# Possible Role of p-Adic Numbers in Bio-Systems

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## Contents

**Abstract**

In this chapter p-adic physics, p-adic length scale hypothesis, and the special features of p-adic numbers are discussed from the point of view of biosystems. The identification of p-adic physics as physics of cognition tentatively identified as a cognitive simulation of real physics is the basic philosophical quide line. Second key idea is that living matter in very general sense lives in the intersection of real and p-adic worlds making among other things possible negentropic entanglement so that Negentropy Maximization Principle drives the formation of increasingly larger structures with negentropic entanglement.

The justification of the p-adic length scale hypothesis in zero energy ontology (ZEO) is discussed and the application of the hypothesis is discussed: both primary p-adic length scales and secondary p-adic length scales emerging naturally in zero energy ontology are discussed and it is found that the secondary p-adic scales assignable to elementary particles are in general macroscopic so that a connection between elementary particle physics and macroscopic physics suggests itself. Small-p p-adicity is also highly attractive idea and it is demonstrated that dark matter hierarchy characterized by hierarchy of Planck constants provides a first principle realization of this idea.

The characteristic features of p-adic physics are due to p-adic ultra-metricity, p-adic non-determinism, and to some exotic properties of p-adic probability and are expected to characterize also cognition. It is however too early to take too strong views concerning the interpretation of p-adics. Therefore also speculative ideas about the role of p-adic numbers in biology, which are only marginally consistent with the cognitive interpretation, are discussed in the sequel.

Also some speculations about possible role of so called exotic representations of super-conformal algebra are included. These speculations rely heavily on the assumption that canonical correspondence between p-adic and real masses holds true in full generality. The prediction is the existence of a hierarchy of p-adic states for which p-adic masses have extremely small real counterparts whereas the corresponding real states have super-astronomical masses. These strange states have huge degeneracies and the original speculation was that they are crucial for the understanding of biological life. Later however came the realization that the states of the super-symplectic representations associated with the light-like boundaries of massless extremals (MEs) have also gigantic almost-degeneracies. In particular, there is no need to assume the highly questionable p-adic–real correspondence at the level of masses for them. Therefore the cautious conclusion is that biology can do without the exotic super-conformal representations.

# 1 Introduction

In this chapter p-adic physics, p-adic length scale hypothesis, and the special features of p-adic numbers are discussed from the point of view of biosystems. The identification of p-adic physics as physics of cognition tentatively identified as a cognitive simulation of real physics is the basic philosophical quide line. Second key idea is that living matter in very general sense lives in the intersection of real and p-adic worlds making among other things possible negentropic entanglement so that Negentropy Maximization Principle drives the formation of increasingly larger structures with negentropic entanglement.

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The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L1].

# 2 General Vision About Fusion Of RealandP-Adic Physics

## 2.1 P-Adic Mass Calculations As Original Motivation For P-Adic Physics

The basic motivation for p-adic physics was provided by successful p-adic mass calculations based on p-adic thermodynamics which is thermodynamics for conformal weight to which p-adic mass squared is proportional. The p-adic mass squared is mapped to a real number by canonical identification.

p-adic thermodynamics is justified by the randomness of the motion of partonic 2-surfaces restricted only by light-likeness of the orbit. It is essential that the conformal symmetries associated with the light-like coordinates of parton and light-cone boundary are not gauge symmetries but dynamical symmetries.

In p-adic thermodynamics scaling generator $L_0$ having conformal weights as its eigen values replaces energy and Boltzmann weight $exp(H/T)$ is replaced by $p^{L_0/T_p}$. The quantization $T_p = 1/n$ of conformal temperature and thus quantization of mass squared scale is implied by number theoretical existence of Boltzmann weights. p-Adic length scale hypothesis states that primes $p \simeq 2^k$, $k$ integer. A stronger hypothesis is that $k$ is prime (in particular Mersenne prime or Gaussian Mersenne) makes the model very predictive and fine tuning is not possible.

The basic mystery number of elementary particle physics defined by the ratio of Planck mass and proton mass follows thus from number theory once $CP_2$ radius is fixed to about $10^4$ Planck lengths. Mass scale becomes additional discrete variable of particle physics so that there is not more need to force top quark and neutrinos with mass scales differing by 12 orders of magnitude to the same multiplet of gauge group. Electron, muon, and $\tau$ correspond to Mersenne prime $k = 127$ (the largest non-super-astrophysical Mersenne), and Mersenne primes $k = 113, 107$. Intermediate gauge bosons and photon correspond to Mersenne $M_{89}$, and graviton to $M_{127}$.

The value of $k$ for quark can depend on hadronic environment [K9] and this would produce precise mass formulas for low energy hadrons. This kind of dependence conforms also with the indications that neutrino mass scale depends on environment [C1]. Amazingly, the biologically most relevant length scale range between 10 nm and 4 $\mu$m contains four Gaussian Mersennes $(1+i)^n - 1$, $n = 151, 157, 163, 167$ and scaled copies of standard model physics in cell length scale could be an essential aspect of macroscopic quantum coherence prevailing in cell length scale.

## 2.2 Questions Raised By The Success Of P-Adic Thermodynamics

p-Adic mass calculations raise several technical questions which in turn help to imagine the interpretation of p-adic physics.

1. Is the canonical identification $I$: $\sum x_n p^n \rightarrow \sum x_n p^{-n}$- the only possible manner to map p-adic mass squared values to real numbers or can one consider also more general mappings? Can one require that p-adic mass calculations are equivalent with their real counterparts with the quantization for the counterpart of the p-adic temperature forced by this equivalence? This requires that a p-adic rational $m/n$ defined as a ratio of finite p-adic integers is mapped to a ratio $I(m)/I(n)$ of the images of these integers under the canonical identification rather than mapping the infinite p-adic power series of the rational to a real number. This would affect p-adic mass calculations but would have no dramatic effects in the case that the lowest contribution to mass squared is integer valued as it indeed is.

2. It is also possible to generalize canonical identification by expanding p-adic numbers in powers of $p^k$ with coefficients being non-negative integers $n < p^k$. This form of canonical identification applied to the numerator and denominator of rational $m/n$ to give $I_k(m)/I_k(n)$is especially suitable when the p-adic temperature is $T = 1/k$. Could one interpret the hierarchy of canonical identifications $I_k$ defined in this manner in terms of a measurement resolution for mass squared (IR cutoff) defined as the p-adic length scale corresponding to $p^k$? p-Adic integer points $n < p^k$ correspond indeed as such to real integers as also do the rationals formed from this kind of integers. Quite generally, for $T = 1/k$ the mass scale of particles is $p^{-k/2}$ and very small.

These questions inspire further questions.

1. Canonical correspondence between p-adics and reals and its possible generalizations apply to probabilities. Could similar correspondence relate also p-adic and real space-time sheets? Could symmetries allow to identify preferred coordinates of the imbedding space so that the general coordinate invariance would not be lost. Could it be enough for the generalized canonical identification to respect the fundamental space-time symmetries in the IR resolution identified in terms of the pinary cutoff defined by p-adic length scale associated with $p^k$?

2. If both real and p-adic space-time physics makes sense what is the correspondence between them? Is it via common rational points of imbedding space plus common algebraic points in preferred coordinates of the imbedding space. This correspondence would be extremely discontinuous and the intersection of the p-adic and real worlds would be discrete. Or should one apply canonical correspondence or some of its generalizations to the coordinates of the points in the preferred coordinate system forced by symmetries.

   Could real physics in finite length and time scale resolution allow an elegant description in terms of p-adic physics in the sense that the lack of the well-ordering of p-adic numbers would be allowed below the resolution scale? Could one apply identification $I_k$ applied to rational valued points in preferred coordinates so that one would have correspondence via common rationals below IR resolution scale and continuous map above this scale: this would mean a compromize between continuity requirement and space-time symmetries. These maps map arbitrarily distant common rational points of real and p-adic space-time sheets arbitrarily near to each other if their differ by a large power of $p$. Does this mean that canonical identification maps have interpretation in terms of holography?

3. What could be the interpretation of p-adic physics if it is a genuine part of physics at the space-time level? Could p-adic physics relate to cognition and intentionality, which are characteristics of living matter? If so, could living matter in some sense correspond to the intersection of p-adic and real worlds?

## 2.3 Zero Energy Ontology And P-Adic Length Scale Hypothesis

### 2.3.1 Zero energy ontology classically

In TGD inspired cosmology [K19] the imbeddings of Robertson-Walker cosmologies are vacuum extremals. Same applies to the imbeddings of Reissner-Nordström solution [K20] and in practice to all solutions of Einstein's equations imbeddable as extremals of Kähler action. Since four-momentum currents define a collection of vector fields rather than a tensor in TGD, both positive and negative signs for energy corresponding to two possible assignments of the arrow of the geometric time to a given space-time surface are possible. This leads to the view that all physical states have vanishing net energy classically and that physically acceptable universes are creatable from vacuum.

The result is highly desirable since one can avoid unpleasant questions such as "What are the net values of conserved quantities like rest mass, baryon number, lepton number, and electric charge for the entire universe?", "What were the initial conditions in the big bang?", "If only single solution of field equations is selected, isn't the notion of physical theory meaningless since in principle it is not possible to compare solutions of the theory?". This picture fits also nicely with the view that entire universe understood as quantum counterpart 4-D space-time is recreated in each quantum jump and allows to understand evolution as a process of continual re-creation.

### 2.3.2 Zero energy ontology at quantum level

Also the construction of $S$-matrix [K2] leads to the conclusion that all physical states possess vanishing conserved quantum numbers. Furthermore, the entanglement coefficients between positive and negative energy components of the state define an M-matrix which can be seen as a "complex" square root of density matrix decomposable to a square root of diagonal positive definite density matrix and a unitary $S$-matrix. $S$-matrix thus becomes a property of the zero energy state and physical states code by their structure what is usually identified as quantum dynamics. The square root of the density matrix means taking square root of thermodynamics which thus becomes genuine part of quantum theory with thermodynamical ensembles realized at single particle level rather than being a useful fiction of theoretician. Also the transitions between zero energy states are possible and described by $U$ matrix which would have natural identification as characterized of intentional action.

At space-time level this would mean that positive energy component and negative energy component are at a temporal distance characterized by an appropriate p-adic time scale and the integer characterizing the value of Planck constant for the state in question. The scale in question would also characterize the geometric duration of quantum jump and the size scale of space-time region contributing to the contents of conscious experience. The interpretation in terms of a mini bang followed by a mini crunch suggests itself also.

### 2.3.3 How do p-adic coupling constant evolution and p-adic length scale hypothesis emerge from zero energy ontology?

In zero energy ontology zero energy states have as imbedding space correlates causal diamonds for which the distance between the tips of the intersecting future and past directed light-cones comes as integer multiples of a fundamental time scale: $T_n = n \times T_0$. p-Adic length scale hypothesis allows to consider a stronger hypothesis $T_n = 2^n T_0$ and its generalization a slightly more general hypothesis $T_n = p^n T_0$, $p$ prime. It however seems that these scales are dynamically favored but that also other scales are possible.

Could the coupling constant evolution in powers of 2 implying time scale hierarchy $T_n = 2^n T_0$ induce p-adic coupling constant evolution and explain why p-adic length scales correspond to $L_p \propto \sqrt{p}R$, $p \simeq 2^k$, $R$ $CP_2$ length scale? This looks attractive but there is a problem. p-Adic length scales come as powers of $\sqrt{2}$ rather than 2 and the strongly favored values of $k$ are primes and thus odd so that $n = k/2$ would be half odd integer. This problem can be solved.

1. The observation that the distance traveled by a Brownian particle during time $t$ satisfies $r^2 = Dt$ suggests a solution to the problem. p-Adic thermodynamics applies because the partonic 3-surfaces $X^2$ are as 2-D dynamical systems random apart from light-likeness of their orbit. For $CP_2$ type vacuum extremals the situation reduces to that for a one-dimensional random light-like curve in $M^4$. The orbits of Brownian particle would now correspond to light-like geodesics $\gamma_3$ at $X^3$. The projection of $\gamma_3$ to a time=constant section $X^2 \subset X^3$ would define the 2-D path $\gamma_2$ of the Brownian particle. The $M^4$ distance $r$ between the end points of $\gamma_2$ would be given $r^2 = Dt$. The favored values of $t$ would correspond to $T_n = 2^n T_0$ (the full light-like geodesic). p-Adic length scales would result as $L^2(k) = DT(k) = D2^k T_0$ for $D = R^2/T_0$. Since only $CP_2$ scale is available as a fundamental scale, one would have $T_0 = R$ and $D = R$ and $L^2(k) = T(k)R$.

2. p-Adic primes near powers of 2 would be in preferred position. p-Adic time scale would not relate to the p-adic length scale via $T_p = L_p/c$ as assumed implicitly earlier but via $T_p = L_p^2/R_0 = \sqrt{p}L_p$, which corresponds to secondary p-adic length scale. For instance, in the case of electron with $p = M_{127}$ one would have $T_{127} = .1$ second which defines a fundamental biological rhythm. Neutrinos with mass around.1 eV would correspond to $L_e(169) \simeq 5$ $\mu$m (size of a small cell) and $T(169) \simeq 1. \times 10^4$ years. A deep connection between elementary particle physics and biology becomes highly suggestive.

3. In the proposed picture the p-adic prime $p \simeq 2^k$ would characterize the thermodynamics of the random motion of light-like geodesics of $X^3$ so that p-adic prime $p$ would indeed be an inherent property of $X^3$. For $T_p = pT_0$ the above argument is not enough for p-adic length scale hypothesis and p-adic length scale hypothesis might be seen as an outcome of a process analogous to natural selection. Resonance like effect favoring octaves of a fundamental frequency might be in question. In this case, $p$ would a property of CD and all light-like 3-surfaces inside it and also that corresponding sector of WCW .

## 2.4 How To Fuse P-Adic And Real Physics?

### 2.4.1 Generalization of number concept and fusion of real and p-adic physics

The unification of real physics of material work and p-adic physics of cognition and intentionality leads to the generalization of the notion of number field. Reals and various p-adic number fields are glued along their common rationals (and common algebraic numbers too) to form a fractal book like structure. Allowing all possible finite-dimensional extensions of p-adic numbers brings additional pages to this "Big Book".

This generalization leads to a generalization of the notion of manifold as a collection of a real manifold and its p-adic variants glued together along common rationals (see **Fig.** `http://tgdtheory.fi/appfigures/book.jpg` or bf Fig. **??** in the appendix of this book). The precise formulation involves of course several technical problems. For instance, should one glue along common algebraic numbers and Should one glue along common transcendentals such as $e^p$? Are algebraic extensions of p-adic number fields glued together along the algebraics too?

This notion of manifold implies a generalization of the notion of imbedding space. p-Adic transcendentals can be regarded as infinite numbers in the real sense and thus most points of the p-adic space-time sheets would be at infinite distance and real and p-adic space-time sheets would intersect in a discrete set consisting of rational points. This view in which cognition would be literally cosmic phenomena is in a sharp contrast with the often held belief that p-adic topology emerges below Planck length scale.

### 2.4.2 What number theoretical universality might mean?

Number theoretic universality has been one of the basic guide lines in the construction of quantum TGD. There are two forms of the principle.

1. The strong form of number theoretical universality states that physics for any system should effectively reduce to a physics in algebraic extension of rational numbers at the level of $M$-matrix so that an interpretation in both real and p-adic sense (allowing a suitable algebraic extension of p-adics) is possible. One can however worry whether this principle only means that physics is algebraic so that there would be no need to talk about real and p-adic physics at the level of $M$-matrix elements. It is not possible to get rid of real and p-adic numbers at the level of classical physics since calculus is a prerequisite for the basic variational principles used to formulate the theory. For this option the possibility of completion is what poses conditions on $M$-matrix.

2. The weak form of principle requires only that both real and p-adic variants of physics make sense and that the intersection of these physics consist of physics associated with various algebraic extensions of rational numbers. In this rational physics would be like rational numbers allowing infinite number of algebraic extensions and real numbers and p-adic number fields as its completions. Real and p-adic physics would be completions of rational physics. In this framework criticality with respect to phase transitions changing number field becomes a viable concept. This form of principle allows also purely p-adic phenomena such as p-adic pseudo non-determinism assigned to imagination and cognition. Genuinely p-adic physics does not however allow definition of notions like conserved quantities since the notion of definite integral is lacking and only the purely local form of real physics allows p-adic counterpart.

Experience has taught that it is better to avoid too strong statements and perhaps the weak form of the principle is enough. It is however clear that number theoretical criticality could provide important insights to quantum TGD. p-Adic thermodynamics is excellent example of this. In consciousness theory the transitions transforming intentions to actions and actions to cognitions would be key applications. Needless to say, zero energy ontology is absolutely essential: otherwise this kind of transitions would not make sense. In the original version of this chapter number theoretical universality was identified as number theoretical criticality and this leads to so strong conditions that they might not be possible to satisfy.

### 2.4.3 p-Adicization by algebraic continuation

The basic challenges of the p-adicization program are following.

1. The first problem -the conceptual one- is the identification of preferred coordinates in which functions are algebraic and for which algebraic values of coordinates are in preferred position.

This problem is encountered both at the level of space-time, imbedding space, and WCW . Here the group theoretical considerations play decisive role and the selection of preferred coordinates relates closely to the selection of quantization axes. This selection has direct physical correlates at the level of imbedding space and the hierarchy of Planck constants has interpretation as a correlate for the selection of quantization axes [K3].

Algebraization does not necessarily mean discretization at space-time level: for instance, the coordinates characterizing partonic 2-surface can be algebraic so that algebraic point of the WCW results and surface is not discretized. If this kind of function spaces are finite-dimensional, it is possible to fix $X^2$ completely data for a finite number of points only.

2. Local physics generalizes as such to p-adic context (field equations, etc...). The basic stumbling block of this program is integration already at space-time (Kähler action etc..). The problem becomes really horrible looking at WCW level (functional integral). Algebraic continuation could allow to circumvent this difficulty. Needless to say, the requirement that the continuation exists must pose immensely tight constraints on the physics. For instance, at WCW level radiative corrections to the functional integral should vanish and the resulting perturbation theory using propagators and vertices could make sense p-adically.

One general idea which results as an outcome of the generalized notion of number is the idea of a universal function continuable from a function mapping rationals to rationals or to a finite extension of rationals to a function in any number field. This algebraic continuation is analogous to the analytical continuation of a real analytic function to the complex plane.

1. Rational functions with rational coefficients are obviously functions satisfying this constraint. Algebraic functions with rational coefficients satisfy this requirement if appropriate finite-dimensional algebraic extensions of p-adic numbers are allowed. Exponent function is such a function.

2. For instance, residue calculus essential in the construction of N-point functions of conformal field theory might be generalized so that the value of an integral along the real axis could be calculated by continuing it instead of the complex plane to any number field via its values in the subset of rational numbers forming the rim of the book like structure having number fields as its pages. If the poles of the continued function in the finitely extended number field allow interpretation as real numbers it might be possible to generalize the residue formula. One can also imagine of extending residue calculus to any algebraic extension. An interesting situation arises when the poles correspond to extended p-adic rationals common to different pages of the “Big book”. Could this mean that the integral could be calculated at any page having the pole common. In particular, could a p-adic residue integral be calculated in the ordinary complex plane by utilizing the fact that in this case numerical approach makes sense.

3. Algebraic continuation is the basic tool of p-adicization program. Entire physics of the TGD Universe should be algebraically continuable to various number fields. Real number based physics would define the physics of matter and p-adic physics would describe correlates of cognition.

4. For instance, the idea that number theoretically critical partonic 2-surfaces are expressible in terms of rational functions with rational or algebraic coefficients so that also p-adic variants of these surfaces make sense, is very attractive.

5. Finite sums and products respect algebraic number property and the condition of finiteness is coded naturally by the notion of finite measurement resolution in terms of the notion of (number theoretic) braid. This simplifies dramatically the algebraic continuation since WCW reduces to a finite-dimensional space and the space of WCW spinor fields reduces to finite-dimensional function space.

The real WCW can well contain sectors for which p-adicization does not make sense. For instance, if the exponent of Kähler function and Kähler are not expressible in terms of algebraic functions with rational or at most algebraic functions or more general functions making sense p-adically, the continuation is not possible. p-Adic non-determinism in p-adic sectors makes also impossible the continuation to real sector. All this is consistent with vision about rational and algebraic physics as as analog of rational and algebraic numbers allowing completion to various continuous number fields.

Due to the fact that real and p-adic topologies are fundamentally different, ultraviolet and infrared cutoffs in the set of rationals are unavoidable notions and correspond to a hierarchy of different physical phases on one hand and different levels of cognition on the other hand. For instance, most points p-adic space-time sheets reside at infinity in real sense and p-adically infinitesimal is infinite in real sense. Two types of cutoffs are predictedp-adic length scale cutoff and a cutoff due to phase resolution related to the hierarchy of Planck constants. Zero energy ontology provides natural realization for the p-adic length scale cutoff. The latter cutoff seems to correspond naturally to the hierarchy of algebraic extensions of p-adic numbers and quantum phases $exp(i2\pi/n)$, $n \geq 3$, coming as roots of unity and defining extensions of rationals and p-adics allowing to define p-adically sensible trigonometric functions These phases relate closely to the hierarchy of quantum groups, braid groups, and $II_1$ factors of von Neumann algebra.

## 2.5 P-Adic Physics And Consciousness

The original vision was that p-adic physics is physics of cognition. It has however turned out that it is only physics of cognition and that the attempt to describe intentional action in terms of p-adic physics leads to mathematical difficulties and is also un-necessary. This view is also in nice concordance with the existing mathematics and adeles provide a natural approach to the unification of reals and p-adics.

### 2.5.1 p-Adic physics and cognition

p-Adic physics as physics of cognition provides one of the key elements of TGD inspired theory of consciousness. At the fundamental level light-like 3-surfaces are basic dynamical objects in TGD Universe and have interpretation as orbits of partonic 2-surfaces. The generalization of the notion of number concept by fusing real numbers and various p-adic numbers to a more general structure makes possible to assign to real parton a p-adic prime $p$ and corresponding p-adic partonic 3-surface obeying same algebraic equations. The almost topological QFT property of quantum TGD is an essential prerequisite for this. The intersection of real and p-adic 3-surfaces would consists of a discrete set of points with coordinates which are algebraic numbers. p-Adic partons would relate to both intentionality and cognition.

Real fermion and its p-adic counterpart forming a pair would represent matter and its cognitive representation being analogous to a fermion-hole pair resulting when fermion is kicked out from Dirac sea. The larger the number of points in the intersection of real and p-adic surfaces, the better the resolution of the cognitive representation would be. This would explain why cognitive representations in the real world are always discrete (discreteness of numerical calculations represent the basic example about this fundamental limitation).

All transcendental p-adic integers are infinite as real numbers and one can say that most points of p-adic space-time sheets are at spatial and temporal infinity in the real sense so that intentionality and cognition would be literally cosmic phenomena. If the intersection of real and p-adic space-time sheet contains large number of points, the continuity and smoothness of p-adic physics should directly reflect itself as long range correlations of real physics realized as p-adic fractality. It would be possible to measure the correlates of cognition and in the framework of zero energy ontology [K2] the success of p-adic mass calculations can be seen as a direct evidence for the role of intentionality and cognition even at elementary particle level: all matter would be basically created by intentional action as zero energy states.

### 2.5.2 Generalization of the notion of information

TGD inspired theory of consciousness, in particular the formulation of Negentropy Maximization Principle (NMP) in p-adic context, has forced to rethink the notion of the information concept. In TGD state preparation process is realized as a sequence of self measurements. Each self measurement means a decomposition of the sub-system involved to two unentangled parts. The decomposition is fixed highly uniquely from the requirement that the reduction of the entanglement entropy is maximal.

The additional assumption is that bound state entanglement is stable against self measurement. This assumption is somewhat ad hoc and it would be nice to get rid of it. The only manner to achieve this seems to be a generalized definition of entanglement entropy allowing to assign a negative value of entanglement entropy to the bound state entanglement, so that bound state entanglement would actually carry information, in fact conscious information (experience of understanding). This would be very natural since macro-temporal quantum coherence corresponds to a generation of bound state entanglement, and is indeed crucial for ability to have long lasting non-entropic mental images.

The generalization of the notion of number concept leads immediately to the basic problem. How to generalize the notion of entanglement entropy that it makes sense for a genuinely p-adic entanglement? What about the number-theoretically universal entanglement with entanglement probabilities, which correspond to finite extension of rational numbers? One can also ask whether the generalized notion of information could make sense at the level of the space-time as suggested by quantum-classical correspondence.

In the real context Shannon entropy is defined for an ensemble with probabilities $p_n$ as

$$S = -\sum_n p_n log(p_n) \ . \tag{2.1}$$

As far as theory of consciousness is considered, the basic problem is that Shannon entropy is always non-negative so that as such it does not define a genuine information measure. One could define information as a change of Shannon entropy and this definition is indeed attractive in the sense that quantum jump is the basic element of conscious experience and involves a change. One can however argue that the mere ability to transfer entropy to environment (say by aggressive behavior) is not all that is involved with conscious information, and even less so with the experience of understanding or moment of heureka. One should somehow generalize the Shannon entropy without losing the fundamental additivity property.

*p-Adic entropies*

The key observation is that in the p-adic context the logarithm function $log(x)$ appearing in the Shannon entropy is not defined if the argument of logarithm has p-adic norm different from 1. Situation changes if one uses an extension of p-adic numbers containing $log(p)$: the conjecture is that this extension is finite-dimensional. One might however argue that Shannon entropy should be well defined even without the extension.

p-Adic thermodynamics inspires a manner to achieve this. One can replace $log(x)$ with the logarithm $log_p(|x|_p)$ of the p-adic norm of $x$, where $log_p$ denotes p-based logarithm. This logarithm is integer valued ($log_p(p^n) = n$), and is interpreted as a p-adic integer. The resulting p-adic entropy

$$\begin{aligned} S_p &= \sum_n p_n k(p_n) \ , \\ k(p_n) &= -log_p(|p_n|) \ . \end{aligned} \tag{2.2}$$

is additive: that is the entropy for two non-interacting systems is the sum of the entropies of composites. Note that this definition differs from Shannon's entropy by the factor $log(p)$. This entropy vanishes identically in the case that the p-adic norms of the probabilities are equal to one. This means that it is possible to have non-entropic entanglement for this entropy.

One can consider a modification of $S_p$ using p-adic logarithm if the extension of the p-adic numbers contains $log(p)$. In this case the entropy is formally identical with the Shannon entropy:

$$S_p = -\sum_n p_n log(p_n) = -\sum_n p_n \left[-k(p_n)log(p) + p^{k_n} log(p_n/p^{k_n}\right] \ . \tag{2.3}$$

It seems that this entropy cannot vanish.

One must map the p-adic value entropy to a real number and here canonical identification can be used:

$$\begin{aligned} S_{p,R} &= (S_p)_R \times log(p)) \ , \\ (\sum_n x_n p^n)_R &= \sum_n x_n p^{-n} \ . \end{aligned} \tag{2.4}$$

The real counterpart of the p-adic entropy is non-negative.

*Number theoretic entropies and bound states*

In the case that the probabilities are rational or belong to a finite-dimensional extension of rationals, it is possible to regard them as real numbers or p-adic numbers in some extension of p-adic numbers for any $p$. The visions that rationals and their finite extensions correspond to islands of order in the seas of chaos of real and p-adic transcendentals suggests that states having entanglement coefficients in finite-dimensional extensions of rational numbers are somehow very special. This is indeed the case. The p-adic entropy entropy $S_p = -\sum_n p_n log_p(|p_n|) log(p)$ can be interpreted in this case as an ordinary rational number in an extension containing $log(p)$.

What makes this entropy so interesting is that it can have also negative values in which case the interpretation as an information measure is natural. In the real context one can fix the value of the value of the prime $p$ by requiring that $S_p$ is maximally negative, so that the information content of the ensemble could be defined as

$$I \equiv Max\{-S_p, \ p \ \text{prime}\} \ . \tag{2.5}$$

This information measure is positive when the entanglement probabilities belong to a finite-dimensional extension of rational numbers. Thus kind of entanglement is stable against NMP, and has a natural interpretation as bound state entanglement. The prediction would be that the bound states of real systems form a number theoretical hierarchy according to the prime $panddimension$ of algebraic extension characterizing the entanglement.

Number theoretically state function reduction and state preparation could be seen as information generating processes projecting the physical states from either real or p-adic sectors of the state space to their intersection. Later an argument that these processes have a purely number theoretical interpretation will be developed based on the generalized notion of unitarity allowing the $U$-matrix to have matrix elements between the sectors of the state space corresponding to different number fields.

### 2.5.3 Does living matter reside in the intersection of real and p-adic worlds?

Number theoretic entanglement entropies make sense only if entanglement probabilities are real or algebraic numbers in the extension of p-adic numbers considered. This is implied by number theoretic universality in the intersection of real and p-adic variants of the imbedding space which at QFT limit of TGD correspond to discrete points of partonic 2-surfaces carrying elementary particle numbers. Their motion along light-like 3-surfaces gives rise to number theoretic braids [K2].

At WCW level the intersection of real and p-adic worlds would correspond to a more abstract intersection with the counterpart of rationals identified as light-like 3-surfaces represented by rational functions with rational coefficients identifiable as common to real and p-adic worlds. State function reduction to the intersection of p-adic and real worlds would induce also the rationality (or algebraic number property) of the entanglement probabilities since they must make sense both p-adically and in the real sense. One might say that the enlightenment means living in both real and p-adic world simultaneously.

One manner to understand the special role of rationals and algebraics relies on the observation that rationals represent islands of order in the sea of chaos defined by reals since their pinary expansion is predictable and analogous to a periodic orbit of a dynamical system whereas for a generic real number there is no manner to predict the pinary expansion.

The phase transitions transforming p-adic space-time sheets to real ones could be understood as a tunnelling through the intersection of the p-adic and real worlds: here zero energy ontology is absolutely essential in order to avoid the problems caused by the impossibility to compare directly real and p-adic quantum numbers and by the non-existence of p-adic conserved charges caused by the lack of definite integral (field equations however make sense). This would provide one candidate for the formation of cognitive representation on one hand and for the transformation of intention to action on the other hand. Only living matter could carry negentropic entanglement and evolution would take place in the intersection of p-adic and real worlds. This has rather far reaching implications also for understanding the evolution of consciousness if one accepts Negentropy Maximization Principle as the basic variational principle of consciousness. These implications are discussed in [K16].

## 2.6 P-Adic Length Scale Hypothesis And Biosystems

In the following a brief summary about biologically relevant p-adic length scales is given.

### 2.6.1 p-Adic coupling constant evolution

Could the time scale hierarchy $T_n = 2^n T_0$ defining hierarchy of measurement resolutions in time variable induce p-adic coupling constant evolution and explain why p-adic length scales correspond to $L_p \propto \sqrt{p}R$, $p \simeq 2^k$, $R$ $CP_2$ length scale? This looks attractive but there is a problem. p-Adic length scales come as powers of $\sqrt{2}$ rather than 2 and the strongly favored values of $k$ are primes and thus odd so that $n = k/2$ would be half odd integer. This problem can be solved.

1. The observation that the distance traveled by a Brownian particle during time $t$ satisfies $r^2 = Dt$ suggests a solution to the problem. p-Adic thermodynamics applies because the partonic 3-surfaces $X^2$ are as 2-D dynamical systems random apart from light-likeness of their orbit. For $CP_2$ type vacuum extremals the situation reduces to that for a one-dimensional random light-like curve in $M^4$. The orbits of Brownian particle would now correspond to light-like geodesics $\gamma_3$ at $X^3$. The projection of $\gamma_3$ to a time=constant section $X^2 \subset X^3$ would define the 2-D path $\gamma_2$ of the Brownian particle. The $M^4$ distance $r$ between the end points of $\gamma_2$ would be given $r^2 = Dt$. The favored values of $t$ would correspond to $T_n = 2^n T_0$ (the full light-like geodesic). p-Adic length scales would result as $L^2(k) = DT(k) = D2^k T_0$ for $D = R^2/T_0$. Since only $CP_2$ scale is available as a fundamental scale, one would have $T_0 = R$ and $D = R$ and $L^2(k) = T(k)R$.

2. p-Adic primes near powers of 2 would be in preferred position. p-Adic time scale would not relate to the p-adic length scale via $T_p = L_p/c$ as assumed implicitly earlier but via $T_p = L_p^2/R_0 = \sqrt{p}L_p$, which corresponds to secondary p-adic length scale. For instance, in the case of electron with $p = M_{127}$ one would have $T_{127} = .1$ second which defines a fundamental biological rhythm. Neutrinos with mass around.1 eV would correspond to $L_e(169) \simeq 5$ $\mu$m (size of a small cell) and $T(169) \simeq 1. \times 10^4$ years. A deep connection between elementary particle physics and biology becomes highly suggestive.

3. In the proposed picture the p-adic prime $p \simeq 2^k$ would characterize the thermodynamics of the random motion of light-like geodesics of $X^3$ so that p-adic prime $p$ would indeed be an inherent property of $X^3$.

4. The fundamental role of 2-adicity suggests that the fundamental coupling constant evolution and p-adic mass calculations could be formulated also in terms of 2-adic thermodynamics. With a suitable definition of the canonical identification used to map 2-adic mass squared values to real numbers this is possible, and the differences between 2-adic and p-adic thermodynamics are extremely small for large values of for $p \simeq 2^k$. 2-adic temperature must be chosen to be $T_2 = 1/k$ whereas p-adic temperature is $T_p = 1$ for fermions. If the canonical identification is defined as

$$\sum_{n\geq 0} b_n 2^n \rightarrow \sum_{m\geq 1} 2^{-m+1} \sum_{(k-1)m\leq n<km} b_n 2^n \quad .$$

   It maps all 2-adic integers $n < 2^k$ to themselves and the predictions are essentially same as for p-adic thermodynamics. For large values of $p \simeq 2^k$ 2-adic real thermodynamics with

$T_R = 1/k$ gives essentially the same results as the 2-adic one in the lowest order so that the interpretation in terms of effective 2-adic/p-adic topology is possible.

### 2.6.2 Biologically relevant primary p-adic length scales

The identification of p-adic length scales above electron length scale involves a systematic error in all writings before 2004. This deserves some comments.

1. The wrong identification was $L(151) \simeq 10$ nm implying wrong identification of other scales above $L(127)$ since I have calculated them by scaling $L(151)$ by an appropriate power of two. What I have denoted by $L(151)$ is actually obtained by scaling the Compton length $L_e(127) = \hbar/m_e$ by $2^{(151-127)/2}$ and therefore electrons Compton scale if it would correspond to $k = 151$. Since the mass of electron from p-adic mass calculations is given by $m_e = \sqrt{5+X}\hbar/L(127)$, the correct identification of $L(151)$ would be

$$L(151) = 2^{(151-127)/2}L(127) = 2^{(151-127)/2}L_e(151)/\sqrt{5+X} = 10/\sqrt{5+X} \ \ nm \ \ , \ \ 0 \leq X \leq 1 \ \ .$$

   Here $X$ denotes the unknown second order contribution of form $X = n/M_{127}$, $n$ integer, to the electron mass, and in the first approximation one can take $X = 0$ - the approximation is excellent unless $n$ is very large. In the sequel I will try to use the shorthand $L_e(k) = \sqrt{5}L(k)$ but cannot guarantee that the subscript "$e$" is always present when needed: it is rather difficult to identify all places where the earlier erratic definition appears. I can only apologise for possible confusions.

2. This mistake has no fatal consequences for TGD inspired quantum biology. Its detection however provides a further support for the speculated central role of electron in living matter. Since the scales obtained by scaling the electron Compton scale seem to be important biologically (scaled up Comton scale $\sqrt{5}L(151)$ corresponds to cell membrane thickness), the conclusion is that electrons - or perhaps their Cooper pairs - play a fundamental role in living matter. The correct value of $L(151)$ is $L(151) = 4.5$ nm, which is slightly below the p-adic length scale $L_e(149) = 5$ nm assigned with the lipid layer of cell membrane.

3. I have also assigned to electron the time scale $T = .1$ seconds defining a fundamental biorhythm as a secondary p-adic time scale $T_2(127) = \sqrt{M_{127}}T(127)$. The correct assignment of $T = .1$ seconds is as the secondary Compton time $T_{2,e}(127) = \sqrt{M_{127}}T_e(127)$ of electron: secondary p-adic time scale is $T_2(127) = \sqrt{M_{127}}T(127)$ and corresponds to $T_{2,e}(127)/\sqrt{5} = .045$ seconds and to $f(127) = 22.4$ Hz.

**Table 1** lists the p-adic length scales $L_p$, $p$ near prime power of 2, which might be interesting as far as biosystems are considered.

Some overall scaling factor $r$ of order one is present in the definition of the length scale and it is interesting to look whether with a suitable choice of $r$ it is possible to identify p-adic length scales as biologically important length scales. The requirement that p-adically scaled up electron Compton scale $L_e(151) \simeq \sqrt{5}L_e(151)$ corresponds to the thickness of the cell membrane about $10^{-8}$ meters gives $r \simeq 1.2$.

The study of the table supports the idea that p-adic length scale hypothesis might have explanatory power in biology. What is remarkable is the frequent occurrence of twin length scales related by a factor 2 in the range of biologically interesting p-adic length scales: only 3 of 15 primes in the range do not belong to a twin pair! The fact that these length scales seem to correspond to biologically interesting length scales suggests that twins might be related to replication phenomenon and to the possible 2-adicity in biology: for a given twin pair the smaller length scale would define basic 2-adic length scale. In the following the scales denoted by $\hat{L}_e(k)$ are related by a factor $r = 1.2$ to the length scales $L_e(k)$ appearing in **Table 1**

1. $\hat{L}_e(137) \simeq 7.84E-11\ m$, $\hat{L}_e(139) \simeq 1.57E-10m$ form a twin pair. This length scales might be associated with atoms and small molecules.

| k | 127 | 131 | 137 | 139 | 149 |
|---|---|---|---|---|---|
| $L_e(k)/10^{-10}m$ | .025 | .1 | .8 | 1.6 | 50 |
| k | 151 | 157 | 163 | 167 | 169 |
| $L_e(k)/10^{-8}m$ | 1 | 8 | 64 | 256 | 512 |
| k | 173 | 179 | 181 | 191 | 193 |
| $L_e(k)/10^{-4}m$ | .2 | 1.6 | 3.2 | 100 | 200 |
| k | 197 | 199 | 211 | 223 | 227 |
| $L_e(k)/m$ | .08 | .16 | 10 | 640 | 2560 |

**Table 1:** Primary p-adic length scales $L_e(k) = \sqrt{5}L_(k) = 2^{k-151}L_e(151)$, $p \simeq 2^k$, $k$ prime, possibly relevant to biophysics. The last 3 scales are included in order to show that twin pairs are very frequent in the biologically interesting range of length scales. The length scale $L_e(151)$ is take to be thickness of cell scale, which is $10^{-8}$ meters in good approximation.

2. The secondary scale $\hat{L}_e(71,2) \simeq .44$ nm corresponds to the thickness of the DNA strand which is about.5 nm. Both DNA strand and double helix must correspond to this length scale. The secondary p-adic length scale $L_e(\hat{73},2) \simeq 1.77\ nm$ is longer than the thickness of DNA double strand which is roughly 1.1 nm. Whether one could interpret this length scale as that associated with DNA double strand remains an open question. alpha helix, the basic building block of proteins provides evidence for has radius $1.81\ nm \sim \hat{L}_e(139)$ and the height of single step in the helix is $.544\ nm$.

3. $\hat{L}_e(149) \simeq .5.0\ nm$ and $\hat{L}_e(151) \simeq 10.0\ nm$ form also a twin pair. The thickness of cell membrane of order $10^{-8}\ m \sim \hat{L}_e(151)$. Cell membrane consists of two separate membranes and the thickness of single membrane therefore corresponds to $\hat{L}_e(149)$. Microtubules, which are basic structural units of the cytoskeleton, are hollow cylindrical surfaces having thickness $d \sim 11\ nm$, which is not too far from the length scale $\hat{L}_e(151)$. It has been suggested that microtubules might play key role in the understanding of biosystem as macroscopic quantum system [J5, J1].

4. If neutrinos have masses of order one $eV$ as suggested by recent experiments then the primary condensation level of neutrinos could correspond to $k_Z = 167$ or $k_Z = 13^2 = 169$ and would be the level at which nuclei feed their $Z^0$ gauge charges. This level is many particle quantum system in p-adic sense and p-adic effects are expected to important at this condensation level. Chirality selection should take place via the breaking of neutrino superconductivity at this level and involve the generation of $Z^0$ magnetic fields at some level $k < k_Z$, too. $k = 151$ is a good candidate for the level in question.

5. In the previous version of this chapter it was stated that $\hat{L}_e(167) = 2.73\ \mu m$, $\hat{L}_e(169 = 13^2) = 5.49\ \mu m$ form a twin pair and correspond to typical length scales associated with cellular structures. Neutrino mass calculations give best predictions for $k = 169$ and this suggests that the generalization of "$k$ = prime" to "$k$ = power of prime" should be considered: generalization would allow also $k = 169$ as basic length scale. Also blackhole elementary particle analogy suggests the generalization of the length scale hypothesis. Furthermore, only $k = 169$ would appear as a new length scale between electron length scale and astrophysical length scales ($k = 3^5, 2^8, 17^2$)! This suggests that the length scales $L_e(167)$ and $L_e(169)$ might form effective twin pair. That this could be the case is suggested by the fact that so called epithelial sheets appearing in skin, glands, etc., consisting of two layers of cells play in biosystems same role as cell membranes and are generally regarded as a step of bioevolution analogous to the formation of cell membrane.

6. $\hat{L}_e(173) = 2.20 \cdot 10^{-5}\ m$ might correspond to a size of some basic cellular structure (A structure consisting of 64 cell layers?). $\hat{L}_e(179) = 1.75 \cdot 10^{-5}\ m$ and $\hat{L}_e(181) = 3.52 \cdot 10^{-4}\ m$ form a twin pair. Later it will be found that the pair $k = 179, 181$ might correspond to basic structures associated with cortex.

7. Length scales $\hat{L}_e(191) = 1.12\ cm, \hat{L}_e(193) = 2.24\ cm$ and $\hat{L}_e(197) = 9.0\ cm$. $\hat{L}_e(199) = 18.0\ cm$ are again twins.

### 2.6.3 Secondary p-adic time scales and biology

The basic implication of zero energy ontology is the formula $T_2(k) = T(k) \simeq 2^{k/2}L_e(k)/c = L_e(2,k)/c$ for the secondary p-adic time scale for $p \simeq 2^k$. This would be the analog of $E = hf$ in quantum mechanics and together hierarchy of Planck constants would imply a direct connection between elementary particle physics and macroscopic physics. Especially important this connection would be in macroscopic quantum systems, say for Bose Einstein condensates of Cooper pairs, whose signature the rhythms with $T(k)$ as period would be. The presence of this kind of rhythms might even allow to deduce the existence of Bose-Einstein condensates of hitherto unknown particles.

Unfortunately, the mistake in the identification of the p-adic length scales above electron scale forces to modify the definition of $T(k)$ by introducing a $\sqrt{5+X}$ factor so that it becomes the secondary Compton time scale of electron in the p-adic length scale considered. Writing this explicitly, one has $T_e(k) \equiv T_{2,e}(k) = 2^{k-127}T_{2,e}(127) \equiv 2^{k-127}T_e(127)$. Apologies for a loose notation replacing subscript "2, e" with "e".

1. For electron secondary Compton time equal to $T_e(k) = .1$ seconds defines the fundamental $f_e = 10$ Hz bio-rhythm appearing as a peak frequency in alpha band. This could be seen as a direct evidence for a Bose-Einstein condensate of Cooper pairs of high $T_c$ super-conductivity. That transition to "creative" states of mind involving transition to resonance in alpha band might be seen as evidence for formation of large BE condensates of electron Cooper pairs.

2. TGD based model for atomic nucleus [K10] predicts that nucleons are connected by flux tubes having at their ends light quarks and anti-quarks with masses not too far from electron mass. The corresponding p-adic frequencies $f_q = 2^k f_e$ could serve as a biological signature of exotic quarks connecting nucleons to nuclear strings. $k_q = 118$ suggested by nuclear string model would give $f_q = 2^{18}f_e = 26.2$ Hz. Schumann resonances are around 7.8, 14.3, 20.8, 27.3 and 33.8 Hz and $f_q$ is not too far from 27.3 Hz Schumann resonance and the cyclotron frequency $f_c(^{11}B^+) = 27.3$ Hz for $B = .2$ Gauss explaining the effects of ELF em fields on vertebrate brain.

3. For a given $T_e(k)$ the harmonics of the fundamental frequency $f = 1/T(k)$ are predicted as special time scales. Also resonance like phenomena might present. In the case of cyclotron frequencies they would favor values of magnetic field for which the resonance condition is achieved. The magnetic field which in case of electron gives cyclotron frequency equal to 10 Hz is $B_e \simeq 3.03$ nT. For ion with charge $Z$ and mass number $A$ the magnetic field would be $B_I = \frac{A}{Z}(m_p/m_e)B_e$. The $B = .2$ Gauss magnetic field explaining the findings about effects of ELF em fields on vertebrate brain is near to $B_I$ for ions with $f_c$ alpha band. Hence the value of $B$ could be understood in terms of resonance with electronic B-E condensate.

4. The hierarchy of Planck constants predicts additional time scales $T_e(k)$. The prediction depends on the strength of the additional assumptions made. One could have scales of form $nT(k)$. Integers $n$ could correspond to ruler and compass integers expressible as products of first powers of Fermat primes and power of 2. There are only four known Fermat primes so that one has $n = 2^n \prod_i F_i$, $F_i \in \{3, 5, 17, 257, 2^{16}+1\}$. In the first approximation only 3- and 5- and 17-multiples of 2-adic length scales would result besides 2-adic length scales.

5. Mersenne primes are expected to define the most important fundamental p-adic time scales. The list of real and Gaussian (complex) Mersennes $M_n$ possibly relevant for biology is given by n=89, 107, 113*, 127, 151*, 157*, 163*, 167* ("*" tells that Gaussian Mersenne is in question).

$$\begin{array}{lllll} n & 89 & 107 & 113 & 127 \\ f_e/Hz & 2.7\times 10^{12} & 1.0\times 10^{7} & 1.6\times 10^{5} & 10 \\ n & 151 & 157 & 163 & 167 \\ T & 19.4\ d & 3.40\ y & 218.0\ y & 3.49\times 10^{3}\ y \end{array} \quad (2.6)$$

# 3 P-Adic Ultra-Metricy And Biosystems

Ultra-metricity is what distinguishes p-adic notion of distance and topology from the real one and makes the latter coarser than the real topology.

## 3.1 Spin Glasses And Ultra-Metricity

Spin glasses [B2, B6, B1] are spin systems with the property that the couplings $J_{kl}$ between neighboring spins $\sigma_k$ and $\sigma_l$ are random variables although the characteristic scale of time variation of $J_{kl}$ is very long as compared to the corresponding time scale associated with the dynamics of the spins. The characteristic property of spin glasses is their infinite ground state degeneracy. More precisely, the dynamics of the spin glasses is non-ergodic and there is infinite number of pure states, which correspond to the local minima of free energy. For the purposes of comparison it should be recalled that for ferromagnet above critical temperature only one pure state exists and below the critical temperature there are two pure states corresponding to two possible directions of magnetization.

The space of pure states possesses a very general property called ultra-metricity, which means that one can define in this space distance function $d(x,y)$ with the property

$$d(x,z) \leq max\{d(x,y),d(y,z)\} . \tag{3.1}$$

The properties of the distance function make it possible to decompose the space into a union of disjoint sets using the criterion that $x$ and $y$ belong to same class if the distance between $x$ and $y$ satisfies the condition

$$d(x,y) \leq D . \tag{3.2}$$

This division of the metric space into classes has following properties:

1. Distances between the members of two different classes $X$ and $Y$ do not depend on the choice of points $x$ and $y$ inside classes. One can therefore speak about distance function between classes.

2. Distances of points $x$ and $y$ inside single class are smaller than distances between different classes.

3. Classes form a hierarchical tree.

These properties of ultra-metric spaces suggest several biological applications.

1. Parisi [B2] has suggested that ultra-metricity might be used in taxonomy. Individuals of various species correspond to points of the ultra-metric space and ultra-metric distance gives mathematical description for the classification criterion: in practice ultra-metric distance might correspond to some genetic measure for the difference between individuals.

2. The representation of biological information seems to take place using a hierarchy of categories. Lowest and most important categories are very rough (friend/ enemy?, black/white?, etc... ). Higher levels correspond more refined classifications (what kind of enemy?, does enemy move or not?, ....). This kind of representation has obvious value in the struggle for survival. The hypothesis that biosystems save information into variables, which define points of ultra-metric space, leads automatically to a hierarchical structure of information storage. The simplest model assumes that states of brain or at least memories correspond to free energy minima of a spin glass like statistical system [B6].

3. Statistical models of memory and learning process, which share with spin glasses the property that the minima of free energy form ultra-metric space are proposed [B6], the main idea being that memories correspond to the minima of free energy. Learning takes place in these models via a slow change (slow as compared to the time scale of the dynamics associated with the spin variables) of the field $J_{kl}$ associated with bond connecting $k$: th and $l$: th cell.

## 3.2 P-Adic Ultra-Metricity

p-Adic numbers [A4] are a natural candidate for a basic tool in the description of higher dimensional critical systems since the distance function defined by p-adic norm is ultra-metric. The verification of the ultra-metricity is elementary task using the definition of the p-adic norm [A4]

$$|x|_p = |\sum_{k\geq k_0} x_k p^k| = p^{-k_0} \quad . \tag{3.3}$$

Since p-adic norm possesses discrete set of values, the values of the parameter $D$ in the classification criterion $|x-y|_p \leq D$ can be chosen to be belong to the set $\{D_k = p^{-k}\}$, $k$ integer. p-Adic numbers belonging to same class have same $k$: th pinary digit p-adic cutoff

$$\begin{aligned} x &= x_0 + x_1 \ , \\ x_0 &= \sum_{k_1<m<k} x_m p^m \ , \\ x_1 &= \sum_{m\geq k} x_m p^m \ , \end{aligned} \tag{3.4}$$

so that the set of classes corresponds to p-adic numbers with cutoff in $k$: th pinary digit. In this picture the p-adic power expansion of any p-adic observable defines a tree. The levels of the tree correspond to various pinary digits of p-adic number and each branching point gives rise to $p$ branches. In p-adic case one can regard the root of the either as the highest cutoff pinary digit or lowest non-vanishing pinary digit of the p-adic number. In the first case, the tree is infinite: in the latter case the tree is always finite.

For $x, y$ with same p-adic norm (same class) and $z$ with different p-adic norm as $x, y$ (different class) the distance function satisfies the condition

$$\begin{aligned} d_p(x,y) &\leq d_p(x,z) \;\; p>2 \ , \\ d_2(x,y) &< d_2(x,z) \ , \end{aligned} \tag{3.5}$$

so that there is important difference between $p = 2$ and $p > 2$ cases. Ultra-metricity (or non-Archimedean property as it is called in p-adic context) holds also true for the algebraic extensions of p-adic numbers with distance defined by the canonical norm [A4].

It has become clear that p-adicity emerges in TGD at the level of space-time topology and that one can identify p-adic space-time regions as cognitive representations of matter regions. Thus p-adic dynamics is predicted to be the dynamics of cognition and thus p-adic ultra-metricity, the exotic features of p-adic probability concept, and non-determinism of p-adic differential equations are predicted to characterize the physics of cognition.

There is however also a second manner how p-adic ultra-metricity might emerge in the description of biological systems. TGD Universe is quantum critical and critical systems [B5] are characterized typically by a large degeneracy of metastable states and resemble in this respect spin glasses. The vacuum degeneracy of the Kähler action defining the Kähler function in the WCW is highly analogous to the ground state degeneracy of the spin glasses in [K6, K1]. One cannot therefore exclude the possibility that p-adicity, in particular, small-p p-adicity for which there is also evidence, could emerge at the level of energy landscape of spin glass. According to the arguments of quantum TGD the reduced WCW $CH_{red}$, consisting of the maxima of Kähler function as a function of zero modes characterizing the shape, size and induced Kähler fields on 3-surface, can be regarded as a spin glass energy landscape. Hence one can define ultra-metric distance function, and it is possible that this distance function could be regarded as being induced from p-adic norm.

## 3.3 P-Adic Ultra-Metricity And Information Processing In Biosystems

This picture suggest a general model for the information processing in biosystems. Observations made by biosystem correlate with the variables characterizing the possibly conscious knowledge of the system about itself. The p-adic expansions of these variables give intrinsically hierarchical coding of the information associated with the observation. The lowest pinary digit is the most significant pinary digit and gives the roughest description for the observation. Higher pinary digits add details to the observation. The number of pinary digits in the p-adic representation of the observation measures the amount of information associated with it. The time order in which the information is stored or retrieved is from lowest to highest pinary digit. The Slaving Hierarchy associated with the topological condensation might have counterpart at the level of biosystems: this would mean the existence of a hierarchy $p_1 < p_2 < ..p_n < ..$ of p-adic dynamics each with its own characteristic time scale satisfying $T_1 < T_2 < .... < T_n < ...$ and the relationship between two consecutive dynamics is that of master and slave. The higher the level $p_n$ in the p-adic Slaving Hierarchy the higher is the intelligence associated with that level as measured in the number of possible conceptual categories.

A highly nontrivial prediction is that the number of conceptual categories is same at all levels and equal to prime $p$. This means that $p = 2$ case provides most primitive (but much used!) classification of type black/white. A well known mystery of cognitive science is the so called $7 \pm 2$ rule [B4]: human mind tends to classify observations into $p = 7$ categories and the classification using more categories than this is difficult. One possibility to test applicability of the p-adic ideas to biosystems is to check whether bio-systems obey small-$p$ p-adic rather than ordinary statistics. The nondeterminism and fractality of biosystems might be in better accordance with p-adic rather than ordinary statistics.

The analogy with spin glass models of learning suggests a microscopic TGD inspired physical model for learning. Short term learning is believed to correspond to slow changes in synaptic connections between neighboring cells. Long term learning probably involves the formation of new contacts between neighoring cells and according to suggestion of [K7] topological storage of information. In TGD inspired model for brain it was suggested that cells correspond to "topological field quanta", 3-surfaces possessing outer boundary and having size of cell One mechanism for the formation of macroscopic quantum systems is as a formation of bonds connecting boundaries of neighbouring "topological field quanta" (now 3-surfaces associated with cells). A possible identification for the counterpart of the spin glass coupling parameter $J_{kl}$ is as Kähler electric interaction energy between neighbouring topological field quanta associated with this kind of bond. Therefore it would be the Kähler electric fluxes through the bonds, which change primarily in the short term learning. This change can be partially nondeterministic process since p-adic dynamics allows partial non-determinism and this nondeterminism is related to freedom to choose the low pinary digits of the dynamical variables arbitrarily.

# 4 P-Adic Non-Determinism And Biosystems

The non-determinism of quantum jump, the classical non-determinism associated with the maximization of Kähler action, and p-adic non-determinism form a trinity of independent non-determinisms. Classical non-determinism of Kähler action can be assigned with volition whereas p-adic non-determinism is naturally the geometric correlate of imagination.

## 4.1 Could P-Adic Differential Equations Simulate Quantum Jump Sequence?

In practical applications one must idealize the biosystem with a system of differential or partial differential equations. Since cognition is basic aspect of living systems one might expect that the general properties of p-adic differential equations might be useful for modelling not only cognition but also the behavior of living matter.

1. The non-determinism associated with p-adic differential and partial differential equations is due to the presence of arbitrary functions depending on finite number of pinary digits of p-adic coordinates, which are in the role of the integration constants. p-Adic integration constants are actual constants below some p-adic time scale. Solution of field equations typically consists of regions which are deterministic in the ordinary sense of the world glued to each other. Various conserved quantities are pseudo constants. This means that p-adic

reality is somewhat like the reality of dreams consisting of fragments which could be realized also in everyday reality.

2. p-Adic space-time sheets could provide a simulation for the time development occurring via quantum jumps. p-Adic space-time surface would consist of fragments for which p-adic integration constants are ordinary constants. These pieces would represent the conscious information obtained about various real space-time surfaces in the sequence of quantum jumps (the space-time surfaces appearing in the quantum superpositions defined by the final states of quantum jumps are macroscopically equivalent). The lack of well-orderedness of the p-adic topology could reflect the fact that the arrow of time associated with $t$ is only statistical.

3. p-Adic realization of the Slaving Hierarchy [B3] roughly means that there is a hierarchy $.. < p_1 < p_2 < ...$ of p-adic dynamics and that the integration constants at level $p_1$ (slave) obey some dynamic equations at some higher p-adic level $p_2 > p_1$ (master) and are actual constants below length scale $L_{p_2} > L_{p_1}$ for each pair in the sequence. This hierarchy of dynamics need not be completely deterministic. If Kähler action allows non-unique classical histories, the p-adic integration constants can be chosen to some degree freely at each level of the Slaving Hierarchy. The free choice of p-adic integration constants has interpretation as a plan of an intelligent system for its future behavior. At p- adic length and time scales (macroscopic!) it is possible to "break physical laws": Universe learns engineering skills and begins to plan its own future!

4. The real counterparts for the solutions of p-adic differential equations have characteristic large jumps followed by small scale zig-zag type behavior. This zig-zag behavior is observed also for analytic solutions containing only ordinary integration constants, say for $x = At^2, y = Bt$ at values $t = 2^n$. Since p-adic integration constants are actual constants only for time scales smaller than $\Delta t = 2^{-n}$, the nondeterminism appears also as sudden jumps concentrated at multiples of $\Delta t$: $\Delta t$ defines clearly a natural unit of time and therefore biological clock. In [B4] the generality of this zig-zag motion in all length scales was emphasized as one of the characteristics of biosystems and the sudden jumps were identified as jumps from strange attractor to another and small scale motion as motion along attractor. A good example of this kind of motion is the motion of eye [B4]. The fractal property of solutions of p-adic differential equations implies an infinite number of time scales corresponding to $\Delta_m = p^{-m}$, $m \leq n$. This in turn implies characteristic $1/f$ spectrum for the Fourier transform of orbit, which is quite general feature of biosystems [B4].

5. An important property of p-adic differential equations suggested by the iteration of simpls p-adic maps (say $Z \rightarrow Z^2$ ) in algebraic extensions of $R_p$ is that critical orbits form a set ($|Z| = 1$), which possesses same dimension as the WCW so that critical metastable orbits are therefore not rare occurrences like in ordinary dynamics based on real topology. The small scale zig-zag motion between large jumps in the motion described by p-adic differential equations could correspond to motion near metastable orbit and be analogous to the motion along strange attractor in the strange attractor model of information processing proposed in [B4].

## 4.2 Information Filtering And P-Adics

Intelligent systems are extremely effective information filters. Only an extremely small amount of information is absorbed from the incoming information. As far as visual observations are considered it is the angles and boundaries, which receive most attention [B4]. An interesting possibility is that intelligent system concentrates its attention to p-adic super-conformal invariants such as angles. This would apply quite generally: any observation correspond to an orbit in some internal WCW simulating the observed system. The correspondence between the observation and simulation is determined only modulo p-adic super-conformal transformations of the configuration space.

One can even consider a simple model for the coupling between internal WCW and outer world using "Newton's equations" assuming that acceleration corresponds to the sensory experience:

$$\frac{d^2x^k}{dt^2} = F^k(t) - k\frac{dx^k}{dt} \quad . \tag{4.1}$$

$F^k(t)$ describes observation as an external force acting on system. In the absence of $F^k(t)$ motion is linear (no angles!) and only when $F^k(t)$ is non-vanishing the direction of motion changes direction (angle). The friction term guarantees that constant stimulus leads to no sensory experience situation (adaptation). Idealized case corresponds to delta pulses causing zig-zag type motion. Note that p-adic indeterminacy brings in certain degree of "subjectivity" and could provide a phenomenological model for the quantum nondeterminacy.

This kind of model could serve as a model of language analogous to that considered in [B4]. Individual phonemes correspond to linear part on the orbit in some WCW and the duration of phoneme doesn't matter. The change of phoneme to another one corresponds to angle on the orbit (external force) and different angles correspond to different phoneme pairs. The hierarchy of structures (phonemes, hyphens, worlds, sentences, ..) might correspond to p-adic slaving hierarchy (say, $p = 2, 3, 7, 127..$) associated with the differential equations governing the orbit in internal WCW .

# 5 P-Adic Probabilities And Biosystems

p-Adic probabilities can be defined in a manner analogous to that used to define ordinary probabilities [A3]. One can consider sufficiently large number of observations $N$ chosen by some criterion since conditional probabilities are considered in practice and observe possible mutually exclusive outcomes $N_i$ labeled by integer $i$. The relative frequencies $N_i/N$ are estimates for p-adic probabilities. Probability conservation corresponds to the condition $\sum_i N_i = N$. The feature, which differentiates between ordinary and p-adic probabilities is related to the large $N$ limit, which must exist in p-adic rather than ordinary sense. This means that the values of $N$, which differ by large powers of $p$ are p-adically near to each other. For example, $N$ and $N + 1$ are in general not near each other p-adically! For large values of $p$ say $p = M_{127} \simeq 10^{38}$ the value of $N$ rarely exceeds the critical value $p$ and there is no practical difference between p-adic and ordinary probabilities. For small values of $p$ the situation changes.

## 5.1 Does P-Adic Probability Apply Only To Cognition?

p-Adic probability concept is expected to apply in quantum statistical models of cognition. If p-adic space-time sheets indeed model sequences of quantum jumps by replacing consciously observed pieces of real space-time appearing in the sequence of quantum jumps by finite space-time regions glued to each other in p-adically continuous manner, then p-adic statistics might apply as a model of self-organization resulting from a dissipative time development by quantum jumps.

p-Adic probabilities might be natural in the statistical description of fractal structures resulting in the self-organization, and which by definition can contain same structural detail with all possible sizes.

1. Consider counting of conformally invariant structural details of a p-adic fractal. A simple biologically interesting example is the solution curve of p-adic differential equations in some configuration space associated with biosystem (say the space of average chemical concentrations). The angles associated with the kinks of the curve measured with some finite precision are the structural details in question.

2. One can count how many times $i$: th structural detail appears in a finite region of the fractal structure: although this number is infinite as real number it might possess (and probably does so!) finite norm as p-adic number and provides a useful p-adic invariant of the fractal. One can calculate also the total number of structural details defined as $N = \sum_i N_i$ and also define p-adic probability for the appearance of $i$: th structural detail as relative frequency $p_i = N_i/N$. The real, "renormalized" counterparts of $N_i$ and $P_i$ obtained via the canonical correspondence define real valued invariants of the fractal structure.

3. The evaluation of the p-adic probabilities of occurrence can be done by evaluating the required numbers $N_i$ and $N$ in a given resolution. Better estimate is obtained by increasing resolution and counting the numbers of the hitherto unobserved structural details. The increase in the resolution greatly increases the number of observations in case of p-adic fractal and the fluctuations in the values of $N_i$ and $N$ increase with resolution so that $N_i/N$ has no well defined limit as real number although one can define the probabilities of occurrence as resolution dependence concept. In p-adic sense the increase in the values of $N_i$ and fluctuations is small and the procedure should converge rapidly so that reliable estimates should result with quite a reasonable resolution.

## 5.2 Is Small-P P-Adic Statistics Possible?

There is a distinct possibility that p-adic statistics with small $p$ might be a unique testable signature of intelligent systems! The replication property of biosystems suggests that the lowest level in topological condensate of biosystem has $p = 2$. The quantization of the number of observations in biological experiment could be understood in the following manner. A natural choice for $N$ in biosystem corresponds to all individuals that have existed or exist in the biosystem during some time interval. For an ideally replicating biosystem this number develops during time in the following manner. $N = 1$ for zeroth generation, $N = 1 + 2 = 3$ for the second generation, $N = 1 + 2 + ....2^k = 2^{k+1} - 1$ for $k + 1$: th generation. The expression for the relative frequency is

$$P = \frac{\sum_k N_k}{\sum_k 2^k} . \tag{5.1}$$

The dominating contribution to p-adic probability comes from the lowest generations. For p-adic probability to make sense the behavior of the system must be sufficiently deterministic during the earliest stages of the development. Non-determinism becomes possible for large of $N$. The development of the embryo during the first cell divisions is indeed highly deterministic process.

An interesting feature of the ideally replicating biosystem is that $N = 2^{k+1} - 1$ is Mersenne prime for certain values of the generation number $k + 1$. If the topological condensate associated with biosystem contains also higher levels $p$ then these values of $N$ might mean the emergence of something new since the value of $N$ exceeds the critical value $p = M_{k+1}$, when the number generations becomes $k + 1$ and p-adic probability concept begins to apply at p: th level. This suggests that the values of the total cell number $N_{cell} = 2^{k-1}$ associated with the Mersenne primes $M_k$ are critical cell numbers. Some of the lowest critical generation numbers are $k = 2$: $N_{cell} = 2$, $k = 3$: $N_{cell} = 4$, $k = 7$: $N_{cell} = 64$, ...

## 5.3 The Concept Of Monitoring

In p-adic quantum theory expected to provide a model for cognition one must somehow associate real probabilities to p-adic probabilities. This problem has been already discussed and leads to the conclusion that the transition probabilities of p-adic quantum system depend on how it is monitored. p-Adic sum of transition probabilities corresponds to the experimental situation, when one does not monitor individual transitions but using some common experimental signature only looks whether the transition leads to this set of finals states or not. When one looks each transition separately or effectively performs different experiment by considering only one transition channel in each experiment one must use the sum of real probabilities. More precisely, the choice of experimental signatures divides the set $U$ of the final states to disjoint union $U = \cup_i U_i$ and one must define the real counterparts for transition probabilities $P_{iU_k}$ as

$$\begin{aligned} P_{iU_k} &= \sum_{j \in U_k} P_{ij} , \\ P_{iU_k} &\rightarrow (P_{iU_k})_R , \\ (P_{iU_k})_R &\rightarrow \frac{(P_{iU_k})_R}{\sum_l (P_{iU_l})_R} \equiv P^R_{iU_k} . \end{aligned} \tag{5.2}$$

Similar resolution can be defined also for initial states by decomposing them into a union disjoint subsets. The assumption means deep difference with respect to the ordinary probability theory.

p-Adic probability conservation implies that the lowest order terms for p-adic probabilities satisfy the condition $\sum_j P^0_{ij} = 1 + O(p)$. The general solution to the condition is $P^0_{ij} = n_{ij}$. If the number of the final states is much smaller than $p$ this alternative implies that real transition rates are enormous: typically of order $p$! Therefore it seems that one must assume

$$P^0_{ij} \;=\; \delta(i,j) \ . \tag{5.3}$$

As a consequence the probability for anything to happen (no monitoring of different events) is given by

$$\sum_j (P_{ij} - \delta(i,j)) \;=\; 0 \ , \tag{5.4}$$

and vanishes identically! This is not so peculiar as it looks first since there must be some signature for anything to happen in order that it can be measured and signature always distinguishes between two different events at least: it is difficult to imagine what the statement "anything did not happen" might mean! Of course, in real context this philosophy would imply the triviality of $S$-matrix.

If biosystems are indeed quantum systems and p-adic probabilities apply to their description then the unavoidable prediction is that the behavior of biosystems depends on how it is monitored (remembering all anecdotes about experimentation with living matter, one might somewhat light-heartedly argue that this is just the case!). For small values of p, in particular for $p = 2$, the deviations from the standard probability theory are especially large. In particular, the resolution of the monitoring is essential factor. It must be stressed that this peculiar behavior seems not to be related with the predictions of standard quantum measurement theory and this supports the view that p-adic probabilities apply only to the statistical modelling of cognition.

An alternative interpretation for the degenerate eigenvalues appearing in the definition of monotoring has emerged years after writing this. The sub-spaces corresponding to given eigenvalue of density matrix represent entangled states resulting in state function reduction interpreted as measurement of density matrix. This entanglement would be negentropic and represent a rule/concept, whose instances the superposed state pairs are. The information measure would Shannon entropy based on the replacement of the probability appearing as argument of logarithm with its p-adic norm. This entropy would be negative and therefore measure the information associated with the entanglement. This number theoretic entropy characterizes two particle state rather than single particle state and has nothing to do with the ordinary Shannon entropy.

Maybe one could say that finite measurement resolution implies automatically conceptualization and rule building. Abstractions are indeed obtained by dropping out the details.

# 6 Is Small-P P-Adicity Possible?

A longstanding problem of TGD inspired theory of consciousness and p-adic TGD in general has been whether small-p p-adicity is present in macroscopic length scales. The basic form of p-adic length scale hypothesis suggests that small-p p-adicity should be present only in length scales near $CP_2$ size about $10^4$ Planck lengths, which defines the fundamental p-adic length scale. On the other hand, p-adic fractality suggests that also the scaled up versions of entire p-adic length scale hierarchy might be possible in the sense that $CP_2$ size is effectively replaced with p-adic length scale $L_p$ for any prime $p$ and most probably for primes $p \simeq 2^k$, $k$ power of prime. In particular, the realization of genetic code, which corresponds to p-adic prime $p = 127$, at the level of DNA molecules suggests, that small-p p-adicity is realized in Nature and involves transmutation of the fundamental p-adic length scale to atomic length scale. This expectation conforms also with the idea that Universe is infinite-sized self-organizing quantum computer emulating itself in all possible scales and building scaled up simulations of the lower levels. Even science could be regarded as one such emulation.

There are two manners to achieve this transmutation. Either there is some mechanism making this transmutation possible dynamically or the scaled up variants of the p-adic length scales are present from the beginning. I constructed long ago an argument suggesting that the first option might be possible. If one accepts the hierarchy of Planck constants, this hierarchy is present from the beginning at the level of dark matter but manifesting itself also in the behavior of the visible matter since dark matter and visible matter interact in TGD framework via the standard interactions such as classical em fields and photon exchange. What darkness means that particles at different pages of the Big Book realizing dark matter hierarchy cannot appear in the same local interaction vertex of Feynman diagram so that in particle physics laboratory these interactions cannot be observed.

## 6.1 Hierarchy Of Planck Constants And Small-P P-Adicity

The hierarchy of Planck constants [K18, K18, K13, K3] realizes small-p p-adicity in a very natural manner.

1. p-Adic length scale hypothesis states that the hierarchy of primary p-adic length scales $L_p = \sqrt{p}R$, where $R$ is $CP_2$ size is fundamental. The primes near power of 2 are favored so that the primary p-adic length and time scales would come as half octaves. The justification for the hypothesis came originally from p-adic mass calculations and Uncertainty Principle.

2. The secondary p-adic time (and length) scales $T_{p,2}$ associated with primes $p \simeq 2^k$ coming as octaves of $CP_2$ scale define the proper time temporal distances between the tips of CDs (and their spatial sizes). The secondary p-adic length scales are analogous to the horizon sizes in cosmology. p-Adic length scale hypothesis follows from a simple argument using the light-like randomness of 3-surfaces implying that primary p-adic length scale is proportional to a square root of the temporal distance between the tips of CD.

3. The basic prediction of the generalization of quantum theory by allowing a hierarchy of Planck constants is that for $r = \hbar/\hbar_0$ the primary p-adic length scale $L_p$ is scaled to $\sqrt{r}L_p$ and secondary p-adic time scale to $rL_{p,2}$. In principle all rational values of $r$ are possible but certain rationals such as ratios and products and inverses of products of ruler-and-compass integers are favored. These integers are expressible as products of a power of two and product of different Fermat primes $F_k = 2^{2^k} + 1$. Only $k = 1, 2, 3, 4$ are known to give rise to prime.

4. Second interesting hierarchy of values of $r$ are ratios, products and inverses of products of primes. The reason is that the quantum phases $exp(i2pi/p)$ behave as primes under multiplication in the sense that more general phases can be expressed as products of powers of these prime phases. This would give as a special case small prime multiples of the secondary p-adic length scales.

## 6.2 Hierarchy Of Planck Constants And Small-P P-Adicity In Gravitational Sector

The hierarchy of Planck constants [K18, K18, K13, K3] realizes small-p p-adicity in a very natural manner. The basic prediction is that for $r = \hbar/\hbar_0$ p-adic length scale $L_p$ is scaled to $\sqrt{r}L_p$. In principle all rational values of $r$ are possible but certain rationals such as ratios and products and inverses of products of ruler-and-compass integers are favored. These integers are expressible as products of a power of two and product of different Fermat primes $F_k = 2^{2^k} + 1$. Only $k = 1, 2, 3, 4$ are known to give rise to prime.

Gravitational Planck constant expressible as $\hbar_{gr} = GM_1M_2/v_0$, wehere $v_0/c < 1$ is not too far from unity, is extremely large. Using $L_p = \sqrt{p}R$, and $R = k10^4\sqrt{G/\hbar}$ one obtains that for the scaled up p-adic length scale the expression

$$L_p \rightarrow \sqrt{\frac{GM_1M_2}{v_0}}L_p = \sqrt{\frac{p}{v_0}G\sqrt{M_1M_2}} \ .$$

For $M_1 = M_2 = M$ which makes sense if one consider self-gravitation one has

$$L_p \rightarrow \sqrt{\frac{GM^2}{v_0}}L_p = \frac{1}{2}\sqrt{\frac{p}{v_0(S)}}r_S \ .$$

where $r_S = 2GM$ is Schwartschild radius. One can ask whether the well-known Titius-Bode law [E1] stating an approximate quantization of orbital radii via formula $r = r_0 + r_1 2^k$ might relate to the p-adic length scale hypothesis for small primes. The powers $2^k$, $k = 1, 2, ..., 8$ correspond to primes $p = 2^k + \epsilon$ for either sign of $k$. The radii of Bohr orbits come as $n^2 r_S/v_0$ and produce for $v_0 \simeq 2^{-11}$ reasonable fit for the orbital radii of the inner planets for $n = 3, 4, 5$. For outer planets the scaling $v_0 \rightarrow v_0/5$ is required. This would give the approximate formula

$$p(n) \simeq log(\frac{4n^4 v_0(S))}{v_0^2})$$

for the inner planets and

$$p(n) \simeq log(\frac{100n^4 v_0(S))}{v_0^2})$$

for the outer planets. The corresponding time scales would come as approximate octaves of the same basic time scale and would be of order few minutes.

Since electron corresponds to a huge prime $p = 2^{127} - 1$, one can consider the possibility that relatively small p-adic primes in this scale give rise to biological time scales and that the periodicities which appear in living matter as prime multiples of year might be understood in terms of dark matter at space-time sheets mediating gravitational interaction.

For Earth the Schwartschild radius is $r_S \simeq .9$ cm so that for $v_0(E) = 2^{-11}$ one would have the basic scale of.4 m and p-adic length scale hypothesis for small values of $p$ would give half octaves of this scale. These scales need not have anything to do with biology.

## 6.3 Small-P P-Adicity And Hydrodynamics

Hydrodynamic turbulence in the atmosphere involves generation of coherent macroscopic structures which are typically structures appearing in excitable media. One example are spiral waves which represent spiral like convective roll pattern such that the radius of the rolling vortex increases exponentially when one moves away from the apex of the spiral wave. Tornadoes and hurricanes are also well known self-sustaining structures. The generation of these structures is difficult to understand in ordinary hydrodynamics and Indian meteorologists Mary Selvam [H1] takes as her challenge to understand the microscopic mechanism leading to the generation of these structures. TGD suggests quite generally the reduction of the hydrodynamical turbulence and chaos in excitable media to magnetic or $Z^0$ magnetic turbulence. The work of Selvam related to the turbulent atmospheric flows inspires also additional very interesting insight to p-adic length scale hypothesis and suggests that n-ary p-adic length scales $L_e(n, k)$ corresponding to very large values of $n$ are realized in hydrodynamical turbulence, and that hydrodynamical vortices could be regarded as elementary particle like objects on the space-time sheets at which they are condensed topologically.

### 6.3.1 Spiral waves and magnetic turbulence

Self-sustaining spiral waves are known to be characteristic for all excitable media [A1] and typical results of self-organization. The growth of plants leads quite generally to the generation of logarithmic spirals; spiral $Ca_{++}$ waves are known to be crucial for intracellular communications [A7] ; spiral waves appear also in heart [A2] [A2, A5].

*1. Logarithmic spiral and Penrose tilings*

Spiral waves (say roll-vortices with vortex core along spiral) are waves for which the center of the wave defined by logarithmic spiral

$$\frac{R}{r} = exp(b\theta) \ .$$

The values of $R/r$ are Fibonacci numbers $F(n + 1) = F(n) + F(n - 1)$ for certain values of the angular variable $\theta$. At the limit of large Fibonacci numbers one has $F_n \simeq \tau^n$ and substituting to the equation one obtains $\theta \simeq n\theta_0$, $\theta_0 = log(\tau)/b$, $\tau = \frac{1+\sqrt{5}}{2}$.

Logarithmic spirals form a one-parameter family and especially interesting is the logarithmic spiral for which the line connecting the points $r = F_n$ and $r = F(n+1)$ has length $F_n$. In this case

$$\theta_0 = \frac{2\pi}{10} = 36 \text{ degrees} .$$

This particular logarithmic spiral leads to a generation of Penrose tiling [A8]: this occurs in both 2- and 3-dimensional case. This particular logarithmic spiral is very general in botany. Rather interestingly, the angle of 36 degrees happens to be the angle between two subsequent DNA nucleotides in DNA helix, which encourages to consider the possibility that the helical structure of DNA rather concretely codes is in some sense fractal growth defined by the logarithmic spiral with this value of $b$. Note that this kind of growth preserves shape and this is probably one reason for why logarithmic spirals appear so often in botany. In fact, the notion of many-sheeted DNA [K8] suggests that genes in DNA helix in some sense represent contracted versions of the organism preserving 1-dimensional homology: perhaps the contraction preserves also spiral structure. A further interesting point to notice is that the shortest sequence of DNA: s for which the net winding angle along helix is multiple of $2\pi$ and which codes for an entire protein consisting of 30 DNA nucleotides, has thickness of cell membrane as already found.

*2. Reduction of chaos to magnetic turbulence?*

TGD suggests that quite generally spiral waves are accompanied by the underlying magnetic and $Z^0$ magnetic flux tube structures. Spiral wave would correspond to $Z^0$ flux tube around which ordinary matter rotates so that rolling vortex results. At the apex magnetic flux tube apparently ends. The conservation of ($Z^0$) magnetic flux requires that flux tube leaves the space-time sheet at the apex and continues at the second space-time sheet. This suggests the fascinating possibility that macroscopic structures in hydrodynamic wormhole magnetic fields [K22] associated with pairs of space-time sheets and be generated by rotating wormholes at the boundaries of the structure. If time orientation is negative at second space-time sheet, this space-time sheet carries negative energy density which can be very small if only the energy of $Z^0$ magnetic field is in question. If wormhole magnetic fields (besides MEs) represent mind-like space-time sheets of finite time duration, one could perhaps (rather loosely) speak about interaction of matter and mind. The same mechanism might be at work also at cell level.

*3. Magnetic turbulence and loss of macroscopic quantum coherence*

For superconductors quantization conditions imply that the increment of the phase of the complex order parameter of the supra phase around the circuit along boundary of the flux tube equals to the magnetic flux through the tube. Thus magnetic turbulence implies turbulence of superconductor and probably destruction of the supraphase. If ionic superconductors are responsible for biocontrol, then magnetic turbulence would be reflected as chaotic functioning of organ. This loss of quantum coherence would be caused by the leakage oft he supra currents from flux tubes via flux tubes. This in turn would imply dissipation at the non-superconducting space-time sheets by particle collisions. This leakage would be forced by the inertia when the local curvature of the flux tube becomes too large: this is indeed expected to occur in a chaotic situation when flux tubes have very Brownian shapes.

Heart failure, known to involve the generation of decaying spiral waves modellable using Hodkin-Huxley equations or their variants [A2, A5], might be one example of this mechanism. The reduction of this model to quantum level is required by internal consistency if one takes seriously TGD based model of nerve pulse activity in terms of ionic and electronic superconductors relying crucially on Josephson junctions associated with axons [K15]. In case of heart, normal situation would in ideal case correspond to spatially constant phase wave of Josephson current oscillating in time with basic frequency (there is precise analogy with a rotating mathematical pendulum) so that the Josephson currents associated with all heart cells oscillate in unisono, perhaps at the rhythm of heart beat. During heart failure magnetic turbulence destroys this coherence. Interestingly, the time period of fibrillation is .1 seconds, the time scale of the memetic code [A2].

*4. Atmosphere as cortex of Mother Gaia?*

In TGD framework self-organization means the presence of conscious selves and suggests that even atmosphere is in some sense part of Mother Gaia. Perhaps it is of some significance that the ratio of the thickness of atmosphere (10 km) to the radius of Earth radius is of order 1/100 and is same as the ratio of cell membrane thickness to cell size. Fractality indeed suggests this ratio if atmosphere is regarded as scaled-up version of the cell membrane. Note however that the thickness of flora is about 10 m: in case of cell membrane this would suggest a layer of thickness of order $10^{-11}$ meters, which happens to correspond to the p-adic length scale $L_{M_{127}}$ associated with electron. The p-adic prime associated with the memetic code pops up again and one could wonder whether the MEs with length of $L_2(127)$ could have thickness equal to $L_e(127)$ and form structure analogous to biosphere at surface of Earth. The fact, that the frequency distribution of so called sferics, em perturbations induced by lightnings resemles at low frequencies delta band in sfericsbrain, suggests that these exotic levels of life might be there and interact with animal brains.

### 6.3.2 Selvam's model and claims

Selvam studies a model for hydrodynamical spiral waves by assuming that these waves are vortices with core at logarithmic spiral

$$z \equiv \frac{R}{r} = b \times exp(b\theta) \ .$$

Selvam assumes also that the radius $\rho$ of rolling convective vortex grows with $z$ and that also this growth obeys similar law: that is $\rho = exp(b\theta)$. Selvam assumes that the parameter $\theta_0$ corresponds to the angle of 36 degree associated with equilateral Fibonacci triangle having short sides $F_n$ and long side $F(n+1)$ at the limit $n \rightarrow \infty$. As noticed, this logarithmic spiral gives rise to Penrose tiling.

Selvam does not specify precisely this growth law: for instance, whether there is phase lag between $R$ characterizing position of growing vortex and $r$ characterizing its size. Selvam does not either clearly specify how $R$ develops with time: for instance, whether growth occurs linearly in which case $\theta$ would grow logarithmically. One possible manner to obtain the proposed growth is to assume that the growth is analogous to biological growth such that turbulent eddies are in the role of cells and replicate. If the growing vortex decomposes of radius $\rho(n)$ to an inner cylinder of thickness $\rho(n-2)$ and outer annulus of thickness $r(n-1)$ such that outer annulus replicates to annulus of same thickness at $n+1$ :th step of growth process one indeed obtains $\rho(n+1) = \rho(n) + \rho(n-1)$ giving rise to Fibonacci sequence asymptotically.

Selvam claims that the dominating temporal periodicities $T_n$ of flow are Fibonacci numbers in suitable units:

$$T_n = F(n) \simeq \tau^n \ , \quad \tau = \frac{1+\sqrt{5}}{2} \ .$$

This claim can be understood if vortex structures with radius $F_n$ form special structures and if there are standing waves moving with constant velocity $v$ along these structures: this gives

$$T_n = \frac{F_n r}{v}$$

for the periodicities of these waves. Selvam argues that Fibonacci numbers reflect also the periodicities of prime number distribution but I find it difficult to understand the motivations for this claim.

Selvam also studies the distribution for the ratio $z = R/r$ of large vortex radius $R$ to smallest vortex radius $r$, and, as far as I have understood correctly, claims that this distribution is the same as the distribution of primes in region of rather small primes. This could be understood if vortex radii are prime multiples of $r$

$$R = kr \ , \quad k \ \text{prime} \ ,$$

and if each prime appears with the same probability. This assumption can be actually loosened: one can also interpret $r$ as the p-adic length scale associated with minimum size vortex interpreted as space-time sheet. Even the assumption that vortices sizes are given by primes might be too strong: only one-one correspondence with the distribution of primes is needed. Selvam also argues that vortex dynamics has quantal features and that vortices could in some aspects be regarded as quantum objects: this is certainly what TGD approach strongly suggests.

It must be emphasized that the arguments of Selvam do not satisfy the requirement of mathematical rigour and it is only my personal feeling that something deep is involved and I just take Selvam's claims as inspiration for studying whether small-p p-adicity suggested strongly by fractality might be realized in hydrodynamical flows. Certainly, TGD predicts p-adic evolution and this evolution should reflect itself directly in biological growth and perhaps even in hydrodynamical self-organization. Also Matthew Watkins has proposed a connection between evolution and prime numbers [A6].

p-Adic evolution and quantum classical correspondence (classical dynamics should provide a Bohr orbit type representation for quantum dynamics) suggests that growth processes quite generally corresponds to p-adic evolution. First pop-up structures with $p = 2$, then structures with $p = 3$, and so on. In hydrodynamics case these structures correspond to stable vortices with prime-valued radius $R/r = p$. If the growth of spiral wave is linear in time then vortices with prime valued radio pop-up for the first time at time values which are prime multiples of basic time unit. If the emergence of these vortices reflects itself as some kind of distinguishable feature in the temporal behavior of dynamical quantities, as one might expect, the Fourier spectrum should reflect the properties of the spectrum of prime numbers. This is clearly a strong and testable prediction.

### 6.3.3 Why vortices with prime radii are stable?

The first question to be answered is why vortices with radii which are prime valued are stable. Suppose that there is fundamental length scale $r$ identifiable as the radius of turbulent eddy. This radius would result from the quantization of $Z^0$ magnetic flux if one assumes that there is a preferred value for the strength of the $Z^0$ magnetic field. Flux quantization would imply that the radii of the vortices are quantized as $r \propto \sqrt{n}$, $n$ integer. The problem is to understand why $n$ is square of prime rather than arbitrary integer.

One could however correspond the possibility that prime valued radii correspond to secondary p-adic lengths scales with a scaled-up fundamental p-adic length scale defined by the $Z^0$ magnetic flux quantization (a possible mechanism leading to transmutation of the fundamental p-adic length scale will be discussed later). This implies that all vortices (cylindrical and annular) have radius which is integer multiple of this length scale: $z = n$. Vortices consists of turbulent eddies or tend to decay to vortices $z = 1 < m < n$. The wavelengths of the radial perturbations tending to induce the decay of the vortices to smaller ones, are integer multiples of $r$. One has effectively aperiodic lattice, Penrose tiling known to be associated with logarithmic spirals [A8]. Also in the periodic lattice only integer multiples of the basic wave vector propagate. Turbulent eddy defines the equivalent of fundamental lattice cell.

As a consequence, only vortices with prime-valued radii are stable. For instance, $n = p_1 \times p_2$, $p_1$ and $p_2$ primes, the vortex can decompose to $p_1$ cylindrical or annular vortices with radius $p_2$ or vice versa by a perturbation with wavelength $\lambda = p_1 r$ $(p_2 r)$. The impossibility to generate radial periodic perturbations with wavelength which is nontrivial multiple of the fundamental length, explains why prime vortices are stable against decay. Note that in [K17] precisely the same argument was used to explain why some retarded persons are able to "see" factorization of 8-digit numbers into prime factors (see the book "The man who mistook his wife for hat" of Oliver Sacks [J3] ). Mental images representing number n, is represented by some structure, perhaps vortex(!), and if n is not prime it has tendency to decay to some number of identical smaller structures! Thus non-primeness is directly visible property: perhaps higher levels selves spend their time by monitoring the factorization of very large integers.

### 6.3.4 How the transmutation of a fundamental p-adic length scale to macroscopic length scale could occur?

What might be the mechanism effectively leading to the transmutation of the fundamental p-adic length scale $l \simeq 10^4$ Planck lengths to a macroscopic length scale? Hierarchy of Planck constants represents one solution to the problem. A possible p-adic explanation for these length scales would be as secondary p-adic length scales for Planck constants, which correspond to a prime multiples of the ordinary Planck constant: $r = \hbar/\hbar_0 = p$. Since electron corresponds to a secondary p-adic length scale of order Earth's radius the primes in question must be smaller than $M_{127}$. Some examples are in order.

1. $k = 113$, which corresponds to nuclear p-adic length scale and Gaussian Mersenne, would correspond to a secondary p-adic length scale 1.831 km. Prime multiples of this scale identified in terms of hierarchy of Planck constants might have something do with the radii of vortices reported by Selvam.

2. The p-adic length scale defined by $M_{107}$ assignable to the hadronic space-time sheets would correspond to a secondary p-adic length scale of 28.6 meters. The secondary p-adic length scale assignable to $M_{89}$ characterizing intermediate gauge bosons would be.1 millimeters defining the size scale of a large neuron and also the size of water blob having Planck mass. The mapping of elementary particle p-adic length scales to secondary p-adic length scales defining size scales of CDs would mean a correlation between elementary particle physics and macroscopic physics in human length and time scales, which has remained hidden.

3. The primary p-adic length scale $k = 137$ assignable to atom corresponds to the secondary p-adic time scale of 102.4 seconds. The corresponding length scale, which is $2^{10}$ times the circumference of Earth, is $r = 61$ Gm. The distance of Earth from Sun is $AU = 149$ Gm and about 5/2 times this distance.

## 6.4 2-Adic Psychophysics?

Music metaphor has turned out to be of crucial importance for the theory of qualia. The most natural explanation for this is that music metaphor reflects underlying 2-adicity of our sensory experience. Perhaps at least some aspects of our experience result from a mimickry of the lowest level of the p-adic self-hierarchy. Taking 2-adicity seriously, one is forced to ask for the possible consequences of 2-adicity. For instance, could it be that at the level of primary qualia the intensity of sensation as function of stimulus depends on the 2-adic norm of the 2-adic counterpart of the stimulus and is thus a piecewise constant function if sensory input?

An observation supporting this speculation is following. When over-learning occurs in tasks involving temporal discrimination, the intensity of sensation as a function of stimulus deviates from smooth logarithmic form in small scales by becoming piecewise continuous function [J2] such that the plateaus where response remains constant are octaves of each other. This observation suggests a generalization inspired by 2-adic version of music metaphor. Primary quale has multiple of cyclotron frequency as its correlate and, being integer valued, is essentially 2-based logarithm of the 2-adic norm for the 2-adic counterpart of the intensity of the sensory input. Hence the increase of intensity of the sensory input by octave correspond to a jump-wise replacement of the n: th harmonic by n+1: th one and should be seen in EEG. Our experience usually corresponds to the average over a large number of this kind of primary experiences so that underlying 2-adicity is smoothed out. In case of over-learning or neurons involved act unisono and the underlying 2-adicity is not masked anymore. At the level of ELF selves this would mean generation of higher harmonic when the number of nerve pulses per unit of time achieves threshold value allowing the amplification of corresponding frequency by the mechanism discussed already earlier.

## 6.5 Small-P P-Adicity In Biosystems And Psychophysics

There are several hints for small-p p-adicity in macroscopic length and time scales from biology and psychophysics besides this decisive result of Selvam.

1. 2-Adicity of music experience suggests that 2-adicity present in macro-temporal scales [K17]. Also the general form of the p-adic length scale hypothesis and the concrete appearance of 2-adic fractals [K11] suggests that 2-adicity is realized also in macroscopic length scales. The topological model for thoughts as association sequences suggests strongly small-p p-adicity and this idea was in fact one of the first ones relating p-adic numbers with consciousness. The 2-adicity of music experience is relatively easy to understand if any p-adic time scale can serve as effective fundamental time scale for 2-adicity of music experience. Note however that by p-adic length scale hypothesis the fundamental time scales come as powers of 2.

The apparently complete freedom to choose the fundamental time scale can be understood if practically any p-adic time scale $L_p$ replacing $l$ can serve as effective fundamental time scale.

2. Genetic code corresponds to $p = 127 = 2^7 - 1$ in TGD inspired model of abstraction process predicting infinite hierarchy of "genetic codes" [K5]. It should be however realized in macrotemporal scales rather than near $CP_2$ time scale and if the proposed mechanism scales $l$ to p-adic length scale of order atomic length scale this is indeed realized.

3. Memetic code corresponds to $p = 2^{127} - 1$ and to a unique p-adic time scale of .1 seconds [K5]. Codeword has 126 bits and single bit corresponds to the time scale of nerve pulse. What is disturbing that this would make time scale of human brain unique. Situation changes if any p-adic time scale can take the role of fundamental p-adic time scale so that .1 seconds would become lower limit for time duration of memetic code word. Hence brain would represents the first step in the evolution creating memetic codes in longer time scales. In light of p-adic fractality the idea that the time scale associated with $M_{127}$ is the only possible duration of memetic codon, does not sound plausible. One can indeed imagine a hierarchy of scaled-up versions of $M_{127}$ code. This would suggest that $M_{127}$ could be also realized at time scales $k \times T_2(127)$, $k$ prime, $T_2(127) = .1$ s. $T_2(127)$ would be the smallest p-adic time scale, where memetic code is possible and the distribution of longer time scales would obey distribution of primes. This distribution should reflect itself in the EEG spectrum at very low frequencies.

## 6.6 Is Evolution 3-Adic?

I received an interesting email from Jose Diez Faixat giving a link to his blog (`http://tinyurl.com/ycesc5mq` ). The title of the article in the blog is "Bye-bye Darwin" and tells something about his proposal. The sub-title "The Hidden rhythm of evolution" tells more. Darwinian view is that evolution is random and evolutionary pressures select the randomly produced mutations. Rhythm does not fit with this picture. Faixat published 1993 the first article about his observations in the journal World Futures Vol. 36, pp. 31-56, edited by Ervin Lazlo with the title "A hypothesis on the rhythm of becoming" [**?**, **?**].

The observation challenging Darwinian dogma is that the moments for evolutionary breakthroughs - according to Faixat's observation - seems to come in powers of 3 for some fundamental time scale. There would be precise 3-fractality and accompanying cyclicity - something totally different from Darwinian expectations.

By looking at the diagrams demonstrating the appearance of powers of 3 as time scales of evolution, it became clear that the interpretation in terms of underlying 3-adicity could make sense. I have speculated with the possibility of small-p p-adicity. In particular, p-adic length scale hypothesis stating that primes near powers of 2 are especially important physically could reflect underlying 2-adicity. One can indeed have for each p entire hierarchy of p-adic length scales coming as powers of $p^{1/2}$. $p = 2$ would give p-adic length scale hypothesis. The observations of Faixat suggest that also powers p=3 are important - at least in evolutionary time scales.

**Note**: The p-adic primes characterizing elementary particles are gigantic. For instance, Mersenne prime $M_{127} = 2^{127} - 1$ characterizes electron. This scale could relate to the 2-adic scale $L_2(127) = 2^{127/2} \times L_2(1)$. The hierarchy of Planck constants coming as $h_{eff} = n \times h$ also predicts that the p-adic length scale hierarchy has scaled up versions obtained by scaling it by $n$.

The interpretation would be in terms of p-adic topology as an effective topology in some discretization defined by the scale of resolution. In short scales there would be chaos in the sense of real topology: this would correspond to Darwinian randomness. In long scales p-adic continuity would imply fractal periodicities in powers of p and possibly its square root. The reason is that in p-adic topology system's states at $t$ and $t + kp^n$, $k = 0, 1, ...p - 1$, would not differ much for large values of $n$.

A possible interpretation relies on p-adic fractality [K11] (`http://tgdtheory.fi/figu.html` ). p-Adic fractals are obtained by assigning to real function its p-adic counterpart by mapping real point by canonical identification

$$\sum_n x_n p^n \rightarrow \sum_n x_n p^{-n}$$

to a p-adic number, assigning to it the value of p-adic variant of real function with a similar analytic form and mapping the value of this function to a real number by the inverse of the canonical identification, the powers of $p$ correspond to a fractal hierarchy of discontinuities.

A possible concrete interpretation is that the moments of evolutionary breakthroughs correspond to criticality and the critical state is universal and very similar for moments which are p-adically near each other.

The amusing co-incidence was that I have been working with a model for 12-note scale [L2], [K14, K21] (http://tinyurl.com/y7csuxaw ), which to my opinion is highly interesting from the point of view of consciousness theory. Already the mathematicians of ancient Greece speculated with a connection with the geometry of Platonic solid and music scale [J4].

The basic observation is that icosahedron is a Platonic solid containing 12 vertices. The scale is represented as a closed non-self-intersecting curve - Hamiltonian cycle - connecting all 12 vertices: octave equivalence is the motivation for closedness. The cycle consists of edges connecting two neighboring vertices identified as quints - scalings of fundamental by factor 3/2 in Platonic scale. What is amusing that scale is obtained essentially powers of 3 are in question scaled down (octave equivalence) to the basic octave by a suitable power of 2. There is of course slight discrepancy due to the fact that $(3/2)^{12} = 2^7$ is not quite true. This motivated the transition to the well temperated scale with half note corresponding to the scaling by $2^{1/12}$.

The faces of icosahedron are triangles and define naturally basic 3-chords. Triangle can contain either 0, 1, 2 edges of the cycle meaning that the 3-chords defined by faces and defining the notion of harmony contain 0, 1, or 2 quints. One obtains large number of different harmonies partially characterized by the numbers of 0-, 1-, and 2-quint icosahedral triangles since the total number of Hamiltonian cycles at icosahedron is $2^{10}$. One must howe$h_{eff} = n \times h$ver notice that those related by an isometry of icosahedron are equivalent.

The connection with 3-adicity comes from the fact that Pythagorean quint cycle is nothing but scaling by powers of 3 followed by suitable downwards scaling by 2 bringing the frequency to the basic octave so that 3-adicity might be realized also at the level of music!

There is also another strange co-incidence. Icosahedron has 20 faces, which is the number of amino-acids. This suggests a connection between fundamental biology and 12-note scale. This leads to a concrete geometric model for amino-acids as 3-chords and for proteins as music consisting of sequences of 3-chords. Amino-acids can be classified into 3 classes using polarity and basic - acid/neutrality character of side chain as basic criteria. DNA codons would would define the notes of this music with 3-letter codons coding for 3-chords. One ends up also to a model of genetic code relying on symmetries of icosahedron from some intriguing observations about the symmetries of the code table.

At the level of details the icosahedral model is able to predict genetic code correctly for 60 codons only, and one must extend it by a fusion it with a tetrahedral code. The fusion of the two codes corresponds geometrically to the fusion of icosahedron with tetrahedron along common face identified as punct (punct) and coded by 2 stopping codons in icosahedral code and 1 stopping codon in tetrahedral code. Tetrahedral code brings in 2 additional amino-acids identified as so called 21st and 22nd amino-acid discovered for few years ago and coded by stopping codons. These stopping codons certainly differ somehow from the ordinary ones - it is thought that context defines somehow the difference. In TGD framework magnetic body of DNA could define the context.

The addition of tetrahedron brings one additional vertex, which correlates with the fact that rational scale does not quite closed. 12 quints gives a little bit more than 7 octaves and this forces to introduce 13 note for instance, $A_b$ and $G_\#$ could differ slightly. Also micro-tubular geometry involves number 13 in an essential manner.

# 7 $L_0$ $Mod$ $P^M = 0$ Excitations Of Super Virasoro Algebra As Higher Forms Of Life?

Topological field quanta can have all possible sizes. Uncertainty Principle suggests that the size of the topological field quantum corresponds to the p-adic length scale of the corresponding 3-surface. This would mean that the vibrational excitations of even macroscopic 3-surfaces could correspond to Super Virasoro representations. Indeed, the states of real super-symplectic representations associated with the light-like boundaries of MEs have gigantic almost-degeneracies and provide excellent candidates for representing biological information [K12]. These representations realize the idea of quantum hologram in the sense of quantum gravity and quantum information theory concretely and emerge naturally also in the TGD based theory of qualia [K4] .

Besides this there are also what might be called exotic p-adic representations of super-conformal Super Virasoro algebra for which the real counterparts of the p-adic masses are extremely small although the masses of the corresponding real states are super-astronomical. These states have enormous quaternion-conformal (rather than only super-symplectic degeneracies) degeneracies and this raises the question about the possible biological relevance of these states. Thus it seems (at least now when I am writing this!) that the exotic states are not relevant for the understanding of biosystems. Despite this, and also because I ended up with super-symplectic representations via exotic p-adic representations, I do not have heart to throw away the discussion of the properties and possible biological significance of these representations. The reader can however safely skip this section if she wishes.

## 7.1 Exotic P-Adic Super-Conformal Representations

The eigenvalues of Super Virasoro generator $L_0$ are non-negative integers $n$. In p-adic context one can naturally decompose these eigenvalues into classes such that in class $m$ eigen-values are of form $n = kp^m$, $k = 1, 2, ...$, $k \ mod \ p \neq 0$. In class $m$ the real counterpart of the mass squared is of order $1/p^m$ and hence extremely small for large values of $m$. Does this predict the existence of light excitations for all particles, even fermions?

1. The answer “No” is suggested by the fact that p-adic representations of super-conformal algebras should describe the physics of cognition rather than real physics so that these exotic states need not correspond to real physics states.

2. One might of course argue that every every p-adic state (imaginable state!) must have a real counterpart with essentially the same real physics properties. In recent case the real counterparts of the p-adic masses obtained by canonical identification are extremely small whereas the masses of the corresponding real states are super-astronomical if the value of the string tension is formally the same and of order $O(p^0)$. String tension is however a dynamical quantity and one can consider the possibility that the real counterpart of the p-adic string tension for the super-conformal representations is such that the real and p-adic mass scales are mutually consistent. Admittedly, this argument does not satisfy the requirement of mathematical elegance.

## 7.2 Elementary Particles Cannot Correspond To Exotic Super-Conformal States

If the real counterparts of the exotic states are created in pairs with vanishing total quantum numbers and having super-astronomical real masses, they certainly cannot have any relevance for elementary particle physics. If one assumes that string tension for real states is such that real masses of exotic states are of same order as p-adic mass situation can change. For instance, intermediate gauge bosons would have also excitations with mass $1/\sqrt{p}$ and one can wonder whether these excitations could correspond to the observed intermediate gauge bosons. One could even consider the possibility of understanding the entire elementary particle mass spectrum in terms of these $n = 0$ and $n = p$ excitations assuming that the vacuum weight of the Super Virasoro representations is vanishing. There are quite a number of consistency conditions, which definitely exclude this possibility.

1. Photon, graviton and gluon correspond to a ground state created by vanishing conformal weight. This happens to be the case. By a suitable choice for the coefficient of modular contribution and with a suitable choice of mass scale one might be able to reproduce charged lepton mass ratios correctly.

2. All states with non-vanishing ground state vacuum weight should correspond to $n = p$ states and would have same non-vanishing mass equal to $1/\sqrt{p}$ in natural units for given $p$. For quarks no mass splitting would result in first order approximation and the experience with

CKM matrix suggests very strongly that it is not possible to achieve correct CKM matrix for mass degenerate $u$ and $d$ quarks.

3. A strong counter argument against the scenario is the huge ground state degeneracy of the states expected. As well known the degeneracy of states with eigenvalue $n$ of $L_0$ increases exponentially as a function of $n$. For instance, huge number of color, electro-weak and spin excitations would have same mass and this does not seem to make sense. Thus it seems that p-adic thermodynamics giving extremely small probability for all large $n$ excitations must be correct for elementary particles at least. Again there is however loophole involved. Low energy hadron physics corresponds to non- perturbative QCD like theory and one might wonder whether these exotic states of Super Virasoro algebra could become important at low hadron momentum transfers and whether some kind of phase transition from the dominance of the ordinary Super Virasoro representations to that of exotic Super Virasoro representations might take place. Amazingly, this hypothesis predicts the mass of pion and Regge slope correctly as fundamental constants of Nature [K23].

## 7.3 Could Exotic P-Adic Counterparts Of Elementary Particles Be Relevant For Living Systems?

Previous arguments do not exclude the appearance of $n\ mod\ p^k = 0$ p-adic states. Also their zero energy pairs could appear as real states. If the couplings of these excitations obey the conservation of $L_0$ charge (conformal weight), the states in class $m$ couple only to the states in same class or to $n = 0$ massless states and therefore these particles could probably emit and absorb ordinary $n = 0$ elementary particles. The possibility of pair creation seems to be excluded (it would require that antiparticles have negative spectrum of $L_0$, which looks peculiar). If this is true then $m = 1$ states are not be created in ordinary elementary particle reactions. It must be emphasized that the matrix elements for emission of exotic states could be small for other reasons: for instance, because the conformal weights of states involved differ so much.

An interesting possibility is that $m > 1$ excitations of known elementary particles could be present in macroscopic length scales.

1. For hadrons $m = 2$ excitations correspond by Uncertainty Principle to the length scale $L_e(k = 2 \times 107) \sim .4$ meters whereas for electron one has length scale $L_e(k = 2 \times 127) \sim 10^7$ meters. The corresponding time scale is .1 seconds, which is the fundamental time scale of brain consciousness defining the duration of psychological moment. This time scale is crucial in the TGD based model of memetic code. The model derives from a model of abstraction process leading to a hierarchy of “genetic codes” labelled by Mersenne numbers: $M(n) = M_{M(n-1)}$. $M_7 = 127$ corresponds to genetic code and $M_{127}$, which is the next level of the hierarchy, corresponds to the memetic code.

2. For $m = 2$ excitations of $Z^0$ and $W$ (also other states could be present) the corresponding length scale is $L_e(k = 2 \times 89 = 178) \sim 10^{-4}$ meters, which is $2^{4.5}$ times larger than the p-adic length scale $L_e(k = 169)$ associated with neutrinos. Is this a pure accident or could it be that there are exotic $Z^0$ bosons in cell length scale and that this explains the primary condensation level of neutrinos? In this picture it would be perhaps easier to understand also why classical $Z^0$ fields appear dominantly above cell length scale as required by the arguments based on the smallness of parity breaking effects. It should be mentioned that $k = 178$ corresponds to the size of the largest neurons.

The super astronomical degeneracy $D \sim exp(p)$, $p = M_{89}$ (!) associated with these excitations plus Negentropy Maximization Principle could make biosystems with size larger that the critical size of $10^{-4}$ meters something quite special, to put it very mildly! The same argument applies to the $p = M_{127}$ associated with the memetic code. The p-adic length scale nearest to $L_e(178)$ corresponds to the secondary condensation level for the $m = 2$ particles. It is $k = 179$ and in fact forms twin prime with $k = 181$. As a rule, twin primes in bio-systems seem to be associated with two-layered structures and this particular twin prime corresponds to ocular dominance columns, the largest known two-layered structure in the cortex (in fact this twin prime is the first one in the series of three twin primes $(179, 181)$, $(191, 193)$, $(197, 199)$!).

This raises the question whether the physics based explanation for the huge qualitative and quantitative differences in the behavior of higher primates and more primitive life forms could be based on the huge entanglement entropy resources provided by these exotic particles? It seems that this question becomes more or less obsolete with the realization that the immense super-symplectic almost-degeneracies for the massless states of super-conformal representations explain very naturally the huge information resources of biosystems without need to introduce exotic representations.

One can end up to the similar speculations via a different route by starting from the TGD based reduction of the notion of potential energy to space-time topology (potential energy unlike kinetic energy does not allow any visualization in standard physics and thus remains a fictive concept).

1. In TGD framework the sign of energy depends on the time orientation of the space-time sheet and can be negative. Topological field quanta of negative energy represent negative energy virtual particles. The generation of negative potential energy corresponds to the emission of negative energy virtual bosons condensing on larger space-time sheets and in this manner one can understand potential energy as the total energy emitted by particle in form of low energy topological field quanta condensed on larger space-time sheets. In particular, the huge energy densities in strong gravitational fields of early cosmology result via the emission of negative energy virtual gravitons: only in this manner one can understand in TGD framework how conservation of energy can be consistent with gravitational interaction. For instance, gravitational redshift, which in GRT means non-conservation of energy, results in TGD framework from the absorption of negative energy virtual gravitons.

2. An objection against this interpretation is provided by long range classical $Z^0$ fields: attractive classical $Z^0$ potential energy should also correspond to topological field quanta of negative energy at larger space-time sheets. This is certainly possible. These topological field quanta cannot however correspond to the ordinary quanta of $Z^0$ field which are extremely massive and propagate only over range of order $10^{-17}$ meters. Thus the correspondence *quanta↔topological quanta* seems to fail.

3. There is however a loophole allowed by p-adic mathematics. As already noticed, the secondary almost massless excitations $n\ mod\ p = 0$ of Super Virasoro algebra have mass of order $m(CP_2)/p$ and possess huge exponential degeneracy of states characteristic for the Super Virasoro algebra. For $p = M_{89} = 2^{89} - 1$ the mass of these excitations is of order $m \sim m_W 2^{-89/2} \sim 10^{-2}$ eV, which happens to be rather near to the thermal energy associated with the room temperature, which is the critical temperature for the higher forms of biological life. The corresponding length scale is by Uncertainty Principle $10^{-4}$ meters and would represent the range of the $Z^0$ forces based on the exchange of the secondary quanta. Thus the exchange of these quanta between nuclei and neutrinos could be an essential element of what it is to be biosystem. These excitations having huge ground state degeneracy could also provide a quantum level description for the huge degeneracy of states certainly characteristic for biosystems. This degeneracy might also explain dynamically why neutrinos topologically condense on cell length scale.

4. A further objection is that classical $Z^0$ force seems to be not restricted to biological length scales but is present also in the planetary length scales. This objection can be circumvented too. Higher secondary excitations of Super Virasoro algebra satisfying $n\ mod\ p^3 = 0$ with mass of order $m(CP_2)/p^{3/2}$ should be also present. This mass would correspond to $m \sim 2^{-89} m_W$ and to the length scale of $2 \times 10^9$ meters characterizing solar system. The corresponding time scale is 8 seconds, which is also an important length scale in biosystems as is also the time scale of .1 seconds associated with the second power of $p = M_{127}$, which is the p-adic length scale of electron and characterizes memetic code. This hypothesis is consistent with the idea that ELF em and $Z^0$ fields give rise to a new form of life, "culture", living in symbiosis with biological life.

5. This would suggest a hierarchy of lifeforms whose intelligence quotient is roughly characterized by the degeneracy of the Super Virasoro states involved and thus by the power and value of the p-adic prime $p$ to which they correspond. Since Mersenne primes are fundamental for elementary particle physics, one expects that the powers of the Mersenne primes

$M_{89}$, $M_{107}$ and $M127$ should label the most important higher lifeforms. $M_{89}$ would give rise to two higher levels already discussed whereas $M_{127}$ gives rise to the menetic code. The $n\ mod\ p^2 = 0$ excitations associated with $M_{107}$, the Mersenne prime characterizing hadrons, would correspond to the length scale of about 25 meters and time scale of order $10^{-7}$ seconds. $n\ mod\ p^3 = 0$ excitations associated with $M_{107}$ would in turn correspond to the time scale of $10^9$ seconds, or 30 years in more natural units: this is of the same order as human life span!

6. A further observation of possible relevance is that if Super Algebra representation has vanishing conformal vacuum weight, the subalgebra consisting of generators having conformal weights $n$ proportional to $p^m$ forms sub-algebra of entire Super algebra. Thus the exotic states correspond to sub-algebra of Super Virasoro and become therefore even more interesting in light of fractality suggesting strongly hierarchical breaking of supersymmetry to subalgebras of Super Virasoro algebra isomorphic with the entire algebra.

Because of their physical properties MEs provide excellent candidate for a model of mind-like space-time sheets and one can assign to the light-like boundaries of MEs super-symplectic representations defining quantum holograms. Thus MEs could carry also exotic p-adic Super Virasoro representations but as already noticed, they are not needed in order to understand the information sources associated with living matter.

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