms- is not topological invariant and one obtains a large number of framings. For links in S^3 the framing giving vanishing self-linking number is the unique option and Atyiah has shown that also in more general case it is possible to identify a unique framing.

For 2-D surfaces self-linking is replaced with self-intersection. This is well-defined notion even without framing and indeed a key invariant. One might hope that framing is not needed also for string world sheets. If needed, this framing would induce the framing at the space-like and light-like 3-surfaces. The restriction of the section of the normal bundle of string world sheet to the 3-surfaces must lie in the tangent space of 3-surfaces. It is not clear whether this is enough to resolve the non-uniqueness problem.

6.3 Khovanov Homology Briefly

Khovanov homology involves three charges Q, F, and P. Q is analogous to super charge and satisfies $Q^2 = 0$ for the elements of homology. The basic commutation relations between the charges are $[F, Q] = Q$ and $[P, Q] = 0$. One can show that the Khovanov homology $\kappa(L)$ for link can be expressed as a bi-graded direct sum of the eigen-spaces $V_{m,n}$ of F and P, which have integer valued spectra. Obviously Q increases the eigenvalue of F and maps $V_{m,n}$ to $V_{m+1,n}$ just as exterior derivative in de-Rham comology increases the degree of differential form. P acts as a symmetry allowing to label the elements of the homology by an integer valued charge n.

Jones polynomial can be expressed as an index assignable to Khovanov homology:

$$\mathcal{J}(q|L) = Tr((-1)^F q^P \quad . \tag{6.1}$$

Here q defining the argument of Jones polynomial is root of unity in Chern-Simons theory but can be extended to complex numbers by extending the positive integer valued Chern-Simons coupling k to a complex number. The coefficients of the resulting Laurent polynomial are integers: this result does not follow from Chern-Simons approach alone. Jones polynomial depends on the spectrum of F only modulo 2 so that a lot of information is lost as the homology is replaced with the polynomial.

Both the need to have a more detailed characterization of links and the need to understand why the Wilson loop expectation is Laurent polynomial with integer coefficients serve as motivations of Witten for searching a physical approach to Khovanov polynomial.

The replacement of $D = 2$ in braid group approach to Jones polynomial with $D = 3$ for Chern-Simons approach replaced by something new in $D = 4$ would naturally correspond to the dimensional hierarchy of TGD in which partonic 2-surfaces plus their 2-D tangent space data fix the physics. One cannot quite do with partonic 2-surfaces and the inclusion of 2-D tangent space-data leads to holography and unique space time surfaces and perhaps also unique string world sheets serving as duals for partonic 2-surfaces. This would realize the weak form of electric magnetic duality at the level of homology much like Poincare duality relates cohomology and homology.

6.4 Surface Operators And The Choice Of The Preferred 2-Surfaces

The choice of preferred 2-surfaces and the identification of surface operators in $\mathcal{N} = 4$ YM theory is discussed in [A14]. The intuitive picture is that preferred 2-surfaces- now string world sheets defining braid cobordisms and 2-knots- correspond to singularities of classical gauge fields. Surface operator can be said to create this singularity. In functional integral this means the presence of the exponent defining the analog of Wilson loop.

1. In [A14] the 2-D singular surfaces are identified as poles for the magnitude r of the Higgs field. One can assign to the 2-surface fractional magnetic charges defined for the Cartan algebra part A_C of the gauge connection as circulations $\oint A_C$ around a small circle around the axis of singularity at $r = \infty$. What happens that 3-D $r = constant$ surface reduces to a 2-D surface at $r = \infty$ whereas A_C and entire gauge potential behaves as $A = A_C = \alpha d\phi$ near singularity. Here ϕ is coordinate analogous to angle of cylindrical coordinates when t-z plane represents the singular 2-surface. α is a linear combination of Cartan algebra generators.

2. The phase factor assignable to the circulation is essentially $exp(i2\pi\alpha)$ and for non-fractional magnetic charges it differs from unity. One might perhaps say that string word sheets correspond to singularities for the slicing of space-time surface with 3-surfaces at which 3-surfaces reduce to 2-surfaces.

Consider now the situation in TGD framwork.

1. The gauge group is color gauge group and gauge color gauge potentials correspond to the quantities H_AJ. One can also consider a generalization by allowing all Hamiltonians generating symplectic transformations of CP_2. Kähler gauge potential is in essential role since color gauge field is proportional to Kähler form.

2. The singularities of color gauge fields can be identified by studing the theory locally as a field theory from CP_2 to M^4. It is quite possible to have space-time surfaces for which M^4 coordinates are many-valued functions of CP_2 coordinates so that one has a covering of CP_2 locally. For singular 2-surfaces this covering becomes singular in the sense that separate sheets coincide. These coverings do not seem to correspond to those assignable to the hierarchy of Planck constants implied by the many-valuedness of the time derivatives of the embedding space coordinates as functions of canonical momentum densities but one must be very cautious in making too strong conclusions here.

3. To proceed introduce the Eguchi-Hanson coordinates

$$(\xi^1, \xi^2) = [rcos(\theta/2)exp(i(\Psi + \Phi)/2), rsin(\theta/2)exp(i(-\Psi + \Phi)/2]$$

for CP_2 with the defining property that the coordinates transform linearly under $U(2) \subset SU(3)$. In QFT context these coordinates would be identified as Higgs fields. The choice of these coordinates is unique apart from the choice of the $U(2)$ subgroup and rotation by element of $U(2)$ once this choice has been made. In TGD framework the definition of CD involves the fixing of these coordinates and the interpretation is in terms of quantum classical correspondence realizing the choice of quantization axes of color at the level of the WCW geometry.

r has a natural identification as the magnitude of Higgs field invariant under $U(2) \subset SU(3)$. The $SU(2) \times U(1)$ invariant 3-sphere reduces to a homologically non-trivial geodesic 2-sphere at $r = \infty$ so that for this choice of coordinates this surface defines in very natural manner the counterpart of singular 2-surface in CP_2 geometry. At this sphere the second phase associated with CP_2 coordinates- Φ - becomes a redundant coordinate just like the angle Φ at the poles of sphere. There are two other similar spheres and these three spheres are completely analogous to North and South poles of 2-sphere.

4. One possibility is that the singular surfaces correspond to the inverse images for the projection of the embedding map to $r = \infty$ geodesic sphere of CP_2 for a CD corresponding to a given choice of quantization axes. Also the inverse images of all homological non-trivial geodesic spheres defining the three poles of CP_2 can be considered. The inverse images of this geodesic 2-sphere under the embedding-projection map would naturally correspond to 2-D string world sheets for the preferred extremals for a generic space-time surface. For cosmic strings and massless extremals the inverse image would be 4-dimensional but this problem can be circumvented easily. The identification turned out to be somewhat ad hoc and later a much more convincing unique identification of string world sheets emerged and will be discussed in the sequel. Despite this the general aspects of the proposal deserves a discussion.

5. The existence of dual slicings of space-time surface by 3-surfaces and lines on one hand and by string world sheets Y^2 and 2-surfaces X^2 with Euclidian signature of metric on one hand, is one of the basic conjectures about the properties of preferred extremals of Kähler action. A stronger conjecture is that partonic 2-surfaces represent particular instances of X^2. The proposed picture suggests an amazingly simple and physically attractive identification of these slicings.

   (a) The slicing induced by the slicing of CP_2 by $r = constant$ surfaces defines an excellent candidate for the slicing by 3-surfaces. Physical the slices would correspond to equivalence classes of choices of the quantization axes for color group related by $U(2)$. In gauge theory context they would correspond to different breakings of $SU(3)$ symmetry labelled by the vacuum expectation of the Higgs field r which would be dynamical for CP_2 projections and play the role of time coordinate.

   (b) The slicing by string world sheets would naturally correspond to the slicing induced by the 2-D space of homologically non-trivial geodesic spheres (or triplets of them) and could be called "CP_2/S^2". One has clearly bundle structure with S^2 as base space and "CP_2/S^2" as fiber. Partonic 2-surfaces could be seen locally as sections of this bundle like structure assigning a point of "CP_2/S^2" to each point of S^2. Globally this does not make sense for partonic 2-surfaces with genus larger than $g = 0$.

6. In TGD framework the Cartan algebra of color gauge group is the natural identification for the Cartan algebra involved and the fluxes defining surface operators would be the classical fluxes $\int H_A J$ over the 2-surfaces in question restricted to Cartan algebra. What would be new is the interpretation as gerbe gauge potentials so that flux becomes completely analogous to Abelian circulation.

   If one accepts the extension of the gauge algebra to a symplectic algebra, one would have the Cartan algebra of the symplectic algebra. This algebra is defined by generators which depend on the second half P_i or Q_i of Darboux coordinates. If P_i are chosen to be functions of the coordinates (r, θ) of CP_2 coordinates whose Poisson brackets with color isospin and hyper charge generators inducing rotations of phases (Ψ, Φ) of CP_2 complex coordinates vanish, the symplectic Cartan algebra would correspond to color neutral Hamiltonians. The spherical harmonics with non-vanishing angular momentum vanish at poles and one expects that same happens for CP_2 spherical harmonics at the three poles of CP_2. Therefore Cartan algebra is selected automatically for gauge fluxes.

   This subgroup leaves the ends of the points of braids at partonic 2-surfaces invariant so that symplectic transformations do not induce braiding.

If this picture -resulting as a rather straightforward translation of the picture applied in QFT context- is correct, TGD would predict uniquely the preferred 2-surfaces and therefore also the braids as inverse images of CP_2 geodesic sphere for the embedding of space-time surface to $CD \times CP_2$. Also the conjecture slicings by 3-surfaces and string world sheets could be identified. The identification of braids and slicings has been indeed one of the basic challenges in quantum TGD since in quantum theory one does not have anymore the luxury of topological invariance and I have proposed several identifications. If one accepts only these space-time sheets then the stringy content for a given space-time surface would be uniquely fixed.

The assignment of singularities to the homologically non-trivial geodesic sphere suggests that the homologically non-trivial space-time sheets could be seen as 1-dimensional idealizations of magnetic flux tubes carrying Kähler magnetic flux playing key role also in applications of TGD, in particular biological applications such as DNA as topological quantum computer and bio-control and catalysis.

6.5 The Identification Of Charges Q, P And F Of Khovanov Homology

The challenge is to identify physically the three operators Q, F, and P appearing in Khovanov homology. Taking seriously the proposal of Witten [A16] and looking for its direct counterpart in TGD leads to the identification and physical interpretation of these charges in TGD framework.

1. In Witten's approach P corresponds to instanton number assignable to the classical gauge field configuration in space-time. In TGD framework the instanton number would naturally correspond to that assignable to CP_2 Kähler form. One could consider the possibility of assigning this charge to the deformed CP_2 type vacuum extremals assigned to the space-like regions of space-time representing the lines of generalized Feynman diagrams having elementary particle interpretation. P would be or at least contain the sum of unit instanton numbers assignable to the lines of generalized Feynman diagrams with sign of the instanton number depending on the orientation of CP_2 type vacuum extremal and perhaps telling whether the line corresponds to positive or negative energy state. Note that only pieces of vacuum extremals defined by the wormhole contacts are in question and it is somewhat questionable whether the rest of them in Minkowskian regions is included.

2. F corresponds to $U(1)$ charge assignable to R-symmetry of $N = 4$ SUSY in Witten's theory. The proposed generalization of twistorial approach in TGD framework suggests strongly that this identification generalizes to TGD. In TGD framework all solutions of Kähler-Dirac equation at wormhole throats define super-symmetry generators but the supersymmetry is badly broken. The covariantly constant right handed neutrino defines the minimally broken supersymmetry since there are no direct couplings to gauge fields. This symmetry is however broken by the mixing of right and left handed M^4 chiralities present for both Dirac actions for induced gamma matrices and for Kähler-Dirac equations defined by Kähler action and Chern-Simons action at parton orbits. It is caused by the fact that both the induced and Kähler-Dirac gamma matrices are combinations of M^4 and CP_2 gamma matrices. F would therefore correspond to the net fermion number assignable to right handed neutrinos and antineutrinos. F is not conserved because of the chirality mixing and electroweak interactions respecting only the conservation of lepton number.

   Note that the mixing of M^4 chiralities in sub-manifold geometry is a phenomenon characteristic for TGD and also a direct signature of particle massivation and SUSY breaking. It would be nice if it would allow the physical realization of Q operator of Khovanov homology.

3. Witten proposes an explicit formula for Q in terms of 5-dimensional time evolutions interpolating between two 4-D instantons and involving sum of sign factors assignable to Dirac determinants. In TGD framework the operator Q should increase the right handed neutrino number by one unit and therefore transform one right-handed neutrino to a left handed one in the minimal situation. In zero energy ontology Q should relate to a time evolution either between ends of CD or between the ends of single line of generalized Feynman diagram. If instanton number can be assigned solely to the wormhole contacts, this evolution should increase the number of CP_2 type extremals by one unit. 3-particle vertex in which right handed neutrino assignable to a partonic 2-surface transforms to a left handed one is thus a natural candidate for defining the action of Q.

4. Note that the almost topological QFT property of TGD together with the weak form of electric-magnetic duality implies that Kähler action reduces to Abelian Chern-Simons term. Ordinary Chern-Simons theory involves imaginary exponent of this term but in TGD the exponent would be real. Should one replace the real exponent of Kähler function with imaginary exponent? If so, TGD would be very near to topological QFT also in this respect. This would also force the quantization of the coupling parameter k in Chern-Simons action. On the other hand, the Chern-Simons theory makes sense also for purely imaginary k [A16].

6.6 What Does The Replacement Of Topological Invariance With Symplectic Invariance Mean?

One interpretation for the symplectic invariance is as an analog of diffeo-invariance. This would imply color confinement. Another interpretation would be based on the identification of symplectic group as a color group. Maybe the first interpretation is the proper restriction when one calculates invariants of braids and 2-knots.

The replacement of topological symmetry with symplectic invariance means that TGD based invariants for braids carry much more refined information than topological invariants. In TGD approach M^4 diffeomorphisms act freely on partonic 2-surfaces and 4-D tangent space data but the action in CP_2 degrees of freedom reduces to symplectic transformations. One could of course consider also the restriction to symplectic transformations of the light-cone boundary and this would give additional refinements.

It is is easy to see what symplectic invariance means by looking what it means for the ends of braids at a given partonic 2-surface.

1. Symplectic transformations respect the Kähler magnetic fluxes assignable to the triangles defined by the finite number of braid points so that these fluxes defining symplectic areas define some minimum number of coordinates parametrizing the moduli space in question. For topological invariance all n-point configurations obtained by continuous or smooth transformations are equivalent braid end configurations. These finite-dimensional moduli spaces would be contracted with point in topological QFT.

2. This picture led to a proposal of what I call symplectic QFT [K4] in which the associativity condition for symplectic fusion rules leads the hierarchy of algebras assigned with symplectic triangulations and forming a structures known as operad in category theory. The ends of braids at partonic 2-surfaces would would define unique triangulation of this kind if one accepts the identification of string like 2-surfaces as inverse images of homologically non-trivial geodesic sphere.

Note that both diffeomorphisms and symplectic transformations can in principle induce braiding of the braid strands connecting two partonic 2-surfaces. Should one consider the possibility that the allow transformations are restricted so that they do not induce braiding?

1. These transformations induce a transformation of the space-time surface which however is not a symplectic transformation in the interior in general. An attractive conjecture is that for the preferred extremals this is the case at the inverse images of the homologically non-trivial geodesic sphere. This would conform with the proposed duality between partonic 2-surfaces and string world sheets inspired by holography and also with quantum classical correspondence suggesting that at string world sheets the transformations induced by symplectic transformations at partonic 2-surfaces act like symplectic transformations.

2. If one allows only the symplectic transformations in Cartan algebra leaving the homologically non-trivial geodesic sphere invariant, the infinitesimal symplectic transformations would affect neither the string word sheets nor braidings but would modify the partonic 2-surfaces at all points except at the intersections with string world sheets.

7 Algebraic Braids, Sub-Manifold Braid Theory, And Generalized Feynman Diagrams

Ulla send me a link to an article by Sam Nelson about very interesting new-to-me notion known as algebraic knots (see http://tinyurl.com/yauy7asy) [A22, A19], which has initiated a revolution in knot theory. This notion was introduced 1996 by Louis Kauffmann [A20] so that it is already 15 year old concept. While reading the article I realized that this notion fits perfectly the needs of TGD and leads to a progress in attempts to articulate more precisely what generalized Feynman diagrams are.

In the following I will summarize briefly the vision about generalized Feynman diagrams, introduce the notion of algebraic knot, and after than discuss in more detail how the notion of algebraic knot could be applied to generalized Feynman diagrams. The algebraic structrures kei, quandle, rack, and biquandle and their algebraic modifications as such are not enough. The lines of Feynman graphs are replaced by braids and in vertices braid strands redistribute. This poses several challenges: the crossing associated with braiding and crossing occurring in non-planar Feynman diagrams should be integrated to a more general notion; braids are replaced with sub-manifold braids; braids of braids....of braids are possible; the redistribution of braid strands in vertices should be algebraized. In the following I try to abstract the basic operations which should be algebraized in the case of generalized Feynman diagrams.

One should be also able to concretely identify braids and 2-braids (string world sheets) as well as partonic 2-surfaces and I have discussed several identifications during last years. Legendrian braids turn out to be very natural candidates for braids and their duals for the partonic 2-surfaces. String world sheets in turn could correspond to the analogs of Lagrangian sub-manifolds or to minimal surfaces of space-time surface satisfying the weak form of electric-magnetic duality. The latter option turns out to be more plausible. Finite measurement resolution would be realized as symplectic invariance with respect to the subgroup of the symplectic group leaving the end points of braid strands invariant. In accordance with the general vision TGD as almost topological QFT would mean symplectic QFT. The identification of braids, partonic 2-surfaces and string world sheets - if correct - would solve quantum TGD explicitly at string world sheet level in other words in finite measurement resolution.

Irrespective of whether the algebraic knots are needed, the natural question is what generalized Feynman diagrams are. It seems that the basic building bricks can be identified so that one can write rather explicit Feynman rules already now. Of course, the rules are still far from something to be burned into the spine of the first year graduate student.

7.1 Generalized Feynman Diagrams, Feynman Diagrams, And Braid Diagrams

7.1.1 How knots and braids a la TGD differ from standard knots and braids?

TGD approach to knots and braids differs from the knot and braid theories in given abstract 3-manifold (4-manifold in case of 2-knots and 2-braids) is that space-time is in TGD framework identified as 4-D surface in $M^4 \times CP_2$ and preferred 3-surfaces correspond to light-like 3-surfaces defined by wormhole throats and space-like 3-surfaces defined by the ends of space-time sheets at the two light-like boundaries of causal diamond CD.

The notion of finite measurement resolution effectively replaces 3-surfaces of both kinds with braids and space-time surface with string world sheets having braids strands as their ends. The 4-dimensionality of space-time implies that string world sheets can be knotted and intersect at discrete points (counterpart of linking for ordinary knots). Also space-time surface can have self-intersections consisting of discrete points.

The ordinary knot theory in E^3 involves projection to a preferred 2-plane E^2 and one assigns to the crossing points of the projection an index distinguishing between two cases which are transformed to each other by violently taking the first piece of strand through another piece of strand. In TGD one must identify some physically preferred 2-dimensional manifold in embedding space to which the braid strands are projected. There are many possibilities even when one requires maximal symmetries. An obvious requirement is however that this 2-manifold is large enough.

1. For the braids at the ends of space-time surface the 2-manifold could be large enough sphere S^2 of light-cone boundary in coordinates in which the line connecting the tips of CD defines a preferred time direction and therefore unique light-like radial coordinate. In very small knots it could be also the geodesic sphere of CP_2 (apart from the action of isometries there are two geodesic spheres in CP_2).

2. For light-like braids the preferred plane would be naturally M^2 for which time direction corresponds to the line connecting the tips of CD and spatial direction to the quantization axis of spin. Note that these axes are fixed uniquely and the choices of M^2 are labelled by the points of projective sphere P^2 telling the direction of space-like axis. Preferred plane M^2 emerges naturally also from number theoretic vision and corresponds in octonionic pictures to hyper-complex plane of hyper-octonions. It is also forced by the condition that the choice of quantization axes has a geometric correlate both at the level of embedding space geometry and the geometry of the "world of classical worlds".

The braid theory in TGD framework could be called sub-manifold braid theory and certainly differs from the standard one.

1. If the first homology group of the 3-surface is non-trivial as it when the light-like 3-surfaces represents an orbit of partonic 2-surface with genus larger than zero, the winding of the braid strand (wrapping of branes in M-theory) meaning that it represents a homologically non-trivial curve brings in new effects not described by the ordinary knot theory. A typical new situation is the one in which 3-surface is locally a product of higher genus 2-surface and line segment so that knot strand can wind around the 2-surface. This gives rise to what are called non-planar braid diagrams for which the projection to plane produces non-standard crossings.

2. In the case of 2-knots similar exotic effects could be due to the non-trivial 2-homology of space-time surface. Wormhole throats assigned with elementary particle wormhole throats are homologically non-trivial 2-surfaces and might make this kind of effects possible for 2-knots if they are possible.

The challenge is to find a generalization of the usual knot and braid theories so that they apply in the case of braids (2-braids) imbedded in 3-D (4-D) surfaces with preferred highly symmetry sub-manifold of $M^4 \times CP_2$ defining the analog of plane to which the knots are projected. A proper description of exotic crossings due to non-trivial homology of 3-surface (4-surface) is needed.

7.1.2 Basic questions

The questions are following.

1. How the mathematical framework of standard knot theory should be modified in order to cope with the situation encountered in TGD? To my surprise I found that this kind of mathematical framework exists: so called algebraic knots [A22, A19] define a generalization of knot theory very probably able to cope with this kind of situation.

2. Second question is whether the generalized Feynman diagrams could be regarded as braid diagrams in generalized sense. Generalized Feynman diagrams are generalizations of ordinary Feynman diagrams. The lines of generalized Feynman diagrams correspond to the orbits of wormhole throats and of wormhole contacts with throats carrying elementary particle quantum numbers.

   The lines meet at vertices which are partonic 2-surfaces. Single wormhole throat can describe fermion whereas bosons have wormhole contacts with fermion and anti-fermion at the opposite throats as building bricks. It seems however that all fermions carry Kähler magnetic charge so that physical particles are string like objects with magnetic charges at their ends.

   The short range of weak interactions results from the screening of the axial isospin by neutrinos at the other end of string like object and also color confinement could be understood in this manner. One cannot exclude the possibility that the length of magnetic flux tube is of order Compton length.

3. Vertices of the generalized Feynman diagrams correspond to the partonic 2-surfaces along which light-like 3-surfaces meet and this is certainly a challenge for the required generalization of braid theory. The basic objection against the reduction to algebraic braid diagrams is that reaction vertices for particles cannot be described by ordinary braid theory: the splitting of braid strands is needed.

   The notion of bosonic emergence however suggests that 3-vertex and possible higher vertices correspond to the splitting of braids rather than braid strands. By allowing braids which come from both past and future and identifying free fermions as wormhole throats and bosons as wormhole contacts consisting of a pair of wormhole throats carrying fermion and anti-fermion number, one can understand boson excanges as recombinations without anyneed to have splitting of braid strands. Strictly and technically speaking, one would have tangles like objects instead of braids. This would be an enormous simplification since $n > 2$-vertices which are the source of divergences in QFT: s would be absent.

4. Non-planar Feynman diagrams are the curse of the twistor approach and I have already earlier proposed that the generalized Feynman amplitudes and perhaps even twistorial amplitudes could be constructed as analogs of knot invariants by recursively transforming non-planar Feynman diagrams to planar ones for which one can write twistor amplitudes. This forces to answer two questions.

   (a) Does the non-nonplanarity of Feynman diagrams - completely combinatorial objects identified as diagrams in plane - have anything to do with the non-planarity of algebraic knot diagrams and with the non-planarity of generalized Feynman diagrams which are purely geometric objects?

   (b) Could these two kind of non-planarities be fused to together by identifying the projection 2-plane as preferred $M^2 \subset M^4$. This would mean that non-planarity in QFT sense is defined for entire braids: braid A can have virtual crossing with B. Non-planarity in the sense of knot theory would be defined for braid strands inside the braids. At vertices braid strands are redistributed between incoming lines and the analog of virtual crossing be identifiable as an exchange of braid strand between braids. Several kinds of non-planarities would be present and the idea about gradual unknotting of a non-planar diagram so that a planar diagram results as the final outcome might make sense and allow to generalize the recursion recipe for the twistorial amplitudes.

   (c) This approach could be combined with the number theoretic vision that amplitudes correspond to sequences of computations with vertices identified as product and co-product for a Yangian variant of super-symplectic algebra [A12] [B4, B2, B3]. When incoming and outgoing algebraic objects are specified there would be unique smallest diagram leading from input to output. This diagram would be tree diagram in ordinary Feynman diagrammatics. This would mean huge generalization of the duality symmetry of string models if all diagrams connecting initial and final collections of algebraic objects correspond to the same amplitude.

       Non-planar diagrams of quantum field theories should have natural counterpart and linking and knotting for braids defines it naturally. This suggests that the amplitudes can be interpreted as generalizations of braid diagrams defining braid invariants: braid strands would appear as legs of 3-vertices representing product and co-product. Amplitudes could be constructed as generalized braid invariants transforming recursively braided tree diagram to an un-braided diagram using same operations as for braids. In [L11] I considered a possible breaking of associativity occurring in weak sense for conformal field theories and was led to the vision that there is a fractal hierarchy of braids such that braid strands themselves correspond to braids. This hierarchy would define an operad with subgroups of permutation group in key role. Hence it seems that various approaches to the construction of amplitudes converge.

   (d) One might consider the possibility that inside orbits of wormhole throats defining the lines of Feynman diagrams the R-matrix for integrable QFT in M^2 (only permutations of momenta are allowed) describes the dynamics so that one obtains just a permutation of momenta assigned to the braid strands. Ordinary braiding would be described by existing braid theories. The core problem would be the representation of the exchange of a strand between braids algebraically.

One can consider different and much simpler general approach to the non-planarity problem. In twistor Grassmannian approach [K14] generalized Feynman diagrams correspond to TGD variants of stringy diagrams. In stringy approach one gets rid of non-planarity problem altogether.

7.2 Brief Summary Of Algebraic Knot Theory

7.2.1 Basic ideas of algebraic knot theory

In ordinary knot theory one takes as a starting point the representation of knots of E^3 by their plane plane projections to which one attach a "color" to each crossing telling whether the strand goes over or under the strand it crosses in planar projection. These numbers are fixed uniquely as one traverses through the entire knot in given direction.

The so called Reidermeister moves are the fundamental modifications of knot leaving its isotopy equivalence class unchanged and correspond to continuous deformations of the knot. Any algebraic invariant assignable to the knot must remain unaffected under these moves. Reidermeister moves as such look completely trivial and the non-trivial point is that they represent the minimum number of independent moves which are represented algebraically.

In algebraic knot theory topological knots are replaced by typographical knots resulting as planar projections. This is a mapping of topology to algebra. It turns out that the existing knot invariants generalize and ordinary knot theory can be seen as a special case of the algebraic knot theory. In a loose sense one can say that the algebraic knots are to the classical knot theory what algebraic numbers are to rational numbers.

Virtual crossing is the key notion of the algebraic knot theory. Virtual crossing and their rules of interaction were introduced 1996 by Louis Kauffman as basic notions [A1]. For instance, a strand with only virtual crossings should be replaceable by any strand with the same number of virtual crossings and same end points. Reidermeister moves generalize to virtual moves. One can say that in this case crossing is self-intersection rather than going under or above. I cannot be eliminated by a small deformation of the knot. There are actually several kinds of non-standard crossings: examples listed in figure 7 of [A22]) are virtual, flat, singular, and twist bar crossings.

Algebraic knots have a concrete geometric interpretation.

(a) Virtual knots are obtained if one replaces E^3 as embedding space with a space which has non-trivial first homology group. This implies that knot can represent a homologically non-trivial curve giving an additional flavor to the unknottedness since homologically non-trivial curve cannot be transformed to a curve which is homologically non-trivial by any continuous deformation.

(b) The violent projection to plane leads to the emergence of virtual crossings. The product $(S^1 \times S^1) \times D$, where $(S^1 \times S^1)$ is torus D is finite line segment, provides the simplest example. Torus can be identified as a rectangle with opposite sides identified and homologically non-trivial knots correspond to curves winding n_1 times around the first S^1 and n_2 times around the second S^1. These curves are not continuous in the representation where $S^1 \times S^1$ is rectangle in plane.

(c) A simple geometric visualization of virtual crossing is obtained by adding to the plane a handle along which the second strand traverses and in this manner avoids intersection. This visualization allows to understand the geometric motivation for the virtual moves.

This geometric interpretation is natural in TGD framework where the plane to which the projection occurs corresponds to $M^2 \subset M^4$ or is replaced with the sphere at the boundary of S^2 and 3-surfaces can have arbitrary topology and partonic 2-surfaces defining as their orbits light-like 3-surfaces can have arbitrary genus.

In TGD framework the situation is however more general than represented by sub-manifold braid theory. Single braid represents the line of generalized Feynman diagram. Vertices represent something new: in the vertex the lines meet and the braid strands are redistributed but do not disappear or pop up from anywhere. That the braid strands can come both from the future and past is also an important generalization. There are physical argments suggesting that there are only 3-vertices for braids but not higher ones [K5]. The challenge is to represent algebraically the vertices of generalized Feynman diagrams.

7.2.2 Algebraic knots

The basic idea in the algebraization of knots is rather simple. If x and y are the crossing portions of knot, the basic algebraic operation is binary operation giving "the result of x going under y", call it $x \triangleright y$ telling what happens to x. "Portion of knot" means the piece of knot between two crossings and $x \triangleright y$ denotes the portion of knot next to x. The definition is asymmetrical in x and y and the dual of the operation would be $y \triangleleft x$ would be "the result of y going above x". One can of course ask, why not to define the outcome of the operation as a pair $(x \triangleleft y, y \triangleright x)$. This operation would be bi-local in a well-defined sense. One can of course do this: in this case one has binary operation from $X \times X \rightarrow X \times X$ mapping pairs of portions to pairs of portions. In the first case one has binary operation $X \times X \rightarrow X$.

The idea is to abstract this basic idea and replace X with a set endowed with operation $\triangleright$ or $\triangleleft$ or both and formukate the Reidermeister conditions given as conditions satisfied by the algebra. One ends up to four basic algebraic structures kei, quandle, rack, and biquandle.

(a) In the case of non-oriented knots the kei is the algebraic structure. Kei - or invontary quandle-is a set X with a map $X \times X \rightarrow X$ satisfying the conditions

   i. $x \triangleright x = x$ (idenpotency, one of the Reidemeister moves)
   ii. $(x \triangleright y) \triangleright y$ =x (operation is its own right inverse having also interpretation as Reidemeister move)
   iii. $(x \triangleright y) \triangleright z = (x \triangleright z) \triangleright (y \triangleright z)$ (self-distributivity)

   $Z([t])/(t^2)$ module with $x \triangleright y = tx + (1-t)y$ is a kei.

(b) For orientable knot diagram there is preferred direction of travel along knot and one can distinguish between $\triangleright$ and its right inverse $\triangleright^{-1}$. This gives quandle satisfying the axios

   i. $x \triangleright x = x$
   ii. $(x \triangleright y) \triangleright^{-1} y = (x \triangleright^{-1} y) \triangleright y = x$
   iii. $(x \triangleright y) \triangleright z = (x \triangleright z) \triangleright (y \triangleright z)$

   $Z[t^{\pm 1}]$ nodule with $x \triangleright y = tx + (1-t)y$ is a quandle.

(c) One can also introduce framed knots: intuitively one attaches to a knot very near to it. More precise formulation in terms of a section of normal bundle of the knot. This makes possible to speak about self-linking. Reidermeister moves must be modified appropriately. In this case rack is the appropriate structure. It satisfied the axioms of quandle except the first axiom since corresponding operation is not a move anymore. Rack axioms are eqivalent with the requirement that functions $f_y : X \rightarrow X$ defined by $f_y(x) x \triangleright y)$ are automorphisms of the structure. Therefore the elements of rack represent its morphisms. The modules over $Z[t^{\pm 1}, s]/s(t+s-1)$ are racks. Coxeter racks are inner product spaces with $x \triangleright y$ obtained by reflecting x across y.

(d) Biquandle consists of arcs connecting the subsequent crossings (both under- and over-) of oriented knot diagram. Biquandle operation is a map $B : X \times X \rightarrow X \times X$ of order pairs satisfying certain invertibility conditions together with set theoretic Yang-Baxter equation:

$$(B \times I)(I \times B)(B \times I) = (I \times B)(B \times I)(I \times B) \ .$$

Here $I : X \to X$ is the identity map. The three conditions to which Yang-Baxter equation decomposes gives the counterparts of the above discussed axioms. Alexander biquandle is the module $Z(t^{\pm 1}, s^{\pm}1$ with $B(x,y) = (ty + (1-ts)x, sx)$ where one has $s \neq 1$. If one includes virtual, flat and singular crossings one obtains virtual/singular aundles and semiquandles.

7.3 Generalized Feynman Diagrams As Generalized Braid Diagrams?

Zero energy ontology suggests the interpretation of the generalized Feynman diagrams as generalized braid diagrams so that there would be no need for vertices at the fundamental braid strand level. The notion of algebraic braid (or tangle) might allow to formulate this idea more precisely.

7.3.1 Could one fuse the notions of braid diagram and Feynman diagram?

The challenge is to fuse the notions of braid diagram and Feynman diagram having quite different origin.

(a) All generalized Feynman diagrams are reduced to sub-manifold braid diagrams at microscopic level by bosonic emergence (bosons as pairs of fermionic wormhole throats). Three-vertices appear only for entire braids and are purely topological whereas braid strands carrying quantum numbers are just re-distributed in vertices. No 3-vertices at the really microscopic level! This is an additional nail to the coffin of divergences in TGD Universe.

(b) By projecting the braid strands of generalized Feynman diagrams to preferred plane $M^2 \subset M^4$ (or rather 2-D causal diamond), one could achieve a unified description of non-planar Feynman diagrams and braid diagrams. For Feynman diagrams the intersections have a purely combinatorial origin coming from representations as 2-D diagrams.

For braid diagrams the intersections have different origin and non-planarity has different meaning. The crossings of entire braids analogous to those appearing in non-planar Feynman diagrams should define one particular exotic crossing besides virtual crossings of braid strands due to non-trivial first homology of 3-surfaces.

(c) The necessity to choose preferred plane M^2 looks strange from QFT point of view. In TGD framework it is forced by the number theoretic vision in which M^2 represents hyper-complex plane of sub-space of hyper-octonions which is subspace of complexified octonions. The choice of M^2 is also forced by the condition that the choice of quantization axes has a geometric correlate both at the level of embedding space geometry and the geometry of the "world of classical worlds".

(d) Also 2-braid diagrams defined as projections of string world sheets are suggestive and would be defined by a projections to the 3-D boundary of CD or to $M^3 \subset M^4$. They would provide a more concrete stringy illustration about generalized Feynman diagram as analog of string diagram. Another attractive illustration is in terms of dance metaphor with the boundary of CD defining the 3-D space-like parquette. The duality between space-like and light-like braids is expected to be of importance.

The obvious conjecture is that Feynman amplitudes are a analogous to knot invariants constructible by gradually reducing non-planar Feynman diagrams to planar ones after which the already existing twistor theoretical machinery of $\mathcal{N} = 4$ SYMs would apply [K7, K14, K2].

7.3.2 Does 2-D integrable QFT dictate the scattering inside the lines of generalized Feynman diagrams

The preferred plane M^2 (more precisely, 2-D causal diamond having also interpretation as Penrose diagram) plays a key role as also the preferred sphere S^2 at the boundary of CD. It is perhaps not accident that a generalization of braiding was discovered in integrable quantum field theories in M^2. The S-matrix of this theory is rather trivial looking: particle moving with different velocities cross each other and suffer a phase lag and permutation of 2-momenta which has physical effects only in the case of non-identical particles. The R-matrix describing this process reduces to the R-matrix describing the basic braiding operation in braid theories at the static limit.

I have already earler conjectured that this kind of integrable QFT is part of quantum TGD [K6]. The natural guess is that it describes what happens for the projections of 4-momenta in M^2 in scattering process inside lines of generalized Feynman diagrams. If integrable theories in M^2 control this scattering, it would cause only phase changes and permutation of the M^2 projections of the 4-momenta. The most plausible guess is that M^2 QFT characterized by R-matrix describes what happens to the braid momenta during the free propagation and the remaining challenge would be to understand what happens in the vertices defined by 2-D partonic surfaces at which re-distribution of braid strands takes place.

7.3.3 How quantum TGD as almost topological QFT differs from topological QFT for braids and 3-manifolds

One must distinguish between two topological QFTs. These correspond to topological QFT defining braid invariants and invariants of 3-manifolds respectively. The reason is that knots are an essential element in the procedure yielding 3-manifolds. Both 3-manifold invariants and knot invariants would be defined as Wilson loops involving path integral over gauge connections for a given 3-manifold with exponent o non-Abelkian f Chern-Simons action defining the weight.

(a) In TGD framework the topological QFT producing braid invariants for a given 3-manifold is replaced with sub-manifold braid theory. Kähler action reduces Chern-Simons terms for preferred extremals and only these contribute to the functional integral. What is the counterpart of topological invariance in this framework? Are general isotopies allowed or should one allow only sub-group of symplectic group of CD boundary leaving the end points of braids invariant? For this option Reidermeister moves are undetectable in the finite measurement resolution defined by the subgroup of the symplectic group. Symplectic transformations would not affect 3-surfaces as the analogs of abstract contact manifold since induced Kähler form would not be affected and only the embedding would be changed.

    In the approach based on inclusions of HFFs gauge invariance or its generalizations would represent finite measurement resolution (the action of included algebra would generate states not distiguishable from the original one).

(b) There is also ordinary topological QFT allowing to construct topological invariants for 3-manifold. In TGD framework the analog of topological QFT is defined by Chern-Simons-Kähler action in the space of preferred 3-surfaces. Now one sums over small deformations of 3-surface instead of gauge potentials. If extremals of Chern-Simons-Kähler action are in question, symplectic invariance is the most that one can hope for and this might be the situation quite generally. If all light-like 3-surfaces are allowed so that only weak form of electric-magnetic duality at them would bring metric into the theory, it might be possible to have topological invariance at 3-D level but not at 4-D level. It however seems that symplectic invariance with respect to subgroup leaving end points of braids invariant is the realistic expectation.

7.3.4 Could the allowed braids define Legendrian sub-manifolds of contact manifolds?

The basic questions concern the identification of braids and 2-braids. In quantum TGD they cannot be arbitrary but determined by dynamics providing space-time correlates for quantum dynamics. The deformations of braids should mean also deformations of 3-surfaces which as topological manifolds would however remain as such. Therefore topological QFT for given 3-manifold with path integral over gauge connections would in TGD correspond to functional integral of 3-surfaces corresponding to same topology even symplectic structure. The quantum fluctuating degrees of freedom indeed correspond to symplectic group divided by its subgroup defining measurement resolution.

What is the dynamics defining the braids strands? What selects them? I have considered this problem several times. Just two examples is enough here.

(a) Could they be some special light-like curves? Could the condition that the end points of the curves correspond to rational points in some preferred coordinates allow to select these light-like curves? But what about light-like curves associated with the ends of the space-time surface?

(b) The solutions of Kähler-Dirac equation [K15] are localized to curves by using the analog of periodic boundary conditions: the length of the curve is quantized in the effective metric defined by the Kähler-Dirac gamma matrices. Here one however introcuced a coordinate along light-like 3-surface and it is not clear how one should fix this preferred coordinate.

1. Legendrian and Lagrangian sub-manifolds

A hint about what is missing comes from the observation that a non-vanishing Chern-Simons-Kähler form A defines a contact structure (see `http://tinyurl.com/yblj4hlq`) [A3] at light-like 3-surfaces if one has $A \wedge dA \neq 0$. This condition states complete non-intebrability of the distribution of 2-planes defined by the condition $A_\mu t^\mu = 0$, where t is tangent vector in the tangent bundle of light-like 3-surface. It also states that the flow lines of A do *not* define global coordinate varying along them.

(a) It is however possible to have 1-dimensional curves for which $A_\mu t^\mu = 0$ holds true at each point. These curves are known as Legendrian sub-manifolds to be distinguished from Lagrangian manifolds for which the projection of symplectic form expressible locally as $J = dA$ vanishes. The set of this curves is discrete so that one obtains braids. Legendrian knots are the simplest example of Legendrian sub-manifolds and the question is whether braid strands could be identified as Legendrian knots. For Legendrian braids symplectic invariance replaces topological invariance and Legendrian knots and braids can be trivial in topological sense. In some situations the property of being Legendrian implies un-knottedness.

(b) For Legendrian braid strands the Kähler gauge potential vanishes. Since the solutions of the Kähler-Dirac equation are localized to braid strands, this means that the coupling to Kähler gauge potential vanishes. From physics point of view a generalization of Legendre braid strand by allowing gauge transformations $A \rightarrow A + d\Phi$ looks natural since it means that the coupling of induced spinors is pure gauge terms and can be eliminated by a gauge transformation.

2. 2-D duals of Legendrian sub-manifolds

One can consider also what might be called 2-dimensional duals of Legendrian sub-manifolds.

(a) Also the one-form obtained from the dual of Kähler magnetic field defined as $B^\mu = \epsilon^{\mu\nu\gamma} J_{\nu\nu}$ defines a distribution of 2-planes. This vector field is ill-defined for light-like surfaces since contravariant metric is ill-defined. One can however multiply B with the square root of metric determining formally so that metric would disappear completely just as it disappears from Chern-Simons action. This looks however somewhat tricky mathematically. At the 3-D space-like ends of space-time sheets at boundaries of CD B^μ is however well-defined as such.

(b) The distribution of 2-planes is integrable if one has $B \wedge dB = 0$ stating that one has Beltrami field: physically the conditions states that the current dB feels no Lorentz force. The geometric content is that B defines a global coordinate varying along its flow lines.

For the preferred extremals of Kähler action Beltrami condition is satisfied by isometry currents and Kähler current in the interior of space-time sheets. If this condition holds at 3-surfaces, one would have an global time coordinate and integrable distribution of 2-planes defining a slicing of the 2-surface. This would realize the conjecture that space-time surface has a slicing by partonic 2-surfaces. One could say that the 2-surfaces defined by the distribution are orthogonal to B. This need not however mean that the projection of J to these 2-surfaces vanishes. The condition $B \wedge dB = 0$ on the space-like 3-surfaces could be interpreted in terms of effective 2-dimensionality. The simplest option posing no additional conditions would allow two types of braids at space-like 3-surfaces and only Legendrian braids at light-like 3-surfaces.

These observations inspire a question. Could it be that the conjectured dual slicings of space-time sheets by space-like partonic 2-surfaces and by string world sheets are defined by A_μ and B^μ respectively associated with slicings by light-like 3-surfaces and space-like 3-surfaces? Could partonic 2-surfaces be identified as 2-D duals of 1-D Legendrian sub-manifolds?

The identification of braids as Legendrian braids for light-like 3-surfaces and with Legendrian braids or their duals for space-like 3-surfaces would in turn imply that topological braid theory is replaced with a symplectic braid theory in accordance with the view about TGD as almost topological QFT. If finite measurement resolution corresponds to the replacement of symplectic group with the coset space obtained by dividing by a subgroup, symplectic subgroup would take the role of isotopies in knot theory. This symplectic subgroup could be simply the symplectic group leaving the end points of braids invariant.

7.3.5 An attempt to identify the constraints on the braid algebra

The basic problems in understanding of quantum TGD are conceptual. One must proceed by trying to define various concepts precisely to remove the many possible sources of confusion. With this in mind I try collect essential points about generalized Feynman diagrams and their relation to braid diagrams and Feynman diagrams and discuss also the most obvious constraints on algebraization.

Let us first summarize what generalized Feynman diagrams are.

(a) Generalized Feynman diagrams are 3-D (or 4-D, depends on taste) objects inside $CD \times CP_2$. Ordinary Feynman diagrams are in plane. If finite measurement resolution has as a space-time correlate discretization at the level of partonic 2-surfaces, both space-like and light-like 3-surfaces reduce to braids and the lines of generalized Feynman diagrams correspond to braids. It is possible to obtain the analogs of ordinary Feynman diagrams by projection to $M^2 \subset M^4$ defined uniquely for given CD. The resulting apparent intersections would represent ne particular kind of exotic intersection.

(b) Light-like 3-surfaces define the lines of generalized Feynman diagrams and the braiding results naturally. Non-trivial first homology for the orbits of partonic 2-surfaces with genus $g > 0$ could be called homological virtual intersections.

(c) It zero energy ontology braids must be characterized by time orientation. Also it seems that one must distinguish in zero energy ontology between on mass shell braids and off mass shell braid pairs which decompose to pairs of braids with positive and negative energy massless on mass shell states. In order to avoid confusion one should perhaps speak about tangles insie CD rather than braids. The operations of the algebra are same except that the braids can end either to the upper or lower light-like boundary of CD. The projection to M^2 effectively reduces the CD to a 2-dimensional causal diamond.

(d) The vertices of generalized Feynman diagrams are partonic 2-surfaces at which the light-like 3-surfaces meet. This is a new element. If the notion of bosonic emergence is accepted no $n > 2$-vertices are needed so that braid strands are redistributed in the reaction vertices. The redistribution of braid strands in vertices must be introduced as an additional operation somewhat analogous to $\triangleright$ and the challenge is to reduce this operation to something simple. Perhaps the basic operation reduces to an exchange of braid strand between braids. The process can be seen as a decay of of braid with the conservation of braid strands with strands from future and past having opposite strand numbers. Also for this operation the analogs of Reidermeister moves should be identified. In dance metaphor this operation corresponds to a situation in which the dancer leaves the group to which it belongs and goes to a new one.

(e) A fusion of Feynman diagrammatic non-planarity and braid theoretic non-planarity is needed and the projection to M^2 could provide this fusion when at least two kinds of virtual crossings are allowed. The choice of M^2 could be global. An open question is whether the choice of M^2 could characterize separately each line of generalized Feynman diagram characterized by the four-momentum associated with it in the rest system defined by the tips of CD. Somehow the theory should be able to fuse the braiding matrix for integrable QFT in M^2 applying to entire braids with the braiding matrix for braid theory applying at the level of single braid.

Both integral QFTs in M^2 and braid theories suggest that biquandle structure is the structure that one should try to generalized.

(a) The representations of resulting bi-quandle like structure could allow abstract interesting information about generalized Feynman diagrams themselves but the dream is to construct generalized Feynman diagrams as analogs of knot invariants by a recursive procedure analogous to un-knotting of a knot.

(b) The analog of bi-quandle algebra should have a hierarchical structure containing braid strands at the lowest level, braids at next level, and braids of braids...of braids at higher levels. The notion of operad would be ideal for formulating this hierarchy and I have already proposed that this notion must be essential for the generalized Feynman diagrammatics. An essential element is the vanishing of total strand number in the vertex (completely analogous to conserved charged such as fermion number). Again a convenient visualization is in terms of dancers forming dynamical groups, forming groups of groups forming .....

I have already earlier suggested [K6] that the notion of operad [A10] relying on permutation group and its subgroups acting in tensor products of linear spaces is central for understanding generalized Feynman diagrams. $n \to n_1 + n_2$ decay vertex for n-braid would correspond to "symmetry breaking" $S_n \to S_{n_1} \times S_{n_2}$. Braid group represents the covering of permutation group so that braid group and its subgroups permuting braids would suggest itself as the basic group theoretical notion. One could assign to each strand of n-braid decaying to n_1 and n_2 braids a two-valued color telling whether it becomes a strand of n_1-braid or n_2-braid. Could also this "color" be interpreted as a particular kind of exotic crossing?

(c) What could be the analogs of Reidermaster moves for braid strands?

i. If the braid strands are dynamically determined, arbitrary deformations are not possible. If however all isotopy classes are allowed, the interpretation would be that a kind of gauge choice selecting one preferred representation of strand among all possible ones obtained by continuous deformations is in question.

ii. Second option is that braid strands are dynamically determined within finite measurement resolution so that one would have braid theory in given length scale resolution.

iii. Third option is that topological QFT is replaced with symplectic QFT: this option is suggested by the possibility to identify braid strands as Legendrian knots or their duals. Subgroup of the symplectic group leaving the end points of braids invariant would act as the analog of continous transformations and play also the role of gauge group. The new element is that symplectic transformations affect partonic 2-surfaces and space-time surfaces except at the end points of braid.

(d) Also 2-braids and perhaps also 2-knots could be useful and would provide string theory like approach to TGD. In this case the projections could be performed to the ends of CD or to M^3, which can be identified uniquely for a given CD.

(e) There are of course many additional subtleties involved. One should not forget loop corrections, which naturally correspond to sub-CDs. The hierarchy of Planck constants and number theoretical universality bring in additional complexities.

All this looks perhaps hopelessly complex but the Universe around is complex even if the basic principles could be very simple.

7.4 About String World Sheets, Partonic 2-Surfaces, And Two-Knots

String world sheets and partonic 2-surfaces provide a beatiful visualization of generalized Feynman diagrams as braids and also support for the duality of string world sheets and partonic 2-surfaces as duality of light-like and space-like braids. Dance metaphor is very helpful here.

(a) The projection of string world sheets and partonic 2-surfaces to 3-D space replaces knot projection. In TGD context this 3-D of space could correspond to the 3-D light-like boundary of CD and 2-knot projection would correspond to the projection of the braids associated with the lines of generalized Feynman diagram. Another identification would be as $M^1 \times E^2$, where M^1 is the line connecting the tips of CD and E^2 the orthogonal complement of M^2.

(b) Using dance metaphor for light-like braiding, braids assignable to the lines of generalized Feynman diagrams would correspond to groups of dancers. At vertices the dancing groups would exchange members and completely new groups would be formed by the dancers. The number of dancers (negative for those dancing in the reverse time direction) would be conserved. Dancers would be connected by threads representing strings having braid points at their ends. During the dance the light-like braiding would induce space-like braiding as the threads connecting the dancers would get entangled. This would suggest that the light-like braids and space-like braidings are equivalent in accordance with the conjectured duality between string-world sheets and partonic 2-surfaces. The presence of genuine 2-knottedness could spoil this equivalence unless it is completely local.

Can string world sheets and partonic 2-surfaces get knotted?

(a) Since partonic 2-surfaces (wormhole throats) are imbedded in light-cone boundary, the preferred 3-D manifolds to which one can project them is light-cone boundary (boundary of CD). Since the projection reduces to inclusion these surfaces cannot get knotted. Only if the partonic 2-surfaces contains in its interior the tip of the light-cone something non-trivial identifiable as virtual 2-knottedness is obtained.

(b) One might argue that the conjectured duality between the descriptions provided by partonic 2-surfaces and string world sheets requires that also string world sheets represent trivial 2-braids. I have shown earlier that nontrivial local knots glued to the string world sheet require that M^4 time coordinate has a local maximum. Does this mean that 2-knots are excluded? This is not obvious: TGD allows also regions of space-time surface with Euclidian signature and generalized Feynman graphs as 4-D space-time regions are indeed Euclidian. In these regions string world sheets could get knotted.

What happens for knot diagrams when the dimension of knot is increased to two? According to the articles of Nelson (see `http://tinyurl.com/yauy7asy`) [A22] and Carter (see `http://tinyurl.com/yclgj739`) [A19] the crossings for the projections of braid strands are replaced with more complex singularities for the projections of 2-knots. One can decompose the 2-knots to regions surrounded by boxes. Box can contain just single piece of 2-D surface; it can contain two intersection pieces of 2-surfaces as the counterpart of intersecting knot strands and one can tell which of them is above which; the box can contain also a discrete point in the intersection of projections of three disjoint regions of knot which consists of discrete points; and there is also a box containing so called cone point. Unfortunately, I failed to understand the meaning of the cone point.

For 2-knots Reidemeister moves are replaced with Roseman moves. The generalization would allow virtual self intersections for the projection and induced by the non-trivial second homology of 4-D embedding space. In TGD framework elementary particles have homologically non-trivial partonic 2-surfaces (magnetic monpoles) as their building bricks so that even if 2-knotting in standard sense might be not allowed, virtual 2-knotting would be possible. In TGD framework one works with a subgroup of symplectic transformations defining measurement resolution instead of isotopies and this might reduce the number of allowed mov

7.4.1 The dynamics of string world sheets and the expression for Kähler action

The dynamics of string world sheets is an open question. Effective 2-dimensionality suggests that Kähler action for the preferred extremal should be expressible using 2-D data but there are several guesses for what the explicit expression could be, and one can only make only guesses at this moment and apply internal consistency conditions in attempts to kill various options.

1. Could weak form of electric-magnetic duality hold true for string world sheets?

If one believes on duality between string world sheets and partonic 2-surfaces, one can argue that string world sheets are most naturally 2-surfaces at which the weak form of electric magnetic duality holds true. One can even consider the possibility that the weak form of electric-magnetic duality holds true only at the string world sheets and partonic 2-surfaces but not at the preferred 3-surfaces.

(a) The weak form of electric magnetic duality would mean that induced Kähler form is non-vanishing at them and Kähler magnetic flux over string world sheet is proportional to Kähler electric flux.

(b) The flux of the induced Kähler form of CP_2 over string world sheet would define a dimensionless "area". Could Kähler action for preferred extremals reduces to this flux apart from a proportionality constant. This "area" would have trivially extremum with respect to symplectic variations if the braid strands are Legendrian sub-manifolds since in this case the projection of Kähler gauge potential on them vanishes. This is a highly non-trivial point and favors weak form of electric-magnetic duality and the identification of Kähler action as Kähler magnetic flux. This option is also in spirit with the vision about TGD as almost topological QFT meaning that induced metric appears in the theory only via electric-magnetic duality.

(c) Kähler magnetic flux over string world sheet has a continuous spectrum so that the identification as Kähler action could make sense. For partonic 2-surfaces the magnetic flux would be quantized and give constant term to the action perhaps identifiable as the contribution of CP_2 type vacuum extremals giving this kind of contribution.

The change of space-time orientation by changing the sign of permutation symbol would change the sign in electric-magnetic duality condition and would not be a symmetry. For a given magnetic charge the sign of electric charge changes when orientation is changed. The value of Kähler action does not depend on space-time orientation but weak form of electric-magnetic duality as boundary condition implies dependence of the Kähler action on space-time orientation. The change of the sign of Kähler electric charge suggests the interpretation of orientation change as one aspect of charge conjugation. Could this orientation dependence be responsible for matter antimatter asymmetry?

2. Could string world sheets be Lagrangian sub-manifolds in generalized sense?

Legendrian sub-manifolds (see `http://tinyurl.com/yblj4hlq`) can be lifted to Lagrangian sub-manifolds [A3] Could one generalize this by replacing Lagrangian sub-manifold with 2-D sub-manifold of space-times surface for which the projection of the induced Kähler form vanishes? Could string world sheets be Lagrangian sub-manifolds?

I have also proposed that the inverse image of homologically non-trivial sphere of CP_2 under embedding map could define counterparts of string world sheets or partonic 2-surfaces. This conjecture does not work as such for cosmic strings, massless extremals having 2-D projection since the inverse image is in this case 4-dimensional. The option based on homologically non-trivial geodesic sphere is not consistent with the identification as analog of Lagrangian manifold but the identification as the inverse image of homologically trivial geodesic sphere is.

The most general option suggested is that string world sheet is mapped to 2-D Lagrangian sub-manifold of CP_2 in the embedding map. This would mean that theory is exactly solvable at string world sheet level. Vacuum extremals with a vanishing induced Kähler form would be exceptional in this framework since they would be mapped as a whole to Lagrangian sub-manifolds of CP_2. The boundary condition would be that the boundaries of string world sheets defined by braids at preferred 3-surfaces are Legendrian sub-manifolds. The generalization would mean that Legendrian braid strands could be continued to Lagrangian string world sheets for which induced Kähler form vanishes. The physical interpretation would be that if particle moves along this kind of string world sheet, it feels no covariant Lorentz-Kähler force and contra variant Lorentz forces is orthogonal to the string world sheet.

There are however serious objections.

(a) This proposal does not respect the proposed duality between string world sheets and partonic 2-surfaces which as carries of Kähler magnetic charges cannot be Lagrangian 2-manifolds.

(b) One loses the elegant identification of Kähler action as Kähler magnetic flux since Kähler magnetic flux vanishes. Apart from proportionality constant Kähler electric flux

$$\int_{Y^2} *J$$

is as a dimensionless scaling invariant a natural candidate for Kähler action but need not be extremum if braids are Legendrian sub-manifolds whereas for Kähler magnetic flux this is the case. There is however an explicit dependence on metric which does not conform with the idea that almost topological QFT is symplectic QFT.

(c) The sign factor of the dual flux which depends on the orientation of the string world sheet and thus changes sign when the orientation of space-time sheet is changed by changing that of the string world sheet. This is in conflict with the independence of Kähler action on orientation. One can however argue that the orientation makes itself actually physically visible via the weak form of electric-magnetic duality. If the above discussed duality holds true, the net contribution to Kähler action would vanish as the total Kähler magnetic flux for partonic 2-surfaces. Therefore the duality cannot hold true if Kähler action reduces to dual flux.

(d) There is also a purely formal counter argument. The inverse images of Lagrangian sub-manifolds of CP_2 can be 4-dimensional (cosmic strings and massless extremals) whereas string world sheets are 2-dimensonal.

7.4.2 String world sheets as minimal surfaces

Effective 2-dimensionality suggests a reduction of Kähler action to Chern-Simons terms to thearea of minimal surfaces defined by string world sheets holds true [K9]. Skeptic could argue that the expressibility of Kähler action involving no dimensional parameters except CP_2 scaled does not favor this proposal. The connection of minimal surface property with holomorphy and conformal invariance however forces to take the proposal seriously and it is easy to imagine how string tension emerges since the size scale of CP_2 appears in the induced metric [K9].

One can ask whether the mimimal surface property conforms with the proposal that string worlds sheets obey the weak form of electric-magnetic duality and with the proposal that they are generalized Lagrangian sub-manifolds.

(a) The basic answer is simple: minimal surface property and possible additional conditions (Lagrangian sub-manifold property or the weak form of electric magnetic duality) poses only additional conditions forcing the space-time sheet to be such that the imbedded string world sheet is a minimal surface of space-time surface: minimal surface property is a condition on space-time sheet rather than string world sheet. The weak form of electric-magnetic duality is favored because it poses conditions on the first derivatives in the normal direction unlike Lagrangian sub-manifold property.

(b) Any proposal for 2-D expression of Kähler action should be consistent with the proposed real-octonion analytic solution ansatz for the preferred extremals [K3]. The ansatz is based on real-octonion analytic map of embedding space to itself obtained by algebraically continuing real-complex analytic map of 2-D sub-manifold of embedding space to another such 2-D sub-manifold. Space-time surface is obtained by requiring that the "imaginary" part of the map vanishes so that image point is hyper-quaternion valued. Wick rotation allows to formulate the conditions using octonions and quaternions. Minimal surfaces (of space-time surface) are indeed objects for which the embedding maps are holomorphic and the real-octonion analyticity could be perhaps seen as algebraic continuation of this property.

(c) Does Kähler action for the preferred exremals reduce to the area of the string world sheet or to Kähler magnetic flux or are the representations equivalent so that the induced Kähler form would effectively define area form? If the Kähler form form associated with the induced metric on string world sheet is proportional to the induced Kähler form the Kähler magnetic flux is proportional to the area and Kähler action reduces to genuine area. Could one pose this condition as an additional constraint on string world sheets? For Lagrangian sub-manifolds Kähler electric field should be proportional to the area form and the condition involves information about space-time surface and is therefore more complex and does not look plausible.

7.4.3 Explicit conditions expressing the minimal surface property of the string world sheet

It is instructive to write explicitly the condition for the minimal surface property of the string world sheet and for the reduction of the area Kähler form to the induced Kähler form. For string world sheets with Minkowskian signature of the induced metric Kähler structure must be replaced by its hyper-complex analog involving hyper-complex unit e satisfying $e^2 = 1$ but replaced with real unit at the level hyper-complex coordinates. e can be represented as antisymmetric Kähler form J_g associated with the induced metric but now one has $J_g^2 = g$ instead of $J_g^2 = -g$. The condition that the signed area reduces to Kähler electric flux means that J_g must be proportional to the induced Kähler form: $J_g = kJ$, $k = constant$ in a given space-time region.

One should make an educated guess for the embedding of the string world sheet into a preferred extremal of Kähler action. To achieve this it is natural to interpret the minimal surface property as a condition for the preferred Kähler extremal in the vicinity of the string world sheet guaranteeing that the sheet is a minimal surface satisfying $J_g = kJ$. By the weak form of electric-magnetic duality partonic 2-surfaces represent both electric and magnetic monopoles. The weak form of electric-magnetic duality requires for string world sheets that the Kähler magnetic field at string world sheet is proportional to the component of the Kähler electric field parallel to the string world sheet. Kähler electric field is assumed to have component only in the direction of string world sheet.

1. Minkowskian string world sheets

Let us try to formulate explicitly the conditions for the reduction of the signed area to Kähler electric flux in the case of Minkowskian string world sheets.

(a) Let us assume that the space-time surface in Minkowskian regions has coordinates coordinates $(u, v, w, \overline{w})$ [K3]. The pair (u, v) defines light-like coordinates at the string world sheet having identification as hyper-complex coordinates with hyper-complex unit satisfying $e = 1$. u and v need not - nor cannot as it turns out - be light-like with respect to the metric of the space-time surface. One can use (u, v) as coordinates for string world sheet and assume that $w = x^1 + ix^2$ and $\overline{w}$ are constant for the string world sheet. Without a loss of generality one can assume $w = \overline{w} = 0$ at string world sheet.

(b) The induced Kähler structure must be consistent with the metric. This implies that the induced metric satisfies the conditions

$$g_{uu} = g_{vv} = 0 \ . \tag{7.1}$$

The analogs of these conditions in regions with Euclidian signature would be $g_{zz} = g_{\overline{z}\overline{z}} = 0$.

(c) Assume that the embedding map for space-time surface has the form

$$s^m = s^m(u, v) + f^m(u, v, x^m)_{kl} x^k x^l \ , \tag{7.2}$$

so that the conditions

$$\partial_l k s^m = 0 \ , \ \partial_k \partial_u s^m = 0, \ \partial_k \partial_v s^m = 0 \tag{7.3}$$

are satisfies at string world sheet. These conditions imply that the only non-vanishing components of the induced CP_2 Kähler form at string world sheet are J_{uv} and $J_{w\overline{w}}$. Same applies to the induced metric if the metric of M^4 satisfies these conditions (no non-vanishing components of form m_{uk} or m_{vk}).

(d) Also the following conditions hold true for the induced metric of the space-time surface

$$\partial_k g_{uv} = 0 \ , \ \partial_u g_{kv} = 0 \ , \ \partial_v g_{ku} = 0 \ . \tag{7.4}$$

at string world sheet as is easy to see by using the ansatz.

Consider now the minimal surface conditions stating that the trace of the four components of the second fundamental form whose components are labelled by the coordinates $\{x^\alpha\} \equiv (u, v, w, \overline{w})$ vanish for string world sheet.

(a) Since only g_{uv} is non-vanishing, only the components H^k_{uv} of the second fundamental form appear in the minimal surface equations. They are given by the general formula

$$\begin{aligned} H^\alpha_{uv} &= H^\gamma P^\alpha_\gamma \ , \\ H^\alpha &= \left(\partial_u \partial_v x^\alpha + \left({}_\beta{}^\alpha{}_\gamma\right) \partial_u x^\beta \partial_v x^\gamma\right) \ . \end{aligned} \tag{7.5}$$

Here P^α_γ is the projector to the normal space of the string world sheet. Formula contains also Christoffel symbols $\left({}_\beta{}^\alpha{}_\gamma\right)$.

(b) Since the embedding map is simply $(u, v) \rightarrow (u, v, 0, 0)$ all second derivatives in the formula vanish. Also $H^k = 0$, $k \in \{w, \overline{w}\}$ holds true. One has also $\partial_u x^\alpha = \delta^\alpha_u$ and $\partial_v x^\beta = \delta^\beta_v$. This gives

$$H^\alpha = \left({}_u{}^\alpha{}_v\right) \ . \tag{7.6}$$

All these Christoffel symbols however vanish if the assumption $g_{uu} = g_{vv} = 0$ and the assumptions about embedding ansatz hold true. Hence a minimal surface is in question.

Consider now the conditions on the induced metric of the string world sheet

(a) The conditions reduce to

$$g_{uu} = g_{vv} = 0 \ . \tag{7.7}$$

The conditions on the diagonal components of the metric are the analogs of Virasoro conditions fixing the coordinate choices in string models. The conditions state that the coordinate lines for u and v are light-like curves in the induced metric.

(b) The conditions can be expressed directly in terms of the induced metric and read

$$\begin{aligned} m_{uu} + s_{kl}\partial_u s^k \partial_u s^l &= 0 \ , \\ m_{vv} + s_{kl}\partial_v s^k \partial_v s^l &= 0 \ . \end{aligned} \tag{7.8}$$

The CP_2 contribution is negative for both equations. The conditions make sense only for $(m_{uu} > 0, m_{vv} > 0)$. Note that the determinant condition $m_{uu}m_{vv} - m_{uv}m_{vu} < 0$ expresses the Minkowskian signature of the (u, v) coordinate plane in M^4.

The additional condition states

$$J^g_{uv} = kJ_{uv} \ . \tag{7.9}$$

It reduces signed area to Kähler electric flux. If the weak form of electric-magnetic duality holds true one can interpret the area as magnetic flux defined as the flux of the dual of induced Kähler form over space-like surface and defining electric charge. A further condition is that the boundary of string world sheet is Legendrean manifold so that the flux and thus area is extremized also at the boundaries.

2.Conditions for the Euclidian string world sheets

One can do the same calculation for string world sheet with Euclidian signature. The only difference is that (u, v) is replaced with $(z, \overline{z})$. The embedding map has the same form assuming that space-time sheet with Euclidian signature allows coordinates $(z, \overline{z}, w, \overline{w})$ and the local conditions on the embedding are a direct generalization of the above described conditions. In this case the vanishing for the diagonal components of the string world sheet metric reads as

$$\begin{aligned} h_{kl}\partial_z s^k \partial_z s^l &= 0 \ , \\ h_{kl}\partial_{\overline{z}} s^k \partial_{\overline{z}} s^l &= 0 \ . \end{aligned} \tag{7.10}$$

The natural ansatz is that complex CP_2 coordinates are holomorphic functions of the complex coordinates of the space-time sheet.

3. Wick rotation for Minkowskian string world sheets leads to a more detailed solution ansatz

Wick rotation is a standard trick used in string models to map Minkowskian string world sheets to Euclidian ones. Wick rotation indeed allows to define what one means with real-octonion analyticity. Could one identify string world sheets in Minkowskian regions by using Wick rotation and does this give the same result as the direct approach?

Wick rotation transforms space-time surfaces in $M^4 \times CP_2$ to those in $E^4 \times CP_2$. In $E^4 \times CP_2$ octonion real-analyticity is a well-defined notion and one can identify the space-time surfaces surfaces at which the imaginary part of of octonion real-analytic function vanishes: imaginary part is defined via the decomposition of octonion to two quaternions as $o = q_1 + Iq_2$ where I is a preferred octonion unit. The reverse of the Wick rotation maps the quaternionic surfaces to what might be called hyper-quaternionic surfaces in $M^4 \times CP_2$.

In this picture string world sheets would be hyper-complex surfaces defined as inverse images of complex surfaces of quaternionic space-time surface obtained by the inverse of Wick rotation. For this approach to be equivalent with the above one it seems necessary to require that the treatment of the conditions on metric should be equivalent to that for which hyper-complex unit e is not put equal to 1. This would mean that the conditions reduce to independent conditions for the real and imaginary parts of the real number formally represented as hyper-complex number with $e = 1$.

Wick rotation allows to guess the form of the ansatz for CP_2 coordinates as functions of space-time coordinates In Euclidian context holomorphich functions of space-time coordinates are the natural ansatz. Therefore the natural guess is that one can map the hypercomplex number $t \pm ez$ to complex coordinate $t \pm iz$ by the analog of Wick rotation and assume that CP_2 complex coordinates are analytic functions of the complex space-time coordinates obtained in this manner.

The resulting induced metric could be obtained directly using real coordinates (t, z) for string world sheet or by calculating the induced metric in complex coordinates $t \pm iz$ and by mapping the expressions to hyper-complex numbers by Wick rotation (by replacing i with $e = 1$). If the diagonal components of the induced metric vanish for $t \pm iz$ they vanish also for hyper-complex coordinates so that this approach seem to make sense.

7.4.4 Electric-magnetic duality for flux Hamiltonians and the existence of Wilson sheets

One must distinguish between two conjectured dualities. The weak form of electric-magnetic duality and the duality between string world sheets and partonic 2-surfaces. Could the first duality imply equivalence of not only electric and magnetic flux Hamiltonians but also electric and magnetic Wilson sheets? Could the latter duality allow two different representations of flux Hamiltonians?

(a) For electric-magnetic duality holding true at string world sheets one would have non-vanishing Kähler form and the fluxes would be non-vanishing. The Hamiltonian fluxes

$$Q_{m,A} = \int_{X^2} JH_A dx^1 dx^2 = \int_{X^2} H_A J_{\alpha\beta} dx^\alpha \wedge dx^\beta \tag{7.11}$$

for partonic 2-surfaces X^2 define WCW Hamiltonians playing a key role in the definition of WCW Kähler geometry. They have also interpretation as a generalization of Wilson loops to Wilson 2-surfaces.

(b) Weak form of electric magnetic duality would imply both at partonic 2-surfaces and string world sheets the proportionality

$$Q_{m,A} = \int_{X^2} JH_A dx^1 \wedge dx^2 \propto Q^*_{m,A} = \int_{X^2} H_A * J_{\alpha\beta} dx^\alpha \wedge dx^\beta \ . \tag{7.12}$$

Therefore the electric-magnetic duality would have a concrete meaning also at the level of WCW geometry.

(c) If string world sheets are Lagrangian sub-manifolds Hamiltonian fluxes would vanish identically so that the identification as Wilson sheets does not make sense. One would lose electric-magnetic duality for flux sheets. The dual fluxes

$$*Q_A = \int_{Y^2} *JH_A dx^1 \wedge dx^2 = \int_{Y^2} \epsilon_{\alpha\beta}{}^{\gamma\delta} J_{\gamma\delta} = \int_{Y^2} \frac{\sqrt{det(g_4)}}{det(g_2^\perp)} J_{34}^\perp dx^1 \wedge dx^2$$

for string world sheets Y^2 are however non-vanishing. Unlike fluxes, the dual fluxes depend on the induced metric although they are scaling invariant.

Under what conditions the conjectured duality between partonic 2-surface and string world sheets hold true at the level of WCW Hamiltonians?

(a) For the weak form of electric-magnetic duality at string world sheets the duality would mean that the sum of the fluxes for partonic 2-surfaces and sum of the fluxes for string world sheets are identical apart from a proportionality constant:

$$\sum_i Q_A(X_i^2) \quad \propto \quad \sum_i Q_A(Y_i^2) \ . \tag{7.13}$$

Note that in zero ontology it seems necessary to sum over all the partonic surfaces (at both ends of the space-time sheet) and over all string world sheets.

(b) For Lagrangian sub-manifold option the duality can hold true only in the form

$$\sum_i Q_A(X_i^2) \quad \propto \quad \sum_i Q_A^*(Y_i^2) \ . \tag{7.14}$$

Obviously this option is less symmetric and elegant.

7.4.5 Summary

There are several arguments favoring weak form of electric-magnetic duality for both string world sheets and partonic 2-surfaces. Legendrian sub-manifold property for braid strands follows from the assumption that Kähler action for preferred extremals is proportional to the Kähler magnetic flux associated with preferred 2-surfaces and is stationary with respect to the variations of the boundary. What is especially nice is that Legendrian sub-manifold property implies automatically unique braids. The minimal option favored by the idea that 3-surfaces are basic dynamical objects is the one for which weak form of electric-magnetic duality holds true only at partonic 2-surfaces and string world sheets. A stronger option assumes it at preferred 3-surfaces. Duality between string world sheets and partonic 2-surfaces suggests that WCW Hamiltonians can be defined as sums of Kähler magnetic fluxes for either partonic 2-surfaces or string world sheets.

7.5 What Generalized Feynman Rules Could Be?

After all these explanations the skeptic reader might ask whether this lengthy discussion gives any idea about what the generalized Feynman rules might look like. The attempt to answer this question is a good manner to make a map about what is understood and what is not understood. The basic questions are simple. What constraints does zero energy ontology (ZEO) pose? What does the necessity to projecti the four-momenta to a preferred plane M^2 mean? What mathematical expressions one should assign to the propagator lines and vertices? How does one perform the functional integral over 3-surfaces in finite measurement resolution? The following represents tentatative answers to these questions but does not say much about exact role of algebraic knots.

7.5.1 Zero energy ontology

Zero energy ontology (ZEO) poses very powerful constraints on generalized Feynman diagrams and gives hopes that both UV and IR divergences cancel.

(a) ZEO predicts that the fermions assigned with braid strands associated with the virtual particles are on mass shell massless particles for which the sign of energy can be also negative: in the case of wormhole throats this can give rise to a tachyonic exchange.

(b) The on mass shell conditions for each wormhole throat in the diagram involving loops are very stringent and expected to eliminate very large classes of diagrams. If however given diagonal diagram leading from n-particle state to the same n-particle state -completely analogous to self energy diagram- is possible then the ladders form by these diagrams are also possible and one one obtains infinite of this kind of diagrams as generalized self energy correction and is excellent hopes that geometric series gives a closed algebraic function.

(c) IR divergences plaguing massless theories are cancelled if the incoming and outgoing particles are massive bound states of massless on mass shell particles. In the simplest manner this is achieved when the 3-momenta are in opposite direction. For internal lines the massive on-mass shell-condition is not needed at all. Therefore there is an almost complete separation of the problem how bound state masses are determined from the problem of constructing the scattering amplitudes.

(d) What looks like a problematic aspect ZEO is that the massless on-mass-shell propagators would diverge for wormhole throats. The solution comes from the projection of 4-momenta to M^2. In the generic the projection is time-like and one avoids the singularity. The study of solutions of the Kähler-Dirac equation [K15] and number theoretic vision [K13] indeed suggests that the four-momenta are obtained by rotating massless M^2 momenta and their projections to M^2 are in general integer multiples of hyper-complex primes or light-like. The light-like momenta would be treated like in the case of ordinary Feynman diagrams using $i\epsilon$-prescription of the propagator and would also give a finite contributions corresponding to integral over physical on mass shell states. This guarantees also the vanishing of the possible IR divergences coming from the summation over different M^2 momenta.

There is a strong temptation to identify - or at least relate - the M^2 momenta labeling the solutions of the Kähler-Dirac equation with the region momenta of twistor approach [K14]. The reduction of the region momenta to M^2 momenta could dramatically simplify the twistorial description. It does not seem however plausible that $\mathcal{N} = 4$ super-symmetric gauge theory could allow the identification of M^2 projections of 4-momenta as region momenta. On the other hand, there is no reason to expect the reduction of TGD certainly to a gauge theory containing QCD as part. For instance, color magnetic flux tubes in many-sheeted space-time are central for understanding jets, quark gluon plasma, hadronization and fragmentation [L9] but cannot be deduced from QCD. Note also that the splitting of parton momenta to their M^2 projections and transversal parts is an ad hoc assumption motivated by parton model rather than first principle implication of QCD: in TGD framework this splitting would emerge from first principles.

(e) ZEO strongly suggests that all particles (including photons, gluons, and gravitons) have mass which can be arbitrarily small and could be perhaps seen as being due to the fact that particle "eats" Higgs like states giving it the otherwise lacking polarization states. This would mean a generalization of the notion of Higgs particle to a Higgs like particle with spin. It would also mean rearrangmenet of massless states at wormhole throat level to massives physical states. The slight massication of photon by p-adic thermodynamics does not however mean disappearance of Higgs from spectrum, and one can indeed construct a model for Higgs like states [K8].

The projection of the momenta to M^2 is consistent with this vision. The natural generalization of the gauge condition $p \cdot \epsilon = 0$ is obtained by replacing p with the projection of the total momentum of the boson to M^2 and ϵ with its polarization so that one has $p_{||} \cdot \epsilon$. If the projection to M^2 is light-like, three polarization states are possible in the generic case, so that massivation is required by internal consistency. Note that if intermediate states in the unitary condition were states with light-like M^2-momentum one could have a problematic situation.

(f) A further assumption vulnerable to criticism is that the M^2 projections of all momenta assignable to braid strands are parallel. Only the projections of the momenta to the orthogonal complement E^2 of M^2 can be non-parallel and for massive wormhole throats they must be non-parallel. This assumption does not break Lorentz invariance since in the full amplitude one must integrate over possible choices of M^2. It also interpret the gauge conditions either at the level of braid strands or of partons. Quantum classical correspondence in strong form would actually suggests that quantum 4-momenta should co-incide with the classical ones. The restriction to M^2 projections is however necessary and seems also natural. For instance, for massless extremals only M^2 projection of wave-vector can be well-defined: in transversal degrees of freedom there is a superposition over Fourier components with diffrent transversal wave-vectors. Also the partonic description of hadrons gives for the M^2 projections of the parton momenta a preferred role. It is highly encouraging that this picture emerged first from the Kähler-Dirac equation and purely number theoretic vision based on the identification of M^2 momenta in terms of hyper-complex primes.

The number theoretical approach also suggests a number theoretical quantization of the transversal parts of the momenta [K13]: four-momenta would be obtained by rotating massless M^2 momenta in M^4 in such a way that the components of the resulting 3-momenta are integer valued. This leads to a classical problem of number theory which is to deduce the number of 3-vectors of fixed length with integer valued components. One encounters the n-dimensional generalization of this problem in the construction of discrete analogs of quantum groups (these "classical" groups are analogous to Bohr orbits) and emerge in quantum arithmetics [K11], which is a deformation of ordinary arithmetics characterized by p-adic prime and giving rigorous justification for the notion of canonical identification mapping p-adic numbers to reals.

(g) The real beauty of Feynman rules is that they guarantee unitarity automatically. In fact, unitarity reduces to Cutkosky rules which can be formulated in terms of cut obtained by putting certain subset of interal lines on mass shell so that it represents on mass shell state. Cut analyticity implies the usual $iDisc(T) = TT^\dagger$. In the recent context the cutting of the internal lines by putting them on-mass-shell requires a generalization.

  i. The first guess is that on mass shell property means that M^2 projection for the momenta is light-like. This would mean that also these momenta contribute to the amplitude but the contribution is finite just like in the usual case. In this formulation the real particles would be the massless wormhole throats.

  ii. Second possibility is that the internal lines on on mass shell states corresponding to massive on mass-shell-particles. This would correspond to the experimental meaning of the unitary conditions if real particles are the massive on mass shell particles. Mathematically it seems possible to pick up from the amplitude the states which correspond to massive on mass shell states but one should understand why the discontinuity should be associated with physical net masses for wormhole contacts or many-particle states formed by them. General connection with unitarity and analyticity might allow to understand this.

(h) CDs are labelled by various moduli and one must integrate over them. Once the tips of the CD and therefore a preferred M^1 is selected, the choice of angular momentum quantization axis orthogonal to M^1 remains: this choice means fixing M^2. These choices are parameterized by sphere S^2. It seems that an integration over different choices of M^2 is needed to achieve Poincare invariance.

7.5.2 How the propagators are determined?

In accordance with previous sections it will be assumed that the braid are Legendrian braids and therefore completely well-defined. One should assign propagator to the braid. A good guess is that the propagator reduces to a product of three terms.

(a) A multi-particle propagator which is a product of collinear massless propagators for braid strands with fermionin number $F = 0, 1-1$. The constraint on the momenta is $p_i = \lambda_i p$ with $\sum_i \lambda_i = 1$. So that the fermionic propagator is $\frac{1}{\prod_i \lambda_i} p^k \gamma_k$. If one gas $p = nP$, where P is hyper-complex prime, one must sum over combinations of $\lambda_i = n_i$ satisfying $\sum_i n_i = n$.

(b) A unitary S-matrix for integrable QFT in M^2 in which the velocities of particles assignable to braid strands appear for which fixed by R-matrix defines the basic 2-vertex representing the process in which a particle passes through another one. For this S-matrix braids are the basic units. To each crossing appearing in non-planar Feynman diagram one would have an R-matrix representing the effect of a reconnection the ends of the lines coming to the crossing point. In this manner one could gradually transform the non-planar diagram to a planar diagram. One can ask whether a formulation in terms of a suitable R-matrix could allow to generalize twistor program to apply in the case of non-planar diagrams.

(c) An S-matrix predicted by topological QFT for a given braid. This S-matrix should be constructible in terms of Chern-Simons term defining a sympletic QFT.

There are several questions about quantum numbers assignable to the braid strands.

(a) Can braid strands be only fermionic or can they also carry purely bosonic quantum numbers corresponding to WCW Hamiltonians and therefore to Hamiltonians of $\delta M^4_\pm \times CP_2$? Nothing is lost if one assumes that both purely bosonic and purely fermionic lines are possible and looks whether this leads to inconsistencies. If virtual fermions correspond to single wormhole throat they can have only time-like M^2-momenta. If virtual fermions correspond to pairs of wormhole throats with second throat carrying purely bosonic quantum numbers, also fermionic can have space-like net momenta. The interpretation would be in terms of topological condensation. This is however not possible if all strands are fermionic. Situation changes if one identifies physical fermions wormhole throats at the ends of Kähler magnetic flux tube as one indeed does: in this case virtual net momentum can be space-like if the sign of energy is opposite for the ends of the flux tube.

(b) Are the 3-momenta associated with the wormholes of wormhole contact parallel so that only the sign of energy could distinguish between them for space-like total momentum and M^2 mass squared would be the same? This assumption simplifies the situation but is not absolutely necessary.

(c) What about the momentum components orthogonal to M^2? Are they restricted only by the massless mass shell conditions on internal lines and quantization of the M^2 projection of 4-momentum?

(d) What kind of braids do elementary particles correspond? The braids assigned to the wormhole throat lines can have arbitrary number n of strands and for $n = 1, 2$ the treatment of braiding is almost trivial. A natural assumption is that propagator is simply a product of massless collinear propagators for M^2 projection of momentum [**?**]. Collinearity means that propagator is product of a multifermion propagator $\frac{1}{\lambda_i p_k \gamma_k}$, znd multiboson propagator $\frac{1}{\mu_i p_k \gamma_k}$, $\sum \lambda_i + \sum_i \mu_i = 1$. There are also quantization conditions on M^2 projections of momenta from Kähler-Dirac equation implying that multiplies of hyper-complex prime are in question in suitable units. Note however that it is not clear whether purely bosonic strands are present.

(e) For ordinary elementary particles with propagators behaving like $\prod_i \lambda_i^{-1} 1 p^{-n}$, only $n \leq 2$ is possible. The topologically really interesting states with more than two braid strands are something else than what we have used to call elementary particles. The proposed interpretation is in terms of anyonic states [K12]. One important implication is that $\mathcal{N} = 1$ SUSY generated by right-handed neutrino or its antineutrino is SUSY for which all members of the multiplet assigned to a wormhole throat have braid number smaller than 3. For $\mathcal{N} = 2$ SUSY generated by right-handed neutrino and its antiparticle the states containing fermion and neutrino-antineutrino pair have three braid strands and SUSY breaking is expected to be strong.

7.5.3 Vertices

Conformal invariance raises the hope that vertices can be deduced from super-conformal invariance as n-point functions. Therefore lines would come from integrable QFT in M^2

and topological braid theory and vertices from confofmal field theory: both theories are integrable.

The basic questions is how the vertices are defined by the 2-D partonic surfaces at which the ends of lines meet. Finite measurement resolution reduces the lines to braids so that the vertices reduces to the intersection of braid strands with the partonic 2-surface.

(a) Conformal invariance is the basic symmetry of quantum TGD. Does this mean that the vertices can be identified as n-point functions for points of the partonic 2-surface defined by the incoming and outgoing braid strands? How strong constraints can one pose on this conformal field theory? Is this field theory free and fixed by anti-commutation relations of induced spinor fields so that correlation function would reduce to product of fermionic two points functions with standard operator in the vertices represented by strand ends. If purely bosonic vertices are present, their correlation functions must result from the functional integral over WCW .

(b) For the fermionic fields associated with each incoming braid the anti-commutators of fermions and anti-fermions are trivial just as the usual equal time anti-commutation relations. This means that the vertex reduces to sum of products of fermionic correlation functions with arguments belonging to different incoming and outgoing lines. How can one calculate the correlators?

   i. Should one perform standard second quantization of fermions at light-like 3-surface allowing infinite number of spinor modes, apply a finite measurement resolution to obtain braids, for each partonic 2-surface, and use the full fermion fields to calculate the correlators? In this case braid strands would be discontinuous in vertices. A possible problem might be that the cutoff in spinor modes seems to come from the theory itself: finite measurement resolution is a property of quantum state itself.

   ii. Could finite measurement resolution allow to approximate the braid strands with continuous ones so that the correlators between strands belonging to different lines are given by anti-commutation relations? This would simplify enormously the situation and would conform with the idea of finite measurement resolution and the vision that interaction vertices reduce to braids. This vision is encouraged by the previous considerations and would mean that replication of braid strands analogous to replication of DNA strands can be seen as a fundamental process of Nature. This of course represents an important deviation from the standard picture.

(c) Suppose that one accepts the latter option. What can happen in the vertex, where line goes from one braid to another one?

   i. Can the direction of momentum changed as visual intuition suggests? Is the total braid momentum conservation the only constraint so that the velocities assignable braid strands in each line would be constrained by the total momentum of the line.

   ii. What kind of operators appear in the vertex? To get some idea about this one can look for the simplest possible vertex, namely FFB vertex which could in fact be the only fundamental vertex as the arguments of [K5] suggest. The propagator of spin one boson decomposes to product of a projection operator to the polarization states divited by p^2 factor. The projection operator sum over products $\epsilon_i^k\gamma_k$ at both ends where γ_k acts in the spinor space defined by fermions. Also fermion lines have spinor and its conjugate at their ends. This gives rise to $p^k\gamma_k/p^2$. $p^k\gamma_k$ is the analog of the bosonic polarization tensor factorizing into a sum over products of fermionic spinors and their conjugates. This gives the BFF vertex $\epsilon_i^k\gamma_k$ slashed between the fermionic propagators which are effectively 2-dimensional.

   iii. Note that if H-chiralities are same at the throats of the wormhole contact, only spin one states are possible. Scalars would be leptoquarks in accordance with general view about lepton and quark number conservation. One particular implication is that Higgs in the standard sense is not possible in TGD framework. It can appear only as a state with a polarization which is in CP_2 direction. In any case, Higgs like states would be eaten by massless state so that all particles would have at least a small mass.

7.5.4 Functional integral over 3-surfaces

The basic question is how one can functionally integrate over light-like 3-surfaces or space-like 3-surfaces.

(a) Does effective 2-dimensionality allow to reduce the functional integration to that over partonic 2-surfaces assigned with space-time sheet inside CD plus radiative corrections from the hierarchy of sub-CDs?

(b) Does finite measurement resolution reduce the functional integral to a ordinary integral over the positions of the end points of braids and could this integral reduce to a sum? Symplectic group of $\delta M^4_\pm \times CP_2$ basically parametrizes the quantum fluctuating degrees of freedom in WCW . Could finite measurement resolution reduce the symplectic group of $\delta M^4_\pm \times CP_2$ to a coset space obtained by dividing with symplectic transformations leaving the end points invariant and could the outcome be a discrete group as proposed? Functional integral would reduce to sum.

(c) If Kähler action reduces to Chern-Simons-Kähler terms to surface area terms in the proposed manner, the integration over WCW would be very much analogous to a functional integral over string world sheets and the wisdom gained in string models might be of considerable help.

7.5.5 Summary

What can one conclude from these argument? To my view the situation gives rise to a considerable optimism. I believe that on basis of the proposed picture it should be possible to build a concrete mathematical models for the generalized Feynman graphics and the idea about reduction to generalized braid diagrams having algebraic representations could pose additional powerful constraints on the construction. Braid invariants could also be building bricks of the generalized Feynman diagrams. In particular, the treatment of the non-planarity of Feynman diagrams in terms of M^2 braiding matrix would be something new and therefore can be questioned.

Few years after writing these lines a view about generalized Feynman diagrams as a stringy generalization of twistor Grassmannian diagrams has emerged [K14]. This approach relies heavily on the localization of spinor modes on 2-D string world sheets (covariantly constant right-handed neutrino is an exception) [K15]. This approach can be regarded as an effective QFT (or rather, effective string theory) approach: all information about the microscopic character of the fundamental particle like entities has been integrated out so that a string model type description at the level of imbdding space emerges. The presence of gigantic symmetries, in particular, the Yangian generalization of super-conformal symmetries, raises hopes that this approach could work. The approach to generalized Feynman diagrams considered above is obviously microscopic.

8 Electron As A Trefoil Or Something More General?

The possibility that electron, and also other elementary particles could correspond to knot is very interesting. The video model (see `http://tinyurl.com/ycz4jm48`) [B5] was so fascinating (I admire the skills of the programmers) that I started to question my belief that all related to knots and braids represents new physics (say anyons, see `http://tinyurl.com/y89xp4bu`) [K12] and that it is hopeless to try to reduce standard model quantum numbers with purely group theoretical explanation (except family replication) to topological quantum numbers.

Electroweak and color quantum numbers should by quantum classical correspondence have geometric correlates in space-time geometry. Could these correlates be topological? As a matter of fact, the correlates existing if the present understanding of the situation is correct but they are not topological.

Despite this, I played with various options and found that in TGD Universe knot invariants do not provide plausible space-time correlates for electroweak quantum numbers. The knot invariants and many other topological invariants are however present and mean new physics. As following arguments try to show, elementary particles in TGD Universe are characterized by extremely rich spectrum of topological quantum numbers, in particular those associated with knotting and linking: this is basically due to the 3-dimensionality of 3-space.

For a representation of trefoil knot by R.W. Gray see `http://tinyurl.com/ycz4jm48`. The homepage of Louis Kauffman (see `http://tinyurl.com/y7r3w5jq`) [A6] is a treasure trove for anyone interested in ideas related to possible applications of knots to physics. One particular knotty idea is discussed in the article "Emergent Braided Matter of Quantum Geometry" (see `http://tinyurl.com/y7lnn3wa`) by Bilson-Thompson, Hackett, and Kauffman [B1].

8.1 Space-Time As 4-Surface And The Basic Argument

Space-time as a 4-surface in $M^4 \times CP_2$ is the key postulate. The dynamics of space-time surfaces is determined by so called Kähler action - essentially Maxwell action for the Kähler form of CP_2 induced to X^4 in induced metric. Only so called preferred extremals are accepted and one can in very loose sense say that general coordinate invariance is realized by assigning to a given 3-surface a unique 4-surface as a preferred extremal analogous to Bohr orbit for a particle identified as 3-D surface rather than point-like object.

One ends up with a radical generalization of space-time concept to what I call many-sheeted space-time. The sheets of many-sheeted space-time are at distance of CP_2 size scale (10^4 Planck lengths as it turns out) and can touch each other which means formation of wormhole contact with wormhole throats as its ends. At throats the signature of the induced metric changes from Minkowskian to Euclidian. Euclidian regions are identified as 4-D analogs of lines of generalized Feynman diagrams and the M^4 projection of wormhole contact can be arbitrarily large: macroscopic, even astrophysical. Macroscopic object as particle like entity means that it is accompanied by Euclidian region of its size.

Elementary particles are identified as wormhole contacts. The wormhole contacts born in mere touching are not expected to be stable. The situation changes if there is a monopole magnetic flux (CP_2 carries self dual purely homological monopole Kähler form defining Maxwell field, this is not Dirac monopole) since one cannot split the contact. The lines of the Kähler magnetic field must be closed, and this requires that there is another wormhole contact nearby. The magnetic flux from the upper throat of contact A travels to the upper throat of contact B along "upper" space-time sheet, goes to "lower" space-time sheet along contact B and returns back to the wormhole contact A so that closed loop results.

In principle, wormhole throat can have arbitrary orientable topology characterized by the number g of handles attached to sphere and known as genus. The closed flux tube corresponds to topology $X^2_g \times S^1$, g=0, 1, 2, ... Genus-generation correspondence (see `http://tinyurl.com/ybowqm5v`) [K5] states that electron, muon, and tau lepton and similarly quark generations correspond to $g = 0, 1, 2$ in TGD Universe and CKM mixing is induced by topological mixing.

Suppose that one can assign to this flux tube a closed string: this is indeed possible but I will not bother reader with details yet. What one can say about the topology of this string?

(a) X^2_g has homology Z^{2g} and S^1 homology S^1. The entire homology is Z^{2g+1} so that there are $2g + 1$ additional integer valued topological quantum numbers besides genus. Z^{2g+1} obviously breaks topologically universality stating that fermion generations are exact copies of each other apart from mass. This would be new physics. If the size of the flux loop is of order Compton length, the topological excitations need not be too heavy. One should however know how to excite them.

(b) The circle S^1 is imbedded in 3-surface and can get knotted. This means that all possible knots characterize the topological states of the fermion. Also this means extremely rich spectrum of new physics.

8.2 What Is The Origin Of Strings Going Around The Magnetic Flux Tube?

What is then the origin of these knotted strings? The study of the Kähler-Dirac equation [K15] determining the dynamics of induced spinor fields at space-time surface led to a considerable insight here. This requires however additional notions such as zero energy ontology (ZEO), and causal diamond (CD) defined as intersection of future and past directed light-cones (double 4-pyramid is the M^4 projection. Note that CD has CP_2 as Cartesian factor and is analogous to Penrose diagram.

(a) ZEO means the assumption that space-time surfaces for a particular sub- WCW ("world of classical worlds") are contained inside given CD identifiable as a the correlate for the "spotlight of consciousness" in TGD inspired theory of consciousness. The space-time surface has ends at the upper and lower light-like boundaries of CD. The 3-surfaces at the ends define space-time correlates for the initial and final states in positive energy ordinary ontology. In ZEO they carry opposite total quantum numbers.

(b) General coordinate invariance (GCI) requires that once the 3-D ends are known, space-time surface connecting the ends is fixed (there is not path integral since it simply fails). This reduces ordinary holography to GCI and makes classical physics defined by preferred extremals an exact part of quantum theory, actually a key element in the definition of Kähler geometry of WCW .

Strong form of GCI is also possible. One can require that 3-D light-like orbits of wormhole throats at which the induced metric changes its signature, and space-like 3-surfaces at the ends of CD give equivalent descriptions. This implies that quantum physics is coded by the their intersections which I call partonic 2-surfaces - wormhole throats - plus the 4-D tangent spaces of X^4 associated with them. One has strong form of holography. Physics is almost 2-D but not quite: 4-D tangent space data is needed.

(c) The study of the Kähler-Dirac equation [K15] leads to further results. The mere conservation of electromagnetic charge defined group theoretically for the induced spinors of $M^4 \times CP_2$ carrying spin and electroweak quantum numbers implies that for all other fermion states except right handed neutrino (, which does not couple at all all to electroweak fields), are localized at 2-D string world sheets and partonic 2-surfaces.

String world sheets intersect the light-like orbits of wormhole throats along 1-D curves having interpretation as time-like braid strands (a convenient metaphor: braiding in time direction si created by dancers in the parquette).

One can say that dynamics automatically implies effective discretization: the ends of time like braid strands at partonic 2-surfaces at the ends of CD define a collection of discrete points to each of which one can assign fermionic quantum numbers.

(d) Both throats of the wormhole contact can carry many fermion state and known fermions correspond to states for which either throat carries single braid strand. Known bosons correspond to states for which throats carry fermion and anti-fermion number.

(e) Partonic 2-surface is replaced with discrete set of points effectively. The interpretation is in terms of a space-time correlate for finite measurement resolution. Quantum correlate would be the inclusion of hyperfinite factors of type II_1.

This interpretation brings in even more topology!

(a) String world sheets - present both in Euclidian and Minkowskian regions - intersect the 3-surfaces at the ends of CD along curves - one could speak of strings. These strings give rise to the closed curves that I discussed above. These strings can be homologically non-trivial - in string models this corresponds to wrapping of branes.

(b) For known bosons one has two closed loop but these loops could fuse to single. Space-like 2-braiding (including linking) becomes possible besides knotting.

(c) When the partonic 2-surface contains several fermionic braid ends one obtains even more complex situation than above when one has only single braid end. The loops associated with the braid ends and going around the monopole flux tube can form space-like N-braids. The states containing several braid ends at either throat correspond to exotic particles not identifiable as ordinary elementary particles.

8.3 How Elementary Particles Interact As Knots?

Elementary particles could reveal their knotted and even braided character via the topological interactions of knots. There are two basic interactions.

(a) The basic interaction for single string is by self-touching and this can give to a local connected sum or a reconnection. In both cases the knot invariants can change and it is possible to achieve knotting or unknotting of the string by this mechanism. String can also split into two pieces but this might well be excluded in the recent case.

The space-time dynamics for these interactions is that of closed string model with 4-D target space. The first guess would be topological string model describing only the dynamics of knots. Note that string world sheets define 2-knots and braids.

(b) The basic interaction vertex for generalized Feynman diagrams (lines are 4-D space-time regions with Euclidian signature) is join along 3-D boundaries for the three particles involved: this is just like ordinary 3-vertex for Feynman diagrams and is not encountered in string models. The ends of lines must have same genus g. In this interaction vertex the homology charges in Z^{2g+1} is conserved so that these charges are analogous to U(1) gauge charges. The strings associated with the two particles can touch each other and connected sum or reconnection is the outcome.

Consider now in more detail connected sum and reconnection vertices responsible for knotting and un-knotting.

(a) The first interaction is connected sum (see `http://tinyurl.com/lye7pvp`) of knots [A2]. A little mental exercise demonstrates that a local connected sum for the pieces of knot for which planar projections cross, can lead to a change in knotted-ness. Local connected sum is actually used to un-knot the knot in the construction of knot invariants.

In dimension 3 knots form a module with respect to the connected sum. One can identify unique prime knots and construct all knots as products of prime knots with product defined as a connected sum of knots. In particular, one cannot have a situation on which a product of two non-trivial knots is un-knot so that one could speak about the inverse of a knot (indeed, the inverse of ordinary prime is not an integer!). For higher-dimensional knots the situation changes (string world sheets at space-time surface could form 2-knots but instead of linking they intersect at discrete points).

Connected sum in the vertex of generalized Feynman graph (as described above) can lead to a decay of particle to two particles, which correspond to the summands in the connected sum as knots. Could one consider a situation in which un-knotted particle decomposes via the time inverse of the connected sum to a pair of knotted particles such that the knots are inverses of each other? This is not possible since knots do not have inverse.

(b) Touching knots can also reconnect. For braids the strands $A \to B$ and $C \to D$ touch and one obtains strands $A \to D$ and $C \to B$. If this reaction takes place for strands whose planar projections cross, it can also change the character of the knot. One one can transform knot to un-knot by repeatedly applying connected sum and reconnection for crossing strands (the Alexandrian way).

(c) In the evolution of knots as string world sheets these two vertices corresponds to closed string vertices. These vertices can lead to topological mixing of knots leading to a quantum superposition of different knots for a given elementary particle. This mixing would be analogous to CKM mixing understood to result from the topological mixing of fermion genera in TGD framework. It could also imply that knotted particles decay rapidly to un-knots and make the un-knot the only long-lived state.

A naïve application of Uncertainty Principle suggests that the size scale of string determines the life time of particular knot configuration. The dependence on the length scale would however suggest that purely topological string theory cannot be in question. Zero energy ontology suggests that the size scale of the causal diamond assignable to elementary particle determines the time scale for the rates as secondary p-adic time scale: in the case of electron the time scale would be.1 seconds corresponding to Mersenne prime $M_{127} = 2^{127} - 1$ so that knotting and unknotting would be very slow processes. For electron the estimate for the scale of mass differences between different knotted states would be about $10^{-19} m_e$: electron mass is known for certain for 9 decimals so that there is no hope of detecting these mass differences. The pessimistic estimate generalizes to all other elementary particles: for weak bosons characterized by M_{89} the mass difference would be of order $10^{-13} m_W$.

(d) A natural guess is that p-adic thermodynamics can be applied to the knotting. In p-adic thermodynamics Boltzmann weights in are of form $p^{H/T}$ (p-adic number) and the allowed values of the Hamiltonian H are non-negative integer powers of p. Clearly, H representing a contribution to p-adic valued mass squared must be a non-negative integer valued invariant additive under connected sum. This guarantees extremely rapid convergence of the partition function and mass squared expectation value as the number of prime knots in the decomposition increases.

An example of an knot invariant (see `http://tinyurl.com/ya6pdykc`) [A9] additive under connected sum is knot genus (see `http://tinyurl.com/y8nfykh3`) [A8] defined as the minimal genus of 2-surface having the knot as boundary (Seifert surface). For trefoil and figure eight knot one has $g = 1$. For torus knot $(p, q) \equiv (q, p)$ one has $g = (p - 1)(q - 1)/2$. Genus vanishes for un-knot so that it gives the dominating contribution to the partition function but a vanishing contribution to the p-adic mass squared.

p-Adic mass scale could be assumed to correspond to the primary p-adic mass scale just as in the ordinary p-adic mass calculations. If the p-adic temperature is $T = 1$ in natural units (highest possible), and if one has $H = 2g$, the lowest order contribution corresponds to the value $H = 2$ of the knot Hamiltonian, and is obtained for trefoil and figure eight knot so that the lowest order contribution to the mass would indeed be about $10^{-19} m_e$ for electron. An equivalent interpretation is that $H = g$ and $T = 1/2$ as assumed for gauge bosons in p-adic mass calculations.

There is a slight technical complication involved. When the string has a non-trivial homology in $X^2_g \times S^1$ (it always has by construction), it does not allow Seifert surface in the ordinary sense. One can however modify the definition of Seifert surface so that it isolates knottedness from homology. One can express the string as connected sum of homologically non-trivial un-knot carrying all the homology and of homologically trivial knot carrying all knottedness and in accordance with the additivity of genus define the genus of the original knot as that for the homologically trivial knot.

(e) If the knots assigned with the elementary particles have large enough size, both connected sum and reconnection could take place for the knots associated with different elementary particles and make the many particle system a single connected structure. TGD based model for quantum biology is indeed based on this kind of picture. In this case the braid strands are magnetic flux tubes and connect bio-molecules to single coherent whole. Could electrons form this kind of stable connected structures in condensed matter systems? Could this relate to super-conductivity and Cooper pairs somehow? If one takes p-adic thermodynamics for knots seriously then knotted and braided magnetic flux tubes are more attractive alternative in this respect.

What if the thermalization of knot degrees of freedom does not take place? One can also consider the possibility that knotting contributes only to the vacuum conformal weight and thus to the mass squared but that no thermalization of ground states takes place. If the increment Δm of inertial mass squared associated with knotting is of from kgp^2, where k is positive integer and g the above described knot genus, one would have $\Delta m/m \simeq 1/p$. This is of order $M_{127}^{-1} \simeq 10^{-38}$ for electron.

Could the knotting and linking of elementary particles allow topological quantum computation at elementary particle level? The huge number of different knottings would give electron a huge ground state degeneracy making possible negentropic entanglement. For negentropic entanglement probabilities must belong to an algebraic extension of rationals: this would be the case in the intersection of p-adic and real worlds and there is a temptation to assign living matter to this intersection. Negentropy Maximization Principle could stabilize negentropic entanglement and therefore allow to circumvent the problems due to the fact that the energies involved are extremely tiny and far below thus thermal energy. In this situation bit would generalize to "nit" corresponding to N different ground states of particle differing by knotting.

A very naïve dimensional analysis using Uncertainty Principle would suggest that the number changes of electron state identifiable as quantum computation acting on q-nits is of order $1/\Delta t = \Delta m/hbar$. More concretely, the minimum duration of the quantum computation would be of order $\Delta t = \hbar/\Delta m$. Single quantum computation would take an immense amount time: for electron single operation would take time of order 10^{17} s, which is of the order of the recent age of the Universe. Therefore this quantum computation would be of rather limited practical value!

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