%\begin{abstract}

Riemann hypothesis states that the nontrivial zeros of Riemann Zeta function lie on the critical line $Re(s)=1/2$. Since Riemann zeta function allows a formal interpretation as thermodynamical partition function for a quantum field theoretical system consisting of bosons labeled by primes, it is interesting to look Riemann hypothesis from the perspective of physics.
The complex value of temperature is not however consistent with thermodynamics. In zero energy ontology one obtains quantum theory as a square root of thermodynamics and this objection can be circumvented and a nice argument allowing to interpret RH physically emerges.

Conformal invariance leads to a beautiful generalization of Hilbert-Polya conjecture allowing to interpret RH in terms of coherent states rather than energy eigenstates of a Hamiltonian. In zero energy ontology the interpretation is that the coherent states in question represent Bose-Einstein condensation at criticality. Zeros of zeta correspond to coherent states orthogonal to the coherent state characterized by $s=0$, which has finite norm, and therefore does not represent Bose-Einstein condensation.

Quantum TGD and also TGD inspired theory of consciousness provide additional view points to the hypothesis and suggests sharpening of Riemann hypothesis, detailed strategies of proof of the sharpened hypothesis, and heuristic arguments for why the hypothesis is true. These considerations are however highly speculative and are represented at the end of the chapter.

\vm{\it 1. Super-conformal invariance and generalization of Hilbert-Polya hypothesis}\vm

Super-conformal invariance inspires a strategy for proving the Riemann hypothesis. The vanishing of the Riemann Zeta reduces to an orthogonality condition for the eigenfunctions of a non-Hermitian operator D^+

having
the zeros of Riemann Zeta as its eigenvalues. The construction of D^+ is
inspired by the conviction that Riemann Zeta is associated with a physical
system allowing super-conformal transformations as its symmetries and
second quantization in terms of the representations of the super-conformal
algebra. The eigenfunctions of D^+ are analogous to coherent states of a
harmonic oscillator and in general they are not orthogonal to each other.
The states orthogonal to a vacuum state (having a negative norm squared)
correspond to the zeros of Riemann Zeta. The physical states having a
positive norm squared correspond to the zeros of Riemann Zeta at the
critical line. Riemann hypothesis follows both from the hermiticity and
positive definiteness of the metric in the space of states corresponding to
the zeros of ζ. Also conformal symmetry in appropriate sense implies
Riemann hypothesis and after one year from the discovery of the basic idea
it became clear that one can actually construct a rigorous twenty line long
analytic proof for the Riemann hypothesis using a standard argument from
Lie group theory.

\vm{\it 2. Zero energy ontology and RH}\vm

A further approach to RH is based on zero energy ontology and is consistent
with the approach based on the notion of coherent state. The postulate that
all zero energy states for Riemann system are zeros of zeta and critical
in the sense being non-normalizable (Bose-Einstein condensation) combined
with the fact that $s=1$ is the only pole of ζ implies that the all
zeros of ζ correspond to $Re(s)=1/2$ so that RH follows from purely
physical assumptions. The behavior at $s=1$ would be an essential element
of the argument. The interpretation as a zero energy counterpart of a
coherent state seems to makes sense also now. Note that in ZEO coherent

state property is in accordance with energy conservation. In the case of
coherent states of Cooper pairs same applies to fermion number conservation. With this interpretation the condition would state orthogonality with respect to the coherent zero energy state characterized
by $s=0$, which has finite norm and does not represent Bose-Einstein
condensation. This would give a connection for the proposal for the strategy for proving Riemann Hypothesis by replacing eigenstates of energy
with coherent states.

\vm{\it 3. Miscellaneous ideas}\vm

During years I have also considered several ideas about Riemann hypothesis
which I would not call miscellaneous. I have moved them to the end of the
chapter because of the highly speculative nature.