%\begin{abstract}

The mathematical aspects of p-adicization of quantum TGD are discussed. In a well-defined sense Nature itself performs the p-adicization and p-adic physics can be regarded as physics of cognitive regions of space-time which in turn provide representations of real space-time regions. Cognitive representations presumably involve the p-adicization of the geometry at the level of the   space-time and imbedding space by a mapping of a real space time region to a p-adic one. One can differentiate between two kinds of maps: the identification induced by the common rationals of real and p-adic space time region and the representations of the external real world to internal p-adic world induced by a canonical identification type maps.

Only the identification by common rationals respects general coordinate invariance, and it leads to a generalization of the number concept. Different number fields form a book like structure with number fields and their extensions representing the pages of the book glued together along common rationals representing the rim of the book. This generalization forces also the generalization of the manifold concept: both imbedding space and WCW are obtained as union of copies corresponding to various number fields glued together along common points, in particular rational ones. Space-time surfaces decompose naturally to real and p-adic space-time sheets. In this framework the fusion of real and various p-adic physics reduces more or less to to an algebraic continuation of rational number based physics to various number fields and their extensions.

The definition of p-adic manifold is not discussed although it has turned out to be highly non-trivial. The feasible definition of p-adic sub-manifold emerged two decades after the emergence of the notion of of p-adic space-time sheet. The definition relies on the idea that p-adic

space-time surfaces serve as p-adic charts - cognitive maps - for real space-time surfaces and vice versa and that both real and p-adic space-time sheets are preferred extremals of K\"ahler action and defined only modulo finite measurement/cognitive resolution

The program makes sense only if also extensions containing transcendentals are allowed: the p-dimensional extension containing powers of e is perhaps the most important transcendental extension involved. Entire cognitive hierarchy of extension emerges and the dimension of extension can be regarded as a measure for the cognitive resolution and the higher the dimension the shorter the length scale of resolution. Cognitive resolution provides also number theoretical counterpart for the notion of length scale cutoff unavoidable in quantum field theories: now the length scale cutoffs are part of the physics of cognition rather than reflecting the practical limitations of theory building.

There is a lot of p-adicizing to do.

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\item The p-adic variant of classical TGD must be constructed. Field equations make indeed sense also in the p-adic context. The strongest assumption is that real space time sheets have the same functional form as real space-time sheet so that there is non-uniqueness only due to the hierarchy of dimensions of extensions.

\item Probability theory must be generalized. Canonical identification playing central role in p-adic mass calculations using p-adic thermodynamics maps genuinely p-adic probabilities to their real counterparts. p-Adic entropy can be defined and one can distinguish between three kinds of entropies: real entropy, p-adic entropy mapped to its real counterpart by canonical identification, and number theoretical entropies applying when probabilities are in finite-dimensional extension of rationals. Number theoretic entropies can be negative and provide genuine information measures, and it turns that bound states should correspond in TGD framework to entanglement coefficients which belong to a finite-dimensional extension of rationals and have negative number theoretic entanglement entropy. These information measures generalize by quantum-classical correspondence to space-time level.

\item p-Adic quantum mechanics must be constructed. p-Adic unitarity differs in some respects from its real counterpart: in particular, p-adic cohomology allows unitary S-matrices $S=1+T$ such that T is hermitian and nilpotent matrix. p-Adic quantum measurement theory based on Negentropy Maximization Principle (NMP) leads to the notion of monitoring, which might have relevance for the physics of cognition.

\item Generalized quantum mechanics results as fusion of quantum mechanics in various number fields using algebraic continuation from the field of rational as a basic guiding principle. It seems possible to generalize the notion of unitary process in such a manner that unitary matrix leads from rational Hilbert space H_Q to a formal superposition of states in all Hilbert spaces H_F, where F runs over number fields. The basic objection is that p-adic numbers allow non-vanishing zero norm states. If one can avoid this objection, state function reduction could be seen as a number theoretical necessity and involves a reduction to a particular number field followed by state function reduction and state preparation leading ultimately to a state containing only entanglement which is rational or finitely-extended rational and because of its negative number theoretic entanglement entropy identifiable as bound state entanglement stable against NMP.

It has later turned out that negentropic entanglement must correspond to density matrix proportional to a unit matrix in order to achieve consistency with the ordinary quantum measurement theory. Unitary entanglement coefficients characterizing topological quantum computation give rise to negentropic entanglement, which would be stable with

respect
to NMP.

\item Generalization of the configuration space (\blockquote{world of classical worlds}
(WCW)) and related concepts is also
necessary and again gluing along common rationals and algebraic
continuation is the basic guide line also now. WCW is a
union of symmetric spaces and this allows an algebraic construction of the
WCW K\"ahler metric and spinor structure, whose definition
reduces to the supersymplectic algebra defined by the function basis at the
light cone boundary. Hence the algebraic continuation is relatively
straightforward. Even WCW functional integral could allow
algebraic continuation. The reason is that symmetric space structure
together with Duistermaat Hecke theorem suggests strongly that
WCW integration with the constraints posed by
infinite-dimensional symmetries on physical states is effectively
equivalent to Gaussian functional integration in free field theory around
the unique maximum of K\"ahler function using contravariant configuration
space metric as a propagator. Algebraic continuation is possible for a
subset of rational valued zero modes if K\"ahler action and K\"ahler
function are rational functions of WCW coordinates for
rational values of zero modes.

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