This chapter is second one in a multi-chapter devoted to the vision about TGD as a generalized number theory. The basic theme is the role of classical number fields in quantum TGD. A central notion is M^8-H duality which might be also called number theoretic compactification. This duality allows to identify imbedding space equivalently either as M^8 or $M^4\times CP_2$ and explains the symmetries of standard model number theoretically. This duality has been recently extended to a $H-H$ duality making sense if the dualism respects associativity (co-associativity). This would make the space of preferred extremals category with dualism representing the fundamental arrow.

These number theoretical symmetries induce also the symmetries dictating the geometry of the \blockquote{world of classical worlds} (WCW) as a union of symmetric spaces. This infinite-dimensional K\"ahler geometry is expected to be highly unique from the mere requirement of its existence requiring infinite-dimensional symmetries provided by the generalized conformal symmetries of the light-cone boundary $\delta M^4_+\times S$ and of light-like 3-surfaces and the answer to the question what makes 8-D imbedding space and $S=CP_2$ so unique would be the reduction of these symmetries to number theory.

Zero energy ontology (ZEO) has become the corner stone of both quantum TGD and number theoretical vision. In ZEO either light-like or space-like 3-surfaces can be identified as the fundamental dynamical objects, and the extension of general coordinate invariance leads to effective 2-dimensionality (strong form of holography) in the sense that the data associated with partonic 2-surfaces and the distribution of 4-D tangent spaces at them located at the light-like boundaries of causal diamonds (CDs) defined as intersections of future and past directed light-cones code for quantum physics and the geometry of WCW.

The basic number theoretical structures are complex numbers, quaternions and octonions, and their complexifications obtained by introducing additional commuting imaginary unit $\sqrt{-1}$. Hyper-octonionic

(-quaternionic,-complex) sub-spaces for which octonionic imaginary units are multiplied by commuting $\sqrt{-1}$ have naturally Minkowskian signature of metric. The question is whether and how the hyper-structures could allow to understand quantum TGD in terms of classical number fields. The answer which looks the most convincing one relies on the existence of octonionic representation of 8-D gamma matrix algebra.

\begin{enumerate}

\item The first guess is that associativity condition for the sub-algebras of the local Clifford algebra defined in this manner could select 4-D surfaces as surface having as tangent spaces associative (co-associative) sub-spaces of this algebra and define WCW purely number theoretically. The associative sub-spaces in question would be spanned by space-time tangent vectors spanning associative (co-associative) sub-algebra of complexified octonions generated by imbedding space tangent vectors. A more concrete representation of vectors of complexified tangent space as imbedding space gamma matrices is not necessary. One can consider also octonionic representation of imbedding space gamma matrices but whether it has any physical content, remains an open question. The answer to the question whether octonions could correspond to the K\"ahler-Dirac gamma matrices associated with K\"ahler-Dirac action turned out to be \blockquote{No}.

\item This condition is quite not enough: one must strengthen it with the condition that a preferred commutative (co-commutative) sub-algebra is contained in the tangent space of the space-time surface. This condition actually generalizes somewhat since one can introduce a family of so called Hamilton-Jacobi coordinates for M^4 allowing an integrable distribution of decompositions of tangent space to the space of non-physical and physical polarizations. The physical interpretation is as a number theoretic realization of gauge invariance selecting a preferred local commutative plane of non-physical polarizations.

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\item  As has become clear, one can define also the notions of
co-associativity and co-commutativity applying in the regions of
space-time
surface with Euclidian signature of  the induced metric.  The basic
unproven conjecture is that  the decomposition of space-time
surfaces to
associative and co-associative regions  containing preferred
commutative
{\it resp.} co-commutative 2-plane in the 4-D tangent plane is
equivalent
with the preferred extremal property of K\"ahler action and the
hypothesis
that space-time surface allows a slicing by string world sheets and
by
partonic 2-surfaces.

\end{enumerate}




%\end{abstract}
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