The original focus of this chapter was p-adic icosahedron. The discussion of attempt to define this notion however leads to the challenge of defining the concept of p-adic sphere, and more generally, that of p-adic manifold, and this problem soon became the main target of attention since it is one of the key challenges of also TGD.

There exists two basic philosophies concerning the construction of both real and p-adic manifolds: algebraic and topological approach. Also in TGD these approaches have been competing: algebraic approach relates real and p-adic space-time points by identifying the common rationals. Finite pinary cutoff is however required to avoid totally wild fluctuations and has interpretation in terms of finite measurement resolution. Canonical identification maps p-adics to reals and vice versa in a continuous manner but is not consistent with p-adic analyticity nor field equations unless one poses a pinary cutoff. It seems that pinary cutoff reflecting the notion of finite measurement resolution is necessary in both approaches.
This represents a new notion from the point of view of mathematics.

\begin{enumerate}

\item One can try to generalize the theory of real manifolds to p-adic context. The basic problem is that p-adic balls are either disjoint or nested so that the usual construction by gluing partially overlapping spheres fails. One attempt to solve the problem relies on the notion of Berkovich disk obtained as a completion of p-adic disk having path connected topology (non-ultrametric) and containing p-adic disk as a dense subset. This plus the complexity of the construction is heavy price to be paid for path-connectedness. A related notion is Bruhat-Tits tree defining kind of skeleton making p-adic manifold path connected. The notion makes sense for the p-adic counterparts of projective spaces, which suggests that p-adic projective spaces (S^2 and CP_2 in TGD framework) are physically very special.

\item Second approach is algebraic and restricts the consideration to
algebraic varieties for which also topological invariants have algebraic
counterparts. This approach looks very natural in TGD framework - at
least for imbedding space. Preferred extremals of K\"ahler action can be
characterized purely algebraically - even in a manner independent of the
action principle - so that they might make sense also p-adically.
\end{enumerate}

Number theoretical universality is central element of TGD. Physical
considerations force to generalize the number concept by gluing reals and
various p-adic number fields along rationals and possible common algebraic
numbers. This idea makes sense also at the level of space-time and of
\blockquote{world of classical worlds} (WCW).

Algebraic continuation between different number fields is the key notion.
Algebraic continuation between real and p-adic sectors takes place along
their intersection , which at the level of WCW (\blockquote{world of classical
worlds}) correspond to surfaces allowing interpretation both as real and
p-adic surfaces for some value(s) of prime p. The algebraic continuation
from the intersection of real and p-adic WCWs is not possible for all
p-adic number fields. For instance, real integrals as functions of
parameters need not make sense for all p-adic number fields. This apparent
mathematical weakness can be however turned to physical strength: real
space-time surfaces assignable to elementary particles can correspond only
some particular p-adic primes. This would explain why elementary particles
are characterized by preferred p-adic primes. The p-adic prime determining
the mass scale of the elementary particle could be fixed number
theoretically rather than by some dynamical principle formulated in real
context (number theoretic anatomy of rational number does not depend
smoothly on its real magnitude!).

Although Berkovich construction of p-adic disk does not look promising in TGD framework, it suggests that the difficulty posed by the total disconnectedness of p-adic topology is real. TGD in turn suggests that the difficulty could be overcome without the completion to a non-ultrametric topology. Two approaches emerge, which ought to be equivalent.

The TGD inspired solution to the construction of path connected effective p-adic topology is based on the notion of canonical identification mapping reals to p-adics and vice versa in a continuous manner. The trivial but striking observation was that canonical identification satisfies triangle inequality and thus defines an Archimedean norm allowing to induce real topology to p-adic context. Canonical identification with finite measurement resolution defines chart maps from p-adics to reals and vice versa and preferred extremal property allows to complete the discrete image to hopefully space-time surface unique within finite measurement resolution so that topological and algebraic approach are combined. Finite resolution would become part of the manifold theory. p-Adic manifold theory would also have interpretation in terms of cognitive representations as maps between realities and p-adicities.