The vision about a number theoretic formulation of quantum TGD is based on the gradual accumulation of wisdom coming from different sources. The attempts to find a formulation allowing to understand real and p-adic physics as aspects of some more general scenario have been an important stimulus and generated a lot of, not necessarily mutually consistent ideas, some of which might serve as building blocks of the final formulation.

The first part of the 3-part chapter is devoted to the p-adicization program attempting to construct physics in various number fields as an algebraic continuation of physics in the field of rationals (or appropriate extension of rationals). The program involves in essential manner the generalization of number concept obtained by fusing reals and p-adic number fields to a larger structure by gluing them together along common rationals. Highly non-trivial number theoretic conjectures are an i outcome of the program.

\vm{\it 1. Real and p-adic regions of the space-time as geometric correlates of matter and mind}\vm

The solutions of the equations determining space-time surfaces are restricted by the requirement that the imbedding space coordinates are real. When this is not the case, one might apply instead of a real completion with some rational-adic or p-adic completion: this is how rational-adic p-adic physics could emerge from the basic equations of the theory. One could interpret the resulting rational-adic or p-adic regions as geometrical correlates for \blockquote{mind stuff}.

p-Adic non-determinism implies extreme flexibility and therefore makes the identification of the p-adic regions as seats of cognitive representations very natural. Unlike real completion, p-adic completions preserve the information about the algebraic extension of rationals and algebraic coding of quantum numbers must be associated with \blockquote{mind like} regions of space-time. p-Adics and reals are in the same relationship as map and territory.

The implications are far-reaching and consistent with TGD inspired theory of consciousness: p-adic regions are present even at elementary particle level and provide some kind of geometric model of \blockquote{self} and external world. In fact, p-adic physics must model the p-adic cognitive regions representing real elementary particle regions rather than elementary particles themselves!

\vm{\it 2. The generalization of the notion of number}\vm

The unification of real physics of material work and p-adic physics of cognition and intentionality leads to the generalization of the notion of number field. Reals and various p-adic number fields are glued along their common rationals (and common algebraic numbers too) to form a fractal book like structure. Allowing all possible finite-dimensional extensions of p-adic numbers brings additional pages to this \blockquote{Big Book}.

At space-time level the book like structure corresponds to the decomposition of space-time surface to real and p-adic space-time sheets. This has deep implications for the view about cognition. For instance, two points infinitesimally near p-adically are infinitely distant in real sense so that cognition becomes a cosmic phenomenon.

\vm{\it 3. Number theoretical Universality and number theoretical criticality}\vm

Number theoretic universality has been one of the basic guide lines in the
construction of quantum TGD. There are two forms of the principle.

\begin{enumerate} \item The strong form of number theoretical universality
states that physics for any system should effectively reduce to a physics
in algebraic extension of rational numbers at the level of M-matrix
(generalization of S-matrix) so that an interpretation in both real and
p-adic sense (allowing a suitable algebraic extension of p-adics) is
possible. One can however worry whether this principle only means that
physics is algebraic so that there would be no need to talk about real and
p-adic physics at the level of M-matrix elements. It is not possible to
get rid of real and p-adic numbers at the level of classical physics since
calculus is a prerequisite for the basic variational principles used to
formulate the theory. For this option the possibility of completion is
what poses conditions on M-matrix.

\item The weak form of principle requires only that both real and p-adic
variants of physics make sense and that the intersection of these physics
consist of physics associated with various algebraic extensions of rational
numbers. In this rational physics would be like rational numbers allowing
infinite number of algebraic extensions and real numbers and p-adic number
fields as its completions. Real and p-adic physics would be completions of
rational physics. In this framework criticality with respect to phase
transitions changing number field - number theoretical criticality -
becomes a viable concept. This form of principle allows also purely p-adic
phenomena such as p-adic pseudo non-determinism assigned to imagination
and cognition. Genuinely p-adic physics does not however allow definition
of notions like conserved quantities since the notion of definite integral
is lacking and only the purely local form of real physics allows p-adic

counterpart. \end{enumerate}

Experience has taught that it is better to avoid too strong statements and perhaps the weak form of the principle is enough.

\vm{\it 4. p-Adicization by algebraic continuation}\vm

One general idea which results as an outcome of the generalized notion of number is the idea of a universal function continuable from a function mapping rationals to rationals or to a finite extension of rationals to a function in any number field. It must be however emphasized that for weaker form of number theoretical universality this restriction applies only at number theoretical quantum criticality. This algebraic continuation is analogous to the analytical continuation of a real analytic function to the complex plane. Rational functions with rational coefficients are obviously functions satisfying this constraint. Algebraic functions with rational coefficients satisfy this requirement if appropriate finite-dimensional algebraic extensions of p-adic numbers are allowed. Exponent function is such a function.

For instance, residue calculus might be generalized so that the value of an integral along the real axis could be calculated by continuing it instead of the complex plane to any number field via its values in the subset of rational numbers forming the rim of the book like structure having number fields as its pages. If the poles of the continued function in the finitely extended number field allow interpretation as real numbers it might be possible to generalize the residue formula. One can also imagine of extending residue calculus to any algebraic extension. An interesting situation arises when the poles correspond to extended p-adic rationals common to different pages of the \blockquote{great book}. Could this mean that the integral could be calculated at any page having the pole common.

In particular, could a p-adic residue integral be calculated in the ordinary complex plane by utilizing the fact that in this case numerical approach makes sense.

Algebraic continuation is the basic tool of p-adicization program. Entire physics of the TGD Universe should be algebraically continuable to various number fields. Real number based physics would define the physics of matter and p-adic physics would describe correlates of cognition and intentionality. The basic stumbling block of this program is integration and algebraic continuation should allow to circumvent this difficulty. Needless to say, the requirement that the continuation exists must pose immensely tight constraints on the physics.

Due to the fact that real and p-adic topologies are fundamentally different, ultraviolet and infrared cutoffs in the set of rationals are unavoidable notions and correspond to a hierarchy of different physical phases on one hand and different levels of cognition on the other hand. Two types of cutoffs are predicted: p-adic length scale cutoff and a cutoff due to phase resolution. Zero energy ontology provides a natural realization for the p-adic length scale cutoff. The latter cutoff seems to correspond naturally to the hierarchy of algebraic extensions of p-adic numbers and quantum phases $exp(i2\pi/n)$, $n\geq 3$, coming as roots of unity and defining extensions of rationals and p-adics allowing to define p-adically sensible trigonometric functions These phases relate closely to the hierarchy of quantum groups, braid groups, and II$_1$ factors of von Neumann algebra.

\vm{\it 5. Number theoretic democracy}\vm

The interpretation allows all finite-dimensional extensions of p-adic number fields and perhaps even infinite-P p-adics. The notion arithmetic

quantum theory generalizes to include Gaussian and Eisenstein variants of infinite primes and corresponding arithmetic quantum field theories. Also the notion of p-adicity generalizes: it seems that one can indeed assign to Gaussian and Eisenstein primes what might be called G-adic and E-adic numbers.

p-Adicization by algebraic continuation gives hopes of continuing quantum TGD from reals to various p-adic number fields. The existence of this continuation poses extremely strong constraints on theory.

%\end{abstract}